UNIVERSITY OF COLORADO - BOULDER

ASEN 2003 - DYNAMICS & SYSTEMS

DESIGN LAB FIVE

Yo-Yo Despinner Experiment

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Dynamics Laboratory 5: Yo-Yo Despinner

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ASEN 2003 Section 11 Group 9

This lab will evaluate the theory and practice behind a de-spin mechanism. Conservation of energy and angular momentum are the main theories behind this. Deriving basic conservation equations will calculate a string length with external masses to eliminate rotation on a device rotating at 130 RPM. Angular velocity and acceleration equations will help model the experiment where trials will evaluate the rotation of a de-spin module with radial release. With the current moment of inertia and masses, the string length for radial release was 0.177 m. This was not quite long enough to bring the device to a stop.

Contents

[1]SID: 109364560
[2]SID: 105752610
[3]SID: 106536744
[4]SID: 107587626

I. Nomenclature

ω	=	angular velocity $[rad/s^2]$
ω_0	=	initial angular velocity $[rad/s^2]$
α	=	angular acceleration $[rad/s^2]$
t	=	time $[s]$
m	=	mass of a single ball $[kg]$
R	=	Radius of satellite $[m]$
T	=	tension force on cables from extra mass $[N]$
$L_(T)$	=	length of cables with tangential release$[N]$
$L_(r)$	=	length of cables from radial release $[N]$
I_s	=	Moment of inertia of the satellite $[Kg * m^2]$
V_m	=	Velocity of the center of mass of the mass added $[Kg * m^2]$
H_B	=	Angular momentum before release $[kg * m^2 * s^-1]$
H_A	=	Angular momentum after release $[kg * m^2 * s^-1]$

II. Theory

Derivations for equations were based on the lab document and technical NASA report. Based on the conservation of momentum (complete derivations can be found in appendix C), the tangential despin deployment uses initial angular velocity ($\omega 0$), the moment of inertia of the satellite (Is), the outer radius of the satellite (R), and the combined despin mass (m). Using the intermediate value C, the angular velocity with respect to length and time can be found similarly.

$$C = \frac{I}{M * R^2} + 1$$

$$\omega(t) = \omega_0 * \frac{C - \omega_0^2 * t^2}{C + \omega_0^2 * t^2} \tag{1}$$

$$\omega(l) = \omega_0 * \frac{C * R^2 - l^2}{C * R^2 + l^2} \tag{2}$$

Taking the derivative of the base equations of angular velocity in terms of time and length gives us the following equations. This allows us to model how quickly the satellite stop spinning. The tension force of each cable can be found by evaluating the dynamics and kinematics. Total centripetal acceleration multiplied by the total mass gives us the total force tension force on the system in a free body diagram.

$$\alpha(t) = \frac{-4C * \omega_0^3 * t}{(C + \omega_0^2 * t^2)^2} \tag{3}$$

$$\alpha(l) = \frac{-4C * R^2 * \omega_0 * l}{(C * R^2 + l^2)^2} \tag{4}$$

$$T_{tangential} = \frac{m * v^2}{r} = m * \omega^2 * r \tag{5}$$

The length of cord required to bring the satellite to a stop with tangential release is found below. Using the derivation for angular velocity with respect to length, the terminal angular velocity is set to zero and algebra is used to solve for the final length (0.1771m). Plugging in our constants for radius, inertia, and mass, the final length is computed (although radial length will be used in the experiment, not tangential). Time for deployment was found in a similar way where terminal angular velocity with respect to time was set to zero and the time was solved producing the value below (0.19s).

$$L_{tangential} = \sqrt{R^2 + \frac{I}{m}} \to \sqrt{0.0762^2 + \frac{0.0063}{2 * 0.054}} \to 0.2533m \tag{6}$$

$$t_{deployment} = \frac{\sqrt{\frac{I}{mR^2} + 1}}{\omega_0} \to \frac{\sqrt{\frac{0.0063}{0.054 * 0.0762^2} + 1}}{13.6} \to 0.19s \tag{7}$$

