

Semantic Analysis and Type checking

- A compiler has to do semantic checks in addition to syntactic checks.
- Semantic checks
 - Static – done during compilation
 - Dynamic – done during run-time
- *Type checking* is one of these static checking operations.
 - we may not do all type checking at compile-time.
 - Some systems also use dynamic type checking too.



The Compiler So Far

- **Lexical analysis**
 - **Detects inputs with illegal tokens**
- **Parsing**
 - **Detects inputs with ill-formed parse trees**
- **Semantic analysis**
 - **Last “front end” phase**
 - **Catches more errors**

Errors

let y: Int in x + 3

Error?

let y: String ← “abc” in y + 3

Error?

Why a Separate Semantic Analysis?

- **Parsing cannot catch some errors**
- **Some language constructs are not context-free**
 - **Example: All used variables must have been declared (i.e. scoping)**
 - **Example: A method must be invoked with arguments of proper type (i.e. typing)**

What Does Semantic Analysis Do?

- **Checks of many kinds . . . cool checks:**
 - 1. All identifiers are declared**
 - 2. Types**
 - 3. Inheritance relationships**
 - 4. Classes defined only once**
 - 5. Methods in a class defined only once**
 - 6. Reserved identifiers are not misused**

And others ...
- **The requirements depend on the language**

Scope

- **Matching identifier declarations with uses**
 - **Important static analysis step in most languages**
 - **Including COOL!**

```
fn main() {  
  // Parent scope  
  let x = 1; {  
    // `x` in this nested scope shadows `x` in the parent scope.  
    let x = "Hello, world";  
    assert_eq!(x, 1);  
  }  
}
```

Scope (Cont.)

- The scope of an **identifier** is the portion of a program in which that identifier is accessible
- The same identifier may refer to different things in different parts of the program
 - Different scopes for same name don't overlap
- An identifier may have restricted scope

Static vs. Dynamic Scope

- Most languages have **static** scope
 - Scope depends only on the program text, not runtime behavior
 - Cool has static scope
- A few languages are **dynamically** scoped
 - Lisp, Perl
 - Lisp has changed to mostly static scoping
 - Scope depends on execution of the program

Static Scope

```
let x: Int <- 0 in
{
  x;
  let x: Int <- 1 in
    x;
  x;
}
```

Uses of **x** refer to **closest enclosing definition**

Dynamic Scope

- A dynamically-scoped variable refers to the closest enclosing binding in the execution of the program

Example

bar(x) = x+a

foo(y) = let a ← 4 in bar(3);

baz(z) = let a ← 5 in bar(3)

Scope in Cool

- **Cool identifier names are introduced by**
 - 1. Class declarations (introduce class names)**
 - 2. Method definitions (introduce method names)**
 - 3. Let expressions (introduce object id's)**
 - 4. Formal parameters (introduce object id's)**
 - 5. Attribute definitions in a class (introduce object id's)**
 - 6. Case expressions (introduce object id's)**

Scope in Cool (Cont.)

- **Not all kinds of identifiers follow the most closely nested rule**
- **For example, class definitions in Cool**
 - **Cannot be nested**
 - **Are globally visible throughout the program**
 - **In other words, a class name can be used before it is defined**

```
Class Foo {  
    ...  
    let y: Bar in ...  
};  
Class Bar {  
    ...  
};
```

```
Class Foo{  
    f(): Int { a };  
    a: Int ← 0;  
}
```

More More Scope in Cool

- Method and attribute names have complex rules
- A method need not be defined in the class in which it is used, but in some parent class
- Methods may also be redefined (overridden)

SYMBOL TABLE AND SCOPE

- Symbol tables typically need to support multiple declarations of the same identifier within a program.
- We shall implement scopes by setting up a separate symbol table for each scope.

Who Creates Symbol Table??

- Identifiers and attributes are entered by the analysis phases when processing a definition (declaration) of an identifier
- In simple languages with only global variables and implicit declarations:
 - ✓ The scanner can enter an identifier into a symbol table if it is not already there
- In block-structured languages with scopes and explicit declarations:
 - ✓ The parser and/or semantic analyzer enter identifiers and corresponding attributes

USE OF SYMBOL TABLE

- Symbol table information is used by the analysis and synthesis phases
- To verify that used identifiers have been defined(declared)
- To verify that expressions and assignments are semantically correct ——type checking
- To generate intermediate or target code

IMPLEMENTATION OF SYMBOL TABLE

- Each entry in the symbol table can be implemented as a record consisting of several fields.
- These fields are dependent on the information to be saved about the name
- But since the information about a name depends on the usage of the name, the entries in the symbol table records will not be uniform.
- Hence to keep the symbol tables records uniform some information are kept outside the symbol table and a pointer to this information is stored in the symbol table record.

SYMBOL TABLE ORGANIZATION

Int x,y;

Procedure P:

Bool x, a ;

Procedure Q:

Real x,y,z ;

 begin

 end

begin

end

Top →

z	Real
y	Real
x	Real

Symbol table
for Q

Q	Proc
x	Bool
a	Bool

Symbol table
for P

P	Proc
x	Int
y	Int

Symbol table
for global

SYMBOL TABLE DATA STRUCTURES

Issues to consider : Operations required

- Insert :Add symbol to symbol table
- Look UP: Find symbol in the symbol table (and get its attributes)
- Insertion is done only once
- Look Up is done many times
- Need Fast Look Up
- The data structure should be designed to allow the compiler to find the record for each name quickly and to store or retrieve data from that record quickly.

A Fancier Symbol Table

1. **enter_scope()** start a new nested scope
2. **find_symbol(x)** finds current x (or null)
3. **add_symbol(x)** add a symbol x to the table
4. **check_scope(x)** true if x defined in current scope
5. **exit_scope()** exit current scope

We will supply a symbol table manager for your project ,e.g., **syntab.h**, a list of scope, a scope is a list of entries <id, data>

Scopes - Summary

- **Scoping rules match uses of identifiers with their declarations**
 - **Static scoping is the most common form**
- **Scoping rules can be implemented using symbol tables**
 - **In one or more passes over the AST**

Class Definitions

- **Class names can be used before being defined**
- **We can't check class names**
 - using a symbol table
 - or even in one pass
- **Solution**
 - **Pass 1: Gather all class names**
 - **Pass 2: Do the checking**
- **Semantic analysis requires multiple passes**
 - **Probably more than two**

Types

- **What is a type?**
 - **The notion varies from language to language**
- **Consensus**
 - **A set of values**
 - **A set of operations on those values**
- **Classes are one instantiation of the modern notion of type**

Types and Operations

- **Certain operations are legal for values of each type**
 - **It doesn't make sense to add a function pointer and an integer in C**
 - **It does make sense to add two integers**
 - **But both have the same assembly language implementation!**

E.g.

add \$r1, \$r2, \$r3

Type Systems

- A language's type system specifies **which operations are valid for which types**
- The goal of type checking is to **ensure that operations are used with the correct types**
 - Enforces intended interpretation of values, because nothing else will!
- Type systems provide a concise formalization of the semantic checking rules

What Can Types do For Us?

