Isomorphism of Graphs

Two graphs G and G' are said to be isomorphic if there is a function

$f: V(G) \rightarrow V(G')$ Such that

i) 'f' is a bijection

ii) for each pair of vertices u and v of G, $\{\langle u,v\rangle\} \in E(G)$ iff $\{\langle f(u), f(v)\rangle\} \in E(G')$.

i.e The function preserves the adjacency.

Note:-① Suppose G and G' are two graphs and that $f: V(G) \rightarrow V(G')$ is a bijection.

Let A be the Adjacency matrix for the vertix Ordering $v_1, v_2 \dots v_n$ of the vertices of 'G'.

Let A' be the Adjacency matrix for the vertix Ordering $f(v_1), f(v_2) \dots f(v_n)$, Then f is an Isomorphism from V(G) to V(G') iff the Adjacency matrices A and A' are equal.

Note ②: Two simple graphs are Isomorphic iff their Complements are Isomorphic

Note 3:- If G and G' are Isomorphic with each other then following Conditions must hold.

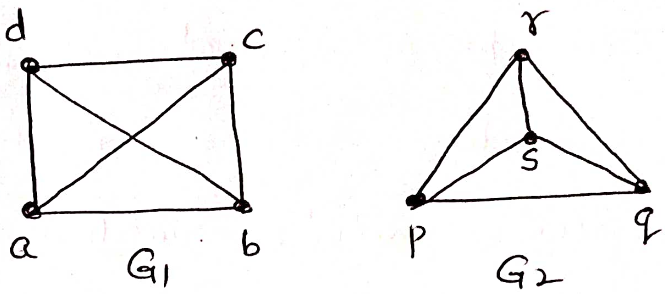
1) $|V(G)| = |V(G')|$

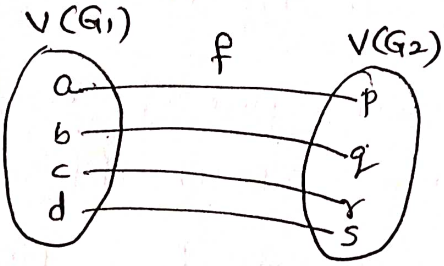
2) $|E(G)| = |E(G')|$

3) The degree sequence of G and G' are same

4) The number of simple circuits of a given length must be same in both graphs.

Ex1 The following two graphs are Isomorphic





i) $f : V(G_1) \rightarrow V(G_2)$ is a bijection

ii) for each pair of vertices u, v in 'G'

$\langle u, v \rangle \in E(G)$ iff $\langle f(u), f(v) \rangle \in E(G')$

In our example

$\langle a, b \rangle \in E(G)$ iff $\langle f(a), f(b) \rangle \in E(G')$

$\langle a, c \rangle \in E(G)$ iff $\langle f(a), f(c) \rangle \in E(G')$

$\langle a, d \rangle \in E(G)$ iff $\langle f(a), f(d) \rangle \in E(G')$

$\langle b, c \rangle \in E(G)$ iff $\langle f(b), f(c) \rangle \in E(G')$

$\langle b, d \rangle \in E(G)$ iff $\langle f(b), f(d) \rangle \in E(G')$

$\langle c, d \rangle \in E(G)$ iff $\langle f(c), f(d) \rangle \in E(G')$

where $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$

$\therefore$ $G_1 \cong G_2$

Work Book
QNo: 28
G: 2015

Let $G(V,E)$ be a graph
it is given that $G \cong \overline{G}$ —— (1)

but we know that $|E(G)| + |E(\overline{G})| = |E(K_n)|$

$\Longrightarrow \quad |E(G)| + |E(G)| = |E(K_n)|$ [$\because$ from (1)]

$\Rightarrow \quad 2|E(G)| = \frac{n(n-1)}{2}$

$\Rightarrow \quad |E(G)| = \frac{n(n-1)}{4}$

i.e either 'n' is divided by 4 (i.e $n \equiv 0 \pmod 4$)
(or) $(n-1)$ is divided by 4 (i.e $n \equiv 1 \pmod 4$)