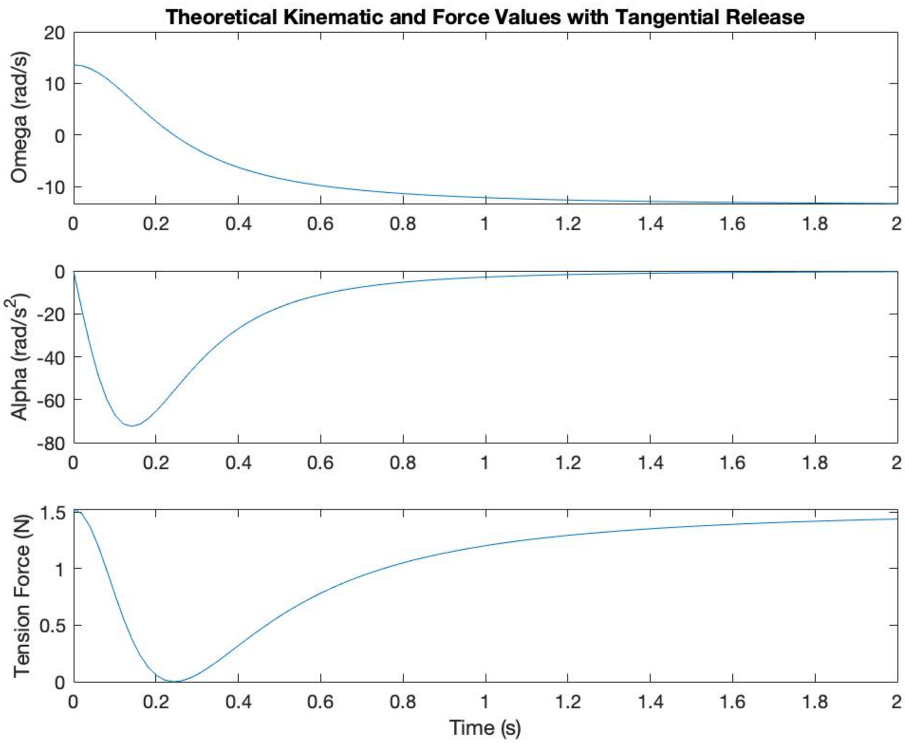


Fig. 1 Angular velocity, Angular acceleration, and Tension with Time

From an initial angular velocity of around 13.6 rad/sec, Fig. 1 shows the theoretical relationships of velocity, acceleration, and tension over time. Since there is no variable in the angular velocity equation for chord length, the plot shows omega decreasing until it approaches negative 13.6 rad/sec. This is because the chord length is continuously unraveling and increasing as the spin changes direction switches because the total moment of inertia increases. The angular acceleration show the satellite decreases in speed quickly before approaching a constant angular velocity. The peak of this change with maximum angular acceleration is the maximum limit for release before the chord length is to long to effectively transfer angular momentum in the masses. This is also visualized in the tension force as it reaches a minimum around the same time because angular velocity is approximately 0 and the satellite will begin to spin the other direction.

From equation 13 in the NASA document [4], the radial length needed for proper release was derived. Similar to tangential release, radius, inertia, and mass are known and give us a final value of 0.2533m. This length for radial release is slightly less than tangential release.

$$L_{radial} = R(\sqrt{\frac{I}{M * R^2} + 1} - 1) \rightarrow 0.1771m \tag{8}$$

III. Experiment

Yo-yo de-spin is a technique used to transfer angular momentum, typically in satellites. In the final stages of missions, spin stabilization is required to balance the satellite during rocket firing and reduce trajectory errors. This increase in angular momentum has to be decreased to adjust to the satellite's attitude control. Angular momentum can be manipulated by adjusting moment of inertia and angular velocity. In a Yo-yo de-spin, two masses attached to strings are ejected in the final stages of the satellite. With the attached masses spinning around the rotation axis, the moment of inertia would increase and the angular velocity would proportionally decrease.