Can detect certain kinds of errors

- **Memory errors (Rust):**

- ✓ Data races
- ✓ Dereferencing a null/dangling raw pointer
- ✓ Reads of undef (uninitialized) memory
- ✓ Etc.

- **Violation of abstraction boundaries:**

```
class FileSystem {  
    open(x : String) : File {  
        ...  
    }  
...  
}
```

```
class Client {  
    f(fs : FileSystem) {  
        File fdesc = fs.open("foo")  
        ...  
    } -- f cannot see inside fdesc !  
}
```

Type Checking Overview

Three kinds of languages:

- **Statically typed**: All or almost all checking of types is done as part of compilation (C, Java, Rust, Cool,)
- **Dynamically typed**: Almost all checking of types is done as part of program execution (Scheme, Python)
- **Untyped**: No type checking (machine code)

```
>>> x = 1
>>> def f():
    print(x)
    x(1)
>>> f()
1
TypeError: 'int' object is not callable
```

The Type Wars

- Competing views on **static** vs. **dynamic** typing
- Static typing proponents say:
 - Static checking catches many Programming errors at compile time
 - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
 - Static type systems are restrictive
 - Rapid prototyping easier in a dynamic type system
- In practice:
 - most code is written in statically typed languages with an “escape” mechanism
 - Unsafe casts in C, native methods in Java, unsafe modules in Modula-3, unsafe code in Rust
 - Some dynamically typed languages support “pragmas” or “advice”
 - type declarations

Type Checking in Cool

- Type concepts in COOL
- Notation for type rules
 - Logical rules of inference
- COOL type rules
- General properties of type systems

Cool Types

- The types are:
 - Class names
 - Base classes: object, IO, Int, String, Bool
 - `SELF_TYPE`
 - Note: there are no base types (as int in C)
- **The user declares types for all identifiers**
- **The compiler infers types for expressions**
 - Infers a type for every sub-expression

Type Checking and Type Inference

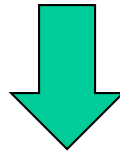
- **Type Checking** is the process of verifying fully typed programs
- **Type Inference** is the process of filling in missing type information
- The two are different, but the terms are often used interchangeably
- We have seen two examples of formal notation specifying parts of a compiler
 - Regular expressions
 - Context-free grammars
- The appropriate formalism for type checking is **logical rules of inference**

Why Rules of Inference?

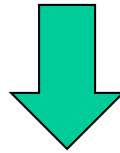
- Inference rules have the form If Hypothesis is true, then Conclusion is true
- Type checking computes via reasoning
 - If E_1 and E_2 have certain types, then E_3 has a certain type
- Rules of inference are a compact notation for “If-Then” statements
- Start with a simplified system and gradually add features by type inference
- **Building blocks**
 - Symbol \wedge is “and”, \vee is “or”,
 - Symbol \Rightarrow is “if-then”
 - $x:T$ is “x has type T”

From English to an Inference Rule

If e_1 has type **Int** and e_2 has type **Int**,
then $e_1 + e_2$ has type **Int**



$(e_1 \text{ has type } \mathbf{Int} \wedge e_2 \text{ has type } \mathbf{Int}) \Rightarrow (e_1 + e_2 \text{ has type } \mathbf{Int})$



$(e_1 : \mathbf{Int} \wedge e_2 : \mathbf{Int}) \Rightarrow (e_1 + e_2 : \mathbf{Int})$

General inference rule:

$\text{Hypothesis}_1 \wedge \text{Hypothesis}_2 \wedge \dots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$

Notation for Inference Rules

- General inference rule:

Hypothesis₁ ∧ Hypothesis₂ ∧ ... ∧ Hypothesis_n → Conclusion

- By tradition inference rules are written

| - Hypothesis₁ ... | - Hypothesis_n

| - Conclusion

- | - means “we can prove that...”

Basic Rules

$\vdash i:\text{Int}$ **[int]**

$\vdash e_1:\text{Int}$

$\vdash e_2:\text{Int}$

[Add]

$\vdash e_1+e_2:\text{Int}$

$\vdash 1:\text{Int}$

$\vdash 2:\text{Int}$

$\vdash 1+2:\text{Int}$

- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete types for expressions
- Example: $1+2$

Soundness

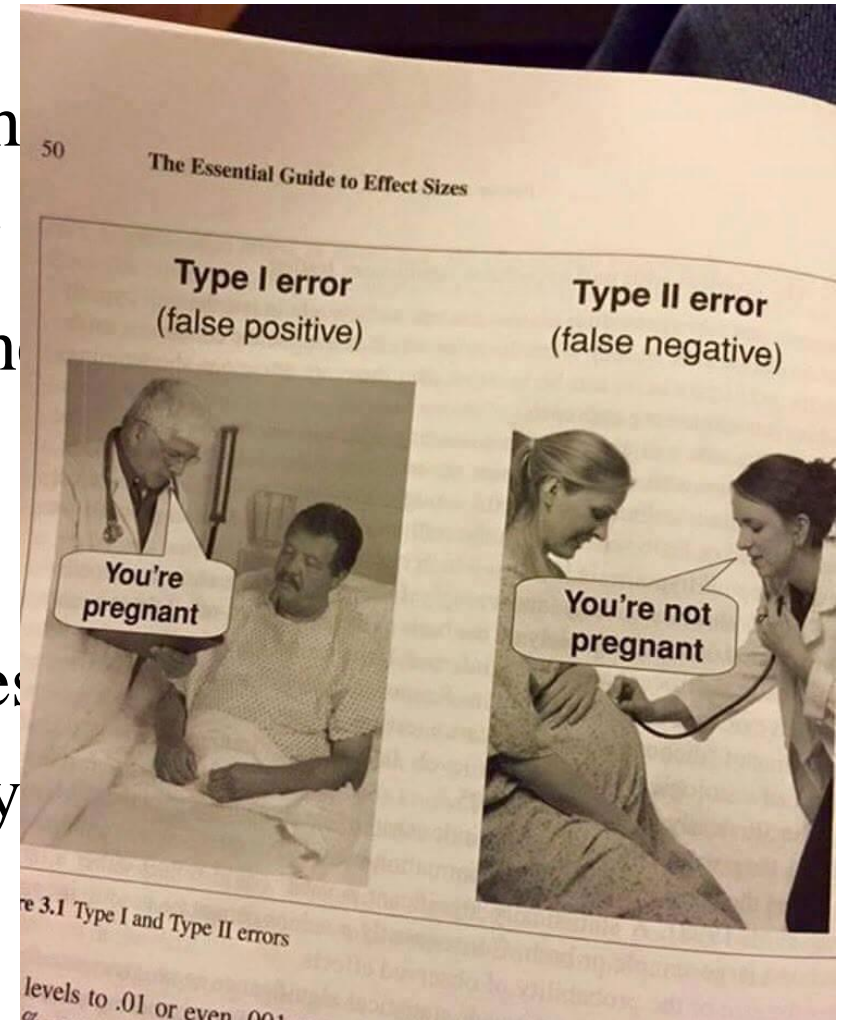
- A type system is **sound** if
 - Whenever $\vdash e : T$
 - Then e evaluates to a value of type T
- We only want sound rules
 - But some sound rules are better than others

$\frac{}{\vdash i : \text{Object}}$ (i is an integer constant)

In Cool, class Int inherits from Object

Soundness and Completeness

- A type-system is **sound** implies that all programs accepted by the type system are correct (in the other words, an incorrect program can't be type checked). There won't be any *false positive*.
- A type-system is **complete** implies that all correct programs can be accepted by the type system. There won't be any *false negative*.



