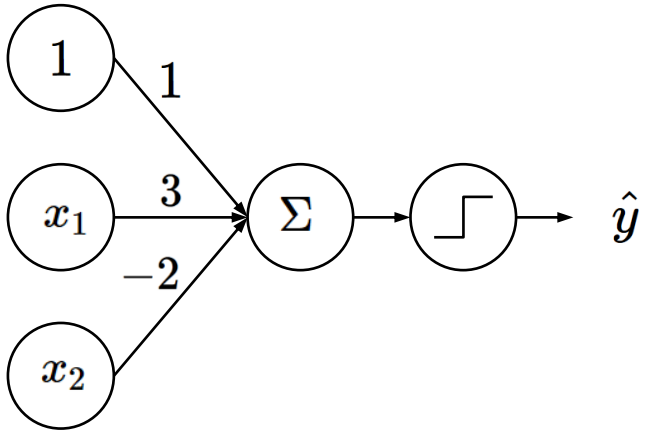
# (Artificial) Neural Networks: From Perceptron to MLP

**Prof. Seungchul Lee**

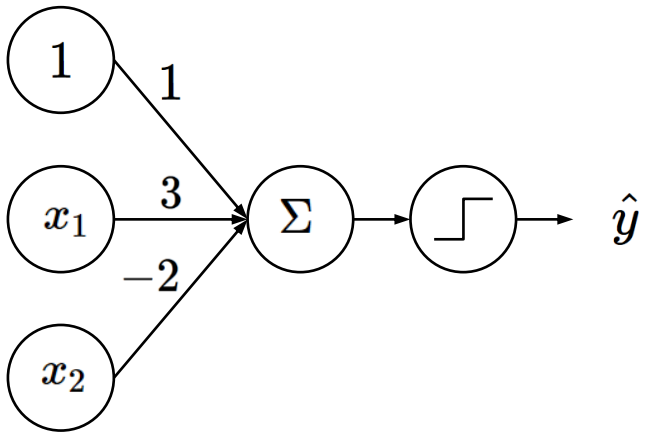
**Industrial AI Lab.**

# Perceptron: Example

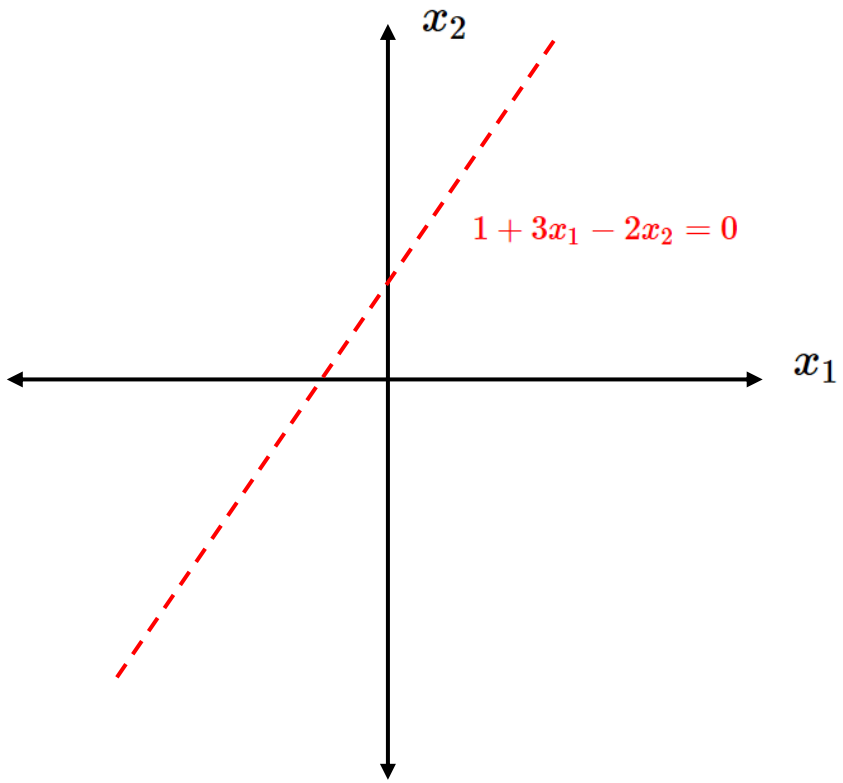



$$
\begin{aligned}
\hat{y} &= g\left(\omega_0 + X^T \omega\right) \\
&= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right) \\
&= g\left(1 + 3x_1 - 2x_2\right)
\end{aligned}
$$

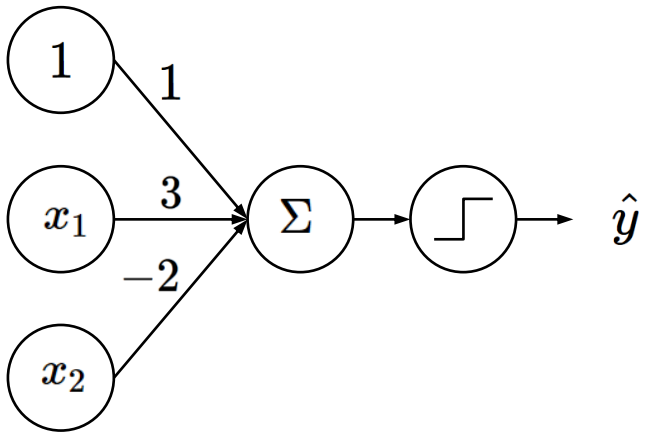
# Perceptron: Example




$$\hat{y} = g(1 + 3x_1 - 2x_2)$$

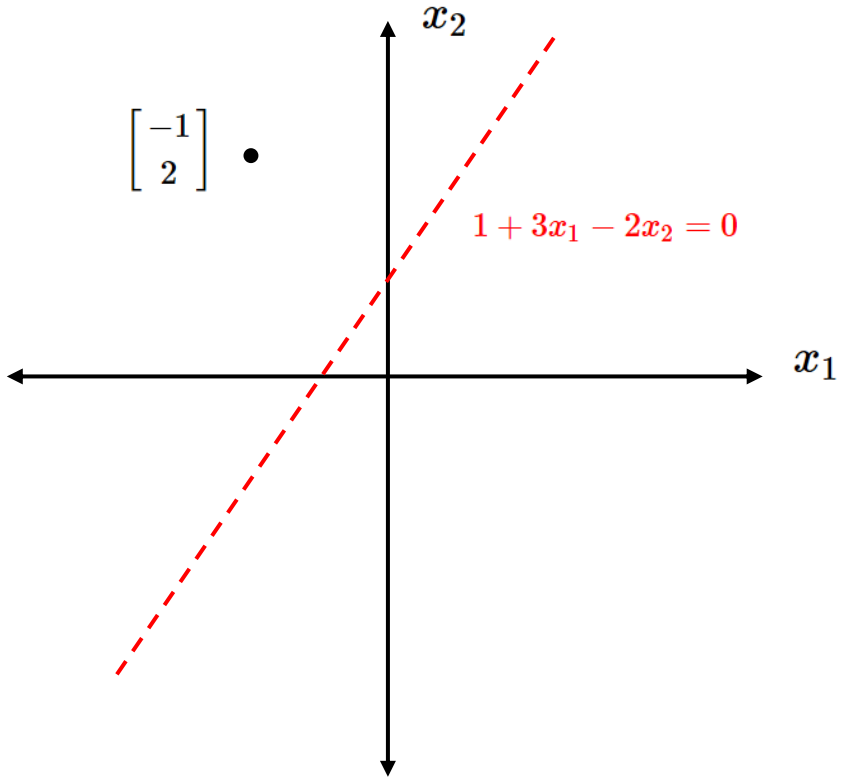


# Perceptron: Example

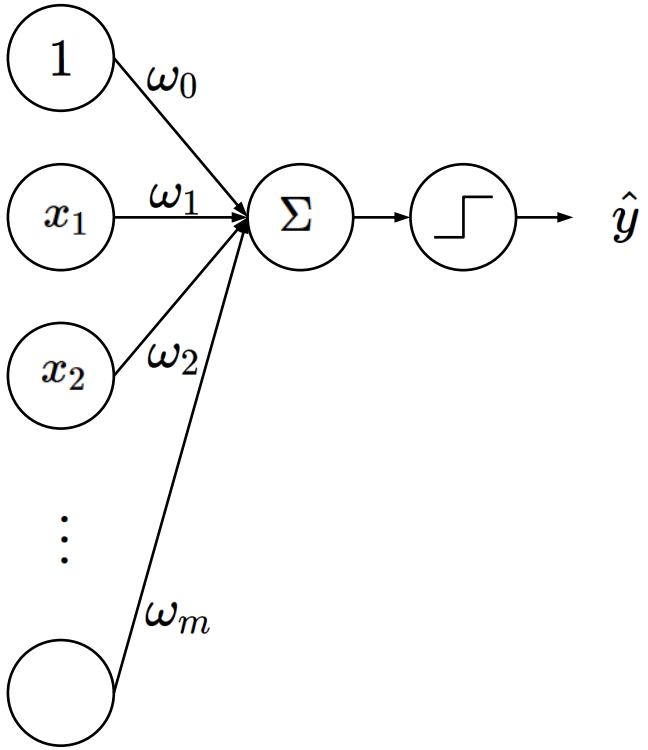



$$\hat{y} = g(1 + 3 \times (-1) - 2 \times 2) = g(-6) = -1$$

$$\hat{y} = g(1 + 3x_1 - 2x_2)$$



# Perceptron: Forward Propagation




$$\hat{y} = g\left(\omega_0 + X^T\omega\right)$$

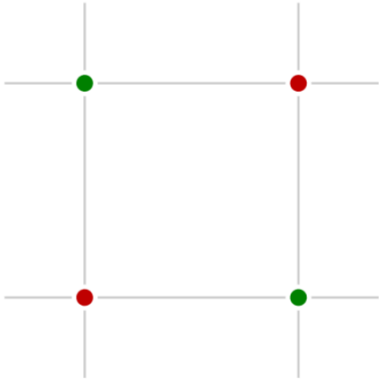
$$= g\left(\omega_0 + \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}^T \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_m \end{bmatrix}\right)$$

