ASEN 3112 STRUCTURES

Lab 3 DESCRIPTION

Fall 2019

Version: November 5, 2019

I Summary

In the third Structures Lab, you will investigate frequency resonances of a highly simplified, scaled model of an airplane, vibrating under prescribed harmonic base excitation produced by a shaker device. The objectives of the lab are:

- To expose you to the phenomenon of resonance of a flexible dynamic system subject to harmonic excitation applied through a shaker.
- To give you a hands-on feeling of an experimental technique (frequency sweep) widely used in industry to locate natural frequencies and thus expose possibly dangerous resonance ranges.
- To compare a subset of the observed resonant frequencies with natural frequencies predicted by two very coarse Bernoulli-Euler beam finite element models.

This document provides a description of the experimental setup, measurement procedure, and provides detailed information on how to prepare the two FEM models.

II Timetable, Groups and Logistics

II.1 Timetable

This lab spans three weeks, with reports due on Monday December 2nd, 2019. The experimental procedure demos will be held on November 4, 2019, in the Co-PILOT at 12:30PM for students in Section 011 and at 2:30PM for students in Section 012. Students must attend the experimental demonstration during their respective lab/recitation times. Attendance to a different section needs prior written approval from the instructors and will only be granted on account of a bona-fide reason, such as a medical emergency. In case of an emergency or unavoidable absence, students should contact the instructors as soon as possible. Attendance is mandatory and will be taken at the start of each demo.

Group tests will be done in the Module Testing 141B, in the PILOT. The groups are formed based on students signing up for a group test time slot; see Section II.2. Groups should meet before or after their session and get organized. All student must attend both the demo and and group test. Failure to do so will result in grade deduction, see Section VI.3 for more details.

II.2 Groups

The groups are formed by the students who sign up for the same time slot, using SignUpGenius. Each slot has a limit of 7 students. As soon as this limit is reached, a student is no longer permitted to sign up for that slot. The sign up is available at the Sign-up Genius website:

https://www.signupgenius.com/go/9040D4FA5A929A6FC1-asen

As in previous labs, each group should select a leader (the Group Leader, or GL) who will have the following responsibilities:

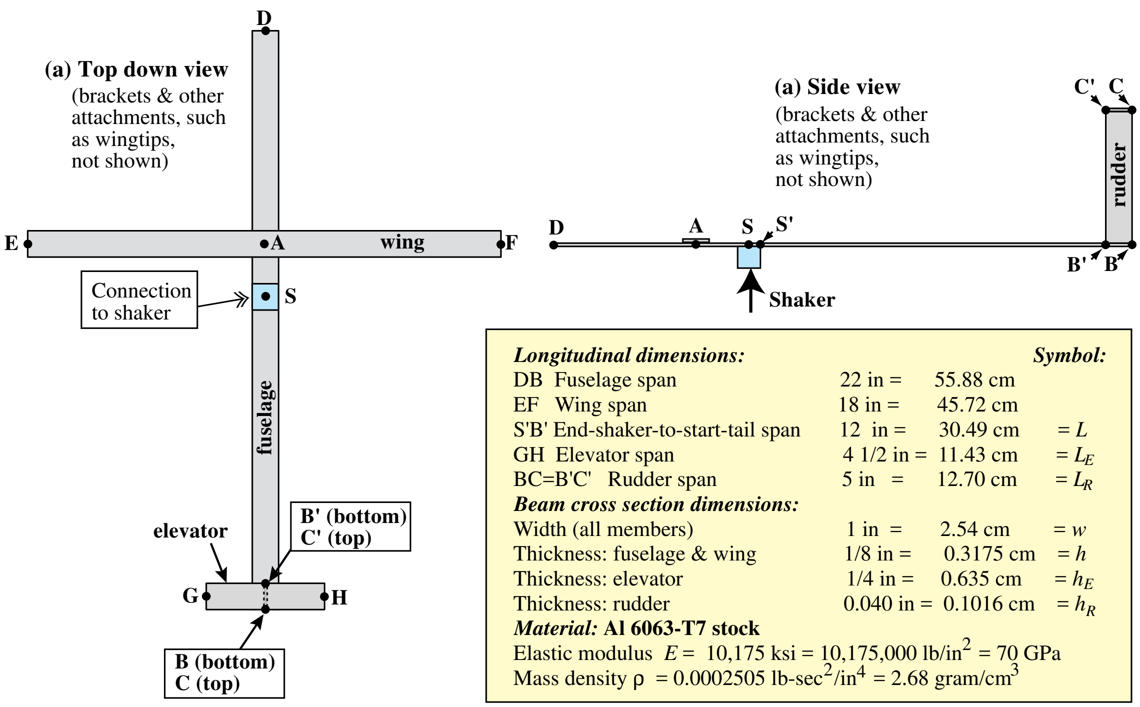
- Divide tasks to be accomplished in writing the report (e.g. writing specific sections, analyzing data and producing necessary plots/figures).

