A Gaussian Variational Inference Approach to Motion Planning

Hongzhe Yu and Yongxin Chen
School of Aerospace Engineering, Georgia Tech

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Background – Motion Planning under uncertainty

- Robot facing uncertainty in
  - System model and **sensor** noise
  - Imperfect controller and **input** noise
  - Model uncertainty as a stochastic process

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{F}(t)\mathbf{w}(t) + \mathbf{b}(t)$$

- Trajectory optimization formulated as
  - **Probabilistic inference** *(Mukadam et al. (2018))*

$$\boxed{\begin{aligned} \mathbf{x}^\star &= \arg\max_{\mathbf{x}} p(\mathbf{x}|\mathbf{z}) \\ &= \arg\max_{\mathbf{x}} p(\mathbf{z}|\mathbf{x})p(\mathbf{x}) \end{aligned}}$$

Solve inference problem – Variational Inference

- Probabilistic motion planning
  - Posterior p(x|z): probability of a feasible trajectory
  - Prior p(x): trajectory joint probability induced by uncontrolled SDE
  - Likelihood p(z|x): probabilistic collision-free factor
- In the case of linear SDE
  - Dynamics: $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{F}(t)\mathbf{w}(t) + \mathbf{b}(t)$
  - Prior: $p(\mathbf{X}) = \tilde{p}(\mathbf{X}|O_{t_N}) \propto \boxed{\exp(-\frac{1}{2}(\|\mathbf{X} - \boldsymbol{\mu}\|^2_{\mathbf{K}^{-1}}))}$
  - Likelihood: $p(\mathbf{z}|\mathbf{x}) \propto \boxed{\exp(-\|\mathbf{h}(\mathbf{x})\|^2_{\boldsymbol{\Sigma}^{-1}_{obs}})}$

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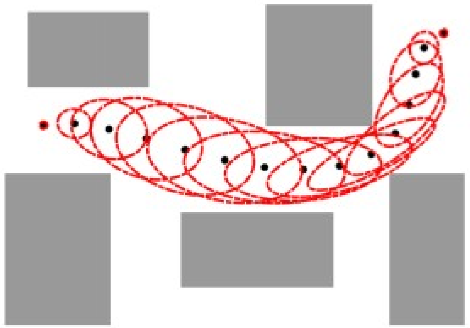
Gaussian VI – Gaussian trajectory distribution

- **Objective:**
  - To **sample from** or to **solve** the target distribution with density p(x|z)
- **Methods:**
  - MAP: find the state that **maximizes the probability** p(x|z)
  - VI: find a **proposal** distribution that is close to p(x|z) by minimizing

$$\boxed{q^\star = \arg\min_{q \in \mathcal{Q}} \mathrm{KL}[q(\mathbf{x}) || p(\mathbf{x}|\mathbf{z})]}$$

    - Gaussian VI: Gaussian proposal



Algorithm – Natural Gradient Descent GVI

- Parameterize the Gaussian proposal $\mathcal{Q} \triangleq \mathcal{N}(\mu_\theta, \Sigma_\theta)$
- NGD updates

$$\boxed{\mu_\theta \leftarrow \mu_\theta + \eta^R \, \delta\mu_\theta, \quad \Sigma_\theta^{-1} \leftarrow \Sigma_\theta^{-1} + \eta^R \, \delta\Sigma_\theta^{-1}}$$

- **Key** computations: **3 Expectations** w.r.t. the proposal Gaussian

$$\boxed{\frac{\partial J(q)}{\partial \mu_\theta}} = \Sigma_\theta^{-1} \boxed{\mathbb{E}[(\mathbf{X} - \mu_\theta)\psi]} \qquad \boxed{\frac{\partial^2 J(q)}{\partial \mu_\theta \partial \mu_\theta^T}} = \Sigma_\theta^{-1} \boxed{\mathbb{E}[(\mathbf{X} - \mu_\theta)(\mathbf{X} - \mu_\theta)^T \psi]} \Sigma_\theta^{-1} - \Sigma_\theta^{-1} \boxed{\mathbb{E}[\psi]}$$

- How: **Decomposition**

$$\psi(\mathbf{X}) = -\log p(\mathbf{X}|Z) = \boxed{\|\mathbf{h}(\mathbf{X})\|^2_{\Sigma_{\text{obs}}}} + \frac{1}{2}\boxed{\|\mathbf{X} - \boldsymbol{\mu}\|^2_{\mathbf{K}^{-1}}}$$

