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March 22, 2019

Sports Team Ranking with Perron and Frobrenius

Sports are an integral part of the culture of many countries, particularly that of America. Each team or player battles it out in their respective challenge, and earns a rank compared to others. Often, this rank is determined solely by the win to loss ratio. However, this doesn’t always give the full picture for how good or bad a team is. Particularly, it ignores how difficult the team’s schedule may be. In this report, we will analyze team rankings without focusing on the games they win. We will use the direct route and the nonlinear method, both utilizing the Perron-Frobenius theorem to some degree. Oskar Perron and Georg Frobenius stated that a real square matrix holding positive entries will have one unique eigenvalue that is the largest of the eigenvalues, and the resulting eigenvector can be found using only positive components. In regard to sports, the matrix can be designed to cause this vector have values corresponding to the rank of each team. The first method, the direct method, begins by choosing a rank vector containing only ones followed by the sum of the wins and losses of each respective team, multiplied by the rank of the other teams. This number is then divided by the number of games played which results in an average score for the team. The method uses the equation where si is the score of team i , aij represents the outcome of the game between teams i and j, N is the total number of participants, and ni is games played by team i. . In this case, the wins are represented by a value of 1, ties receive a value of ½, and the losses use a value of 0.

The other method, the non-linear method, takes into account both schedule difficulty and performance within the games, even if they are lost. Each team is given the rank where e­­ij is the outcome of the game between teams i and j, rj is the positive rank of team j, and . The outcome of each game, in particular, is found by , where Sij is the score of team i in the game against team j.

Although the theorems take into account different aspects of gameplay and use different equations, both require the Perron-Frobenius theorem in order to work in any way at all. Without this theorem, there would be no guarantee of a positive eigenvector to use as the team ranks, thus rendering these systems useless.