Release Type	Equation	R [m]	I [$kg * m^2$]	M [g]	Length [m]
Radial (experiment)	$\sqrt{\frac{I}{M} + R^2} - R$	0.0762	0.0063	0.054	0.1771
Tangential	$\sqrt{R^2 + \frac{I}{m}}$	0.0762	0.0063	0.054	0.2533

Table 1 Comparison between different types of releases

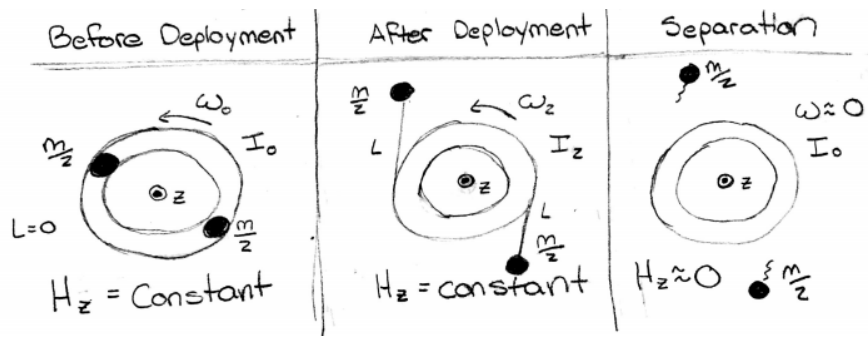


Fig. 2 Despinner apparatus sketch

$$H_{Satellite} = I \times \omega \tag{9}$$

At this point, angular momentum is about the same (slightly less with mass decrease) before and after the release of the masses, but when the masses are released, the satellite loses energy in the form kinetic energy by momentum. Whether the jettisoned masses are released, radially or tangentially, the satellite stops spinning because most of its momentum (if not all) is taken with the masses.

In our experiment and model, we used radial release techniques instead of tangential release. The high speed video showed that the masses released slightly before theorized, so it was not completely radial. Assumptions made in the theory include no gravity, mass-less strings, no friction, and extra masses are point mass. These are not true in the actual experiment and error was introduced with extra moments, making the actual release not perfectly radial. The equations show that with the same mass, the radial chord length mass release in the experiment was slightly less than tangential deployment.

When the de-spinner runs without the weights being released, it still slows to a stop due to friction. This takes about 40 seconds as seen in figure 4. Because of this friction force, a moment is applied on the craft. If we assumed that the angular acceleration is constant, then it follows from Newton's laws that the moment applied on the body is constant too. Hence, an estimation for the moment can be obtained by numerically differentiating the angular velocity data obtained, and then multiply it by the moment of inertia of the satellite. This happens to be a moment of -0.0021 N.m in this case. This value may seems to be very small, which is reflected in the long time (relative to when the mass was used) that it took the satellite to come to full stop when friction moment was the only thing trying to stop it.

The functional block diagram gives an overall view about the experimental setup and can be seen in figure 3.

IV. Results and Analysis

The two tests done with the despinner included a test where the spinner was allowed to stop solely due to friction and a test that the masses were used to stop it. The angular velocity of the craft over time for the test with no mass is shown in figure 4. The vertical line in the beginning can be ignored because it represents the beginning of the experiment where the motor starts to rotate.

