ASEN 3112

STRUCTURES

LAB 1

Torsion of Circular Thin Walled Sections

Author 1 :
Cameron HUMPHREYS SID: [redacted]

Author 2:
Sam HARTMAN SID: [redacted]

Author 3:
Joseph BUESCHER SID: [redacted]

Author 4:
Megan JONES SID: [redacted]

Author 5:
Abdulla Al AMERI SID: [redacted]

Author 6:
Conner MARTIN SID: [redacted]

Author 7:
Zak DMITRIYEV SID: [redacted]

Professors:
FRANCISCO LOPEZ JIMENEZ
and
KURT MAUTE

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Engineering & Applied Science
UNIVERSITY OF COLORADO BOULDER

In this laboratory, the shear strains of two specimens were measured using an extensometer to compare the differences in response to an applied torque. The first specimen had a uniform circular cross section, while the second specimen had a circular cross section with a slot cut along the length. After analyzing the data, the closed-thin-wall section was much more resistant to the torque than the open-thin-walled section. This observation is quantified via the calculations of torsional rigidity of each specimen, with the closed-thin-wall specimen having a higher torsional rigidity and a lower twist angle than the open-thin-wall specimen.

Contents

Nomenclature

γ	Shear Strain $[inches/inches]$
ϕ	Twist Angle $[radians]$
τ	Shear Stress $[psi]$
A_e	Area Enclosed $[in]$
D_e	Outer Diameter $[inches]$
G	Shear Modulus $[psi]$
J	Polar Second Moment of Inertia $[inches^4]$
P_m	Midline Perimeter $[in]$
R_e	Outer Radius $[in]$
R_i	Inner Radius $[in]$
R_m	Midline Radius $[in]$
T	Torque $[lbs - inch]$
t	Thickness $[inches]$

I. Introduction

In this lab, we will be testing two circular thin wall sections using an MTS Torsional Testing Machine. This machine will apply a torque and an extensometer will measure shear strain/twist angle of the specimen. These measurements will lead to an analysis to verify and correlate the theory of open-thin-wall (OTW) and closed-thin-wall (CTW) sections with strain and twist-angle measurements. The OTW section will differ by the CTW section by having a score down the longitudinal axis with an assumed negligible width compared to the radius of the section.

II. Results and Analysis

A. Analysis of the Closed Thin Wall Specimen

The first section tested was the closed thin wall specimen, a stock aluminum tube. The dimensions were found, exterior diameter was $D_e = \frac{11}{16}$ in, exterior radius $R_e = \frac{1}{2}D_e = \frac{11}{32}$ in, length $L = 12\frac{3}{4}$, and uniform wall thickness $t = \frac{1}{16}$ in. The interior radius can be calculated by $R_i = R_e - t = \frac{9}{32}$ in. The shear modulus of the material is $G = 3.75$ Psi.

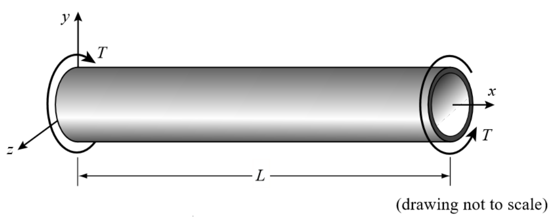

Fig. 1 CTW Specimen

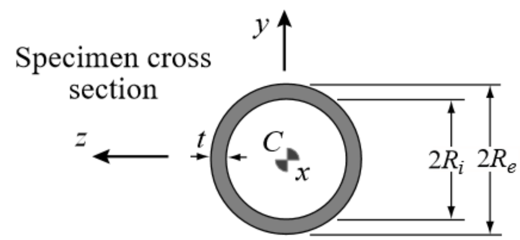

Fig. 2 CTW Cross Section

The specimen was then subjected to torquing to observe shear strain and twist angle. The applied torque ranged from $T_0 = 0$ lbs-in to $T_{max} = 400$ lbs-in. The max shear stress reaches around 8620 psi, providing a safety factor of 2.3-2.6 against yielding. During testing, three sets of measurements were recorded; the shear strain (γ), the total rotation applied to the specimen, and the torque recorded by the testing machine. The twist angle over the length is $\phi = \frac{\gamma L}{R_e}$, with R_e being the exterior tube radius.

