moment generating function: $\psi(t)=E\left[e^{tX}\right]=\int e^{iX}dF(x)$

Discrete Probability Distribution	Probability Mass Function, $p(x)$	Moment Generating Function, $\phi(t)$	Mean	Variance
Binomial with parameters n, p, $0 \leq p \leq 1$	$\binom{n}{x} p^x(1-p)^{n-x}$ $x = 0, 1, \quad , n$	$(pe^t + (1-p))^n$	np	$np(1-p)$
Poisson with parameter $\lambda > 0$	$e^{-\lambda}\frac{\lambda^x}{x!}$, $x = 0, 1, 2,$	$\exp\{\lambda(e^t - 1)\}$	λ	λ
Geometric with parameter $0 \leq p \leq 1$	$p(1-p)^{x-1}$, $x = 1, 2,$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative binomial with parameters r, p	$\binom{x-1}{r-1} p^r(1-p)^{x-r}$, $x = r, r+1,$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Conditional Expectation: $P\{X=x \mid Y=y\}=\frac{P\{X=x, Y=y\}}{P\{Y=y\}}$

Exponential Distribution:

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x \geq 0 \\ 0 & x<0\end{cases}$$

$$F(x)=\int_{-\infty}^{x} f(y) dy=\begin{cases}1-e^{-\lambda x} & x \geq 0 \\ 0 & x<0\end{cases}$$

$$E\left[e^{\prime X}\right]=\int_{0}^{\infty} e^{ix} \lambda e^{-\lambda x} dx=\frac{\lambda}{\lambda-t}$$

$$E[X]=1 / \lambda, \quad \operatorname{Var}(X)=1 / \lambda^{2}$$

Lemma 1.7.1 Markov's Inequality

If X is a nonnegative random variable, then for any $a > 0$

$$P\{X \geq a\} \leq E[X]/a$$

PROPOSITION 1.7.2 Chernoff Bounds

Let X be a random variable with moment generating function $M(t) = E[e^{tX}]$ Then for $a > 0$

$$P\{X \geq a\} \leq e^{-ta}M(t) \qquad \text{for all } t > 0$$
$$P\{X \leq a\} \leq e^{-ta}M(t) \qquad \text{for all } t < 0.$$

PROPOSITION 1.7.3 Jensen's Inequality

If f is a convex function, then

$$E[f(X)] \geq f(E[X])$$

Poisson Process:

$$P\{N(t+s)-N(s)=n\}=e^{-\lambda t} \frac{(\lambda t)^{n}}{n!}, \quad n=0,1$$

$$P\left\{X_{1}>t\right\}=P\{N(t)=0\}=e^{-\lambda t}$$

$$S_{n}=\sum_{i=1}^{n} X_{t}, \quad n \geq 1, f(t)=\lambda e^{-\lambda'} \frac{(\lambda t)^{n-1}}{(n-1)'}, \quad t \geq 0$$

Nonhomogeneous Poisson Process:

$m(t) = \int_0^t \lambda(s) ds$

$P\{N(t+s) - N(t) = n\} = \exp\{-(m(t+s) - m(t))\}[m(t+s) - m(t)]^n / n', n \geq 0$

Compound Poisson Process:

$W = \sum_{i=1}^N X_t, E[W] = \lambda E[X] \operatorname{Var}(W) = \lambda E\left[X^2\right]$

$E[Wh(W)] = \lambda E[Xh(W+X)]$

Conditional Poisson Process:

$P\{\Lambda \leq x \mid N(t) = n\} = \frac{\int_0^x e^{-\lambda t}(\lambda t)^n dG(\lambda)}{\int_0^\infty e^{-\lambda t}(\lambda t)^n dG(\lambda)}$

Markov Chain:

$P_{ij}^{n+m} = \sum_{k=0}^\infty P_{ik}^n P_{kj}^m$

$i \leftrightarrow j$: same class, *irreducible*: only one class

aperiodic: $d(i) = 1$

$f_{ij}^n = P\left\{X_n = j, X_k \neq j, k = 1, \ldots, n-1 \mid X_0 = i\right\}, f_{ij} = \sum_{n=1}^\infty f_{ij}^n$

recurrent: $f_{jj} = 1 \Leftrightarrow \sum_{n=1}^\infty P_{jj}^n = \infty$ *transient*: otherwise

$\mu_{jj} = \begin{cases} \infty & \text{if } j \text{ is transient} \\ \sum_{n=1}^\infty n f_{jj}^n & \text{if } j \text{ is recurrent.} \end{cases}$

$\pi_j = \lim_{n \to \infty} P_{jj}^{nd(j)}$

positive recurrent: $\mu_{jj} < \infty \Leftrightarrow \pi_j > 0$

null recurrent: $\mu_{jj} = \infty \Leftrightarrow \pi_j = 0$

class property: $d(i) = d(j)$, *Positive(null) recurrent*

stationary: $P_j = \sum_{i=0}^\infty P_i P_{ij}, \quad j \geq 0$

ergodic: irreducible aperiodic Markov chain, all states ate positive recurrent:
$\pi_j = \lim_{n \to \infty} P_{ij}^n > 0$

$$\pi_j = \sum_i \pi_i P_{ij}$$

$$\sum_j \pi_j = 1$$