V

Discrete & Engineering Mathematics

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1 Propositional Logic

Multiple Choice Questions

Q.1 Which of the following is a proposition?
(a) What is your name?
(b) Do your assignments.
(c) 1 + 1 = 4
(d) What a great day it is!

Q.2 Which of the following is not a proposition?
(a) Delhi is in India
(b) London is the capital of France
(c) This sentence is true
(d) This sentence is false

Q.3 Which of the following is true?
(a) 2 + 2 = 4 and x^2 is negative for a real x
(b) 2 + 2 = 5 follows from $2x$ is always even
(c) x is divisible by 2 or x is not divisible by 2
(d) $2x$ is divisible by 2 iff $3x$ is not divisible by 3

Q.4 Which of the following is the negation of x is even iff x is divisible by 2
(a) (x is even or x is not divisible by 2) and (x is not even or x is divisible by 2)
(b) (x is even and x is not divisible by 2) or (x is not even and x is divisible by 2)
(c) x is not even iff x is not divisible by 2
(d) x is even if x is divisible by 2

Q.5 Which of the following is not equivalent to x is even iff x is divisible by 2
(a) x is divisible by 2 iff x is even
(b) x is not even iff x is not divisible by 2
(c) x is not divisible by 2 iff x is not even
(d) x is even if x is divisible by 2 or x is divisible by 2 if x is even

Q.6 Which of the following is a functionally complete set of connectives?
(a) $\{\neg\}$ (b) $\{\vee\}$
(c) $\{\wedge, \vee\}$ (d) $\{\wedge, \neg\}$

Q.7 Which of the following is not a functionally complete set of connectives?
(a) $\{\wedge, \neg\}$ (b) $\{\vee, \neg\}$
(c) $\{\Rightarrow\}$ (d) $\{\uparrow\}$

Q.8 Which of the following is a smallest minimal functionally complete set of connective?
(a) $\{\wedge, \neg\}$ (b) $\{\downarrow\}$
(c) $\{\wedge, \vee\}$ (d) $\{\vee, \neg\}$

Q.9 Which of the following is commutative but not associative
(a) NAND (b) OR
(c) EX-OR (d) Implication

Q.10 Which of the following is false?
(a) $p \Leftrightarrow q \equiv \sim p \Leftrightarrow \sim q$
(b) $p \Leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$
(c) $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$
(d) $p \Leftrightarrow q \equiv pq' + p'q$

Q.11 Which of the following is false?
(a) $p \Rightarrow q \equiv \sim p \wedge q$
(b) $p \bar{\vee} q \equiv \sim(p \Leftrightarrow q)$
(c) $p \bar{\vee} q \equiv pq' + p'q$
(d) $p \Rightarrow q \equiv \sim p \vee q$

Q.12 Which of the following equivalences does not hold?
(a) $p \uparrow q \Leftrightarrow \neg p$
(b) $(p \downarrow q) \downarrow (p \downarrow q) \Leftrightarrow p \vee q$
(c) $(p \uparrow p) \uparrow (q \uparrow q) \Leftrightarrow p \vee q$
(d) $(p \uparrow q) \uparrow (p \uparrow q) \Leftrightarrow p \wedge q$

Q.13 Which of the following is an equivalent formula for $p \Rightarrow (q \Leftrightarrow r)$ which contains neither the biconditional nor the conditional?
(a) $p \wedge (\neg q \vee r) \wedge (\neg r \vee q)$
(b) $p \vee (\neg q \wedge r) \vee (\neg r \vee q)$
(c) $\sim p \vee (\neg q \vee r) \wedge (\neg r \vee q)$
(d) $p \vee (\neg q \vee r) \vee (\neg r \vee q)$

Q.14 Which of the following is the correct order of precedence of the connectives? (in descending order of precedence)
(a) $\neg, \vee, \wedge, \Rightarrow, \Leftrightarrow$ (b) $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
(c) $\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$ (d) $\wedge, \vee, \neg, \Leftrightarrow, \Rightarrow$

Q.15 Which one of the following is the dual of $\neg(p \wedge q) \vee (p \wedge \neg (q \vee \neg s))$
(a) $(p \vee q) \wedge (p \vee (q \wedge s))$
(b) $\neg(p \vee q) \wedge (p \vee \neg (q \wedge \neg s))$
(c) $(p \wedge q) \vee (p \wedge (q \vee \neg s))$
(d) Does not have a dual

Q.16 Which of the following is the dual of $p \vee T \equiv T$
(a) $p \vee T \equiv F$ (b) $p \wedge T \equiv F$
(c) $p \wedge F \equiv F$ (d) $p \vee T \not\equiv T$

Q.17 Which of the following is the dual of $\neg(\neg p) \equiv p$
(a) $\neg p \equiv p$ (b) $\neg(\neg p) \equiv p$
(c) $\neg p \equiv \neg p$ (d) $p \equiv \neg p$

Q.18 Which of the following is false?
(a) If a logical equivalence is true then its dual is also true. (If $A \Leftrightarrow B$, then $A^* \Leftrightarrow B^*$)
(b) If A^* represents the dual of A, then $(A^*)^* = A$
(c) $\neg A(P_1, P_2, \ldots, P_n) \Leftrightarrow A^*(\neg P_1, \neg P_2, \ldots, \neg P_n)$ where $P_1, P_2, \ldots, P_n$ are the atomic variables that occur in A and A^*
(d) $A = \neg A^*$

Q.19 If $A(P, Q, R) = \neg P \wedge \neg (Q \vee R)$ what is $A^*(\neg P, \neg Q, \neg R)$?
(a) $P \wedge Q \wedge R$ (b) $P \vee Q \vee R$
(c) $P \vee Q \wedge R$ (d) $P \wedge Q \vee R$

Q.20 Which of the following is a tautology?
(a) $[(p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (p \vee r)] \Rightarrow (q \vee s)$
(b) $[(p \Rightarrow q) \vee (q \Rightarrow p)] \Leftrightarrow (p \Leftrightarrow q)$
(c) $(p \Rightarrow q) \Leftrightarrow (\sim q \vee p)$
(d) $(p \Rightarrow q) \wedge p \Rightarrow \sim q$

Q.21 If $a \equiv F$ and $b \Leftrightarrow c$ is a tautology then which of the following is equivalent to $a \vee (b \wedge c)$
(a) a (b) F
(c) T (d) b

Q.22 If p is false, then $p \wedge q \Rightarrow r$ is a
(a) tautology
(b) contingency
(c) contradiction
(d) true only if q and r are both false

Q.23 If $p \Rightarrow q$ is true, then the truth value of which of the following cannot be determined
(a) $\sim p \vee q$ (b) $\sim q \Rightarrow \sim p$
(c) $\sim p \Rightarrow \sim q$ (d) $\sim(p \wedge \sim q)$

Q.24 If $p \Rightarrow q$ is true, then the truth value of which of the following can be determined?
(a) p (b) q
(c) $p \wedge q$ (d) $q \vee \sim p$

Q.25 If $p \Rightarrow q$ is true and $q \Rightarrow p$ is true then which of the following is not possible?
(a) p and q are both true
(b) p and q are both false
(c) one of p or q is true and the other is false
(d) $p \oplus q = F$

Q.26 Which of the following is a contradiction?
(a) $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$
(b) $(p \Leftrightarrow q) \wedge (p \oplus q)$
(c) $p \vee (p \Rightarrow q)$
(d) $p \vee q$

Q.27 If $p \Rightarrow q$ is false, which of the following is not true?
(a) $q \Rightarrow p$ is satisfiable
(b) $q \Rightarrow \sim p$ is unsatisfiable
(c) $q \Rightarrow (p \vee r)$ is satisfiable
(d) $q \Rightarrow (p \vee r)$ is a tautology

Q.28 $F_1 : p \vee (p \wedge q)$
$F_2 : p \wedge (p \vee q)$
Which of the following is false?
(a) F_1 and F_2 are equivalent
(b) F_1 and F_2 are duals of each other
(c) F_1 and F_2 are both tautologies
(d) F_1 and F_2 are both contingencies

Q.29 Which of the following is the inverse of the statement "If today is Monday, tomorrow is Tuesday"?
(a) If today is Monday, then tomorrow is not Tuesday
(b) Tomorrow is not Tuesday follows from today is not Monday
(c) Today is Monday is sufficient for tomorrow is Tuesday
(d) Today is not Monday is necessary for tomorrow is not Tuesday

Q.30 What is the converse of $p = 0$ or $q = 0$, if $pq = 0$?
(a) $p \neq 0 \wedge q \neq 0$; if $pq \neq 0$
(b) If $pq = 0$, $p = 0$ or $q = 0$
(c) $pq = 0$, if $p = 0$ or $q = 0$
(d) $pq = 0$ sufficient for $p = 0$ or $q = 0$

Q.31 Which of the following is the negation of "All triangles are polygons"?
(a) There is some triangle which is a polygon
(b) All triangles are not polygons
(c) All polygons are triangles
(d) There is some triangle which is not a polygon

Q.32 What is the negation of $\forall x\ \forall y\ x + y = 10$?
(a) $\exists x\ \forall y\ x + y = 10$
(b) $\exists x\ \exists y\ x + y \neq 10$
(c) $\exists x\ \exists y\ x + y = 10$
(d) $\forall x\ \forall y\ x + y \neq 10$

Q.33 What is the negation of x is prime iff x is not composite?
(a) x is prime and x is not composite or x is not prime and x is composite
(b) x is prime if x is not composite
(c) x is not composite iff x is prime
(d) x is prime and x is composite or x is not prime and x is not composite

Q.34 Which of the following well-formed formulas are equivalent?
(a) $P \rightarrow Q$ (b) $\sim P \rightarrow Q$
(c) $\sim P \vee Q$ (d) $\sim Q \rightarrow \sim P$
[GATE-1988]

Q.35 If F_1, F_2 and F_3 are propositional formulae such that $F_1 \wedge F_2 \rightarrow F_3$ and $F_1 \wedge F_2 \rightarrow \sim F_3$ are both tautologies, then which of the following is true
(a) Both F_1 and F_2 are tautologies
(b) The conjunction $F_1 \wedge F_2$ is not satisfiable
(c) Neither is tautologous
(d) Neither is satisfiable
(e) None of the above
[GATE-1991]

Q.36 The operation which is commutative but not associative is
(a) AND (b) OR
(c) EX-OR (d) NAND
[GATE-1992]

Q.37 Which of the following predicate calculus statements is/are valid
(a) $(\forall x)\ P(x) \vee (\forall x)\ Q(x) \rightarrow (\forall x)\ \{P(x) \vee Q(x)\}$
(b) $(\exists x)\ P(x) \wedge (\exists x)\ Q(x) \rightarrow (\exists x)\ \{P(x) \wedge Q(x)\}$
(c) $(\forall x)\ \{P(x) \vee Q(x)\} \rightarrow (\forall x)\ P(x) \vee (\forall x)\ Q(x)$
(d) $(\exists x)\ \{P(x) \vee Q(x)\} \rightarrow \sim(\exists x)\ P(x) \vee (\exists x)\ Q(x)$
[GATE-1992]

Q.38 Which of the following is/are tautology
(a) $a \vee b \rightarrow b \wedge c$ (b) $a \wedge b \rightarrow b \vee c$
(c) $a \vee b \rightarrow (b \rightarrow c)$ (d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$
[GATE-1992]

Q.39 The proposition $p \wedge (\sim p \vee q)$ is
(a) a tautology
(b) equivalent to $p \wedge q$
(c) equivalent to $p \vee q$
(d) a contradiction
(e) none of the above
[GATE-1993]

Q.40 If the proposition $\sim p \Rightarrow q$ is true, then the truth value of the proposition $\sim p \vee (p \Rightarrow q)$, where $\sim$ is negative, '$\vee$' is inclusive or and $\Rightarrow$ is implication, is
(a) true
(b) multiple-valued
(c) false
(d) cannot be determined
[GATE-1995]

Q.41 Which of the following propositions is a tautology?

(a) $(p \vee q) \rightarrow p$ (b) $p \vee (q \rightarrow p)$

(c) $p \vee (p \rightarrow q)$ (d) $p \rightarrow (p \rightarrow q)$

[GATE-1997]

Q.42 What is the converse of the following assertion?
I stay only if you go

(a) I stay if you go

(b) If I stay then you go

(c) If you do not go then I do not stay

(d) If I do not stay then you go

[GATE-1998]

Q.43 Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \wedge b) \rightarrow ((a \wedge c) \vee d)$ is always

(a) True

(b) False

(c) Same as the truth value of b

(d) Same as the truth value of d

[GATE-2000]

Q.44 Consider two well-formed formulas in propositional logic

$F_1 : P \Rightarrow \neg P$

$F_2 : (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct?

(a) F_1 is satisfiable, F_2 is valid

(b) F_1 is unsatisfiable, F_2 is satisfiable

(c) F_1 is unsatisfiable, F_2 is valid

(d) F_1 and F_2 are both satisfiable

[GATE-2001]

Q.45 "If X then Y unless Z" is represented by which of the following formulas in propositional logic? ("¬" is negative, "∧" is conjunction, and "→" is implication)

(a) $(X \wedge \neg Z) \rightarrow Y$ (b) $(X \wedge Y) \rightarrow \neg Z$

(c) $X \rightarrow (Y \wedge \neg Z)$ (d) $(X \rightarrow Y) \wedge \neg Z$

[GATE-2002]

Q.46 Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable)

(a) $((\forall x)[\alpha] \Rightarrow (\forall x)[\beta]) \Rightarrow (\forall x)[\alpha \Rightarrow \beta]$

(b) $(\forall x)[\alpha] \Rightarrow (\exists x)[\alpha \wedge \beta]$

(c) $(\forall x)[\alpha \vee \beta] \Rightarrow (\exists x)[\alpha] \Rightarrow (\forall x)[\alpha]$

(d) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha] \Rightarrow (\forall x)[\beta])$

[GATE-2003]

Q.47 The following resolution rule is used in logic programming: Derive clause (P ∨ Q) from clauses (P ∨ R), (Q ∨ ¬ R)

Which of the following statements related to this rule is FALSE?

(a) (P ∨ R) ∧ (Q ∨ ¬ R) ⇒ (P ∨ Q) is logically valid

(b) (P ∨ Q) ⇒ (P ∨ R) ∧ (Q ∨ ¬ R) is logically valid

(c) (P ∨ Q) is satisfiable if and only if (P ∨ R) ∧ (Q ∨ ¬ R) is satisfiable

(d) (P ∨ Q) ⇒ FALSE if and only if both P and Q are unsatisfiable

[GATE-2003]

Q.48 Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\neg s \rightarrow q)$

Q: $[(\neg p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\neg q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \neg r)] \rightarrow q$

Which of the above arguments are valid?

(a) P and Q only (b) P and R only

(c) P and S only (d) P, Q, R and S

[GATE-2004]

Q.49 What is the first order predicate calculus statement equivalent to the following? Every teacher is liked by some student

(a) $\forall(x)$ [teacher $(x) \rightarrow \exists(y)$ [student $(y) \rightarrow$ likes (y, x)]]

(b) $\forall(x)$ [teacher $(x) \rightarrow \exists(y)$ [student $(y) \wedge$ likes (y, x)]]

(c) $\exists(y) \forall(x)$ [teacher $(x) \rightarrow$ [student $(y) \wedge$ likes (y, x)]]

(d) $\forall(x)$ [teacher $(x) \wedge \exists(y)$ [student $(y) \rightarrow$ likes (y, x)]]

[GATE-2005]

Q.50 Let $P(x)$ and $Q(x)$ be arbitrary predicates. Which of the following statements is always TRUE?

(a) $(\forall x(P(x) \vee Q(x))) \Rightarrow ((\forall x P(x)) \vee (\forall x Q(x)))$

(b) $(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$

(c) $(\forall x(P(x) \Rightarrow (\forall x Q(x)))) \Rightarrow (\forall x(P(x) \Rightarrow Q(x)))$

(d) $((\forall x(P(x)) \Leftrightarrow (\forall x Q(x))) \Rightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

[IT-2005]

Q.51 A logical binary relation $\odot$, is defined as follows:

A	B	A $\odot$ B
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation (NOT) operator, with higher precedence, than $\odot$. Which one of the following is equivalent to A $\wedge$ B?

(a) (~ A $\odot$ B) (b) ~ (A $\odot$ ~ B)

(c) ~ (~ A $\odot$ ~ B) (d) ~ (~ A $\odot$ B)

[GATE-2006]

Q.52 Let Graph (x) be a predicate which denotes that x is a graph. Let Connected (x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement; "Not every graph is connected"?

(a) $\neg \forall x$ (Graph$(x) \Rightarrow$ Connected (x))

(b) $\exists x$ (Graph$(x) \wedge \neg$ Connected (x))

(c) $\neg \forall x$ ($\neg$ Graph$(x) \vee$ Connected (x))

(d) $\forall x$ (Graph$(x) \Rightarrow \neg$ Connected (x))

[GATE-2007]

Q.53 Let *fsa* and *pda* be two predicates such that *fsa*(x) means x is a finite state automaton, and *pda*(y) means, that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statement represents the following:

Each finite state automaton has an equivalent pushdown automaton.

(a) $(\forall x\ fsa(x) \Rightarrow (\exists y\ pda(y) \wedge$ equivalent $(x, y))$

(b) $\sim \forall y(\exists x\ fsa(x) \Rightarrow pda(y) \wedge$ equivalent $(x, y))$

(c) $\forall x\ \exists y\ (fsa(x) \wedge pda(y) \wedge$ equivalent $(x, y))$

(d) $\forall x\ \exists y\ (fsa(y) \wedge pda(x) \wedge$ equivalent $(x, y))$

[GATE-2008]

Q.54 Which of the following first order formulae is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable.

(a) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$

(b) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$

(c) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

(d) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

[IT-2008]

Q.55 Which of the following is the negation of

$[(\forall x \alpha \rightarrow (\exists y \beta \rightarrow (\forall u \exists v \gamma))]$

(a) $[\exists x \alpha \rightarrow (\forall y \beta \rightarrow (\exists u \forall v \gamma))]$

(b) $[\exists x \alpha \rightarrow (\forall y \beta \rightarrow (\exists u \forall v \neg\gamma))]$

(c) $[\forall x \neg\alpha \rightarrow (\exists y \neg\beta \rightarrow (\forall u \exists v \neg\gamma))]$

(d) $[\forall x \alpha \wedge (\exists y \beta \wedge (\exists u \forall v \neg\gamma))]$

[IT-2008]

Q.56 Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t\ (\neg F(x, y, t))$?

(a) Everyone can fool some person at some time

(b) No one can fool everyone all the time

(c) Everyone cannot fool some person all the time

(d) No one can fool some person at some time

[GATE-2010]

Q.57 What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(a) $\exists x$ (real $(x) \vee$ rational $(x))$

(b) $\forall x$ (real $(x) \rightarrow$ rational $(x))$

(c) $\exists x$ (real $(x) \wedge$ rational $(x))$

(d) $\exists x$ (rational $(x) \rightarrow$ real $(x))$

[GATE-2012]

Q.58 What is the logical translation of the following statements?

"None of my friends are perfect"

(a) $\exists x\,(F(x) \wedge \neg P(x))$ (b) $\exists x\,(\neg F(x) \wedge P(x))$

(c) $\exists x\,(\neg F(x) \wedge \neg P(x))$ (d) $\neg\exists x\,(F(x) \wedge P(x))$

[GATE-2013]

Q.59 Consider the statement:

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(a) $\forall x : \text{glitters}(x) \Rightarrow \neg\text{gold}(x)$

(b) $\forall x : \text{gold}(x) \Rightarrow \text{glitters}(x)$

(c) $\exists x : \text{gold}(x) \wedge \neg\text{glitters}(x)$

(d) $\exists x : \text{glitters}(x) \wedge \neg\text{gold}(x)$

[GATE-2014 (Set-1)]

Q.60 Which one of the following propositional logic formulas is TRUE when exactly two of p, q and r are **TRUE**?

(a) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(b) $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(c) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(d) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

[GATE-2014 (Set-1)]

Q.61 Consider the following statements:

P : Good mobile phones are not cheap

Q : Cheap mobile phones are not good

L : P implies Q

M : Q implies P

N : P is equivalent to Q

Which one of the following about L, M and N is **CORRECT**?

(a) Only L is TRUE

(b) Only M is TRUE

(c) Only N is TRUE

(d) L, M and N are TRUE

[GATE-2014 (Set-3)]

Q.62 The CORRECT formula for the sentence, "not all rainy days are cold" is

(a) $\forall d(\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

(b) $\forall d(\sim\text{Rainy}(d) \rightarrow \text{Cold}(d))$

(c) $\exists d(\sim\text{Rainy}(d) \rightarrow \text{Cold}(d))$

(d) $\exists d(\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

[GATE-2014 (Set-3)]

Q.63 The binary operator $\neq$ is defined by the following truth table.

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator $\neq$?

(a) Both commutative and associative

(b) Commutative but not associative

(c) Not commutative but associative

(d) Neither commutative nor associative

[GATE-2015 (Set-1)]

Q.64 Consider the following two statements:

S_1: If a candidate is known to be corrupt, then he will not be elected.

