# Lecture 07: CNNs III – Model Training and Optimization

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# Outline

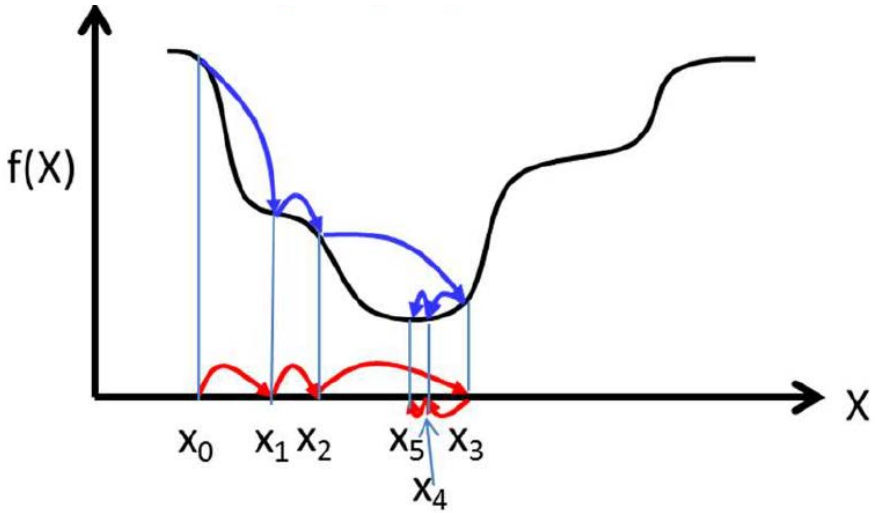
- Overview of CNN training
- CNN training as optimization
  - Data preprocessing
  - Batch normalization
  - Weight initialization
  - First-order update
- Reading: DL book Chapter 8

*Acknowledgement: UofT, CMU & Feifei Li's cs231n notes*

# Training overview

- Supervised learning paradigm
- Mini-batch SGD

  Loop:
  - Sample a (mini-)batch of data
  - Forward propagation it through the network, compute loss
  - Backpropagation to calculate the gradients
  - Update the parameters using the gradient



# Training overview

- Two aspects of training networks
  - Optimization
    - How do we minimize the loss function effectively?
  - Generalization
    - How do we avoid overfitting?
- Optimization – this lecture
  - Data pre-processing, weight initialization, parameter updates, batch normalization
- Generalization – next lecture
  - Data augmentation, dropout, model ensembles, hyper-parameter optimization

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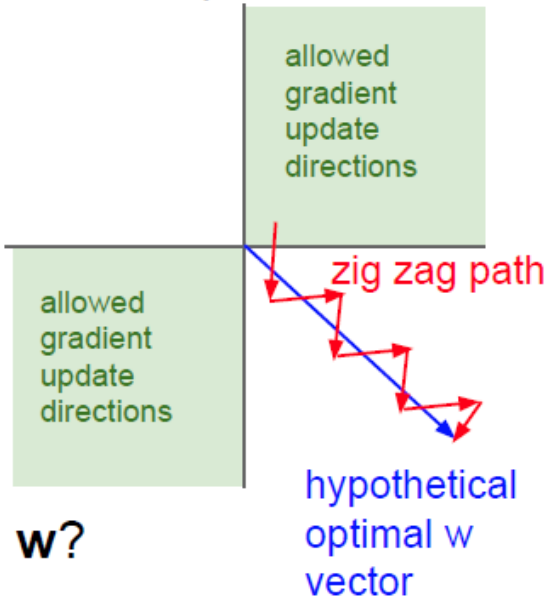
# Data Preprocessing

- Motivation
  - Remember: Consider what happens when the input to a neuron is always positive...

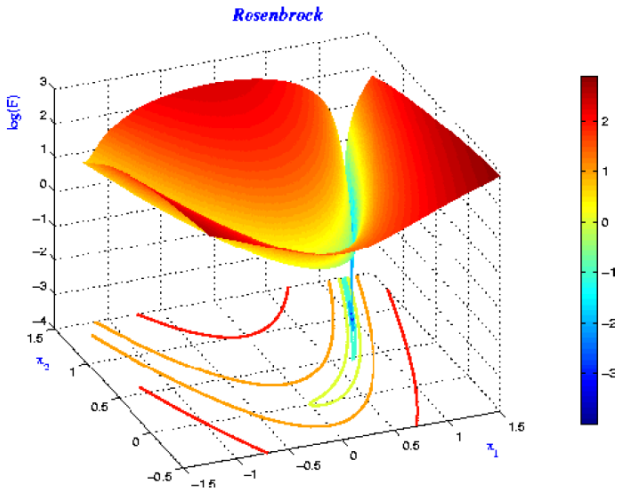
$$f\left(\sum_i w_i x_i + b\right)$$

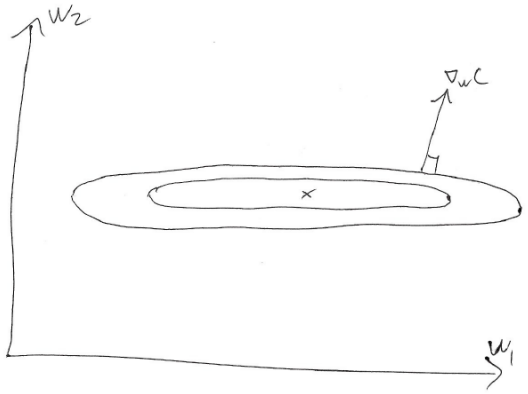



What can we say about the gradients on **w**?
Always all positive or all negative :(
(this is also why you want zero-mean data!)

# Data Preprocessing

- Motivation
  - Error surfaces with long, narrow ravines



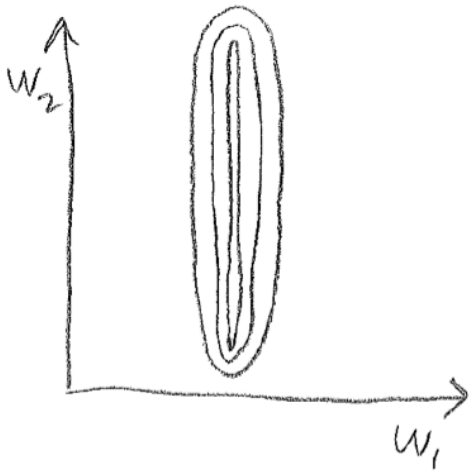



# Data Preprocessing

- Motivation
  - Example of linear regression

| $x_1$ | $x_2$ | $t$ |
|---|---|---|
| 114.8 | 0.00323 | 5.1 |
| 338.1 | 0.00183 | 3.2 |
| 98.8 | 0.00279 | 4.1 |
| ⋮ | ⋮ | ⋮ |

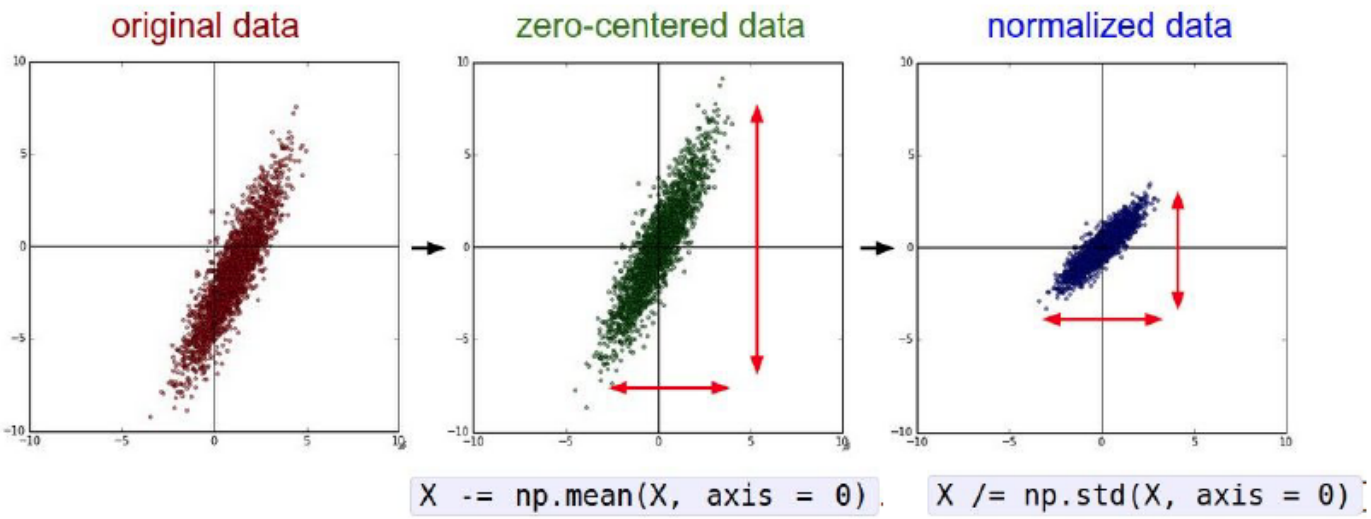
$$\overline{w_i} = \overline{y}\, x_i$$




  - Which direction of weights has a larger gradient updates?
  - Which one do you want to receive a larger update?