Type Checking Proofs

- Type checking proves facts $e:T$
 - Proof is on the structure of the AST e
 - Proof has the shape of the AST
 - One type rule is used for each AST node (sub-expressive)
- In the type rule used for a node e
 - The hypotheses are the proofs of types of e 's subexpressions
 - The conclusion is the proof of type of e
- Types are computed in a bottom-up pass over the AST

Rules for Constants

————— [False]
|- false:Bool

————— [True]
|- True:Bool

————— [String]
|- s:String

————— [Int]
|- i:Int

Rule for New

- `new T` produces an object of type `T`
 - Ignore `SELF_TYPE` for now . . .

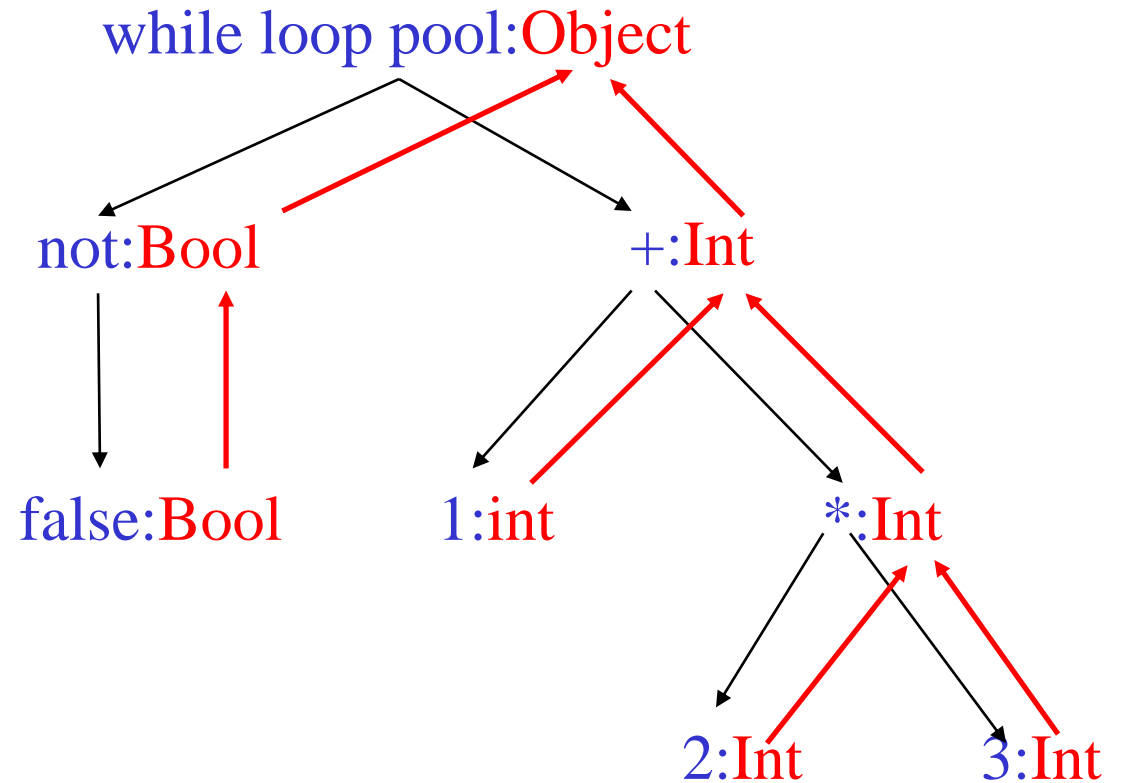
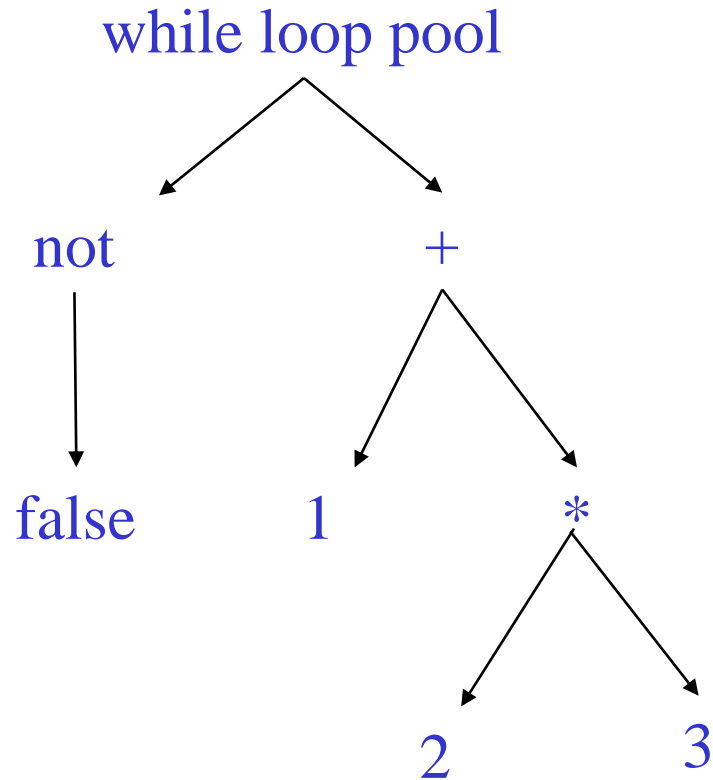
————— [New]
|- new T: T

Two More Rules

$$\frac{|- e:\text{Bool}}{\quad} [\text{Not}]$$
$$|- \text{not } e: \text{Bool}$$
$$\frac{\begin{array}{l} |- e_1:\text{Bool} \\ |- e_2:T \end{array}}{\quad} [\text{While}]$$
$$|- \text{while } e_1 \text{ loop } e_2 \text{ pool:Object}$$

Typing: Example

- Typing for `while not false loop 1 + 2 * 3 pool`



Typing Derivations

- The typing reasoning can be expressed as an inverted tree:
 - The root of the tree is the whole expression
 - Each node is an instance of a typing rule
 - Leaves are the rules with no hypotheses

$$\frac{\frac{\frac{}{|- \text{false:Bool}}{|- \text{not false: Bool}}}{|- \text{while not false loop } 1 + 2 * 3 \text{ pool:Object}} \quad \frac{\frac{\frac{}{|- 1:\text{Int}}{|- 1:\text{Int}} \quad \frac{\frac{}{|- 2:\text{Int}}{|- 3:\text{Int}}}{|- 2*3:\text{Int}}}{|- 1+2*3:\text{Int}}}{|- 1+2*3:\text{Int}}}$$

A Problem

- What is the type of a variable reference?

$$\frac{}{|- x : ?} \text{ [Var]}$$

- The local, structural rule does not carry enough information to give a type.
- We need a hypothesis of the form “we are in the scope of a declaration of x with type T ”)

A Solution: Put more information in the rules!

