Digital Logic and Design

Mr. Abdul Ghafoor

Lecture No. 09

Recap

- Examples of Boolean Analysis of Logic Circuits

- Examples of Simplification of Boolean Expressions

- Standard form of SOP and POS expressions

Recap

- Need for Standard SOP and POS expressions
- Converting standard SOP-POS
- Minterms & Maxterms
- Converting SOP & POS to truth table format

Karnaugh Map

- Simplification of Boolean Expressions
  - Doesn't guarantee simplest form of expression
  - Terms are not obvious
  - Skills of applying rules and laws
- K-map provides a systematic method
  - An array of cells
  - Used for simplifying 2, 3, 4 and 5 variable expressions

3-Variable K-map

AB\C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

A\BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

4-Variable K-map

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Mapping of Standard SOP expression

- Selecting n-variable K-map
- 1 marked in cell for each minterm
- Remaining cells marked with 0

Mapping of Standard SOP expression

- SOP expression

$$A\overline{BC} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$$

AB\C	0	1
00	0	0
01	1	0
11	1	0
10	1	0

A\BC	00	01	11	10
0	0	0	0	1
1	1	0	0	1

Mapping of Standard SOP expression

- SOP expression

$$\overline{A}.\overline{B}.\overline{C}.D + \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D + \overline{A}.B.C.\overline{D} + A.\overline{B}.\overline{C}.\overline{D} + A.B.\overline{C}.D + A.B.C.\overline{D}$$

AB\CD	00	01	11	10
00	0	1	0	0
01	1	1	0	1
11	0	1	0	1
10	1	0	0	0

Mapping of Non-Standard SOP expression

- Selecting n-variable K-map

- 1 marked in all the cells where the non-standard product term is present

- Remaining cells marked with 0

Mapping of Non-Standard SOP expression

- SOP expression $A + B\overline{C}$

AB\C	0	1
00		
01		
11	1	1
10	1	1

A\BC	00	01	11	10
0				
1	1	1	1	1

Mapping of Non-Standard SOP expression

- SOP expression $A + B\overline{C}$

AB\C	0	1
00	0	0
01	1	0
11	1	1
10	1	1

A\BC	00	01	11	10
0	0	0	0	1
1	1	1	1	1

Mapping of Non-Standard SOP expression

- SOP expression $D + A\overline{C} + BC$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

Mapping of Non-Standard SOP expression

- SOP expression $D + A\overline{C} + BC$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	1	1	0
10	1	1	1	0

Mapping of Non-Standard SOP expression

- SOP expression $D + A\overline{C} + BC$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	1	1	1	1
10	1	1	1	0

Simplification of SOP expressions using K-map

- Mapping of expression

- Forming of Groups of 1s

- Each group represents product term

- 3-variable K-map
  - 1 cell group yields a 3 variable product term
  - 2 cell group yields a 2 variable product term
  - 4 cell group yields a 1 variable product term
  - 8 cell group yields a value of 1 for function

Simplification of SOP expressions using K-map

- 4-variable K-map
  - 1 cell group yields a 4 variable product term
  - 2 cell group yields a 3 variable product term
  - 4 cell group yields a 2 variable product term
  - 8 cell group yields a 1 variable product term
  - 16 cell group yields a value of 1 for function