S_2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S_1 and S_2 as per sound inference rules of logic?

(a) If a person is known to be corrupt, he is kind

(b) If a person is not known to be corrupt, he is not kind

(c) If a person is kind, he is not known to be corrupt

(d) If a person is not kind, he is not known to be corrupt

[GATE-2015 (Set-2)]

Q.65 Which one of the following well formed formulae is a tautology?

(a) $\forall x\,\exists y\,R(x, y) \leftrightarrow \exists y\,\forall x\,R(x, y)$

(b) $(\forall x\,[\exists y\,R(x, y) \rightarrow S(x, y)]) \rightarrow \forall x\,\exists y\,S(x, y)$

(c) $[\forall x\,\exists y\,(P(x, y) \rightarrow R(x, y))]$
$\leftrightarrow [\forall x\,\exists y\,(\neg P(x, y) \vee R(x, y))]$

(d) $\forall x\,\forall y\,P(x, y) \rightarrow \forall x\,\forall y\,P(y, x)$

[GATE-2015 (Set-2)]

Q.66 Which one of the following well-formed formulae in predicate calculus is **NOT** valid?

(a) $(\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x\, q(x))$

(b) $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x(p(x) \vee q(x))$

(c) $\exists x(p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

(d) $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

[GATE-2016 (Set-2)]

Q.67 Consider the first-order logic sentence $F : \forall x(\exists y R(x,y))$. Assuming non-empty logical domains, which of the sentences below are implied by F?

I. $\exists y(\exists x\, R(x,y))$ II. $\exists y(\forall x\, R(x,y))$

III. $\forall y(\exists x\, R(x,y))$ IV. $\neg\exists x(\forall y \neg R(x,y))$

(a) IV only (b) I and IV only

(c) II only (d) II and III only

[GATE-2017 (Set-1)]

Q.68 Let p, q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is

(a) A tautology

(b) A contradiction

(c) Always TRUE when p is FALSE

(d) Always TRUE when q is TRUE

[GATE-2017 (Set-1)]

Q.69 Let p, q, r denote the statements "It is raining", "It is cold", and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(a) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

(b) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$

(c) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$

(d) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$

[GATE-2017 (Set-2)]

Q.70 The statement $(\neg\, p) \Rightarrow (\neg\, q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$ II. $q \Rightarrow p$

III. $(\neg\, q) \vee p$ IV. $(\neg\, p) \vee q$

(a) I only (b) I and IV only

(c) II only (d) II and III only

[GATE-2017]

Q.71 Consider the first order predicate formula φ:

$\forall x[(\forall z\, z \mid x \Rightarrow ((z = x) \vee (z = 1))) \Rightarrow \exists w\,(w > x) \wedge (\forall z\, z \mid w \Rightarrow ((w = z) \vee (z = 1)))]$

Here '$a \mid b$' denotes that 'a divides b', where a and b are integers. Consider the following sets:

S_1: {1, 2, 3, ..., 100}

S_2: Set of all positive integers

S_3: Set of all integers

Which of the above sets satisfy φ?

(a) S_1 and S_3 (b) S_2 and S_3

(c) S_1, S_2 and S_3 (d) S_1 and S_2

[GATE-2019]

Q.72 Which one of the following is true for the following propositional logic formula?

$(P \Rightarrow Q) \Rightarrow [(R \vee P) \Rightarrow (R \vee Q)]$

(a) Tautology (b) Inconsistent

(c) Satisfiable (d) Unsatisfiable

[JNU]

Q.73 Which one of the following inferences is logically correct?

(a) If it is a clear day then humidity is normally and if humidity is normal then children can play outdoors therefore if children can play outdoors then it is a clear day

(b) If it is a clear day then humidity is normal and on a given day it is clear therefore humidity is normal

(c) If it is a clear day then humidity is normal and given that humidity is not normal on a particular day therefore it is not a clear day

(d) If it is a clear day and if humidity is normal then children can play outdoors therefore children can play outdoors

[JNU]

Q.74 The negation of the statement $\forall x\, \exists y(p\,(x, y) \rightarrow q(x, y))$ is

(a) $\forall x\, \exists y(\neg p(x, y) \rightarrow q(x, y))$

(b) $\forall x\, \exists y(\neg p(x, y) \rightarrow \neg q(x, y))$

(c) $\exists x\, \forall y \neg(p(x, y) \rightarrow q(x, y))$

(d) $\exists x\, \forall y(\neg p(x, y) \rightarrow q(x, y))$

[JNU]

Q.75 Any set of Boolean operators that is sufficient to represent all Boolean expressions is said to be complete. Which of the following is not complete?

(a) {AND, NOT} (b) {NOT, OR}
(c) {AND, OR} (d) {NAND}

[JNU]

Q.76 Consider the following WFFs

1. $\forall x$ Purple $(x) \wedge$ Mushroom $(x) \rightarrow$ Poisonous (x)
2. $\forall x$ Purple $(x) \rightarrow$ (Mushroom $(x) \rightarrow$ Poisonous (x))
3. $\forall x$ Mushroom $(x) \rightarrow$ (Purple $(x) \rightarrow$ Poisonous (x))

Which of the following statements is false?

(a) 1 and 3 are equivalent
(b) 2 and 3 are equivalent
(c) 1, 2 and 3 are not equivalent
(d) 1 and 2 are equivalent

[JNU]

Q.77 Consider the following WFFs (Well Formed Formulas)

1. $\forall x$ Purple $(x) \wedge$ Mushroom $(x) \Rightarrow$ Poisonous (x)
2. $\forall x$ Purple $(x) \Rightarrow$ (Mushroom $(x) \Rightarrow$ Poisonous (x))
3. $\forall x$ Mushroom $(x) \Rightarrow$ (Purple $(x) \Rightarrow$ Poisonous (x))

How many of the above WFFs is logically equivalent to the statement, 'All purple mushrooms are poisonous'?

(a) 3 (b) 2
(c) 1 (d) 0

[JNU]

Q.78 Let $B(x, y)$ be the statement '*y is the best friend of x*' and let z be a person other than y. Then the logical expression of the statement 'Everyone has exactly one best friend' will be equal to

(a) $\forall x \forall y \forall z (B(x, y) \wedge ((z \neq y) \rightarrow \neg B(x, z)))$
(b) $\forall x \exists y \forall z (B(x, z) \wedge ((z \neq y) \rightarrow \neg B(x, y)))$
(c) $\forall x \exists y \forall z (B(x, y) \wedge ((z \neq y) \rightarrow \neg B(x, z)))$
(d) None of the above

[JNU]

Q.79 Identify the false statement:

(a) Let $Q(x, y)$ denote "$x + y = 0$". The truth value of the quantification $\exists y \forall x Q(x, y)$ is true
(b) The statements $\neg \exists x \forall y P(x, y)$ and $\forall x \exists y \neg P(x, y)$ have the same truth value
(c) $\neg (P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent
(d) $\forall x \forall y P(x, y) \Rightarrow \exists x \forall y P(x, y)$

[JNU]

Multiple Select Questions

Q.80 Which of the following are proposition?

(a) "Today is Monday".
(b) "What day is today"?
(c) "n is less than 100".
(d) "Two is less than four".

Q.81 Which of the following are correct for the statement "Everyone has exactly one relative"? Predicate $R(a, b)$ denotes b is the relative of a.

(a) $\exists a R(a, b) \wedge \forall c ((c \neq b) \rightarrow R(a, c))$
(b) $\forall a \exists! b R(a, b)$
(c) $\forall a \exists b (R(a, b) \wedge \forall c ((c \neq b) \rightarrow \neg R(a, c)))$
(d) $\forall a \exists b (R(a, b) \wedge \forall c ((a \neq c) \vee (a \neq b)))$

Q.82 Let $s(x)$ be "x in this society", $p(x)$ be "x has parking space" and $c(x)$ be "x has car". Which of the following can be inferred for the premises "A person in this society has no car and everyone in the society has parking space?"

(a) $\forall x (s(x) \wedge c(x) \rightarrow \neg s(x))$
(b) $\exists x (s(x) \wedge \neg c(x)) \wedge \forall x (s(x) \rightarrow p(x))$
(c) $\forall x (p(x) \rightarrow c(x))$
(d) $\neg \forall x (\neg s(x) \vee c(x)) \wedge \forall x (s(x) \rightarrow p(x))$

Q.83 Which of the following logical expression are correctly translated for the given statement?

(a) "There are atmost two apples" : $\forall x \forall y \forall z$ ((Apple$(x) \wedge$ Apple$(y) \wedge$ Apple$(z)) \rightarrow (x = y \vee x = z \vee y = z)$).
(b) "There are exactly two apples" : $\exists x \exists y$ (Apple$(x) \wedge$ Apple$(y) \wedge (x \neq y) \wedge \forall z$(Apple$(z)$ $\rightarrow ((z = x) \vee (z = y))))$.

(c) "There are atmost one apple" : $\forall x\ \forall y$ ((Apple(x) $\wedge$ Apple(y)) $\rightarrow$ ($x = y$).
(d) "There are exactly one apple" : $\exists x$ (Apple(x) $\wedge \forall y$ (Apple(y) $\rightarrow$ ($x = y$))).

Q.84 Choose the correct choice(s) regarding the following propositional logic assertion S:

S : $((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$

(a) S is a tautology.
(b) S is neither a tautology nor a contradiction.
(c) The antecedent of S is logically equivalent to the consequent of S.
(d) S is a contradiction.

[GATE : 2021 (Set-2)]

Try Yourself

T1. The proposition $[(p \wedge q) \rightarrow (p \vee q)] \vee \sim p \vee q$ is
(a) a tautology
(b) satisfiable but not tautology
(c) a contradiction
(d) None of the above

[Ans: (a)]

T2. Consider the following predicate statements:

P_1 : $\neg\forall x \neg(P(x) \rightarrow \exists y\ Q(y))$

P_2 : $\exists x(\neg P(x) \vee \exists y\ Q(y))$

P_3 : $\exists x(\neg\exists y Q(y) \rightarrow \neg P(x))$

P_4 : $\neg\forall x(P(x) \wedge \neg\exists y\ Q(y))$

Which of the above predicates are equivalent to the predicate statement:

$$\exists x(P(x) \rightarrow \exists y\ Q(y))$$

(a) P_1, P_2, P_3 (b) P_1, P_3, P_2
(c) P_2, P_3, P_4 (d) All of these

[Ans: (d)]

T3. Which one of the first order predicate calculus statements given below correctly expresses the following English statement? (Domain consists of all people; $F(x)$: x is a female, $M(x, y)$: x is mother of y, $P(x)$: x is a parent).
"If a person is female and is a parent, then this person is someone's mother"

(a) $\exists x\big((F(x) \wedge P(x)) \rightarrow \forall y\, M(x,y)\big)$
(b) $\forall x \exists y\big((F(x) \wedge P(x)) \rightarrow M(x,y)\big)$
(c) $\forall x\big((F(x) \vee P(x)) \rightarrow \exists y\, M(x,y)\big)$
(d) $\forall x \exists y\big((F(x) \vee P(x)) \rightarrow M(x,y)\big)$

[Ans: (b)]

T4. Identify the contrapositive for the following statement. "If a real number is greater than 2, then its square is greater than 4."
(a) $\forall x \in$ R, if $x^2 > 4$ then $x < 2$
(b) $\forall x \in$ R, if $x^2 \le 4$ then $x \le 2$
(c) $\forall x \in$ R, if $x \le 2$ then $x^2 \le 4$
(d) None of these

[Ans: (b)]

T5. Find the predicate logic for the following statement. There are atmost two apples.
(a) $\forall x \forall y$((Apple (x) $\wedge$ Apple (y)) $\rightarrow$ ($x = y \vee y = x$))
(b) $\exists x \exists y$ (Apple (x) $\wedge$ Apple (y) $\wedge\ x \neq y \wedge \forall z$(Apple ($z$) $\rightarrow$ ($z = x \vee z = y$)))
(c) $\forall x \forall y \forall z$((Apple ($x$) $\wedge$ Apple (y) $\wedge$ Apple (z)) $\rightarrow$ ($x = y \vee x = z \vee y = z$))
(d) None of these

[Ans: (c)]

T6. Consider the following logical inferences
I_1 : If Socrates is human, then Socrates is mortal. Socrates is human.
Inference: Socrates is mortal.
I_2 : If it rains today, MADE EASY will close. MADE EASY is not closed today.
Inference: It will not rain today.

Which of the following is true?
(a) Both I_1 and I_2 are correct inferences.
(b) I_1 is correct but I_2 is not a correct inference.
(c) I_1 is not correct but I_2 is a correct inference.
(d) Both I_1 and I_2 are not correct inferences.

[Ans: (a)]

T7. Which of the following is/are tautology?
(a) $(p \rightarrow q) \rightarrow (q \rightarrow p)$
(b) $\sim(p \rightarrow q) \rightarrow \sim q$
(c) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
(d) Both (b) and (c)

[Ans: (d)]

T8. Which of the following is not correct.

(a) $\sim(\forall x)\,(\exists y)\,P(x, y) \equiv (\exists x)\,(\forall y)\,[\sim P(x, y)]$

(b) $\sim(\forall x)\,P(x) \equiv \exists x\,[\sim P(x)]$

(c) $\sim(\exists x)\,(\forall y)\,[P(x, y) \vee Q(x, y)] \equiv (\forall x)\,(\exists y)\,[\sim P(x, y) \wedge \sim Q(x, y)]$

(d) None of these

[Ans: (d)]

T9. The predicate statement $\neg\forall z\,[P(z) \rightarrow (\neg\, Q(z) \rightarrow P(z))]$ is _____.

(a) Satisfiable (b) Tautology

(c) Contradiction (d) None of the above

[Ans: (c)]

T10. Which of the following statement is not correct?

(a) Every valid propositional formula is satisfiable

(b) Every satisfiable is not tautology

(c) Every contradiction is not satisfiable

(d) None of the above

[Ans: (b)]

T11. Match the following Lists:

List-I

A. There are atmost two apples

B. There are exactly two apples

C. There is atmost one apple

D. There is exactly one apple

List-II

1. $\forall x\,\forall y\,\forall z\,((\text{Apple}\,(x) \wedge \text{Apple}\,(y) \wedge \text{Apple}\,(z)) \rightarrow (x = y \vee x = z \vee y = z))$
2. $\forall x\ \forall y\ ((\text{Apple}\ (x) \wedge \text{Apple}\ (y)) \rightarrow (x = y \vee y = x))$
3. $\exists x\,\exists y\,(\text{Apple}\,(x) \wedge \text{Apple}\,(y) \wedge (x \neq y) \wedge \forall z\,(\text{Apple}\,(z) \rightarrow ((z = x) \vee (z = y))))$
4. $\exists x\,(\text{Apple}\,(x) \wedge \forall y\,(\text{Apple}\,(y) \rightarrow (x = y)))$

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	2	1	4
(c)	1	3	2	4
(d)	3	1	2	4

[Ans: (c)]

T12. Consider the following statement over the domain of natural number.

"No prime number except 7 is divisible by 7"

Find the equivalent predicate logic for the above statement.

(a) $\forall x \in \text{N}[x \neq 7 \wedge \text{Prime}\,(x) \rightarrow (\neg\text{Divisible by 7}\,(x))]$

(b) $\neg\exists x \in \text{N}[x \neq 7 \wedge \text{Prime}\,(x) \wedge \text{Divisible by 7}\,(x)]$

(c) Both (a) and (b)

(d) None of these

[Ans: (c)]

T13. Match the following Lists:

List-I

A. Everybody loves Mahesh

B. Everybody loves somebody

C. There is somebody whom everybody loves

D. There is somebody whom no one loves

List-II

1. $\forall x$ Loves $(x,$ Mahesh$)$
2. $\forall x\,\exists y$ Loves (x, y)
3. $\exists y\,\forall x$ Loves (x, y)
4. $\exists y\,\forall x \neg$ Loves (x, y)

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	1	4	3	2
(d)	1	2	4	3

[Ans: (a)]

T14. Which of the following predicate arguments is valid?

(a) $\forall x\ P(x) \rightarrow \forall x[P(x) \vee Q(x)]$

(b) $\exists x\ \exists y\ P(x, y) \rightarrow \exists y\ \exists x\ P(x, y)$

(c) $\exists x\,[R(x) \vee S(x)] \rightarrow \exists x\ R(x) \vee \exists x\ S(x)$

(d) All of these

[Ans: (d)]

T15. Consider the following formula:

$$\neg\,(\neg\, p \vee q) \vee (r \rightarrow \neg\, s)$$

Which of the following is equivalent to the above CNF formula?

(a) $(p \wedge \neg\, r \vee \neg\, s) \wedge (\neg\, q \vee \neg\, r \wedge \neg\, s)$

(b) $(p \vee \neg\, r \vee \neg\, s) \wedge (\neg\, q \vee \neg\, r \vee \neg\, s)$

(c) $(p \wedge \neg\, r \wedge \neg\, s) \vee (\neg\, q \wedge \neg\, r \wedge \neg\, s)$

(d) None of these

[Ans: (b)]

T16. Consider the following statements:

P_1 : $\exists x\ \exists y\ (x \neq y \wedge \forall z\ (\text{Apple}\ (z) \leftrightarrow ((z = x) \vee (z = y)))$

P_2 : $\exists x\ \exists y\ (\text{Apple}(x) \wedge \text{Apple}\ (y) \wedge x \neq y) \wedge \forall x\ \forall y\ \forall z\ ((\text{Apple}(x) \wedge \text{Apple}\ (y) \wedge \text{Apple}\ (z)) \rightarrow (x = y \vee x = z \vee y = z))$

P_3 : $\exists x\ \exists y\ (\text{Apple}\ (x) \wedge \text{Apple}\ (y) \wedge (x \neq y) \wedge \forall z\ (\text{Apple}\ (z) \rightarrow (z = x) \vee (z = y)))$

Which of the predicate logic statements represent the following statement.

"there are exactly two apples".

(a) P_1 and P_2 only
(b) P_1 and P_3 only
(c) P_2 and P_3 only
(d) P_1, P_2 and P_3

[Ans: (d)]

T17. Consider the following predicates for the domain of real numbers

$P(x, y) : x > y$
$Q(x, y) : x \leq y$
$R(x) : x - 7 = 2$
$S(x) : x > 9$

Which of the following proposition gives the False as the truth value?

(a) $\exists x\ R(x) \vee \forall y\ [\sim S(y)]$
(b) $\forall x\ \exists y\ P(x,y)$
(c) $\forall x\ \exists y\ [P(x, y) \vee Q(x, y)]$
(d) None of these

[Ans: (d)]

T18. Let p, q, r, s represent the following propositions.

p : $x \in \{8, 9, 10, 11, 12\}$
q : x is a composite number
r : x is a perfect square
s : x is a prime number

The integer $x \geq 2$ which satisfies $\neg ((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is _____.

[GATE-2016, Ans: (11)]

■■■■

2 Combinatorics and Recurrence Relations

Multiple Choice Questions

Q.1 In how many ways can seven girls be seated at a round table so that two particular girls are separated?
(a) 120 (b) 260
(c) 320 (d) 480
(e) 560

Q.2 A team of 11 is to be selected out of 14 players of whom 5 are bowlers. Find the number of ways in which this can be done so as to include at least 4 bowlers.
(a) 264 (b) 180
(c) 84 (d) 50
(e) 17

Q.3 How many odd number having four digits and with no repeated digits can be formed from {1, 2, 3, 4, 5, 6, 7, 8, 9}?
(a) 1860 (b) 1680
(c) 8610 (d) 6801
(e) 1068

Q.4 The m^{th} term of an A.P. is n and the n^{th} term is m. The $(m + n)^{th}$ term would be
(a) m (b) n
(c) $(m + n)$ (d) $(m - n)$
(e) zero

Q.5 The n^{th} term of a series is given to be $(3 + n)/4$. Find the sum of 105 terms of this series.
(a) 1575 (b) 2730
(c) 1470 (d) 2835
(e) 5460

Q.6 A candidate is required to answer 7 out of 12 questions, which are divided in to two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. In how many different ways can he choose the 7 questions?
(a) 180 (b) 210
(c) 600 (d) 780
(e) 792

Q.7 Consider the three places Hyderabad, Nanded and Nagapur. If there are 3 routes from Hyderabad to Nanded and 2 from Nanded to Nagapur in how many different ways can a person travel from Hyderabad to Nagapur via Nanded?
(a) 1 (b) 3
(c) 2 (d) 5
(e) 6

Q.8 A lock contains three rings, each ring containing four letters. If the lock opens in only one arrangement of three letters, how many unsuccessful events are possible?
(a) 144 (b) 63
(c) 24 (d) 121
(e) 17

Q.9 Find the number of ways in which 8 boys and 6 girls are to sit for a lunch at a round table so that no two girls are to sit together.
(a) $7! \, {}^8P_6$ (b) $8! \, 6!$
(c) $8! \, {}^8C_6$ (d) $7!$
(e) $8!$

Q.10 In how many different ways can 8 different shirts be distributed among 4 different people so that each receives 2 shirts?

