Combinatorics – Workbook
Questions

Q1: (Ans d)

No of ways in which 7 people can be seated around a round table without any condition is 6!

Now, let us assume that two particular people always sit together and let us consider them as one unit.

No of ways 6 people can be arranged around a table is 5! and particular people can be arranged between them selves in 2!

$\therefore \quad 6! - (5! \times 2)$

$= 480$

[Note: No of permutations of 'n' objects around table $= (n-1)!$]

Q2: (Ans: a)

$= {}^5C_4 \times {}^9C_7 + {}^5C_5 \times {}^9C_6$

$= 264$

Q NO: 3 (Ans: b)

There are total of '9' digits. {1,2,3,4,5,6,7,8,9}

Unit digit: Odd number can not have unit digit so there are 5 digits/choice (1, 3, 5, 7, 9)

$\therefore {}^5C_1$

Thousands $\quad {}^8C_1$

hundreds: $\quad {}^7C_1$

Tens: $\quad {}^6C_1$

$\therefore$ Total 4 digit odd integer with different digits

$= {}^5C_1 \times {}^8C_1 \times {}^7C_1 \times {}^6C_1$

$= 1680.$

Q NO: 04 Ans e

Let t_m, t_n are denoting m^{th} term, n^{th} term of A.P respectively

now $\quad t_m = a+(m-1)d, \quad t_n = a+(n-1)d$

but given that $t_m = n$ and $t_n = m$

$\therefore \quad n = a+(m-1)d \quad$ and $\quad m = a+(n-1)d$

$\Rightarrow \quad a = n-(m-1)d \quad$ ① and $\quad a = m-(n-1)d \quad$ ②

Now ① = ② $\quad n - md + \cancel{d} = m - nd + \cancel{d}$

$d = -1 \quad$ now from ①, $a = n+m-1$

now $\quad t_{m+n} = a+(m+n-1)d = (n+m-1)+(m+n-1)(-1)$

$= 0.$

QNO:05
Ans (C)

$$T_n = \frac{3+n}{4}$$

$$\sum_{n=0}^{K} T_n = \sum_{n=0}^{K} \frac{3}{4} + \sum_{n=0}^{K} \frac{n}{4}$$

$$= \frac{3}{4}(K) + \frac{1}{4}\left(\frac{K(K+1)}{2}\right)$$

$$\sum_{n=0}^{K} T_n = \frac{7K}{8} + \frac{K^2}{8}$$

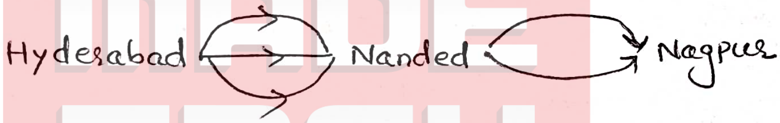
$$\sum_{n=0}^{105} T_n = \frac{7(105)}{8} + \frac{(105)^2}{8} = 1470.$$

QNO:06
Ans (d)

$$= (6C_3 \times 6C_4) + (6C_4 \times 6C_3) + (6C_2 \times 6C_5) + (6C_5 \times 6C_2)$$

$$= 780.$$

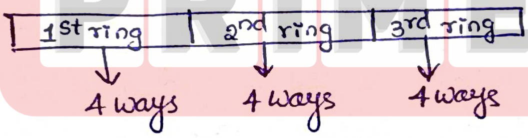
QNO: 07
Ans (e)



$\therefore \; 3C_1 \times 2C_1$

$= 6$

QNO:08
Ans (b)



By Product Rule these three rings can be arranged in $4C_1 \times 4C_1 \times 4C_1 = 64$ ways

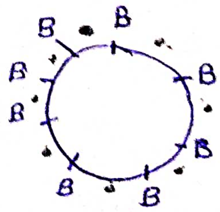
$\therefore$ No of unsuccessful events possible $= 64 - 1$
$= 63.$

QNO:09
Ans (a)

8 boys can sit for a lunch at a round table

$= (8-1)!$ ways

$= 7!$ ways.



We have 8 Gaps among boys to arrange '6' girls. So $8P_6$

$\therefore$ Total no of ways $= 7! \times 8P_6$

QNO: 10
Ans (c)

Divide '8' different shirts into '4' groups and each group Contains '2' Shirts and then distribute among '4' persons

$$= \frac{8!}{(2!\,2!\,2!\,2!) \times 4!} \times 4!$$

$$= 2520$$

QNO: 11
Ans (e)

$$S_n = 2n^2 + 3n$$

$$S_1 = 2(1)^2 + 3(1) = 5$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$2n^2 + 3n = \frac{n}{2}[5 + a_n]$$

$$4n^2 + 6n = n[5 + a_n]$$

$$a_n = 4n + 6 - 5$$

$$a_n = 4n + 1$$

QNO: 12
Ans (d)

Let there were 'n' teams participating in the games. The total no of matches $= {}^nC_2 = 153$