# Data Preprocessing

- Data normalization
  - To avoid these problems, center your inputs to zero mean and unit variance

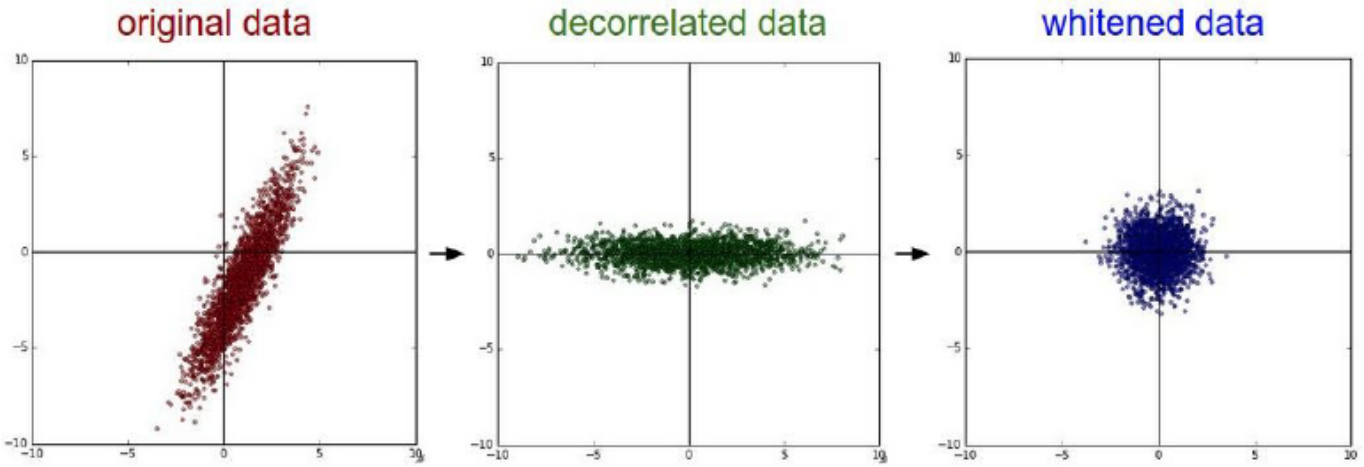



(Assume X [NxD] is data matrix,
each example in a row)

# Data Preprocessing

- More advanced methods

In practice, you may also see **PCA** and **Whitening** of the data



# Data Preprocessing

- For visual recognition tasks
  - In practice for images: centering only
  - Not common to normalize variance, do PCA or whitening

e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet)
  (mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
  (mean along each channel = 3 numbers)

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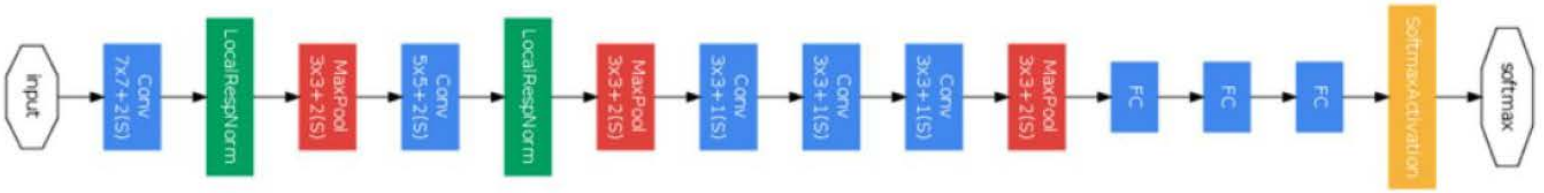
# Batch Normalization

- Problem in deep network learning: Internal Covariance Shift




$$\ell = F_2(F_1(\mathrm{u}, \Theta_1), \Theta_2)$$

  - Change of distribution in activation across layers
  - Considering previous layer output as input to the next layer

# Batch Normalization

- Normalize the inputs to a layer:

"you want unit gaussian activations? just make them so."

consider a batch of activations at some layer.
To make each dimension unit gaussian, apply:

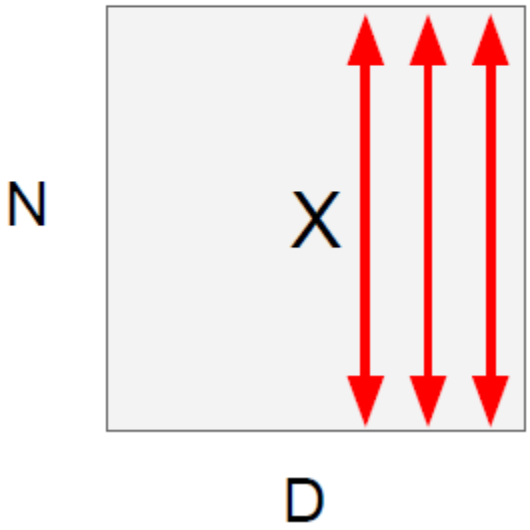
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

# Batch Normalization

- Layer details

"you want unit gaussian activations?
just make them so."



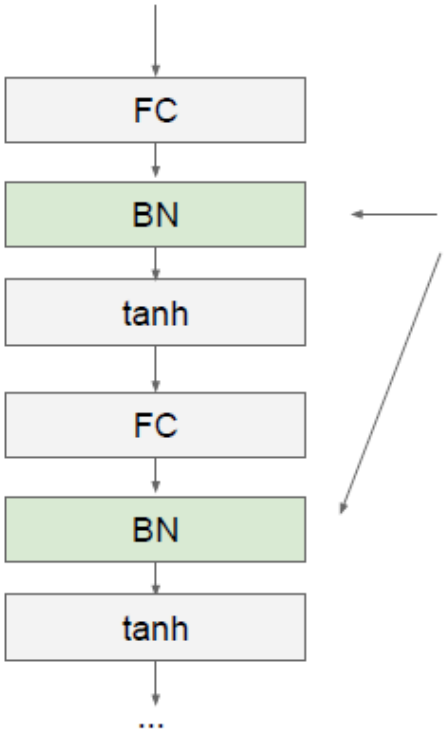

1. compute the empirical mean and variance independently for each dimension.

2. Normalize

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

# Batch Normalization

- Layer details




Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

# Batch Normalization

- Extra capacity:

Normalize:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

And then allow the network to squash the range if it wants to:

$$y^{(k)} = \gamma^{(k)}\widehat{x}^{(k)} + \beta^{(k)}$$

Note, the network can learn:

$$\gamma^{(k)} = \sqrt{\mathrm{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathrm{E}[x^{(k)}]$$

to recover the identity mapping.

# Batch Normalization

- Algorithm

**Input:** Values of $x$ over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: $\gamma$, $\beta$

**Output:** $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe

# Batch Normalization

- Algorithm

**Input:** Values of $x$ over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
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$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

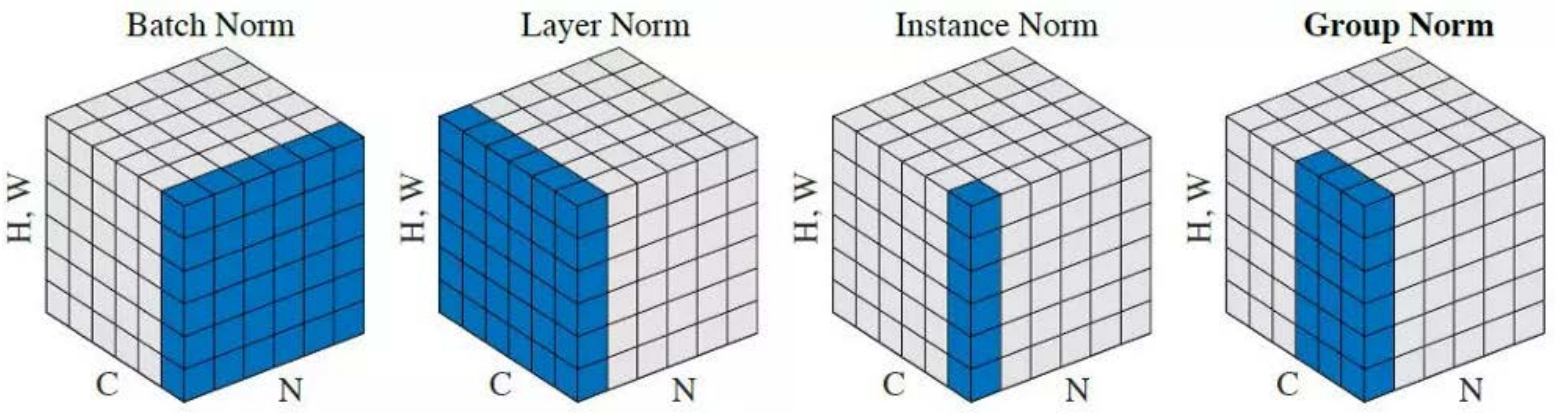
**Note: at test time BatchNorm layer functions differently:**

The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

# Other Batch-like Normalization

- Layer normalization (Ba, Kiros, Hinton, 2016)
- Instance normalization (Ulyanov, Vedaldi, Lempitsky, 2016)
- Group normalization (Wu and He, 2018)



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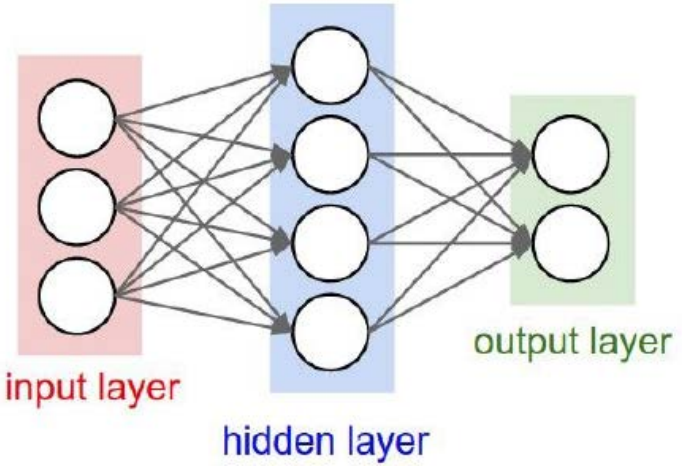
# Weight Initialization

- Non-convex objective functions
  - Neural nets have a weight symmetry: permute all the hidden units in a given layer and obtain an equivalent solution.
  - Q: What happens when W=0 initialization is used?



# Weight Initialization

- First idea: Small random numbers
  - Gaussian with zero mean and 1e-2 std

```
W = 0.01* np.random.randn(D,H)
```

  - Simpler models to start
  - Outputs are close to uniform for classification

Works ~okay for small networks, but problems with deeper networks.