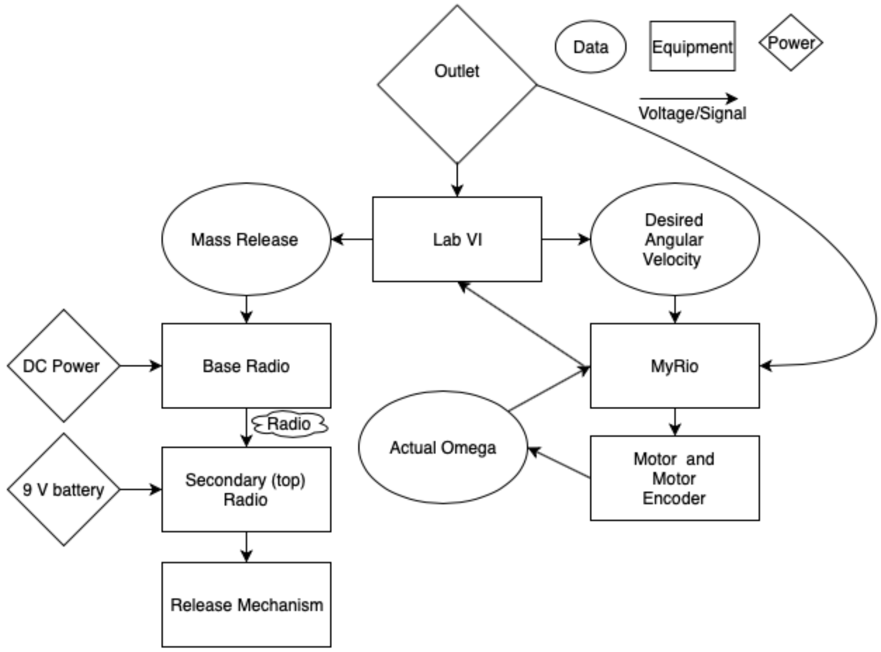


Fig. 3 Functional Block Diagram of the Experiment setup

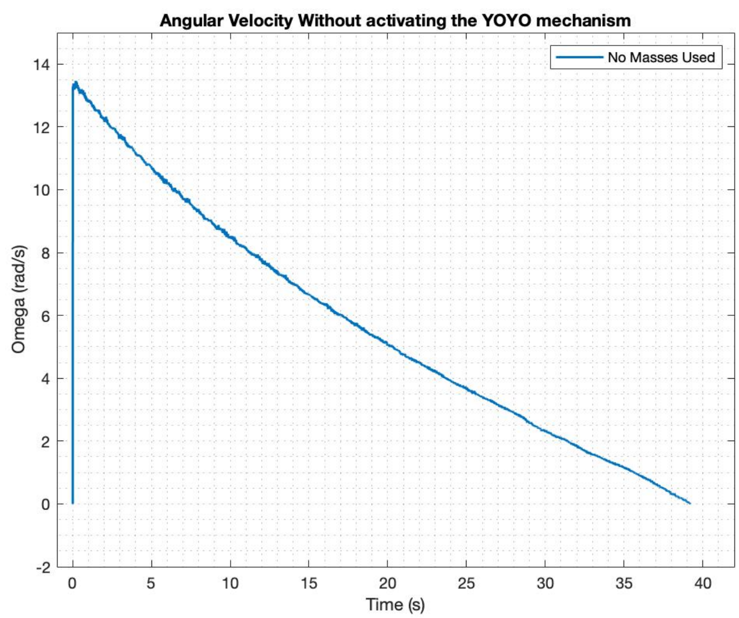


Fig. 4 Angular Velocity profile when the YOYO mechanism is not activated

By taking the derivative of the angular velocities, the angular accelerations during the two tests can be found. The angular acceleration drops to an absolute max and then decreases in magnitude to zero. The tension and acceleration is greatest when the string is at max length, right before release. The model and the experiment has much different values for acceleration. While the experiments largest value was around -0.1, the model reached past -70.

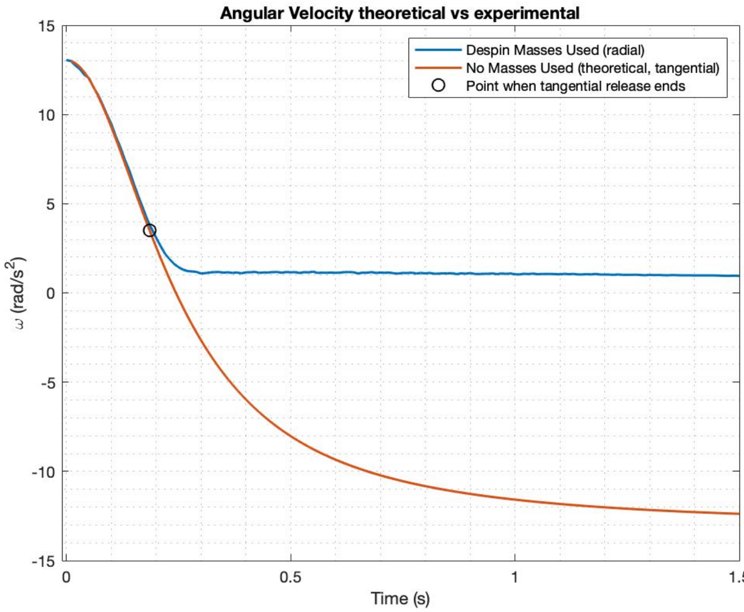


Fig. 5 Theoretical and experimental angular velocities

The experimental tests with masses took about 6 seconds to come to a full stop, unlike the theoretical, which took 0.2 seconds. The angular velocity profile of this test is in figure 5, along with the theoretical ω. The main speed drop occurs within 0.45, which lines up better with the time seen in the theoretical. This drop does not go all the way to zero because of a faulty string length. In order to decrease speed to zero in shorter time, a longer string will be needed. The speed is slowed to zero only because of friction after these 0.45 seconds. This time is still different than the theoretical due to the use of a radial release rather than the theoretical tangential and a delay before the masses were released. On figure 5 it can be seen that the model starts to diverge away from experimental data at an angular velocity that is estimated by 0.185 rad/s^2. This value was estimated by visually inspecting the difference between the model and the x erimept.en