(a) 2850 (b) 2680
(c) 2520 (d) 1908
(e) 1800

Q.11 If the sum of first n terms of an A.P. is $2n^2 + 3n$, then n^{th} term is
(a) n (b) $n + 1$
(c) $2n + 1$ (d) $3n + 1$
(e) $4n + 1$

Q.12 In a football championship, 153 matches were played. Every two teams played one match with each other. The number of teams participating in the championship is
(a) 15 (b) 16
(c) 17 (d) 18
(e) 19

Q.13 A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. The probability that one of them is black and the other is white is
(a) $\frac{2}{3}$ (b) $\frac{4}{5}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

[GATE-1995]

Q.14 The number of functions from an m elements set to an n element set is
(a) $m + n$ (b) m^n
(c) n^m (d) $m * n$

[GATE-1998]

Q.15 In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada whereas 13 persons speak both Kannada and English. How many people speak all three languages?
(a) 9 (b) 8
(c) 7 (d) 6

[GATE-1998]

Q.16 The number of binary relations on a set with n elements is
(a) n^2 (b) 2^n
(c) 2^{n^2} (d) None of these

[GATE-1999]

Q.17 The number of binary strings of n zeroes and k ones that no two ones are adjacent is
(a) ${}^{n-1}C_k$ (b) ${}^nC_k$
(c) ${}^nC_{k+1}$ (d) None of these

[GATE-1999]

Q.18 Two girls have picked 10 lilies, 15 sunflowers and 14 daffodils. What is the number of ways they can divide the flowers amongst themselves?
(a) 1638 (b) 2100
(c) 2640 (d) None of these

[GATE-1999]

Q.19 The minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from some same suit is
(a) 3 (b) 8
(c) 9 (d) 12

[GATE-2000]

Q.20 How many 4 digit even numbers have all 4 digits distinct?
(a) 2240 (b) 2296
(c) 2620 (d) 4536

[GATE-2001]

Q.21 The solution to the recurrence equation
$T(2^k) = 3T(2^{k-1}) + 1$,
$T(1) = 1$ is
(a) 2^k (b) $(3^{k+1} - 1)/2$
(c) $3^{\log_2 k}$ (d) $2^{\log_3 k}$

[GATE-2002]

Q.22 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that (i) each is sorted in ascending order, (ii) B has 5 and C has 3 elements, and (iii) the result of merging B and C gives A?
(a) 2 (b) 30
(c) 56 (d) 256

[GATE-2003]

Q.23 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

(a) $\binom{2n}{n} * 2^n$ (b) 3^n

(c) $\frac{(2n)!}{2^n}$ (d) $\binom{2n}{n}$

[GATE-2003]

Q.24 Mala has a colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour-pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of k that satisfies this requirement?

(a) 9 (b) 8

(c) 7 (d) 6

[GATE-2004]

Q.25 In how many ways can we distribute 5 distinct balls, B_1, B_2,..., B_5 in 5 distinct cells, C_1, C_2, ..., C_5 such that Ball B_i is not in cell $C_i, \forall_i = 1, 2,..., 5$ and each cell contains exactly one ball?

(a) 44 (b) 96

(c) 120 (d) 3125

[IT-2004]

Q.26 Let $n = p^2q$, where p and q are distinct prime numbers. How many numbers m satisfy $1 \le m \le n$ and $gcd(m, n) = 1$? Note that $gcd(m, n)$ is the greatest common divisor of m and n.

(a) $p(q-1)$ (b) pq

(c) $(p^2-1)(q-1)$ (d) $p(p-1)(q-1)$

[IT-2005]

Q.27 What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that $a \equiv c \bmod 3$ and $b \equiv d \bmod 5$

(a) 4 (b) 6

(c) 16 (d) 24

[GATE-2005]

28. Let $G(x) = 1/(1-x)^2 = \sum_{i=0}^{\infty} g(i)x^i$ where $|x| < 1$. What is $g(i)$?

(a) i (b) $i + 1$

(c) $2i$ (d) 2^i

[GATE-2005]

Common Data for Q.29 & Q.30

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i + 1, j)$ or $(i, j + 1)$.

Q.29 How many distinct paths are there for the robot to reach the point (10, 10) starting from the initial position (0, 0)?

(a) $\binom{20}{10}$ (b) 2^{20}

(c) 2^{10} (d) None of these

[GATE-2007]

Q.30 Suppose that the robot is not allowed to traverse the line segment from (4, 4) to (5, 4). With this constraint, how many distinct paths are there for the robot to reach (10, 10) starting from (0, 0)?

(a) 2^9 (b) 2^{19}

(c) $\binom{8}{4} \times \binom{11}{5}$ (d) $\binom{20}{10} - \binom{8}{4} \times \binom{11}{5}$

[GATE-2007]

Q.31 The exponent of 11 in the prime factorization of 300! is

(a) 27 (b) 28

(c) 29 (d) 30

[IT-2008]

Q.32 In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

(a) $\frac{(n+b-1)!(n+r-1)!}{(n-1)!\,b!(n-1)!\,r!}$

(b) $\frac{(n+(b+r)-1)!}{(n-1)!(n-1)!\,(b+r)!}$

(c) $\frac{n!}{b!\,r!}$

(d) $\frac{(n+b+r)-1)!}{n!\,(b+r-1)}$

[GATE-2008]

Q.33 Let a_n be the number of n-bit strings that do NOT contain two consecutive 1's. Which one of the following is the recurrence relation for a_n?

(a) $a_n = a_{n-1} + 2a_{n-2}$
(b) $a_n = a_{n-1} + a_{n-2}$
(c) $a_n = 2a_{n-1} + a_{n-2}$
(d) $a_n = 2a_{n-1} + 2a_{n-2}$

[GATE-2016 (Set-1)]

Q.34 Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, ...$?

(a) $\frac{3}{(1-x)^2}$ (b) $\frac{3x}{(1-x)^2}$

(c) $\frac{2-x}{(1-x)^2}$ (d) $\frac{3-x}{(1-x)^2}$

[GATE-2018]

Q.35 A florist sells roses of five different colours. How many bunches of a half-dozen roses can be formed?

(a) 196 (b) 210
(c) 236 (d) 300

[JNU]

Q.36 Evaluate the sum $1 \times 3 + 2 \times 4 + 3 \times 5 + ...$upto n terms

(a) $\frac{n(n+1)(2n+7)}{6}$ (b) $\frac{n(n+1)}{3}$

(c) $n^2(n+1)^2$ (d) $\frac{n^2(2n+1)}{6}$

[JNU]

Q.37 The number of ways, the numbers 1 to 15 are assigned to 25 squares of 5 by 5 grid having 10 squares empty, is

(a) 25! (b) 15!

(c) $\frac{25!}{15!}$ (d) $\frac{25!}{10!}$

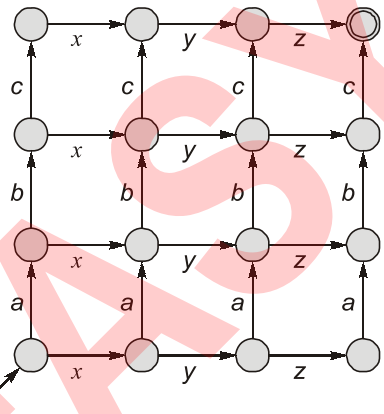
[JNU]

Q.38 Let $A = \frac{(2n)!}{(n!)^2 2!}, n \geq 1$, $B = \frac{(2n)!}{(2!)^n n!}, n \geq 1$ which of these is always an integer?

(a) A only (b) B only
(c) A and B (d) Neither

[JNU]

Q.39 The following finite automaton recognizes a set of strings of length 6. What is the total number of strings in the set?



(a) 18 (b) 16
(c) 32 (d) None of these

[JNU]

Q.40 Consider the following recurrence relation $S(k) - 10\,S(k-1) + 9\,S(k-2) = 0$, with the initial conditions $S(0) = 3$, $S(1) = 11$.
The solution of the recurrence relation is

(a) $1 + 2^k + 9^{k-1}$ (b) $3 + 8^k$
(c) $2 + 9^k$ (d) $1 + 3^k + 11^{k-1}$

[JNU]

Q.41 The maximum number of distinct subwords of the word AXIOMATIZABLE is

(a) 183 (b) 111
(c) 92 (d) 88

[JNU]

Q.42 Suppose the population of a colony of ants doubles every six months successively. If a colony is established with an initial population of 37 ants, how many ants will the colony have after 5 years?

(a) 9475 (b) 21439
(c) 37888 (d) None of these

[JNU]

Q.43 The number of positive integral solutions of $x + y + z + t = 20$ is

(a) $^{19}C_2$ (b) $^{19}C_3$
(c) $^{18}C_3$ (d) $^{18}C_2$

[JNU]

Q.44 Let A be a finite non-empty set with cardinality n. The number of subsets $S \subseteq A$ having odd cardinality is

(a) n (b) $2^{n/2}$
(c) 2^{n-1} (d) None of these

[JNU]

Q.45 How many routes are there from the lower-left corner of an 8×8 square-grid to the upper-right corner if we are restricted to travelling only to the right or upwards?

(a) 64 (b) 12870
(c) 12800 (d) 512

[JNU]

Q.46 How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

(a) 128 (b) 160
(c) 174 (d) None of these

[JNU]

Q.47 How many positive integers not exceeding 1000 are divisible by 7 or 11?

(a) 192 (b) 216
(c) 220 (d) 228

[JNU]

Numerical Answer Type Questions

Q.48 Compute the coefficient value of $x^3 y^2 z$ in the $(2x - y + 3z)^6$

Q.49 Suppose the group of twelve consists of 5 men and 7 women. How many five-person teams contain atleast one man?

Q.50 How many number of 5 letter words that use letters from the 3 letter set $\{a, b, c\}$ in which each letter occurs at least once?

Q.51 Find the largest integer value of x satisfying the inequality $^{10}C_{x-1} < 2(^{10}C_x)$.

Q.52 A group consists of 4 women and 7 men. In how many ways can a team of 5 members be selected when the team has atleast 3 women?

Q.53 Find the number of seven digit integers with sum of the digits equal to 11 and formed by using the digits 1, 2 and 3 only.

Q.54 Find the number of odd integers between 1000 and 9999 have distinct digits.

Q.55 Find the number of ways 4 fruits can be chosen out of 8 fruits so as to exclude the largest fruit.

Q.56 A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennants. The set of all possible 1-pennant is {(1)}, the set of all possible 2-pennants is {(2), (1,1)} and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10-pennants is _____.

[GATE-2014 (Set-1)]

Q.57 The number of distinct positive integral factors of 2014 is _____.

[GATE-2014 (Set-2)]

Q.58 $\sum_{x=1}^{99} \frac{1}{x(x+1)} =$ _____.

[GATE-2015 (Set-1)]

Q.59 Let a_n represent the number of bit strings of length n containing two consecutive 1s. What is the recurrence relation for a_n?

(a) $a_{n-2} + a_{n-1} + 2^{n-2}$
(b) $a_{n-2} + 2a_{n-1} + 2^{n-2}$
(c) $2a_{n-2} + a_{n-1} + 2^{n-2}$
(d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

[GATE-2015 (Set-1)]

Q.60 The number of divisors of 2100 is _____.

[GATE-2015 (Set-2)]

Q.61 The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + \ldots)^3$ is _____.

[GATE-2016 (Set-1)]

Q.62 Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is _____.

[GATE-2016 (Set-1)]

Q.63 If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to _____.

[GATE-2017]

Multiple Select Questions

Q.64 Which of the following integer value of x satisfying the inequality ${}^{10}C_{x-1} < 2 * {}^{10}C_x$?

(a) 6 (b) 7
(c) 8 (d) 9

Q.65 Which of the following are correct?

(a) If N object are placed into k boxes then there is at least one box containing $\left\lfloor \frac{N}{k} \right\rfloor + 1$ objects.

(b) $\sum_{k=0}^{n} {}^nC_k = 2^{n-1}$

(c) ${}^{n+1}C_k = {}^nC_{k-1} + {}^nC_k$, n and k positive integers with $n \geq k$.

(d) ${}^{m+n}C_k = \sum_{r=0}^{k} {}^mC_{k-r} \times {}^nC_r$, m, n and r non-negative integers $r \leq m$ and n.

Q.66 Which of the following are correct?

(a) In an MCQ exam of 150 questions each questions can be answered with any one of four choices then total $(150)^4$ ways in which they can be answered.

(b) A group consists of 7 male and 4 female. There are only 91 ways in which a team of 5 members be selected provided the team has atleast 3 women.

(c) There are total 1260 ways of arranging the letters "GGGAATTECSS" in a row when no two G's are together.

(d) None of the above are correct.

Try Yourself

T1. The number of ways of arranging the letters GGGGGAAATTTECCS in a row when no two T's are together is

(a) $\frac{12!}{5!3!2!} \times {}^{13}P_3$ (b) $\frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}$

(c) $\frac{15!}{5!3!2!} - \frac{13!}{5!3!2!}$ (d) $\frac{15!}{5!3!3!2!} - 3!$

[Ans: (b)]

T2. $\sum_{k=0}^{n} \binom{n}{k} (-1)^k \cdot 3^{n-k}$ = _____.

(a) 1 (b) 2^n
(c) 4^n (d) None of these

[Ans: (b)]

T3. In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is _____.

(a) $\frac{9!}{2} + 1$ (b) $\frac{9!}{2} - 1$
(c) 2^8 (d) $2^8 - 1$

[Ans: (c)]

T4. Consider the following recurrence relation $T(n) - 4T(n-1) + 3T(n-2) = 0$, $T(0) = 0$, $T(1) = 2$. What is the solution of $T(n)$?

(a) $3^n + 4^n$ (b) $3^n - 1$
(c) $4^n - 1$ (d) None of these

[Ans: (b)]

T5. How many solutions are there of $x + y + z = 17$ in positive integers?

(a) 120 (b) 171
(c) 180 (d) 221

[Ans: (a)]

T6. A set of *mn* objects can be partitioned into 'm' sets of size '*n*' in _____ different ways.

(a) $\frac{m!}{(n!)^m}$ (b) $\frac{(mn)!}{(n!)^m m!}$

(c) $\frac{(mn)!}{(n!)^m}$ (d) None of these

[Ans: (b)]

T7. Solve the following recurrence relation.

$T(n) = 3\,T(n-1) + 2^n$, $n > 0$ and $T(0) = 1$

(a) $3^{n+1} - 2^{n+1}$ (b) $3^n - 2^n$

(c) $3^{n+1} - 2^n$ (d) $3^{n+1} + 2^n$

[Ans: (a)]

T8. There are 65 questions and each question can be answered with any one of four choices. Find the number of ways, in which they can be answered.

(a) 65 (b) 65^4

(c) 4^{65} (d) None of these

[Ans: (c)]

T9. Solve the following recurrence relation.

$T(n) = 10.T(n-1) - 25\,T(n-2)$; if $n > 1$

$= 5$; if $n = 0$ or $n = 1$

(a) $5^n + 2n5^n$ (b) $5^{n+1} - 2n5^n$

(c) $5^{n+1} - 4n5^n$ (d) $5^n - 2n5^n$

[Ans: (c)]

T10. The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + \ldots)^3$ is _____.

[GATE-2016, Ans: (10)]

3 Set Theory and Algebra

Multiple Choice Questions

Q.1 Let A be a finite set of size n. The number of elements in the power set A × A is
(a) 2^{2^n} (b) 2^{n^2}
(c) $(2^n)^2$ (d) $(2^2)^n$
(e) None of the above
[GATE-1993]

Q.2 The less-than relation, <, on reals is
(a) a partial ordering since it is asymmetric and reflexive
(b) a partial ordering since it is antisymmetric and reflexive
(c) not a partial ordering because it is not asymmetric and not reflexive
(d) not a partial ordering because it is not antisymmetric and reflexive
(e) None of the above
[GATE-1993]

Q.3 Let A and B be sets with cardinalities m and n respectively. The number of one-one mappings (injections) from A to B, when $m < n$, is
(a) m^n (b) ${}^nP_m$
(c) ${}^mC_n$ (d) ${}^nC_m$
(e) ${}^mP_n$
[GATE-1993]

Q.4 Let S be an infinite set and S_1 ..., S_n be sets such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$. Then,
(a) at least one of the set S_i is a finite set
(b) not more than one of the sets S_i can be finite
(c) at least one of the sets S_i is an infinite set
(d) not more than one of the sets S_i can be infinite
(e) None of the above
[GATE-1993]

Q.5 Some group (G, o) is known to be abelian. Then, which one of the following is true for G?
(a) $g = g^{-1}$ for every $g \in G$
(b) $g = g^2$ for every $g \in G$
(c) $(goh)^2 = g^2 o h^2$ for every $g, h \in G$
(d) G is of finite order
[GATE-1994]

Q.6 Let R be a symmetric and transitive relation on a set A. Then
(a) R is reflexive and hence an equivalence relation
(b) R is reflexive and hence a partial order
(c) R is reflexive and hence not an equivalence relation
(d) None of the above
[GATE-1995]

Q.7 The number of elements in the power set P(S) of the set S = {(ϕ), 1, (2, 3)} is
(a) 2 (b) 4
(c) 8 (d) None of these
[GATE-1995]

Q.8 Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. The set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
(a) $A \cup B$ (b) $A^c \cup B^c$
(c) $A \cap B$ (d) $A^c \cap B^c$
[GATE-1996]

Q.9 Let X = {2, 3, 6, 12, 24}, Let ≤ be the partial order defined by X ≤ Y iff X/Y. The number of edges in the Hasse diagram of (X, ≤) is

(a) 3 (b) 4
(c) 9 (d) None of these

[GATE-1996]

Q.10 Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this one can conclude that
(a) $|X| = 1, |Y| = 97$
(b) $|X| = 97, |Y| = 1$
(c) $|X| = 97, |Y| = 97$
(d) None of the above

[GATE-1996]

Q.11 Which of the following statements is false?
(a) The set of rational numbers is an Abelian group under addition
(b) The set of integers is an Abelian group under addition
(c) The set of rational numbers form an Abelian group under multiplication
(d) The set of real numbers excluding zero in an Abelian group under multiplication

[GATE-1996]

Q.12 Let R denotes the set of real numbers. Let $f: R \times R \to R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by

(a) $f^{-1}(x, y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

(b) $f^{-1}(x, y) = (x - y, x + y)$

(c) $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

(d) $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

[GATE-1996]

Q.13 Let R be a non-empty relation on a collection of sets defined by $A\ R\ B$ if and only if $A \cap B = \phi$. Then, (pick the true statement)
(a) R is reflexive and transitive
(b) R is symmetric and not transitive
(c) R is an equivalence relation
(d) R is not reflexive and not symmetric

[GATE-1996]

Q.14 Which one of the following is false?
(a) The set of all bijective functions on a finite set forms a group under function composition
(b) The set $\{1, 2 \ldots, p-1\}$ forms a group under multiplication mod p where p is a prime number
(c) The set of all strings over a finite alphabet forms a group under concatenation
(d) A subset $s \neq \phi$ of G is a subgroup of the group <G, *> if and only if, for any pair of elements $a, b \in s$, $a * b^{-1} \in s$

[GATE-1996]

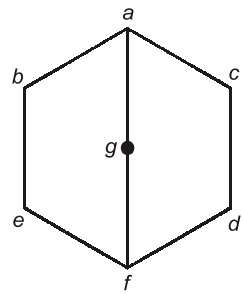
Q.15 Let $*$ be defined as $x * y = \bar{x} + y$. Let $z = x * y$. Value of $z * x$ is
(a) $\bar{x} + y$ (b) x
(c) 0 (d) 1

[GATE-1997]

Q.16 Let $(Z, *)$ be an algebraic structure, where Z is the set of integers and the operation $*$ is defined by $n * m$ = maximum (n, m). Which of the following statements is true for $(Z, *)$?
(a) $(Z, *)$ is a monoid
(b) $(Z, *)$ is an Abelian group
(c) $(Z, *)$ is a group
(d) None of the above

[GATE-1997]

Q.17 In the lattice defined by the Hasse diagram given in following Figure, how many complements does the element 'e' have?