- Compile and edit the final report (ensure consistency between sections and make sure that other member's contributions are satisfactory).
- Provide internal deadlines to group members so that the lab report project stays on schedule.
- Keep a record of delegated tasks, internal deadlines, and confirmations from team members. This record can simply be a thread of emails between the group leader and group members. This will not be turned in but will be used by the TAs to resolve any disputes about participation scores.
- Provide a participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. It is the group leader's responsibility to organize a peer evaluation process to determine each group member's contribution grade; see Section VI.3. Note, the group leader should not assign a participation score without the input of the entire group.

Group leaders do not have to write their own section of the report (though they can if they want to). Group members are responsible for timely communication with the group leader. If a student is assigned a task to complete with a deadline, the student should confirm that she/he will do so. If the student does not agree to the task or has difficulty with it and needs more time or help with the task, this should also be communicated to the group leader (well before the deadline).

II.3 Lab Reports

Each group prepares and submits one hard copy of the report, which is due before class on Monday, December 2nd, 2019. Instructions for preparing this report are given in Section VI of this document.



Longitudinal dimensions:			*Symbol:*
DB Fuselage span	22 in =	55.88 cm	
EF Wing span	18 in =	45.72 cm	
S'B' End-shaker-to-start-tail span	12 in =	30.49 cm	$= L$
GH Elevator span	4 1/2 in =	11.43 cm	$= L_E$
BC=B'C' Rudder span	5 in =	12.70 cm	$= L_R$
Beam cross section dimensions:			
Width (all members)	1 in =	2.54 cm	$= w$
Thickness: fuselage & wing	1/8 in =	0.3175 cm	$= h$
Thickness: elevator	1/4 in =	0.635 cm	$= h_E$
Thickness: rudder	0.040 in =	0.1016 cm	$= h_R$
***Material:* Al 6063-T7 stock**			
Elastic modulus E = 10,175 ksi = 10,175,000 lb/in^2 = 70 GPa			
Mass density ρ = 0.0002505 lb-sec^2/in^4 = 2.68 gram/cm^3			

Figure 1: Data for test article of Airplane Shaker demo. Values with specific names such as E, ρ, L, L_E, etc., are those used in the FEM analysis to predict a natural frequency subset. They are also restated in Table 1, along with other information.

III Experimental Procedure Summary

In the experiments you will characterize the dynamical response of a simplified scaled model of an airplane, shown in Figure 1). It consists of nose (section AD), wings (section EAF) and tail (section AB). The experimental procedure is detailed in a separate document; we only present a brief summary here.

The structure will be instrumented with four accelerometers, one of which will always be used to measure the input acceleration provided by the shaker. A laser vibrometer will be used to monitor the horizontal displacement of the rudder, since the weight of an accelerometer will be enough to alter its dynamics.

First, you will perform a frequency sweep, in which the excitation frequency will be increased slowly, in order to excite all modes. All sensors (laser vibrometer and accelerators) will be placed in "standard" locations, so that the vibration of all elements (nose, wings and tail) are monitored.

Then, you will explore the two modes that can be predicted with the finite element model described in Section IV. These are the modes that excite the tail of the model airplane. You will need to decide where to relocate the accelerometers to obtain the best data possible to characterize the mode shape.

IV Finite Element Analysis

The objective of the computer portion of this Lab is to predict, using very simple FEM models, two of the five natural frequencies observed experimentally. These two involve *cantilever modes* of the aft-fuselage (the portion of the fuselage behind the shaker plus the tail assembly, see Figure 1). They appear as modes #2 and #5 in the shaker frequency sweep and are identified as **Horizontal tail vertical + "T" section sideways**, and **Horizontal tail vertical second mode vibrations**,respectively.

