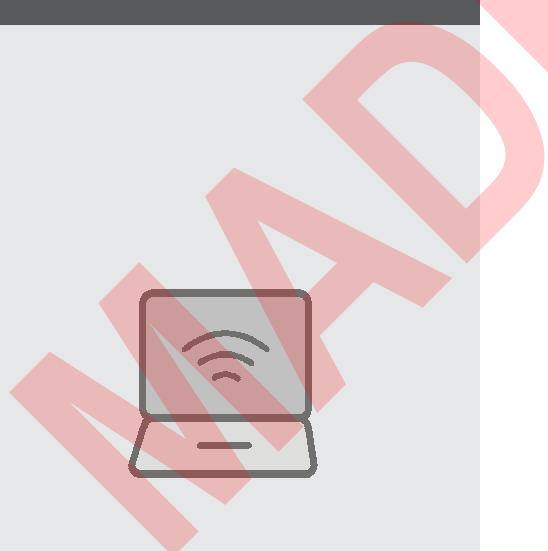


Theory of Computation

I



Contents

1. Regular Languages and Finite Automata	5
2. Context Free Languages and Pushdown Automata	35
3. Recursive, Recursively Enumerable Languages and Turing Machines	51

DESCRIPTION SHEET

THEORY OF COMPUTATION

Chapter 1 : Regular Languages and Finite Automata

- Introduction
- Chomsky Hierarchy
- Automaton
- Grammars
- Equivalence of Languages
- Expressive Power of Automata
- Applications of Automata
- Deterministic Finite Automata (DFA)
- Non-deterministic Finite Automata
- Epsilon NFA (e-NFA)
- Regular Expressions
- Equivalence between Finite Automata and Regular Expressions
- Regular Grammar
- Closure Properties of Regular Languages
- Decision Properties of Regular Language
- Moore and Mealy Machines
- Pumping Lemma
- Minimization of Finite Automata
- Myhill - Nerode Theorem

Chapter 2 : Context Free Languages and Push Down Automata

- Context Free Grammars
- Context Free Language
- Push Down Automata (PDA)
- Equivalence between CFL and CFG
- Closure Properties of CFL's
- Closure Properties of DCFL's
- Decision Properties of CFL's
- Non-Context Free Languages
- Non-Regular Languages and CFL's

Chapter 3 : Recursive, Recursively Enumerable Languages and Turing Machines

- Turing Machine (TM)
- Variation of Turing Machines
- Turing Machine Computation
- Turing Machine Recognizable Languages
- Recursive Enumerable Grammars
- Closure Properties
- Restricted Turing Machines
- Turing Machine Construction
- Post Correspondence Problem (PCP)
- Languages (Decidable, Semi-decidable and Totally not decidable)
- Chomsky Hierarchy Vs Other Classes



1

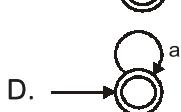
Regular Languages and Finite Automata



Multiple Choice Questions

- If $S = \{aa, aba, baa\}$ then which of the following strings does not belong to S^* ?
 - λ
 - baaaaaaba
 - abaababaaa
 - baabaaaabaaaa
- If $S = \{ab, aa\}$ and $T = \{ab, aa, abab\}$ are two languages on $\Sigma = \{a, b\}$, then which of the following is true?
 - $S^* \subset T^*$
 - $T^* \subset S^*$
 - $S^* = T^*$
 - $(S \cup T)^* = \Sigma^*$
- If $S = \{ab, ba\}$, which of the following is true?
 - S^* contains finite number of strings of infinite length
 - S^* has no strings having "aaa" or "bbb" as substring
 - S^* has no strings having "aa" as substring
 - If $T = \{a, b\}$, then $S^* \not\leq T^*$
- Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:
Assume $\Sigma = \{a, b\}$

List-I

- A. 
- B. 
- C. 
- D. 

List-II

1. NFA accepting nothing
2. DFA accepting nothing
3. NFA accepting everything
4. DFA accepting everything
5. NFA accepting " ϵ " only
6. DFA accepting " ϵ " only

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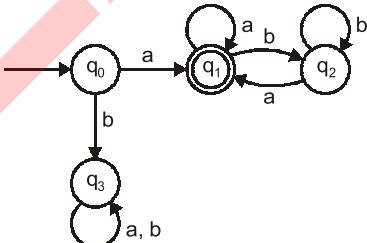
Codes:

A	B	C	D
1	2, 1	1	6
1	2, 1	5	4
1	2, 1	5	4, 3
1	2	5	4

- 1 2, 1 1 6
- 1 2, 1 5 4
- 1 2, 1 5 4, 3
- 1 2 5 4

Linked Question 5, 6, 7 and 8

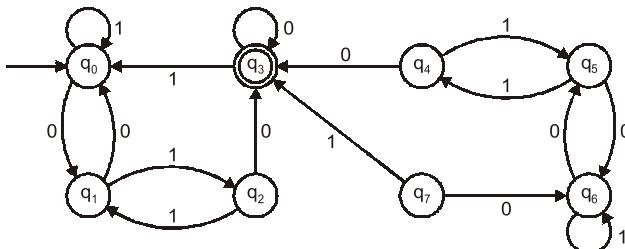
- What is the language accepted by DFA given below:



```

graph LR
    q0((q0)) -- a --> q1((q1))
    q1 -- "a, b" --> q1
    q1 -- a --> q2((q2))
    q2 -- b --> q1
    q2 -- a --> q2
    q3((q3)) -- b --> q1
    q3 -- "a, b" --> q3
    
```

 - strings starting with "a"
 - strings starting with "a" and ending with "b"
 - strings not starting with "b"
 - strings starting with and ending with "a"
- How many states are there in the minimum state DFA accepting above language?
 - 2
 - 3
 - 4
 - 5
- How many states are there in the minimum state FA accepting above language?
 - 2
 - 3
 - 4
 - 5
- Which of the following is not a regular expression for above language?
 - $aa^*(bb^*a)^*$
 - $a(a + bb^*a)^*$
 - $a(a + b)^*a + a$
 - $a(a + aa + bb^*a)^*$



Linked Answer Q.12 & Q.13

12. A 2-DFA whose behavior is given below:

State	0	1
q_0	(q_0, R)	(q_1, R)
q_1	(q_1, R)	(q_2, L)
q_2	(q_0, R)	(q_2, L)

$$\delta^*(q_0, 101001) = ?$$

- (a) q_0
 (b) q_1
 (c) q_2
 (d) Cannot be determined

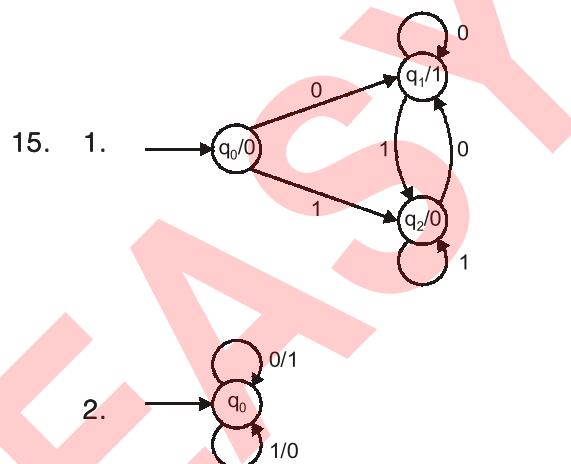
13. The string “101001” is

 - (a) accepted by above DFA
 - (b) rejected by above DFA
 - (c) neither accepted nor rejected
 - (d) not accepted since the dfa goes into an infinite loop when processing above string

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14. Which of the following is false?

