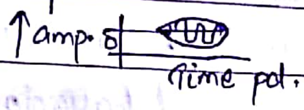
Quantum Physics

Dual Nature of light

It shows wave nature as well as particle nature.

wave : It has amplitude 'a', freq. ν; momentum 'p' energy $(E = h\nu)$



It also show particle nature, It has a particle of mass 'm', velocity 'v', momentum $(p = mv)$ & energy $(E = \frac{1}{2}mv^2)$

de-Broglie Hypothesis

From wave theory, $E = h\nu$ —(i) .... Plank's law

$E = mc^2$ —(ii) .... Eienstein Eqn

$h\nu = mc^2 \Rightarrow c = \nu\lambda, \; \nu = c/\lambda \Rightarrow \frac{hc}{\lambda} = mc^2$

$h/\lambda = mc$ or $\lambda = \frac{h}{mc}$

$$\lambda = h/p$$

Matter Waves: A wave associated with material particle or matter eg. e^-, proton, neutron called matter wave.

de-Broglie wavelength associated with electron.

energy of electron, $E = eV$

$\lambda = h/\sqrt{2mE}$ .... $p = \sqrt{2mE} \Rightarrow \frac{h}{\sqrt{2meV}} \Rightarrow \lambda = \frac{12.27}{\sqrt{V}} A^0$

$$\Rightarrow \lambda = \frac{12.27}{\sqrt{V}} A^0$$

$m = 9.1 \times 10^{-31}$ Kg, $e = 1.6 \times 10^{-19}$ C, $h = 6.626 \times 10^{-34}$ J sec

→ Davisson & Germer Experiment

It is used to proof validity of matter waves. It shows diffraction pattern. It consist of

(i) an electron gun

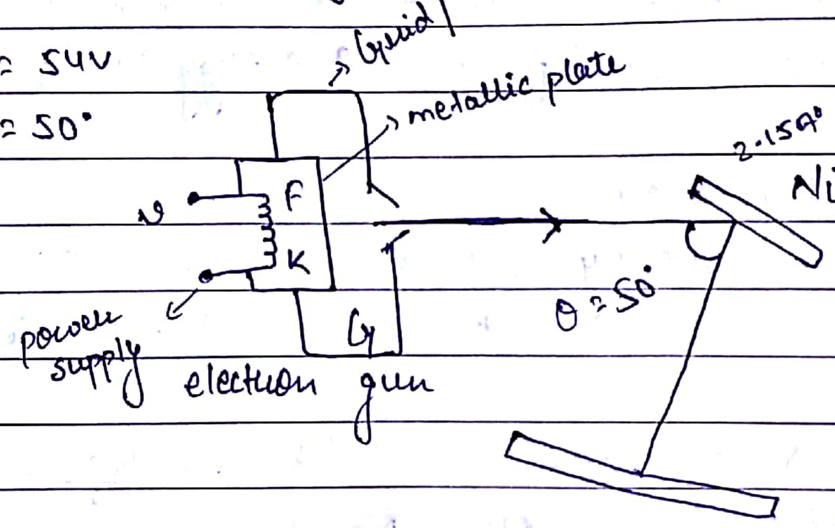
(ii) Nickel crystal as a slit.

(iii) Vernier circular scale with photographic plate (Faraday's cylinder).

bez its ~~inter nucleus~~ distance is equal to 2.15Å.

V = 54V

θ = 50°



Q: If $v = 1kV$ cal. de-Broglie wavelength for an e^-, proton & α particle.

<u>Electron,</u> $\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1000}}$

$= \frac{6.6 \times 10^{-34}}{\sqrt{29.12 \times 10^{-50} \times 1000}} = \frac{6.6 \times 10^{-34}}{\sqrt{291 \times 10^{-48}}}$

$= \frac{6.6 \times 10^{-34}}{17 \times 10^{-24}} = \frac{6.6 \times 10^{-10}}{17}$

$\Rightarrow$ $\underline{\lambda = 3.8 \times 10^{-9} m}$

Proton, $m = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times 1000}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{5.344 \times 10^{-43}}} = \frac{6.6 \times 10^{-34}}{7.28 \times 10^{-22}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 10^{22} \times 100}{7.28 \times 10} = 0.906 \times 10^{-11}$$

$$\Rightarrow \underline{\lambda = 9.06 \times 10^{-12}\ m}$$

α-particle, $m = 6.6 \times 10^{-27}$ kg (4 times of mass of proton)