A diagram of the test article for use in the FEM model is shown in Figure 2. That figure provides geometric dimensions and material properties. It also defines the symbols used in this Lab. Values are listed in both English and metric units. For this Lab we will use the following *English units*: length in inches (in), force in pounds (lb), and time in seconds (sec). The mass density ρ is a derived unit: lb-sec^2/in^4.

Note: the element stiffness and mass matrices supplied below to carry out the FEM analysis are given as recipe. Their derivation requires the Method of Virtual Displacements and the Lagrange equations applied to beams; the latter topic is beyond the scope of this course.

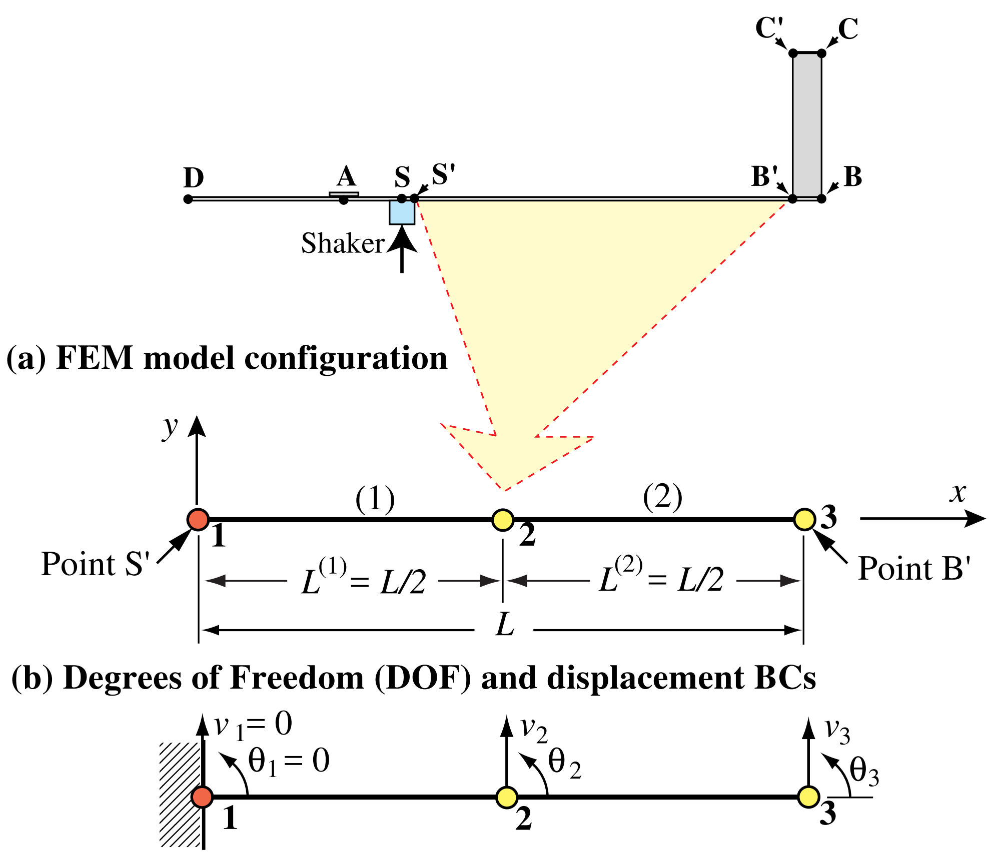


Figure 2: Two-element FEM model of aft-fuselage plus "lumped tail" assembly.

IV.1 Two-Element Model

A very coarse FEM model is shown in Figure 2. The aft-fuselage span S'B' between the end of the shaker and the start of the tail assembly is modeled with two Bernoulli-Euler plane beam elements as shown. The tail assembly is modeled as a combination of point-mass, point-first-mass-moment-of-inertia and point-second-mass-moment-of-inertia. Those three quantities were evaluated using a symbolic computation program, e.g., *Mathematica*, to carry out straightforward but error-prone integrations. They are lumped to node 3 (located at B').

Since the FEM model is two-dimensional, *it can only account for aft-shake vertical cantilever modes that occur in the fuselage-rudder plane.* As such, it misses three of the five observed natural frequencies, because their associated mode shapes do not fit the stated 2D motion restrictions.