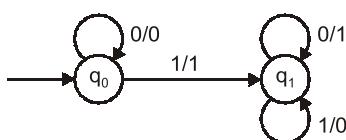
 - (a) 2-DFA's have same power as DFA's
 - (b) 2-DFA's accept regular sets
 - (c) The class of 2-DFA's are equivalent to class of NFA's but not equivalent to the class of DFA's
 - (d) 2-DFA's have less power compared to PDA



Which of the following is true?

- (a) Machines 1 and 2 given above are Moore and Mealy machines respectively that produce 1's complement of a given binary number
 - (b) Machines 1 and 2 given above are Mealy and Moore machines respectively that produce 1's complement of a given binary number
 - (c) Machines 1 and 2 given above are Moore and Mealy machines respectively that produce 2's complement of a given binary number
 - (d) Machines 1 and 2 given above are Mealy and Moore machines respectively that produce 2's complement of a given binary number

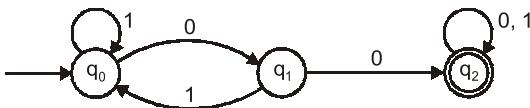
- 16.** The machine given below is



17. The below FA accepts which language?

 - (a) A Mealy machine to find 2's complement of a given binary number read from least significant bit (LSB)
 - (b) A Moore machine to find 2's complement of a given binary number read from least significant bit (LSB)
 - (c) A Mealy machine to find 2's complement of a given binary number read from most significant bit (MSB)
 - (d) A Moore machine to find 2's complement of a given binary number read from most significant bit (MSB)

17. The below FA accepts which language?



19. Which of the following are equivalent?

1. $a^+ b^+ c^+$
2. $aa^* bb^* cc^*$
3. $\{a^n b^n c^n; n \geq 0\}$
4. $\{a^m b^n c^p; m, n, p \geq 0\}$

(a) 1 and 2 only (b) 1, 2 and 4
(c) 2 and 4 only (d) 1, 2, 3 and 4

$$20. \quad R_1 = 11(0 + 1)^*$$

$$R_2 = (0 + 1)^* 11$$

$$R_3 = 11(0 + 1)^* 11 + 111 + 11$$

Which is true?

- (a) $L(R_1) = L(R_2) = L(R_3)$
 (b) $L(R_1) \cup L(R_2) = L(R_3)$
 (c) $L(R_1) \subseteq L(R_3)$
 (d) $L(R_1) \cap L(R_2) = L(R_3)$

$$\begin{aligned}
 21. \quad R_1 &= 00(0+1)^* \\
 R_2 &= 01(0+1)^* \\
 R_3 &= 10(0+1)^* \\
 R_4 &= 11(0+1)^* \\
 \Sigma &= \{0, 1\}
 \end{aligned}$$

Which of the following is true?

- (a) $L(R_1 + R_2 + R_3 + R_4) = \Sigma^*$
 (b) $L(R_2) = [L(R_3)]^R$
 (c) $L(R_1) \cap L(R_2) = \{\lambda\}$
 (d) $L(R_1 + R_2 + R_3 + R_4) = \Sigma^* - \{\lambda, 0, 1\}$

22. The regular expression $(0+1)^*1(0+1)(0+1)$ represent which of the following languages?

- (a) strings of 0's and 1's containing at least one 1
 - (b) strings of 0's and 1's containing at least three 1's
 - (c) strings of 0's and 1's ending with at least 3 1's
 - (d) string of 0's and 1's in which the 3rd bit from the right is a "1"

23. The regular expression for even number of zeroes is

- (a) $(1^* 01^* 01^*)^*$
 (b) $(1^* 001^*)^*$
 (c) $1^* + (1^* 01^* 01^*)^*$
 (d) $((0+1)^* 0(0+1)^* 0(0+1)^*)^*$

24. Which of the following represents binary strings with no two consecutive zeros's?

$$r_1 = (0 + \lambda) (1 + 10)^*$$

$$r_2 = (1 + 01)^* (0 + \lambda)$$

$$r_3 = (1 + \lambda)(0 + 01)^*$$

$$r_4 = (0 + 10)^* (1 + \lambda)$$

- (a) r_1 and r_2 only (b) r_3 and r_4 only
 (c) r_1, r_2, r_3 and r_4 (d) None of these

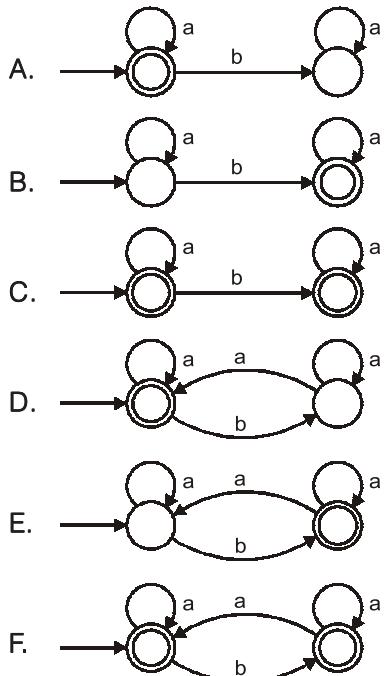
25. Consider the following regular expressions:

- | | |
|--------------------|----------------------|
| 1. $(a^* + b^*)^*$ | 2. $(a^* b^*)^*$ |
| 3. $(a + b)^*$ | 4. $(a^* + b)^*$ |
| 5. $(a + b^*)^*$ | 6. $(b^*a + a^*b)^*$ |
| 7. $(a + ab)^*$ | 8. $(ba + ab)^*$ |

Which of the following is true?