# Weight Initialization

- Motivating example

Lets look at some activation statistics

E.g. 10-layer net with 500 neurons on each layer, using tanh non-linearities, and initializing as described in last slide.

```
# assume some unit gaussian 10-D input data
D = np.random.randn(1000, 500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)
```

```
act = {'relu':lambda x:np.maximum(0,x), 'tanh':lambda x:np.tanh(x)}
Hs = {}
for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1] # input at this layer
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) * 0.01 # layer initialization

    H = np.dot(X, W) # matrix multiply
    H = act[nonlinearities[i]](H) # nonlinearity
    Hs[i] = H # cache result on this layer
```

```
# look at distributions at each layer
print 'input layer had mean %f and std %f' % (np.mean(D), np.std(D))
layer_means = [np.mean(H) for i,H in Hs.iteritems()]
layer_stds = [np.std(H) for i,H in Hs.iteritems()]
for i,H in Hs.iteritems():
    print 'hidden layer %d had mean %f and std %f' % (i+1, layer_means[i], layer_stds[i])

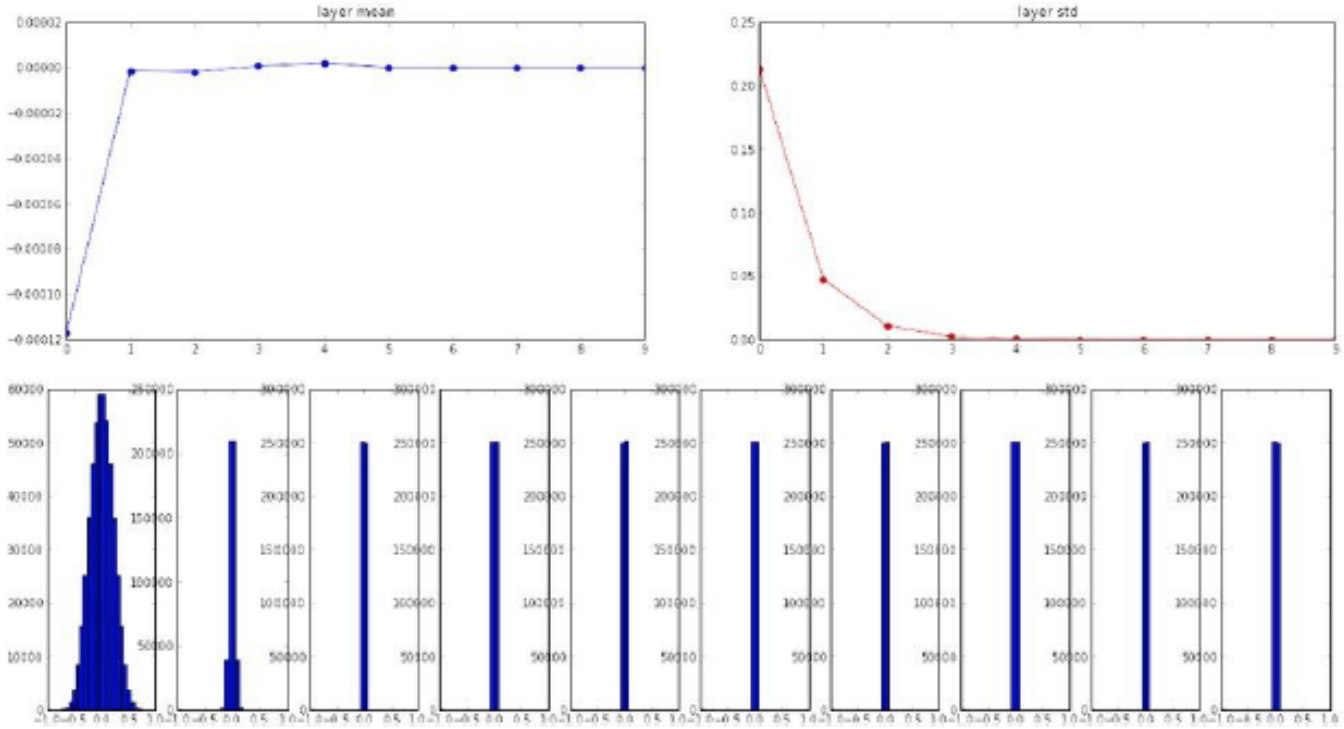
# plot the means and standard deviations
plt.figure()
plt.subplot(121)
plt.plot(Hs.keys(), layer_means, 'ob-')
plt.title('layer mean')
plt.subplot(122)
plt.plot(Hs.keys(), layer_stds, 'or-')
plt.title('layer std')

# plot the raw distributions
plt.figure()
for i,H in Hs.iteritems():
    plt.subplot(1,len(Hs),i+1)
    plt.hist(H.ravel(), 30, range=(-1,1))
```

# Weight Initialization

- Motivating example

```
input layer had mean 0.000927 and std 0.998388
hidden layer 1 had mean -0.000117 and std 0.213081
hidden layer 2 had mean -0.000001 and std 0.047551
hidden layer 3 had mean -0.000002 and std 0.010630
hidden layer 4 had mean 0.000001 and std 0.002378
hidden layer 5 had mean 0.000002 and std 0.000532
hidden layer 6 had mean -0.000000 and std 0.000119
hidden layer 7 had mean 0.000000 and std 0.000026
hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
```



# Weight Initialization

- Motivating example

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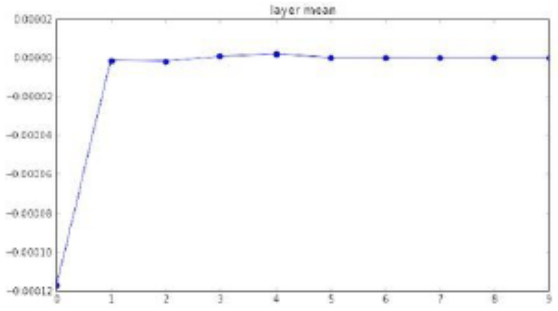
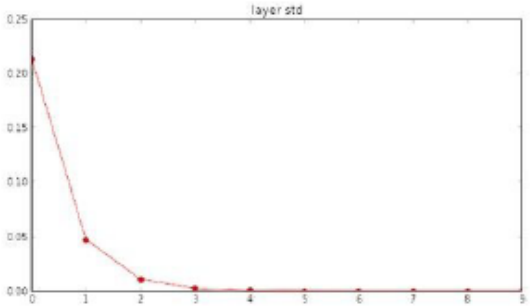
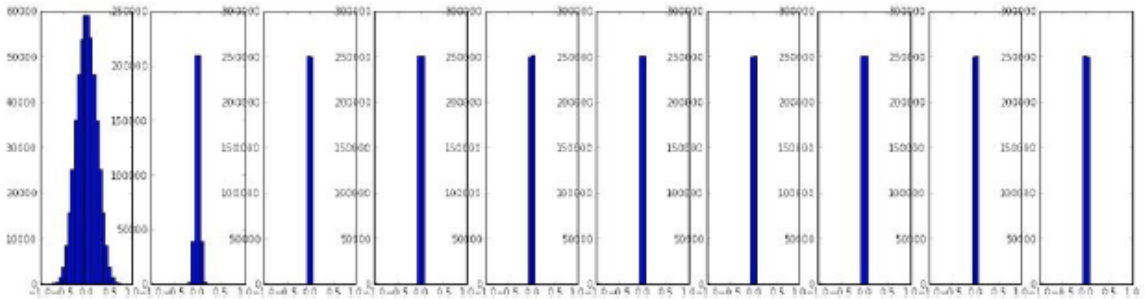
```



All activations become zero!

Q: think about the backward pass. What do the gradients look like?

Hint: think about backward pass for a W*X gate.

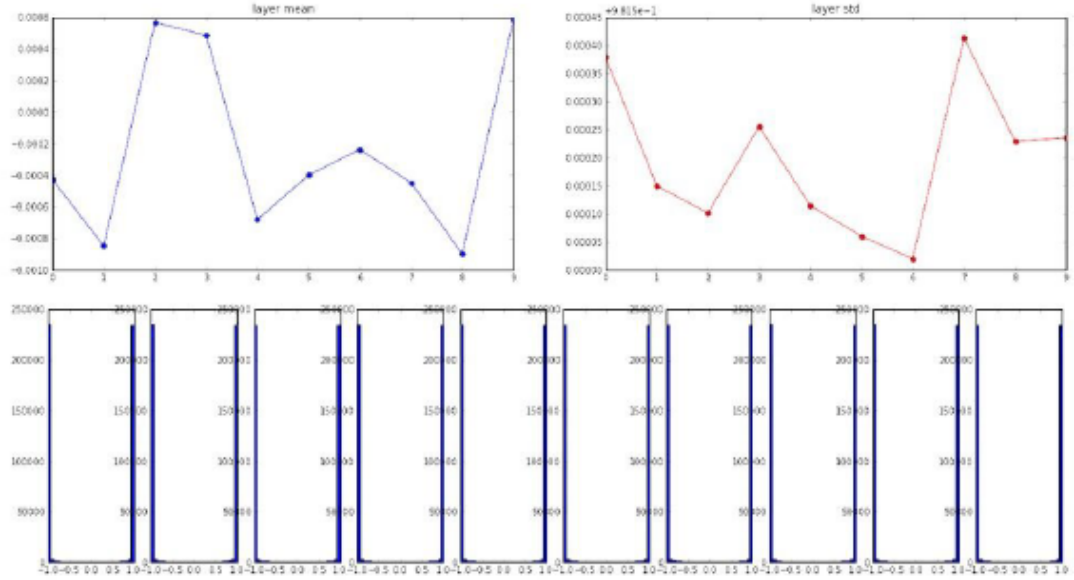
# Weight Initialization

- Motivating example

```
W = np.random.randn(fan_in, fan_out) * 1.0 # layer initialization
```

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean -0.000430 and std 0.981879
hidden layer 2 had mean -0.000849 and std 0.981649
hidden layer 3 had mean 0.000566 and std 0.981601
hidden layer 4 had mean 0.000483 and std 0.981755
hidden layer 5 had mean -0.000682 and std 0.981614
hidden layer 6 had mean -0.000401 and std 0.981560
hidden layer 7 had mean -0.000237 and std 0.981520
hidden layer 8 had mean -0.000448 and std 0.981913
hidden layer 9 had mean -0.000899 and std 0.981728
hidden layer 10 had mean 0.000584 and std 0.981736
```

*1.0 instead of *0.01




Almost all neurons completely saturated, either -1 and 1. Gradients will be all zero.