Before applying displacement boundary conditions this FEM model has six DOF collected in the vector

$$\mathbf{u} = \begin{bmatrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3. \end{bmatrix}^T \tag{1}$$

Here v_i denotes the displacement of node i normal to the beam while θ_i is the rotation of the beam cross section about z, as sketched in Figure 3. The master mass and stiffness matrices for this model are

$$\mathbf{M}_2 = c_{M2} \begin{bmatrix} 19272 & 1458L & 5928 & -642L & 0 & 0 \\ 1458L & 172L^2 & 642L & -73L^2 & 0 & 0 \\ 5928 & 642L & 38544 & 0 & 5928 & -642L \\ -642L & -73L^2 & 0 & 344L^2 & 642L & -73L^2 \\ 0 & 0 & 5928 & 642L & 19272 & -1458L \\ 0 & 0 & -642L & -73L^2 & -1458L & 172L^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_T & S_T \\ 0 & 0 & 0 & 0 & S_T & I_T \end{bmatrix},$$

$$\mathbf{K}_2 = c_{K2} \begin{bmatrix} 24 & 6L & -24 & 6L & 0 & 0 \\ 6L & 2L^2 & -6L & L^2 & 0 & 0 \\ -24 & -6L & 48 & 0 & -24 & 6L \\ 6L & L^2 & 0 & 4L^2 & -6L & L^2 \\ 0 & 0 & -24 & -6L & 24 & -6L \\ 0 & 0 & 6L & L^2 & -6L & 2L^2 \end{bmatrix},$$

$$\text{in which} \quad c_{M2} = \rho A\, L/100800, \quad c_{K2} = 4EI_{zz}/L^3, \quad A = wh, \quad I_{zz} = wh^3/12. \tag{2}$$

Symbols E, ρ, L, w, h, M_T, S_T and I_T, which appear above, are defined in Table 1 on page 6 of this document. (For reference convenience that Table also lists tail assembly member dimensions; these were used to compute the lumped values M_T, S_T and I_T, but do not appear in the matrices above.)

Denoted by $\hat{\mathbf{M}}_2$ and $\hat{\mathbf{K}}_2$, the reduced mass and stiffness matrices are obtained by removing rows 1,2 and columns 1,2 from $\mathbf{M}_2$ and $\mathbf{K}_2$, respectively, on account of the fixed-end displacement BC at node 1. Those reduced matrices are of order 4×4. The vibration eigen problem for this model is

$$\boxed{\hat{\mathbf{K}}_2 \mathbf{U} = \omega^2\, \hat{\mathbf{M}}_2\, \mathbf{U}.} \tag{3}$$

Build a code to solve the Eigenvalue problem in Equation 3 and report only the smallest three frequencies obtained, i.e., $f_i = \omega_i/(2\pi)$, $i = 1, 2, 3$ in Hz. Compare the mode shapes to the partial reference results listed in Figure 4. Then compare f_1 and f_2 to frequencies #2 and #5 observed in the shaker demo.

To plot the eigenvectors (mode shapes), use the procedure described in Section V. Those are shown only for informational purposes; they need not be compared with experimental results.

IV.2 Four-Element Model

A more refined FEM model is shown in Figure 3. Here the fuselage span S'B' is modeled with four Bernoulli-Euler plane beam elements as shown. The tail assembly is modeled exactly as done for the two-element model, with lumped values assigned to node 5. Before applying displacement boundary conditions, the model has ten DOF collected in the vector

$$\mathbf{u} = \begin{bmatrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 & v_4 & \theta_4 & v_5 & \theta_5 \end{bmatrix}^T. \tag{4}$$