- (a) only 1, 2, 3 and 4 are equivalent
 - (b) all the regular expressions except 6, 7 and 8 are equivalent
 - (c) all except 8 are equivalent
 - (d) all except 7 and 8 are equivalent

26. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

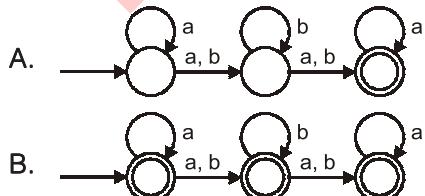
List-I**List-II**

1. $(a + ba^*a)^*$
2. $a^*(\lambda + ba^*)$
3. a^*ba^*
4. a^*
5. $(a + ba^*a)^* + a^*b(a + aa^*b)^*$
6. $a^*b(a + aa^*b)^*$

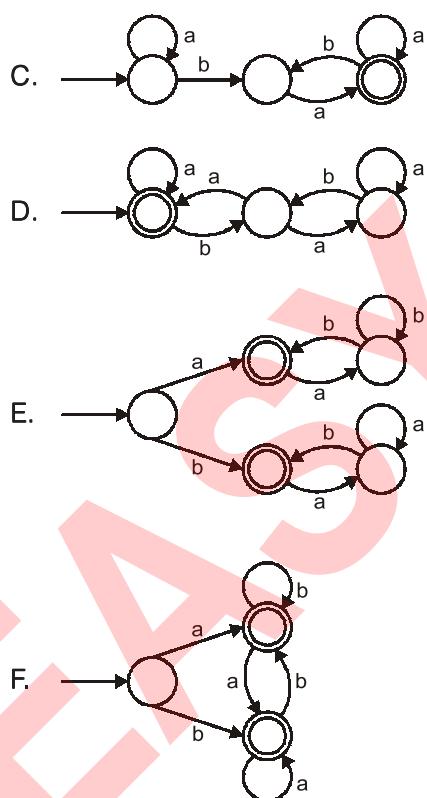
Codes:

	A	B	C	D	E	F
(a)	1	2	3	4	5	6
(b)	4	3	2	1	6	5
(c)	4	3	2	6	1	5
(d)	4	3	1	2	5	6

27. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

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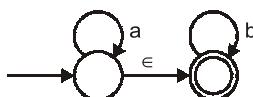
**List-II**

1. $a^*(a + b) b^*(a + b)a^*$
2. $a^*ba(a + ba)^*$
3. $a(ab^*b)^* + b(aa^*b)^*$
4. $a(b + aa^*b)^*a^* + b(a + bb^*a)^*b^*$
5. $(a + b(aa^*b)^*)a^*$
6. $a^* + a^*(a + b)b^* (\lambda + (a + b)a^*)$

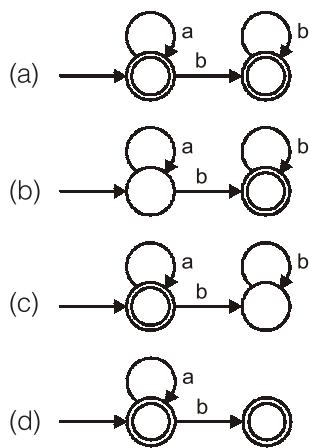
Codes:

	A	B	C	D	E	F
(a)	1	6	2	5	3	4
(b)	1	2	3	4	5	6
(c)	2	3	1	4	5	6
(d)	1	3	2	5	4	6

28. Consider the following FA:

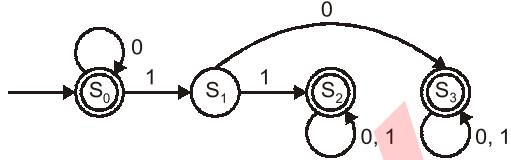


Which of the following is an equivalent NFA without ϵ -moves?



29. The expression $a^* b^* (a^* b^*)^*$ is equivalent to which one?
 (a) $(ab + bb)^*$ (b) $(a + bab)^*$
 (c) $(ab + ba)^*$ (d) $(a + b)^*$
30. If L_1 and $L_1 \cap L_2$ are regular, then
 (a) L_2 has to be regular
 (b) L_2 need not be regular
 (c) L_2 has to be context free
 (d) None of these
31. If L_1 and L_3 are regular and $L_3 = L_1 L_2$ then
 (a) L_2 has to be regular
 (b) L_2 need not be regular
 (c) L_2 has to be context free
 (d) None of these
32. If L_1 and $L_1 \cup L_2$ are regular then which of the following is true?
 (a) L_2 has to be regular
 (b) L_2 need not be regular
 (c) L_2 has to be context free
 (d) None of these
33. If L_1 is regular and $L_2 \subseteq L_1$, then which of the following has to be regular?
 (a) L_2 (b) $L_1 \cap L_2$
 (c) L_2^n (d) L_1^n
34. If $L_1 = \{a^i b^i c^i \mid i \geq 0\}$
 $L_2 = \{a b^i c^i \mid i \geq 0\}$
 Then $L_1 \cap L_2 =$
 (a) $\{a b^i c^i \mid i \geq 0\}$ (b) $\{a b^i c^i \mid i \geq 1\}$
 (c) $\{a^i b^i c^i \mid i \geq 0\}$ (d) $\{abc\}$

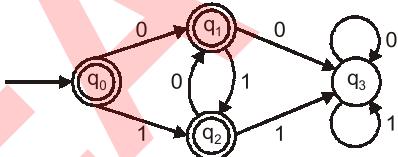
35. The language recognized by automata given below is



- (a) $0^*(\lambda + 1(0+1)^*)$
 (b) $0^* 1(0+1)^+$
 (c) $0^*(0+1)^*$
 (d) $0^*(\lambda + 1(0+1)^+)$

Linked Answer Q.36 & Q.37

36. The language accepted by the machine given below is



- (a) $\lambda + 0(10)^*(\lambda + 1) + 1(01)^*(\lambda + 0)$
 (b) $(0+1)(0+1)^*$
 (c) $(0(10)^* 0 + 1(01)^* 1)(0+1)^*$
 (d) $(0+1)^*$

37. The language accepted by machine in the previous question is best described as
 (a) strings containing the substring "101"
 (b) strings having no consecutive 0's
 (c) strings having at least one consecutive 0's or one consecutive 1's
 (d) strings consisting of no consecutive 0's or 1's

38. $S \rightarrow aB$
 $B \rightarrow bC$
 $C \rightarrow xB$
 $C \rightarrow c$

The above grammar generates which language?