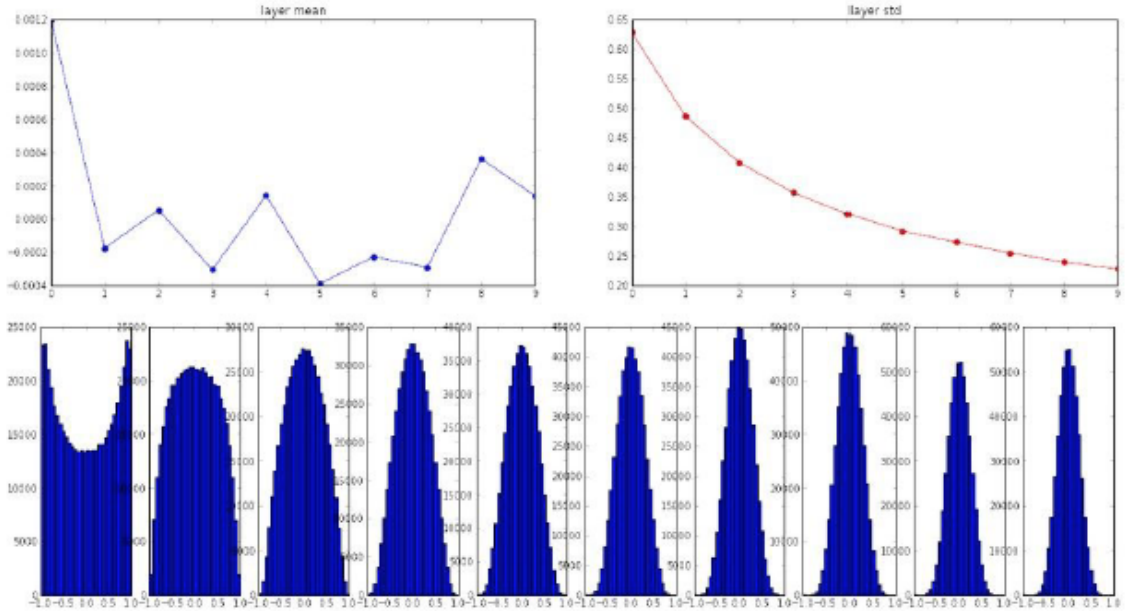
# Weight Initialization

- Xavier initialization

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean 0.001198 and std 0.627953
hidden layer 2 had mean -0.000175 and std 0.486051
hidden layer 3 had mean 0.000055 and std 0.407723
hidden layer 4 had mean -0.000306 and std 0.357108
hidden layer 5 had mean 0.000142 and std 0.320917
hidden layer 6 had mean -0.000389 and std 0.292116
hidden layer 7 had mean -0.000228 and std 0.273387
hidden layer 8 had mean -0.000291 and std 0.254935
hidden layer 9 had mean 0.000361 and std 0.239266
hidden layer 10 had mean 0.000139 and std 0.228008
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

"Xavier initialization"
[Glorot et al., 2010]




**Reasonable initialization.**
(Mathematical derivation assumes linear activations)

# Weight Initialization

- Theoretic analysis

Suppose we have an input $X$ with $n$ components and a fully connected layer (also denoted linear or dense) with random weights $W$ that outputs a number $Y$ such that

$$Y = W_1 X_1 + W_2 X_2 + \ldots + W_n X_n$$

To make sure that the weights remain in a reasonable range, we expect that $Var(Y) = Var(X_i)_{i \in [1,n]}$

We also know how to compute the variance of the product of two random variables. Therefore

$$Var(W_i X_i) = E[X_i]^2 Var(W_i) + E[W_i]^2 Var(X_i) + Var(W_i) Var(X_i)$$

Both our inputs and weights have a mean 0. It simplifies to

$$Var(W_i X_i) = Var(W_i) Var(X_i)$$

Now we make a further assumption that the $X_i$ and $W_i$ are all independent and identically distributed (iid).

$$Var(Y) = Var(W_1 X_1 + W_2 X_2 + \ldots + W_n X_n) = n Var(W_i) Var(X_i)$$

It turns that, if we want to have $Var(Y) = Var(X_i)$, we must enforce the condition $n Var(W_i) = 1$.

$$Var(W_i) = \frac{1}{n} = \frac{1}{n_{in}}$$

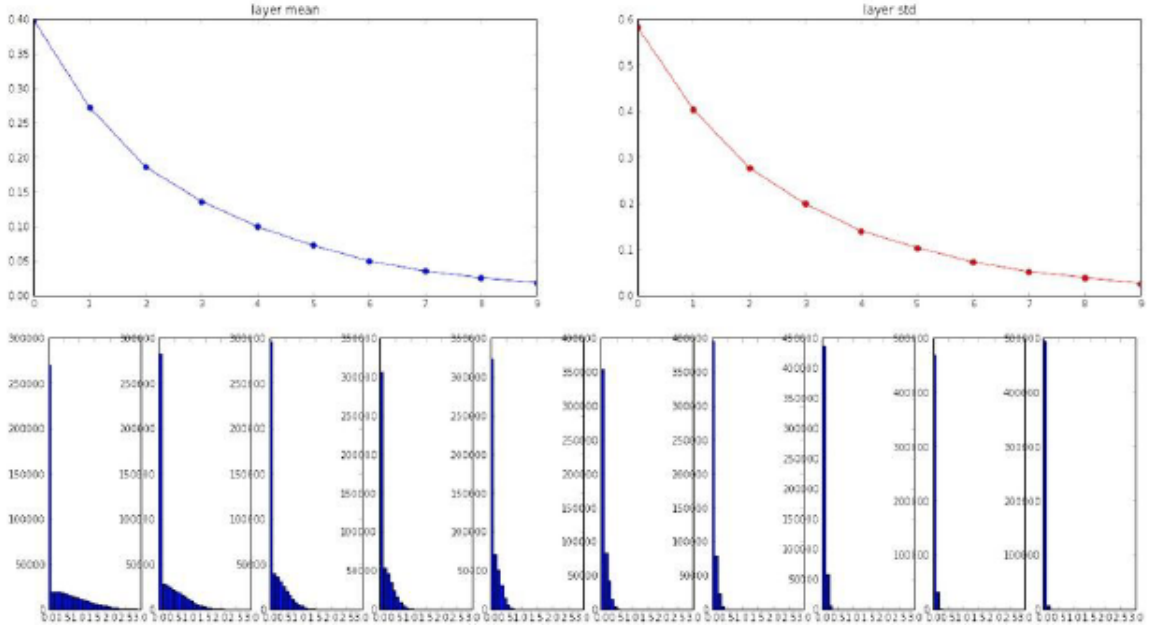
# Weight Initialization

- Problems with ReLU activation

```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.398623 and std 0.582273
hidden layer 2 had mean 0.272352 and std 0.403795
hidden layer 3 had mean 0.186076 and std 0.276912
hidden layer 4 had mean 0.136442 and std 0.198685
hidden layer 5 had mean 0.099568 and std 0.140299
hidden layer 6 had mean 0.072234 and std 0.103280
hidden layer 7 had mean 0.049775 and std 0.072748
hidden layer 8 had mean 0.035138 and std 0.051572
hidden layer 9 had mean 0.025404 and std 0.038583
hidden layer 10 had mean 0.018408 and std 0.026076
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

but when using the ReLU nonlinearity it breaks.



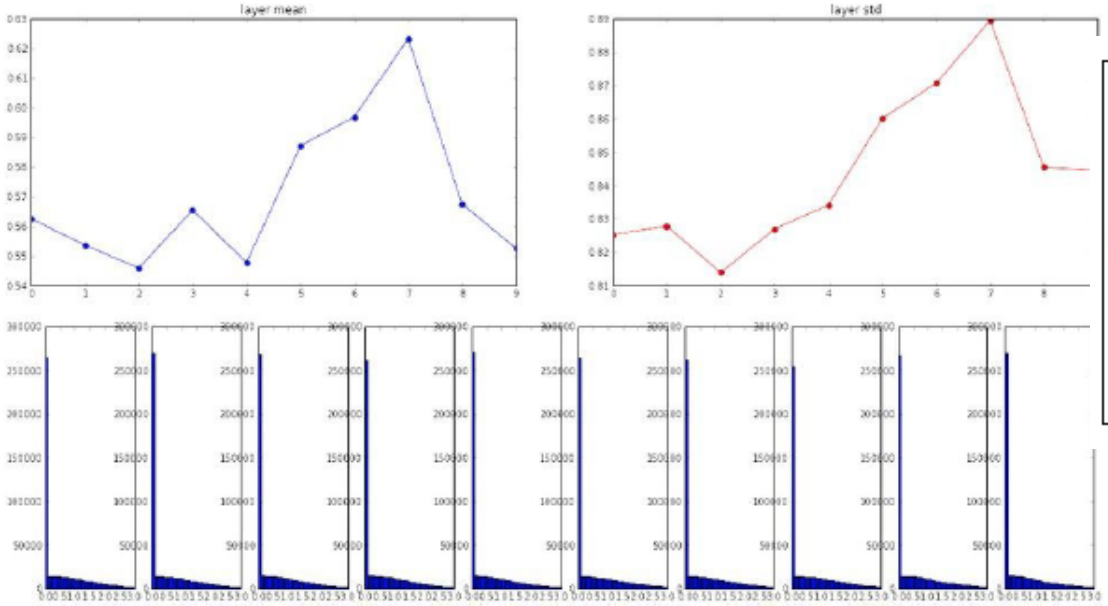
# Weight Initialization

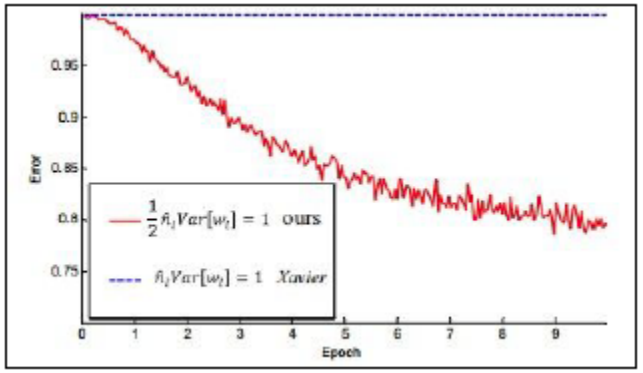
- Initialization for CNNs