The master mass and stiffness matrices for this model are

$$\mathbf{M}_4 = c_{M4}\begin{bmatrix} 77088 & 2916L & 23712 & -1284L & 0 & 0 & 0 & 0 & 0 & 0 \\ 2916L & 172L^2 & 1284L & -73L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 23712 & 1284L & 154176 & 0 & 23712 & -1284L & 0 & 0 & 0 & 0 \\ -1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 23712 & 1284L & 154176 & 0 & 23712 & -1284L & 0 & 0 \\ 0 & 0 & -1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 23712 & 1284L & 154176 & 0 & 23712 & -1284L \\ 0 & 0 & 0 & 0 & -1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 23712 & 1284L & 77088 & -2916L \\ 0 & 0 & 0 & 0 & 0 & 0 & -1284L & -73L^2 & -2916L & 172L^2 \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_T & S_T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_T & I_T \end{bmatrix},$$

$$\mathbf{K}_4 = c_{K4}\begin{bmatrix} 96 & 12L & -96 & 12L & 0 & 0 & 0 & 0 & 0 & 0 \\ 12L & 2L^2 & -12L & L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -96 & -12L & 192 & 0 & -96 & 12L & 0 & 0 & 0 & 0 \\ 12L & L^2 & 0 & 4L^2 & -12L & L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -96 & -12L & 192 & 0 & -96 & 12L & 0 & 0 \\ 0 & 0 & 12L & L^2 & 0 & 4L^2 & -12L & L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -96 & -12L & 192 & 0 & -96 & 12L \\ 0 & 0 & 0 & 0 & 12L & L^2 & 0 & 4L^2 & -12L & L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -96 & -12L & 96 & -12L \\ 0 & 0 & 0 & 0 & 0 & 0 & 12L & L^2 & -12L & 2L^2 \end{bmatrix},$$

$$\text{in which}\quad c_{M4} = \rho A L/806400, \quad c_{K4} = 8EI_{zz}/L^3, \quad A = wh, \quad I_{zz} = wh^3/12. \tag{5}$$

Here E, ρ, L, w, h, M_T, S_T and I_T, which appear above, are the same as for the two-element model. Their values may be retrieved from Table 1.

Denoted by $\hat{\mathbf{M}}_4$ and $\hat{\mathbf{K}}_4$, the reduced mass and stiffness matrices are obtained by removing rows 1,2 and columns 1,2 from $\mathbf{M}_4$ and $\mathbf{K}_4$, respectively. Those reduced matrices are of order 8×8. The vibration eigenproblem for this model is

$$\boxed{\hat{\mathbf{K}}_4\,\mathbf{U} = \omega^2\,\hat{\mathbf{M}}_4\,\mathbf{U}}. \tag{6}$$

Build a code to solve the Eigenvalue problem in Equation 6 and report only the smallest three frequencies obtained, that is, $f_i = \omega_i/(2\pi)$, $i = 1, 2, 3$ in Hz. Compare them to the first three frequencies from the two-element eigenproblem Equation 3. Then compare f_1 and f_2 to frequencies #2 and #5 observed in the shaker demo.

To plot the eigenvectors (mode shapes), use the procedure described in Section V. Those are shown only for informational purposes; they need not be compared with experimental results. To provide a beacon for the computational work, Figure 4 shows partial results obtained using *Mathematica* for the two-finite-element model.