V. Application

One example of a despinner being used in industry is the Dawn Mission, which was launched in 2007. The spacecraft had a mass of 1,218 kg and the despinner used had 2 masses of 1.44 kg on the end of 2 cables with a length of 12.15 m. These weights reduced the RPM from 46 to -3. Despinning the craft was important in this mission because the control system would not work at a high RPM. The despin approach used had some problems, which can be seen because the spacecraft still had rotation afterwords, just in the opposite direction. This was attributed to reduction in mass because of ballast removal. Ultimately, the rotation slowed down to zero after about 8 minutes. Even though it worked out, it still shows a problem in this despinner system. If the amount of ballast in the craft is even slightly different than estimated, the length and mass needed in the despinner changes. Because the craft is in space, this is not a problem that can be fixed when it occurs. However, it is a very simple way to despin a craft. Another alternative that could be used is despinning the craft with thrusters on the side of the craft. This would be more expensive, but more control would be given over rotation.

VI. Conclusion

During this lab a Yo-Yo Despinner was studied, modeled, and tested. The functionality of the Despinner was learned then the physics of the system were derived. Both radial and tangential release mechanisms were studied and modeled. The tangential model gave us theoretical predictions for the angular acceleration and velocity, length of cord required, and time of deployment. Friction in the Despinner system caused the experimental data to have minor discrepancies with the modeled system. The length of cord found by the model worked very well in the experiment. This lab taught the team about the transfer of angular momentum, mathematical modeling, and real world spin control systems. The full despin experiment performed moderately well, with the spinner coming to a full stop longer.than modeled The massless despin experiment did not perform as the model predicted because of the moment caused by friction. In future labs, this should be taken into account for more accurate results.

VII. Acknowledgements

We would like to thank Professor Frew and all of the TAs and LAs that assisted with the completion of this lab. A special thank you to the ITLL staff, for building and providing the instrumentation and guidance on its use.

References

[1] Hibbeler, R. C., *Engineering Mechanics: Dynamics. 5th Ed.* Upper Saddle River, NJ: Pearson/Prentice Hall, 2004. Print.

[2] Frew, Eric. *ASEN 2003 Lab 5: Yo-Yo Despinner Experiment* Canvas. University of Colorado Boulder, Web. Mar. 2019.

[3] Rayman, Marc D. *September 12, 2007* NASA Jet Propulsion Labratory. Web. Sept. 2007.

[4] Eide, Donald *NASA Technical Note* NASA, Jan. 1962.

VIII. Appendix A: Team Table

Team-member contributions table
Lab: Design Lab 3: Locomotive Crankshaft
Date: February, 2019

Name	Plan	Model	Experiment	Result	Report	Code
Abdulla Ameri	1	1	2	1	2	1
Brandon Cummings	1	2	1	1	1	1
Axel Haugland	1	1	1	1	1	2
Kevin Yevak	2	1	1	2	1	0

IX. Appendix B: Code

lab5.m *(Main)*

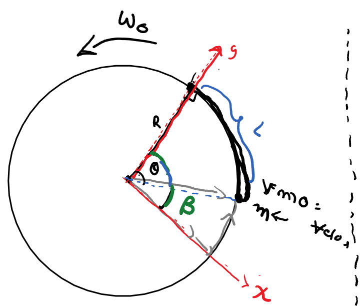
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Lab 5
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Housekeeping and Constants
clc; clear all; close all;

omega0 = 130; % RPM
I = 0.0063; %+/- 0.0001 kg*m^2
R = 0.0762; %m
m = .054*2; %kg

%% Theoretical Tangential
omega0 = omega0*2*pi/60; %rad/s
C = I/(m*R^2)+1;
t = linspace(0,2,100);
```