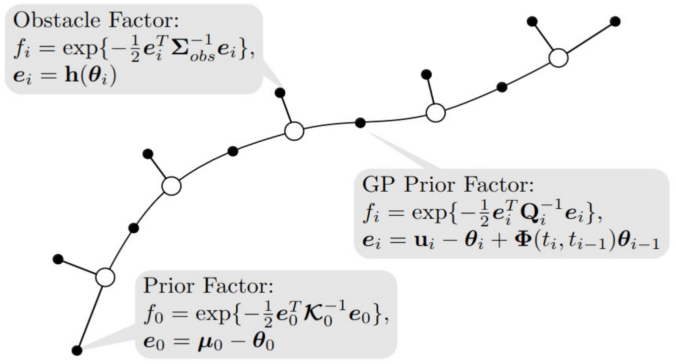
collision Factor: Nonlinear → **Gauss-Hermite quadratures**

prior factor: Gaussian → **Known closed-form**

Algorithm – Sparsity and Factor graph

- Motion Planning problem has a sparsity pattern
  - Prior factors: only consecutive **2 states**
  - Collision factors: only **1 state**
- **Write the joint probability into factors**

$$\boxed{\psi(\mathbf{X}) = \sum_{l=1}^{L} \psi_\ell(\mathbf{X}) \triangleq -\log p(\mathbf{X}_\ell | Z)}$$

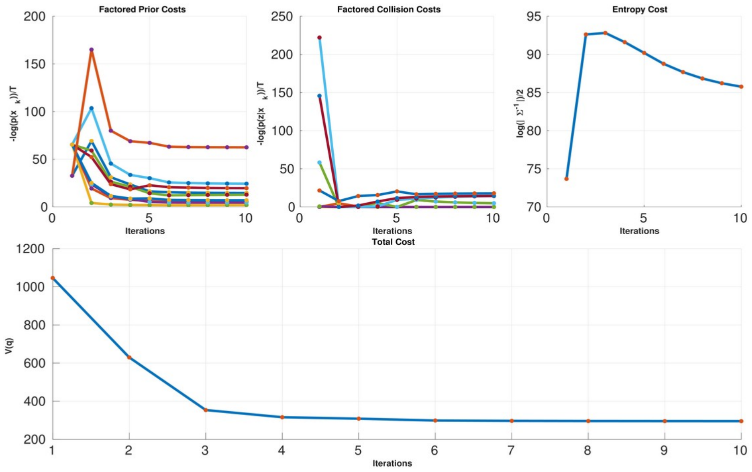


Factor graph *(Mukadam et al. (2018))*

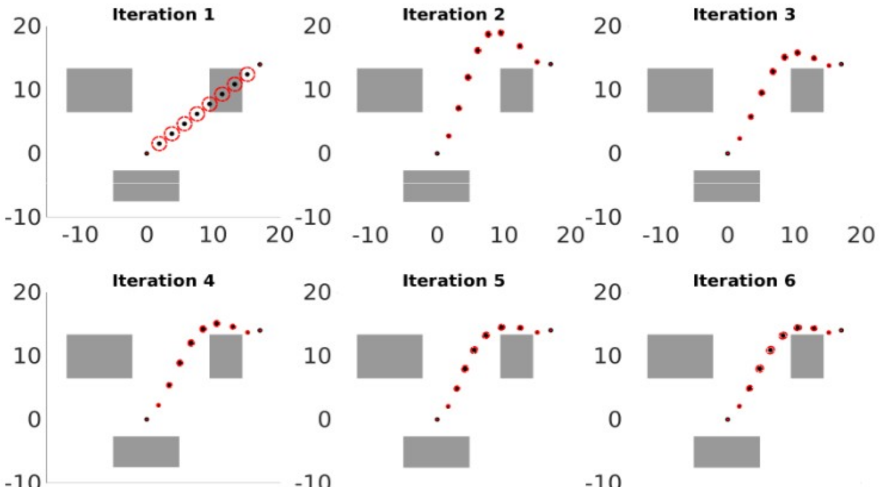
- Marginals have the same prior and collision factors as the joint level
  - Why? Compute the joint-level **expectation** is costly!