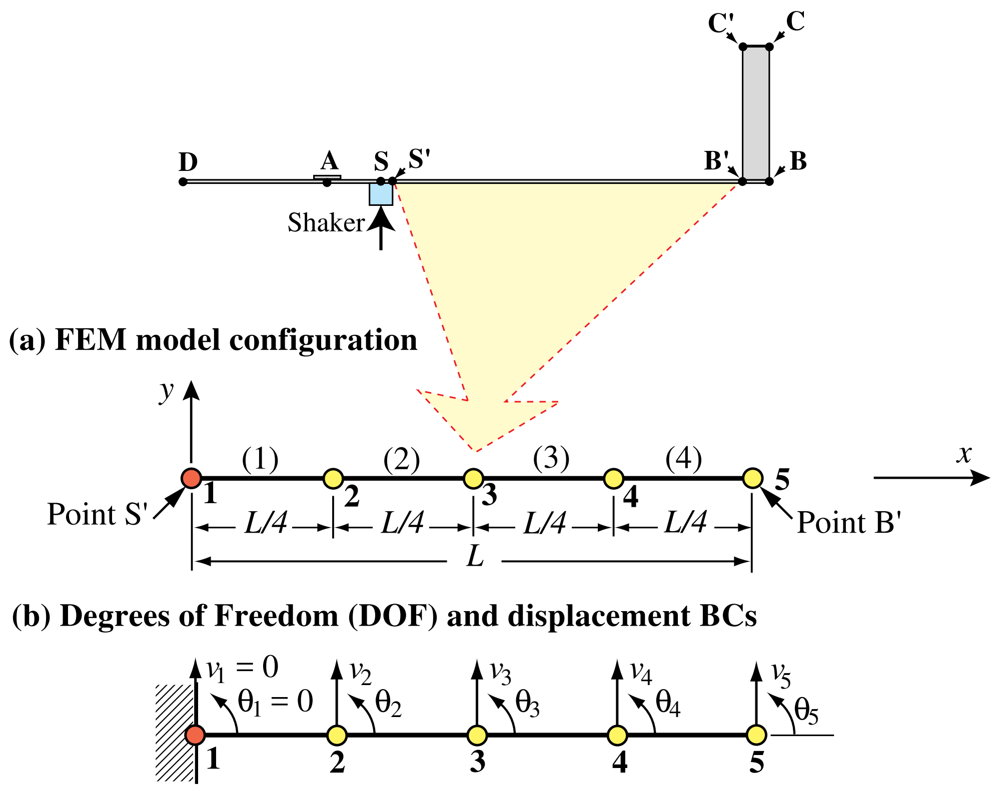


Figure 3: Four-element FEM model of aft-fuselage plus "lumped tail" assembly.

Description	Symbol	Value in English units
Cantilever span S'B'	L	12 in
Elevator span	L_E	4.5 in
Rudder span	L_R	5 in
Width of all members	w	1 in
Thickness of fuselage member	h	1/8 in
Thickness of elevator member	h_E	1/4 in
Thickness of rudder member	h_R	0.040 in
Material elastic modulus	E	10,175 ksi = 10,175,000 psi
Material mass density	ρ	0.0002505 lb-sec^2/in^4
Mass of tail assembly	M_T	(1.131 in^3) ρ
First mass-moment of tail assembly wrt B'	S_T	(0.5655 in^4) ρ
Second mass-moment of tail assembly wrt B'	I_T	(23.124 in^5) ρ

Note 1. Values listed for M_T, S_T and I_T must be scaled by the mass density ρ, as shown.
Note 2. Calculations may be done either using English units: in, lb and sec, or metric units: cm, N and sec. Use of English units, as given above, is recommended.

Table 1: Numerical values to be used in FEM models of aft-fuselage.