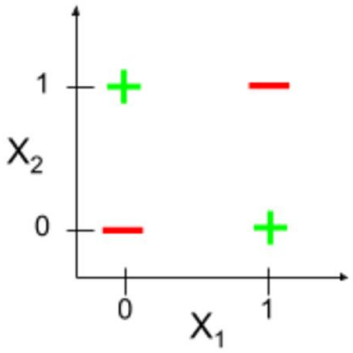
# From Perceptron to MLP

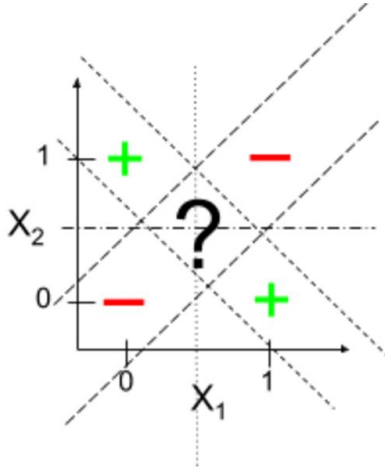
# XOR Problem

- Minsky-Papert Controversy on XOR
  - Not linearly separable
  - Limitation of perceptron

| $x_1$ | $x_2$ | $x_1$ **XOR** $x_2$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



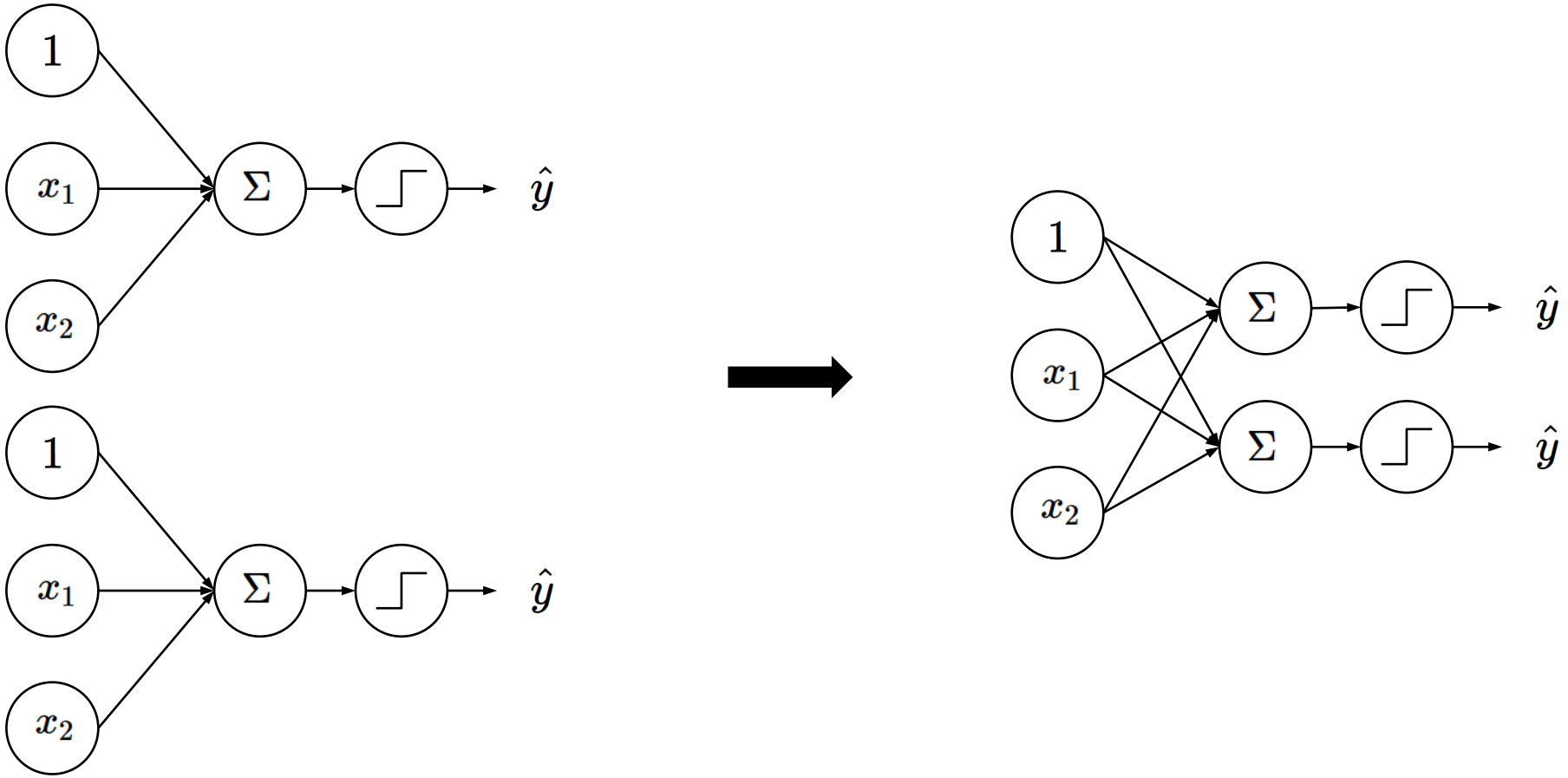



- Single neuron = one linear classification boundary

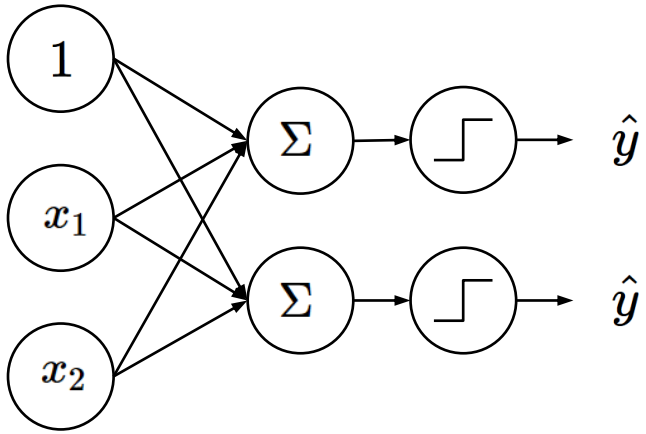
# Artificial Neural Networks: MLP

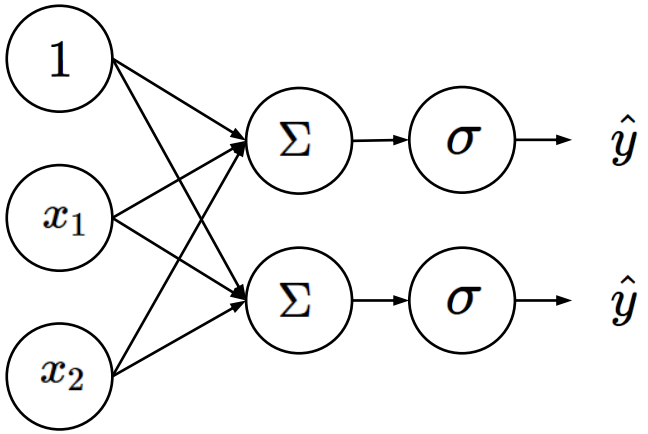
- Multi-layer Perceptron (MLP) = Artificial Neural Networks (ANN)
  - Multi neurons = multiple linear classification boundaries

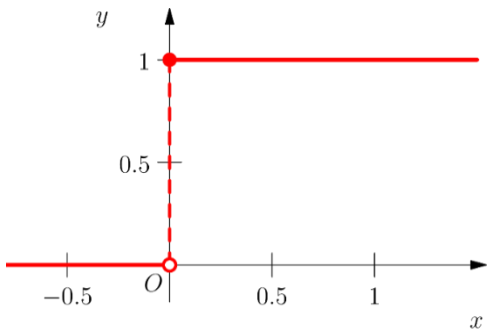


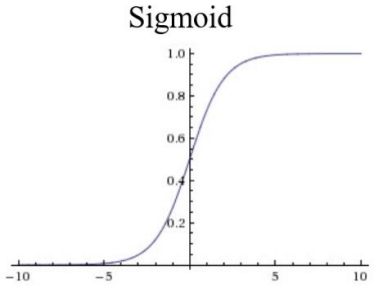
# Artificial Neural Networks: Activation Function