- (a) abc (b) $a(bx + bc)$
 (c) $a(bx)^*bc$ (d) $(bx)^*bc$

39. Which of the following is true?
 (a) regular sets are closed under infinite union
 (b) subsets of regular sets are always regular
 (c) subsets of regular sets are never regular
 (d) subsets of finite sets are always regular

40. Consider the grammar G :
 $S \rightarrow aA \mid \lambda, A \rightarrow aB, B \rightarrow aS$
 Which of the following strings do not belong to $L(G)$?
 (a) aaa (b) λ
 (c) aaaaaa (d) aaab
41. Which of the following language is not regular?
 (a) $L = \{a^n b^n \mid n \geq 1\}$
 (b) $L = \{\omega \in [a, b]^* \mid \omega \text{ has at least 3 } a's\}$
 (c) $L = \{\omega \in [a, b]^* \mid \omega \text{ has the second last symbol as } b\}$
 (d) $L = \{\omega \in [a, b]^* \mid \omega \text{ begin with a and ends with } b\}$
42. Which of the following regular expression over $\{0, 1\}$ denotes the set of all strings not containing 100 as sub string
 (a) $0^* (10)^*$ (b) $0^* 10 \ 10^*$
 (c) $0^* 1^* 01^*$ (d) $0^* (10 + 1)^*$
43. The language $L = \{0^i 1^j \mid \gcd(i, j) = 1\}$ is
 (a) Regular (b) Not regular
 (c) Recursive (d) Context-free
44. If $S = \{0, 1\}$, then the number of possible strings of length ' n ' is
 (a) 2^n (b) n^2
 (c) $n!$ (d) n^n
45. Pumping Lemma is generally used for proving
 (a) A given grammar is regular
 (b) A given grammar is not regular
 (c) Whether two given regular expressions are equivalent or not
 (d) None of the above
46. Which of the following are regular?
 (a) String of 0's whose length is a perfect square
 (b) Set of all palindromes made up of 0's and 1's
 (c) String of 0's, whose length is a prime number
 (d) String of odd number of zeros
47. The regular expression is given by
 $R = (ab \mid abb)^* bbab$ which of the following expression does not belong to R?
 (a) ab bb ab (b) ab ab ab
 (c) ababbabbab (d) abbabbab

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48. The expression $\lambda + 1^*(011)^*(1^*(011)^*)^*$ is equivalent to
 (a) $(10 + 11)^*$ (b) $(0 + 101)^*$
 (c) $(01 + 10)^*$ (d) $(1 + 011)^*$
49. Which of the following pairs of regular expressions are not equivalent?
 (a) $(a^* + b)^*$ and $(a + b)^*$
 (b) $(ab)^*a$ and $a(ba)^*$
 (c) $(a + b)^*$ and $(a^*b^*)^*b^*$
 (d) None of the above
50. Which of the following pairs of regular expressions are not equivalent?
 (a) $1(01)^*$ and $(10)^*1$
 (b) $x(xx)^*$ and $(xx)^*x$
 (c) $(ab)^*$ and a^*b^*
 (d) x^+ and x^*x^+
51. If the regular set A is represented by $A = (01 + 1)^*$ and regular set B is represented by $B = ((01)^*1^*)^*$ then
 (a) A and B are incomparable
 (b) $A = B$
 (c) $A \subset B$
 (d) $B \subset A$
52. The language recognized by the following finite automaton is
-
- ```

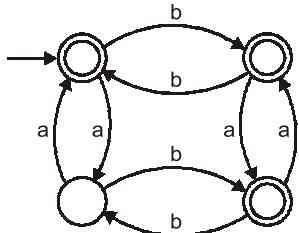
graph LR
 Start((Start)) --> q0((q0))
 q0 -- a --> q2((q2))
 q0 -- b --> q1((q1))
 q1 -- a --> q4((q4))
 q2 -- a --> q3((q3))
 q3 -- b --> q2
 q4 -- b --> q3

```
- (a)  $(aa + \epsilon)(b + ba)(bab)^*$   
 (b)  $(aab + ba)(bab)^*$   
 (c)  $(aab)(bab)^* + (bab)^*$   
 (d)  $(aab)(\epsilon + (bab)^*)^*$
53. The language recognized by the following state diagram is
- 
- ```

graph LR
    Start((Start)) --> q1((q1))
    q1 -- 0 --> q1
    q1 -- 1 --> q2((q2))
    q2 -- 1 --> q2
    q2 -- 0 --> q3((q3))
    q3 -- 0 --> q1
  
```

- (a) $(0 + (1 + 01)^* + 10)$
 (b) $(0 + 1(0 + 10)^* + 11)^*$
 (c) $(0 + 1(1 + 01)^*00)^*$
 (d) $(0 + 0(1 + 01)^*00)^*$

54. The FSM given in the following figure



recognizes

- (a) Any string of even number of a's and odd number of b's
 (b) Any string of even number of a's or odd number of b's
 (c) Any string of odd number of a's and even number of b's
 (d) Any string of odd number of a's or even number of b's
55. The language accepted by the following finite state automaton



- (a) $(a + b)^*$ (b) (a^*b)
 (c) $(a + b)^*a$ (d) $(a + b)^*(a + b)$

56. Consider the production grammar

$$S \rightarrow AB \mid AS, A \rightarrow a \mid aA, B \rightarrow b$$

Which of the following regular expressions corresponding to the production grammar?

- (a) $(ab)^*a$ (b) aa^*b
 (c) ab^+ (d) aa^*ba

57. The set $(a + b)^*$ does not correspond to which grammar?

1. $S \rightarrow aS \mid bS \mid a \mid b$
 2. $S \rightarrow aS \mid bS \mid \lambda$
 3. $S \rightarrow a \mid b \mid SS \mid \lambda$
 4. $S \rightarrow aS \mid bS \mid \lambda \mid a \mid b$
- (a) 1 only (b) 1 and 4
 (c) 1 and 3 (d) 2 and 3

58.

Which of the following CFG cannot be simulated by a Finite State Machine?

- (a) $S \rightarrow Sa \mid a$ (b) $S \rightarrow aSb \mid ab$
 (c) $S \rightarrow abX$ (d) None of these
- $$X \rightarrow cY$$
- $$Y \rightarrow d \mid aX$$

59.

Which sentence can be generated by $S \rightarrow aS \mid bA, A \rightarrow d \mid ccA$

- (a) bccddd (b) ababcccd
 (c) aabcccd (d) aadb

60.

The following grammar

$$\begin{aligned} S &\rightarrow bA \mid aC \\ A &\rightarrow bS \mid aB \mid \lambda \\ B &\rightarrow aA \mid bC \\ C &\rightarrow bB \mid aS \end{aligned}$$

generates strings of terminals that have

- (a) Even number of a's and even numbers of b's
 (b) Even number of a's and odd number of b's
 (c) Odd numbers of a's and odd numbers of b's
 (d) Equal number of a's and b's

61.

Which of the following relationship is true?

- (a) Right Linear \subset Context Free \subset Context Sensitive
 (b) Context Sensitive \subset Right Linear \subset Context Free
 (c) Context Free \subset Right Linear \subset Context Sensitive
 (d) Context Free \subset Context Sensitive \subset Right Linear

62.

If L is a finite subset of Σ^* , then L is

- (a) Context Sensitive
 (b) Context Free
 (c) Regular
 (d) None of the above

63.