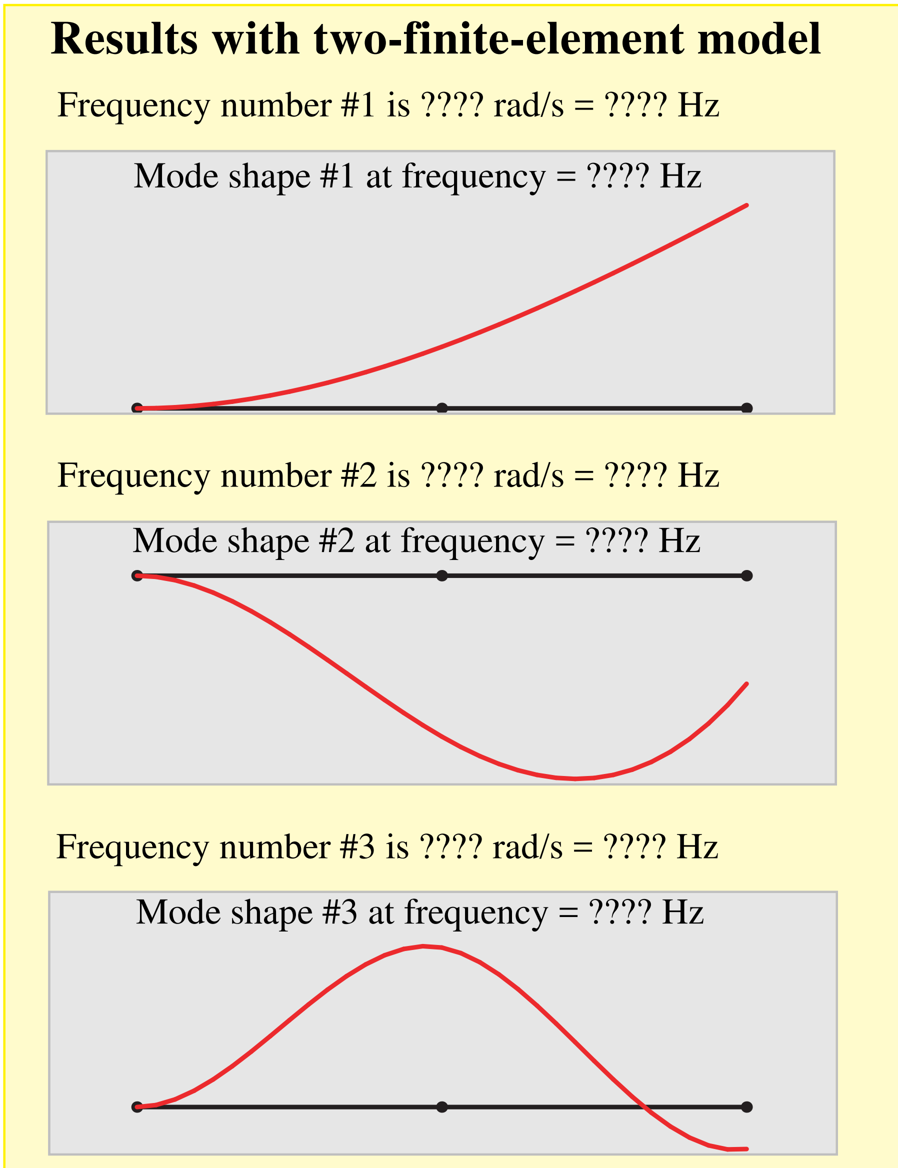


Figure 4: First three vibration frequencies — to be determined by groups — and associated mode shapes predicted by the FEM two-element model. Note: the FEM frequency associated with mode shape #3 is likely out of range of the frequency sweep range in group experiments. It is pictured only for completeness.

V Plotting Natural Frequency Mode Shapes

The frequency mode shapes are the first 3 eigenvectors associated with the 3 lowest computed natural frequencies. To show pictures of those modes, the C-like pseudocode listed in Figure 5 may be used. This code can be readily translated to a higher order language such as *Matlab* or *Mathematica*, which provide built-in graphics. Note, however, that the actual plotting statement will be very language dependent.

The procedure is invoked as

`ploteigenvector(L,ev,ne,nsub,scale)`

The arguments are:

`L`	The length of the FEM mode; see Figure 2 or Figure 3.
`ev`	The eigenvector to be plotted; see below for more details.
`ne`	The number of beam finite elements in the model (2 or 4)
`nsub`	The number of subdivisions of each element for plotting the eigenvector cubic curve as sequence of line segments. For a visually smooth plot 10 or more are recommended.
`scale`	A scaling factor for the eigenvector values. Initially should be set to 1. If the plot aspect ratio is unpleasant (for example, too flat), this argument may be adjusted.

An additional text argument: `title`, could be optionally appended to the foregoing list to write a heading over the plot, for example `"Mode shape #x, for frequency xx.xx Hz"`. But it is just as easy to write the heading before the procedure is called.

Argument `ev` requires some explanation. It is a vector containing `2*ne+2` entries. If the model contains 2 elements (`ne=2`), `ev` has the configuration in Equation 1 in which $v_1 = \theta_1 = 0$. If the reduced eigenproblem Equation 3 is solved by, say, *Matlab*, the program will return the last 4 eigenvector components, and the two zero ones will have to be inserted (prepended) before submitting to the plotting procedure. Similarly, if the model contains 4 beam elements (`ne=4`) , then `ev` has the configuration in Equation 4, with $v_1 = \theta_1 = 0$. Again if the reduced eigenproblem in Equation 6 is solved by *Matlab*, the program will return the last 8 components, and the two zero entries will have to be prepended for plotting.

```
procedure ploteigenvector (L,ev,ne,nsub,scale);
// declare local variables here if required by language
nv=ne*nsub+1; Le=L/ne; dx=Le/nsub; k=1;
x=v=zeroarray(nv); // declare and set to zero plot arrays
for (e=1,e<=ne,e++)  // loop over elements
    xi=Le*(e-1); vi=ev(2*e-1); θi=ev(2*e); vj=ev(2*e+1); θj=ev(2*e+2);
    for (n=1,n<=nsub,n++) // loop over subdivisions
        xk=xi+dx*n; ξ=float(2*n-nsub/nsub); // isoP coordinate
        vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(ξ^2-3)*ξ+
            Le*(ξ^2-1)*(θj-θi+(θi+θj)*ξ))) // Hermitian interpolant
        x(++k)=xk; v(k)=vk;  // build plot functions
    endfor  // end n loop
endfor // end e loop
// plot v (vertical) vs x (horizontal) -- language dependent
endprocedure
```

Figure 5: Pseudo-code for plotting mode shapes.