Using exact theory, the torsional rigidity of the section was calculated using the equations below and the tube measurements provided.

$$J = \frac{1}{2}\pi(R_e^4 - R_i^4) \tag{1}$$

J was found to be 0.0121 in^4 and the torsional rigidity, GJ, was found to be 60311 lb-in^2.

Using closed thin wall theory, the midline radius, enclosed area, and perimeter were found to calculate the torsional rigidity. Because the thickness was constant throughout the tube, the equation for the polar moment of inertia, J, was simplified.

$$R_m = \frac{R_e + R_i}{2} \tag{2}$$

$$A_e = \pi R_m^2 \tag{3}$$

$$P_m = 2\pi R_m \tag{4}$$

$$J = \frac{4A_e^2}{\oint \frac{ds}{t}} = \frac{4A_e^2 t}{P_m} \tag{5}$$

With this method, J was calculated as 0.01198 in^4, and the torsional rigidity, GJ, was found to be 59,816 lb-in^2.

Then, experimentally, the torsional rigidity was calculated utilizing the data collected in lab. The first method was to use the rate of total twist angle to find the max shear strain.

$$\gamma = R_e \frac{d\phi}{dx} = R_e \frac{\phi}{L} \tag{6}$$

$$\gamma = \frac{TR_e}{GJ} \tag{7}$$

$$GT = \frac{T}{\tau} R_e \tag{8}$$

From the twist angle method, the torsional rigidity was calculated to be 18,463 lb-in^2.

The second method utilized the extensometer shear strain data collected. This shear strain was plotted against torsion and the slope, $\frac{T}{\gamma}$, was found and multiplied by the radial distance to get torsional rigidity.

$$\gamma = \frac{TR_e}{GJ} \tag{9}$$

$$GJ_{CTW} = \frac{T}{\gamma} R_e \tag{10}$$

Using the extensometer data, the torsional rigidity was found to be 60,396 lb-in^2.

B. Analysis of the Open Thin Wall Specimen

Next, an open thin wall specimen was tested. This specimen was an aluminum tube with a cut of negligible width compared to the radius down the length of it. The cross section dimensions and material properties are the same as the closed thin wall specimen.

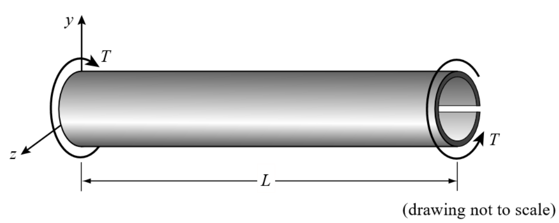

Fig. 3 OTW Specimen

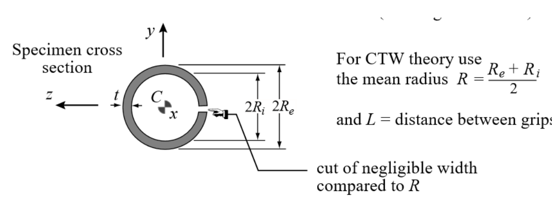

Fig. 4 OTW Cross Section

This specimen was subjected to torque from $T_0 = 0$ lbs-in to $T_{max} = 20$ lbs-in. The max shear stress is around 7800 psi, providing a safety factor of approximately 2.8 against yielding. The twist angle over the length is $\phi = \frac{\gamma L}{t}$.