On Solving it we get $n = -17$ and $n = 18$

but we can not negative so $n = 18$

QNO: 13
[redacted]
Correct Answer $= \frac{1}{2}$

Let A: be the event that first ball is ~~black~~ black

B: be the event that second ball is white

$$P(A) = \frac{15}{25} = \frac{3}{5}$$

$$P(B) = \frac{10}{24} = \frac{5}{12}$$

Case (i) Probability that first ball is ~~white~~ black and Second is ~~black~~ white

$$= \frac{3}{5} \times \frac{5}{12}$$

$$= \frac{1}{4}.$$

Case (ii) probability that first ball is white and second is black

$$= \frac{5}{12} \times \frac{3}{5}$$

$$= \frac{1}{4}$$

$\therefore$ Required probability = Case (i) + Case (ii) $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

QNO: 14 Ans(C)

n^m

QNO: 15 Ans (d)

$28 = 18+15+22-9-11-13+n(K\cap E\cap H)$

$\therefore n(K\cap E\cap H) = 28-55+33$

$\therefore n(K\cap E\cap H) = 6$.

QNO:16 Ans (B)

The no of binary relations on set with 'n' element $= n^2$

QNO:17 Option (A) (Printing Mistake)

$^{(n+1)}C_K$.

QNO: 18 Ans (C)

No of ways of distributing 'r' similar items among 'n' different object $= {}^{n+r-1}C_r$

i) 10 lillies

$G_1+G_2 = 10$ where $G_1 \geq 0$, $G_2 \geq 0$

Let G_1, G_2 are two girls.

No of ways $= {}^{11}C_1$
$= 11$

ii) 15 Sunflowers

$G_1+G_2 = 15$

No of ways $= {}^{16}C_1$

iii) 14 daffodils

$G_1+G_2 = 14$

No of ways $= {}^{15}C_1$

$\therefore$ The Total no of ways distributing
$= 11\times16\times15$
$= 2640$

QNO:19 Ans(C)

$n =$ No of Suits $= 4$ (No of pigeon holes)

atleast $k+1 = 3 \Rightarrow k = 2$

$\therefore$ Minimum $= kn+1$
$= 2(4)+1$
$= 9$.

QNo: 20

4 digit even numbers end with zero

$$= \boxed{\cancel{0}\ |\ \ \ |\ \ \ |\ 0}$$

9 ways (1,2,3,4,5,6,7,8,9)   8 ways   7 ways   1 way

$$= 9 \times 8 \times 7 \times 1$$

$$= 504 \text{ ways}$$

4 digit even numbers ending with 2, 4, 6, 8

$$= \boxed{\cancel{0}\ |\ \ \ |\ \ \ |\ \ \ }$$

8 ways   8 ways   7 ways   4 ways

$$= 1792 \text{ ways}$$

$\therefore$ Total number of 4 digit even numbers

$$= 504 + 1792$$

$$= 2296.$$

QNo: 21 : Ans(b)   Already explained in the class.

QNo: 22 : Ans (c)

$$\frac{8!}{3!\,5!} = 56.$$

QNo: 23 Ans(b)

There 'n' couples and each couple can attend to party in '3' ways

i) Both husband and wife attend the party
ii) wife only attend the party
iii) Neither husband nor wife attend the party

$\therefore$ Total no of possibilities $= 3^n$.

QNo: 24
Ans (d)

The problem reduces to finding how many distinct ordered color pairs (c_1, c_2) are possible with 'K' colors.

Since the first color 'c_1' can be any one of the 'K' colors and second color 'c_2' also any one of the 'K' colors (both prints of a letter can be colored with same color).

∴ The total no of such order color pairs $= K \times K$
$= K^2$

$$\left[\left.\begin{matrix} (c_1, c_1) & (c_1, c_2) & \cdots & (c_1, c_K) \\ (c_2, c_1) & (c_2, c_2) & \cdots & (c_2, c_K) \\ \vdots & & & \\ (c_K, c_1) & (c_K, c_2) & \cdots & (c_K\ c_K) \end{matrix}\right\} = K^2\right]$$

∴ Since each pair of letters must be colored with different color pairs, atleast 26 color pairs are required to do this

∴ $K^2 \geq 26$

The minimum value of $K = 6$.

QNo: 25
Ans (A)

$D_5 = 44$ (Already discussed in the class)

QNo: 26
Ans (d)

Already discussed in the class.

QNo: 27
Ans (C)

QNo: 28
Ans (b)

$$(1-x)^{-n} = \sum_{k=0}^{\infty} {}^{(n+k-1)}C_k \, x^k .$$

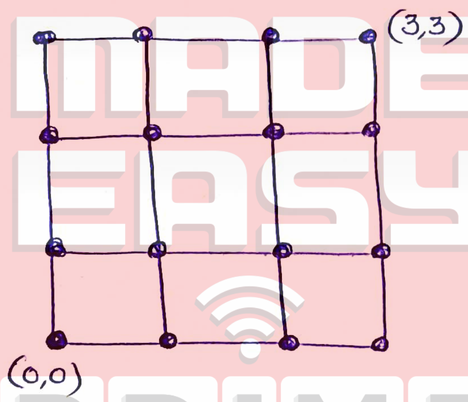
Given that $(1-x)^{-2} = \sum_{i=0}^{\infty} g(i)\, x^i$

here $n = 2$, $k = i$

$$\therefore \quad (1-x)^{-2} = \sum_{i=0}^{\infty} {}^{(i+1)}C_i \, x^i$$

$$\therefore \quad g(i) = {}^{(i+1)}C_i = (i+1)$$

QNo: 29
Ans (C)



The robo can move only right (or) up as defined in the problem.