- A **type environment** gives types for **free variables**
- A variable is **free** in an expression
if it is **not** defined within the expression
- A **type environment** is a function

O: ObjectIds \rightarrow Types

E.g.:

1. **x** and **y** are free in the expression **x * y**
2. **x** is **not** free, **y** is free in **let x: Int x + y**
3. **x** and **y** are free in the expression **x + let x: Int in x + y**

Type Environments

- Let **O** be a **type environment** function **O: ObjectIds \rightarrow Types**

The sentence **O|-e:T**

is read: Under the type environment **O**, it is provable that the expression **e** has the type **T**

Modified Rules for Constants

$$\frac{}{O \vdash \text{false}:\text{Bool}} \quad [\text{False}]$$
$$\frac{}{O \vdash \text{True}:\text{Bool}} \quad [\text{True}]$$
$$\frac{}{O \vdash s:\text{String}} \quad [\text{String}]$$
$$\frac{}{O \vdash i:\text{Int}} \quad [\text{Int}]$$
$$\frac{\begin{array}{l} O \vdash e_1:\text{Int} \\ O \vdash e_2:\text{Int} \end{array}}{O \vdash e_1 + e_2:\text{Int}} \quad [\text{Add}]$$
$$\frac{\begin{array}{l} O \vdash e_1:\text{Bool} \\ O \vdash e_2:T \end{array}}{O \vdash \text{while } e_1 \text{ loop } e_2 \text{ pool}:\text{Object}} \quad [\text{While}]$$

New Rules

- And we can write new rules:

$$\frac{\mathbf{O(x) = T}}{\mathbf{O \vdash x : T}} \quad \mathbf{[Var]}$$

Review

- Semantic analysis
- Scope vs. Symbol Table
- Type systems
- Static type vs. Dynamic Type
- Let **O** be a **type environment** function

$O: \text{ObjectIds} \rightarrow \text{Types}$

The sentence **$O|-e:T$**

is read: Under the type environment **O**, it is provable that the expression **e** has the type **T**

Modified Rules for Constants

$$\frac{}{O \vdash \text{false}:\text{Bool}} \quad [\text{False}]$$
$$\frac{}{O \vdash \text{True}:\text{Bool}} \quad [\text{True}]$$
$$\frac{}{O \vdash s:\text{String}} \quad [\text{String}]$$
$$\frac{}{O \vdash i:\text{Int}} \quad [\text{Int}]$$
$$\frac{\begin{array}{l} O \vdash e_1:\text{Int} \\ O \vdash e_2:\text{Int} \end{array}}{O \vdash e_1 + e_2:\text{Int}} \quad [\text{Add}]$$
$$\frac{\begin{array}{l} O \vdash e_1:\text{Bool} \\ O \vdash e_2:T \end{array}}{O \vdash \text{while } e_1 \text{ loop } e_2 \text{ pool}:\text{Object}} \quad [\text{While}]$$
$$\frac{O(x) = T}{O \vdash x : T} \quad [\text{Var}]$$

Let Rule

$$\frac{O(T_0/x) \vdash e : T_1}{O \vdash \text{let } x: T_0 \text{ in } e : T_1} \quad [\text{Let-No-Init}]$$

$O(T_0/x)$ is a new environment obtained from O by assigning T_0 to x

$$\begin{aligned} O(T_0/x)(x) &= T_0 \\ O(T_0/x)(y) &= O(y) \text{ if } x \neq y \end{aligned}$$

Note that the let-rule enforces variable scope

Let Example

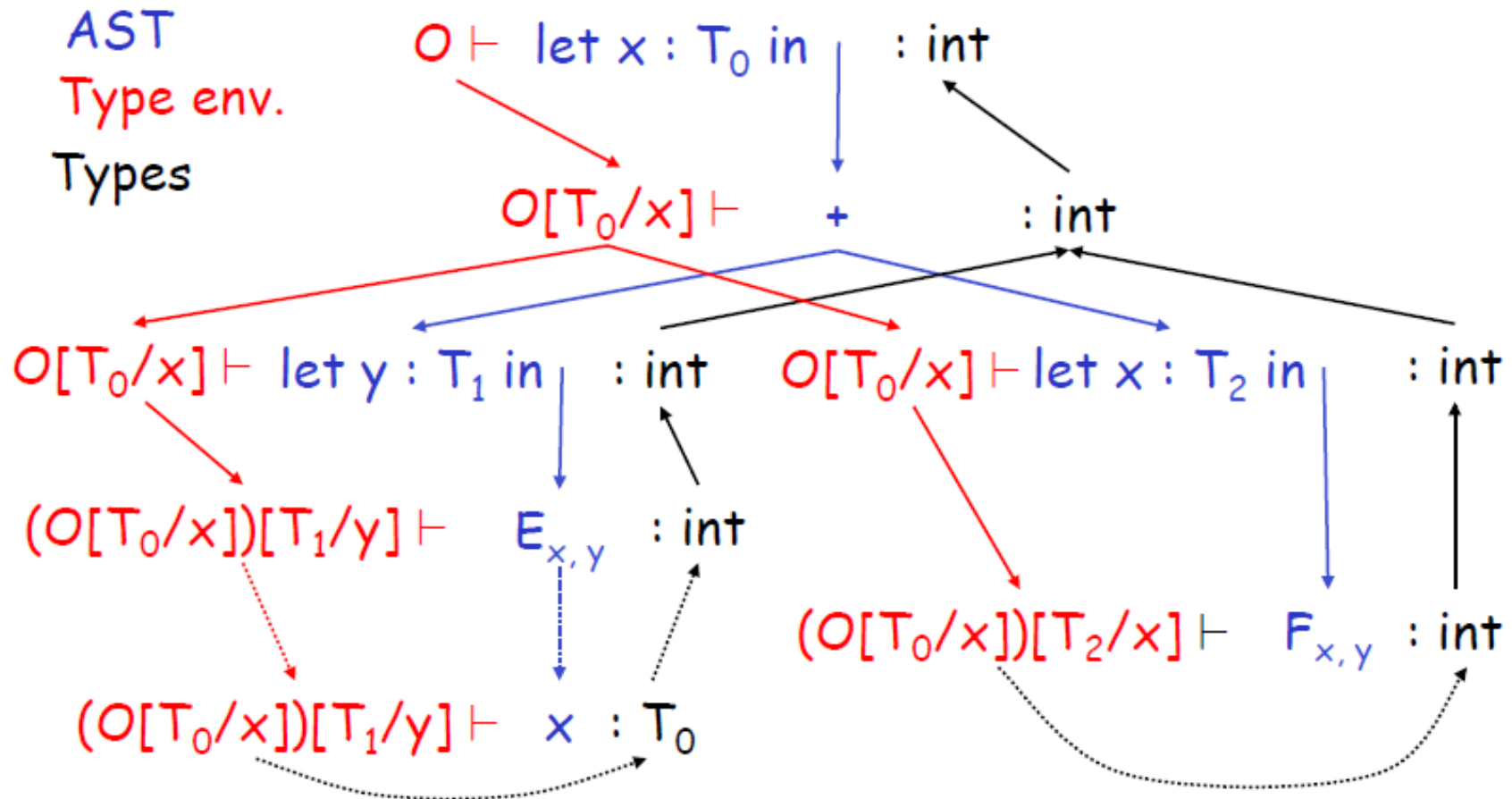
- Consider the Cool expression

let $x : T_0$ in (let $y : T_1$ in $E_{x,y}$) + (let $x : T_2$ in $F_{x,y}$)