- Differentiable nonlinear activation function



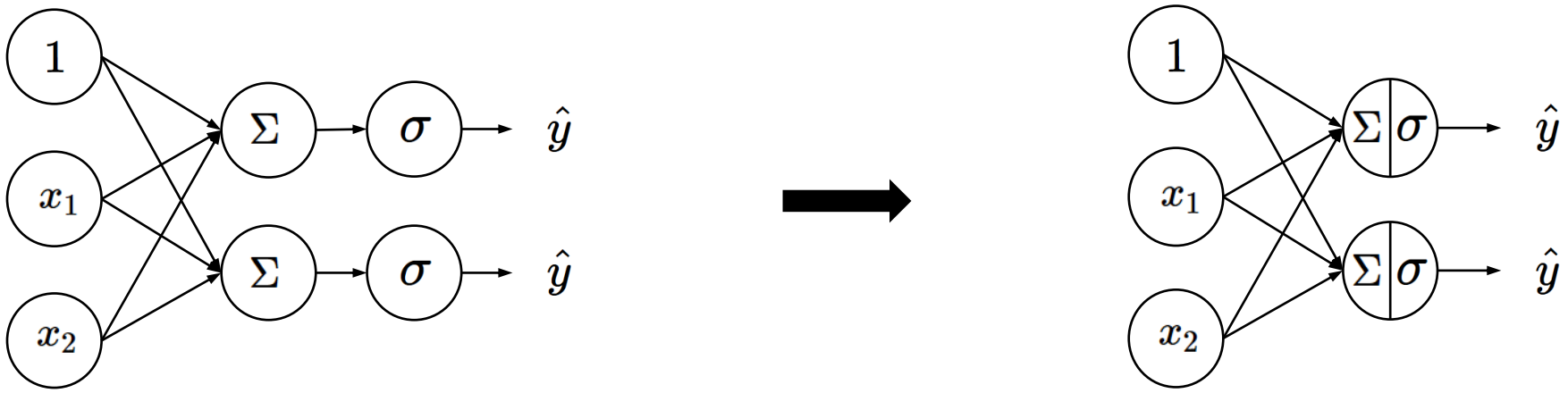




$$g(x) = \frac{1}{1 + e^{-x}}$$

# Artificial Neural Networks

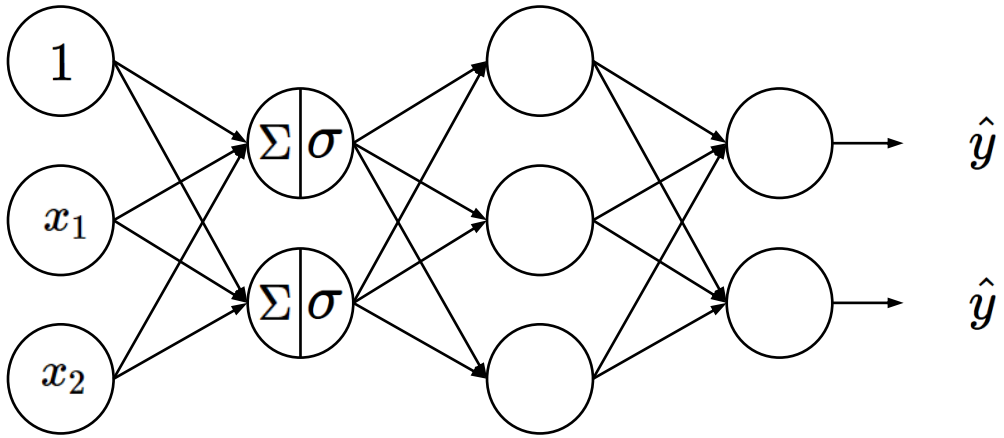
- In a compact representation



# Artificial Neural Networks

- A single layer is not enough to be able to represent complex relationship between input and output

  $\Longrightarrow$ perceptron with many layers and units



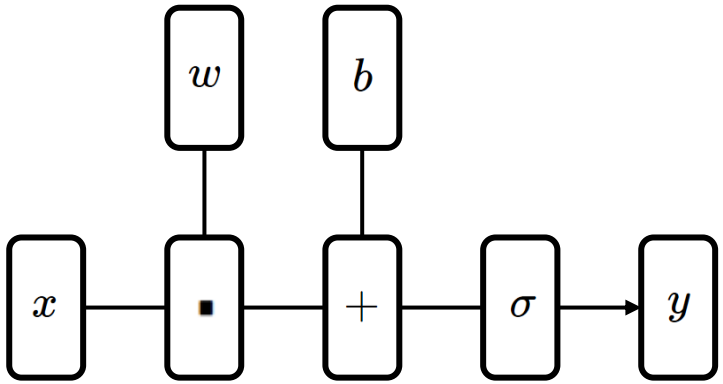

- Multi-layer perceptron
  - Features of features
  - Mapping of mappings

# Another Perspective:
# ANN as Kernel Learning

# Neuron

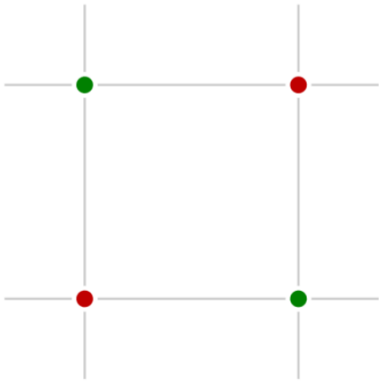
- We can represent this “neuron” as follows:

$$f(x) = \sigma(w \cdot x + b)$$



# XOR Problem

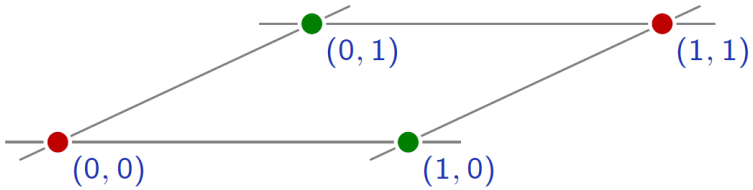
- The main weakness of linear predictors is their lack of capacity.
- For classification, the populations have to be linearly separable.



“xor”

# Nonlinear Mapping

- The XOR example can be solved by pre-processing the data to make the two populations linearly separable.

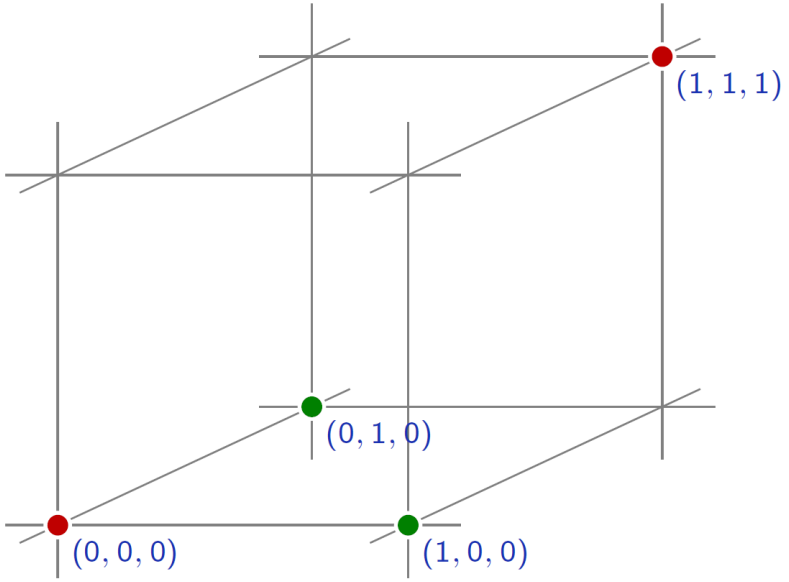



Source: Dr. Francois Fleuret at EPFL

# Nonlinear Mapping

- The XOR example can be solved by pre-processing the data to make the two populations linearly separable.

$$\phi : (x_u, x_v) \rightarrow (x_u, x_v, x_u x_v)$$



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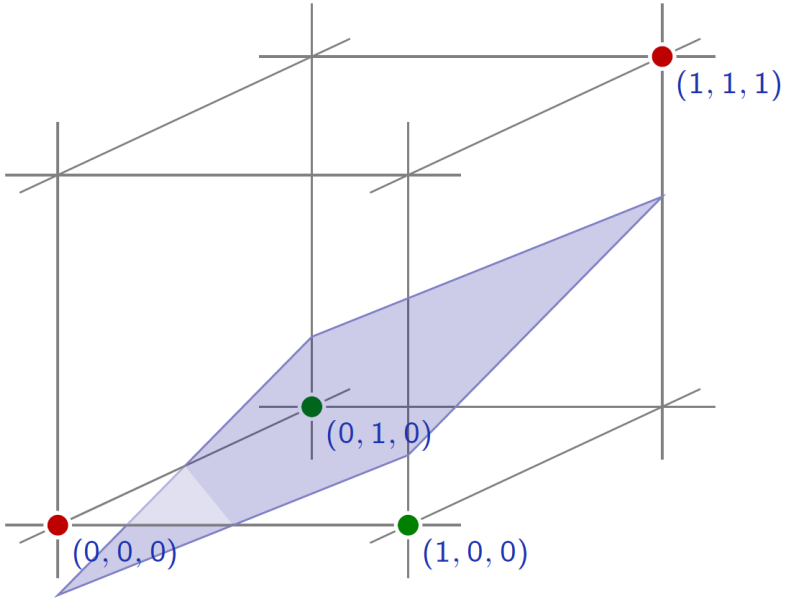