Any given Transition graph has an equivalent

- (a) Deterministic Finite State Machine
 (b) Non-Deterministic Finite State Machine
 (c) Regular Expression
 (d) All of A, B and C

64. $S \rightarrow aA | a$, $A \rightarrow aA | bA$, $A \rightarrow a$
 generates
 (a) set of all strings beginning and ending with a
 (b) set of all strings beginning with a
 (c) set of all strings ending with a
 (d) None of the above
65. L_1 , L_2 and L_3 are three languages. If L_1 and L_3 regular and $L_1 L_3 = L_2$ then
 (a) L_2 has to be regular
 (b) L_2 cannot be regular
 (c) L_2 need not be regular
 (d) L_1 has to be Context Free Language
66. L_1 , L_2 and L_3 are three languages. If L_3 is not regular and $L_1 \cup L_2 = L_3$ then
 (a) Both L_1 and L_2 has to be non-regular
 (b) Atleast one of L_1 or L_2 has to be non-regular
 (c) Atleast one of L_1 or L_2 has to be regular
 (d) Both may be regular
67. Palindromes can't be recognized by any FSM because
 (a) an FSM can't remember arbitrarily large amount of information
 (b) an FSM can't deterministically fix the midpoint
 (c) even if the midpoint is known, an FSM can't find whether the second half of the string matches the first half or not
 (d) All of above
68. Consider the following two languages L_1 and L_2 on the alphabet $\Sigma = \{0, 1\}$
 1. $L_1 = \{wlw \text{ has even no. of } 0's\}$
 2. $L_2 = \{wlw \text{ has odd no. of } 0's\}$
 Which of the following is false?
 (a) The final states in the minimal DFA for L_1 and L_2 is the same.
 (b) The number of states in the minimal DFA for L_1 and L_2 is the same.
 (c) The number of states in the minimal NFA for L_1 and L_2 is the same.
 (d) The Myhill-Nerode equivalence classes for L_1 and L_2 are same.

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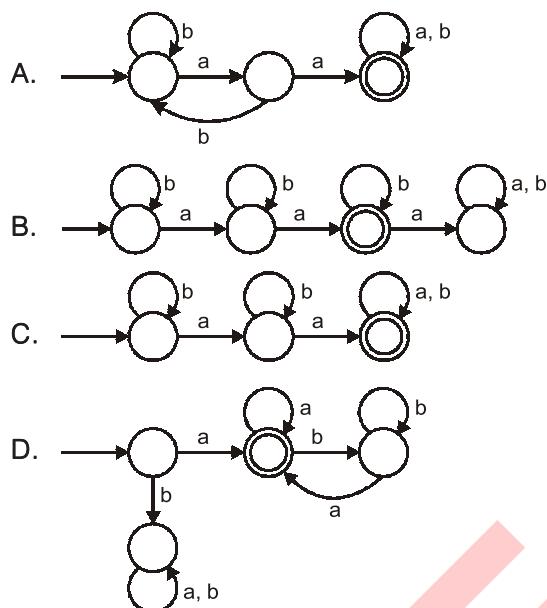
69. The regular expression for all strings in which any number of 0's followed by any number of 1's are followed by any number of 2's
 (a) $(0|1|2)^*$ (b) $0^* 1^* 2^*$
 (c) $0^* + 1 + 2$ (d) $(0+1)^* 2^*$
70. The machine pictured in figure below:
-
- (a) complements a given bit pattern
 (b) finds 2's complement of a given bit pattern
 (c) increments a given bit pattern by 1
 (d) changes the sign bit
71. What is the language generated by $S \rightarrow AS | SA$, $A \rightarrow BA | B$, $B \rightarrow AB | A$
 (a) Strings starting with a
 (b) Strings ending with a
 (c) Palindromes
 (d) \emptyset
72. An FSM has
 (a) no memory
 (b) infinite memory
 (c) finite memory
 (d) None of these
73. The minimum state FA that accepts all strings on $\{a, b\}$ with a^n as a substring has how many states
 (a) $n - 1$ (b) n
 (c) $n + 1$ (d) $n + 2$
74. Which of the following is not a finite state language?
 (a) $L = \{0, 00, 0000, 0000000, \dots\}$ on $\Sigma = \{0\}$
 (b) $L = \{s | s \in \{0, 1\}^*\text{ and } s \text{ when interpreted as a binary string is such that } s \bmod 5 = 0\}$
 (c) $L = \{s | s \in \{0, 1\}^*\text{ and } s \text{ when input from least significant bit to most significant bit}$

is such that its integer interpretation is divisible by 2}

- (d) $L = \{0^{2n} ; n \geq 0\}$ on $\Sigma = \{0\}$

75. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-1



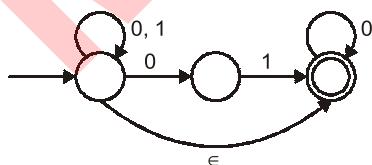
List-II

1. Starts and ends with “a”
 2. Contains exactly 2 a’s
 3. Contains “aa” as a substring
 4. Contains at least 2 a’s

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	2	1	3
(c)	1	2	3	4
(d)	3	2	4	1

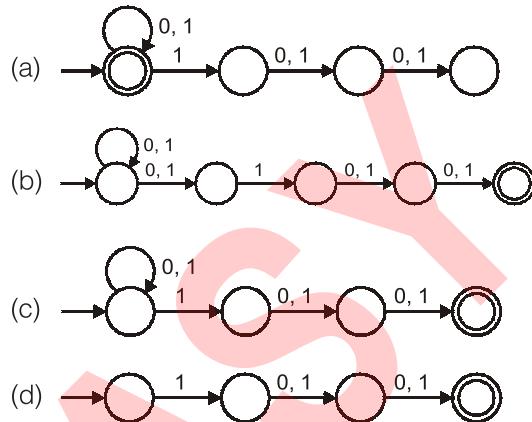
- 76.** Which of the following strings is rejected by the NFA given below:



- (a) 0
 - (b) 101000
 - (c) 0100
 - (d) 01011
 - (e) None of these

Linked Answer Q.77 & Q.78

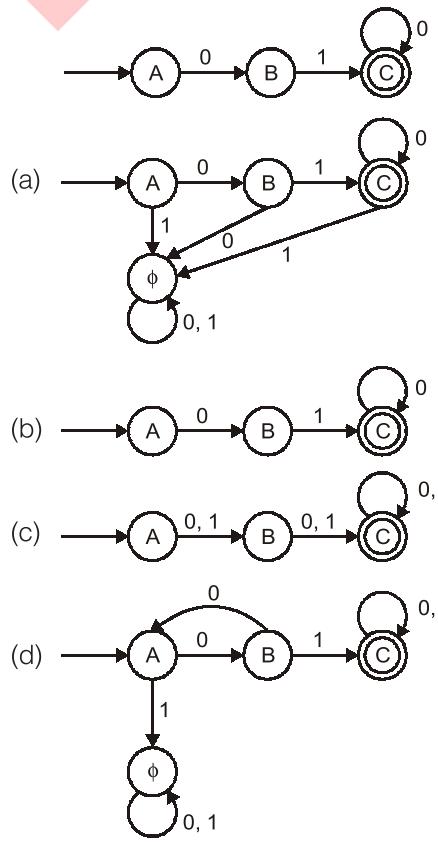
77. The NFA for accepting the set of strings whose 3rd from right bit is 1 is