```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.562488 and std 0.825232
hidden layer 2 had mean 0.553614 and std 0.827835
hidden layer 3 had mean 0.545867 and std 0.813855
hidden layer 4 had mean 0.565396 and std 0.826902
hidden layer 5 had mean 0.547678 and std 0.834092
hidden layer 6 had mean 0.587103 and std 0.860035
hidden layer 7 had mean 0.596867 and std 0.870610
hidden layer 8 had mean 0.623214 and std 0.889348
hidden layer 9 had mean 0.567498 and std 0.845357
hidden layer 10 had mean 0.552531 and std 0.844523
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015
(note additional /2)



# Weight Initialization

- Weight initialization is an active are of research…

***Understanding the difficulty of training deep feedforward neural networks*** by Glorot and Bengio, 2010

***Exact solutions to the nonlinear dynamics of learning in deep linear neural networks*** by Saxe et al, 2013

***Random walk initialization for training very deep feedforward networks*** by Sussillo and Abbott, 2014

***Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification*** by He et al., 2015

***Data-dependent Initializations of Convolutional Neural Networks*** by Krähenbühl et al., 2015

***All you need is a good init***, Mishkin and Matas, 2015

…

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  - Data preprocessing
  - Batch normalization
  - Weight initialization
  - First-order update

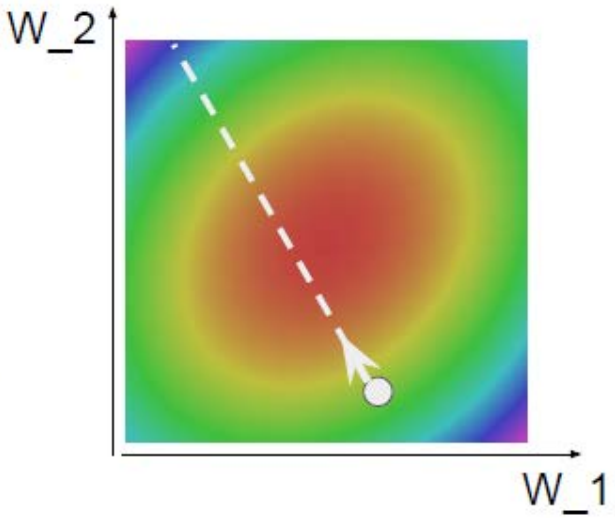
*Acknowledgement: UofT, CMU & Feifei Li's cs231n notes*

# Optimization

- Stochastic Gradient Descent

```
# Vanilla Gradient Descent

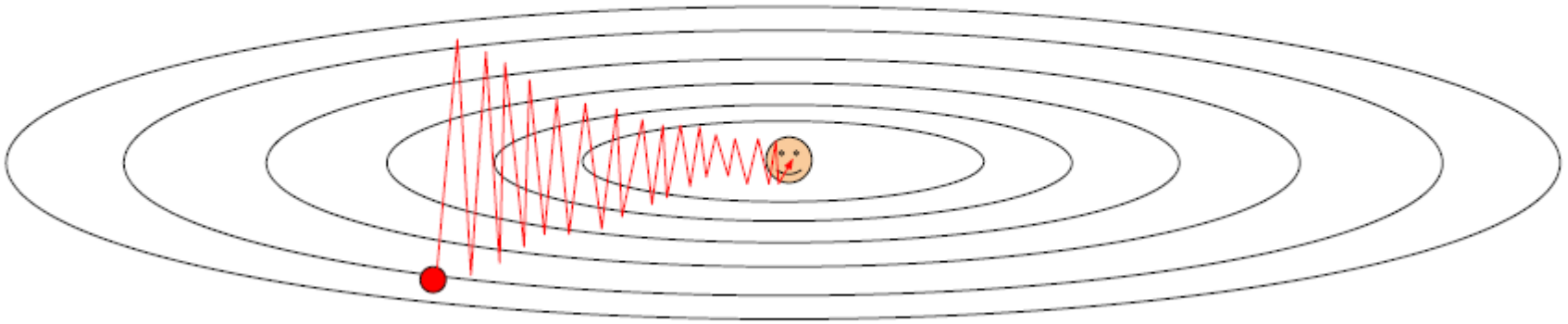
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```



# Optimization

- Problems with SGD

What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?
Very slow progress along shallow dimension, jitter along steep direction



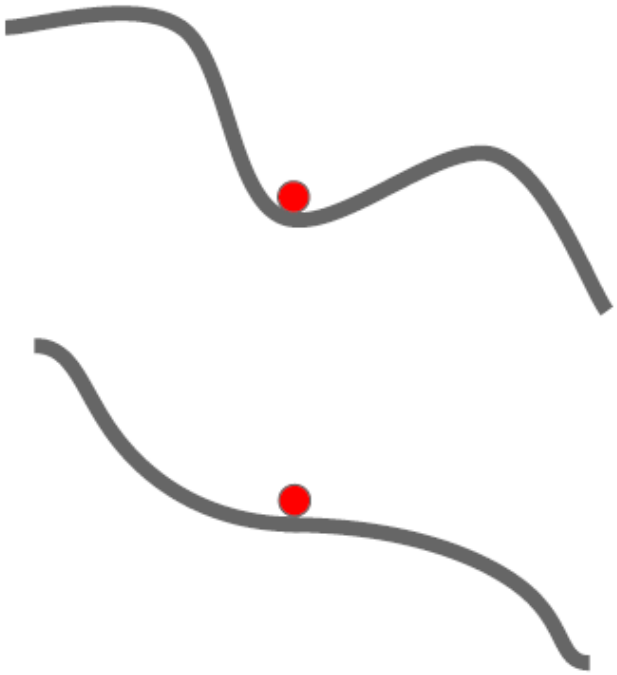
Loss function has high **condition number**: ratio of largest to smallest singular value of the Hessian matrix is large

# Optimization

- Problems with SGD

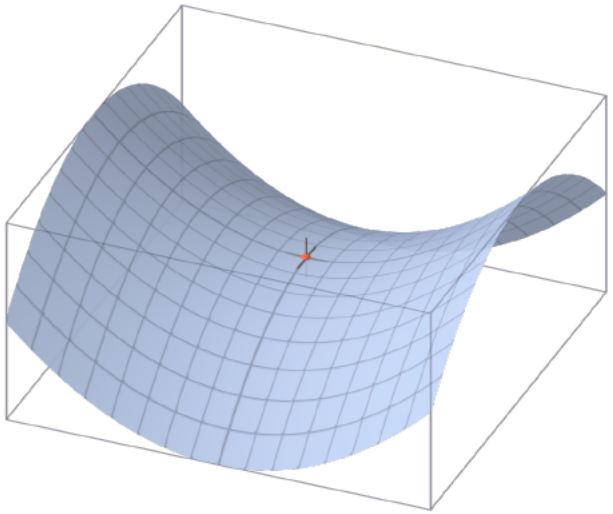
What if the loss function has a **local minima** or **saddle point**?

Zero gradient, gradient descent gets stuck



# Optimization

- Problems with SGD
  - Saddle points are more common in high-dim space



At a saddle point $\frac{\partial \mathcal{E}}{\partial \boldsymbol{\theta}} = 0$, even though we are not at a minimum. Some directions curve upwards, and others curve downwards.

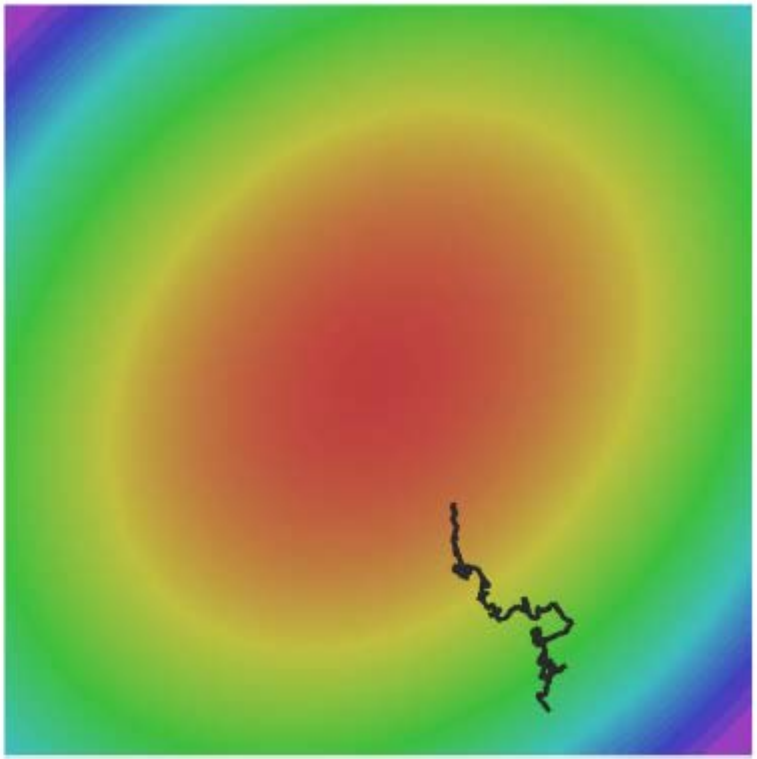
# Optimization

- Problems with SGD

Our gradients come from minibatches so they can be noisy!

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$



# Optimization

- SGD + Momentum

## SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