Source: Dr. Francois Fleuret at EPFL

# Kernel

- Often we want to capture nonlinear patterns in the data
  - nonlinear regression: input and output relationship may not be linear
  - nonlinear classification: classes may note be separable by a linear boundary

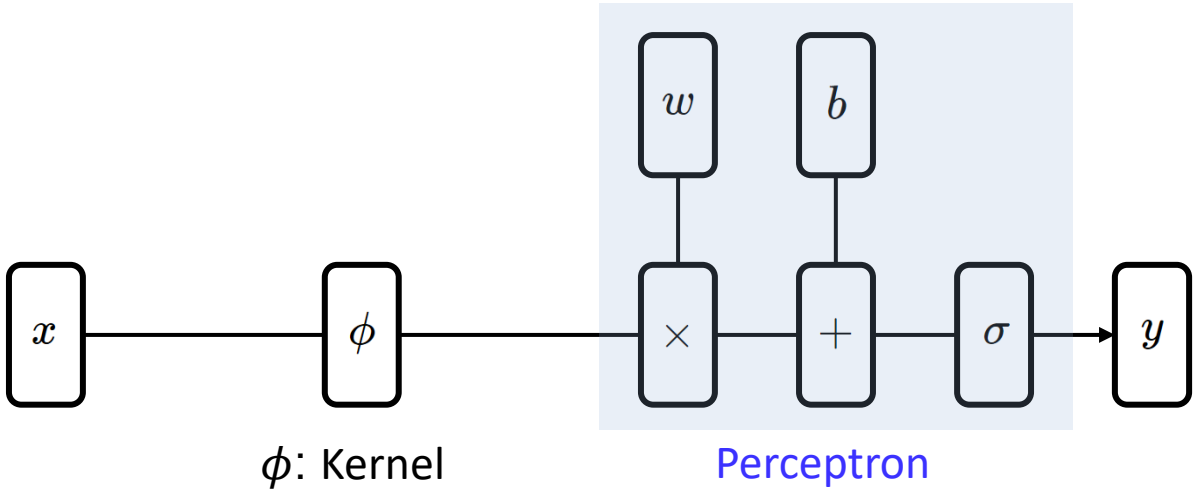
- Linear models (e.g. linear regression, linear SVM) are not just rich enough
  - by mapping data to higher dimensions where it exhibits linear patterns
  - apply the linear model in the new input feature space
  - mapping = changing the feature representation

- Kernels: make linear model work in nonlinear settings

# Kernel + Neuron

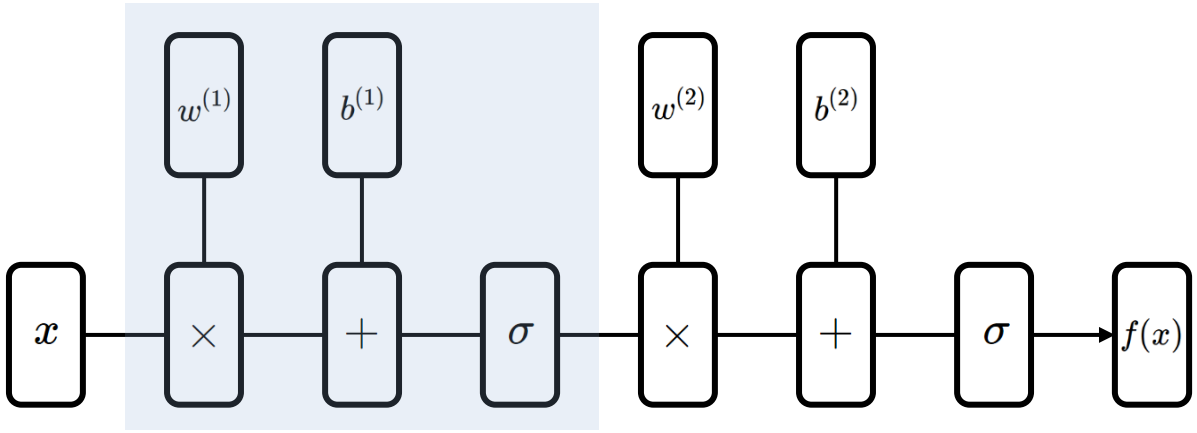
- Nonlinear mapping + neuron

$$\phi : (x_u, x_v) \rightarrow (x_u, x_v, x_u x_v)$$



# Neuron + Neuron

- Nonlinear mapping can be represented by another neurons

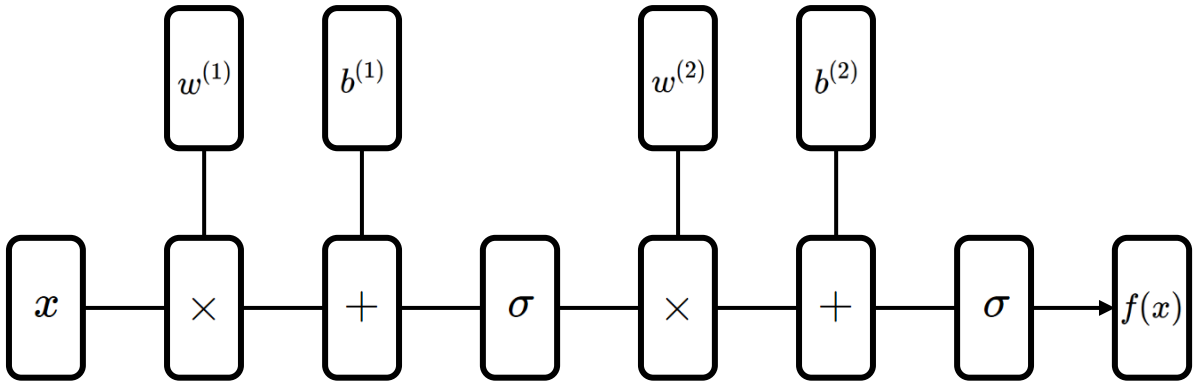



$\phi$: Kernel

- Nonlinear Kernel
  - Nonlinear activation functions

# Multi Layer Perceptron

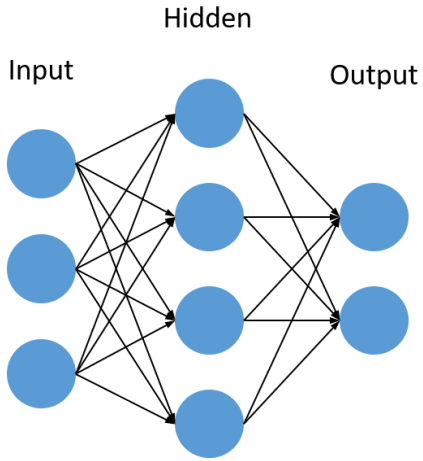
- Nonlinear mapping can be represented by another neurons
- We can generalize an MLP




$\phi$: Kernel

# Summary

- Universal function approximator
- Universal function classifier
- Parameterized

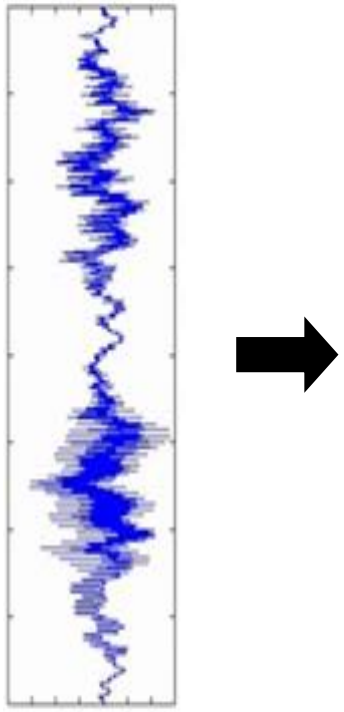



$$x \rightarrow \boxed{\hat{y} = f_{\omega_1,\cdots,\omega_k}(x)} \rightarrow y$$

# Artificial Neural Networks

- Complex/Nonlinear universal function approximator
  - Linearly connected networks
  - Simple nonlinear neurons