(a) 2 (b) 3
(c) 0 (d) 1

[GATE-1997]

Q.18 The number of equivalence relations of the set {1, 2, 3, 4} is
(a) 15 (b) 16
(c) 24 (d) 4
[GATE-1997]

Q.19 A partial order ≤ is defined on the set S = {x, a_1, a_2, ... a_n, y} as $x \le a_i$ for all i and $a_i \le y$ for all i, where $n \ge 1$. The number of total orders on the set S which contain the partial order ≤ is
(a) $n!$ (b) $n + 2$
(c) n (d) 1
[GATE-1997]

Q.20 Suppose A is a finite set with n elements. The number of elements in thelargest equivalence relation of A is
(a) n (b) n^2
(c) 1 (d) $n + 1$
[GATE-1998]

Q.21 Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:
1. $R_1 \cup R_2$ is an equivalence relation
2. $R_2 \cap R_2$ is an equivalence relation
Which of the following is correct?
(a) Both assertions are true
(b) Assertion 1 is true but Assertion 2 is not true
(c) Assertion 2 is true but Assertion 1 is not true
(d) Neither 1 nor 2 is true
[GATE-1998]

Q.22 The binary relation R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)} on the set A = {1, 2, 3, 4} is
(a) reflexive, symmetric and transitive
(b) neither reflexive, nor irreflexive but transitive
(c) irreflexive, symmetric and transitive
(d) irreflexive and antisymmetric
[GATE-1998]

Q.23 Let L be a set with a relation R which is transitive, anti-symmetric and reflexive and for any two elements $a, b \in L$ let the least upper bound lub (a, b) and the greatest lower bound glb (a, b) exist. Which of the following is/are true?
(a) L is a poset
(b) L is a Boolean algebra
(c) L is a lattice
(d) None of the above
[GATE-1999]

Q.24 A relation R is defined on the set of integers as $x\ Ry$ iff $(x + y)$ is even. Which of the following statements is true?
(a) R is not an equivalence relation
(b) R is an equivalence relation having 1 equivalence class
(c) R is an equivalence relation having 2 equivalence classes
(d) R is an equivalence relation having 3 equivalence classes
[GATE-2000]

Q.25 Let P(S) denotes the power set of set S. Which of the following is always true?
(a) P(P(S)) = P(S) (b) $P(S) \cap P(P(S)) = \{\phi\}$
(c) $P(S) \cap S = P(S)$ (d) $S \notin P(S)$
[GATE-2000]

Q.26 Consider the following relations:
R_1 (a, b) iff $(a + b)$ is even over the set of integers
R_2 (a, b) iff $(a + b)$ is odd over the set of integers
R_3 (a, b) iff $a.b > 0$ over the set of non-zero rational numbers
R_4 (a, b) iff $|a - b| \le 2$ over the set of natural numbers
Which of the following statements is corrects?
(a) R_1 and R_2 are equivalent relations, R_3 and R_4 are not
(b) R_1 and R_3 are equivalent relations, R_2 and R_4 are not
(c) R_1 and R_4 are equivalent relations, R_2 and R_3 are not
(d) R_1, R_2, R_3 and R_4 are all equivalence relations
[GATE-2001]

Q.27 Consider the following statements:

S_1: There exist infinite sets A, B, C such that $A \cap (B \cup C)$ is finite.

S_2: There exist two irrational numbers x and y such that $(x + y)$ is rational.

Which of the following is true about S_1 and S_2?

(a) Only S_1 is correct
(b) Only S_2 is correct
(c) Both S_1 and S_2 are correct
(d) None of S_1 and S_2 is correct

[GATE-2001]

Q.28 Let $f : A \rightarrow B$ be a function, and let E and F be subsets of A. Consider the following statements about images.

S_1: $f(E \cup F) = f(E) \cup f(F)$

S_2: $f(E \cap F) = f(E) \cap f(E)$

Which of the following is true about S_1 and S_2?

(a) Only S_1 is correct
(b) Only S_2 is correct
(c) Both S_1 and S_2 are correct
(d) None of S_1 and S_2 is correct

[GATE-2001]

Q.29 Which of the following is true?

(a) The set of all rational negative numbers forms a group under multiplication
(b) The set of all non-singular matrices forms a group under multiplication
(c) The set of all matrices form a group under multiplication
(d) Both (b) and (c) are true

[GATE-2002]

Q.30 The binary relation S = ϕ (empty set) on set A = {1, 2, 3} is

(a) Neither reflexive nor symmetric
(b) Symmetric and reflexive
(c) Transitive and reflexive
(d) Transitive and symmetric

[GATE-2002]

Q.31 Consider the binary relation:

$S = \{(x, y) \mid y = x + 1 \text{ and } x, y \in \{0, 1, 2, ...\}\}$

The reflexive transitive closure of S is

(a) $\{(x, y) \mid y > x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(b) $\{(x, y) \mid y \geq x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(c) $\{(x, y) \mid y < x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(d) $\{(x, y) \mid y \leq x \text{ and } x, y \in \{0, 1, 2, ...\}\}$

[GATE-2004]

Q.32 In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Programming Language and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?

(a) 15 (b) 20
(c) 25 (d) 30

[IT-2004]

Q.33 Let R_1 be a relation from $A = \{1, 3, 5, 7\}$ to $B = \{2, 4, 6, 8\}$ and R_2 be another relation from B to $C = \{1, 2, 3, 4\}$ as defined below:

(i) An element x in A is related to an element y in B (under R_1) if $x + y$ is divisible by 3.
(ii) An element x in B is related to an element y in C (under R_2) if $x + y$ is even but not divisible by 3.

Which is the composite relation R_2R_1 from A to C?

(a) {(1, 2), (1, 4), (3, 3), (5, 4), (7, 3)}
(b) {(1, 2), (1, 3), (3, 2), (5, 2), (7, 3)}
(c) {(1, 2), (3, 2), (3, 4), (5, 4), (7, 2)}
(d) {(3, 2), (3, 4), (5, 1), (5, 3), (7, 1)}

[IT-2004]

Q.34 The inclusion of which of the following sets into $S = \{\{1, 2\}, \{1,2,3\}, \{1, 3, 5\}, \{1, 2, 4\}, \{1, 2, 3, 4, 5\}\}$ is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?

(a) {1}
(b) {1}, {2, 3}
(c) {1}, {1, 3}
(d) {1}, {1, 3}, {1, 2, 3, 4}, {1, 2, 3, 5}

[GATE-2004]

Q.35 Let A, B and C be non-empty sets and let X = (A – B) – C and Y = (A – C) – (B – C) Which one of the following is TRUE?
(a) X = Y (b) X ⊂ Y
(c) Y ⊂ X (d) None of these
[GATE-2005]

Q.36 Let f: B → C and g: A → B be two functions and let $h = f \circ g$. Given that h is an onto function. Which one of the following is TRUE?
(a) f and g should both be onto functions
(b) f should be onto but g need not be onto
(c) g should be onto but f not be onto
(d) Both f and g need not be onto
[GATE-2005]

Q.37 Let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum cardinality of C?
(a) n (b) $n+1$
(c) $2^{n-1}+1$ (d) $n!$
[IT-2005]

Q.38 For the set N of natural numbers and a binary operation $f : N \times N \to N$, an element $z \in N$ is called an identity for f, if $f(a, z) = a = f(z, a)$, for all $a \in N$. Which of the following binary operations have a identity?
I. $f(x, y) = x + y - 3$
II. $f(x, y) = \max(x, y)$
III. $f(x, y) = x^y$
(a) I and II only (b) II and III only
(c) I and III only (d) None of these
[IT-2006]

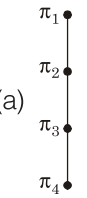
Q.39 Let, X, Y, Z be sets of sizes x, y and z respectively. Let W = X × Y and E be the set of all subsets of W. The number of functions from Z to E is
(a) z (b) $z \times 2^{xy}$
(c) 2^z (d) 2^{xyz}
[GATE-2006]

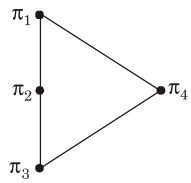
Q.40 The set {1, 2, 3, 5, 7, 8, 9} under multiplication modulo 10 is not a group. Given below are four possible reasons. Which one of them is false?
(a) It is not closed
(b) 2 does not have an inverse
(c) 3 does not have an inverse
(d) 8 does not have an inverse
[GATE-2006]

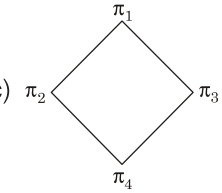
Q.41 A relation R is defined on ordered pairs of integers as follows: $(x, y)R(u, v)$ if $x < u$ and $y > v$. Then R is
(a) Neither a Partial Order nor an Equivalence Relation
(b) A Partial Order but not a Total Order
(c) A Total Order
(d) An Equivalence Relation
[GATE-2006]

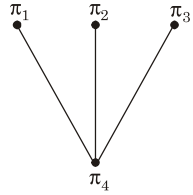
Q.42 Let S = {1, 2, 3,..., m}, $m > 3$. Let X_1, X_2, ..., X_n be subsets of S each of size 3. Define a function f from S to the set of natural numbers as, $f(i)$ is the number of sets X_j that contains the element i. That is $f(i) = |\{j | i \in X_j\}|$.
Then $\sum_{i=1}^{m} f(i)$ is
(a) $3m$ (b) $3n$
(c) $2m + 1$ (d) $2n + 1$
[GATE-2006]

Q.43 Consider the set S = {a, b, c, d}. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on
S : $\pi_1 = \{\overline{abcd}\}, \pi_2\{\overline{ab}, \overline{cd}\}, \pi_3 = \{\overline{abc}, \overline{d}\}$,
$\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$
Let $\prec$ be the partial order on the set of partitions $S' = (\pi_1, \pi_2, \pi_3, \pi_4)$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for $(S', \prec)$ is

(a)


(b)


(c)


(d)


[GATE-2007]

Q.44 If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is

(a) $Q^c \cup R^c$ (b) $P \cup Q^c \cup R^c$

(c) $P^c \cup Q^c \cup R^c$ (d) U

[GATE-2008]

Q.45 How many onto (or surjective) functions are there from an n-element ($n \geq 2$) set to a 2-element set?

(a) 2^n (b) $2^n - 1$

(c) $2^n - 2$ (d) $2(2^n - 2)$

[GATE-2012]

Q.46 Let X and Y be finite sets and $f : X \to Y$ be a function. Which one of the following statements is TRUE?

(a) For any subsets A and B of X,
$|f(A \cup B)| = |f(A)| + |f(B)|$

(b) For any subsets A and B of X,
$f(A \cap B) = f(A) \cap f(B)$

(c) For any subsets A and B of X,
$|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

(d) For any subsets S and T of Y,
$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

[GATE-2014 (Set-3)]

Q.47 Consider the set of all functions $f : \{0,1, \ldots, 2014\} \to \{0,1, \ldots,2014\}$ such that $f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements:

P : For each such function it must be the case that for every i, $f(i) = i$.

Q : For each such function it must be the case that for some i, $f(i) = i$.

R : Each such function must be onto.

Which one of the following is CORRECT?

(a) P, Q and R are true

(b) Only Q and R are true

(c) Only P and Q are true

(d) Only R is true

[GATE-2014 (Set-3)]

Q.48 If $g(x) = 1 - x$ and $h(x) = \dfrac{x}{x-1}$, then $\dfrac{g(h(x))}{h(g(x))}$ is

(a) $\dfrac{h(x)}{g(x)}$ (b) $\dfrac{-1}{x}$

(c) $\dfrac{g(x)}{h(x)}$ (d) $\dfrac{x}{(1-x)^2}$

[GATE-2015 (Set-1)]

Q.49 For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True?

1. $\varnothing \in 2^A$
2. $\varnothing \subseteq 2^A$
3. $\{5,\{6\}\} \in 2^A$
4. $\{5,\{6\}\} \subseteq 2^A$

(a) 1 and 3 only (b) 2 and 3 only

(c) 1, 2 and 3 only (d) 1, 2 and 4 only

[GATE-2015 (Set-1)]

Q.50 Suppose U is the power set of the set $S = \{1, 2, 3, 4, 5, 6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T' denote the complement of T. For any T, $R \in U$, let $T \setminus R$ be the set of all elements in T which are not in R. Which one of the following is true?

(a) $\forall X \in U(|X| = |X'|)$

(b) $\exists X \in U \exists Y \in U(|X| = 5, |Y| = 5$ and $X \cap Y = \varnothing)$

(c) $\forall X \in U \forall Y \in U(|X| = 2, |Y| = 3$ and $X \setminus Y = \varnothing)$

(d) $\forall X \in U \forall Y \in U(X \setminus Y = Y' \setminus X')$

[GATE-2015 (Set-3)]

Q.51 Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R?

(a) Both reflexive and symmetric

(b) Reflexive but not symmetric

(c) Not reflexive but symmetric

(d) Neither reflexive nor symmetric

[GATE-2015 (Set-3)]

Q.52 A binary relation R on N × N is defined as follows: (a, b) R (c, d) if $a \le c$ or $b \le d$. Consider the following propositions:

P : R is reflexive **Q :** R is transitive

Which one of the following statements is TRUE?

(a) Both P and Q are true
(b) P is true and Q are false
(c) P is false and Q are true
(a) Both P and Q are false

[GATE-2016 (Set-2)]

Q.53 Let N be the set of natural numbers. Consider the following sets:

P : Set of Rational numbers (positive and negative).
Q : Set of functions from {0, 1} to N.
R : Set of functions from N to {0, 1}.
S : Set of finite subsets of N.

Which of the sets above are countable?

(a) Q and S only (b) P and S only
(c) P and R only (d) P, Q and S only

[GATE-2018]

Q.54 Let G be an arbitrary group. Consider the following relations on G:

R_1 : $\forall a, b \in G, a\ R_1\ b$ if and only if $\exists g \in G$ such that $a = g^{-1}bg$

R_2 : $\forall a, b \in G, a\ R_2\ b$ if and only if $a = b^{-1}$

Which of the above is/are equivalence relation/ relations?

(a) R_1 and R_2 (b) R_1 only
(c) R_2 only (d) Neither R_1 nor R_2

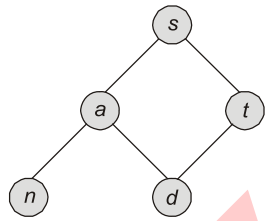
[GATE-2019]

Q.55 Which of the following is not a Boolean algebra?

(a) D_{40} (b) D_{66}
(c) D_{210} (d) D_{646}

[JNU]

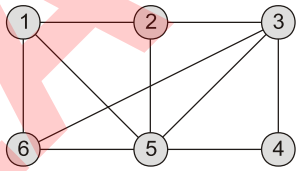
Q.56 Consider the following Hasse diagram in which the labels of the nodes are used only for reference. Labels s, a, t, n and d are the elements (contents) of the poset (A, $\prec$) represented by the following Hasse diagram. What is the result of topological sorting of the poset A?



(a) $s \prec t \prec a \prec n \prec d$ (b) $s \prec a \prec n \prec t \prec d$
(c) $n \prec a \prec d \prec t \prec s$ (d) $n \prec d \prec a \prec t \prec s$

[JNU]

Q.57 Let A_G denote the adjacency matrix of a graph G. Let $A_G^{k+1} = A_G^k \cdot A_G$, where '·' is matrix multiplication operator. For the following graph G and the following statement with blanks, select the correct option to correspond the blanks:



Statement: The $(3, 5)^{th}$ element in A_G^2 is _____, which denotes the number of paths of length _____.

(a) 3, 2 (b) 2, 2
(c) 2, 3 (d) 3, 3

[JNU]

Q.58 Let A = {1, 2, 3, 4, 5, 6}. Given that set A is a group with respect to multiplication mod 7. Is A a cyclic group? If yes, identify its generator.

(a) Yes, and generators are 1, 3, 5
(b) Yes, and generators are 3 and 6
(c) Yes, and the generators are 3 and 5
(d) No, A is not a cyclic group

[JNU]

Q.59 Which of the following statements is false?

(a) The set of rational numbers is an Abelian group under addition
(b) The set of integers is an Abelian group under addition
(c) The set of rational numbers forms an Abelian group under multiplication
(d) The set of real numbers excluding zero is an Abelian group under multiplication

[JNU]

Q.60 The sum $\frac{1}{1!\,9!} + \frac{1}{3!\,7!} + \frac{1}{5!\,5!} + \frac{1}{7!\,3!} + \frac{1}{9!\,1!}$ can be written in the form $\frac{2^a}{b!}$ where a and b are positive integers. The ordered pair (a, b) is

(a) (10, 9) (b) (9, 10)
(c) (7, 9) (d) (9, 7)

[JNU]

Q.61 Let A be adjacency matrix of a graph G. Then $a_K(i, j)$, the $(i, j)^{th}$ entry in matrix A^K gives number of paths of length K from v_i to v_j.

If $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ what will be the value of A^3?

(a) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

[JNU]

Q.62 If R be an equivalence relation on the set $A = \{6, 7, 8, 9, 10\}$ defined by $R = \{(6, 6), (7, 7), (8, 8), (9, 9), (10, 10), (6, 7), (7, 6), (8, 9), (9, 8), (9, 10), (10, 9) (8, 10), (10, 8)\}$, its partitions are

(a) {{6}, {7}, {8, 9}, {10}}
(b) {{6, 7, 8}, {9, 10}}
(c) {{6}, {7, 8}, {9, 10}}
(d) {{6, 7}, {8, 9, 10}}

[JNU]

Q.63 Over the set A = {1, 2, 3, 4, 5} two partitions a and b are defined as = {{1, 2, 3}, {4}, {5}} and b = {{1}, {2, 3}, {4, 5}}. The MEET and JOIN (under refinement relation between partitions) are respectively

(a) {(1), (2, 3), (4), (5)} and {(1, 2, 3), (4, 5)}
(b) {(1, 2, 3), (4, 5)} and {(1, 2), (3), (4, 5)}
(c) {(1, 2), (3, 4), (5)} and {(1, 2), (3, 4), (5)}
(d) {(1, 2, 3, 4, 5)} and {(1), (2), (3), (4), (5)}

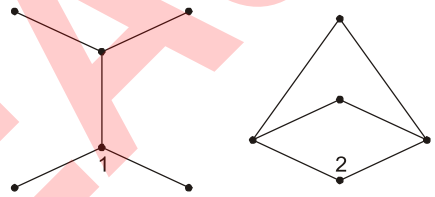
[JNU]

Q.64 A 'chain' is a

(a) Totally ordered set
(b) Partially ordered set
(c) Well-ordered set
(d) None of the above

[JNU]

Q.65 From the following two diagrams 1 and 2, determine



(a) Both 1 and 2 are lattice
(b) 1 is poset and 2 is lattice
(c) 1 is lattice and 2 is poset
(d) Both 1 and 2 are poset not lattice

[JNU]

Q.66 The set of 2 into 2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ with the operation multiplication of matrices is a

(a) group
(b) abelian group
(c) monoid but not group
(d) semigroup but not monoid

[JNU]

Q.67 The following statement S holds for any Boolean algebra [B; ¬, ∨, ∧] S : $(a \wedge b) = a$ if $a \le b$.

Find the dual, S*, of the statement S

(a) S * : $(a \wedge b) = a$ if $a \le b$
(b) S * : $(a \vee b) = a$ if $a \le b$
(c) S * : $(a \wedge b) = a$ if $a \ge b$
(d) S * : $(a \vee b) = a$ if $a \ge b$

[JNU]

Q.68 Let the relation R be defined as, $x\,Ry$ iff $y = x + 2$ and the relation S as, $x\,Sy$ iff $x \le y$. The relation RS on {1, 3, 5} is

(a) {(1,1), (1,3), (1,5)} (b) {(1,3), (1,5), (3,5)}

(c) {(1,3), (3,5), (5,7)} (d) {(1,3), (3,5)}

[JNU]

Q.69 Let A be a set and S be any subset of A consider the function $C_{\tilde{S}} : A \to \{0, 1\}$ as defined below:

$$C_{\tilde{S}}(x) = \begin{cases} 0 & \text{if } x \in S \\ 1 & \text{if } x \notin S \end{cases}$$

If A = {a, b, c, d} and S = {a, b, d}, when find $C_{\tilde{S}}$

(a) $C_{\tilde{S}}$ = {(a, 1), (b, 1), (c, 0), (d, 1)}

(b) $C_{\tilde{S}}$ = {(a, 1), (b, 0), (c, 1), (d, 0)}

(c) $C_{\tilde{S}}$ = {(a, 0), (b, 0), (c, 1), (d, 0)}

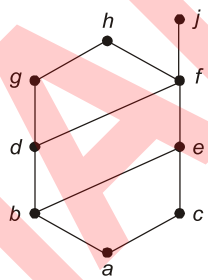
(d) $C_{\tilde{S}}$ = {(a, 0), (b, 1), (c, 0), (d, 1)}

[JNU]

Numerical Answer Type Questions

Q.70 What is the order of the element $\frac{1}{\sqrt{2}}[1+i]$ in the non-zero complex numbers under multiplication?

Q.71 Consider the Hasse diagram of a poset given below:



Find the number of upper bounds of the subset {a, c, d, f}?

Q.72 Let G be a finite group. If A and B are subgroups of G with orders 4 and 5 respectively then $|A \cap B|$ = _____.

Q.73 Assume g is an element of the group G. Consider the following conditions of g with e as identity element.

(i) $g^8 = e$ (ii) $g^2 \neq e$

(iii) Order of g is not 8.

Find the order of g.

Q.74 Let S denote the set of all functions $f : \{0, 1\}^4 \to \{0, 1\}$. Denote by N the number of functions from S to the set {0, 1}. The value of $\log_2 \log_2 N$ is _____.

[GATE-2014 (Set-1)]

Q.75 Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is _____.

[GATE-2014 (Set-3)]

Q.76 The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is _____.

[GATE-2015 (Set-2)]

Q.77 Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X and Y. Let f be randomly chosen from F. The probability of f being one-to-one is _____.

Q.78 The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _____.