```
while True:
  dx = compute_gradient(x)
  x += learning_rate * dx
```

## SGD+Momentum

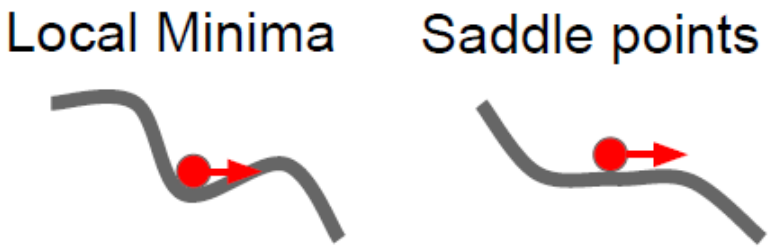
$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

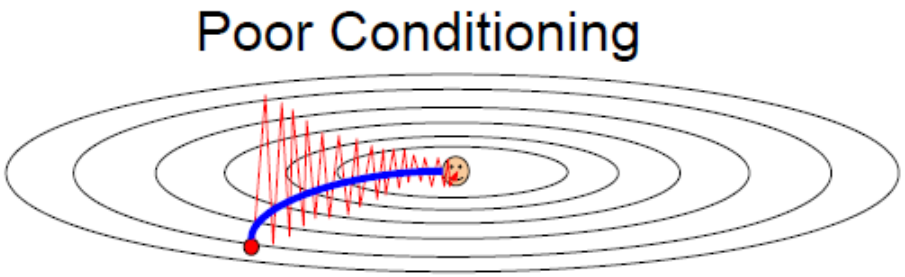
```
vx = 0
while True:
  dx = compute_gradient(x)
  vx = rho * vx + dx
  x += learning_rate * vx
```

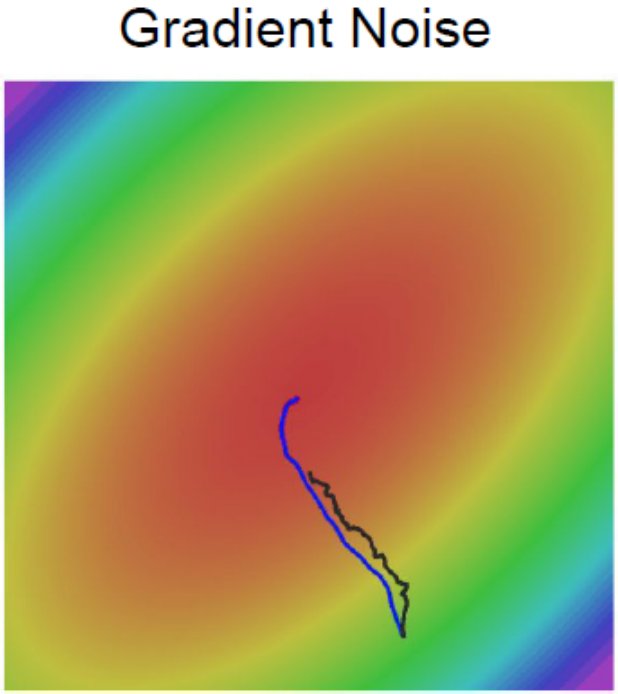
- Build up “velocity” as a running mean of gradients
- Rho gives “friction”; typically rho=0.9 or 0.99

# Optimization

- SGD + Momentum
  - Momentum sometimes helps a lot, and almost never hurts

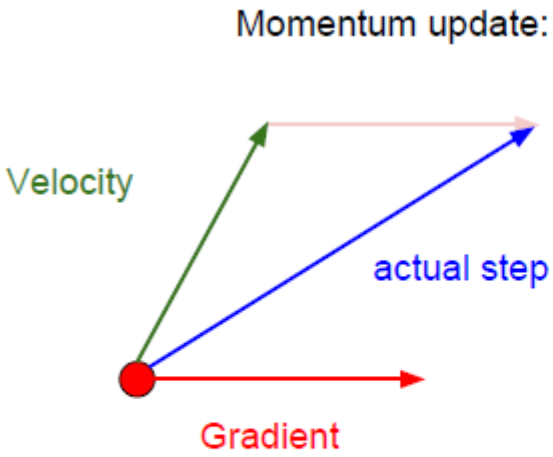


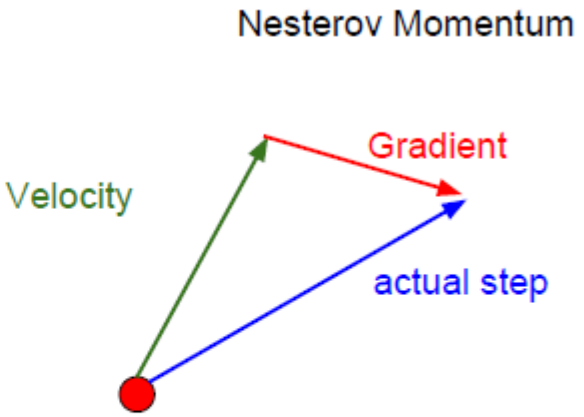



# Optimization

- Nesterov Momentum




Nesterov, "A method of solving a convex programming problem with convergence rate O(1/k^2)", 1983
Nesterov, "Introductory lectures on convex optimization: a basic course", 2004
Sutskever et al, "On the importance of initialization and momentum in deel learning", ICML 2013

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

# Optimization

- Nesterov Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$

# Optimization

- Nesterov Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$

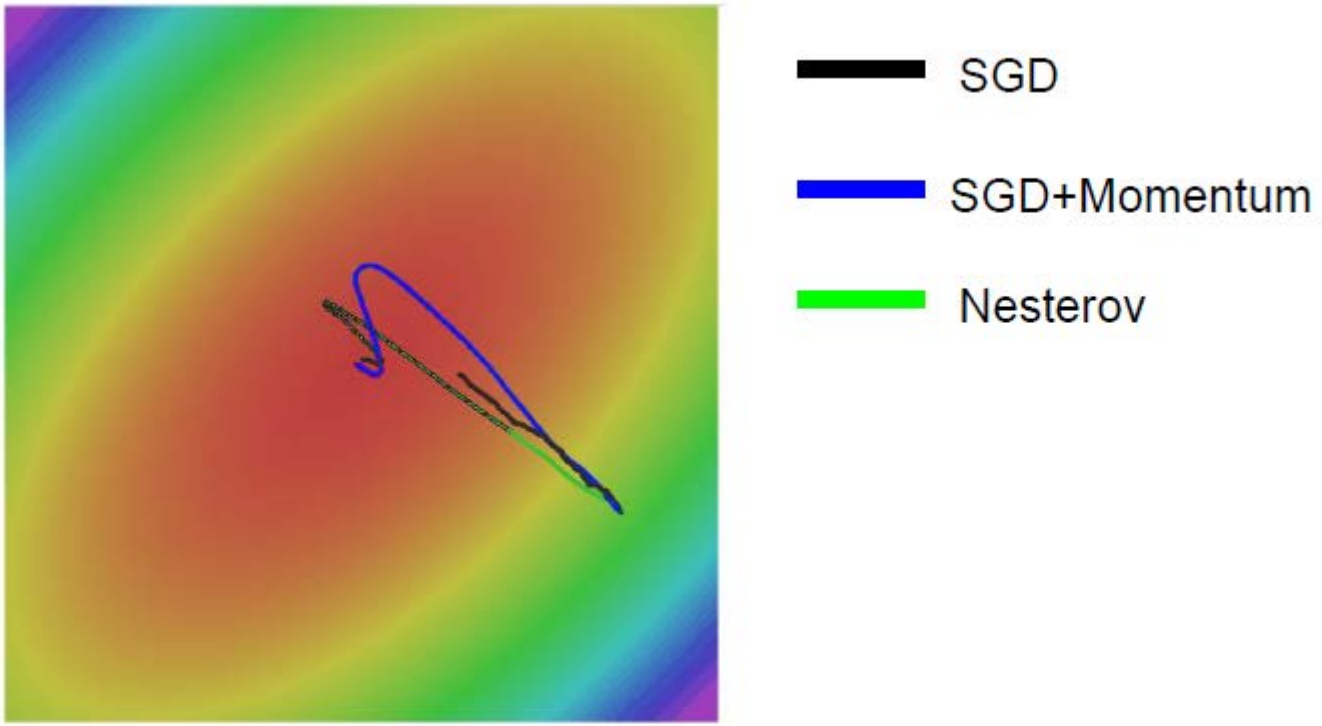
Change of variables $\tilde{x}_t = x_t + \rho v_t$ and rearrange:

$$v_{t+1} = \rho v_t - \alpha \nabla f(\tilde{x}_t)$$
$$\tilde{x}_{t+1} = \tilde{x}_t - \rho v_t + (1+\rho) v_{t+1}$$
$$= \tilde{x}_t + v_{t+1} + \rho(v_{t+1} - v_t)$$