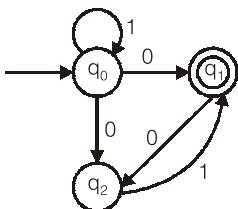
78. The regular expression for above language is

 - (a) $1(0 + 1)(0 + 1)$
 - (b) $(0 + 1)^+ 1 (0 + 1)(0 + 1)$
 - (c) $(0 + 1) 1 (0 + 1)(0 + 1)$
 - (d) $(0 + 1)^* 1 (0 + 1)(0 + 1)$

79. Construct a DFA equivalent to following NFA



- 80.** The number of states in the equivalent DFA obtained by subset construction algorithm from the NFA given below is



- 81.** Which of the following sets are equal

- $L(0^*(0+1))$
 - $\{0, 00, 000, \dots\} \cup \{1, 01, 001, 0001, \dots\}$
 - $\{0^n; n \geq 1\} \cup \{0^n 1; n \geq 0\}$
 - $L((0+1)0^*)$

(a) 1 and 3 only (b) 1, 3 and 4 only
 (c) 1, 2 and 3 only (d) 1, 2, 3 and 4

- 82.** Which of the following regular expressions are equivalent?

- $(a + ba)^* (b + \epsilon)$
 - $(a^* (ba)^*)^* (b + \epsilon) + a^* (b + \epsilon) + (ba)^* (b + \epsilon)$
 - $(a + ba) (a + ba)^* (b + \epsilon)$

(a) 1 and 2 only (b) 1 and 3 only
 (c) 2 and 3 only (d) 1, 2 and 3

83. Which of the following is false?

- (a) Given a Mealy machine with m outputs and n states, a corresponding equivalent Moore machine has a maximum of mn states
 - (b) Given a Moore machine with n states, the corresponding equivalent Mealy machine has a maximum of n states
 - (c) Given a DFA with n states, an equivalent NFA has a maximum of 2^n states
 - (d) If an NFA $M_1 = \{Q_1, \Sigma, \delta_1, q_0, F_1\}$ and its equivalent DFA $M_2 = \{Q_2, \Sigma, \delta_2, q_0', F_2\}$, then $Q_2 \subseteq 2^{Q_1}$ and $F_2 \subseteq 2^{Q_1}$

84. Which of the following is false?

- (a) Every NFA with ϵ -moves can be converted to an equivalent NFA without ϵ -moves
 - (b) Corresponding to every NFA there is an equivalent right-linear grammar

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- (c) Corresponding to every right-linear grammar there is an equivalent left-linear grammar and vice-versa
 - (d) A grammar is regular if and only if it is either right linear or left-linear
 - (e) Every grammar can be converted into an equivalent one which is right linear or left linear

85. Which of the following is false?

 - (a) Every NFA with any number of final states, has an equivalent NFA with a single final state
 - (b) If M is an NFA with single final state accepting language L , then a machine M' , obtained by reversing all arrows and making final state to initial state and initial state to final state, will accept the language L^R
 - (c) If L is regular, then the language $\{uv; u \in L, v \in L^R\}$ is also regular
 - (d) If $w \in L$, where L is regular then the language $\{ww^R; w \in L\}$ is regular

86. If h is a homomorphism from $\Sigma = \{a, b\}$ to $\Gamma^* = \{0, 1\}^*$ and if $L = (a + ba)^*$ ($\in + b$), then $h(L) = ?$

 - (a) $(h(a) + h(b)) . h(a))^*$
 - (b) $(b + ab)^*$
 - (c) $(h(a) + h(b)) . h(a))^* (\in + h(b))$
 - (d) $(0 + 10)^* (\in + 1)$

87. If $L = \{0, 11, 10, 01\}$
 and $h(0) = aa$
 $h(1) = bbc$

When of the following does not belong to $h(L)$?

88. If $\Sigma = \{a, b, c\}$ and $\Gamma = \{0, 1\}$
Let h be defined from Σ to Γ^* as $h(a) = 0$

$$b(b) = 1 \quad b(c) =$$

Let $I = \{1010\}$

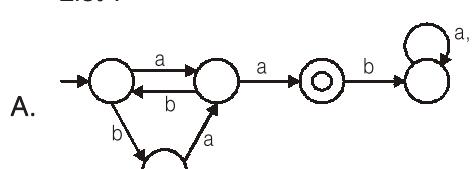
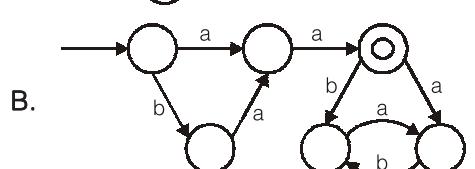
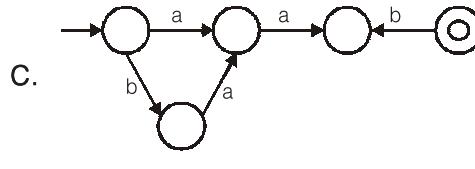
$E_0 \cup E_1 = \{1010\}$

Then $h^{-1}(L_1) =$

- (a) cc
 - (b) {cc, baba, cba}
 - (c) {cc, cba, bac}
 - (d) {cc, baba, cba, bac}

89. Which of the following is false?
- $h(h^{-1}(L))$ can be a subset of L
 - $h^{-1}(h(L))$ can be a superset of L
 - $h^{-1}(L)$ can be empty
 - $h(h^{-1}(L)) = L$
90. Let $\Sigma = \{0, 1\}$ and $\Gamma = \{a, b\}$
Let $h(0) = aa$ and $h(1) = bb$
Let $L_1 = (ab + ba)^*$
 $L_2 = (10 + 01)^*$
 $L_3 = (a^{2n} b^{2n}; n \geq 0)$
- Then $h^{-1}(L_1)$, $h(L_2)$ and $h^{-1}(L_3)$ are respectively,
- $(01 + 10)^*$ and $(bbaa + aabb)^*$ and $\{0^n 1^n; n \geq 0\}$
 - $(10 + 01)^*$ and $(ba + ab)^*$ and $\{0^n 1^n; n \geq 1\}$
 - $(0 + 1)^*$ and $(ba + ab)^*$ and $\{0^n 1^n; n \geq 0\}$
 - $\{\lambda\}$ and $(bbaa + aabb)^*$ and $\{0^n 1^n; n \geq 0\}$
91. If $h(0) = aa$ and $h(1) = \epsilon$ is a Homomorphism from
 $\Sigma = \{0, 1\}$ to $\Gamma^* = \{a, b\}^*$
If $L_1 = L\{1^* 0 1^*\}$
 $L_2 = \{\epsilon\}$
 $L_3 = \emptyset$
 $L_4 = \{1110\}$
- Then $h(L_1)$, $h(L_2)$, $h(L_3)$ and $h(L_4)$ are respectively
- $\{aa\}, \{\epsilon\}, \emptyset, \{aa\}$
 - $\{aa\}, \text{undefined}, \emptyset$ and $\{aa\}$
 - $\{aa\}, \{1\}, \emptyset$ and $\{aa\}$
 - $\{1^* aa 1^*\}, \{\epsilon\}, \emptyset, \{aa\}$
92. Let $L_1 = 0^* 1$
 $L_2 = 1^* 0$
 $L_1/L_2 = ?$
- $0^* 11^* 0$
 - 0^*
 - $0^* 1$
 - \emptyset
93. Let $L_1 = a^* b a^*$ and $L_2 = b^* a$ then $L_1/L_2 =$
- $a^* b a^* + a^*$
 - a^*
 - $a^* b a^*$
 - $a^* b^+ a^+$
94. $L_1 = a^* b$ and $L_2 = b^*$, then $L_1/L_2 = ?$
- $a^* b^+$
 - $a^* (b + \epsilon)$
 - $a^* b$
 - b^*
95. If $L_1 = a^* b^*$ and $L_2 = \{0^n 1^n; n \geq 0\}$
INIT (L_1) and INIT (L_2) are respectively,