To find the torsional rigidity of this specimen, it can be approximated as a rectangular plate with a width equal to the midline circumference. The torsional rigidity is equal to GJ_β, where G is known and J_β is found using the following equations.

$$J_\beta = \beta b t^3 \tag{11}$$

$$b = 2\pi R_m \tag{12}$$

$$R_m = R_i + \frac{t}{2} \tag{13}$$

$$GJ_{OTW} = \frac{T}{\gamma} t \tag{14}$$

β is an experimentally-derived value, and since b is much greater than t, it is assumed to $\alpha = \beta = 1/3$ for the purposes of this lab. Using this method, the torsional rigidity was calculated to be 599.21 lb-in^2. The torsional rigidity was also calculated using the experimentally-collected extensometer and twist angle data. These methods yielded torsional rigidity values of 686.43 lb-in^2 and 3122.5 lb-in^2, respectively.

The torsional rigidity values calculated using both theoretical and experimental methods for both test sections are shown below.

	Exact Theory	CTW/OTW	Extensometer Method	Twist Angle Method
Torsional Rigidity - Closed (lb-in^2)	60311	59816	60396	18463
Torsional Rigidity - Open (lb-in^2)	N/A	599.21	686.43	3122.5

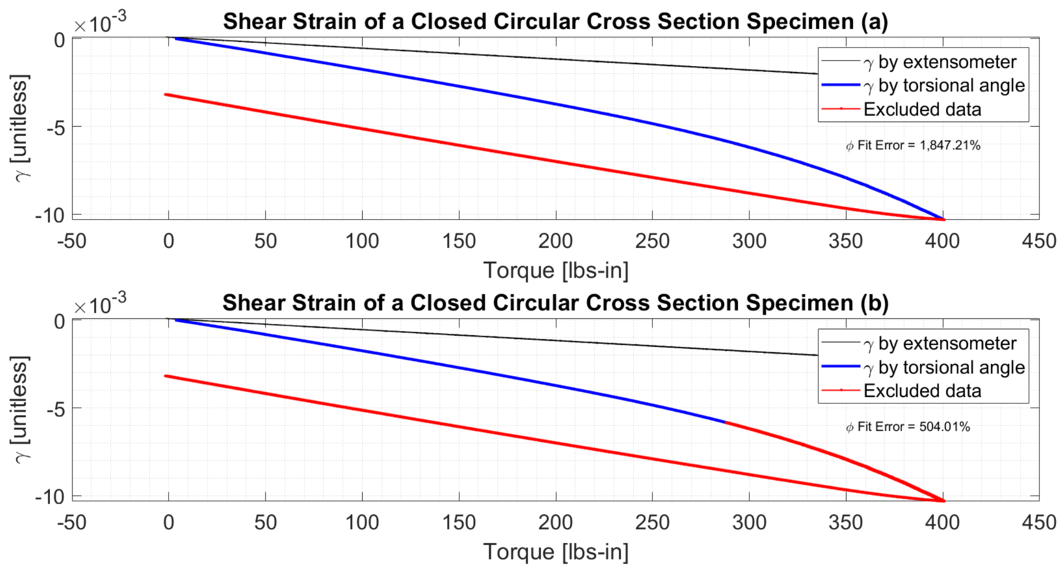

Fig. 5 Shear Strain of CTW with Different Amounts of Excluded Data. 5.a: Top, 5.b: Bottom.

