Page no: 01

Propositional Logic

Proposition: A declarative sentence to which we can assign one and only one of the truth values TRUE (S.) FALSE is called proposition.

Ex: Delhi is a city (T) // here Truth value TRUE (T)

Ex: 2x3=5(F) // here Truth value FALSE(F)

Proposition Can be Classified into two Categories

- 1. Atomic Proposition
- 2. Compound Proposition

Atomic Proposition: A proposition which can not be divided further.

Ex: p: 2+8=11

9: 2×10=20

Compound Proposition: Two (or) more atomic proposition Combe

Ex: Sun Rises in the east and 3+9=12

Ex: Sun Rises in the east (Or) 3+9=12

Connectivities: There are five Connectivities in the logic

mot ~

OY V

and A

implies ->

if and only if

- 1. Conjunction: PAQ is True only when both p and q have
 Truth values are TRUE
- 2. <u>Disjunction</u>: PVQ is False only when both P and 9 have truth values are FALSE
- 3) Implication: if p then q can be written as p→q
 P→q Touth value is False when p is True
 and q is False

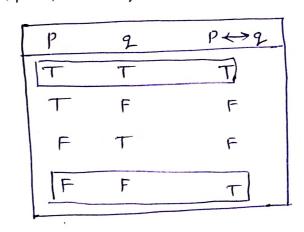
Note: - In P→9, . We can call 'p' as antecedent and 'q' is called Consequent.

- We can also call 'p' as premise (or) hypothesis and 'q' is called Consequent (or) Conclusion.

Note: 1) The Converse of $(P \rightarrow Q)$ is $(Q \rightarrow P)$ 2) The inverse of $(P \rightarrow Q)$ is $\sim P \rightarrow \sim Q$ 3) The Contrapositive of $(P \rightarrow Q)$ is $\sim Q \rightarrow \sim Q$ Note $P \rightarrow Q \cong \sim Q \rightarrow \sim Q$

Page no: 02

BiConditional: Piffq Can be written as P+9 is
a proposition whose Truth value is True
if P and 2 have Same True values



Note:
$$(P \leftrightarrow q) \equiv ((P \rightarrow q) \land (q \rightarrow P))$$

$$\sim (P \leftrightarrow q) \equiv ((P \land wq) \lor (wP \land q))$$

$$\stackrel{\sim}{=} P \oplus q$$

$$\therefore \oplus \text{ is } xor \text{ operation.}$$

Tautology: A Compound proposition which is always True

Ex: (PV~P)

Ex: (pv(p→q))

Ex: $P \rightarrow (PV2)$

 $Ex : (PAQ) \rightarrow (P \leftrightarrow Q)$

Contradiction: A compound proposition which is always False

Ex: PA ~P

Ex: ~ (P->9) -> (~pv9)}

Contingency: A Compound proposition which is neither a tautology not a Contradiction

Ex: $P \rightarrow 2$

Ex: PV2

Ex: P+2

Salisfiability: A Compound proposition which is not a Contradiction is said to be Salisfiable

Ex: P->9

Ex PY2

Ex P+9

Note: 1 Every Contingency is Satisfiable

2) Every tautology is Satisfiable

Tautological Implication

1) If P and Q are two Compound propositions and If $P \rightarrow Q$ is a tautology then we say that "P tautologically implies Q" and written as $P \Rightarrow Q$.

Method 1

Whenever P is True if Q' is also True, then $P \rightarrow Q$ is True hence $P \Rightarrow Q$

Meltod2

*) Whenever Q is False, if P is also False then P⇒ Q.

Equivalence: Let P and Q are two Compound propositions then "P is equivalent to Q" written as

 $P \cong Q$ (or) $P \Leftrightarrow Q$

if 'P' and 'Q' have same Truth Tables.