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96. If L_1 and L_2 are regular, which of the following is not necessarily regular
- $L_1^n \cdot L_2^n$ for some fixed integer n
 - L_2^*
 - $L_1^R L_2^*$
 - A subset or superset of L_1 or L_2
 - Nor (L_1, L_2) = $\{w \mid w \notin L_1 \text{ and } w \notin L_2\}$
97. Which of the following is false?
- If an NFA accepts a string of length at least equal to its number of states, then its language is infinite
 - The set $L \subseteq \Sigma^*$ is accepted by some FA if and only if L is the union of some of the equivalence classes of a right invariant equivalence relation of finite index
 - An FA accepts an infinite language iff there is at least one vertex in the path from initial to final vertex, which is a base of some cycle
 - An FA accepts empty language iff there is no simple path from initial vertex to any final vertex
 - None of these
98. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:
- List-I**
- A. 
- B. 
- C. 

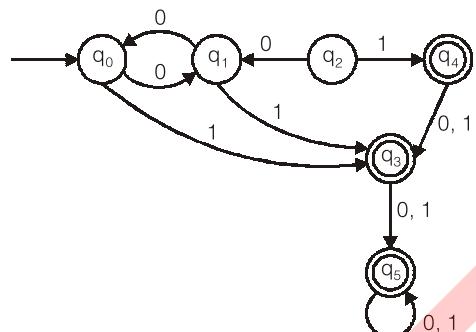
List-II

1. Empty language
 2. Finite language
 3. Infinite language

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	2	3	1
(d)	3	2	1

99. The minimum DFA that accepts the same language as accepted by DFA given below, has how many states?



100. How many strings of at most 4 bits that start with and end with "0" belongs to the language described by $(01 + 10)^* 0 (0 + 1)^*$

 - (a) 8
 - (b) 11
 - (c) 12
 - (d) 14

101. Which of the following is false?

- (a) If p, q, r are 3 regular expressions and $\in \notin L(q)$, then $p = pq + r$ iff $p = rq^*$
 - (b) $((10)^* 1)^* (10)^* = (10)^* (1(10)^*)^*$
 - (c) $(10)^* 0 = 1(00)^*$
 - (d) $(0 + 1)^* 0 (0 + 1)^* + 1^* = (0 + 1)^*$

- 102.** Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

| ist=|

- A. 0's occur only in pairs
 - B. 1's occur only in pairs
 - C. Both 0's and 1's occur only in pairs

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- D. No consecutive 0's
 - E. No consecutive 1's
 - F. Even no of 0's

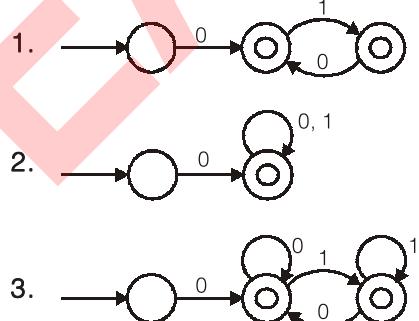
List-II

1. $(00 + 11)^*$
 2. $(0 + 11)^*$
 3. $(1 + 00)^*$
 4. $(\in + 1)(0 + 01)^*$
 5. $(1 + 01)^*(\in + 0)$
 6. $(1^* 01^* 01^*)^* + 1^*$

Codes:

	A	B	C	D	E	F
(a)	1	2	3	4	5	6
(b)	2	1	3	4	5	6
(c)	1	3	2	4	6	5
(d)	3	2	1	5	4	6

- 103.** Consider the following FA's



Which are equivalent?

- (a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

104. Which of the following is regular?

- (a) Strings of 0's, whose length is prime
 - (b) Strings of 0's and 1's, when the number of 0's and 1's are equal
 - (c) $L = \{0^n; n \bmod 3 = 0\}$
 - (d) $L = \{0^{3^n}; n\}$

- 105.** Which of the following is regular?

- (a) $\{0^n 1^m; n + m \text{ is odd}\}$
 (b) $\{0^p 1^q; pq = 64\}$
 (c) $\{0^p 1^q; p \geq 0, q \geq 0, pq \geq 4\}$
 (d) $\{0^p 1^q; p = 5 - q\}$
 (e) All are regular

106. How many states are there in a minimum state FA accepting {0, 01, 011, 001}
 (a) 4 (b) 5
 (c) 6 (d) 7

107. Consider the grammar:

$$G_1 : S \rightarrow Sab \mid AB$$

$$A \rightarrow aA \mid \epsilon$$

$$B \rightarrow bB \mid \epsilon$$

$$G_2 : S \rightarrow aA$$

$$A \rightarrow aAB \mid \epsilon$$

$$B \rightarrow c$$

$$G_3 : S \rightarrow aAb \mid \epsilon$$

$$A \rightarrow c$$

Which of the above grammars generate a regular language?

- (a) G_1 and G_2 only (b) G_1 only
 (c) G_2 and G_3 only (d) G_1 and G_3 only

108. The grammar $\{S \rightarrow aA \mid aB \mid \lambda, A \rightarrow Ab \mid \lambda\}$ is

- (a) Regular and Linear
 (b) Regular but not Linear
 (c) Linear but not Regular
 (d) Neither Linear nor Regular

109. $L_1 = \{0^{3^i} ; i \text{ an integer}\}$

$$L_2 = \{0^{i^3} ; i \text{ an integer}\}$$

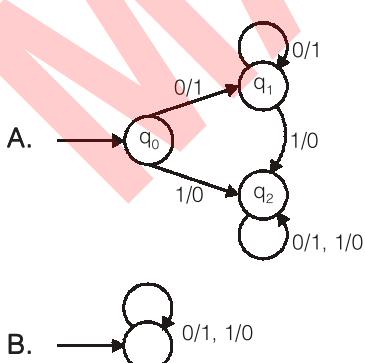
$$L_3 = \{0^{3^i} ; i \text{ an integer}\}$$

Which of them is regular?