Experiment results

- Convergence

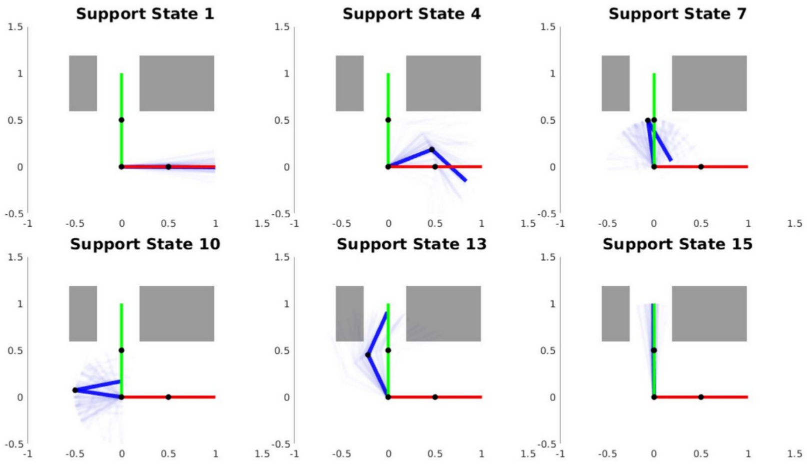


Factor and total costs

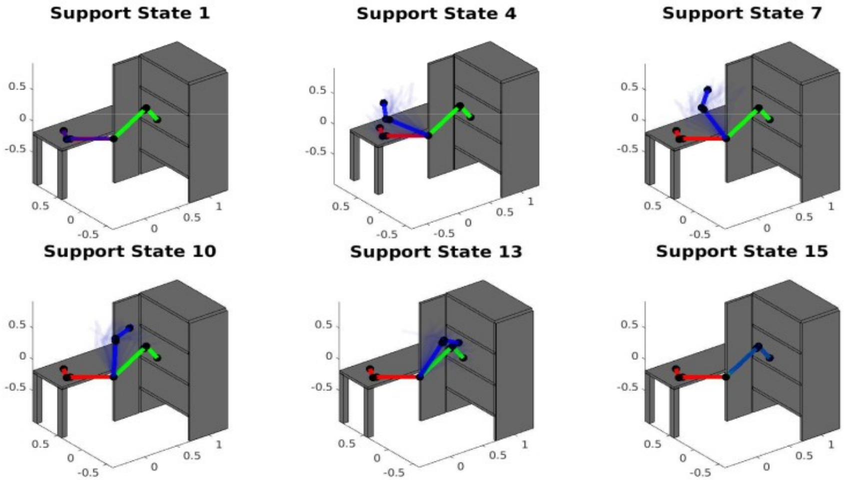


Trajectory distributions

- Arm Robot: **sampled** trajectories



Sampled trajectories



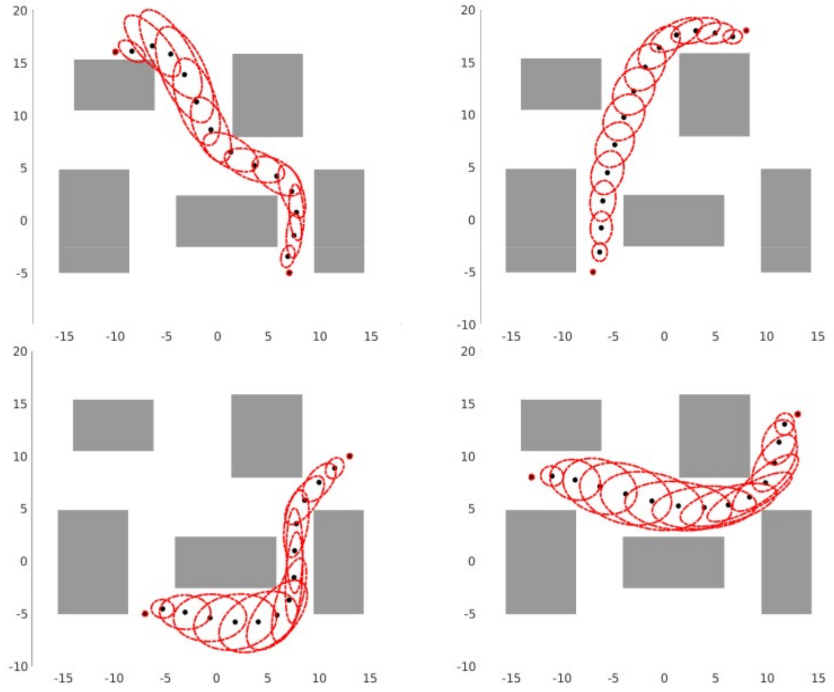
Higher dimension (7-DOF)

Experiment results

- Entropy regularized **robust** motion planning

$$q^\star = \arg\min_{q\in\mathcal{Q}} \mathrm{KL}[q(\mathbf{X})||p(\mathbf{X}|Z)] = \arg\max_{q\in\mathcal{Q}} \mathbb{E}_q[\log p(\mathbf{X}) + \log p(Z|\mathbf{X})] + \boxed{H(q(\mathbf{X}))}$$

  - Higher entropy = wider-spread distribution = probabilistic safer



Max-entropy planning in cluttered environments

Fig. 3: Entropy regularized motion planning [35].

	Prior	Collision	MP	Entropy	Total
Left	**34.4583**	9.1584	**43.6168**	44.1752	87.7920
Right	42.9730	**2.0464**	45.0193	**39.9193**	**84.9387**

Trade-off **risk V.S. distance and smoothness**

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Paper

Thanks!

Contact: hyu419@gatech.edu

Code: https://github.com/hzyu17/VIMP