- Scope:
 - of y is $E_{x,y}$
 - of outer x is $E_{x,y}$
 - of inner x is $F_{x,y}$
- This is captured precisely in the let-rule

Let Example

$\text{let } \mathbf{x} : T_0 \text{ in } (\text{let } \mathbf{y} : T_1 \text{ in } E_{\mathbf{x},\mathbf{y}}) + (\text{let } \mathbf{x} : T_2 \text{ in } F_{\mathbf{x},\mathbf{y}})$



Notes

- **The type environment gives types to the free identifiers in the current scope**
- **The type environment is passed down the AST from the root towards the leaves**
- **Types are computed up the AST from the leaves towards the root**

Let with Initialization

- Now consider **let** with initialization:

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O(T_0/x) \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

This rule is weak.

```
class C inherits P { ... }  
...  
let x : P ← new C in ...  
...
```

The previous let rule does not allow this code

Subtyping

- Define a relation $X \leq Y$ on classes (types) to say that:
 - An object of type X could be used when one of type Y is acceptable, or equivalently
 - X conforms with Y
 - In Cool this means that X is a subclass of Y
- Define a relation \leq on classes (reflexive transitive closure)
 1. $X \leq X$
 2. $X \leq Y$ if X inherits from Y
 3. $X \leq Z$ if $X \leq Y$ and $Y \leq Z$

Let with Initialization

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O(T_0/x) \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T \leq T_0 \\ O(T_0/x) \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

class C inherits P { ... }
 ...
 let x : P \leftarrow new C in ...
 ...

- Both rules for let are sound
 - Flexible rules that do not constrain programming
 - Restrictive rules that ensure safety of execution
- But more programs type check with the latter

Quiz

```
class A {...}  
class B inherits A {...}
```

```
let x : A ← new B in 1+2
```

Draw Type Derivation Tree

Expressiveness of Static Type Systems

- A static type system enables a compiler to detect many common programming errors
- The cost is that some correct programs are disallowed
 - Some argue for dynamic type checking instead
 - Others argue for more expressive static type checking
- But more expressive type systems are also more complex
- There usually does not exist a sound and complete type systems for programming languages

Dynamic and Static Types

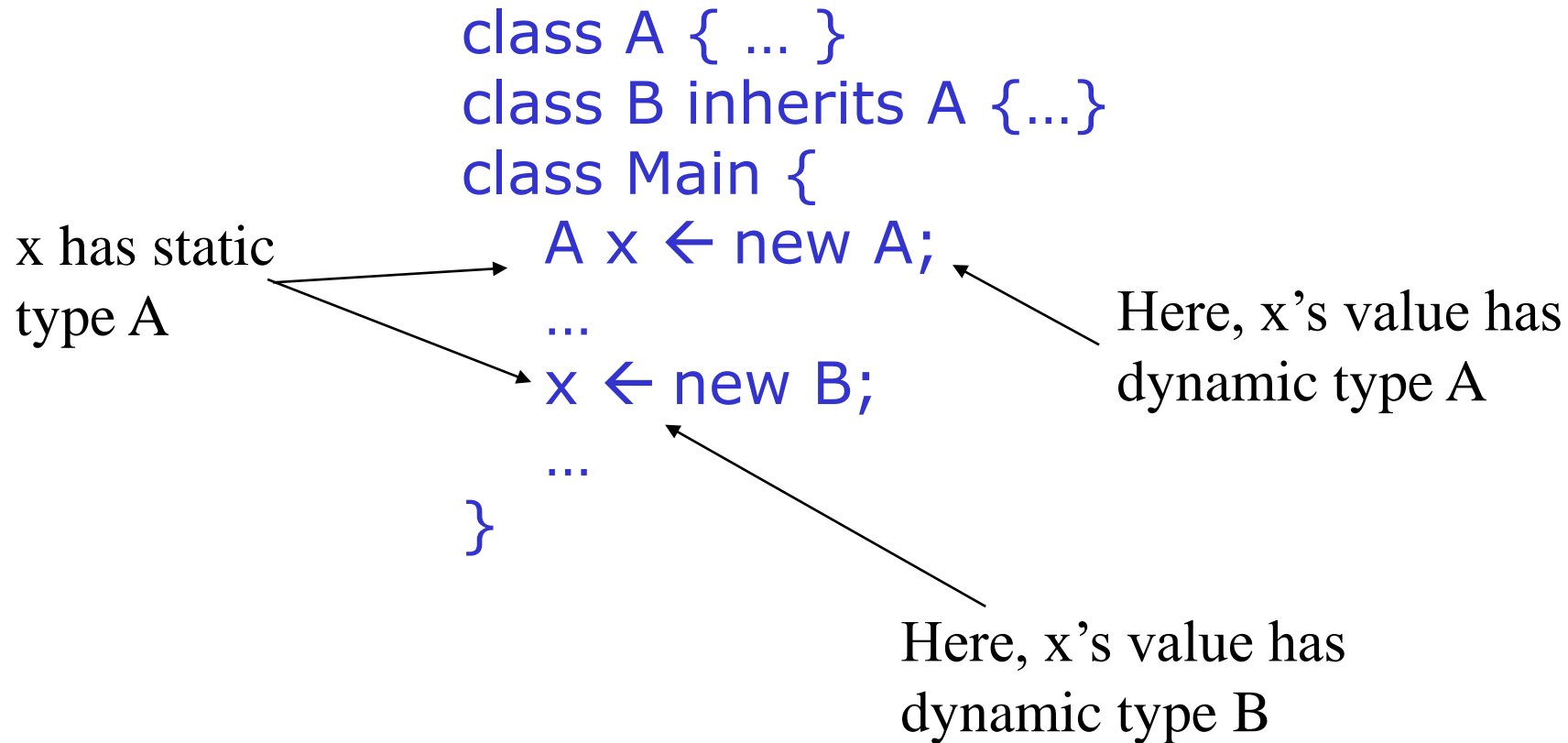
- The **dynamic type** of an object is the class **C** that is used in the “**new C**” expression that creates the object
 - A run-time notion
 - Even languages that are not statically typed have the notion of dynamic type
- The **static type** of an expression is a notation that captures all possible dynamic types the expression could take
 - A compile-time notion

Dynamic and Static Types. (Cont.)

- In early type systems the set of static types correspond directly with the dynamic types
- **Soundness theorem**: for all expressions E
$$\text{dynamic_type}(E) = \text{static_type}(E)$$

(in **all** executions, E evaluates to values of the type inferred by the compiler)
- This gets more complicated in advanced type systems

Dynamic and Static Types in COOL



A variable of static type `A` can hold values of static type `B`,
if $B \leq A$

Dynamic and Static Types

Soundness theorem for the Cool type system:

$$\forall E. \text{dynamic_type}(E) \leq \text{static_type}(E)$$

Why is this Ok?