$e = 3.2 \times 10^{-19}$ C

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 6.6 \times 10^{-27} \times 1.6 \times 10^{-19} \times 2 \times 1000}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{10^{-46} \times 1000 \times 13.2 \times 3.2}} = \frac{6.6 \times 10^{-34}}{10^{-23} \times \sqrt{132 \times 32 \times 10}} \Rightarrow$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-11}}{205.5} = 0.03219 \times 10^{-11} \Rightarrow \underline{3.21 \times 10^{-13}\ m}$$

Q. If K.E of neutron is 1000 eV then cal. de-Broglie wavelength of neutron.

$$\lambda = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 1000 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 10^{22}}{\sqrt{5.12 \times 10}} = \frac{6.6 \times 10^{-12}}{7.2}$$

$$\Rightarrow \lambda = 9.2 \times 10^{-11}\ m.$$

working : In this experiment, electrons emerge out of a hole in the form of a fine beam which is then made to fall on a Nickel crystal. Electrons are scattered in all dir^n by the atom of the crystal. The intensity of the e^-s scattered in a particular dir^n is found by detector. By rotating the detector about an axis the intensity of scattered beam can be measured ~~at~~ for different angles.

For max^m amplitude or ~~pattern~~ diffraction pattern value of V is $= 54V$ & $\theta = 50°$

$V = 54V$
$\theta = 50°$

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \lambda = \frac{12.27}{\sqrt{V}} A^0$$

$$\Rightarrow \lambda = \frac{12.27}{\sqrt{54}} A^0 = 1.67 A^0 \text{ or } 0.167 nm$$

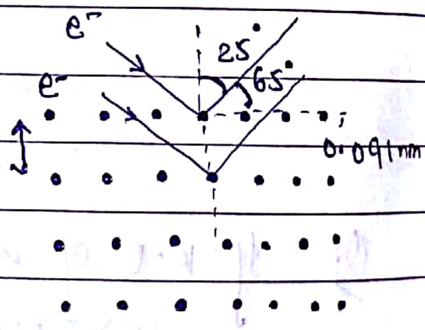
From Bragg's equation

$2d\sin\theta = n\lambda$

$\Rightarrow 2(2.15\sin 25°)\sin 65° = 1\lambda$

$\Rightarrow \lambda = 1.66 A^0$

$65° = \theta =$ glancing angle



→ Properties of Matter wave :-

(i) The particle should have mass

(ii) Particle should be moving.

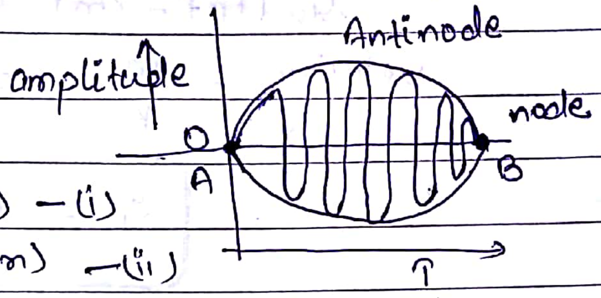
(iii) Matter waves are not Electro Magnetic (EM) waves, because λ is independent of charge.

(iv) Velocity of matter wave is greater than velocity of light.

Group, wave (Phase), Particle velocities

Let ~~we~~ us consider a packet of waves, ~~the waves~~ having diff. angular freq. & propagation constants ω_1, ω_2 & K_1, K_2. The resultant amplitude of the wave is 'A'

The displacement eq^n for the waves are:-



(i) $y_1 = A\sin(\omega_1 t - K_1 x)$ — (i)

$y_2 = A\sin(\omega_2 t - K_2 x)$ — (ii)

the resultant of the two wave

$\Rightarrow \quad y = y_1 + y_2$

$= A\sin(\omega_1 t - K_1 x) + A\sin(\omega_2 t - K_2 x)$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$y = 2A\left[\sin\left(\frac{\omega_1+\omega_2}{2}t - x\frac{(K_1+K_2)}{2}\right)\cos\left(\frac{\omega_1-\omega_2}{2}t - \frac{(K_1-K_2)}{2}x\right)\right]$$

$\frac{\omega_1+\omega_2}{2} \approx \omega$ (Assume), $\frac{K_1+K_2}{2} \approx K$

and

$\frac{\omega_1-\omega_2}{2} = \frac{d\omega}{2}$, $\frac{K_1-K_2}{2} = \frac{dK}{2}$

$$y = 2A\sin(\omega t - Kx)\cos\left(\frac{d\omega\, t}{2} - \frac{dK x}{2}\right) \quad \text{— (iii)}$$

$$y = A\sin(\omega t - Kx) \quad \text{— (iv)} \quad \text{(General Eq}^n\text{)}$$