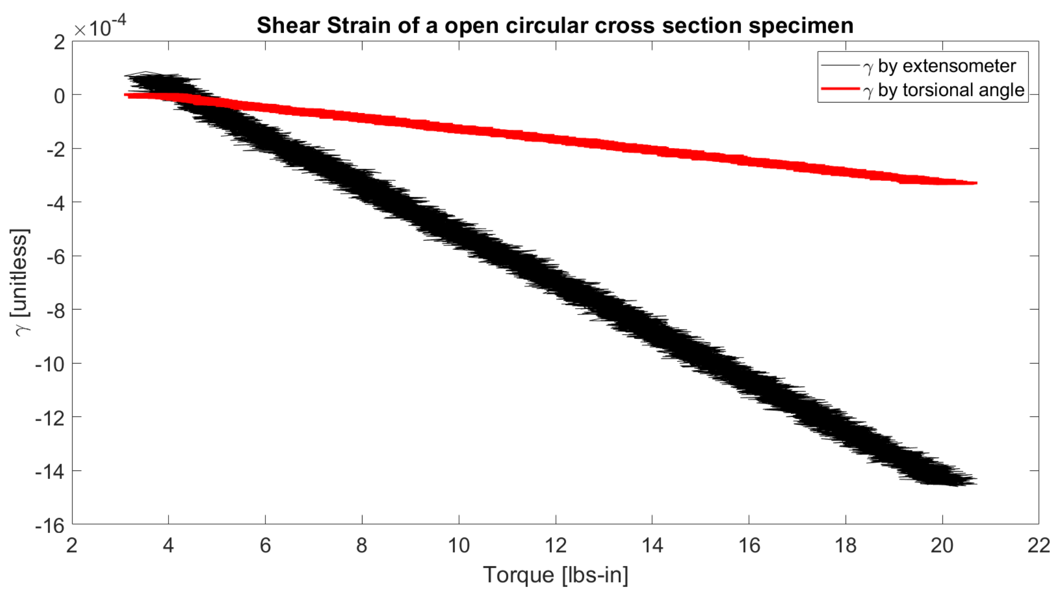

Fig. 6 Shear Strain of OTW

In figures 5 (a) and 5 (b), there is excluded data. The excluded data is data that the MTS machine measured. Theoretically, the shear should be linear and follow the same path when applying the torque and removing the torque. In both (a) and (b), the data from when the MTS machine removes the applied load is excluded and labeled in red. The blue data is the data from the MTS machine that is kept. In (b), more of the curved data is excluded so that only linear data is used in the model. Theoretically there should only be linear data from the MTS machine so the linear region will be used when comparing to the extensometer data. The extensometer is linear as expected so no data was excluded. The twist angle error between the measured data and theory for the CTW was 69.39% and the extensometer error between the measured data and theory for the CTW was 0.14%. The twist angle error between the measured data and theory for the OTW was 421.10% and the extensometer error between the measured data and theory for the OTW was 14.56%. The following errors clearly show that the extensometer is much more precise than using the overall twist angle measured from the MTS machine. It is therefore more precise to measure the torsional rigidity (GJ) using the extensometer than the overall twist angle. The OTW section also has an overall increae in error compared to the CTW section. This could be due to the fairly large score in the physical specimen (non-negligible). There is also error for the least squares fitted data. For CTW extensometer and CTW twist the error was 153.06% and 431.51% respectively. For OTW extensometer and OTW twist the error was 34.66% and 33.37% respectively. The reason the error is lower for the OTW for the least squares fit is because the data is less scattered for OTW than CTW.

C. Importance of the Extensometer

For the closed thin wall specimen, the measured torsional rigidity did not vary wildly between the extensometer method and the twist angle method. Differences in values between the two are likely small because the torsional rigidity is higher for this specimen.

When compared with the differences between measurements for the open thin wall specimen, the torsional rigidity for the open thin wall specimen varied much more between the extensometer and the testing machine. Again, this is likely due to the fact that the torsional rigidity of the open thin wall specimen is much lower, and therefore errors in the machine measurement at the end caps are more likely to appear at the lower torsional rigidity.

The extensometer acts as a sort of encoder for the twist angle of the shaft. The machine's connection to the actual bar may slip and cause an inaccurate angle to be read, but the extensometer theoretically should stay seated at the two points that it is originally attached to. Additionally, it allows us to measure near the center of the specimen as opposed to the ends. This is important because there may be unpredictable deformations and weird end cap effects occurring, but the middle of the specimen should behave as we are able to model using OTW and CTW theory for the respective specimens.

