PIXEL RELATION, CONNECTIVITY & DISTANCE MEASURES

DIGITAL IMAGE PROCESSING

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Pixel Relationship (or Relation)

Pixel relationship refers to how **pixels in an image are related to each other** based on their **position (neighborhood)** and **intensity values (gray level or color).**

It describes whether pixels are **adjacent, similar, or connected.**

Main relationships: Neighborhood, Connectivity/Adjacency**.**

- Images are stored in a **2D matrix** form (rows × columns).
- Pixel relationship & connectivity are essential for:
- Image segmentation
- Boundary detection
- Region labeling

Pixel Neighborhoods

4-Neighborhood (N4):

Pixels directly left, right, above, and below.

Diagonal (ND):

Pixels located at the four diagonal directions.

8-Neighborhood (N8):

Combines N4 and ND → total 8 surrounding pixels.

Pixel Relationships

Adjacency: Two pixels are adjacent if they share a boundary (depending on 4, 8, or m-connectivity).

Path: A sequence of adjacent pixels.

Connected Component: A group of pixels that are all connected.

Region: A set of connected pixels with similar intensity values.

Types of Image

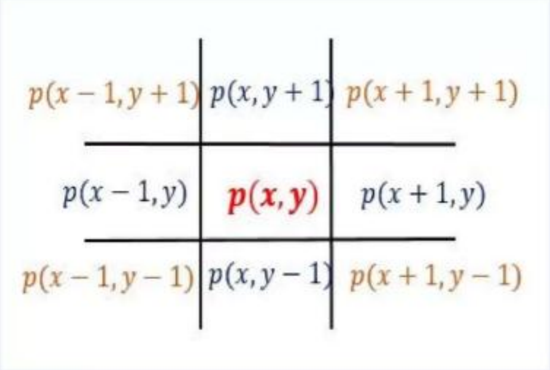
- Binary image (1 bit quantized image) $\{0,1\}$
- Gray scale image (8 bit quantized image) $\{0,255\}$
- Color image : Made with the help of primary color
  1. Red (8 bit)
  2. Blue (8 bit)
  3. Green (8 bit)

  Total 24 bit scale is used for representing the color image.

Note:

- Color image consumes more bandwidth.
- By 24 bit quantizer level, $2^{24} \rightarrow 16$ M colors can be formed.
- Most of the camera claim 16 M color.

Pixels Neighborhood

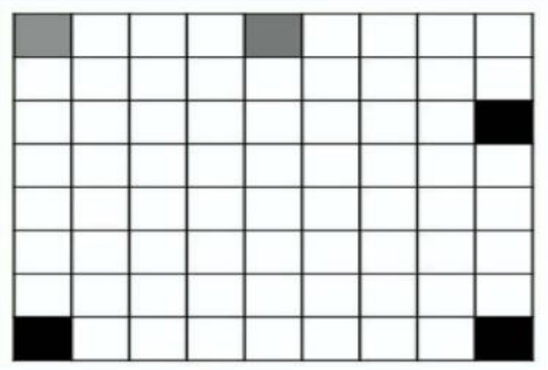


- A pixels $p(x, y)$ has two horizontal and two vertical neighbor.
- $p(x+1, y)$ and $p(x-1, y)$ $\rightarrow$ Horizontal neighbors. $p(x, y+1)$ and $p(x, y-1)$ $\rightarrow$ Vertical neighbors. Denoted as $N_4(p)$
- $p(x-1, y+1)$, $p(x-1, y-1)$ $\rightarrow$, $p(x+1, y-1)$ and $p(x+1, y+1)$ $\rightarrow$ Diagonal neighbors. Denoted as $N_D(p)$

- The points of $N_4(p)$ and $N_D(p)$ is combined, then this pixel has 8 neighbors.

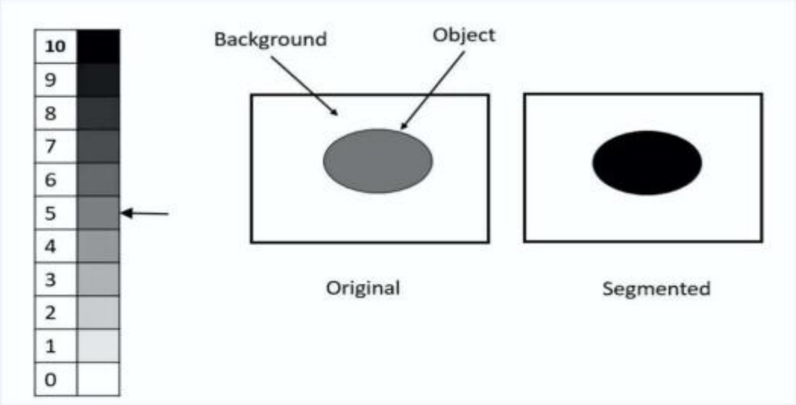
$$N_8(p) = N_4(p) \cup N_D(p)$$

- If pixel $p(x, y)$ is a boundary pixel then number of neighbors is less than 8.



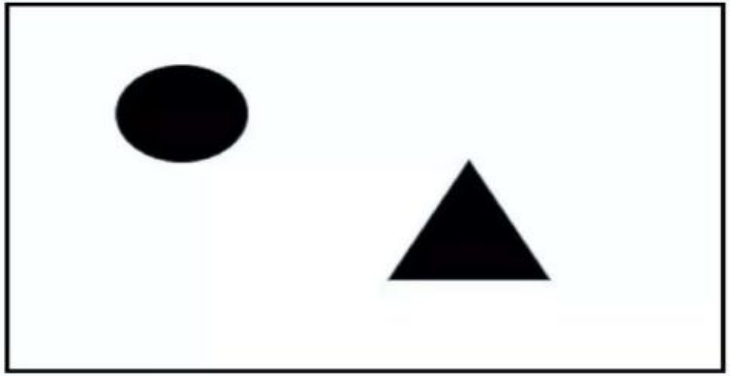
Key Features

- Finding object boundary
- Address the image component/regions etc
- Address the shape, size and other important information to the object.