Standard dis
comparing eq^n (iii) & (iv)

$$\boxed{y = \underbrace{\left\{2A\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right)\right\}}_{\text{Amplitude}} \sin(\omega t - kx)}$$

from, sine term of eq^n (v)

$\sin(\omega t - kx)$

velocity $= \frac{\omega}{k}$ $\Rightarrow$ wave or phase vel. $= \frac{\omega}{k}$

$$\Rightarrow \boxed{v_p = \frac{\omega}{k}} \longrightarrow (vi)$$

from cosine ~~curve~~ term of (v)

$\cos\left(\frac{d\omega}{2}t - \frac{dkx}{2}\right)$

group velocity $= \frac{d\omega/2}{dk/2}$ $\Rightarrow$ $\boxed{v_g = \frac{d\omega}{dk}}$ — (vii)

Particle Velocity (v) :-

Let us consider a particle of mass 'm', velocity 'v' & K.E.

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{2E}{m} \Rightarrow v = \sqrt{\frac{2E}{m}}$$

$E = h\nu$

or $\boxed{v = \sqrt{\frac{2h\nu}{m}}}$ — (viii)

→ **Relation in b/w Group velocity & Particle velocity**

By the definition of group velocity

$$v_g = \frac{d\omega}{dk} \quad \text{— (i)}$$

where, $\omega = 2\pi\nu$ & $k = \frac{2\pi}{\lambda}$

$$\Rightarrow v_g = \frac{d(2\pi\nu)}{d\left(\frac{2\pi}{\lambda}\right)} \Rightarrow \frac{2\pi}{2\pi}\frac{d(\nu)}{d(1/\lambda)} \Rightarrow \frac{d\nu}{d(1/\lambda)}$$

$$\Rightarrow \frac{1}{v_g} = \frac{d(1/\lambda)}{d\nu} \quad \text{— (ii)}$$

from de-broglie hypothesis

$$\lambda = \frac{h}{p} \quad \text{or} \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mh\nu}}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{(2mh\nu)^{1/2}}{h}$$

$$\frac{1}{v_g} = \frac{d}{d\nu}\frac{1}{h}(2mh\nu)^{1/2}$$

$$= \frac{1}{h} \times \frac{1}{2}(2mh\nu)^{-1/2} \times 2mh$$

$$\Rightarrow \frac{1}{v_g} = \frac{m}{\sqrt{2mh\nu}}$$

from $v = \sqrt{\frac{2h\nu}{m}}$

$v_p > c$

Relation in terms of relavistic / non relavistic

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$$\frac{1}{v_g} = \frac{m}{\sqrt{2mh\nu}} = \sqrt{\frac{m^2}{2mh\nu}}$$

$$\Rightarrow \frac{1}{v_g} = \frac{1}{v} \Rightarrow \boxed{v_g = v}$$

Group velocity of wave packets is equal to velocity of particle

→ Relation in b/w wave of phase velocity and particle velocity.

$$\text{wave (phase) velocity} = \frac{\omega}{k}$$

$$\omega = 2\pi\nu \quad , \quad k = \frac{2\pi}{\lambda}$$

$$v_p = \nu\lambda \qquad \cancel{\Rightarrow v_p = \nu\lambda}$$

$$E = h\nu \quad , \quad E = \frac{1}{2}mv^2 \quad , \quad \lambda = \frac{h}{p}$$

$$\nu = \frac{E}{h}$$

$$\therefore v_p = \frac{E}{h}\lambda = \frac{mv^2}{2h}\lambda = \frac{mv^2}{2h} \times \frac{h}{p}$$

$$\Rightarrow v_p = \frac{mv^2}{2p} = \frac{mv^2}{2 \times mv} = \frac{v}{2}$$

$$\Rightarrow \boxed{v_p = \frac{v}{2}}$$

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velocity of electron = nearly about speed of light

$v_e = 0.9c$

→ Prove that velocity of matter wave or wave is greater than speed of light.

$$v_p = \frac{\omega}{K} \qquad \omega = 2\pi\nu, \quad K = \frac{2\pi}{\lambda}$$

$$\Rightarrow v_p = \nu\lambda$$

$E = h\nu$, $\lambda = h/p$

$$v_p = \frac{E}{h} \times \frac{h}{p} \Rightarrow \frac{E}{p} = \frac{mc^2}{mv}$$

$$\Rightarrow v_p \cdot v = c^2$$

$$\Rightarrow \boxed{c^2 = v \cdot v_p}$$

always, $v < c$

$\therefore$, $v_p > c$

→ **Heisenberg's Uncertanity Principle**

It is impossible to determine exact position and momentum of micro particle simultaneously.