[GATE-2017 (Set-1)]

Q.79 Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _____.

[GATE-2018]

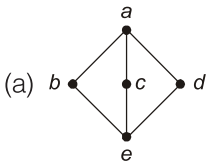
Multiple Select Questions

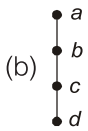
Q.80 Let R_1 = {(a, b), (a, c), (c, a), (a, a)} and Let R_2 = {(c, c), (c, b), (a, d), (d, b), (c, d)}. Where R_1 and R_2 are relations on set A = {a, b, c, d}. Then $R_1 \cap R_2$ is

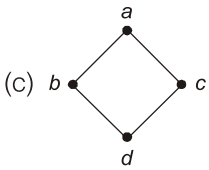
(a) Reflexive (b) Symmetric

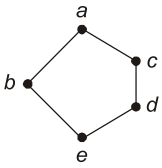
(c) Transitive (d) Asymmetric

Q.81 Identify the distributive lattice from the following.

(a) 

(b) 

(c) 

(d) 

Q.82 Which of the following are not group?

(a) Set of integers under the addition operation.
(b) Set of integers under the multiplication operation.
(c) Set of natural numbers under the multiplication operation.
(d) Set of rational numbers under the multiplication operation.

Q.83 Which of the following statement are true?

(a) A lattice is called complete if each of its non empty subsets has least upper bound and a great lower bound.
(b) Every chain is distributive lattice.
(c) A lattice is said to be a complementary lattice iff every element of L has only one complement.
(d) Let $(L, *, \oplus)$ be a lattice and $S \subseteq L$ be a subset of L. The algebra $(S, *, \oplus)$ is a sub lattice of $(L, *, \oplus)$ if and only if S is closed under both operations $*$ and $\oplus$ and for all $x, y \in S$, $x * y$ and $x \oplus y$ must be same in S as it is in L.

Q.84 Let f is a function from set of integers to the set of integers. Which of the following functions are either one-to-one or onto function?

(a) $f(x) = x^2 + 6$ (b) $f(x) = x^4 + 1$
(c) $f(x) = x^3 + 1$ (d) $f(x) = x + 5$

Q.85 Consider the following sets, where $n \geq 2$:

S_1 : Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$
S_2 : Set of all functions from the set $\{0, 1, 2, \ldots, n^2 - 1\}$ to the set $\{0, 1, 2\}$

Which of the following choice(s) is/are correct?

(a) There exists a surjection from S_1 to S_2.
(b) There does not exist a bijection from S_1 to S_2.
(c) There does not exist an injection from S_1 to S_2.
(d) There exists a bijection from S_1 to S_2.

[GATE : 2021 (Set-2)]

Try Yourself

T1. The function f is mapped from natural numbers to integer numbers and $f(x) = x^2 - 2x + 3$. Consider N = {0, 1, 2, 3, . . .} and Z = {1 ... , –2, –1, 0, 1, 2, ...}. What is the function f?

(a) Injective function (b) Surjective function
(c) Bijective function (d) None of these

[Ans: (d)]

T2. Which of the following set is uncountable.

(a) Set of all natural numbers
(b) Set of all integers
(c) Set of all positive rational numbers
(d) Set of all real numbers in the interval [0, 1]

[Ans: (d)]

T3. Let R be a relation over the set of integers, and $(x, y) \in R$ if and only if $|x - y| \leq 2$. Then R is _____.

(a) Reflexive and transitive relation
(b) Reflexive and symmetric relation
(c) Symmetric and transitive relation
(d) Equivalence relation

[Ans: (b)]

T4. Let $f(x) = (5x + 1)^2$. Then $f^{-1}(x) =$ _____.

(a) $\sqrt{x} - \frac{1}{5}$ (b) $\frac{5\sqrt{x}+1}{2}$
(c) $\frac{1}{5}(\sqrt{x} - 1)$ (d) $\sqrt{x} + \frac{2}{5}$

[Ans: (c)]

T5. The set of real numbers denoted by R and * is an operation on R, such that $x * y = x + y + xy$ for all $x, y \in$ R. If S $\subseteq$ R and (S, *) is an abelian group then which of the following could be S?

(a) R (b) R \ {0}
(c) R \ {1} (d) R \ {–1}

[Ans: (d)]

T6. Consider three finite sets A, B and C.
Let S1 = ((A ∩ B) – (B ∩ C)) – C and
S2 = (A – (A ∩ C))– (A – B).
Sets S1 and S2 are related as
(a) S1 ⊂ S2 (b) S1 ⊃ S2
(c) S1 = S2 (d) None of these

[Ans: (c)]

T7. Range of $f(x) = 3 + 5 \sin x$ is _____.
(a) [–1, 5] (b) [2, 8]
(c) [–2, 8] (d) [1, 5]

[Ans: (c)]

T8. Consider a relation $R = \{(x, y) | x, y$ are positive integers ≤ 4 and $(x + y) \leq 5\}$. Which of the following is true?
(a) R is reflexive, symmetric and transitive
(b) R is symmetric and transitive
(c) R is antisymmetric and transitive
(d) R is symmetric

[Ans: (d)]

T9. Let X = {{}, {a}}. The power set of X is _____.
(a) {{}, {a}} (b) {{}, {a}, {{}}, {{}, a}}
(c) {{}, {{a}}, {{}, a}} (d) {{}, {{a}}, {{}}, {{}, {a}}}

[Ans: (d)]

T10. Find the countable set from the following
(a) Subset of natural numbers
(b) $\mathbb{N} \times \mathbb{N}$
(c) Set of positive rational numbers
(d) All of these

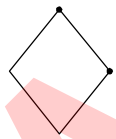
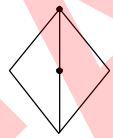
[Ans: (d)]

T11. Identify the correct statement from the following
(a) Commutative property not holds for addition of matrices
(b) Associative property not holds for addition of matrices
(c) Commutative property not holds for multiplication of matrices
(d) None of the above

[Ans: (c)]

T12. A function $f : \mathbb{N}^+ \to \mathbb{N}^+$, defined on the set of positive integers $\mathbb{N}^+$, satisfies the following properties:
$f(n) = f(n/2)$ if n is even
$f(n) = f(n + 5)$ if n is odd
Let $R = \{i | \exists\ j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is _____.

[GATE-2016, Ans: (2)]

T13. Which of the following lattices are complete lattices?
(a) 
(b)
(c) 
(d) All of these

[Ans: (d)]

T14. Let A, B and C be three sets. Then $(A – B) – C$ = _____.
(a) $(A – B) – (B – C)$ (b) $(B – C) – (A – C)$
(c) $(A – B) – (A – C)$ (d) $(A – C) – (B – C)$

[Ans: (d)]

T15. Identify the correct statement.
(a) Every distributive lattice is complemented lattice.
(b) Every complemented lattice is distributive lattice.
(c) Every complemented lattice is bounded.
(d) Both (b) and (c)

[Ans: (c)]

T16. A binary relation R on N × N is defined as follows: $(a, b)\ R\ (c, d)$ if $a \leq c$ or $b \leq d$. Consider the following propositions
P : R is reflexive
Q : R is transitive
Which one of the following statements is **TRUE**?
(a) Both P and Q are true
(b) P is true and Q is false
(c) P is false and Q is true
(d) Both P and Q are false

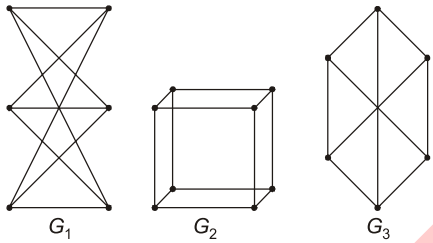
[GATE-2016, Ans: (b)]

■■■■

4 Graph Theory

Multiple Choice Questions

Q.1 Which of the following graphs is/are planar?



(a) G_1 only (b) G_1 and G_2
(c) G_2 only (d) G_2 and G_3

[GATE-1988]

Q.2 A graph is planar if any only if
(a) It does not contain subgraphs homeomorphic to K_5 and $K_{3,3}$
(b) It does not contain subgraphs isomorphic to K_5 and $K_{3,3}$
(c) It does not contain subgraphs isomorphic to K_5 or $K_{3,3}$
(d) It does not contain subgraph homeomorphic to K_5 or $K_{3,3}$

[GATE-1990]

Q.3 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots v_n\}$ of n vertices?

(a) $\frac{n(n-1)}{2}$ (b) 2^n
(c) $n!$ (d) $2^{n(n-1)/2}$

[GATE-2001]

Q.4 Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between.
(a) k and n (b) $k-1$ and $k+1$
(c) $k-1$ and $n-1$ (d) $k+1$ and $n-k$

[GATE-2003]

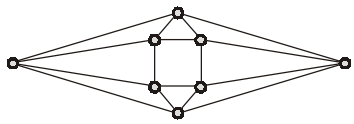
Q.5 How many perfect matchings are there in a complete graph of 6 vertices?
(a) 15 (b) 24
(c) 30 (d) 60

[GATE-2003]

Q.6 A Graph $G = (V, E)$ satisfies $|E| \le 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V}\{\text{degree}(v)\}$. Therefore, min-degree of G cannot be
(a) 3 (b) 4
(c) 5 (d) 6

[GATE-2003]

Q.7 The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is



(a) 2 (b) 3
(c) 4 (d) 5

[GATE-2004]

Q.8 What is the maximum number of edges in an acyclic undirected graph with n vertices?
(a) $n-1$ (b) n
(c) $n+1$ (d) $2n-1$

[IT-2004]

Q.9 What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

(a) 10 (b) 11
(c) 18 (d) 19

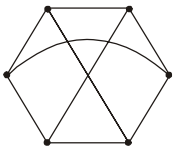
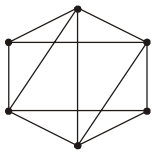
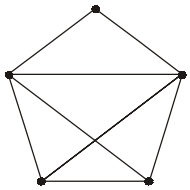
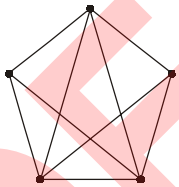
[IT-2004]

Q.10 Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is

(a) 0 (b) 8
(c) 9 (d) 13

[GATE-2005]

Q.11 Which one of the following graphs is NOT planner?

(a) G_1: 
(b) G_2: 
(c) G_3: 
(d) G_4: 

[GATE-2005]

Q.12 If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a

(a) Hamiltonian cycle (b) Grid
(c) Hypercube (d) Tree

[IT-2006]

Q.13 Let G be the non-planar graph with the minimum possible number of edges. Then G has

(a) 9 edges and 5 vertices
(b) 9 edges and 6 vertices
(c) 10 edges and 5 vertices
(d) 10 edges and 6 vertices

[GATE-2007]

Q.14 Which of the following graphs has an Eulerian circuit?

(a) Any k-regular graph where k is an even number
(b) A complete graph on 90 vertices
(c) The complement of a cycle on 25 vertices
(d) None of the above

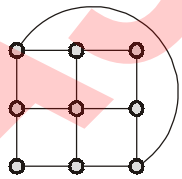
[GATE-2007]

Q.15 What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes?

(a) 5 (b) 4
(c) 3 (d) 2

[IT-2008]

Q.16 What is the chromatic number of the following graph?



(a) 2 (b) 3
(c) 4 (d) 5

[IT-2008]

Q.17 G is a simple, connected, undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be

(a) Regular (b) Complete
(c) Hamiltonian (d) Euler

[IT-2008]

Q.18 What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$.

(a) 2 (b) 3
(c) $n-1$ (d) n

[GATE-2009]

Q.19 Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?

(a) No two vertices have the same degree
(b) At least two vertices have the same degree
(c) At least three vertices have the same degree
(d) All vertices have the same degree

[GATE-2009]

Q.20 Let, $G = (V, E)$ be a graph. Define $\xi(G) = \sum_{d} i_d \times d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

(a) $|S| = 2|T|$ (b) $|S| = |T| - 1$

(c) $|S| = |T|$ (d) $|S| = |T| + 1$

[GATE-2010]

Q.21 The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1 **II.** 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2 **IV.** 8, 7, 7, 6, 4, 2, 1, 1

(a) I and II (b) III and IV

(c) IV only (d) II and IV

[GATE-2010]

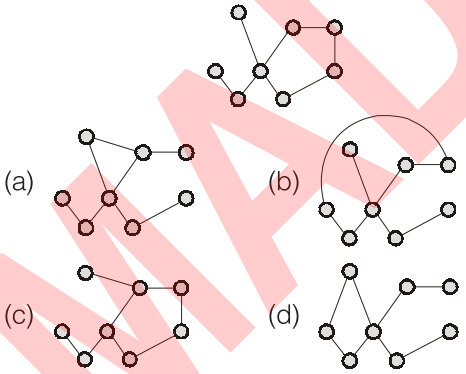
Q.22 Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(a) 3 (b) 4

(c) 5 (d) 6

[GATE-2012]

Q.23 Which of the following graphs is isomorphic to



(a) (b)

(c) (d)

[GATE-2012]

Q.24 Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

(a) 15 (b) 30

(c) 90 (d) 360

[GATE-2012]

Q.25 Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected numbers of unordered cycles of length three?

(a) $\frac{1}{8}$ (b) 1

(c) 7 (d) 8

[GATE-2013]

Q.26 If G is a forest with n-vertices and k connected components, how many edges does G have?

(a) $\left\lfloor \frac{n}{k} \right\rfloor$ (b) $\left\lceil \frac{n}{k} \right\rceil$

(c) $n - k$ (d) $n - k + 1$

[GATE-2014 (Set-3)]

Q.27 Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$ which one of the following is **TRUE**?

(a) In any planar embedding, the number of faces is at least $\frac{n}{2} + 2$

(b) In any planar embedding, the number of faces is less than $\frac{n}{2} + 2$

(c) There is a planar embedding in which the number of faces is less than $\frac{n}{2} + 2$

(d) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta + 1}$

[GATE-2014 (Set-3)]

Q.28 A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is

(a) A multiple of 4

(b) Even

(c) Odd

(d) Congruent to 0 mod 4, or 1 mod 4

[GATE-2015 (Set-2)]

Q.29 Let G be an undirected complete graph, on n vertices, where $n > 2$. Then, the number of different Hamiltonian cycles in G is equal to

(a) $n!$ (b) $(n-1)!$

(c) 1 (d) $\frac{(n-1)!}{2}$

[GATE-2019]

Q.30 The diameter and chromatic number of the star graph ($K_{1,n}$, $n \geq 2$) is

(a) 1, 2 (b) 2, 2

(c) 3, 2 (d) 2, n

Q.31 Which one of the following statements is false?

(a) A spanning tree is a tree that contains all edges of a graph

(b) The degree of a vertex in a simple graph is the number of vertices adjacent to it

(c) A loop is a special case of cycle in which a single are begins and ends with same node

(d) The adjacency matrix of a directed graph with no cycles must have all diagonal elements as 0's

[JNU]

Q.32 A connected multigraph has an Eulerian path (not circuit) if and only if

(a) every vertex in it has even degree

(b) every vertex in it has odd degree

(c) it has one vertex which is of odd degree

(d) it has two vertices which are of odd degree

[JNU]

Q.33 In a directed graph, if there is a directed path from any node u to v or vice versa, for any pair of nodes of the graph, the graph is

(a) weakly connected

(b) unilaterally connected

(c) strongly connected

(d) disconnected

[JNU]

Q.34 What type of graph STAR is?

(a) Multigraph (b) Bipartite

(c) Tripartite (d) Multipartite

[JNU]

Q.35 Euler's formula $v - e + r = 2$ is used to test

(a) an Euler path in the graph

(b) a Hamiltonian path in a graph

(c) planarity of a graph

(d) none of the above

[JNU]

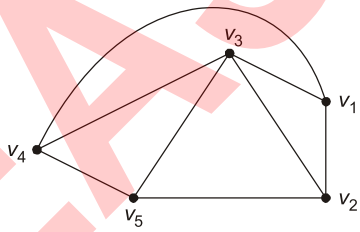
Q.36 If d_{max} is the maximum degree of the vertices in a graph G, Chromatic number C of G is defined as

(a) $C = 1 + d_{max}$ (b) $C < 1 + d_{max}$

(c) $C > 1 + d_{max}$ (d) $C \leq 1 + d_{max}$

[JNU]

Q.37 How many chromatic partition(s) the given graph has?



(a) Three (b) Two

(c) One (d) Zero

[JNU]

Q.38 Consider the following statements:

1. Every tree with a minimum of three vertices is 2-colorable.
2. The complete bipartite graph $K_{3,3}$ is non-planar.
3. If e is an edge in a connected graph with the smallest weight, then e is an edge in every minimum spanning tree of the graph.

Which of the statements is true?

(a) Only 1 and 2 are true

(b) Only 2 and 3 are true

(c) Only 3 and 1 are true

(d) All statements are true

[JNU]

Q.39 A simple graph with n vertices ($n \geq 3$) is Hamiltonian if the sum of degrees of every pair of non-adjacent vertices is at least

(a) $\sqrt{n}$ (b) $n^{2/3}$

(c) n (d) None of these

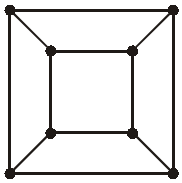
[JNU]

Q.40 Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

(a) 12 (b) 10
(c) 11 (d) 8

[JNU]

Q.41 For the graph G shown in the figure:



Identify the correct answer. G is a

(a) bipartite and regular but not a Hamiltonian graph
(b) regular and Hamiltonian but not a bipartite graph
(c) Hamiltonian and bipartite but not a regular graph
(d) regular, bipartite and Hamiltonian graph

[JNU]

Q.42 A tree has n_2 vertices of degree 2, n_3 vertices of degree 3, ..., and n_k vertices of degree k. How many vertices of degree 1 does it have?

(a) $n_2 + 2n_4 + 3n_5 + \ldots + (k-2)n_k + 1$
(b) $n_2 + 2n_3 + 3n_4 + \ldots + (k-1)n_k$
(c) $n_3 + 2n_4 + 3n_5 + \ldots + (k-2)n_k + 2$
(d) None of the above

[JNU]

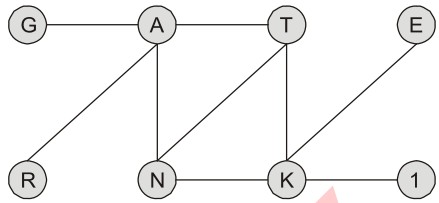
Q.43 If G is a tree with n vertices, then which of the following statements is correct?

(a) G is a connected graph
(b) G has $n-1$ edges
(c) G is a connected graph with $n-1$ edges
(d) None of the above

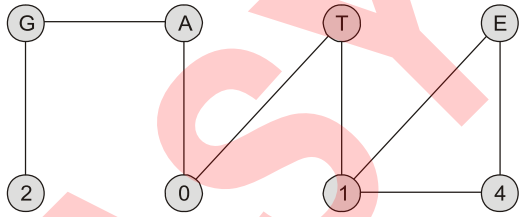
[JNU]

Numerical Answer Type Questions

Q.44 Find the matching number for the following graph.

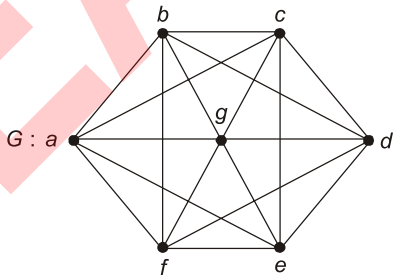


Q.45 Find the matching number for the following graph



Common Data Q.46 & Q.47:

Consider the graph below:

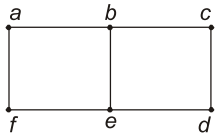


Q.46 What is the chromatic number of G?

Q.47 How many edges are present in complimentary graph $\bar{G}$ of the given graph G?

Q.48 A connected cubic planar graph with no bridges has exactly 5 edges in boundary of each region. Find the number of edges of the graph.

Q.49 Find the number of spanning trees for the following graph.



Q.50 Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j) : 1 \le i \le 12, 1 \le j \le 12\}$. There is an edge between (a, b) and (c, d) if $|a - c| \le 1$ and $|b - d| \le 1$.

The number of edges in this graph is _____.

[GATE-2014 (Set-1)]

Q.51 The maximum number of edges in a bipartite graph on 12 vertices is _____.

[GATE-2014 (Set-2)]

Q.52 A cycle on n vertices is isomorphic to its complement. The value of n is_____.

[GATE-2014 (Set-2)]

Q.53 Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _____.

[GATE-2015 (Set-1)]

Q.54 Consider a binary tree T that has 200 leaf nodes. Then, the number of nodes in T that have exactly two children are _____.

[GATE-2015 (Set-3)]

Q.55 The minimum number of colours that is sufficient to vertex-colour any planar graph is _____.

[GATE-2016 (Set-2)]

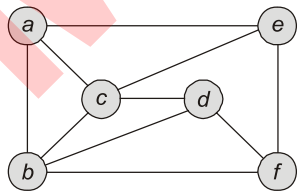
Q.56 Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is _____.

[GATE-2017 (Set-1)]

Q.57 G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is _____.