D. Plastic Deformation

If the samples are tested beyond the elastic regime we will begin to see different material behavior. Figure 7 gives a sketch of the expected response of the system in the form of a $T - \gamma$ plot.

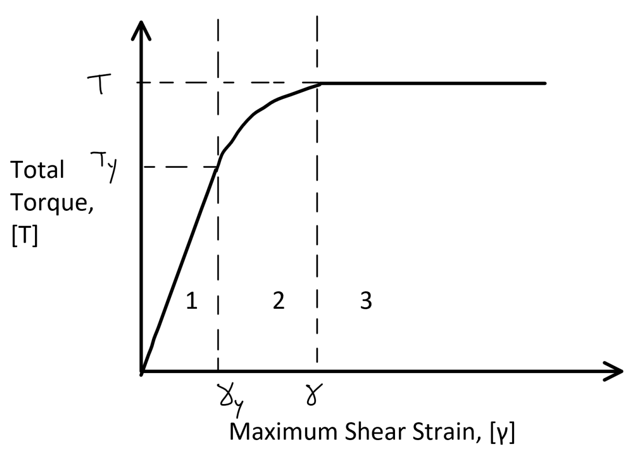

Fig. 7 Expected Response of the Specimen

The three regions are as follows:
1) Linear elastic region: any deformation in this region is fully recoverable.
2) Partially plastic region: strain has reached γ_y.

3) Fully plastic region: the specimen no longer resists any torque and any effort to increase it will simply cause more deformation.

Additionally, assuming a closed wall specimen with length L, external radius R_e, internal radius R_i, and thickness $t = R_e - R_i$, the shear strain γ corresponding to the transition between the regions can be expressed as a function of γ_y and the geometry of the sample. The max shear strain is defined as γ and is the shear strain when the specimen has transitioned to fully plastic deformation. The shear strain γ_y is then the shear strain when the material first starts to experience partial plastic deformation. This occurs when the inner radius remains elastic while the outer radius begins to experience plastic deformation. Then from equation 7.4 in the lecture notes, we can express this as:

$$\gamma = \frac{\rho}{R}\gamma_{max}$$

$$\gamma_y = \frac{R_i}{R_e}\gamma$$

Re-arranging terms we get:

$$\gamma = \frac{R_e}{R_i}\gamma_y$$

This solution makes sense because the outer radius is larger than then inner radius. This ratio then expresses the transition to fully plastic shear as being just slightly above the shear at the end of the linear elastic region. This coincides with what we would expect from a thin-walled specimen.

III. Conclusion

The purpose of this lab is to demonstrate the difference in resistance to torque of two different test specimens. By evaluating the torsional rigidity of the open and close-walled tubes, their discrepancy reveals the importance of cross-sectional shape when designing a shaft. Additionally, the importance of the extensometer was emphasized by the disparity in measurement from the twist angle data. Finally, theoretical values of torsional rigidity were calculated for each cross-section using different methods of assumption. These results show the importance of using proper assumptions when designing structural components. This information can be applied to future projects where we may need to test the torsional rigidity of a composite specimen or make important decisions regarding the structural integrity of the design.

References

[1] "MathWorks - Makers of MATLAB and Simulink." MathWorks - Cleve Moler and makers of MATLAB and Simulink - MATLAB & Simulink, www.mathworks.com/.

[2] University of Colorado Boulder, Department of Aerospace Engineering. "Lab 1 - Description," University of Colorado Boulder Stuctures, 2019.

IV. Acknowledgements

We thank the lab assistant Will Butler and teaching fellow Deven Mhadgut along with Professor Lopez-Jimenez for assistance in the experimental testing procedure.