Ans (d)

Work Book
QNo: 52

A cycle graph on 'n' vertices is Isomorphic to its Complement

$\therefore \quad C_n \cong \overline{C_n}$

$|E(C_n)| + |E(\overline{C_n})| = |E(K_n)|$

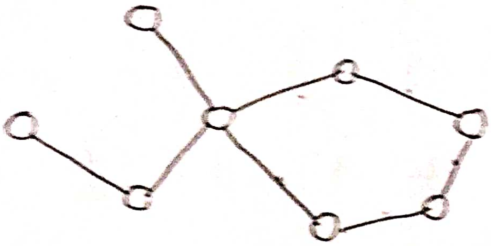
$\Rightarrow \quad 2|E(C_n)| = \frac{n(n-1)}{2} \quad (\because C_n \cong \overline{C_n})$

$\Rightarrow \quad 2n = \frac{n(n-1)}{2} \quad (\because |E(C_n)| = n)$

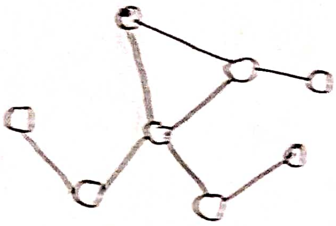
$\Rightarrow \quad n = 5$

QNo: 23
Work Book
G: 2012

Which of the graph is Isomorphic to

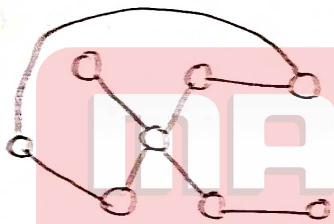


a)



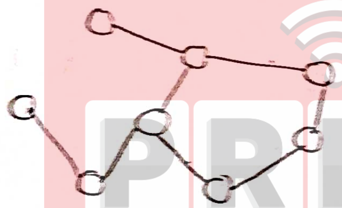
No, It has circuit of length '3' where the given graph doesnot have a circuit of length '3'

b)



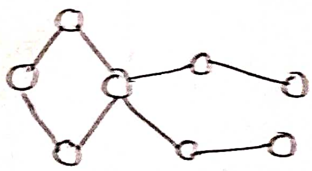
Yes

c)



No Given graph has one vertex of degree '4' but in option (c) we don't have such vertex

D)



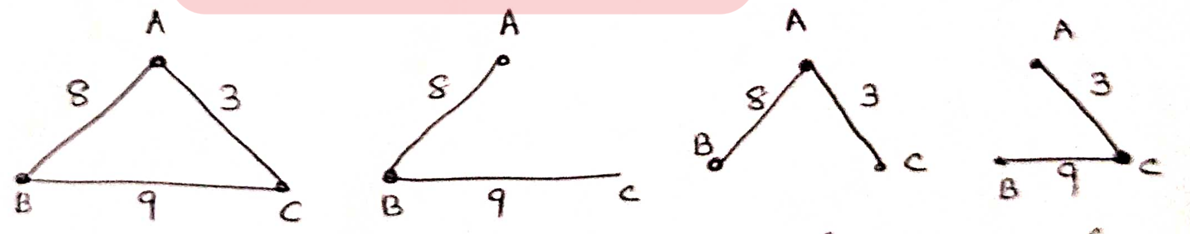
No, It has circuit of length '4' which does not exist in the given graph

Spanning Tree

A Subgraph H of a graph 'G' is called a Spanning tree of 'G' if

i) H is a tree and
ii) H Contains all vertices of G

- In general, if 'G' is a Connected graph with 'n' vertices and 'm' edges, Spanning Tree of 'G' must have $(n-1)$ edges
- The number of edges that must be removed before a spanning tree is obtained must be $m-n+1$. This is called a Circuit rank of 'G'
- The Complete graph 'K_n' has n^{n-2} different Spanning Tree (Caley's Theorem)
- The Complete graph $K_{m,n}$ has $m^{n-1} \times n^{m-1}$ different Spanning Trees.