- For E , compiler uses $\text{static_type}(E)$ (call it C)
- All operations that can be used on an object of type C can also be used on an object of type $C' \leq C$
 - Such as fetching the value of an attribute
 - Or invoking a method on the object
- Subclasses can only add attributes or methods
- Methods can be redefined but with same type !

Let Example

- Consider the following Cool class definitions

```
Class A { a() : Int { 0 }; }
```

```
Class B inherits A { b() : Int { 1 }; }
```

- An instance of **A** has method “a”
- An instance of **B** has methods “a” and “b”
 - A type error occurs if we try to invoke method “b” on an instance of **A**
 - It is OK to invoke method “a” on an instance of **B**

```
Let mya: A ← new B (OK)
```

```
Let myb: B ← new A (error)
```

```
mya.b() (error)
```

```
Let myb: B ← new B (OK)
```

```
Myb.a() (OK)
```


Let Example

Any error?

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T \leq T_0 \\ O \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

Any error?

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T_0 \leq T \\ O(T_0/x) \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

Comments

- The typing rules use very concise notation
- They are very carefully constructed
- Virtually any change in a rule either:
 - Makes the type system unsound
(bad programs are accepted as well typed)
 - Or, makes the type system less usable, although sound
(good programs are rejected)
- But some good programs will be rejected anyway
 - The notion of a good program is undecidable

Assignment

- More uses of subtyping:

$$\frac{\begin{array}{l} O(x)=T_0 \\ T_1 \leq T_0 \\ O \vdash e_1:T_1 \end{array}}{O \vdash x \leftarrow e_1 : T_1} \quad [\text{Assign}]$$

Example

Is there any type error?

```
class A {a():Int {0}; }  
class B inherits A {b():Int {1};}  
class Main {  
  A x ← new A;  
  ...  
  x ← new B;  
  y: Int ← x.b ()  
  ...  
}
```

x has static
type A

Here, x's value has
dynamic type A

Here, x's value has
dynamic type B

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x:T$ in class C
- Attribute initialization is similar to let, except for the scope of names

$$\frac{\begin{array}{l} O_C(x)=T_0 \\ T_1 \leq T_0 \\ O_C \vdash e_1:T_1 \end{array}}{O_C \vdash x:T_0 \leftarrow e_1;} \quad [\text{Attr-Init}] \quad \frac{O_C(x)=T_0}{O_C \vdash x:T_0} \quad [\text{Attr-No-Init}]$$

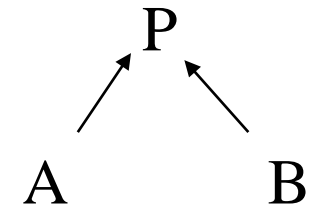
If-Then-Else

- Consider:

if e_0 then e_1 else e_2 fi

- The result can be either e_1 or e_2 ,
- The dynamic type is either e_1 's or e_2 's type
- The best we can do statically is the smallest **supertype larger than** the type of e_1 and e_2

- Consider the class hierarchy



if e_0 then new **A** else new **B** fi

- Its type should allow for the dynamic type to be both **A** or **B**
 - Smallest supertype is **P**

Least Upper Bounds

- $\text{lub}(X, Y)$, the least upper bound of X and Y , is Z if

$$- X \leq Z \wedge Y \leq Z$$

Z is an **upper bound**

$$- X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$$

Z is **least** among upper bounds

- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

$O \vdash e_0 : \text{Bool}$

$O \vdash e_1 : T_1$

$O \vdash e_2 : T_2$

[If-Then-Else]

$O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \text{lub}(T_1, T_2)$

Case

- The rule for **case** expressions takes a **lub** over all branches

$$O \vdash e_0 : T_0$$
$$O[T_1/x_1] \vdash e_1 : T_1'$$
$$\dots\dots$$
$$O[T_n/x_n] \vdash e_n : T_n'$$

[If-Then-Else]

$$O \vdash \text{case } e_0 \text{ of}$$
$$x_1 : T_1 \rightarrow e_1;$$
$$\dots\dots$$
$$x_n : T_n \rightarrow e_n;$$
$$\text{esac} : \text{lub}(T_1', \dots, T_n')$$

Method Dispatch

- There is a problem with type checking method calls:

$$\frac{\begin{array}{c} O \vdash e_0:T_0 \\ O \vdash e_1:T_1 \\ \dots\dots \\ O \vdash e_n:T_n \end{array}}{O \vdash e_0.f(e_1,\dots,e_n):?} \quad [\text{Dispatch}]$$

- We need information about the formal parameters and return type of f

Notes on Dispatch

- In Cool, method and object identifiers live in different name spaces
 - A method `foo` and an object `foo` can coexist in the same scope
- In the type rules, this is reflected by a separate mapping **M** for method signatures

$$M(C, f) = (T_1, \dots, T_n, T)$$

means in class **C** there is a method f : $f(x_1:T_1, \dots, x_n:T_n): T_n$

- Now we have two environments **O** and **M**
- The form of the typing judgment is:

$$O, M \vdash e : T$$

read as: “with the assumption that the object identifiers have types as given by **O** and the method identifiers have signatures as given by **M**, the expression **e** has type **T**”

The Method Environment

- The method environment must be added to all rules
- In most cases, M is passed down but not actually used
 - Example of a rule that does not use M :

$$\begin{array}{c}
 \frac{}{O, M \vdash \text{false} : \text{Bool}} \text{[False]} \qquad \frac{}{O, M \vdash 1 : \text{Int}} \text{[Int]} \\
 \\
 \frac{O, M \vdash e_1 : \text{Int} \quad O, M \vdash e_2 : \text{Int}}{O, M \vdash e_1 + e_2 : \text{Int}} \text{[Add]} \qquad \frac{O, M \vdash e_1 : \text{Bool} \quad O, M \vdash e_2 : T}{O, M \vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \text{[While]}
 \end{array}$$

- Only the dispatch rules use M

Method Dispatch

- There is a problem with type checking method calls:

$$\begin{array}{c} \mathbf{O, M \vdash e_0 : T_0} \\ \mathbf{O, M \vdash e_1 : T_1} \\ \dots\dots\dots \\ \mathbf{O, M \vdash e_n : T_n} \\ \mathbf{M(T_0, f) = (T_1', \dots, T_n', T)} \\ \mathbf{T_i \leq T_i' \text{ for all } 1 \leq i \leq n} \\ \hline \mathbf{O, M \vdash e_0.f(e_1, \dots, e_n) : T} \end{array} \quad \mathbf{[Dispatch]}$$

Static Dispatch

- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

$$\begin{array}{c}
 \mathbf{O, M \vdash e_0 : T_0} \\
 \mathbf{O, M \vdash e_1 : T_1} \\
 \dots\dots \\
 \mathbf{O, M \vdash e_n : T_n} \\
 \mathbf{M(\mathbf{T}, f) = (T_1', \dots, T_n', \mathbf{T_{n+1}})} \\
 \mathbf{T_i \leq T_i' \text{ for all } 1 \leq i \leq n} \\
 \mathbf{T_0 \leq \mathbf{T}} \\
 \hline
 \mathbf{O, M \vdash e_0 @ \mathbf{T}.f(e_1, \dots, e_n) : \mathbf{T_{n+1}}}
 \end{array}
 \quad [\text{StaticDispatch}]$$