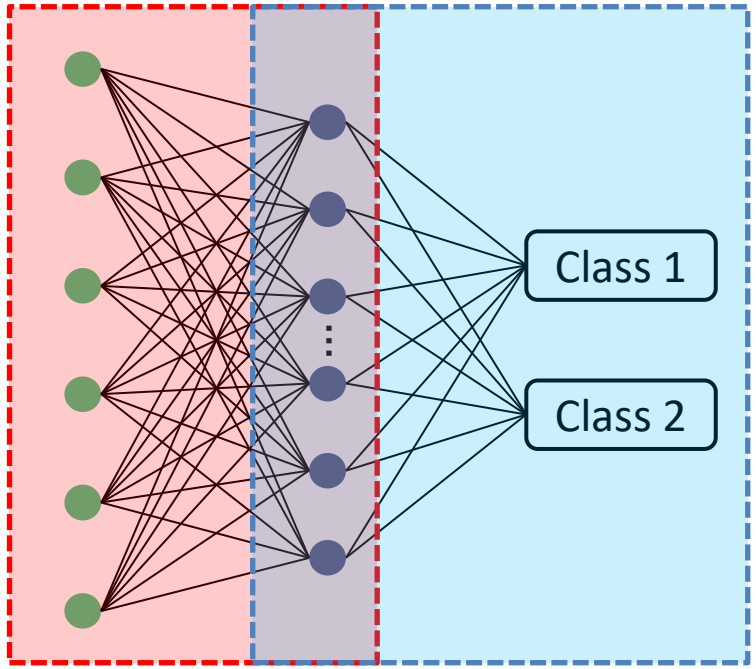
**Input**





**Output**

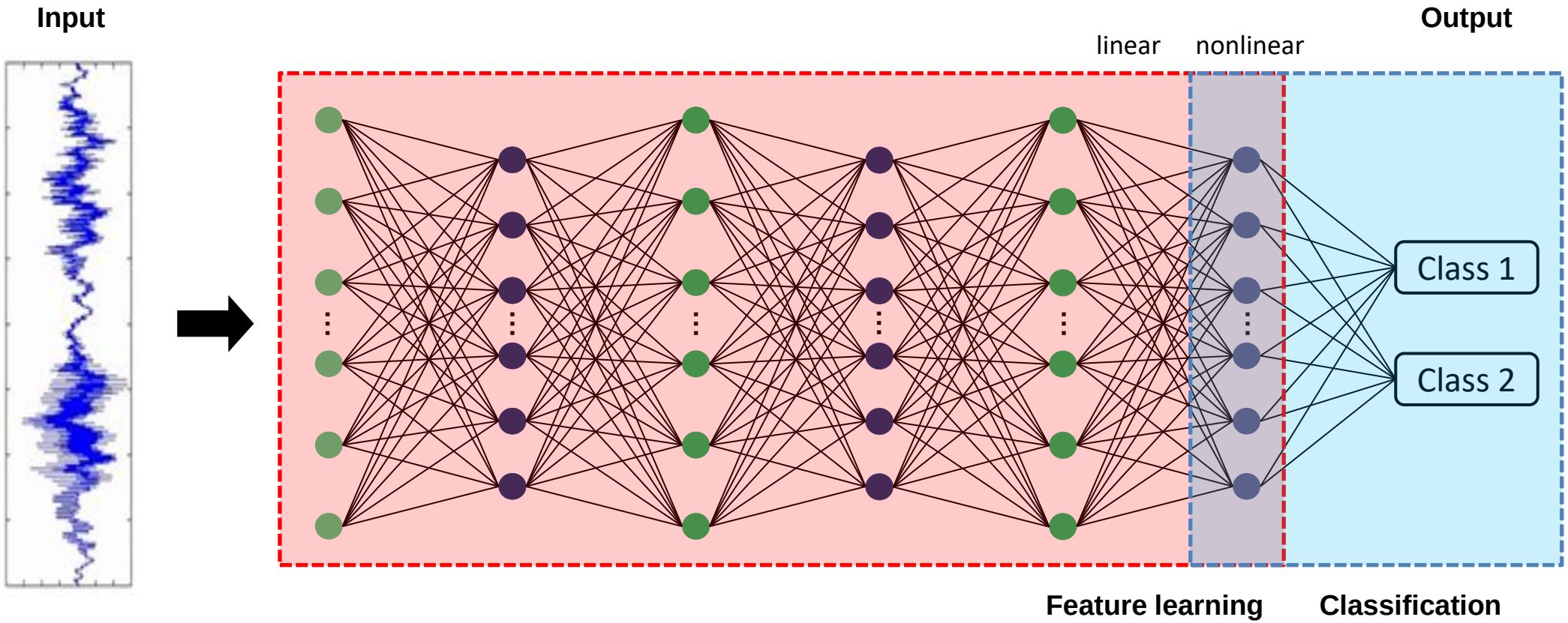
linear nonlinear




**Feature learning** **Classification**

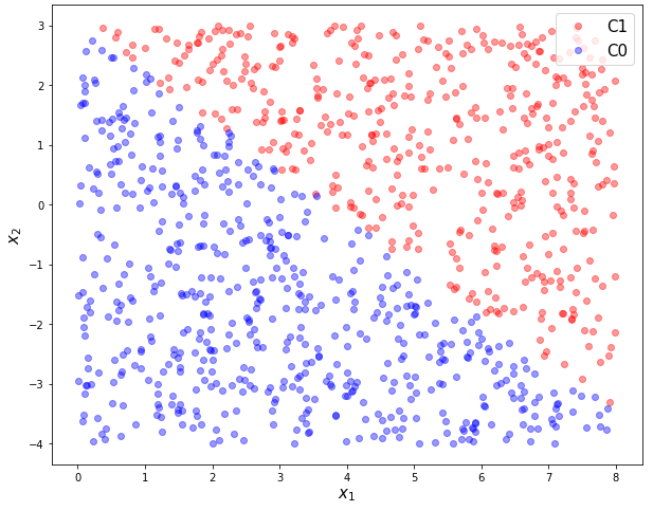
# Deep Artificial Neural Networks

- Complex/Nonlinear universal function approximator
  - Linearly connected networks
  - Simple nonlinear neurons