[GATE-2017 (Set-2)]

Q.58 The chromatic number of the following graph is _____.



[GATE-2018]

Multiple Select Questions

Q.59 Which of the following are correct?

(a) A multigraph with zero odd vertex is traversable.

(b) A multigraph with two odd vertices is traversable.

(c) A multigraph with more than two odd vertices are not traversable.

(d) The sum of degree equal to double the edges is valid for multigraph.

Q.60 Which of the following are correct?

(a) If a graph contains two vertices of odd degree then there must be a path joining them.

(b) The max degree of any vertex in a simple graph with n vertices is $n-1$.

(c) The max number of edges in a simple graph for n-vertices is $\frac{n^2-n}{2}$.

(d) A simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

Q.61 Which of the following are correct?

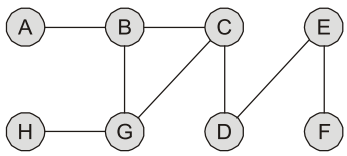
(a) The order of a forest F with 17 edges and 4 components is 13.

(b) Biconnected graph contains no vertex whose removal disconnects the rest of the graph.

(c) The size of a forest F with 17 vertices and 4 components is 13.

(d) The number of distinct simple labelled graph with upto 3 nodes is 11.

Q.62 Which of the following is possible cut vertices for the following graph?



(a) A (b) B
(c) C (d) D

Try Yourself

T1. Let G be a disconnected simple graph with n vertices. Find the maximum number of edges are possible in G.

(a) $\frac{(n-1)}{2}$ (b) $(n-1)$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{(n-1)(n-2)}{2}$

[Ans: (d)]

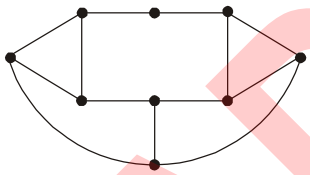
T2. Let G be a planar graph such that every face is bordered by exactly 3 edges. Which of the following can never be the value for $\chi(G)$? [where $\chi(G)$ is chromatic number of G]

(a) 2 (b) 3
(c) 4 (d) None of these

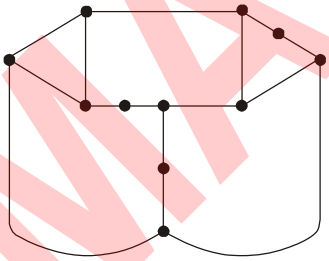
[Ans: (a)]

T3. Consider the following graphs:

G_1 :



G_2 :



Identify the correct statement?

(a) Only G_1 is Hamiltonian
(b) Only G_2 is Hamiltonian
(c) Both G_1 and G_2 are Hamiltonian
(d) Neither G_1 nor G_2 is Hamiltonian

[Ans: (a)]

Linked Answer T4 & T5:

Let G be a graph with V(G) = $\{i | 1 \le i \le 4n, n \ge 1\}$ where V(G) is the set of vertices of G. Such that two numbers x and y in V(G) are adjacent if and only if $(x + y)$ is a multiple of 4.

T4. Find the number of components in graph G.

(a) 2
(b) 4
(c) n
(d) None of these

[Ans: (d)]

T5. From the above k components, assume that each component C_k has m_k vertices, then what is the maximum value of m_k in the graph G?

(a) n (b) $2n$
(c) $3n$ (d) None of these

[Ans: (b)]

T6. Find the number of perfect matchings in a cycle graph with $2n$ vertices is _____.

(a) 0 (b) 2
(c) n (d) $\frac{n}{2}$

[Ans: (b)]

T7. Consider the following statements.
Let G be a connected graph of order n where $n \ge 3$.

S_1: If $d(V) \ge \frac{n}{2}$ for each vertex V in G, then G is Hamiltonian. [$d(V)$ is the degree of a vertex V].

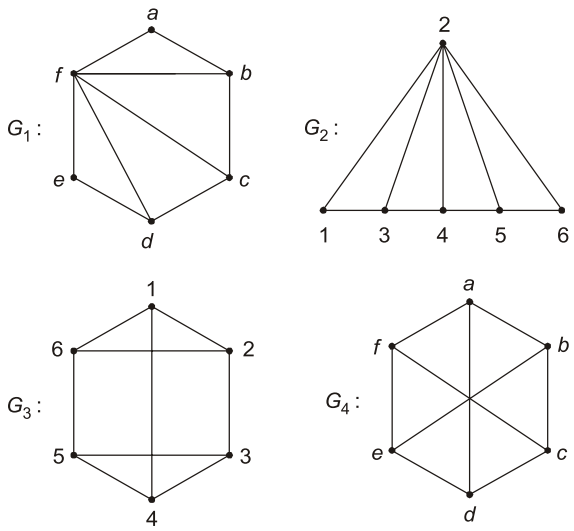
S_2: If every vertex in G has even degree then G is eulerian.

Which of the above statements are correct?

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

[Ans: (c)]

T8. Consider the following graph:



Which of the above graphs are isomorphic?

(a) G_1 and G_2 (b) G_3 and G_4

(c) G_1 and G_3 (d) G_2 and G_4

[Ans: (a)]

T9. Let G be a planar graph such that every face is bordered by exactly three edges. What are the possible values for chromatic number of G?

(a) Only 3 (b) Only 4

(c) 3 or 4 (d) None of these

[Ans: (c)]

T10. Consider the following statements:

S_1 : The maximum number of edges in a disconnected simple graph having n-vertices is $\frac{(n-1)(n-2)}{2}$.

S_2 : A simple graph G (with n-vertices and k-components) is a forest if and only if G has $(n-k)$ edges.

Find which of the above statements are correct

(a) S_1 only

(b) S_2 only

(c) Both S_1 and S_2

(d) Neither S_1 nor S_2

[Ans: (c)]

■■■■

5 Probability and Statistics

Multiple Choice Questions

Q.1 A multiple choice test consists of three problems. For each problem, there are five choices, one of which is correct. One student comes totally unprepared and decides to answer by sheer guessing. What is the probability that he will answer at least one problem correctly?

(a) $\frac{4}{5}$ (b) $\frac{61}{125}$

(c) $\frac{48}{125}$ (d) $\frac{12}{125}$

(e) $\frac{1}{125}$

Q.2 From twenty tickets marked with the first twenty numerals, one ticket is drawn at random. What is the probability that the numeral marked on it is a multiple of 3 or 5?

(a) $\frac{1}{20}$ (b) $\frac{1}{10}$

(c) $\frac{3}{10}$ (d) $\frac{9}{20}$

(e) $\frac{1}{2}$

Q.3 If six fair dice are rolled, what is the probability that each of the six numbers will appear exactly once?

(a) 0.0065 (b) 0.0154

(c) 0.0198 (d) 0.0215

(e) 0.0285

Q.4 One shot is fired from each of the three guns, where G_1, G_2 and G_3 denote the events that the target is hit by the first, second and third guns respectively. If $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and G_1, G_2 and G_3 are independent of each other. The probability that at least two hits to be registered is

(a) 0.50 (b) 0.55

(c) 0.60 (d) 0.65

(e) 0.70

Q.5 If $P(A') = \frac{3}{8}$, $P(B') = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$ then the value of $P\left(\frac{B'}{A'}\right)$ is

(a) 0 (b) 1

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(e) $\frac{1}{5}$

Q.6 A pair of fair dice is thrown. Find the probability of getting a sum of 7, when it is known that the digit in the first die is greater than that of the second.

(a) $\frac{5}{12}$ (b) $\frac{1}{5}$

(c) $\frac{1}{12}$ (d) $\frac{5}{8}$

(e) $\frac{7}{10}$

Q.7 The manufacturing process of an article consists of two parts A and B. The probabilities of defect in parts A and B are 8% and 12% respectively. What is the probability that the assembled product will not have any defect? (Assume that A and B are independent.)

(a) 80.9% (b) 76.5%
(c) 74.02% (d) 70.25%
(e) 68.57%

Q.8 If $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.25$ then the value of $P(\bar{A}) + P(\bar{B})$ is

(a) 0.60 (b) 1.93
(c) 0.10 (d) 1.15
(e) 2.32

Q.9 When five coins are tossed, Heads and tails show up on the coins. Find the probability of showing exactly two heads on the coins.

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$
(c) $\frac{3}{8}$ (d) $\frac{5}{16}$
(e) $\frac{7}{20}$

Q.10 If the occurrence of some events are dependent on the occurrence of an event A then the sum of all the joint probabilities in which the occurrence of event A is considered gives the

(a) Subjective probability of event A
(b) Classical probability of event A
(c) Conditional probability of event A
(d) Marginal probability of event A
(e) Relative frequency of occurrence of event A

Q.11 If events B and C are dependent on event A and P(A and B) = 0.30, P(A and C) = 0.20, and the dependent events B and C are mutually exclusive and collectively exhaustive, then $P\left(\frac{C}{A}\right)$ is equal to

(a) 0.20 (b) 0.40
(c) 0.50 (d) 0.60
(e) 0.80

Q.12 A bin in a hardware store, contains 125 bolts and 200 nuts. One-fifth of the bolts and three-fourth of the nuts are defective. One item is picked randomly from the bin. What is the probability that the item is either a defective item or it is a nut?

(a) 92.3% (b) 84.6%
(c) 69.2% (d) 53.8%
(e) 38.5%

Q.13 The average deviation of all items in the data from zero is equal to the

(a) Arithmetic mean (b) Median
(c) Mode (d) Standard deviation
(e) Variance

Q.14 A negatively skewed distribution curve tails off towards the lower end and for such a curve

(a) A.M > Median > Mode
(b) A.M < Median < Mode
(c) A.M = Median = Mode
(d) A.M = Median < Mode
(e) A.M > Median = Mode

Q.15 What is the variance of first n natural numbers?

(a) $\frac{(n^2+1)}{12}$ (b) $\frac{(n-1)}{12}$
(c) $\frac{(n^2-1)}{2}$ (d) $\frac{(n^2-1)}{12}$
(e) $\frac{(n^2+1)}{2}$

Q.16 A football match is played from 9 A.M. to 11 A.M. A boy arrives to see the match (not before the match starts). What is the probability that he will miss the only goal of the match, which takes place at the 21st minute of the match?

(a) 0.126 (b) 0.935
(c) 0.736 (d) 0.825
(e) 0.901

Q.17 The mean of squares of first 23 natural numbers is

(a) 4324 (b) 188
(c) 1128 (d) 104
(e) 3424

Q.18 A die is loaded so that probability of getting face x is proportional to x. The probability of an odd number occurring when the die is rolled would be

(a) $\frac{1}{21}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

(e) $\frac{5}{7}$

Q.19 A, B and C are mutually exclusive and collectively exhaustive events, resulting from an experiment. B is twice as likely as A and C is 2.5 times as likely as B. Which of the following is correct?

(a) P(A) = 3%, P(B) = 6% and P(C) = 15%
(b) P(A) = 5%, P(B) = 10% and P(C) = 25%
(c) P(A) = 8%, P(B) = 16% and P(C) = 40%
(d) P(A) = 10%, P(B) = 20% and P(C) = 50%
(e) P(A) = 12.5%, P(B) = 25% and P(C) = 62.5%

Q.20 In a certain town, male and female each from 60 and 40 percent of the population respectively. It is known that 25 percent of the males and 15 percent of the females are unemployed. A research student studying the employment situation selects an unemployed person at random. What is the probability that the person so selected is male?

(a) 0.121 (b) 0.714
(c) 0.800 (d) 0.485
(e) 0.735

Q.21 Probability of getting a plumbing contract is 0.5. Probability of getting an electrical contract is 0.3. Probability of getting neither contract is 0.25. What is the probability of getting both types of contract?

(a) 60% (b) 75%
(c) 30% (d) 5%
(e) 85%

Q.22 Experiment one results in the mutually exclusive and collectively exhaustive events A, B and C. Experiment two results in the mutually exclusive and collectively exhaustive events D and E. The joint probabilities of the events that may result from these two experiments are given below:

	A	B	C
D	0.12	0.075	0.12
E	0.28	0.225	0.18

Which of the following statements are correct?

1. P(A) = 0.30
2. P(B) = 0.40
3. P(C) = 0.30
4. P(D) = 0.685
5. P(E) = 0.685
6. $P(D \mid C) = 0.25$
7. $P(E \mid B) = 0.75$
8. $P(D \mid A) = 0.30$
9. D is independent of the events A, B and C respectively.
10. E is dependent on the events A, B and C respectively.

(a) 1, 4 and 6 above
(b) 4, 6, 7 and 10 above
(c) 2, 3, 4, 6 and 9 above
(d) 3, 5, 7, 8 and 10 above
(e) 1, 2, 4, 6 and 9 above

Q.23 Two computers A and B are to be sold. A salesman who is assigned the job of selling these has the chances of 60 percent and 40 percent respectively to get success. The two computers may be sold independently. Given that atleast one computer has been sold, what is the probability that computer A has been sold? (Round off your answer up to the second decimal)

(a) 0.40 (b) 0.64
(c) 0.79 (d) 0.90
(e) 0.24

Q.24 The probability of solving a problem by three students X, Y and Z are $\frac{1}{4}$, $\frac{3}{7}$ and $\frac{2}{9}$ respectively. If all of them try independently then the probability that the problem could not be solved is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{2}{5}$

(e) $\frac{3}{7}$

Q.25 If A is an event, the conditional probability of A given A is equal to
(a) 0 (b) 0.4
(c) 0.5 (d) 0.8
(e) 1

Q.26 Because a new medical procedure has been shown to be effective in early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies someone with the illness as positive is 0.99, and the probability that the test correctly identifies some one without the illness as negative is 0.95. The incidence of illness in the general population is 0.0001. You take the test, and the result is positive. What is the probability that you have the illness? (Round off your answer upto the third decimal)
(a) 0.002 (b) 0.005
(c) 0.995 (d) 0.998
(e) 0.949

Q.27 Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{5}{12}$ (d) $\frac{7}{15}$
(e) $\frac{7}{18}$

Q.28 Which of the following is most appropriate if certain events are mutually exclusive and collectively exhaustive?
(a) The sum of the probabilities of the events will be less than 1
(b) The sum of the probability of the events will be more than 1
(c) The sum of the probabilities of the events will be equal to 1
(d) Some of the events will be definitely have zero probability
(e) Each of the events has a zero probability

Q.29 If A and B are any two events, the probability that exactly one of them occurs is given by
(a) $P(A) + P(B) - 2P(AB)$
(b) $P(A) + P(B) - P(AB)$
(c) $P(\overline{A}) + P(\overline{B}) - 2P(\overline{AB})$
(d) $P(\overline{A}) + P(\overline{B}) - P(\overline{AB})$
(e) $P(\overline{AB})$

Q.30 Which of the following statements are FALSE?
(a) For Poisson distribution, the mean in twice the variance
(b) In queuing theory, if arrivals occurs according to Poisson distribution, then the inter-arrival time is exponentially distributed
(c) If the time between successive arrivals is exponential, then the time between the occurrences of every third arrival is also exponential
(d) None of these

[GATE-1988]

Q.31 Let A and B be any two arbitrary events, then, which one of the following is true?
(a) $P(A \cap B) = P(A)\,P(B)$
(b) $P(A \cup B) = P(A) + P(B)$
(c) $P(A \mid B) = P(A \cap B)\,P(B)$
(d) $P(A \cup B) \leq P(A) + P(B)$

[GATE-1994]

Q.32 The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is
(a) $\frac{16}{25}$ (b) $\left(\frac{9}{10}\right)^3$
(c) $\frac{27}{25}$ (d) $\frac{18}{25}$

[GATE-1995]

Q.33 Two dice are thrown simultaneously. The probability that at least one of them will have 6 facing up in
(a) $\frac{1}{36}$ (b) $\frac{1}{3}$
(c) $\frac{25}{36}$ (d) $\frac{11}{36}$

[GATE-1996]

Q.34 The probability that top and bottom cards of a randomly shuffled deck are both aces is

(a) $\frac{4}{52} \times \frac{4}{52}$ (b) $\frac{4}{52} \times \frac{3}{52}$

(c) $\frac{4}{52} \times \frac{3}{51}$ (d) $\frac{4}{52} \times \frac{4}{51}$

[GATE-1996]

Q.35 The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

(a) 0.3 (b) 0.25

(c) 0.35 (d) 0.4

[GATE-1997]

Q.36 A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is

(a) $\frac{1}{6}$ (b) $\frac{3}{8}$

(c) $\frac{1}{8}$ (d) $\frac{1}{2}$

[GATE-1998]

Q.37 Suppose that the expectation of a random variable X is 5. Which of the following statements is true?

(a) There is a sample point at which X has the value 5

(b) There is a sample point at which X has value greater than 5

(c) There is a sample point at which X has a value greater than or equal to 5

(d) None of the above

[GATE-1999]

Q.38 Consider two events E_1 and E_2 such that probability of E_1, $\Pr[E_1] = \frac{1}{2}$, probability of E_2, $\Pr[E_2] = \frac{1}{3}$ and probability of E_1 and E_2, $\Pr[E_1 \text{ and } E_2] = \frac{1}{5}$. Which of the following statements is/are true?

(a) $\Pr[E_1 \text{ or } E_2]$ is $\frac{2}{3}$

(b) Events E_1 and E_2 are independent

(c) Events E_1 and E_2 are not independent

(d) $\Pr\left[\frac{E_1}{E_2}\right] = \frac{4}{5}$

[GATE-1999]

Q.39 E_1 and E_2 are events in a probability space satisfying the following constraints

$\Pr(E_1) = \Pr(E_2)$

$\Pr(E_1 \cup E_2) = 1$

E_1 and E_2 are independent

The value of $\Pr(E_1)$, the probability of the event E_1, is

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

[GATE-2000]

Q.40 Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day?

(a) $\frac{1}{7^7}$ (b) $\frac{1}{7^6}$

(c) $\frac{1}{2^7}$ (d) $\frac{7}{2^7}$

[GATE-2001]

Q.41 Four fair coins are tossed simultaneously. The probability that at least one head and one tail turn up is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

[GATE-2002]

Q.42 The tailors in a tailoring shop on an average stitch three shirts in an hour, stitching has a Poisson distribution. During a particular hour, what is the probability that the number of shirts stitched is 5 or more?

(a) 0.8571 (b) 0.8152
(c) 0.1848 (d) 0.1439

[JNU]

Q.43 A coin is biased so that the probability of heads is $\frac{2}{3}$. What is the probability that exactly four heads come up when the coin is flipped seven times, assuming that the flips are independent?

(a) $\frac{560}{2281}$ (b) $\frac{521}{2187}$

(c) $\frac{509}{2013}$ (d) None of these

[JNU]

Q.44 An electronic gadget has 5 components of which two are of type-1 and the other three are of type-2. Unless all the components are functional the gadget will not be operative. During some interval, the probability of failure of each of the components of type-1 is 0.04 and the probability of failure of each of the components of type-2 is 0.03. What is the corresponding probability of failure of the gadget?

(a) 15.88% (b) 16.75%
(c) 0.432% (d) 0.956%

[JNU]

Q.45 Three pens are randomly drawn without replacement from a pack of 50 pens of which four are defective. The probability that none of the pens is defective, first one is defective while the other two are not, and third one is defective while the first two are not, respectively are

(a) 75%, 6.62%, 7.19%
(b) 77.44%, 7.04%, 7.04%
(c) 77.86%, 6.77%, 6.62%
(d) 80%, 6.77%, 7.04%

[JNU]

Q.46 What is the probability of obtaining at least four Heads yet not all Heads while tossing an unbiased coin 7 times?

(a) 49.2% (b) 40.08%
(c) 50.00% (d) 50.78%

[JNU]

Q.47 Suppose the random variate X has the probability distribution given below:

x	–2	–1	0	1	2
$P(X = x)$	0.25	0.20	0.15	0.35	0.05

Let $Y = 2X^2 + 6$. The expected value $E(Y)$ is

(a) 9.5 (b) 6
(c) 15.5 (d) 18

[JNU]

Q.48 A discrete random variate X has pmf $P(X = i) = \frac{C}{i^2}$, $i = 1, 2, 3, \ldots$. Find C

(a) $\frac{1}{2}$ (b) 0

(c) π^2 (d) $\frac{6}{\pi^2}$

[JNU]

Q.49 A student takes a multiple-choice examination, where each question has 5 possible answers. He attempts a question correctly if he knows the answer, otherwise he guesses at random. Suppose he knows answer to 70% of the questions, what is the probability that on a question chosen at random, the student gets correct answer?

(a) 0.20 (b) 0.76
(c) 0.70 (d) None of the above

[JNU]

Q.50 In four tests taken by 450 students, marks are found to be normally distributed with mean and variance as given below:

Test No.	Mean	Variance
1	74	121
2	75	100
3	78	196
4	82	169

A has secured 80 in the first test, 81 in the second, 86 in the third and 89 in the fourth. In which test did A actually perform best relative to other students?