Simplification of SOP expressions using K-map

$$B.\overline{C} + A.C + \overline{B}.C$$

AB\C	0	1
00	0	1
01		0
11		
10	0	

A\BC	00	01	11	10
0	0			
1		0	0	0

$$A.\overline{B}.\overline{C} + \overline{A}.\overline{C} + \overline{A}.B$$

Simplification of SOP expressions using K-map

$B + A.C$

AB\C	0	1
00	0	0
01		
11		
10	0	

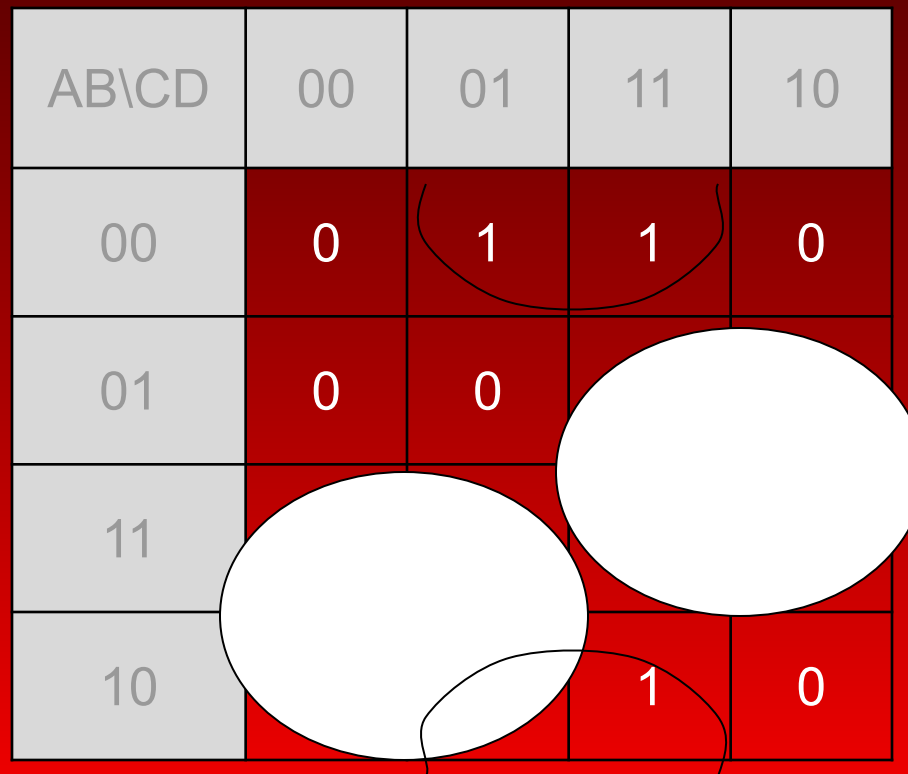| A\BC	00	01	11	10
0	0	0		
1				0

$A.\overline{B} + B.C + \overline{A}.B$

Simplification of SOP expressions using K-map

$$A.\overline{C} + \overline{B}.D + B.C$$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	0		
11				
10			1	0

Simplification of SOP expressions using K-map

$$A.\overline{C}.\overline{D} + C.D + B.C$$

AB\CD	00	01	11	10
00	0	0		0
01	0	0		
11		0		
10		0		0

Simplification of SOP expressions using K-map

$$\overline{B}.\overline{D} + \overline{B}.\overline{C} + A.B.D + \overline{A}.C.\overline{D}$$

AB\CD	00	01	11	10
00	1	0	1	1
01	0	0	0	1
11	0			0
10	1	0	1	1

Mapping Directly from Function Table

- Function of a logic circuit defined by function table

- Function can be directly mapped to K-map

Mapping Directly from Function Table

Inputs				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1

Inputs				Output
A	B	C	D	F
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Mapping Directly from Function Table

$$\overline{A}.D + B.\overline{C}.D + \overline{B}.C.D$$

AB\CD	00	01	11	10
00	0			0
01	0			0
11	0		0	0
10	0	0	1	0

Don't care Conditions

- Some input combinations never occur

- Outputs are assumed to be don't care

- Don't care outputs used as 0 or 1 during simplification.

- Results in simpler and shorter expressions

Don't Care Conditions

Inputs				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1

Inputs				Output
A	B	C	D	F
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don't Care Conditions

$\overline{A}.D$

AB\CD	00	01	11	10
00	0			0
01	0			0
11	x	x	x	x
10	0	0	x	x

Don't Care Conditions

D

AB\CD	00	01	11	10
00	0			0
01	0			0
11	x			x
10	0			x