Let we consider a wave packet, $x_2 - x_1 = \Delta x_1$.

$\therefore$, Change in posn will be Δx and if change in velocity is Δv or change in momentum Δp $(= m\Delta v)$ then, mathematically, we can write

$$\boxed{\Delta x \cdot \Delta p \geq \frac{h}{4\pi}}$$

$$\frac{h}{2\pi} = \hbar \quad (h \text{ not})$$

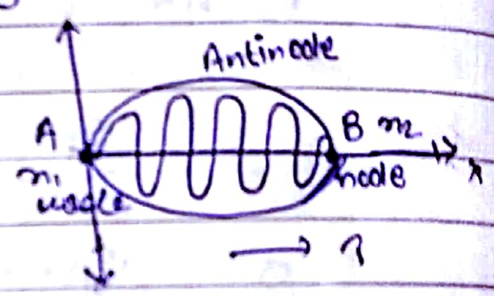
$$\text{or} \quad \Delta x \cdot \Delta p \geq \frac{h}{2}$$

$$h = 6.6 \times 10^{-34} \text{ j sec}$$

→ **Proof of Heisenberg's Uncertainity Principle**

Let ~~we~~ us consider a wave packet, ~~the waves~~ having diff. angular freq. & propagation const. ω_1, ω_2, K_1 & K_2

The resultant amp. of the wave is 'A'.



The displacement eq^n for the waves are :-

(i) $y_1 = A\sin(\omega_1 t - K_1 x)$ — (i)

(ii) $y_2 = A\sin(\omega_2 t - K_2 x)$ — (ii)

$$\left(\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right)$$

$$Y = Y_1 + Y_2$$

$$\Rightarrow Y = A\sin(\omega_1 t - K_1 x) + A\sin(\omega_2 t - K_2 x)$$

$$= 2A\sin\left(\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{K_1+K_2}{2}\right)x\right)$$

$$\cos\left(\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{K_1-K_2}{2}\right)x\right)$$

$$\frac{\omega_1+\omega_2}{2} = \omega \text{ (Assume)} \quad \& \quad \frac{K_1+K_2}{2} = K$$

$$\& \quad \frac{\omega_1-\omega_2}{2} = \frac{d\omega}{2} \quad \& \quad \frac{K_1-K_2}{2} = \frac{dK}{2}$$

$$\therefore \boxed{y = 2A\cos\left(\frac{d\omega t}{2} - \frac{dk\,x}{2}\right)\sin(\omega t - kx)}$$

The amplitude of wave "$\cos\left(\frac{d\omega}{2}t - \frac{dkx}{2}\right)$" is "0" at boundaries (A & B).

It can be written as, ~~at A~~.

at A, $\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x_1\right) = 0$

or $\cos\left(\frac{d\omega t}{2} - \frac{dk}{2}x_1\right) = \cos\pi/2$.

$$\Rightarrow \frac{d\omega}{2}t - \frac{dk}{2}x_1 = \pi/2 \quad \text{—— (i)}$$

Similarly, at B $\frac{d\omega}{2}t - \frac{dk}{2}x_2 = \frac{3\pi}{2}$ —— (ii)

On solving eq^n (i) & (ii)

$$\frac{dk}{2}(x_2 - x_1) = -\pi$$

$$\Rightarrow \boxed{\frac{dk}{2}(x_1 - x_2) = \pi}$$

$$\Rightarrow \frac{dk}{2}\Delta x = \pi \quad \text{—— (iii)}$$

$k = \frac{2\pi}{\lambda}$, $\lambda = \frac{h}{p}$, $k = \frac{2\pi p}{h}$, $dk = \frac{2\pi}{h}dp$ (or Δp)

or $\boxed{\frac{dk}{2} = \frac{1}{2}\times\frac{2\pi}{h}\Delta P}$ —— (iv)

$$\frac{1}{2}\times\frac{2\pi}{h}\Delta P\cdot\Delta x = \pi$$

$\Rightarrow$

$$\Rightarrow \boxed{\Delta x\cdot\Delta p = h} \quad \text{—— (v)}$$

$$E = \frac{7.8 \times 10^{-13}}{1.6 \times 10^{-19}} \ eV = 4.875 \times 10^{6} \ eV$$