(a) Fourth test (b) Third test
(c) Second test (d) First test

[JNU]

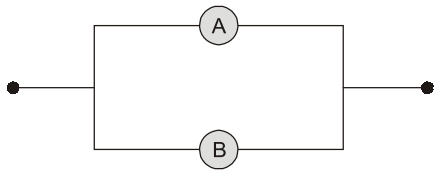
Q.51 Suppose the average number of telephone calls arriving at the switchboard of an organization is 30 calls per hour. Determine the probability that no calls will arrive at a 3-minute period

(a) $e^{-1.5}$ (b) $e^{-0.05}$

(c) $e^{-1.5(1.5)^k}\frac{1}{k!}$ (d) $\sum_{k=3}^{\infty}\frac{e^{-1.5(1.5)^k}}{k!}$

[JNU]

Q.52 A machine contains a component A that is vital for its operation. The reliability of the component A is 80%. To improve the reliability of the machine, a similar component is used in parallel for the system S as shown in the following figure.



The machine works if one of these two components functions. What is the reliability of the system S?

(a) 98% (b) 96%

(c) 94% (d) 92%

[JNU]

Q.53 For answering a question in a multiple choice test a student either knows the answer, or guesses it. Let p be the probability that the answer is known to the student and $(1-p)$ that the answer is guessed. Let m be the number of alternative answers, then $\frac{1}{m}$ be the probability that the answer is guessed correctly. The conditional probability that the student knew the answer to a question given that she/he answered it correctly is

(a) $\frac{p}{1+(m-1)p}$ (b) $p-\frac{1}{m}$

(c) $p+\frac{1}{m}-\frac{p}{m}$ (d) $\frac{mp}{1+(m-1)p}$

[JNU]

Q.54 A die is loaded in such a way that each odd number is twice as likely to occur as each even number. If E is the event that a number greater than or equal to 4 occurs on a single toss of the die, then $P(E)$ is

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

[JNU]

Q.55 A random variate has the following distribution:

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

The value of k is

(a) –1 (b) 0

(c) 0.1 (d) –0.1

[JNU]

Q.56 A point P is chosen at random on a line AB of length $2l$. The probability

$$P\left[(AP\times BP)>\frac{l^2}{2}\right] \text{ is}$$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\left(1-\frac{1}{\sqrt{2}}\right)l$ (d) $\frac{1}{\sqrt{2}}l$

[JNU]

Q.57 Consider any set of 201 observations $x_1, x_2, \ldots, x_{200}, x_{201}$. It is given that $x_1 < x_2 < \ldots < x_{200} < x_{201}$. Then the mean deviation of this set of observations about a point k is minimum when k equals

(a) $\frac{(x_1+x_2+\ldots+x_{200}+x_{201})}{201}$

(b) x_1

(c) x_{101}

(d) x_{201}

[JNU]

Q.58 Three identical dice are rolled. The probability that the same number will appear on each of them is

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$

(c) $\frac{1}{18}$ (d) $\frac{3}{28}$

[JNU]

Q.59 If M and N are any two events, which of the following is not the probability that exactly one of them occurs

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(M \cap N)$

(c) $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$

(d) $P(M \cap N^C) + P(M^C \cap N)$

[JNU]

Q.60 A room has three lamp sockets. From a collection of 12 light bulbs of which 7 are no good, a person selects 3 at random and put them in sockets. What is the probability that he will have light?

(a) $\frac{37}{44}$ (b) $\frac{5}{12}$

(c) $\frac{41}{48}$ (d) None of these

[JNU]

Q.61 Find the variance of the random variable X whose value when two dice are rolled is $X(i, j) = i + j$ where i is the number appearing on the first die and j is the number appearing on the second die

(a) $\frac{29}{12}$ (b) $\frac{23}{17}$

(c) $\frac{35}{6}$ (d) $\frac{31}{8}$

[JNU]

Q.62 Let A and B be any two events with $p(A) = 0.3$, $p(B) = 0.8$. Which of the following is true regarding $p(A \cup B)$ and $p(A \cap B)$?

(a) $0.8 \le p(A \cup B) \le 1$ and $0.1 \le p(A \cap B) \le 0.3$

(b) $0.7 \le p(A \cup B) \le 1$ and $0.1 \le p(A \cap B) \le 0.3$

(c) $0.8 \le p(A \cup B) \le 1$ and $0.1 \le p(A \cap B) \le 0.2$

(d) $0.2 \le p(A \cup B) \le 1$ and $0.7 \le p(A \cap B) \le 0.8$

Q.63 Suppose there are n number of CPUs on a common bus. The probability that any CPU tries to use the bus in a given cycle is p. What is the chance that the bus is idle?

(a) $\frac{p^n}{(1-p)^n}$ (b) $p^n(1-p)^n$

(c) p^n (d) $(1-p)^n$

[JNU]

Q.64 For any discrete random variable X, with probability mass function $P(X = j) = p_j$, $p_j \ge 0$, $j \in \{0 \ldots N\}$, and $\sum_{j=0}^{N} p_j = 1$, define the polynomial function $g_X(z) = \sum_{j=0}^{N} p_j z^j$. For a certain discrete random variable Y, there exists a scalar $\beta \in [0, 1]$ such that $g_Y(z) = (1 - \beta + \beta z)^N$. The expectation of Y is

(a) $N\beta(1-\beta)$

(b) $N\beta$

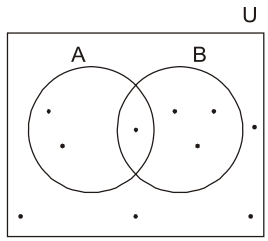
(c) $N(1-\beta)$

(d) Not expressible in terms of N and β alone

[GATE-2017]

Numerical Answer Type Questions

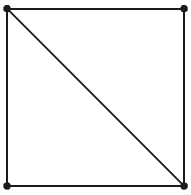
Q.65 Consider the following diagram which shows the results of a game where discs are thrown into two circles A and B

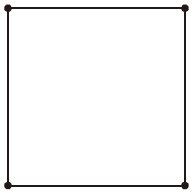


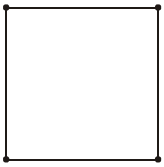
What is the value of $P(\bar{A})$? [Assume P is probability]. Please enter the value in $x \cdot y$ format

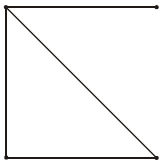
Q.66 5 Students are selected without replacement from a class of 3 boys and 5 girls. The probability of choosing at least 3 girls is _____. [Please enter the value in $x \cdot yz$ format]

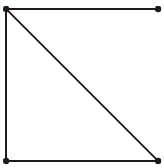
Q.67 A simple graph G has only 5 vertices and by the deletion of every vertex V_i from G the subgraph G_i is obtained by removal of all adjacent edges of V_i of G. Suppose the following graphs are obtained as subgraphs G_1, G_2, G_3, G_4 and G_5 by deletion of vertices V_1, V_2, V_3, V_4 and V_5 respectively, how many edges are present in the given graph G?

G_1: 

G_2: 

G_3: 

G_4: 

G_5: 

Q.68 A fair coin is tossed three times in succession. If the first toss produces a head, then what is the probability of getting exactly two heads in three tosses?

Q.69 A fair six sided die is thrown twice. If the sum of the face values of these two tosses is 5 then what is the probability that the face value of the first toss is less than that of second toss?

Q.70 Let X be a continuous random variable with following probability density function.

$$f(x) = 2x,\ x \in [0,1]$$
$$= 0,\ x \notin [0,1]$$

Find the variance of X. (Round upto three decimal places)

Q.71 If a fair coin is tossed until the same result turns up in succession (both head or both tail) then find the probability when the number of tosses are even. (Round upto two decimal places)

Q.72 Let X be a Gaussian random variable with mean 0 and variance σ^2. Let Y = max (X, 0) where max (a, b) is the maximum of a and b. The median of Y is _____.

[GATE-2017]

Multiple Select Questions

Q.73 Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which one of the following statements are true?

(a) $P(A \cap B) = P(A)\,P(B)$

(b) $P(A/B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

Q.74 Which of the following are correct?

(a) The standard deviation of 6, 8, 10, 12, 14 is $\sqrt{40}$.

(b) The standard deviation of 6, 8, 10, 12, 14 is 2.828.

(c) The probability that the ace of the spades will be drawn from a pack of well-shuffled cards atleast once in 104 consecutive trials is 86.5%.

(d) Mode = 3 × Median – 2 × Mean

Q.75 x is a continuous random variable with probability density function

$$f(x) = \begin{cases} ke^{-|x|^2} & ; \quad -\infty < x < \infty \\ 0 & ; \quad \text{Else} \end{cases}$$

Which of the following is/are true?

(a) $k = \dfrac{1}{\sqrt{\pi}}$ (b) $E(x) = 0$

(c) $V(x) = \dfrac{10}{\sqrt{\pi}}$ (d) $\sigma_x = \sqrt{\dfrac{10}{\pi}}$

Q.76 R.V., x following probability distribution given below:

x	1	2	3
$P(x)$	$\frac{1}{2}$	k	$\frac{1}{3}$

Then which of the following is/are correct?

(a) $k = \frac{1}{6}$ (b) $E(2x+3) = \frac{20}{3}$

(c) $E(x^2) = \frac{25}{6}$ (d) $V(x) = 0$

Q.77 $E(x) = 1$, $Var(x) = 2$, which of the following is/are correct?

(a) $E(6x) = 6$ (b) $Var(6x) = 72$

(c) $E(1-x) = 0$ (d) $Var(1-x) = 0$

Q.78 Consider two events E_1 and E_2 such that $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{3}$ and $P[E_1 n E_2] = \frac{1}{5}$.

Which of the following statements is/are TRUE?

(a) $P[E_1 \cup E_2] = \frac{19}{30}$

(b) E_1, E_2 are independent

(c) E_1, E_2 are not independent

(d) $P\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

Try Yourself

T1. Consider a coin with probability P to be heads. What is the probability that the first head will appear on the even numbered tosses?

(a) $\frac{1-P}{2}$ (b) $\frac{1-P}{2-P}$

(c) $\frac{1-P}{3-P}$ (d) $\frac{1}{P(1-P)}$

[Ans: (b)]

T2. Suppose that X is a random variable for which $E(X) = \mu$ and $Var(X) = \sigma^2$. Let 'c' be an arbitrary constant. Then $E[(X-c)^2]$ = _____.

(a) $(\mu - c)^2$ (b) $(\mu - c)^2 - \sigma^2$

(c) $(\mu - c)^2 + \sigma^2$ (d) None of these

[Ans: (c)]

T3. Assume that connected simple graph 'G' has n vertices ($n \geq 3$) and e edges. Which of the following statement is invalid?

(a) If G is planar then $e \leq (3n-6)$

(b) If $e \leq (3n-6)$ then G is a planar

(c) If $e > (3n-6)$ then G is not planar

(d) If $e = 3n-5$ then G is not planar

[Ans: (b)]

T4. Which one of the following random variable is discrete?

(a) A person's exact weight

(b) Number of questions attempted in a test

(c) Both (a) and (b)

(d) Neither (a) nor (b)

[Ans: (b)]

T5. Match the following Lists:

List-I

A. $\binom{n}{x} \cdot P^x (1-P)^{n-x}$

B. $\frac{e^{-\lambda} \cdot \lambda^x}{x!}$

C. $\lambda \cdot e^{-\lambda x}$

D. $\frac{1}{b-a}$

List-II

1. Discrete distribution
2. Continuous distribution

Codes:

	A	B	C	D
(a)	1	2	1	2
(b)	2	1	2	1
(c)	1	1	2	2
(d)	2	2	1	1

[Ans: (c)]

T6. If the proportion of handicapped people in a large population is 0.006, then what is the probability that there will be atmost one handicapped person in a randomly chosen group of 500 people? Use poisson approximation to compute the probability.

(a) $3e^{-4}$ (b) $2e^{-3}$
(c) $4e^{-3}$ (d) $3e^{-2}$

[Ans: (c)]

T7. Let X be an exponential random variable with rate parameter λ. Then find variance of X.

(a) $\frac{1}{\lambda}$ (b) $\frac{1}{\lambda^2}$
(c) $\frac{2}{\lambda}$ (d) $\frac{2}{\lambda^2}$

[Ans: (b)]

T8. E[X] = 1 and Var[X] = 2, which one of the following is not correct?

(a) E[6 X] = 6 (b) Var[6 X] = 72
(c) E[1 – X] = 0 (d) Var[1 – X] = 3

[Ans: (d)]

T9. Find the value of λ such that function $f(x)$ is valid probability density function.

$f(x) = \lambda(x-1)(2-x)$ for $1 \le x \le 2$
$= 0$ otherwise

[Ans: (6)]

T10. Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output *Y* and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is *Y* is _____ (up to two decimal places).

[GATE-2016, Ans: (0.33)]

■■■■

6 Linear Algebra

Multiple Choice Questions

Q.1 A square matrix is singular whenever
(a) the rows are linearly independent
(b) the columns are linearly independent
(c) the rows are linearly dependent
(d) None of the above

[GATE-1987]

Q.2 The eigen vector(s) of the matrix

$$\begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \alpha \neq 0 \text{ is (are)}$$

(a) $(0, 0, \alpha)$ (b) $(\alpha, 0, 0)$
(c) $(0, 0, 1)$ (d) $(0, \alpha, 0)$

[GATE-1993]

Q.3 If $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{pmatrix}$ the matrix A^4, calculated by the use of Cayley-Hamilton theorem or otherwise, is
(a) O (b) I
(c) A (d) None of these

[GATE-1993]

Q.4 Let A and B real symmetric matrices of size $n \times n$. Then which one of the following is true?
(a) $AA^t = I$ (b) $A = A^{-1}$
(c) $AB = BA$ (d) $(AB)^t = BA$

[GATE-1994]

Q.5 The rank of matrix $\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is
(a) 0 (b) 1
(c) 2 (d) 3

[GATE-1994]

Q.6 If A is an orthogonal a matrix, which of the following statements about A, is false?
(a) $A = A^{-1}$
(b) $A\,A^t = \mathrm{I}$
(c) $A^t = A^{-1}$
(d) The eigen values of A have magnitude 1

Q.7 The rank of the following $(n+1) \times (n+1)$ matrix, where a is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix}$$

(a) 1
(b) 2
(c) n
(d) Depends on the value of a

[GATE-1995]

Q.8 Let A be the set of all non singular matrices over real number and let $*$ be the matrix multiplication operation. Then
(a) A is closed under $*$ but <A, $*$> is not a semi-group

(b) <A, *> is a semi-group but not a monoid
(c) <A, *> is a monoid but not a group
(d) <A, *> is a group but not an Abelian group

[GATE-1995]

Q.9 Let $Ax = b$ be a system of linear equations where A is an $m \times n$ matrix and b is a $m \times 1$ column matrix and X is a $n \times 1$ column vector of unknowns. Which of the following is false?

(a) The system has a solution if and only if, both A and the augmented matrix $[A\ b]$ have the same rank
(b) If $m < n$ and b is the zero vector, then the system has infinitely many solutions
(c) If $m = n$ and b is non-zero vector, then the system has a unique solution
(d) The system will have only a trivial solution when $m = n$, b is the zero vector and rank $(A) = n$

[GATE-1996]

Q.10 The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication

(a) If $a = b$ or $\theta = n\pi$, where n is an integer
(b) Always
(c) Never
(d) If $a \cos\theta \neq b \sin\theta$

[GATE-1996]

Q.11 The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ is

(a) 11 (b) –48
(c) 0 (d) –24

[GATE-1997]

Q.12 Let $A = (a_{ij})$ be an n-rowed square matrix and I_{12} be the matrix obtained by interchanging the first and second rows of the n-rowed identity matrix. Then AI_{12} is such that its first

(a) row is the same as its second row
(b) row is the same as the second row of A
(c) column is the same as the second column of A
(d) row is all zero

[GATE-1997]

Q.13 Consider the following set of equations

$$x + 2y = 5$$
$$4x + 8y = 12$$
$$3x + 6y + 3z = 15$$

This set

(a) has unique solution
(b) has no solutions
(c) has finite number of solutions
(d) has infinite number of solutions

[GATE-1998]

Q.14 The rank of the matrix given is $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$

(a) 3 (b) 1
(c) 2 (d) 4

[GATE-1998]

Q.15 Consider the following determinant

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Which of the following is a factor of Δ?

(a) $a + b$ (b) $a - b$
(c) $a + b + c$ (d) abc

[GATE-1998]

Q.16 The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is

(a) 4 (b) 0
(c) 15 (d) 20

[GATE-2000]

Q.17 Consider the following statements:

S_1: The sum of two singular $n \times n$ matrices may be non-singular.

S_2: The sum of two $n \times n$ non-singular matrices may be singular.

Which of the following statements is correct?

(a) S_1 and S_2 are both true
(b) S_1 is true and S_2 is false
(c) S_1 is false and S_2 is true
(d) S_1 and S_2 are both false

[GATE-2001]

Q.18 The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is

(a) 4 (b) 2
(c) 1 (d) 0

[GATE-2002]

Q.19 For the following matrix A, which one of the given options is true?

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$$

(a) The rank of A^T is 4
(b) The rank of the matrix A is 3
(c) The number of linearly independent row vectors is 2
(d) The matrix A is linearly independent

[JNU]

Q.20 Consider the matrix $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$. Which of the following is two out of its three eigenvalues?

(a) 3, 4 (b) 3, 5
(c) 5, 4 (d) 4, 6

[JNU]

Q.21 Given an $n \times n$ matrix B with determinant D, let B′ be obtained by first multiplying the i^{th} row by k and then interchanging the i^{th} and the j^{th} rows. What is the determinant of B′?

(a) $k^n D$ (b) $k - D$
(c) $-kD$ (d) $\frac{D}{k}$

[JNU]

Q.22 For large n, how many arithmetic operations does the Gauss-Jordan elimination require?

(a) n^3 (b) $\frac{n^3}{6}$
(c) $\frac{n^3}{3}$ (d) $\frac{n^3}{2}$

[JNU]

Q.23 Total number of divisions and multiplications required for solving a system of 5 equations using Gauss Elimination Method is

(a) 113 (b) 208
(c) 65 (d) 45

[JNU]

Q.24 Which one of the following is not a property of eigenvalues and eigenvectors?

(a) The eigenvalues of a real symmetric matrix are real
(b) Eigenvalues of a singular matrix are reciprocal of the eigenvalues of inverse of that matrix
(c) Similar matrices have the same eigenvalues
(d) The matrix and its transpose have the same eigenvalues

[JNU]

Q.25 Let AX = B be a system of n linear equations in n unknown with integer coefficient and the components of B are all integer. Consider the following:

1. det (A) =1
2. det (A) = 0
3. solution X has integer entries
4. solution X does not have all integer entries

For the given system of linear equations which one of the following is correct?

(a) Only 3 unconditionally holds true
(b) If 1 then 3 holds true
(c) If 1, then 4 holds true
(d) If 2, then 3 holds true

[JNU]

Q.26 Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50$. Consider the following statements.

I. One eigenvalue must be in [–5, 5]

II. The eigenvalue with the largest magnitude must be strictly greater than 5.

Which of the above statements about eigenvalues of A is/are necessarily CORRECT?

(a) Both I and II (b) I only

(c) II only (d) Neither I nor II

[GATE-2017]

Numerical Answer Type Questions

Q.27 Find the rank of following matrix A

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 1 \\ 3 & 6 & 2 \end{bmatrix}$$

Q.28 Consider the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

For which value of x is the matrix A not invertible?

Q.29 Suppose 3-digit number is formed as following.

- First digit is any one of 1, 2 and 3
- Second digit is any one of 4, 5 and 6
- Third digit is any one of 7, 8 and 9

How many ways the above 3-digit number can be formed?

Q.30 Find the value of x in the matrix A^{-1} for the following matrix A and A^{-1}.

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & \frac{1}{2} & x \\ 0 & 0 & 1 \end{pmatrix}$$

Q.31 Consider the following system of equations.

$$x + y = 2$$
$$x + py = 2$$

Where p is constant. Find the value of 'p' such that the system has more than two solutions.

Multiple Select Questions

Q.32 Which of the following matrices are invertible?