Let us denote right move by 'R' and up move by 'U'.

Now to reach (3,3) from (0,0) the robo has to make exactly 3 times 'R' and 3 times 'U' moves in any order

Similarly to reach (10,10) from (0,0), the robot has to make 10 times 'R' and 10 times 'U' moves in any order

$$\therefore \quad \text{No of ways} = \underbrace{{}^{20}C_{10}}_{\text{10 time 'R'}} \times {}^{10}C_{10}$$

Total no of paths to reach (0,0) to (10,10)

$$= \frac{20!}{10!\,10!}$$

QNO: 30
Ans (d)

There are $^8C_4$ ways for robot to reach (4,4) from (0,0). and then robot takes the 'U' move from (4,4) to (5,4).

Now from (5,4) to (10,10) the robot has to make 5 times 'U' moves and '6' times 'R' moves in any order which can be done in $^{11}C_5$ ways

∴ The number of ways robot can move from (0,0) to (10,10) via (4,4)−(5,4) move is

$$= {}^8C_4 \times {}^{11}C_5$$

∴ Number of ways robot can move from (0,0) to (10,10) without using (4,4) to (5,4) moves is

$$= {}^{20}C_{10} - ({}^8C_4 \times {}^{11}C_5)$$

QNO: 31 : Already discussed in the class.
Ans (c)

QNO: 32
Ans (a)

$$x_1 + x_2 + \cdots + x_n = b \Rightarrow {}^{(n+b-1)}C_{(n-1)}$$

$$x_1 + x_2 + \cdots + x_n = r \Rightarrow {}^{(n+r-1)}C_{(n-1)}$$

Since both are independent events

$$\therefore \; {}^{(n+b-1)}C_{(n-1)} \times {}^{(n+r-1)}C_{n-1}$$

$$\Rightarrow \frac{(n+b-1)!}{(n-1)!\, b!} \times \frac{(n+r-1)!}{(n-1)!\, r!}$$

Q No: 33 Already discussed in the class.

Q No: 34
Ans (d).

$$a_n = 2n+3 \quad \text{for all } n = 0, 1, 2 \dots$$

$n=0, \quad a_0 = 3$

$n=1 \quad a_1 = 5$

$n=2 \quad a_2 = 7$

$n=3 \quad a_3 = 9$

$\vdots$

$$\langle 3, 5, 7, 9, \dots \rangle = 3 + 5x + 7x^2 + 9x^3 + 11x^4 + \dots$$

Apply Generating function Concept.

$$= \frac{3-x}{(1-x)^2}$$

Q No: 48

of $(ax+by+cz)^m$ then coeff of $x^p y^q z^r$ where $p+q+r=m$

is $\dfrac{m!}{p!\, q!\, r!} (a)^p \cdot (b)^q \cdot (c)^r$

here it is given $(2x-y+3z)^6$, $a=2, b=-1, c=3$
$p=3 \quad q=2, r=1$
$(\because x^p y^q z^r = x^3 y^2 z^1)$

Coeff of $x^3y^2z^1$ is $\dfrac{6!}{3!\, 2!\, 1!} (2)^3 (-1)^2 (3)^1$

$$= \frac{\cancel{6} \times 5 \times 4 \times 3}{\cancel{3 \times 2}} \times (8)(1)(3)$$

$$= 60 \times 24$$

$$= 1440$$

QNo: 49

$^{12}C_5 - {}^{7}C_5$

QNo: 50 :
Ans : 150

It is equivalent to no. of onto functions from $|A| = 5$ to $|B| = 3$.

$\therefore \quad 3^5 - {}^3C_1 (2)^5 + {}^3C_2 (1)^5$ ($\because$ using onto function)

$= 150$

QNo: 51
Ans : 7

$$^{10}C_{x-1} < 2\left({}^{10}C_x\right)$$

$$\Rightarrow \frac{10!}{(x-1)!\,(11-x)!} < 2\left[\frac{10!}{x!\,(10-x)!}\right]$$

$$\Rightarrow \frac{1}{(x-1)!\,(11-x)!} < \frac{2}{x\,(x-1)!\,(10-x)!}$$

$$\Rightarrow \frac{1}{(11-x)\,(10-x)!} < \frac{2}{x\,(10-x)!}$$

$$\Rightarrow \frac{1}{(11-x)} < \frac{2}{x}$$

$$\Rightarrow x < 22 - 2x$$

$$\Rightarrow 3x < 22$$

$$\Rightarrow x < \frac{22}{3}$$

$$\therefore \cancel{x < 7.33}$$

$$\Rightarrow x < 7.33$$

$$\therefore x = 7.$$

Dear students for remaining questions you can refer Made Easy Previous year question bank, wish you all the best. If you face any difficulties then send your analysis along with the question then I can guide to you properly. Thanks for giving oportunity to teach all of you.