V. Appendix

A. MATLAB Code

```matlab
%% Info:

% this code is post-data collection analysis for two tubes, one is open and
% one is closed, for more info, go to /Info

%% ASEN 3112 Experimental Lab 1
% Author: Abdulla Al Ameri
% Created: 09/20/2019
% Collaborators: Cameron Humphreys, Megan Jones

% This code is post-data collection analysis for two test specimens. For more info, go to /Info.


%% housekeeping

clear
clc
close all

%% define onstants:

G = 3.75 * 10 ^ 6 ; % psi (shear modulus);


% closed thin wall specimen:

De_closed = 3/4 ; % in, exterior daiamter
t_closed = 1/16 ; % in, thickness
L_closed = 13 ; % inches, measured in lab.
T_max_closed = 400; % (lbs-in), maximum torque
Tau_max_closed = 8620; % psi, max shear stress
FS_closed = mean([ 2.3 2.6]); % given in lab doc as range, so took the mean.
L_closed = 7/16 * 8 ; % inches, measured in lab.


% open thin wall specimen
% everything same as closed, we assume cut is negligible :)

De_open = 3/4 ; % in, exterior daiamter
De_open = (5/16)*2;
t_open = 1/16 ; % in, thickness
L_open = 13 ; % inches, measured in lab.
T_max_open = 20; % lbs -in
Tau_max_open = 7800; % psi
```

```
FS_open = 2.8; % factor of safety.
L_open = 7/16 * 8;

%% read data:

addpath('./Data'); % add path of data files

% to use importdata, fopen must be issued first
fopen('400inclosed.txt');
fopen('20inopen.txt');

% now import data safely
Data_closed = importdata('400inclosed.txt'); %import data
Data_open = importdata('20inopen.txt'); %import data


% close open handles
fclose('all');

% extract data

time_closed = Data_closed.data(:,1); % in sec
twist_angle_closed = Data_closed.data(:,2); % in deg
shear_strain_closed = Data_closed.data(:,3); % in deg
Torque_closed = Data_closed.data(:,4); % in-lbf
Axial_closed = Data_closed.data(:,5); % in
twist_angle_closed = twist_angle_closed - twist_angle_closed(1) ; % zero torsional angle

time_open = Data_open.data(:,1); % in sec
twist_angle_open = Data_open.data(:,2); % in deg
shear_strain_open = Data_open.data(:,3); % in deg
Torque_open = Data_open.data(:,4); % in-lbf
Axial_open = Data_open.data(:,5); % in
twist_angle_open = twist_angle_open - twist_angle_open(1) ; % zero total torsional angle


% zero time:
time_closed = time_closed - time_closed(1);
time_open = time_open - time_open(1);


Theortical_rigidity_closed_exact = G .* (1/2) * pi * ( (De_closed/2)^4 - ((De_closed/2) - t_closed )^4); % exact theortical rigidity for closed only

J_open = (1/3) * (2*pi*(De_open/2)) * (t_open)^3 ; % polar moment of inertia
J_Closed = ( 4 * ((pi)*(((De_closed - t_closed)/2)^2))^2 * t_closed ) ./ ( 2 * pi * ((De_closed - t_closed)/2)) ; % polar moment of inertia

Theortical_rigidity_CTW = G .* J_Closed; %theortical rigidity for closed thin wall theory
Theortical_rigidity_OTW = G .* J_open; % theortical rigidity for open thin wall theory
```

```
% convert units:

shear_strain_closed = deg2rad(shear_strain_closed);
twist_angle_closed = deg2rad(twist_angle_closed);

shear_strain_open = deg2rad(shear_strain_open);
twist_angle_open = deg2rad(twist_angle_open);



%% backgroun:

%{

No matter what's the theory we're using, it is always the case that we can