VI Writing A Report

VI.1 Organization

Reports are due December 2nd, 2019, before class time. **The report must be electronically processed, e.g. by WORD or LATEX; hard copies only are to be turned in**.

It must include:

- **Title Page**: Describes Lab, lists the name of the team members and identifies the group leader.

- **Results:** The results should address the questions in Section VI.2.
- **Appendix - Code.** A printout of all the code used to produce the results.
- **Appendix - Participation report.** More details on how the grade of individual group members is calculated can be found in Section VI.3.

VI.2 Report Content

The report will be graded on both technical content and presentation. Regarding the content, instead of an open-ended report, you should process the experimental data and computational results to address the specific questions posted below. Regarding presentation, make sure that you follow these guidelines:

- All plots should be readable. This includes using different color or line styles and a suitable font size for the axis labels and all other text in the plot. The range of both axes should be chosen to focus on the region of interest (*i.e.*, the data).
- Show your work, including equations used and partial results.
- All results should be presented with appropriate units.
- Be quantitative when comparing results. Use percentage of error or deviation. Refer back to predicted error or variance when applicable.

VI.2.1 Question 1: Experimental Results

- Provide plots of the response of the system as a function of the excitation frequency. Make sure that you factor out possible changes in the input excitation (*e.g.*, by using the magnification factor instead of just the output of the accelerometers). Provide a detailed explanation of the method used to process the data.
- Use your data to identify the resonant frequency of all five modes.
- For each mode, identify which sensors capture resonance, and use them to describe the shape of the mode.

VI.2.2 Question 2: FEM Results - Resonant frequencies

- Compute the first three resonant frequencies using both finite element models.
- Compare the frequencies, when possible, with the experimental results. Quantify and discuss the errors.

VI.2.3 Question 3: FEM Results - Mode shapes

- Plot the shapes of the first three modes provided by each of the FEM codes.
- Compare, when possible, with the experimental results of the corresponding modes. Plot the experimental results on the same figure as the predictions. Normalize the shapes appropriately.

VI.3 Individual Contribution Evaluation and Deductions for No-Show

The group leader submits along with the report a separate participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. **Please make sure to include this as an Appendix in the report**.

The performance of each group member is rated with a "contribution factor" on a scale of 0 to 100%. A score of 100% indicates that the member contributed the expected share to the experiment and to the preparation of the report. The scores are normally assigned by *peer evaluation*, using the same procedures followed in the ASEN 200x sophomore courses. The group leader is responsible for administering the peer evaluation, tabulating and submitting the results of said peer evaluation.

The individual score will be equal to:

$$\text{Individual score} = \text{Group score} \times \frac{100 + \text{Contribution factor}}{200} \tag{7}$$

For example, if the group receives an overall score of 88.75 and the individual received a "contributing factor" of 90%, the individual score is $88.75 \times (100 + 90)/200 = 84.31$.

ALL students in the group must attend the demo **AND** the group test. A no-show at the lab demos, without justification, will be penalized by a deduction of 50% from the final individual score. A no-show at group experiments, without justification, is deducted 50%. A no-show at both is deducted 100%. Students that miss one or both events on account of a bona-fide reason, such as a medical emergency or unavoidable absence, should contact instructors or TAs as soon as possible.

Addendum I. Report Grading

The score assigned to the lab report includes technical content (75%) and presentation (25%). This is a more detailed breakdown of the weights:

Category	Weight	Score	Contribution
Technical content			
Question 1	0.3		
Question 2	0.15		
Question 3	0.3		
Presentation			
Plots	0.10		
Grammar, style & spelling	0.10		
Formatting	0.05		
Total	1.00		(overall score)

The score within each category ranges from 0 to 100%. For example, if the score for 'Question 1' is 80%, it contributes $0.25 \times 80 = 20\%$ to the overall score. The final score of each team member is then calculated following the procedure detailed in Section VI.3.