$$= 4.875 \ MeV > 4 \ MeV$$

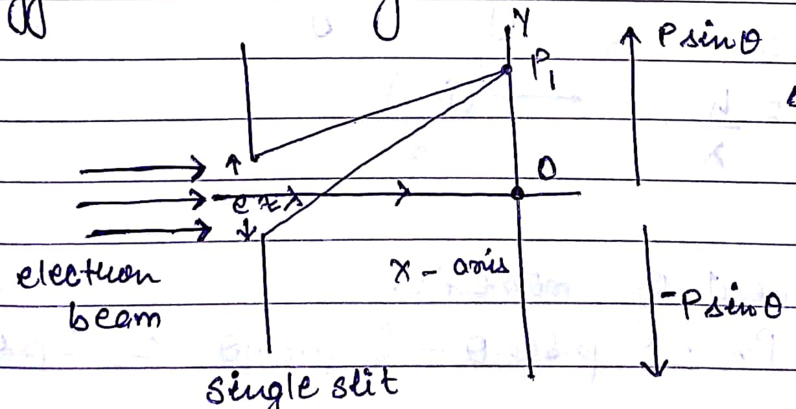
~~It is clear from result the value of K.E of an e⁻ is greater than the K.E of~~

The calculated result of K.E of e⁻ 4.875 MeV > 4 MeV, therefore e⁻ cannot exist in the nucleus.

(ii) With the help of diffraction at a single slit.
We consider a slit whose size $(e) \approx \lambda$ (is in order of λ of e⁻).

An e⁻ beam incident on slit get diffracted in normal dirⁿ to the dirⁿ of incident electron beam

Diffraction at single slit



$$\Delta P = (P\sin\theta) - (-P\sin\theta)$$
$$= 2P\sin\theta \quad \text{— (i)}$$
$$\Delta x = e$$
$$e\sin\theta = n\lambda$$

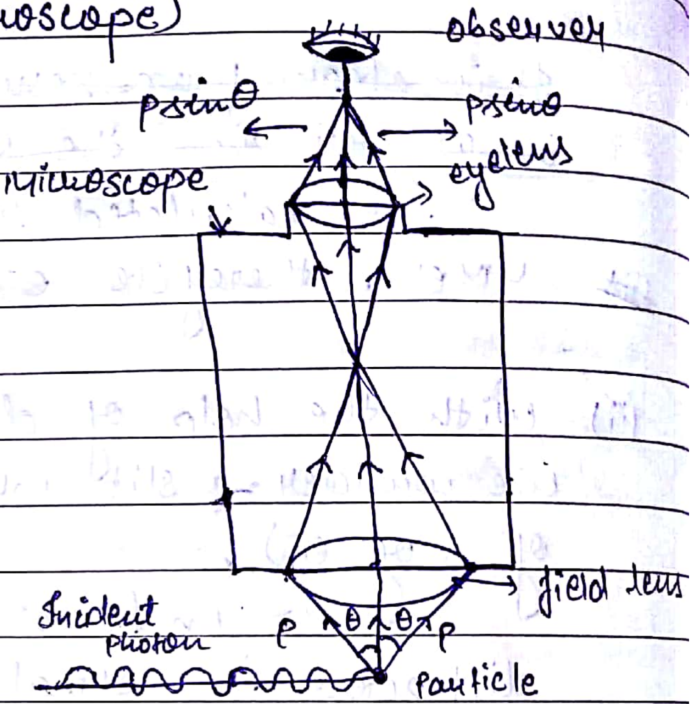
$$\Delta x = \frac{\lambda}{\sin\theta} \quad \text{— (ii)}$$

L.H.S $\quad \Delta x \cdot \Delta p = \frac{\lambda}{\sin\theta} \times 2p\sin\theta = 2\lambda p \quad \Big| \ \lambda = \frac{h}{p}$

$$\Rightarrow \boxed{\Delta x \cdot \Delta p = 2h}$$

Electron propagate in x-dirⁿ & diffracted from single slit

(iii) By microscope (γ- microscope)



The resolving power i.e., the smallest distance b/w the points that can be just resolved by the microscope is given by

$$\Delta x \approx \frac{\lambda}{2\sin\theta} \quad \text{—— (i)}$$

Initial momentum of the photon which strike the electron is given by

$$p = \frac{h}{\lambda} \quad \text{—— (ii)}$$

x component of momentum

i.e. $P_x = p\sin\theta - (-p\sin\theta) = 2p\sin\theta$

$\Rightarrow P_x = 2 \times \frac{h}{\lambda} \sin\theta$

$\Rightarrow P_x = \frac{2h}{\lambda} \sin\theta$

From eq^n (i) & (ii), we have

$$\Delta x \cdot \Delta P_x = \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta = h$$

$$\Rightarrow \boxed{\Delta x \cdot \Delta P_x \propto h}$$

This shows that the product of uncertainties in position and momentum is of the order of planck's constant.