Note: $9 + (P \Rightarrow Q)$ and $(Q \Rightarrow P)$ then $P \Leftrightarrow Q$

EX1 P->Q \ ~PVQ

Ex2 P-Q \ NQ -NP

Argument:

Of a Set of Premises of PI, P2, ... Prof. Yield another proposition Q (Conclusion) then the whole Process is Called an argument (inference) and denoted by

dP1, P2,··· Pof → Q is a Tautology

(Pr NP2 NP3··· NPn) → 9 is a Tautology

(Property (Or)

Prove the following

(Or)

This Symbol is Tautologically implication

implication

From [Conclusion]

 $Ex: q(Pvq), \sim P3 \Rightarrow q$

We want to Show that [(PVQ) 1 NP] -> Q is a Taulology

Now {(PVØ) 1 ~P} -> Q

Assum (PVQ) is True, (OP) is True, at this point of time (PVQ) N(OP) is True, at this point of time 'q' must be True

So according our discussion when we assume L.H.s is True then for those truth values, we have to get R.H.s is There then $L ext{-}H ext{-}S o R ext{-}H ext{-}S$ is Tautology and hence $L ext{-}H ext{-}S o R ext{-}H ext{-}S$ So here whenever $(P ext{-}Q) ext{-} (
ext{-}P ext{-}Q) ext{-} (
ext{-}P ext{-}Q) ext{-}N ext{-}P ext{-}Q,$

Rules of Inference

1) Simple-fication Rule

a)
$$(P \land Q) \Rightarrow P$$

2) Addition Rule

a)
$$P \Rightarrow (PVQ)$$

3) $\sim P \Rightarrow (P \rightarrow Q)$ 4) $Q \Rightarrow (P \rightarrow Q)$



- 6) $\sim (P \rightarrow Q) \Rightarrow \sim Q$
- 7) Disjunctive Syllogism $\{(PVQ) \land NP\} \Rightarrow Q$

It is saying that whenever {(PVQ) 1 op & is True then
iq' is also True

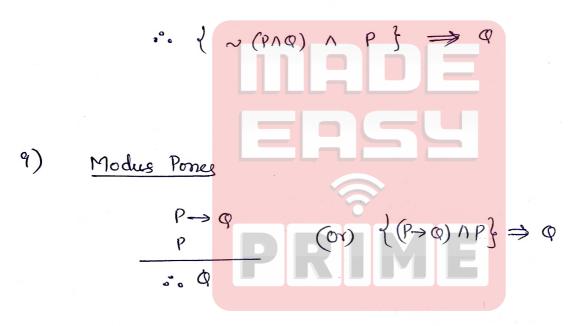
Whenever Q' is False then {(PVQ) 1 2P} also False.

8) Conjunctive Syllogism:

i.e We have to Show that

{~ (PNQ) NP} -> Q is a tautology

Assume L. H. S is True, if we can prove R. H. s also
True then given statement
is valid



No) Modus Tollens

$$\begin{cases}
(P \rightarrow Q) \land Q \end{cases} \xrightarrow{\Rightarrow} P \\
QR)$$

$$P \rightarrow Q \\
P \rightarrow Q \\
P$$

$$\begin{cases} (P \lor Q) \land (P \rightarrow R) \land (Q \rightarrow R) \end{cases} \Rightarrow R$$

Constructive Dilemma
$$\{(PVQ) \land (P \rightarrow R) \land (Q \rightarrow S) \} \Rightarrow (RVS)$$

(P→R)Λ(Q→S)Λ (NRVNS) & ⇒ NPVNQ

- 15) \mathcal{D} emorgan daw: i) $\sim (P \land Q) \Leftrightarrow \sim P \lor \sim Q$ ii) $\sim (P \lor Q) \Leftrightarrow \sim P \land \sim Q$
- 16) <u>Idempotent law</u>: i) PVP ⇒ P ii) PAP ⇔ P
 - 17) Absortion law i) PV (PNQ) ⇔ P

 ii) PN (PVQ) ⇔ P

Pageno:05

* Conditional proof (EP Rule)

of $2P_1, P_2, \dots P_n g \Rightarrow (O \rightarrow R)$ then first convert it into $\{\{P_1, P_2, \dots P_n\}\} / \{P_1, P_2, \dots P_n\} / \{P_n\} \Rightarrow R$ and prove this argument is valid, hence by CoP Rule given argument also valid.

* Indirect proof (Peoof by Contradiction)

Stepi: To apply this rule, first we assume that the argument is not valid.

Step 2:- So now we take negation of the Conclusion as a new premise.

Step3:- Now Combined new premise with ofter premises if get any Contradiction then given asymment is valid.