(a) $A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ (b) $A_2 = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & -1 \\ -1 & 0 & 3 \end{bmatrix}$

(c) $A_3 = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 2 \\ 4 & 7 & 4 \end{bmatrix}$ (d) $A_4 = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Q.33 Which of the following represents the LU decomposition of the given matrix. (Using Doolitles & method)

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} U = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$

(b) $L = \begin{bmatrix} 25 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 0.7 \end{bmatrix} U = \begin{bmatrix} 1 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} U = \begin{bmatrix} 25 & -5 & 1 \\ 0 & 4.8 & 1.56 \\ 0 & 0 & -0.7 \end{bmatrix}$

(d) $L = \begin{bmatrix} 25 & 0 & 0 \\ 2.56 & -4.8 & 0 \\ 5.76 & 3.5 & 0.7 \end{bmatrix} U = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & -1.56 \\ 0 & 0 & 1 \end{bmatrix}$

Q.34 Which of the following are true?
(a) The value of determinant remains unchanged, if the rows are inter changed into column or vice-versa.
(b) A skew symmetric matrix of even order is perfect square.
(c) Determinant of odd order skew symmetric matrix is zero.
(d) None of these

Q.35 A, B are two square matrices of same order such that $AB = B$ and $BA = A$, then
(a) $A^2 = A$
(b) $B^2 = B$
(c) $(A + B)^4 = 2^3 (A + B)$
(d) $A^2B = B$

Q.36 If A, B are non zero matrices such that $AB = 0_{n \times n}$. Then which of the following is/are true?
(a) $|A| = 0$ (b) $|B| = 0$
(c) $|A| \neq 0$ (d) $|B| \neq 0$

Q.37 Let M be an $n \times n$ matrix with real entries such that $M^3 = I$ suppose that $Mx \neq x$ for any non zero vector x. Then which of the following statement is/are true?
(a) M has real Eigen values
(b) $M + M^{-1}$ has real Eigen values
(c) n is divisible by 2
(d) n is divisible by 3

Q.38 Suppose $\alpha, \beta\gamma \in R$. Consider the following system of linear equations.
$x + y + z = \alpha$
$x + \beta y + z = \gamma$
$x + y + \alpha z = \beta$
If this system has atleast one solution, then which of the following statement is/are true?
(a) If $\alpha = 1$ then $\gamma = 1$
(b) If $\beta = 1$ then $\gamma = \alpha$
(c) If $\beta \neq 1$ then $\alpha = 1$
(d) If $\gamma = 1$ then $\alpha = 1$

Q.39 Let $m, n \in N$, $m < n$, $P_{n \times m}$, $Q_{m \times n}$ are two real matrices then which of the following not possible?
(a) Rank $(PQ) = n$
(b) Rank $(QP) = m$
(c) Rank $(PQ) = m$
(d) Rank $(QP) = \left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

Q.40 For the following matrix A, which of the following is/are false?

$$A \begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$$

(a) rank of A^T is 4
(b) rank of A is 3
(c) no. of linearly independent row vector is '2'
(d) the matrix A is linearly independent

Q.41 Let $Ax = b$ be a system of linear equations where A is an $m \times n$ matrix and b is a $m \times 1$ column matrix and x is a $n \times 1$ column vector of unknowns.
Which of the following is/are true?
(a) The system has a solution if and only if, both A and the augmented matrix $[Ab]$ have the same rank
(b) If $m < n$ and b is the zero vector, then the system has infinitely many solutions
(c) If $m = n$ and b is non-zero vector, then the system has a unique solution
(d) The system will have only a trivial solution when $m = n$, b is the zero vector and rank $(A) = n$

Q.42 For real constants a and b, let

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ a & b \end{bmatrix}$$

be an orthogonal matrix. Then which of the following statements is/are always TRUE?
(a) $a + b = 0$ (b) $b = \sqrt{1 - a^2}$
(c) $ab = -\frac{1}{2}$ (d) $M^2 = I_2$

Q.43 Let P be a 2×2 real matrix such that every non-zero vector in R^2 is an eigen vector of P. Suppose that λ_1 and λ_2 denote the eigenvalues of P and $P\begin{bmatrix}\sqrt{2}\\ \sqrt{3}\end{bmatrix} = \begin{bmatrix}2\\ t\end{bmatrix}$ for some $t \in R$. Which of the following statements is (are) TRUE?

(a) $\lambda_1 \neq \lambda_2$
(b) $\lambda_1\lambda_2 = 2$
(c) $\sqrt{2}$ is an eigen value of P
(d) $\sqrt{3}$ is an eiven value of P

Try Yourself

T1. The system of linear equations
$x + y + z = 2,\ 2x + y - z = 3,\ 3x + 2y + Kz = 4$
has unique solution if _____

(a) $K \neq 0$ (b) $K \leq 0$
(c) $K \geq 0$ (d) $K = 0$

[Ans: (a)]

T2. A matrix $P_{3\times3}$ has three eigen values $-1, \frac{1}{2}, 3$.
What will be the eigen values of $P^2 + 2P + I$?
[Where I is identity matrix (3×3 order)]

(a) $0, 16, \frac{9}{4}$ (b) $1, \frac{1}{4}, 9$
(c) $2, \frac{9}{2}, 4$ (d) $2, \frac{3}{2}, -2$

[Ans: (a)]

T3. Find the value of K for which the following matrix A is singular.

$$A = \begin{bmatrix}1 & 2 & K\\ 3 & -1 & 1\\ 5 & 3 & -5\end{bmatrix}$$

(a) $K = -1$
(b) $K = -2$
(c) $K = -3$
(d) None of these

[Ans: (c)]

T4. Compute the inverse of the following matrix A.

$$A = \begin{bmatrix}1 & 3 & -2\\ 0 & 2 & 4\\ 0 & 0 & -1\end{bmatrix}$$

(a) $\begin{bmatrix}1 & -\frac{3}{2} & -8\\ 0 & \frac{1}{2} & 2\\ 0 & 0 & -1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 3 & -2\\ 0 & 2 & 4\\ 0 & 0 & -1\end{bmatrix}$

(c) $\begin{bmatrix}1 & -\frac{3}{2} & 8\\ 0 & 2 & 4\\ 0 & 0 & -1\end{bmatrix}$ (d) None of these

[Ans: (a)]

T5. Let A be a 4×4 matrix with real entries such that $-1, 1, 2, -2$ are its eigen values. If $B = A^4 - 5A^2 + 5I$ where I denotes 4×4 identity matrix, then which of the following is correct? [det(X) represents determinant of X]

(a) det (A + B) = 0
(b) det (B) = 1
(c) trace of A + B is 4
(d) All of these

[Ans: (d)]

T6. Which of the following is a scalar matrix?

(a) $\begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 3\\ 2 & 3 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$

[Ans: (c)]

T7. Find an eigen vector corresponding to largest eigen value of matrix $A = \begin{bmatrix}1 & -1 & 1\\ 1 & -1 & 1\\ -1 & 1 & 1\end{bmatrix}$

(a) $\begin{bmatrix}1\\1\\1\end{bmatrix}$ (b) $\begin{bmatrix}1\\2\\1\end{bmatrix}$

(c) $\begin{bmatrix}2\\1\\1\end{bmatrix}$ (d) $\begin{bmatrix}\frac{1}{2}\\1\\0\end{bmatrix}$

[Ans: (a)]

T8. Identity matrix is _____.

(a) Scalar matrix (b) Diagonal matrix

(c) Both (a) and (b) (d) Neither (a) nor (b)

[Ans: (c)]

T9. Let A be $n \times n$ matrix, $A^n = A^{n-1} A$ for $n \geq 1$ and $A^0 = I_n$. Which of the following is correct

(a) $A^m \cdot A^n = A^{m+n}$ (b) $(A^m)^n = A^{mn}$

(c) Both (a) and (b) (d) Neither (a) nor (b)

[Ans: (c)]

T10. Let $A = \begin{bmatrix}-1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}a & b\\ c & d\\ e & f\end{bmatrix}_{3\times2}$.

If $AB = I$ then element f = _____.

(a) a (b) $a + 1$

(c) b (d) $1 - b$

[Ans: (c)]

T11. If we take n-tuples of the form $(a_1, a_2, ..., a_n)$, maximum how many of these n-tuples can be linearly independent.

(a) $n - 1$ (b) n

(c) $\frac{n}{2}$ (d) $2n$

[Ans: (b)]

T12. Consider the following factorization of a matrix A.

$A = LU$,

where $L = \begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 7 & 2 & 1\end{bmatrix}_{3\times3}$, $U = \begin{bmatrix}a & b & c\\ 0 & d & e\\ 0 & 0 & f\end{bmatrix}_{3\times3}$

and $A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}_{3\times3}$

If $i = 1$ then $a_{ij} = j$ otherwise $a_{ij} = 3 + a_{(i-1)j}$. Find the matrix U.

(a) $\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & -2\\ 0 & 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2 & -6\\ 0 & -3 & 3\\ 0 & 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3\\ 0 & -3 & -6\\ 0 & 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2 & -6\\ 0 & -1 & -6\\ 0 & 0 & 0\end{bmatrix}$

[Ans: (c)]

■■■■

7 Calculus

Multiple Choice Questions

Q.1 What is the value of $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$?

(a) 0 (b) ∞

(c) 1 (d) $\frac{1}{2}$

(e) $\frac{1}{4}$

Q.2 What is the value of $\lim_{x \to \infty} \frac{\sqrt{(3x^2 + 5)}}{5x + 3}$?

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{\sqrt{3}}{5}$ (d) $\frac{\sqrt{5}}{3}$

(e) 0

Q.3 Find the value of $\int \frac{1}{x \log x} dx$

(a) $\log x + c$ (b) $\log(\log x) + c$

(c) $\log(\log(\log x)) + c$ (d) $\frac{1}{\log x} + c$

(e) $x + c$

Q.4 $f(t) = 112\,t - \frac{28}{15}t^2 - 600$, then $f(t)$ is maximum at t = ?

(a) 0 (b) 15

(c) 30 (d) 112

(e) 600

Q.5 Which of the value of $\lim_{x \to \infty} \frac{3^{x+1} + 4}{3^{x+2} + 4}$?

(a) 0 (b) 1

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(e) ∞

Q.6 If $f(x) = 2x^2 + x - 3$, then the value of $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$ is equal to

(a) 9 (b) 5

(c) 2 (d) 1

(e) Does not exist

Q.7 The function $f(x) = x^3 - 3x^2 + 3x + 2$ is

(a) monotonically increasing in [0, 1] and monotonically decreasing in [2, 3]

(b) monotonically decreasing in [0, 1] and monotonically increasing in [2, 3]

(c) monotonically decreasing in every interval

(d) monotonically increasing in every interval

Q.8 The maximum value of $2x^3 - 15x^2 + 36x + 10$ is

(a) 37 (b) 38

(c) 49 (d) 60

Q.9 $f(x) = (x - 1)(x - 2)(x - 3)$ has maximum value when x is equal to

(a) $2 - \frac{1}{\sqrt{3}}$ (b) $2 + \frac{1}{\sqrt{3}}$

(c) $1 + \frac{2}{\sqrt{3}}$ (d) $1 - \frac{2}{\sqrt{3}}$

Q.10 If $f(n) = \int_0^{\pi/4} \tan^n x\, dx$ then $f(3) + f(1)$ equals

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) $\frac{3}{2}$

Q.11 The Langrange's mean value theorem is applicable to $f(x) = \sin \frac{1}{x}$ in the interval.

(a) [–3, 3] (b) [–2, 5]

(c) [2, 3] (d) [–1, 4]

Q.12 What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2]?

(a) 6 (b) 10

(c) 12 (d) 5.5

[GATE-1997]

Q.13 Consider the function $y = |x|$ in the interval [–1, 1]. In this interval, the function is

(a) continuous and differentiable

(b) continuous and not differentiable

(c) differentiable but not continuous

(d) neither continuous nor differentiable

[GATE-1998]

Q.14 Consider the following statements:

1. If f is a function such that a = 9, $f(a) = 9$ and $f'(a) = 4$, then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} = 4$.
2. If $f(x)$ has derivative at the point x_0; then the function $F(x) = f(x)\, g(x)$ will not be differentiable at x_0.
3. Let $f(x) = \begin{cases} -2 & \text{for } -3 \le x \le 0 \\ x - 2 & \text{for } 0 < x \le 3 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$, then $g(x)$ is differentiable in (–3, 3) except at $x = 0$.

Identify the valid option from the following, relating the number of correct statements

(a) All the three statements are correct

(b) Exactly two statements are correct

(c) Exactly one statement is correct

(d) All the statements are incorrect

[JNU]

Q.15 $\int \frac{1}{\sin^2 x + \sin 2x}\, dx$ is equal to

(a) $\frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right|$ (b) $\frac{1}{2}\log\left(\frac{\tan x + 2}{\tan x}\right)$

(c) $\frac{1}{2}\log\left|\frac{\cot x}{\cot x + 2}\right|$ (d) $\frac{1}{2}\log\left(\frac{\cot x + 2}{\cot x}\right)$

[JNU]

Q.16 If $x^{\sin y} = y^{\sin x}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x^2 \cos x \log x - y \sin y}{x^2 \cos x \log x - x \sin x}$

(b) $\frac{y^2 \cos y \log y - x \sin x}{y^2 \cos y \log y - y \sin y}$

(c) $\frac{xy \cos x \cos y - y \sin y}{xy \cos x \cos y - x \sin x}$

(d) $\frac{xy \cos x \log y - y \sin y}{xy \log x \cos y - x \sin x}$

[JNU]

Q.17 Consider the function $f(x) = x^3 - 3x + 2$. Assume that as x moves from left to right through a point $c = 2$, what is the behaviour of the graph of $f(x)$?

(a) Rising

(b) Falling

(c) Growing parallel to x-axis after c

(d) None of the above

[JNU]

Q.18 Consider the following two statements:

1. If $f(x)$ has derivative at the point x_0 and $g(x)$ has no derivative there, then the function $F(x) = f(x)\, g(x)$ will not be differentiable at x_0 where $f(x)$ and $g(x)$ are real-valued functions of the real variables.
2. The function $f(x)$ defined by $f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ -x & \text{when } x \text{ is irrational} \end{cases}$ is Riemann integrable over the interval $[a, b]$.

Which of the following options is correct for the above two statements?

(a) 1 is true but 2 is false
(b) 1 is false but 2 is true
(c) Both 1 and 2 are true
(d) Both 1 and 2 are false

[JNU]

Q.19 If $f(a) = a^2$, $\varphi(a) = b^2$; $f'(a) = \varphi'(a)$, then $\lim_{x \to a} \frac{\sqrt{f(x)} - a}{\sqrt{\varphi(x)} - b}$ is equal to

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$
(c) 0 (d) ∞

[JNU]

Q.20 If $f(x) = \begin{cases} x \sin x(\log x^2); & x \neq 0 \\ 0 & ; x = 0 \end{cases}$, then

(a) $f(x)$ is discontinuous and differentiable at $x = 0$
(b) $f(x)$ is continuous but differentiable at $x = 0$
(c) $f(x)$ is continuous and not differentiable at $x = 0$
(d) $f(x)$ is discontinuous and not differentiable at $x = 0$

[JNU]

Q.21 $\lim_{x \to 0} \left(\frac{\sin(3x)(2^x - 1)}{x^2} \right) =$

(a) 3 log2 (b) 2 log3
(c) 6 (d) 0

Q.22 For $f(x) = \begin{cases} x^2 + 3a; & x \leq 1 \\ bx + 2; & x > 1 \end{cases}$, what are the values of a and b for $f(x)$ to be derivable at $x \in R$?

(a) $a = 1, b = 2$ (b) $a = 2, b = 1$
(c) $a = 1, b = 1$ (d) $a = 2, b = 2$

[JNU]

Q.23 If x and y are real and $x^2 + y^2 = 1$, find the maximum value of $(x + y)^2$

(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

[JNU]

Q.24 The maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$ respectively are

(a) +1, –1 (b) –1, –2
(c) +2, –1 (d) +2, –2

[JNU]

Q.25 An open box is to be made from a rectangular piece of metal sheet, 12 cm by 18 cm, by cutting out equal squares from each corner and folding up the sides. Then the side of the cut-out squares, such that the open box is of largest volume, is equal to

(a) $7 + \sqrt{5}$ cm (b) $7 - \sqrt{5}$ cm
(c) $5 - \sqrt{7}$ cm (d) $5 + \sqrt{7}$ cm

[JNU]

Q.26 If $f(x) = R \sin\left(\frac{\pi x}{2}\right) + S$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are, respectively

(a) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (b) $\frac{2}{\pi}$ and 0
(c) $\frac{4}{\pi}$ and 0 (d) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

[GATE-2017]

Numerical Answer Type Questions

Q.27 What is the Cauchy's mean value for the functions e^x and e^{-x} between (0, 1)? [Please enter the value in $x \cdot y$ format]

Q.28 Let $f(x) = x \cdot \int_1^{x^2} \sqrt{y^2 + 3} \cdot dy$. Find $f(1)$

Multiple Select Questions

Q.29 Consider the function:

$$f(x) = \begin{cases} x^4, & x^2 < 1 \\ x, & x^2 \geq 1 \end{cases}$$

Then which of the following is/are true

(a) $f(x)$ is continuous at $x = -1$.
(b) $f(x)$ is not continuous at $x = -1$.
(c) $f(x)$ is differentiable at $x = -1$.
(d) $f(x)$ is not differentiable at $x = -1$.

Q.30 Which of the following are correct?

(a) For the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at 5.
(b) For the function $y = x^2 - 6x + 9$. The minimum value of y obtained when x varies over the interval 2 to 5 will be at 3.
(c) The absolute maximum value of the function $f(x, y) = 2x + 2y - x^2 - y^2 + 2$ will be at 4.
(d) The absolute maximum value of the function $f(x, y) = 2x + 2y - x^2 - y^2 + 2$ will be at 2.

Q.31 Which of the following are correct?

(a) The value of $\lim_{x \to 0} \frac{e^{ax} - e^{-ax}}{\log(1 + bx)}$ is $\frac{2a}{b}$.

(b) The value of integral $\int \frac{\sin x}{1 - \sin x} dx$ is equal to $(\tan x + \sec x - x + C)$.

(c) The value of integral $\int_0^1 x e^x dx = 1$.

(d) The value of integral $\int_0^1 x e^x dx = 0$.

Q.32 For the function

$f(x) = 2x^3 - 3(a + b)x^2 + 6\ abx \forall\ xR$

which of the following is true?

(a) $f(x)$ has local maximum and a local minimum when $a < b$
(b) $f(x))$ has a local maximum and a local minimum when $a > b$
(c) $f(x)$ is neither maximum nor minimum when $a = b$
(d) $f(x)$ has 2 local minima when $a \neq b$

Q.33 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2 (2 - y) - y^3 + 3y^2 + 9y$, where $(x, y) \in \mathbb{R}^2$.

Which of the following is/are suddle point(s) of f?

(a) $(0, -1)$ (b) $(0, 3)$
(c) $(3, 2)$ (d) $(-3, 2)$

Q.34 Let $f \cdot \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiate on $(-\infty, 0)\ U(0, \infty)$. Which of the following statements is (are) always TRUE?

(a) If f is differential at 0 and $f'(0) = 0$, then f has a local maximum or a local minimum at 0
(b) If f has a local minimum at 0, then f is differentiable at 0 and $f'(0) = 0$
(c) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then f has a global minimum at 0
(d) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then f has a global maximum at 0

Try Yourself

T1. Consider the following statements:

Assertion (A): The function $f(x) = x - [x]$, $x \in Z$ is discontinuous at $x = 1$

Reason (R): $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

Choose the correct option.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true and R is not the correct explanation of A
(c) A is true but R is false
(d) Both A and R are false

[Ans: (d)]

T2. Which of the following is correct?

(a) $\left(1+\frac{1}{n}\right)^{n+1} \to e$ as $n \to \infty$

(b) $\left(1+\frac{1}{n}\right)^{n^2} \to e$ as $n \to \infty$

(c) $\left(1+\frac{1}{n^2}\right)^{n} \to e$ as $n \to \infty$

(d) None of these

[Ans: (a)]

T3. $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3-x}}{x} =$ _____

(a) 0 (b) 1

(c) 2 (d) None of these

[Ans: (d)]

Linked Answer for T4 and T5:

Given a function $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$

T4. The minimal point in the given function is

(a) (1, 0) (b) (1, 0.33)

(c) (0.33, 0) (d) None of these

[Ans: (b)]

T5. The optimal value of $f(x, y)$ at the above minimal point is

(a) $\frac{8}{3}$ (b) 4

(c) $\frac{10}{3}$ (d) None of these

[Ans: (c)]

T6. Find the x values for which the following function is increasing.

$$f(x) = \frac{x}{x^2+4}$$

(a) $x < 2$ (b) $x > 2$

(c) $x < -2$ and $x > 2$ (d) $x > -2$ and $x < 2$

[Ans: (d)]

T7. Consider the function $f(x) = x + \ln x$ and f is differentiable on $(1, e)$ and $f(x)$ is continuous on $[1, e]$. Determine the c value using mean value theorem. [By computing $f'(c) = \frac{f(b)-f(a)}{b-a}$]

(a) e (b) $e - 1$

(c) $\frac{e}{e-1}$ (d) $\frac{e-1}{e}$

[Ans: (b)]

T8. Let $z = x \sin y - y \sin x$. The total differential $dz =$ _____.

(a) $(\sin y + y \cos x)\, dx + (x \cos y + \sin x)\, dy$

(b) $(\sin y - y \cos x)\, dx + (x \cos y + \sin x)\, dy$

(c) $(\sin y + y \cos x)\, dx + (x \cos y - \sin x)\, dy$

(d) $(\sin y - y \cos x)\, dx + (x \cos y - \sin x)\, dy$

[Ans: (d)]

T9. Consider the following statements:

S_1 : $f(x) = x^5 + 3x - 1$ is an increasing function for all values of x.

S_2 : $f(x) = 1 - x^3 - x^9$ is a decreasing function for all values of x where $x \neq 0$.

Which of the above statements are TRUE?

(a) S_1 only (b) S_2 only

(c) Both S_1 and S_2 (d) Neither S_1 nor S_2

[Ans: (c)]

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