Alternate (by Prof. Dr. Meetoo Mam)

From, electron microscope

$$2d\sin\theta = \lambda$$

$$\Delta x = d = \frac{\lambda}{\sin\theta} \quad \text{—— (i)}$$

$$\Delta P = P\sin\theta - (-P\sin\theta) = 2P\sin\theta$$

$$\because P = \frac{h}{\lambda} \Rightarrow h = \lambda P$$

$$\Delta x \cdot \Delta P = \frac{\lambda}{2\sin\theta} \times 2P\sin\theta = \lambda P$$

$$\Rightarrow \boxed{\Delta x \cdot \Delta P = h}$$

The product of change in pos^n & change in momentum is in order of Planck's const. (h).

Wave Function (ψ)

For establishment of differential equation of de-Broglie wave a f^n or wave f^n required.
It is denoted by (ψ).

$$\psi = A \sin(\omega t - kx)$$

$\psi = A + iB$, & its complex congugate is

$$\psi^* = A - iB$$

$$\psi \psi^* = A^2 + B^2$$

$$|\psi|^2 = A^2 + B^2 \quad , \text{ where } |\psi|^2 = \psi \psi^*$$

→ **Born Interpretation :-**

ψ has real as well as imaginary parts, it is not observable or acceptable quantity, whereas $|\psi|^2$ is real, positive, single valued, quantity, it is observable or acceptable.

$|\psi|^2$ is defined as, probability density of finding electron (free particle) any time any where.

→ **Following are Properties / Assumptions of Wave f^n**

(i) It must be finite
(ii) It must be continuous
(iii) It must be positive and single valued.
(iv) It must be normalised.

Normalization: $\boxed{\int_0^{\infty} |\psi|^2 \, dx = 1}$

Probability density of finding electron at any where any time is always equals to 1. This cond^n is known as Normalization.

$$\boxed{\text{Orthogonal Property} \quad \int_0^\infty |\psi|^2 dx = 0}$$

→ **Operators :-** (Derivation of Operator)

E- Operator & P- Operator

A wave of amp. 'A' propagate in x-dir^n with propagation const. 'K' and its angular freq. is 'ω' and wave f^n represented by 'ψ'.

$$\psi = Ae^{-i(\omega t - Kx)} \quad \text{—— (i)}$$

$$\omega = 2\pi\nu \quad \& \quad K = \frac{2\pi}{\lambda}$$

$$\psi = Ae^{-i2\pi(\nu t - \frac{x}{\lambda})}$$

$$E = h\nu, \quad \nu = \frac{E}{h} \quad \text{--- (Planck's law)}$$

$$\lambda = \frac{h}{P}, \quad \frac{1}{\lambda} = \frac{P}{h} \quad \text{--- (de-Broglie hypothesis)}$$

$$\psi = Ae^{-i2\pi\left(\frac{E}{h}t - \frac{P}{h}x\right)}$$

$$= Ae^{\frac{i2\pi}{h}(Et - Px)} \quad \text{---} \quad \hbar = h/2\pi$$

$$\psi = Ae^{\frac{-i}{\hbar}(Et - Px)} \quad \text{—— (ii)}$$

~~Comparing eq^n (i) & (ii)~~

differentiate eq^n (ii) w.r.t 't'

$$\frac{d\psi}{dt} = Ae^{-i/\hbar (Et - Px)} \left\{ -\frac{i}{\hbar}(E-0) \right\}$$

$$\Rightarrow \frac{d\psi}{dt} = -\frac{iE}{\hbar}\psi$$

$$\Rightarrow E\psi = \frac{\hbar}{-i}\frac{d\psi}{dt} \times \frac{i}{i} = i\hbar\frac{d\psi}{dt}$$

$$\Rightarrow E\psi = i\hbar\frac{d\psi}{dt}$$

$$\boxed{E\text{-operator} = i\hbar\frac{d}{dt}} \quad \text{—— (iii)}$$

Again, differentiate eq^n (ii), w.r.t 'x'.