Handling the SELF_TYPE

- Recall that type systems have two conflicting goals:
 - Give flexibility to the programmer
 - Prevent valid programs to “go wrong”

Milner, 1981: “Well-typed programs do not go wrong”

- An active line of research is in the area of inventing more flexible type systems while preserving soundness

An Example

```
class Count {  
  i : Int ← 0;  
  inc () : Count {  
    {  
      i ← i + 1;  
      self;  
    }  
  };  
};
```

Any error?

```
class Stock inherits Count {  
  name() : String { ... };  
};
```

```
class Main {  
  a : Stock ← (new Stock).inc ();  
  ... a.name() ...  
};
```

- `(new Stock).inc()` has dynamic type `Stock`,
- So it is legitimate to write `a : Stock ← (new Stock).inc ()`
- But this is **not well-typed**: `(new Stock).inc()` has static type `Count`
- The type checker “looses” type information
- This makes inheriting `inc` useless
 - So, we must redefine `inc` for each of the subclasses, with a specialized return type

SELF_TYPE to the Rescue

- Insight:
 - `inc` returns “`self`”
 - Therefore the return value has same type as “`self`”
 - Which could be `Count` or any subtype of `Count` !
 - In the case of `(new Stock).inc ()` the type is `Stock`
- We introduce the keyword `SELF_TYPE` to use for the return value of such functions
- `SELF_TYPE` allows the `return type` of `inc` to change when `inc` is inherited
- Modify the declaration of `inc` to read
$$\text{inc}() : \text{SELF_TYPE} \{ \dots \}$$
- The type checker can now prove:
$$\begin{aligned} O, M \vdash (\text{new Count}).\text{inc}() &: \text{Count} \\ O, M \vdash (\text{new Stock}).\text{inc}() &: \text{Stock} \end{aligned}$$
- The program from before is now well typed

Notes About SELF_TYPE

- SELF_TYPE is **not** a **dynamic type**. It is a **static type**
- It helps the type checker to keep better track of types
- It enables the type checker to accept more correct programs
- In short, having **SELF_TYPE** increases the expressive power of the type system
- What can be the dynamic type of the object returned by **inc**?
 - Answer: whatever could be the type of “**self**”

```
class A inherits Count { } ;  
class B inherits Count { } ;  
class C inherits Count { } ;
```

(**inc** could be invoked through any of these classes)
 - Answer: **Count** or any subtype of **Count**

SELF_TYPE and Dynamic Types

- In general, if **SELF_TYPE** appears textually in the class **C** as the declared type of **E** then it denotes the dynamic type of the “**self**” expression:

$$\text{dynamic_type}(E) = \text{dynamic_type}(\text{self}) \leq C$$

- Note: The meaning of **SELF_TYPE** depends on where it appears
 - We write **SELF_TYPE_C** to refer to an occurrence of **SELF_TYPE** in the body of **C**

$$\text{SELF_TYPE}_C \leq C$$

Type Checking

- This suggests a typing rule:

$$\text{SELF_TYPE}_c \leq C$$

- This rule has an important consequence:
 - In type checking it is always safe to replace SELF_TYPE_c by C
- This suggests one way to handle SELF_TYPE :
 - Replace all occurrences of SELF_TYPE_c by C
- This would be correct but it is like not having SELF_TYPE at all

Operations on SELF_TYPE

- Recall the operations on types
 - $T_1 \leq T_2$, T_1 is a subtype of T_2
 - $\text{lub}(T_1, T_2)$ the least-upper bound of T_1 and T_2
- We must extend these operations to handle SELF_TYPE
- Let T and T' be any types but SELF_TYPE
- Four cases:
 1. $\text{SELF_TYPE}_C \leq T$ if $C \leq T$
 - SELF_TYPE_C can be any subtype of C including C itself
 - Thus this is the most flexible rule we can allow
 2. $\text{SELF_TYPE}_C \leq \text{SELF_TYPE}_C$
 3. $T \leq \text{SELF_TYPE}_C$ always false
 - Note: SELF_TYPE_C can denote any subtype of C .
 4. $T \leq T'$ (according to the rules from before)

Extending $\text{lub}(T, T')$

- Let T and T' be any types but SELF_TYPE
- Again there are four cases:
 1. $\text{lub}(\text{SELF_TYPE}_c, \text{SELF_TYPE}_c) = \text{SELF_TYPE}_c$
 2. $\text{lub}(\text{SELF_TYPE}_c, T) = \text{lub}(C, T)$

This is the best we can do because $\text{SELF_TYPE}_c \leq C$

 3. $\text{lub}(T, \text{SELF_TYPE}_c) = \text{lub}(C, T)$
 4. $\text{lub}(T, T')$ defined as before

Where Can SELF_TYPE Appear in COOL?

- The parser checks that SELF_TYPE appears only where a type is expected
- But SELF_TYPE is not allowed everywhere a type can appear:

1. **class T inherits T' {...}: T, T' cannot be SELF_TYPE**

- Because SELF_TYPE is never a dynamic type

2. **m@T(E1,...,En)**

- T cannot be SELF_TYPE

3. **x : T.**

- T can be SELF_TYPE, an attribute whose type is SELF_TYPE_C

4. **let x : T in E**

- T can be SELF_TYPE, x has type SELF_TYPE_C

5. **new T**

- T can be SELF_TYPE, creates an object of the same type as self

6. **m(x : T) : T' { ... } Only T' can be SELF_TYPE !**

Typing Rules for SELF_TYPE

- Since occurrences of SELF_TYPE depend on the enclosing class, we need to know the class in which an expression occurs.
- We need to carry more context during type checking
- New form of the typing judgment:

$O, M, C \vdash e : T$

- A mapping O giving types to object id's
- A mapping M giving types to methods
- The current class C where e occurs

New Rules

- There are two new rules using **SELF_TYPE**

$$O, M, C \vdash \text{self} : \text{SELF_TYPE}_C$$

$$O, M, C \vdash \text{new SELF_TYPE} : \text{SELF_TYPE}_C$$

- There are a number of other places where **SELF_TYPE** is used

Type Checking Rules

- The next step is to design type rules using **SELF_TYPE** for each language construct
- Most of the rules remain the same except that \leq and **lub** are the new ones
- Example:

$$\frac{\begin{array}{l} \text{O,M,C} \vdash e_1:\text{Int} \\ \text{O,M,C} \vdash e_2:\text{Int} \end{array}}{\text{O,M,C} \vdash e_1+e_2:\text{Int}} \text{ [Add]} \quad \frac{\begin{array}{l} \text{O(x)} = \text{T}_0 \\ \text{O,M,C} \vdash e_1:\text{T}_1 \\ \text{T}_1 \leq \text{T}_0 \end{array}}{\text{O,M,C} \vdash x \leftarrow e_1:\text{T}_1} \text{ [Assign]}$$

What's Different?

$O, M, C \vdash e_0 : T_0$

$O, M, C \vdash e_1 : T_1$

.....