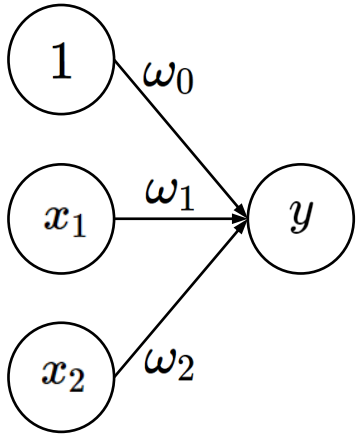


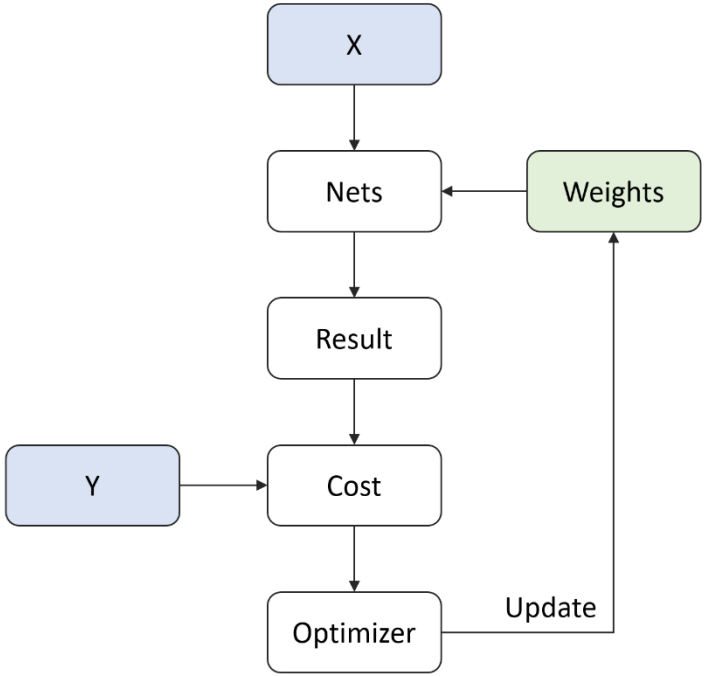
# Looking at Parameters

# Logistic Regression in a Form of Neural Network



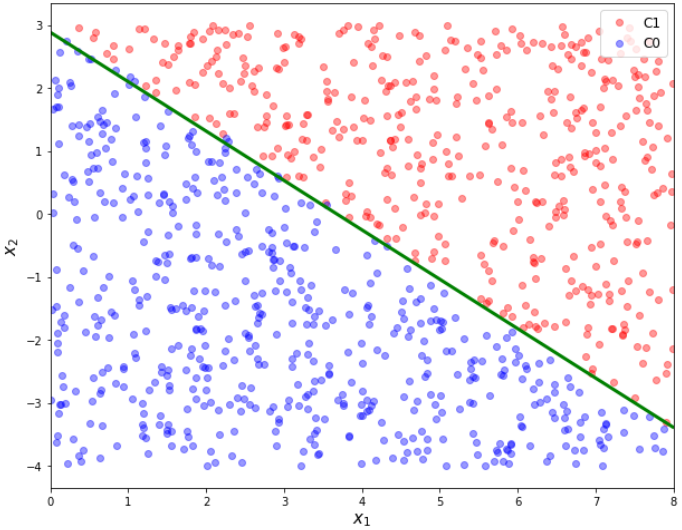

$$y = \sigma\left(\omega_0 + \omega_1 x_1 + \omega_2 x_2\right)$$



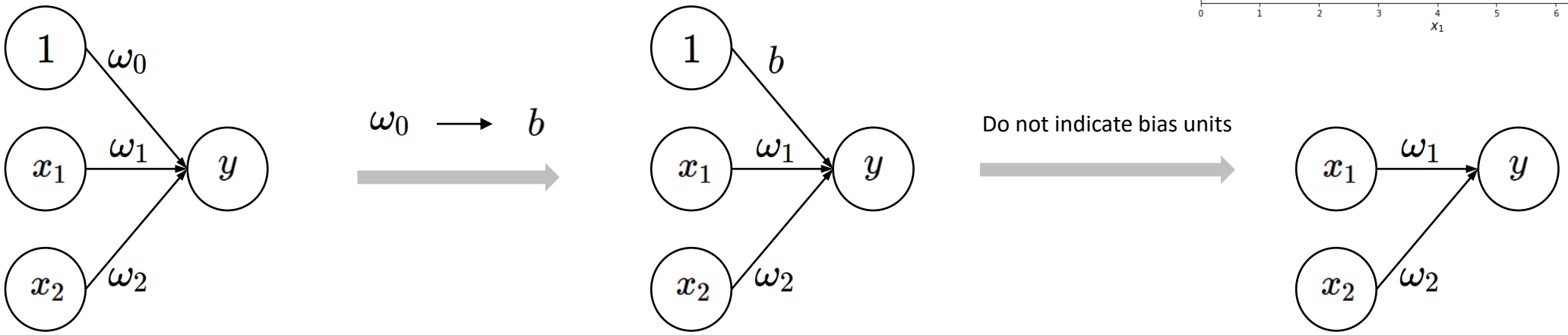
# Logistic Regression in a Form of Neural Network

- Neural network convention




$$y = \sigma\left(\omega_0 + \omega_1 x_1 + \omega_2 x_2\right)$$

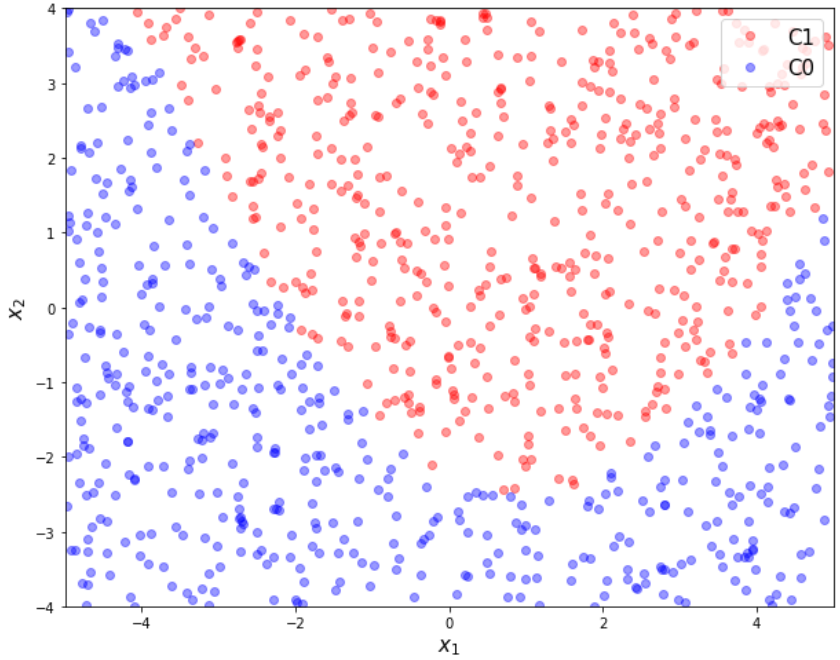
$$y = \sigma\left(b + \omega_1 x_1 + \omega_2 x_2\right)$$

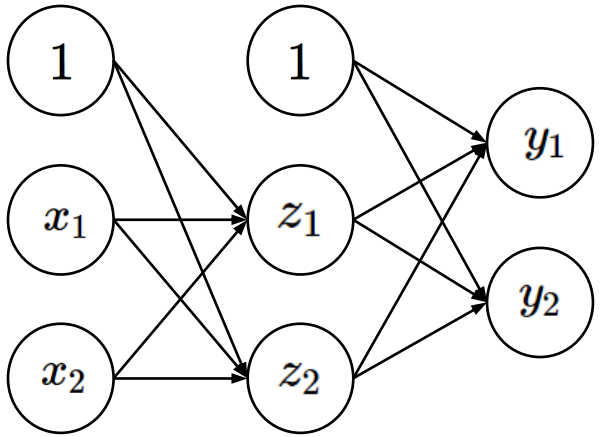



```
n_input = 2
n_output = 1
```

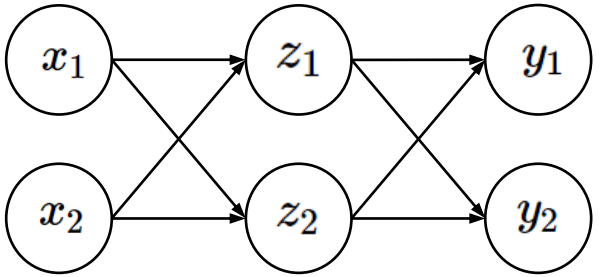
# Nonlinearly Distributed Data

- Example to understand network's behavior
  - Include a hidden layer



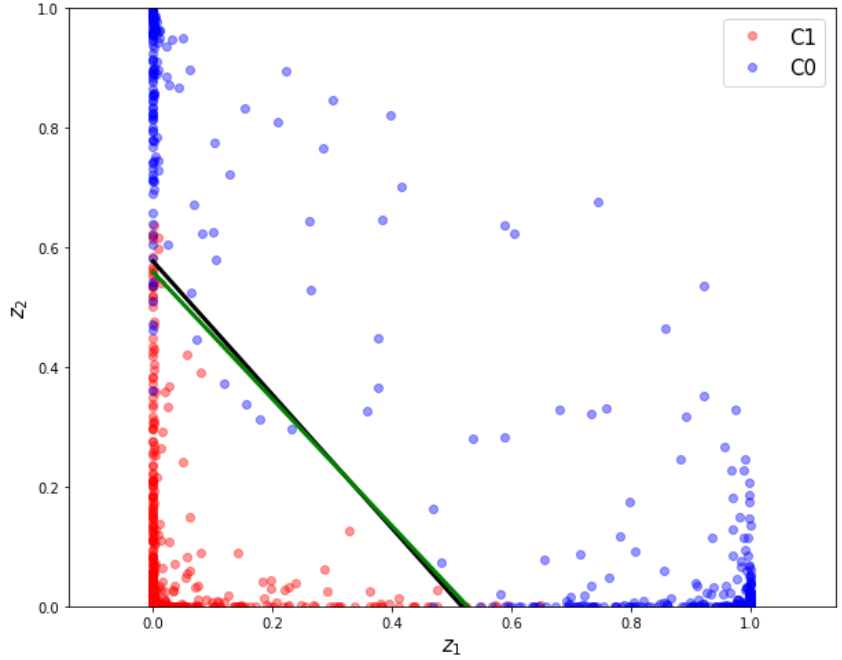

Do not include bias units

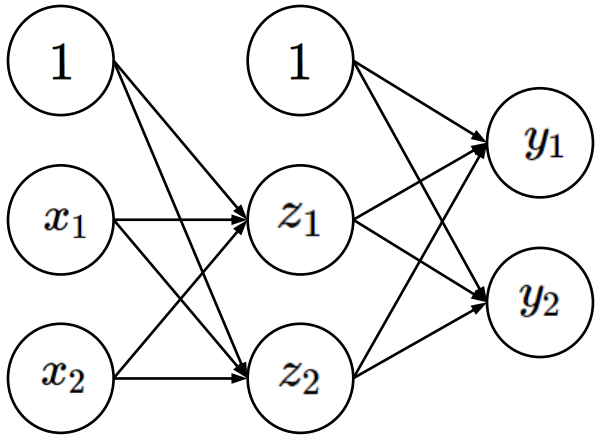



```
n_input = 2
n_hidden = 2
n_output = 2
```