$$\frac{d\psi}{dx} = Ae^{-i/\hbar (Et - Px)} \left\{ \frac{i}{\hbar}(0-P) \right\}$$

$$\frac{d\psi}{dx} = \frac{i}{\hbar}P\psi \quad \Big| \quad P\psi = \frac{\hbar}{i}\frac{d\psi}{dx}$$

$$P\text{-operator} = \frac{\hbar}{i}\frac{d}{dx} \quad \text{—— (iv)}$$

$$\boxed{P\text{-operator} = \frac{\hbar}{i}\frac{d}{dx}} \quad \text{—— (iv)}$$

$$\boxed{E\psi = \frac{p^2}{2m}\psi + V\psi} \quad \text{—— (v)}$$

p. operator $= \frac{\hbar}{i}\frac{d}{dx}$ $\Rightarrow p^2 = \left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 = -\hbar^2 \frac{d}{dx}\frac{d}{dx}$

$$\Rightarrow p^2 = -\hbar^2 \frac{d^2}{dx^2}$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi \quad \text{—— (vi)}$$

$$\Rightarrow \boxed{\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E-V)\psi} = 0$$

divide eq^n (vi) with $\frac{\hbar^2}{2m}$

we have,

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \text{—— (vii)}$$

This above eq^n is known as one-dimensional time independent schrodinger's wave equation. In this eq^n ψ is f^n of pos^n x only, not a f^n of time and known as time independent.

For a free particle P.E 'V = 0' eq^n (vii) can be written as

for free particle

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0} \quad \text{—— (viii)}$$

This above eq^n is known as one-dimensional time independent Schrodinger wave eq^n for free particle.

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for 3D $\rightarrow$ Eq^n no. (vii) can also be written as

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{2m(E-V)\psi}{\hbar^2} = 0}$$

This above eq^n is known as three - dimensional time independent Schrodinger's wave eq^n.

Or

$$\boxed{\Delta^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

where $\Delta^2 = \frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}$

Laplace operator

From eq^n (v) $E\psi = \frac{p^2}{2m}\psi + V\psi$

Time dependent

$$\boxed{i\hbar\frac{d\psi}{dt} = \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi} \quad \text{—— (x)}$$

where E - operator $= i\hbar\frac{d\psi}{dt}$

P - Operator $= \frac{\hbar}{i}\frac{d}{dt}$

Eq^n (x) is known as One - dimensional time dependent Schrodinger's wave f eq^n. In this eq^n ψ is f^n of pos^n 'x' as well as time 't' & known as time dependent.

→ Applications of time Dependent Schrodinger's wave function

→ 1- Dimensional Time Independent Schrodinger's wave Eq^n can be written as :-

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{--- (i)}$$

V = 0 for free particle,

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0} \quad \text{--- (ii)}$$

$$\boxed{\frac{2mE}{\hbar^2} = K^2} \quad \text{(Assume)} \quad \text{--- (iii)}$$

$K = \frac{2\pi}{\lambda}$ | $\lambda = \frac{h}{p}$

$p = \sqrt{2mE}$

$K = \frac{2\pi}{h}\sqrt{2mE}$

$K = \frac{\sqrt{2mE}}{\hbar}$

$\Rightarrow K^2 = \frac{2mE}{\hbar^2}$

So, eq^n (ii) can be written as

$$\boxed{\frac{d^2\psi}{dx^2} + K^2\psi = 0} \quad \text{--- (iv)}$$

ψ has sine or cosine form, or General sol^n of ψ is given as

$$\boxed{\psi = A\sin Kx + B\cos Kx} \quad \text{--- (v)}$$

where, 'A' & 'B' are const. can be calculate with the help of boundary cond^{n}s.

Boundary cond^{n}s are :-

(i) at $x = 0$, $\psi = 0$

(ii) at $x = a$, $\psi = 0$

By applying 1st boundary condn in eq(v)

$$0 = A\sin k \times 0 + B$$

$$\Rightarrow B = 0$$

putting B in eqn (v)

$$\Rightarrow \boxed{\psi = A\sin kx} \text{ ——— (vi)}$$

By applying 2nd boundary condn in eqn (v)

$$0 = A\sin ka + B\cos ka \quad (B=0)$$

$$A\sin ka = 0$$

or $\sin ka = \sin 2\pi \Rightarrow ka = n\pi$

$$\Rightarrow \boxed{k = \frac{n\pi}{a}} \text{ ——— (vii)}$$

From eqn (vii) & (iii)