$O, M, C \vdash e_n : T_n$

$M(T_0, f) = (T_1', \dots, T_n', T_{n+1}')$

$T_i \leq T_i'$ for all $1 \leq i \leq n$

$T_{n+1}' \neq \text{SELF_TYPE}$

$O, M, C \vdash e_0.f(e_1, \dots, e_n) : T_{n+1}'$

$O, M, C \vdash e_0 : T_0$

$O, M, C \vdash e_1 : T_1$

.....

$O, M, C \vdash e_n : T_n$

$M(T_0, f) = (T_1', \dots, T_n', \text{SELF_TYPE})$

$T_i \leq T_i'$ for all $1 \leq i \leq n$

$O, M, C \vdash e_0.f(e_1, \dots, e_n) : T_0$

If the return type of the method is **SELF_TYPE** then the type of the dispatch is the type of the dispatch expression

An Example

```
class Count {  
  i : int ← 0;  
  inc () : SELF_TYPE {  
    {  
      i ← i + 1;  
      self;  
    }  
  };  
};
```

```
class Stock inherits Count {  
  name() : String { ... };  
};  
  
class Main {  
  a : Stock ← (new Stock).inc ();  
  ... a.name() ...  
};
```

- (new Stock).inc() has dynamic type Stock,
- (new Stock).inc() has static type Stock,
- So this is well-typed

Static Dispatch

- Recall the original rule for static dispatch

$$\begin{array}{c}
 O, M, C \vdash e_0 : T_0 \\
 O, M, C \vdash e_1 : T_1 \\
 \dots\dots \\
 O, M, C \vdash e_n : T_n \\
 M(\mathbf{T}, f) = (T_1', \dots, T_n', T_{n+1}') \\
 T_i \leq T_i' \text{ for all } 1 \leq i \leq n \\
 \mathbf{T_0 \leq T} \\
 \mathbf{T_{n+1}' \neq SELF_TYPE}
 \end{array}$$

$$O, M, C \vdash e_0 @ \mathbf{T.f}(e_1, \dots, e_n) : \mathbf{T_{n+1}'}$$

$$\begin{array}{c}
 O, M, C \vdash e_0 : T_0 \\
 O, M, C \vdash e_1 : T_1 \\
 \dots\dots \\
 O, M, C \vdash e_n : T_n \\
 M(\mathbf{T}, f) = (T_1', \dots, T_n', \mathbf{SELF_TYPE}) \\
 T_i \leq T_i' \text{ for all } 1 \leq i \leq n \\
 \mathbf{T_0 \leq T}
 \end{array}$$

$$O, M, C \vdash e_0 @ \mathbf{T.f}(e_1, \dots, e_n) : \mathbf{T_0}$$

Attributes and Methods

$$O_C(x)=T_0 \quad T_1 \leq T_0$$

$$O_C, M, C \vdash e_1 : T_1$$

$$\text{[Attr-Init]}$$

$$O_C, M, C \vdash x : T_0 \leftarrow e_1;$$

$$O_C(x)=T_0$$

$$\text{[Attr-No-Init]}$$

$$O_C, M, C \vdash x : T_0;$$

$$M(C, f) = (T_1, \dots, T_n, \mathbf{T_0}) \quad \mathbf{T_0 \neq SELF_TYPE} \quad \mathbf{T_0' \leq T_0}$$

$$O_C[\mathbf{SELF_TYPE}_C/\text{self}][T_1/x_1] \dots [T_n/x_n], M, C \vdash e : \mathbf{T_0'}$$

[Method]

$$O_C, M, C \vdash f(x_1 : T_1, \dots, x_n : T_n) : \mathbf{T_0} \{ e \}$$

$$M(C, f) = (T_1, \dots, T_n, \mathbf{SELF_TYPE}) \quad \mathbf{T_0' \leq SELF_TYPE}_C$$

$$O_C[\mathbf{SELF_TYPE}_C/\text{self}][T_1/x_1] \dots [T_n/x_n], M, C \vdash e : \mathbf{T_0'}$$

[Method]

$$O_C, M, C \vdash f(x_1 : T_1, \dots, x_n : T_n) : \mathbf{SELF_TYPE} \{ e \}$$

Summary of SELF_TYPE

- The extended \leq and **lub** operations can do a lot of the work. Implement them to handle **SELF_TYPE**
- **SELF_TYPE** can be used only in a few places. Be sure it isn't used anywhere else.
- A use of **SELF_TYPE** always refers to any subtype in the current class
 - The exception is the type checking of dispatch.
 - **SELF_TYPE** as the return type in an invoked method might have nothing to do with the current class

Why Cover SELF_TYPE ?

- SELF_TYPE is a research idea
 - It adds more expressiveness to the type system
- SELF_TYPE is itself not so important
 - except for the project
- Rather, SELF_TYPE is meant to illustrate that type checking can be quite subtle
- In practice, there should be a balance between the complexity of the type system and its expressiveness

Type Systems

- The rules in these lecture were COOL-specific
 - Other languages have very different rules
- General themes
 - Type rules are defined on the structure of expressions
 - Types of variables are modeled by an environment
- Types are a play between flexibility and safety

One-Pass Type Checking

- COOL type checking can be implemented in a single traversal over the AST
- Type environment is passed down the tree
 - From parent to child
- Types are passed up the tree
 - From child to parent

$$\frac{\begin{array}{l} \text{O,M,C} \vdash e_1:\text{Int} \\ \text{O,M,C} \vdash e_2:\text{Int} \end{array}}{\text{O,M,C} \vdash e_1+e_2:\text{Int}} \quad [\text{Add}]$$

```
TypeCheck(Environment, n) // node n denotes the expression e1 + e2
{
    T1 = TypeCheck(Environment, n.leftchild);
    T2 = TypeCheck(Environment, n.rightchild);
    Check T1 == T2 == Int;
    return Int;
}
```

Error Recovery

- As with parsing, it is important to recover from type errors
- Detecting where errors occur is easier than in parsing
 - There is no reason to skip over portions of code
- The Problem:
 - What type is assigned to an expression with no legitimate type?
 - This type will influence the typing of the enclosing expression

Error Recovery Attempt

- Assign type **Object** to ill-typed expressions

$\text{let } y : \text{Int} \leftarrow x + 2 \text{ in } y + 3$

- Since **x** is undeclared its type is **Object**
- But now we have **Object + Int**
- This will generate another typing error
- We then say that **Object + Int = Object**
- Then the initializer's type will not be **Int**

\Rightarrow a workable solution but with cascading errors

Better Error Recovery

- We can introduce a new type called `No_type` for use with ill-typed expressions
- Define `No_type` \leq `C` for all types `C`
- Every operation is defined for `No_type`
 - With a `No_type` result
- Only one typing error for:

`let y : Int \leftarrow x + 2 in y + 3`

Notes

- A “real” compiler would use something like
`No_type`
- However, there are some implementation issues
 - The class hierarchy is not a tree anymore
- The `Object` solution is fine in the class project

Quiz

Write the typing derivation for

i: Int;

x: Int;

i \leftarrow 2;

x \leftarrow 1;

while ~ i < 0 loop{

 i \leftarrow i - 1;

 let y: Int \leftarrow 2 in x \leftarrow x * y;

}

pool;