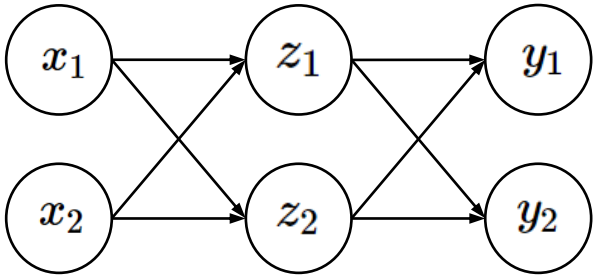
# Multi Layers

- $z$ space



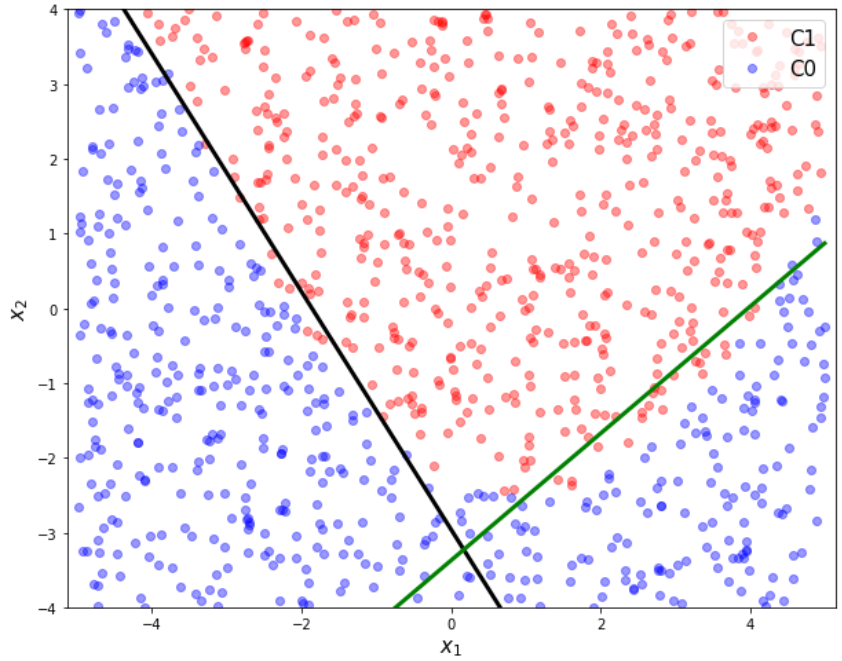

Do not include bias units

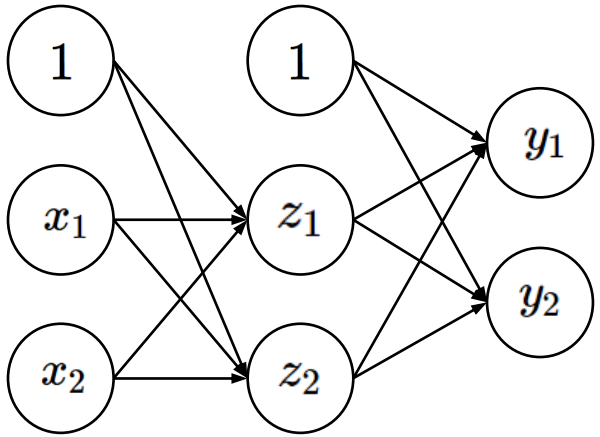



```
n_input = 2
n_hidden = 2
n_output = 2
```

# Multi Layers

- $x$ space

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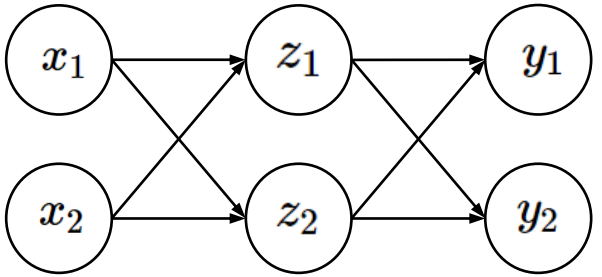



```
n_input = 2
n_hidden = 2
n_output = 2
```

# (Artificial) Neural Networks: Training

**Industrial AI**

**Prof. Seungchul Lee**

# Training Neural Networks: Loss Function

- Measures error between target values and predictions

$$\min_{\omega} \sum_{i=1}^{m} \ell \left( h_{\omega} \left( x^{(i)} \right), y^{(i)} \right)$$

- Example
  - Squared loss (for regression):

$$\frac{1}{m} \sum_{i=1}^{m} \left( h_{\omega} \left( x^{(i)} \right) - y^{(i)} \right)^2$$

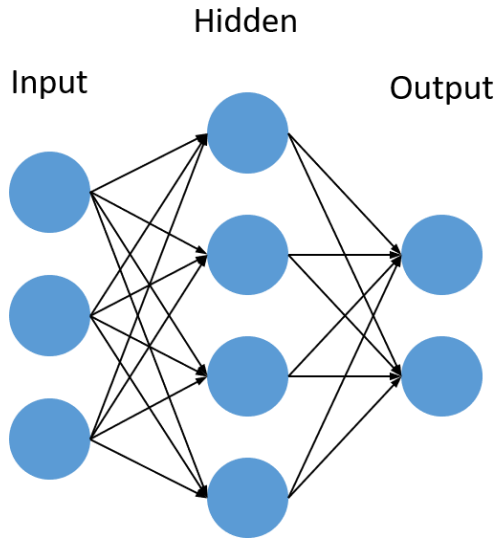
  - Cross entropy (for classification):

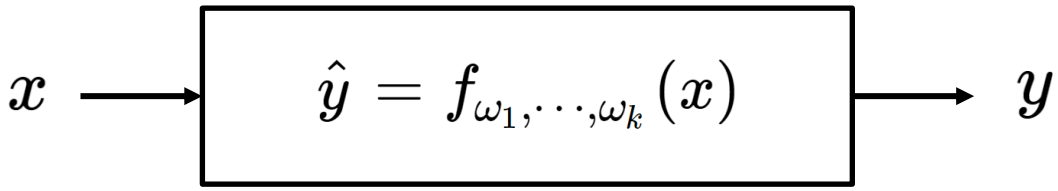
$$-\frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log \left( h_{\omega} \left( x^{(i)} \right) \right) + \left( 1 - y^{(i)} \right) \log \left( 1 - h_{\omega} \left( x^{(i)} \right) \right)$$

# Gradients in ANN

- Learning weights and biases from data using gradient descent
- $\frac{\partial \ell}{\partial \omega}$: too many computations are required for all $\omega$

- Structural constraint of NN:
  - Composition of functions
  - Chain rule
  - Dynamic programming

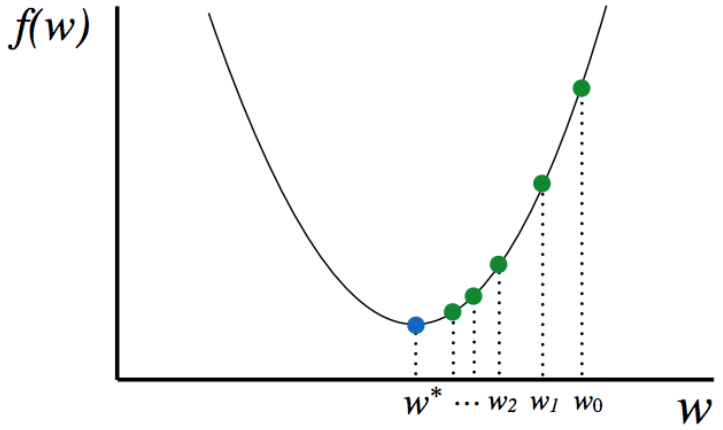


# Training Neural Networks with TensorFlow

- Optimization procedure

Start at a random point
**Repeat**
    Determine a descent direction
    Choose a step size
    Update
**Until** stopping criterion is satisfied



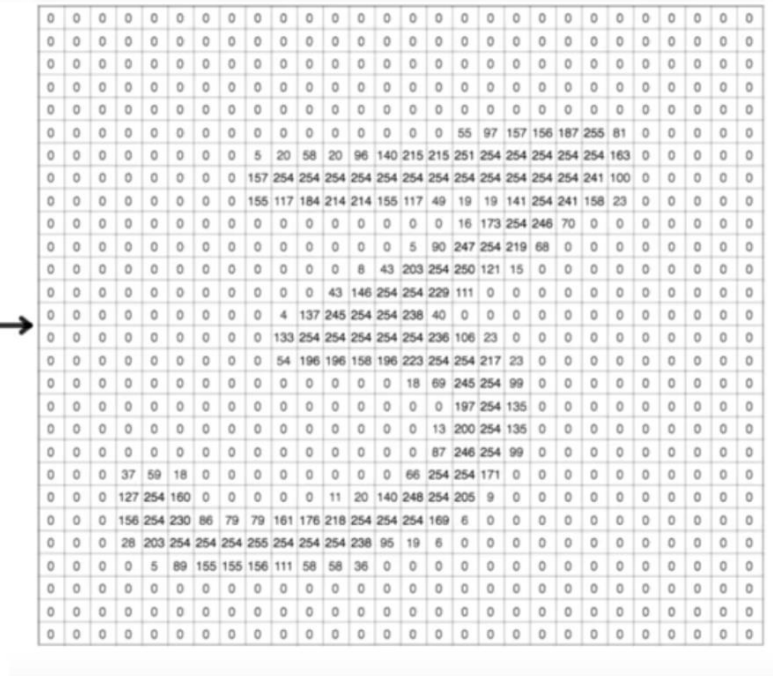

- It is not easy to numerically compute gradients in network in general.
  - The good news: people have already done all the "hard work" of developing numerical solvers (or libraries)
  - There are a wide range of tools → We will use the TensorFlow

# ANN in TensorFlow: MNIST

# MNIST database

- Mixed National Institute of Standards and Technology database
- Handwritten digit database
- $28 \times 28$ gray scaled image
- Flattened matrix into a vector of $28 \times 28 = 784$