estimate the theortical rigitdy by knowing that for both OTW + CTW the rate
of total twist angle (derivative of Phi) is = T / (GJ); so dividing torque
by rate of total twist angle will estimate GJ (rigidity)..

it's also the case that for the exact method (I think it holds for both OTW
and CTW) that the shear strain = ( T*roh ) / (GJ) where roh is radial
distance, so if we have plots for T on Y and Shear strain on X if we take
dslope of that (i.e. T/Y)  * R it'll also be the rigidity :)


UPDATES:

dphi/dx = T / (GJ) always.

Phi = ( shear Strain * L ) / t -- open thin wall
Phi = ( shear Strain * L ) / Re -- closed thin wall

we can get shear strain the above equations.

this means also that for :

shear strain / t = T / (GJ) -- open thin wall
shear strain / Re = T / (GJ) -- closed thin wall

%}
%% Closed specimen:

% it's circular, so we can use exact, or thin wall theory.

% Plot the torque vs.  shear strain provided by the extensometer, as well as
    the torque vs.  shear
% strain calculated using the total rotation angle imposed by the testing
    machine.

% SS = Shear strain. C = closed
SS_C_Epsilon = shear_strain_closed;  % Shear strain, closed;
SS_C_Twist = (twist_angle_closed ./ L_closed) .* (De_closed/2 );
```

```
% Shear strain = SS, O = Open.
SS_O_Epsilon = shear_strain_open;  % Shear strain, closed;
SS_O_Twist = (twist_angle_open ./ L_open) .* (t_open);


% because there are some issues with the data, we will exclude what's not
% used:

% include from 1 all the way up to this index
Exclude_i = find(Torque_closed==max(Torque_closed)) - 12000 ;



figure(1)

plot(Torque_closed,SS_C_Epsilon,'k');
hold('on')
plot(Torque_closed(1:Exclude_i),SS_C_Twist(1:Exclude_i),'b','LineWidth',2);
plot(Torque_closed(Exclude_i:end),SS_C_Twist(Exclude_i:end),'.-r','LineWidth'
    ,1);
grid minor

xlabel('Torque [lbs-in]');
ylabel('\gamma [unitless]');
title('Shear Strain of a closed circular cross section specimen');
legend('\gamma by extensometer','\gamma by torsional angle','Excluded data')


figure(2)

plot(Torque_open,SS_O_Epsilon,'k');
hold on
plot(Torque_open,SS_O_Twist,'r','LineWidth',2);
xlabel('Torque [lbs-in]');
ylabel('\gamma [unitless]');
title('Shear Strain of a open circular cross section specimen');
legend('\gamma by extensometer','\gamma by torsional angle')


%% least squares fit

% polyfit will do lest squares linear fit.

% in polyval Evaluate the first-degree polynomial
% fit in p at the points in x. Specify the error estimation
% structure as the third input so that polyval calculates an
% estimate of the standard error. The standard error estimate is
% returned in delta.

%
[ p S ] = polyfit(SS_C_Epsilon,Torque_closed,1); % get fit
Rigidity_C_Epsilon = abs(p(1))*(De_closed/2); % estimate rigidity
[y_fit,delta] = polyval(p,SS_C_Epsilon,S);
```

```
Err_Rigidity_C_Epsilon = mean(delta);
Fit_SS_C_Epsilon = @(x) p(1)*x + p(2) ;

% [ p S ] = polyfit(SS_C_Twist,Torque_closed,1);
% Rigidity_C_Twist = abs(p(1))*(De_closed/2);
% [y_fit,delta] = polyval(p,SS_C_Twist,S);
% Err_Rigidity_C_Twist = mean(delta);
% Fit_SS_C_Twist = @(x) p(1)*x + p(2) ;

[ p S ] = polyfit(SS_C_Twist(1:Exclude_i),Torque_closed(1:Exclude_i),1);
Rigidity_C_Twist = abs(p(1))*(De_closed/2);