```
a = (C+((omega0^2)*(t.^2)));
b = (C-((omega0^2)*(t.^2)));
omega = omega0*(b./a);
alpha = (-4.*C.*(omega0^3).*t)./((C+(omega0^2).*t.^2).^2);
T = m*((omega.*R).^2)/R;
subplot(3,1,1)
plot(t,omega)
ylabel('Omega (rad/s)')
title('Theoretical Kinematic and Force Values with Tangential Release');
subplot(3,1,2)
plot(t,alpha)
ylabel('Alpha (rad/s^2)')
subplot(3,1,3)
plot(t, T)
ylabel('Tension Force (N)')
xlabel('Time (s)')
Lt = R*sqrt(C); %m tangential
Lr = R*sqrt((I/(m*R^2)+1)-1); %Given
Ltest = sqrt(I/m+R^2)-R; %Derived


%% Experimental omega
data1 = load('YoYo_despin');
data2 = load('YoYo_noweight');
timeDe = data1(:,1);
omegaDe = data1(:,2);




omegaDe = omegaDe.*(0.104719755);%rpm to rad/s
timeNo = data2(:,1);
omegaNo = data2(:,2);
omegaNo = omegaNo.*(0.104719755);%rpm to rad/s


% calculate avg omega0 that the experiment started with so the model starts
% with too!

omega0_avg_exp = mean(omegaDe(1:18));


%Fixing errors in time data
for i = 120:267
    timeDe(i) = timeDe(i)+1180;
end
for i = 268:1001
    timeDe(i) = timeDe(i)+1180+1480;
end
for i = 126:275
    timeNo(i) = timeNo(i)+1240;
end
for i = 276:3922
    timeNo(i) = timeNo(i)+1240+1500;
end

% zero the matrices:
timeDe(1) = [];
omegaDe(1) = [];

% eyeball where we started the mechanism, then zero from there:
timeDe(1:19) = [];
omegaDe(1:19) = [];

% zero time:
```

```
timeDe = timeDe - timeDe(1);

figure(2)
plot(timeNo./1000, omegaNo,'LineWidth',1.5)
legend('No Masses Used')
xlabel('Time (s)')
ylabel('Omega (rad/s)')
title('Angular Velocity Without activating the YOYO mechanism')
grid minor
ylim([ -2 15])
xlim([ -1 42])

% estimate friction moment:

AlphaNo = diff(omegaNo(2:end));

MomentFriciton = (sum(AlphaNo(2:end))/length(AlphaNo(2:end)))/(sum(diff(timeDe(2:end)))/length(
    timeDe(2:end)))*I ;

MomentFriciton = (-I*omegaNo(2)) / (timeNo(end)./1000) ;


%% Experimental alpha

alphaDe = diff(omegaDe)./diff(timeDe);
alphaNo = diff(omegaNo)./diff(timeNo);

%MomentFriciton = mean(alphaNo(2:end))*I ;


omega = @(t) omegaDe(1)*(((C-((omega0^2)*(t.^2)))./((C+((omega0^2)*(t.^2)))));

TimePlug = (timeNo./1000);


% compute when tangential realse ends:


Wtang = omegaDe(1)*((C-((omegaDe(1)^2)*(Lr.^2)))/(C+((omegaDe(1)^2)*(Lr.^2))));