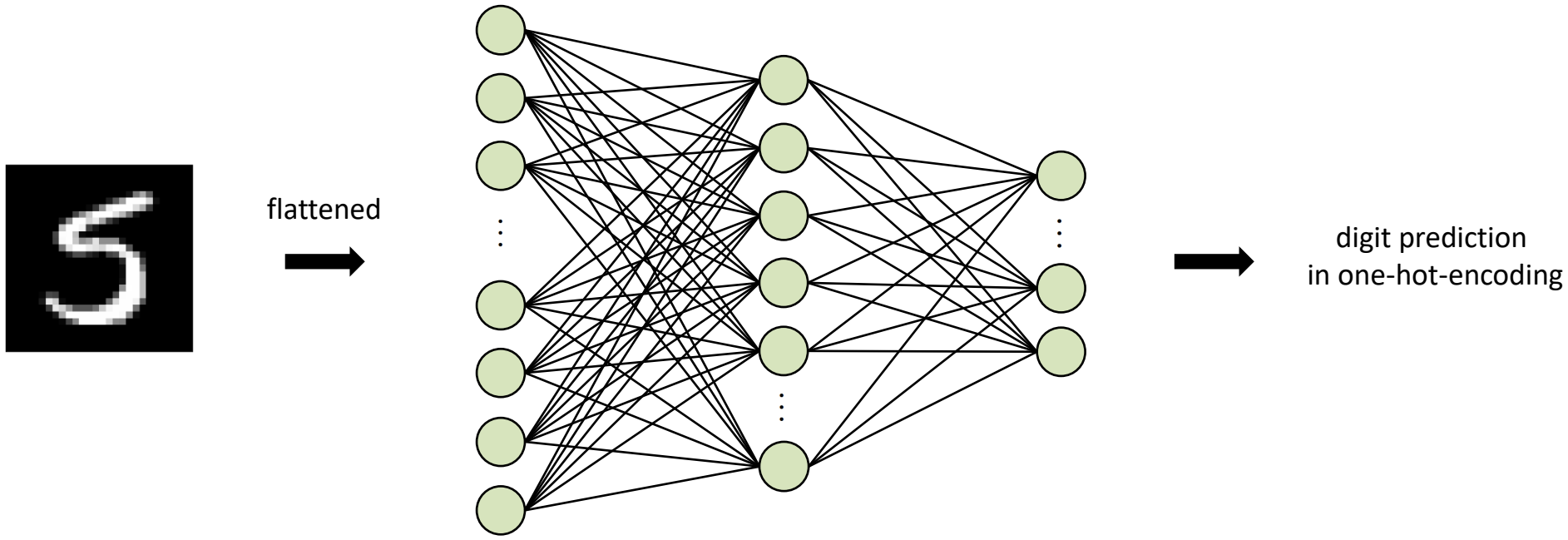
28 x 28
= 784 pixels



pixel 1
pixel 2
pixel 3
pixel 4
pixel 5
pixel 6
pixel 7
pixel 8
pixel 9
pixel 10
pixel 11
pixel 12
pixel 13
pixel 14
pixel 15
pixel 16
pixel 17
pixel 18
pixel 19

pixel 784

# Our Network Model



# Machine Learning and Deep Learning

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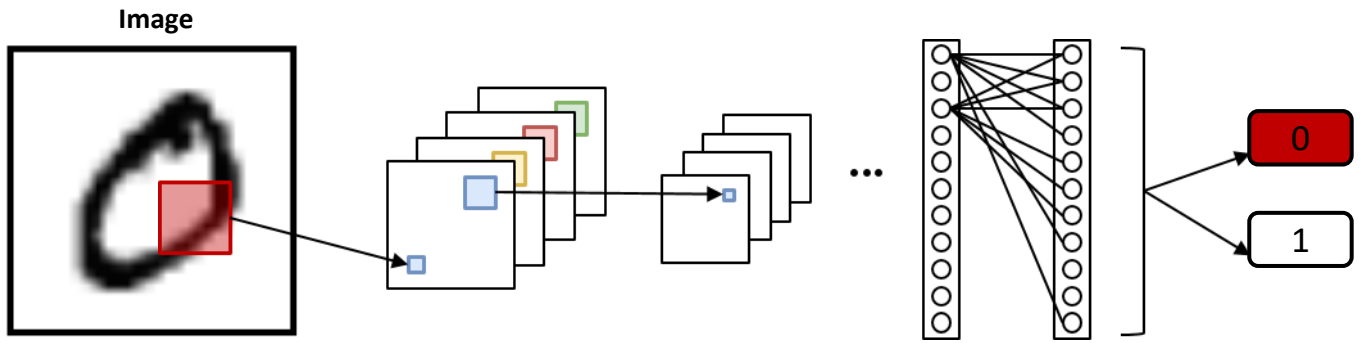
**Industrial AI Lab.**

# Machine Learning

- Image of digit 0
- Image of digit 1

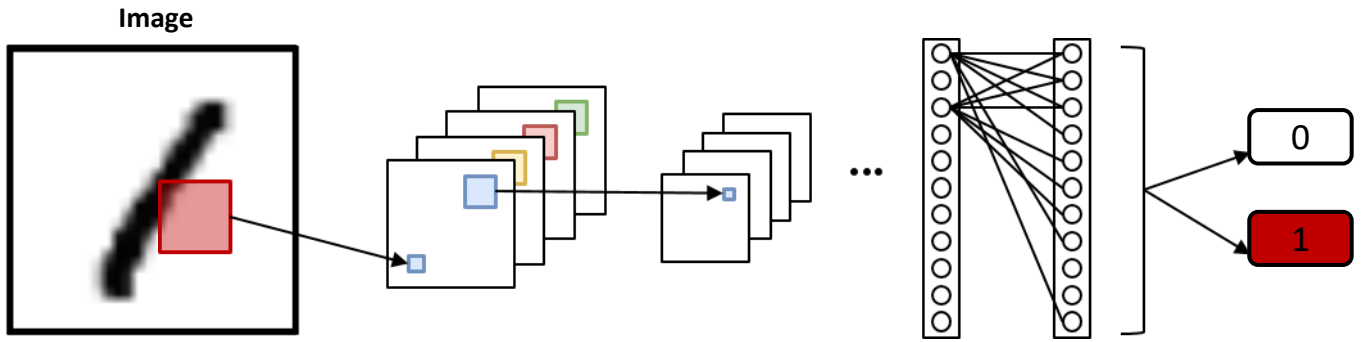
# Deep Learning

- Convolutional Neural Networks (CNN)
- Image pattern recognition problems



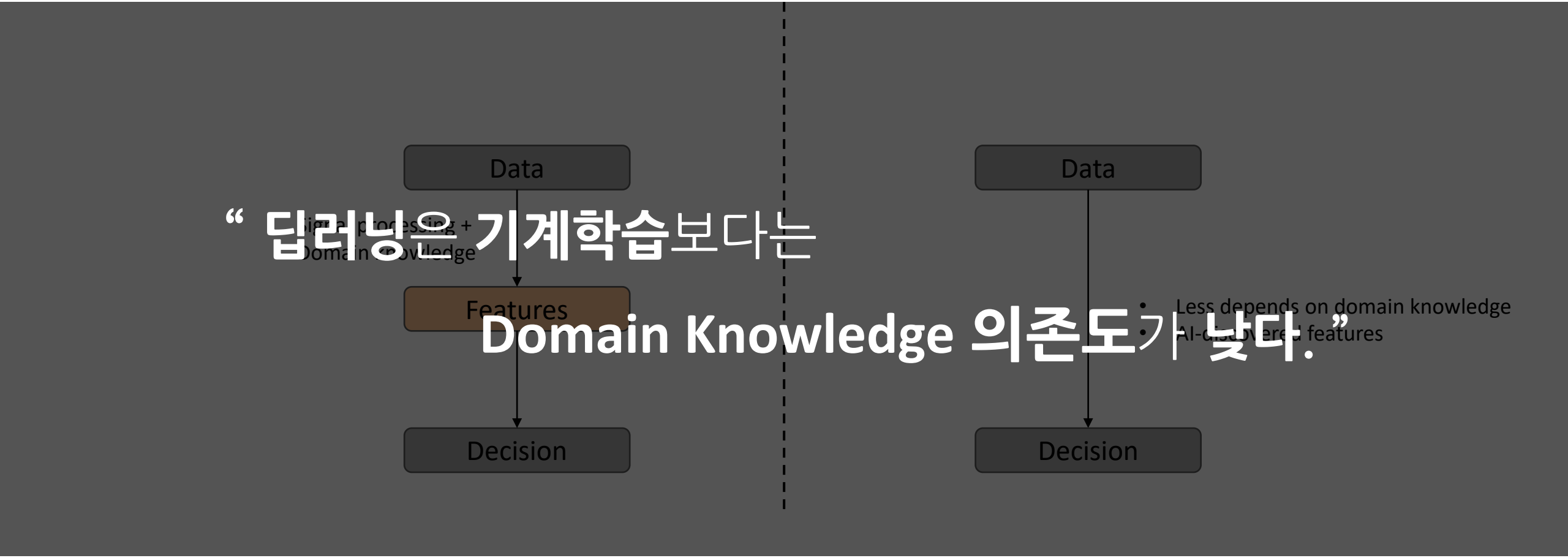
# Deep Learning

- Convolutional Neural Networks (CNN)
- Image pattern recognition problems



## *Machine* Learning

## *Deep* Learning




“ 딥러닝은 기계학습보다는 Domain Knowledge 의존도가 낮다.”