G(3,3) H_1 (3,2) H_2 (3,2) H_3 (3,2)

here H_1, H_2, H_3 are Spanning Tree of the graph.

The Spanning Tree which Contain minimum weight is minimum weight Spanning Tree. (H_2 is MST)

We Can find Minimum weight Spanning Tree using

1) Kruskal's algorithm (or) 2) Prim's algorithm

[Note: Above two algorithms will be discussed in Algorithm subject]

Covering

1. Line Covering (or) Edge Covering

Let $G(V,E)$ be a graph, A subset C of E is called a line Covering of 'G', if every vertex of 'G' is incident with atleast one of edge in C

i.e in ~~a~~ lineCovering C of 'G'

$$deg(v) \geq 1 \quad \forall v \in G$$

Minimal Line Covering:-

A Line Covering from which no edge can be removed without destroying its ability to cover the graph is called minimal line Covering

Minimum Line Covering

A Line Covering with minimum no of edges is called a minimum line Covering

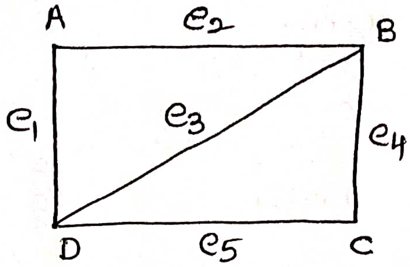
The number of Edges in a minimum line Covering of a graph 'G' is called line Covering number of graph 'G'

Properties:-

1) A line Covering exists for a graph 'G' iff 'G' has no Isolated vertix
2) A line Covering of 'n' vertix graph has atleast $\lceil \frac{n}{2} \rceil$ edges
3) No minimal line Covering can contain circuit.

$\chi \le \Delta + 1$

Ex



Subset of E	Line Covering	Minimal Line Covering	Minimum Line Covering
$C_1 = \{\langle A,D\rangle, \langle C,B\rangle\}$	✓	✓	✓
$C_2 = \{\langle A,B\rangle, \langle C,D\rangle\}$	✓	✓	✓
$C_3 = \{\langle A,B\rangle \langle B,D\rangle \langle C,D\rangle\}$	✓	✗	✗
$C_4 = \{\langle A,B\rangle, \langle B,C\rangle \langle B,D\rangle\}$	✓	✓	✗
$C_5 = \{\langle A,B\rangle \langle B,D\rangle\}$	✗	✗	

$\therefore$ Line Covering Number = No of edges in Minimum line covering

= 2

Vertex Covering :- Let $G = (V, E)$ be a graph

A Subset 'K' of V is called vertex covering of 'G' if every edge of 'G' is incident with a vertex in 'K'

Minimal vertex Covering :- A vertex covering from which no vertex can be removed without destroying its ability to cover the graph

Minimum vertex covering :- A vertex covering with minimum no of vertices is called minimum vertex covering.

The number of vertices in minimum vertex covering of 'G' is called vertex covering number of G.

Independent Set:-

Let $G = (V, E)$ be a graph

- A Subset S of V is called an Independent set of vertices of 'G' if no two vertices of 'S' are adjacent in 'G'

- **Maximal Independent Set:-**

  - An independent vertix set of graph 'G' to which no other vertix of 'G' can be added is called maximal independent set of vertices

- **Maximum independent vertix set**

  A maximal Independent vertix set which contains maximum no of vertices is called maximum Independent vertix set.

  The number of vertices in a maximum Independent set of a graph 'G' is called Independence number of G.

Properties:-

1) A Set 'S' is an independent set of 'G' iff $(V - S)$ is a vertix-Covering of G

2) Vertex Covering number + Vertex Independence number = Number of vertices in G