SEEM 5580 Homework 2

2.11

Problem: Cars pass a certain street location according to a Poission process with rate λ. A person wanting to cross the street at that location waits until she can see that no cars will come by in the next T time units. Find the expected time that the person waits before starting to cross (Note, for instance, that if no cars will be passing in the first T time units then the waiting time is 0)

Solve:

let $W = E(waiting\, time)$

From **proposition 2.2.1**, we get that event X_i is independent identically distributed exponential random variables having mean $\frac{1}{\lambda}$

Thus, we can get the equation:

$$\begin{aligned} W &= \int_0^T (W+x)Pr\{X_i = x\}dx \\ &= \int_0^T (W+x)\lambda e^{-\lambda x}dx \\ &= W + \frac{1}{\lambda} - e^{-\lambda T}(T + W + \frac{1}{\lambda}) \\ W &= \frac{e^{\lambda T - 1}}{\lambda} - T \end{aligned}$$

So we get the expected waiting time $W = \frac{e^{\lambda T - 1}}{\lambda} - T$

2.30

Problem: Let $T_1, T_2, \ldots$ denote the interarrival times of events of a nonhomogeneous Poisson process having intensity function $\lambda(t)$

(a) Are the T_i independent?

(b) Are the T_i identically distributed?

(c) Find the distribution of T_1

(d) Find the distribution of T_2

Solve:

(a)

No, we can calculate T_2's conditional distribution:

$$\begin{aligned} Pr\{T_2 > t | X_1 = s\} &= \lim_{s' \to s} Pr\{N(s+t) - N(s) = 0, N(s') = 0, N(s) - N(s') = 1 | N(s) = 1\} \\ &= Pr\{N(s+t) - N(s) = 0\} \\ &= e^{m(s) - m(t+s)} \end{aligned}$$

we can find this distribution is depends on s, thus T_1, T_2 is not independent and T_i not independent.

(b)

No, with the result of (a), we find that T_2 depends on T_1

(c)

$Pr\{T_1 > t\} = Pr\{N(t) = 0\} = e^{m(0)-m(t)} = e^{-m(t)}$

$F_{X_1}(t) = Pr\{T_1 \leq t\} = 1 - e^{-m(t)}$

thus, $f_{X_1}(t) = \lim_{t' \to t} \frac{Pr\{T_1 > t'\} - Pr\{T_1 > t\}}{t' - t} = \lambda(t)e^{-m(t)}$

(d)

combine the result in (a) and (c), we get

$$\begin{aligned} Pr\{T_2 > t\} &= \int_0^\infty Pr(T_2 > t | T_1 = s) f_{X_1}(s) ds \\ &= \int_0^\infty e^{m(s)-m(t+s)} \lambda(s) e^{-m(s)} ds \\ &= \int_0^\infty \lambda(s) e^{-m(t+s)} ds \\ F_{X_2}(t) &= Pr\{T_2 \leq t\} \\ &= 1 - \int_0^\infty \lambda(s) e^{-m(t+s)} ds \end{aligned}$$