timeDe = timeDe(1:end);
timeNo = timeNo(1:end);
figure(3)
plot(timeDe(1:end)./1000,omegaDe(1:end),'-','LineWidth',1.5)
hold on
plot(TimePlug(2:end),omega(TimePlug(2:end)),'LineWidth',1.5)
plot(0.185,3.5,'ok','MarkerSize',8)
legend('Despin Masses Used (radial)','No Masses Used (theoretical, tangential)','Point when
    tangential release ends')
ylabel('\omega (rad/s^2)')
xlabel('Time (s)')
title('Angular Velocity theoretical vs experimental')
xlim([-0.01 1.5])
grid minor
```

X. Appendix C: Derivations

Lab Derivation: Yo-Yo!



Before Release

Fig 1

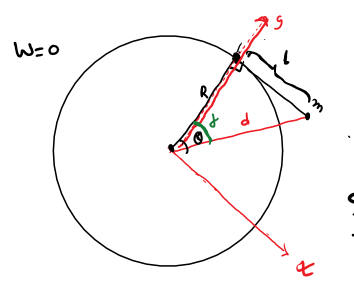
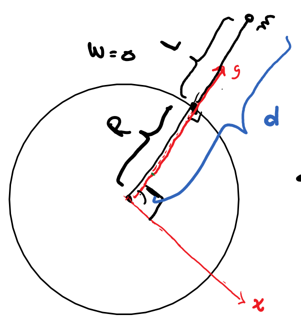


Fig2

After: Tangential release

$d=\sqrt{L^2+R^2}$



$d = R+L$

Fig 3

After: Tangential release

Conservation of Momentum Applies Because there are No External Forces To The System

Before Release: — Assumptions: ① ignore Inertia of Masses

$$H_B = I_s \omega_0 + (\vec r \times m\vec V_{m,o})\cdot\hat k$$

at any given time, in figure 1 $\Phi = L/R$, $\beta = \frac{\pi}{2} - \frac{L}{R}$

$\vec r = \langle R\cos(\beta), R\sin(\beta)\rangle$, using kinematics we can get expression for $\vec V_{m,o}$

$\vec V_{m,o}$ = using kinematics, i = $\langle -\omega_0 R\sin(\frac{\pi}{2}-\frac{L}{R}), \omega_0 R\cos(\beta)\rangle = \langle A, B\rangle$

$\vec r \times V_{m,o} = R\cos(\beta)\cdot B\hat k - A R\sin(\beta)\hat k$

∴ H_B reduces to:

$$H_B = I_s\,\omega_0 + m\omega_0\left(R\cos\left(\frac{\pi}{2}-\frac{L}{R}\right)\right)^2 + m\,\omega_0\left[R\sin\left(\frac{\pi}{2}-\frac{L}{R}\right)\right]^2$$

which reduces to :

$$\boxed{H_B = \omega_0\left[I_s + mR^2\right]}$$

• Angular Momentum Before Release

After Realese :

After :

$$H_a = \cancel{I_m\,\omega_1}^{\,0} + (r \times m v)\cdot k + I_s\,\omega_2$$

mass

$$H_a = I_s\,\omega_s + (r_m \times m\,v_m)\cdot k$$

• $v_m \longrightarrow$ from Conservation of Energy .
" Before AND After Realese

• No Potential energy
• No work Done By Non-Conservative work

$$KE_B = \frac{1}{2} m v_{m,0}^2 + \frac{1}{2}\cancel{I\omega_{m,0}}^{\,0} + \frac{1}{2} I_s\,\omega_0^2$$

Before

Dot Product

$$v_{m,0}^2 = \langle v_{m,0} \mid v_{m,0}\rangle = \omega_0^2 R^2 \sin^2(\beta) + \omega_0^2 R^2 \cos^2(\beta)$$

$$\boxed{= \omega_0^2 R^2}$$

$$KE_{After} = \cancel{\frac{1}{2} I_s \omega_2^2}^{\,0} + \frac{1}{2} m \overline{v}_{m,2}^{\,2} = \frac{1}{2} v_{m,2}^2 m$$

Set $KE_B = KE_{After}$

$$\frac{1}{2} m v_{m,2}^2 = \frac{\omega_0^2}{2} \left[mR^2 + I_s\right]$$

$$v_{m,2}^2 = \frac{\omega_0^2}{m} \left[mR^2 + I_s\right]$$

$$v_{m,2} = \sqrt{\frac{\omega_0^2}{m} \left[mR^2 + I_s\right]}$$ ← Plug Back in H_a

$\omega_2 = 0$, So it goes away

$$H_a = (\vec{r} \times m\vec{v}) \hat{k} = (R+L)(m)\left(\frac{\omega_0^2 \left[mR^2 + I_s\right]}{m}\right)^{1/2}$$

$$H_a = \boxed{(R+L)(\sqrt{m})\sqrt{\omega_0^2 \left[mR^2 + I_s\right]}}$$

Set $H_a = H_B$

$$(R+L)(\sqrt{m})\, \cancel{\omega_0} \sqrt{\left[mR^2 + I_s\right]} = \cancel{\omega_0} \left[I_s + mR^2\right]$$

$$(R+L) = \frac{I_s + mR^2}{\sqrt{mR^2 + I_s} \cdot \sqrt{m}}$$