```
dx = compute_gradient(x)
old_v = v
v = rho * v - learning_rate * dx
x += -rho * old_v + (1 + rho) * v
```

# Optimization

- Nesterov Momentum



# Optimization

- AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

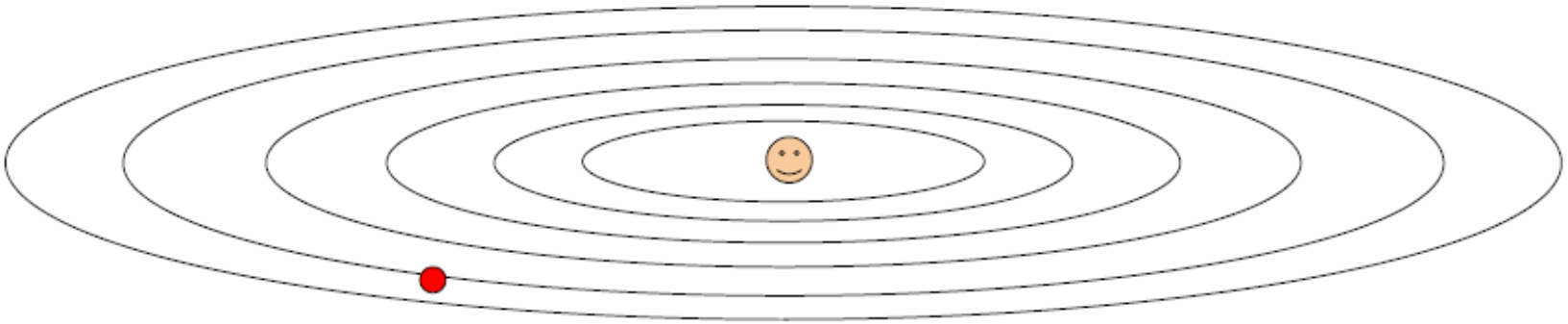
Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

Duchi et al, "Adaptive subgradient methods for online learning and stochastic optimization", JMLR 2011

# Optimization

- AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

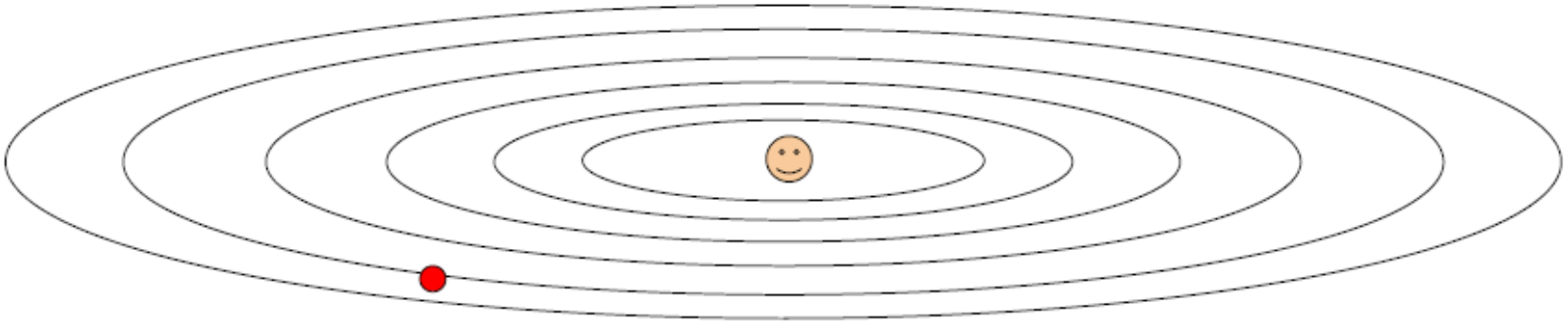


Q: What happens with AdaGrad?

# Optimization

- AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```



Q2: What happens to the step size over long time?

# Optimization

- RMSProp

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

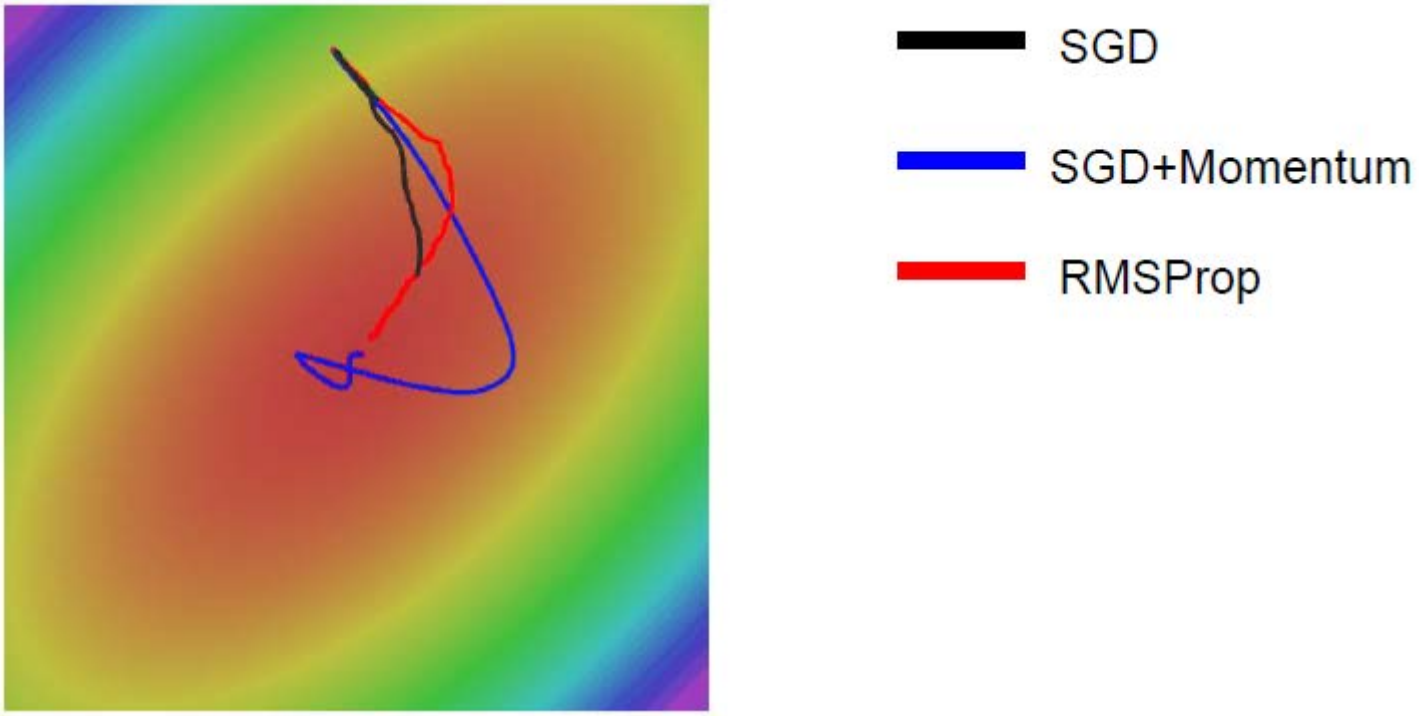
RMSProp

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Tieleman and Hinton, 2012

# Optimization

- RMSProp



# Optimization

- Adam (almost)

```
first_moment = 0
second_moment = 0
while True:
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  x -= learning_rate * first_moment / (np.sqrt(second_moment) + 1e-7))
```

Kingma and Ba, "Adam: A method for stochastic optimization", ICLR 2015

# Optimization

- Adam (almost)

```
first_moment = 0
second_moment = 0
while True:
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  x -= learning_rate * first_moment / (np.sqrt(second_moment) + 1e-7))
```

Momentum

AdaGrad / RMSProp

Sort of like RMSProp with momentum

Q: What happens at first timestep?

# Optimization

- Adam (full form)

```
first_moment = 0
second_moment = 0
for t in range(num_iterations):
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  first_unbias = first_moment / (1 - beta1 ** t)
  second_unbias = second_moment / (1 - beta2 ** t)
  x -= learning_rate * first_unbias / (np.sqrt(second_unbias) + 1e-7))
```

Momentum

Bias correction

AdaGrad / RMSProp

Bias correction for the fact that
first and second moment
estimates start at zero

# Optimization

- Adam (full form)