- If $g(x, y) > Th \Rightarrow (x, y) \in$ **Object** and $g(x, y) < Th \Rightarrow (x, y) \in$ **Background**

Object but not connected

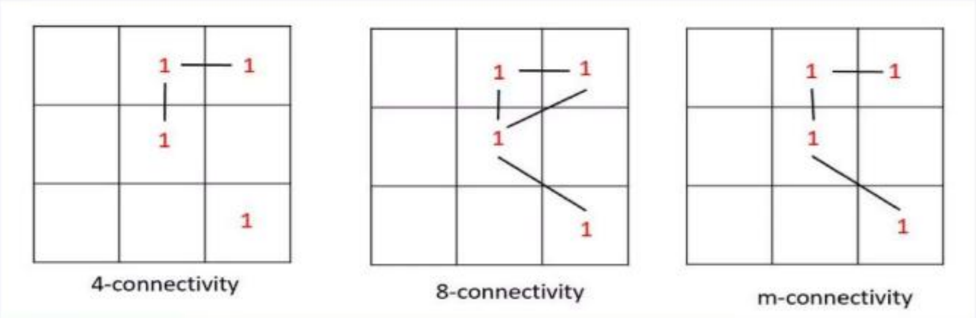


Connectivity:

- Two pixels are said to be connected, if they are adjacent in some sense
  - (a) If they are neighbors ($N_8(p)$, $N_4(p)$ or $N_D(p)$).
  - (b) In gray scale, the intensity level is similar.

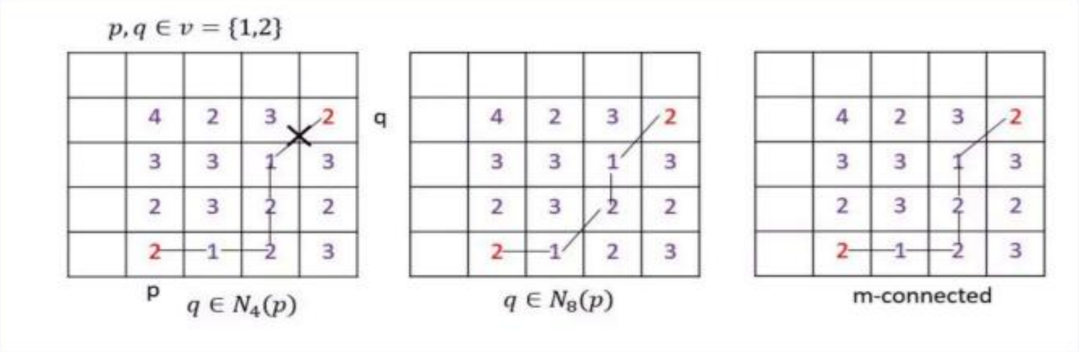
Let $\mathcal{V}$ is the set of gray level to define connectivity between two pixel **p** and **q** then three types of connectivity is used

1. 4-connectivity $\Rightarrow p, q \in \ v \ \& \ p \in N_4(q)$
2. 8-connectivity $\Rightarrow p, q \in \ v \ \& \ p \in N_8(q)$
3. m-connectivity
   i. $q \in N_4(p)$ or
   ii. $q \in N_4(p)$ and $N_4(p) \cap N_4(q) = \phi$
4. Mixed connectivity is a special case of 8-connectivity that eliminates the multiple path.



Two pixels are p and q are adjacent if they are connected

- 4-adjacency
- 8-adjacency
- m-adjacency



Region and Boundary

- Let R represent a subset of pixels in an image
- We call R a region of the image if R is a connected set
- We can specify the region by using 4-adjacency and 8-adjacency
- Region = { Set of all pixels which fulfill adjacency criteria }
- **Boundary (or border)**
  - ❑ The *boundary* of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
  - ❑ If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.
- **Foreground and background**
  - ❑ An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement
  - ❑ All the points in R_u is called foreground;
  - ❑ All the points in $(R_u)^c$ is called background.

Distance measures

•Distance measures quantify how far apart two pixels (or points) are in an image.

•**Importance in DIP:**

•Used in segmentation, clustering, object recognition, and pattern analysis.

Common distance measures:

•Euclidean Distance

•City-block (Manhattan) Distance

•Chessboard Distance

Euclidean Distance

- Formula:

$$D_e(p, q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Measures the straight-line (shortest path) distance.
- Example: distance between points (2, 3) and (5, 7) = $\sqrt{[(2–5)^2 + (3–7)^2]}$ =
- 5.Application: Edge detection, geometric analysis

City Block (Manhattan) Distance

•Formula:

$$D_4(p, q) = |x_1 - x_2| + |y_1 - y_2|$$

•Measures distance along grid lines (like moving in a city grid).

Example:

•Distance between (2, 3) and (5, 7) = |2–5| + |3–7| = 7.

•Application: 4-neighbor connectivity, path planning.

Chessboard Distance

- Formula:

$$D_8(p, q) = \max(|x_1 - x_2|, |y_1 - y_2|)$$

- Measures the distance like a king's move in chess.
- Example:
- Distance between (2, 3) and (5, 7) = max(|2–5|, |3–7|) = 4.
- Application: 8-neighbor connectivity, morphological operations.