First Order Logic

→ To describe arguments that deal with all cases (or) some cases out of many cases, we need first order predicate calculus.

→ The Term 'first order' refers to the fact that quantifiers can quantifies only variables that occur in predicates

Ex: Modi is a politician

Modi ↑ Subject

is a politician ↑ Predicate

Let us denote the subjects by lower case and the predicate by upper case letters.

Let us denote subject Modi with letter 'm'

and is politician with 'P'

∴ P(m): Modi is a politician.

Ex2: If 'd' denote the person 'Dhoni' and 'S' denote the predicate "is a sportsman" then

S(d): Dhoni is a Sportsman

Ex3:- If x and y are two persons and F denote the predicate "is friend of" then

$F(x, y)$: x is a friend of y

Ex4 If 'x' and 'y' are two numbers and G denote "is greater than" then:

$G(x, y)$: 'x is greater than y'

Predicate and Connectivities

Let $P(x)$: x is a politician

$S(x)$: x is a Sportsman

$\sim P(x)$: x is not a politician

$P(x) \vee S(x)$: x is politician (or) x is Sportsman

$P(x) \wedge S(x)$: x is politician cum Sportsman

$P(x) \rightarrow S(x)$: If x is a politician then x is a Sportsman

$P(x) \leftrightarrow S(x)$: x is a politician iff x is a sportsman.

Quantifiers

We have two quantifiers in first order logic

i) Existential quantifier ii) Universal Quantifier

Existential quantifier

$\exists$: There exists

$\exists x$: For some 'x' in the universe of discourse

$\exists x\, P(x)$: There exist atleast one "x" such that 'x' is a politician

$\exists x \{P(x) \wedge S(x)\}$: Some politicians are Sportsman

$\exists x \{P(x) \vee S(x)\}$: Some persons are politicians (or) Sportsman

$\exists x \{P(x) \rightarrow S(x)\}$: Some persons are non politicians (or) Sportsman (Do you know why I am writing like that ?)

Reason is

$$\exists x \{P(x) \rightarrow S(x)\} \cong \exists x \{\sim P(x) \vee S(x)\}$$

(I hope you understood now. ☺)

Universal Quantifier

$\forall$: For all

$\forall x$: For all 'x' in the universe of discourse

$\forall x\, P(x)$: All are politicians

$\forall x \{P(x) \wedge S(x)\}$: All are politicians cum Sportsman

$\forall x \{P(x) \vee S(x)\}$: Every person is a politician (or) Sportsman

$\forall x \{P(x) \rightarrow S(x)\}$: All politicians are Sportsman

Note:-
- The universal quantifier $\forall x$, quantifies a Conditional
- The Existential quantifier $\exists x$ quantifies a conjunction

Well formed formula

1) Any atomic proposition is w.f.f

2) If W and V are w.f.fs and x is a variable, then the following expressions are also wffs.

$$W,\ \sim W,\ W \vee V,\ W \wedge V,\ W \rightarrow V,\ \exists x W \text{ and } \forall x W$$

Scope:- In the Well-Formed Formula $\exists x W$, W is the scope of the quantifier $\forall x$

Bound and free Variables

An Occurence of a variable "x" in a wff is said to be bound if it lies ~~between~~ with in the scope of either $\exists x$ (or) $\forall x$ (or) if it is the quantifier variable 'x' itself.

Ex: In the wff

$$\exists x\, P(x, y) \longrightarrow Q(x)$$

The first two occurence of 'x' are bound because the Scope of $\exists x$ is $P(x, y)$.

In $\exists x\, P(x, y) \longrightarrow Q(x)$

The first two occurrence of "x" are bound because of $\exists x$ is $P(x, y)$

The only occurrence of y is ~~fee~~ free, and the third occurrence of "x" is free

<u>Relationships between Universal and existential</u> ~~quantifier~~

Let $F(x) = x$ is True then

Sentence	Meaning
$\forall x\, F(x)$	all True
$\exists x\, F(x)$	atleast one True
$\sim[\exists x, F(x)]$	none True
$\forall x\, [\sim F(x)]$	all false
$\exists x\, [\sim F(x)]$	atleast one false
$\sim\{\exists x, \sim F(x)\}$	none is false
$\sim\{\forall x, F(x)\}$	not all True
$\sim\{\forall x, \sim F(x)\}$	not all false.

The following equivalence hold good

01. $\forall x\, F(x) \Leftrightarrow \sim[\exists x, \sim F(x)]$
02. $\forall x\, [\sim F(x)] \Leftrightarrow \sim[\exists x, F(x)]$
03. $\sim[\forall x, F(x)] \Leftrightarrow \exists x, [\sim F(x)]$
04. $\sim[\forall x, \sim F(x)] \Leftrightarrow \exists x\, F(x)$

Note: To negate statement formula, in first order logic we have to replace $\forall x$ with $\exists x$, and $\exists x$ with $\forall x$ and finally negate the scope of the quantifier

Statement	Negation
$\forall x\, F(x)$	$\exists x\, [\sim F(x)]$
$\exists x \sim F(x)$	$\forall x\, F(x)$
$\forall x\, [\sim F(x)]$	$\exists x\, F(x)$
$\exists x\, F(x)$	$\forall x \sim F(x)$

Sentences with Multiple quantifiers

Let $P(x, y)$: x likes y (or) (y is liked by x)
where x and y are two persons

i) $\forall x \forall y\, P(x, y)$: Everybody likes everyone

ii) $\forall y \forall x\, P(x, y)$: Everybody is liked by every one

iii) $\exists x \exists y\, P(x, y)$: Somebody likes some one

iv) $\exists y \exists x\, P(x, y)$: Somebody is liked by Somebody

v) $\forall x \exists y\, P(x, y)$: Everybody likes Someone

vi) $\exists y \forall x\, P(x, y)$: Somebody is liked by everyone

vii) $\exists x \forall y\, P(x, y)$: Some body likes everyone

viii) $\forall y \exists x\, P(x, y)$: Everybody is liked by Some one.

Note

i) $\forall x \forall y\, P(x, y) \Leftrightarrow \forall y \forall x\, P(x, y)$

ii) $\exists x \exists y\, P(x, y) \Leftrightarrow \exists y \exists x\, P(x, y)$