```
first_moment = 0
second_moment = 0
for t in range(1, num_iterations):
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  first_unbias = first_moment / (1 - beta1 ** t)
  second_unbias = second_moment / (1 - beta2 ** t)
  x -= learning_rate * first_unbias / (np.sqrt(second_unbias) + 1e-7))
```

Momentum

Bias correction

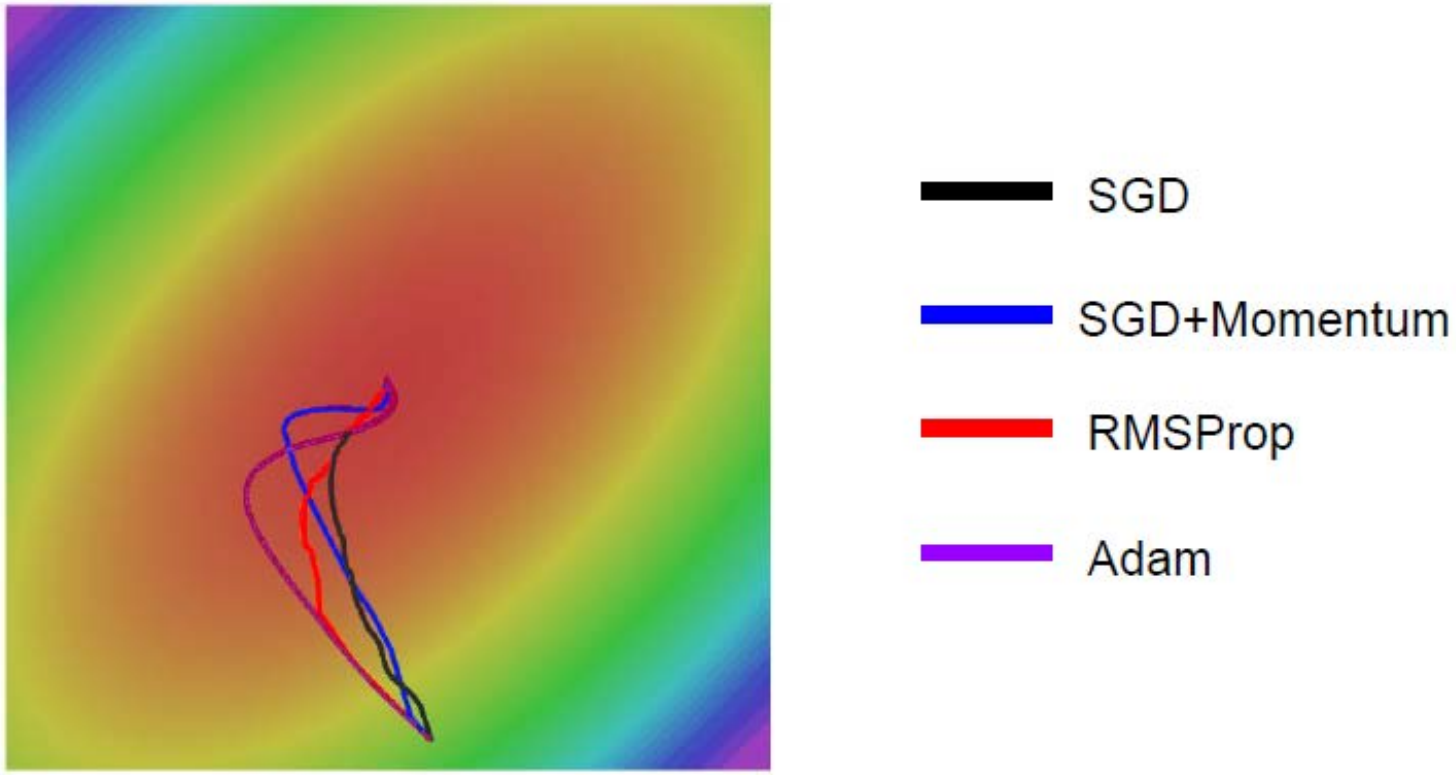
AdaGrad / RMSProp

Bias correction for the fact that first and second moment estimates start at zero

Adam with beta1 = 0.9, beta2 = 0.999, and learning_rate = 1e-3 or 5e-4 is a great starting point for many models!

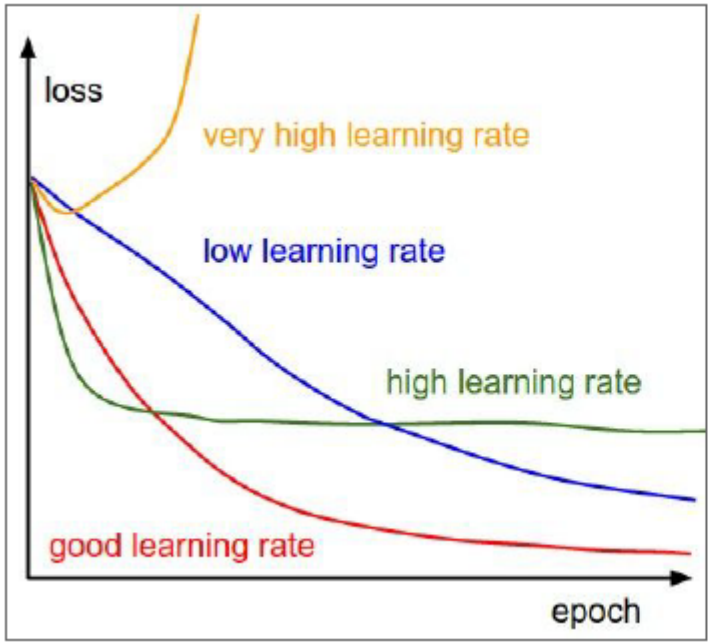
# Optimization

- Adam (full form)



# Learning rate

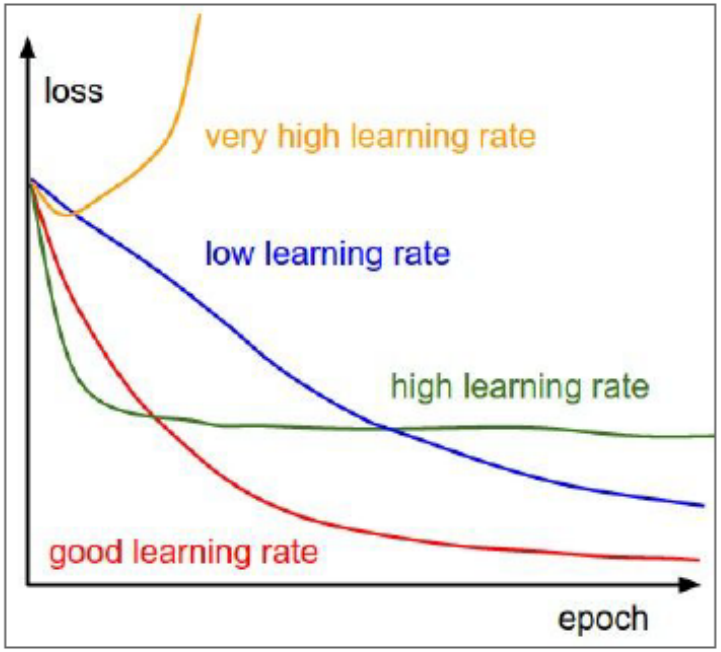
- SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter




Q: Which one of these learning rates is best to use?

# Learning rate

- SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter




**=> Learning rate decay over time!**

**step decay:**
e.g. decay learning rate by half every few epochs.

**exponential decay:**

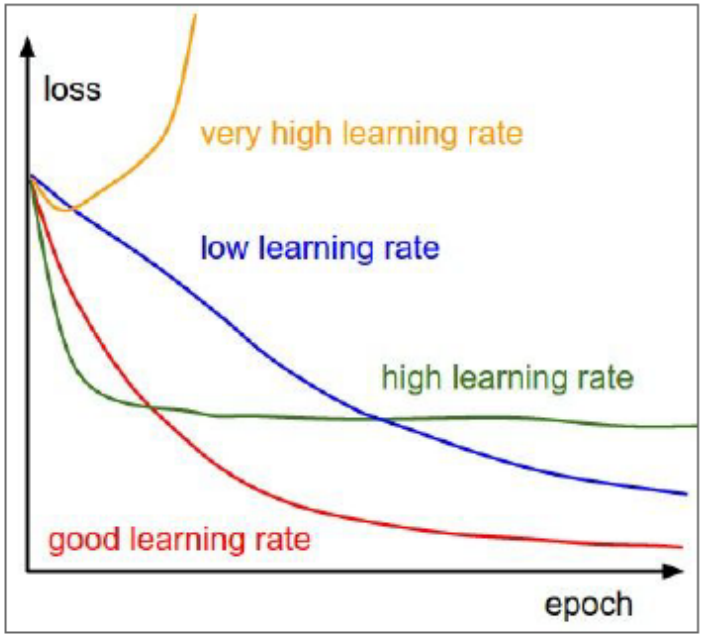
$$\alpha = \alpha_0 e^{-kt}$$

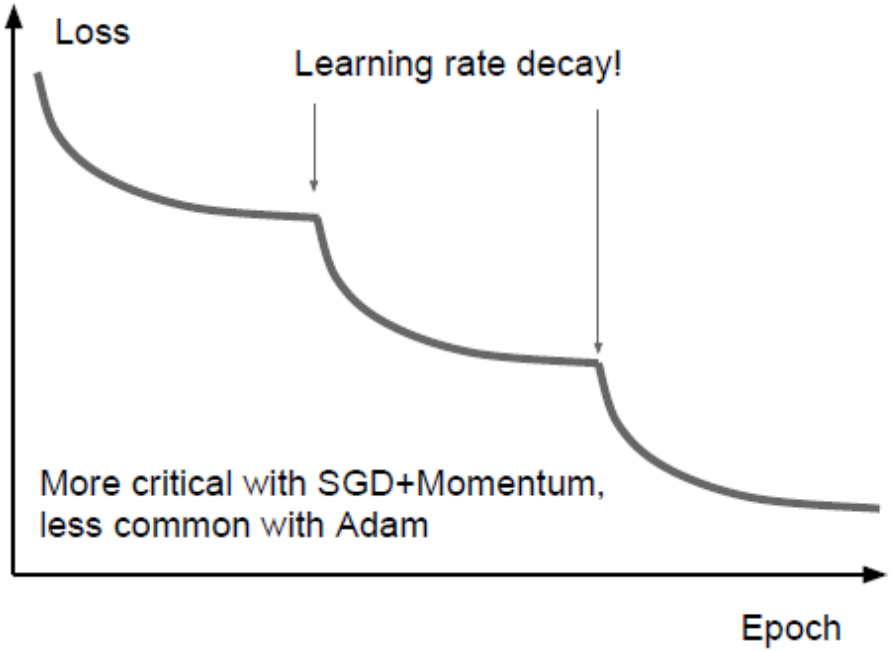
**1/t decay:**

$$\alpha = \alpha_0 / (1 + kt)$$

# Learning rate

- SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter



# What can we find

- Popular hypothesis
  - In large networks, saddle points are far more common than local minima
  - Gradient descent algorithms often get “stuck” in saddle points
  - Most local minima are equivalent and close to global minimum

# What can we find

- Controversial loss surface
  - **Baldi and Hornik (89),** "*Neural Networks and Principal Component Analysis: Learning from Examples Without Local Minima*" : An MLP with a *single* hidden layer has only saddle points and no local Minima
  - **Dauphin et. al (2015),** "*Identifying and attacking the saddle point problem in high-dimensional non-convex optimization*" : An exponential number of saddle points in large networks
  - **Chomoranksa et. al (2015)**, "*The loss surface of multilayer networks*" : For large networks, most local minima lie in a band and are equivalent
    - Based on analysis of spin glass models
  - **Swirscz et. al. (2016)**, "Local minima in training of deep networks", In networks of finite size, trained on finite data, you *can* have horrible local minima

# Summary

- CNN training as optimization task
  - Non-convex and local minimal
  - Overcoming ravines in loss surfaces
- Next time …
  - Regularization to avoid overfitting
- Quiz 3