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2} \quad \because \hbar = \frac{h}{2\pi}$$

$$\Rightarrow E = \frac{n^2\pi^2}{a^2} \times \frac{h^2}{4\pi^2 \times 2m} = \frac{n^2h^2}{8ma^2}$$

$$\Rightarrow E = \frac{n^2h^2}{8ma^2} \text{ ——— (viii)}$$

4/09/19

Value of

$\psi = A \sin kx$ — from eq^n (vi)

from $k = \frac{n\pi}{a}$

$\psi = A \sin \frac{n\pi}{a} x$ — (ix)

From the normalisation

$$\int_0^a |\psi|^2 dx = 1$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Or, $\int_0^a A^2 \sin^2 \frac{n\pi}{a} x \, dx = 1$

$$\frac{A^2}{2} \int_0^a \left[1 - \cos 2n\pi \frac{x}{a}\right] dx = 1$$

$$\frac{A^2}{2} \left[a - \left\{ \sin \frac{2n\pi x}{a} \times \frac{a}{2n\pi} \right\}_0^a \right] = 1$$

$$\Rightarrow \frac{A^2}{2} \left[a - \left\{ \sin 2n\pi \times \frac{a}{2n\pi} \right\} \right] = 1$$

$\because \sin n\pi = 0$

$\therefore \sin 2n\pi = 0$

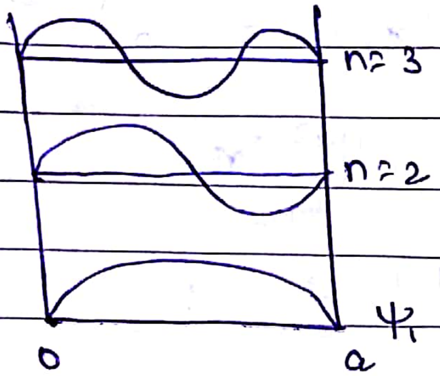
$$\Rightarrow \frac{A^2}{2} [a - 0] = 1$$

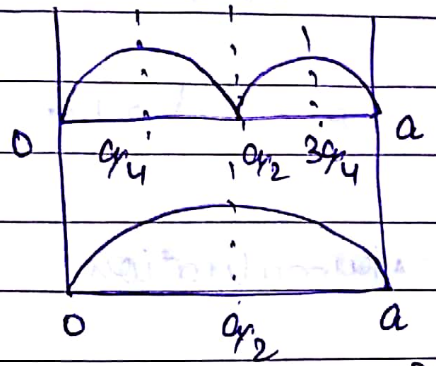
$\Rightarrow \frac{A^2}{2} \quad A^2 = \frac{2}{a} \Rightarrow A = \sqrt{\frac{2}{a}}$ — (ix)

$$\Rightarrow \boxed{\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}} \quad \text{—— (x)}$$

Or $\boxed{\psi^2 = \frac{2}{a} \sin^2 \frac{n\pi x}{a}}$ — (xi)

Acceptance of $|\psi|^2$ with the help of graphical Presentation





$$|\psi_n|^2 = \frac{2}{a}\sin^2\frac{n\pi x}{a}$$

$$|\psi|^2 = \frac{2}{a}\sin^2 \frac{1\, n\pi}{a}$$

$|\psi|^2$ at $x=0$; $|\psi|^2 = 0$

$x = a/2$; $= 2/a$

$x = a$; $= a$

$$|\psi|^2 = \frac{2}{a}\sin^2\left(\frac{2\pi x}{a}\right)$$

at $x = 0$; $|\psi_2|^2 = 0$

$x = a/4$; $= 2/a$

$x = a/2$; $= 0$

$x = \frac{3a}{4}$; $= 2/a$

$x = a$; $= 0$

It is clear from second graph $|\psi|^2$ is single value & always positive and real.

$\therefore$, it is acceptable.

NOTES

Born Interpretation

$$P(x, y, z) = |\psi(x, y, z, t_0)|^2$$

$$P = \int P(x) = \int_a^b |\psi|^2 dx$$

Normalisation $= \int_{-\infty}^{+\infty} |\psi|^2 dV = 1$

Expectation value can be calculated using

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} x|\psi|^2 dx}{\int_{-\infty}^{+\infty} |\psi|^2 dx} = \frac{\int_{-\infty}^{+\infty} \psi^* x \psi dx}{\int_{\infty} \psi^* \psi dx}$$

If ψ is normalised,

$$\int_{-\infty}^{+\infty} \psi^* \psi dx = 1$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} |\psi|^2 dx$$

$\int_a^b |\psi|^2 dx = 0, \infty, -$ is not corrected.