[y_fit,delta] = polyval(p,SS_C_Twist,S);
Err_Rigidity_C_Twist = mean(delta);
Fit_SS_C_Twist = @(x) p(1)*x + p(2) ;



[ p S ] = polyfit(SS_O_Epsilon,Torque_open,1);
Rigidity_O_Epsilon = abs(p(1))*(t_open);
[y_fit,delta] = polyval(p,SS_O_Epsilon,S);
Err_Rigidity_O_Epsilon = mean(delta);
Fit_SS_O_Epsilon = @(x) p(1)*x + p(2) ;


[ p S ] = polyfit(SS_O_Twist,Torque_open,1);
Rigidity_O_Twist = abs(p(1))*(t_open);
[y_fit,delta] = polyval(p,SS_O_Twist,S);
Err_Rigidity_O_Twist = mean(delta);
Fit_SS_O_Twist = @(x) p(2)*x + p(1) ;


%% compute relative error:

% relative error to theortical rigidity for exact method
rel_CTW_Epsilon_exact = abs(Theortical_rigidity_closed_exact-
    Rigidity_C_Epsilon)./Theortical_rigidity_closed_exact;
rel_CTW_Twist_exact = abs(Theortical_rigidity_closed_exact-Rigidity_C_Twist)./
    Theortical_rigidity_closed_exact;

rel_CTW_Epsilon = abs(Theortical_rigidity_CTW-Rigidity_C_Epsilon)./
    Theortical_rigidity_CTW;
rel_CTW_Twist= abs(Theortical_rigidity_CTW-Rigidity_C_Twist)./
    Theortical_rigidity_CTW;

rel_OTW_Epsilon = abs(Theortical_rigidity_OTW-Rigidity_O_Epsilon)./
    Theortical_rigidity_OTW;
rel_OTW_Twist= abs(Theortical_rigidity_OTW-Rigidity_O_Twist)./
    Theortical_rigidity_OTW;


%% output results to table


Method = {'CTW-Exact';'CTW';'OTW'};
```

```
Theortical = [Theortical_rigidity_closed_exact; Theortical_rigidity_CTW;
    Theortical_rigidity_OTW];
Extensometer = { 'N/A';Rigidity_C_Epsilon;Rigidity_O_Epsilon};
TwistAngle = { 'N/A';Rigidity_C_Twist;Rigidity_O_Twist};
Extensometer_fit_err = { 'N/A';Err_Rigidity_C_Epsilon;Err_Rigidity_O_Epsilon};
TwistAngle_fit_err = { 'N/A';Err_Rigidity_C_Twist;Err_Rigidity_O_Twist};



Extensometer_relative_err = { rel_CTW_Epsilon_exact ; rel_CTW_Epsilon ;
    rel_OTW_Epsilon };
TwistAngle_relative_err = { rel_CTW_Twist_exact ; rel_CTW_Twist ;
    rel_OTW_Twist };


Results = table(Method,Theortical ,Extensometer ,TwistAngle ,Extensometer_fit_err
    ,TwistAngle_fit_err ,Extensometer_relative_err ,TwistAngle_relative_err)
fprintf('Note 1: all torsional rigidities are in psi-in^4')
fprintf('\n')
fprintf('Note 2: convert error to %% by multiplying by 100')
fprintf('\n')
```

B. Team-Member Contributions

Table 1 Team Member Participation Report

Member	Tasks	Contributions	Performance
Cameron Humphreys	Data analysis and write up.	Assisted with coding and debugging, OTW theoretical calculations. Wrote abstract and OTW report sections.	100
Sam Hartman	Error calculations and plots relating shear strain and T	Ensured values matched code, plots, error analysis, abstract	100
Joseph Buescher	Lab Document setup, Theoretical Calculations	Calculations for 2.1 and 2.2, Document Setup, Nomenclature, Proofing Document	100
Megan Jones	Experimental setup, theoretical calculations, write up	Write up for 2.1, validating code with calculations	100
Abdulla Al Ameri	Group leader and data analysis (code)	Theoretical rigidity results, shear strain results, and figures.	100
Conner Martin	Plastic deformation section	Question 2.4, second part of question 2.3, introduction, conclusion	100
Zak Dmitriyev	Running experiment procedure and data gathering	Part of question 2.3	100