# SEEM 5580 Homework 2

### 2.11

**Problem**: Cars pass a certain street location according to a Poission process with rate $\lambda$. A person wanting to cross the street at that location waits until she can see that no cars will come by in the next $T$ time units. Find the expected time that the person waits before starting to cross (Note, for instance, that if no cars will be passing in the first $T$ time units then the waiting time is 0)

**Solve**:

let $W = E(waiting\,time)$

From **proposition 2.2.1**, we get that event $X_i$ is independent identically distributed exponential random variables having mean $\frac{1}{\lambda}$

Thus, we can get the equation:

$$\begin{aligned} W &= \int_0^T (W+x)Pr\{X_i = x\}dx \\ &= \int_0^T (W+x)\lambda e^{-\lambda x}dx \\ &= W + \frac{1}{\lambda} - e^{-\lambda T}(T + W + \frac{1}{\lambda}) \\ W &= \frac{e^{\lambda T - 1}}{\lambda} - T \end{aligned}$$

So we get the expected waiting time $W = \frac{e^{\lambda T-1}}{\lambda} - T$

### 2.30

**Problem:** Let $T_1, T_2, \ldots$ denote the interarrival times of events of a nonhomogeneous Poisson process having intensity function $\lambda(t)$

**(a)** Are the $T_i$ independent?

**(b)** Are the $T_i$ identically distributed?

**(c)** Find the distribution of $T_1$

**(d)** Find the distribution of $T_2$

**Solve**:

**(a)**

No, we can calculate $T_2$'s conditional distribution:

$$\begin{aligned} Pr\{T_2 > t | X_1 = s\} &= \lim_{s' \to s} Pr\{N(s+t) - N(s) = 0, N(s') = 0, N(s) - N(s') = 1 | N(s) = 1\} \\ &= Pr\{N(s+t) - N(s) = 0\} \\ &= e^{m(s) - m(t+s)} \end{aligned}$$

we can find this distribution is depends on $s$, thus $T_1, T_2$ is not independent and $T_i$ not independent.

**(b)**

No, with the result of (a), we find that $T_2$ depends on $T_1$

**(c)**

$Pr\{T_1 > t\} = Pr\{N(t) = 0\} = e^{m(0)-m(t)} = e^{-m(t)}$

$F_{X_1}(t) = Pr\{T_1 \leq t\} = 1 - e^{-m(t)}$

thus, $f_{X_1}(t) = \lim_{t' \to t} \frac{Pr\{T_1 > t'\} - Pr\{T_1 > t\}}{t' - t} = \lambda(t)e^{-m(t)}$

**(d)**

combine the result in (a) and (c), we get

$$\begin{aligned}
Pr\{T_2 > t\} &= \int_0^\infty Pr(T_2 > t | T_1 = s) f_{X_1}(s) ds \\
&= \int_0^\infty e^{m(s)-m(t+s)} \lambda(t) e^{-m(t)} ds \\
&= \lambda(t) e^{-m(t)} \int_0^\infty e^{m(s)-m(t+s)} ds \\
F_{X_2}(t) &= Pr\{T_2 \leq t\} \\
&= 1 - \lambda(t) e^{-m(t)} \int_0^\infty e^{m(s)-m(t+s)} ds
\end{aligned}$$