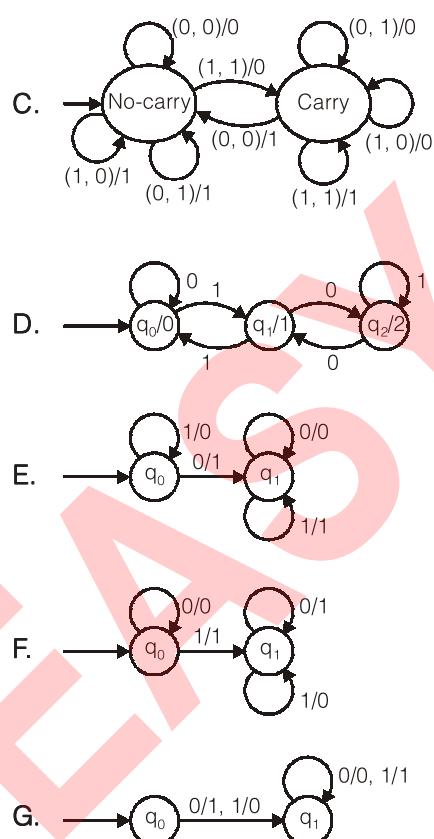
- (a) All of them (b) Only L_1 and L_2
 (c) Only L_3 (d) None of these

110. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I



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List-II

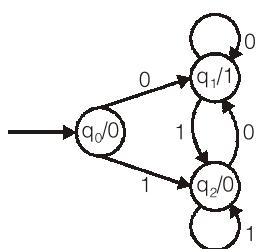
1. Complements a given bit string (1's complement)
2. 2's complement
3. Increments by 1
4. Changes sign bit
5. Binary adder
6. Residue mod 3 of decimal equivalent of a binary number (Divisibility by 3 checker)

Note: In machine E and F the input is read from least significant bit in reverse order. In machine D, actual output is taken to be the last bit of the output generated by the machine.

Codes:

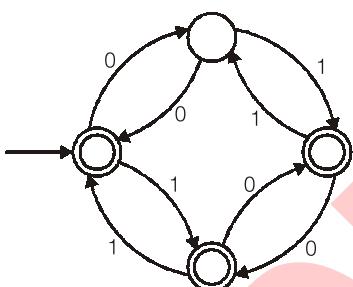
	A	B	C	D	E	F	G
(a)	1	2	3	4	5	6	7
(b)	3	2	1	5	4	6	6
(c)	1	1	5	6	3	2	4
(d)	1	1	5	6	2	4	3

111. The below machine is



- (a) a mealy machine to find 2's complement of a number
- (b) a Moore machine to find 2's complement of a number
- (c) a mealy machine to find 1's complement of a number
- (d) a Moore machine to find 1's complement of a number

112. The below fm recognizes



- (a) a string with even nos of zeros and odd nos of 1's
- (b) even number of 0's or odd number's of 1's
- (c) odd number of 0's or even nos of 1's
- (d) even number of 0's and even nos of 1's

Linked Answer for Q.113 to Q.115

113. Which production should be added so that the resulting grammar will generate all strings starting and ending with 0.

$$\begin{aligned} S &\rightarrow 0A \\ A &\rightarrow 0A \mid 1B \mid \lambda \\ B &\rightarrow 1B \end{aligned}$$

- (a) $B \rightarrow 1S$
- (b) $B \rightarrow 0A$
- (c) $S \rightarrow 1C$
- (d) $B \rightarrow 1B$

114. A minimum DFA for accepting the language generated by resulting grammar requires how many states?

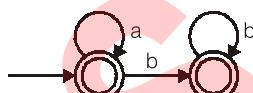
- (a) 2
- (b) 3
- (c) 4
- (d) 5

115. A minimum FA for accepting the above language requires how many states?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

Linked Answer for Q.116 & Q.117

116. The NFA give below, accepts which language?



- (a) $a^+ b^+$
- (b) $a^* bb^*$
- (c) $a^* + bb^*$
- (d) $a^* b^*$

117. A DFA for the above language will have minimum how many states?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

118. Regular languages are closed under intersection because they are closed under

- (a) union
- (b) complement and concatenation
- (c) union and complement
- (d) union and Kleene star

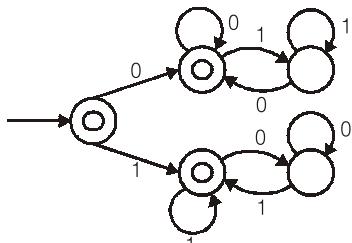
119. Which of the following are equivalent?

1. $(010)^* 01$
 2. $(1^* 0)^* 1^*$
 3. $01(001)^*$
 4. $1^* (01^*)^*$
 5. $(0 + 1)^*$
- (a) 1 and 5, 2 and 3
 - (b) 1 and 2, 3 and 4
 - (c) 1 and 3, 2 and 4
 - (d) 1, 2, 3 and 5

120. Which of the following is true?

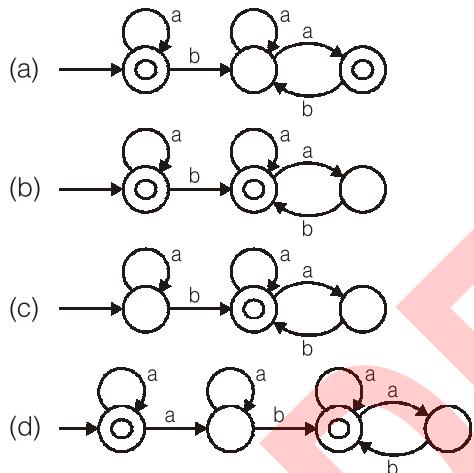
- (a) A grammar which is regular is either right linear or left linear
- (b) Linear grammars are either right linear or left linear
- (c) A regular grammar is one which is either right linear or left linear or neither
- (d) $S \rightarrow aS \mid Bb \mid a$ is a regular grammar

121. The language recognized by the DFA given below is



- (a) equal occurrence of substrings "0" and "1"
- (b) equal occurrence of substrings "01" and "10"
- (c) $\{w \mid n_0(v) = n_1(v) \text{ in every prefix } v \text{ of } w\}$
- (d) None of these

122. Which of the following machines accept $a^* + a^* b$ ($a + ab$) *



123. Let R_1 and R_2 be regular sets defined over the alphabet Σ then

- (a) $R_1 \cap R_2$ is not regular
- (b) $R_1 \cup R_2$ is regular
- (c) $\Sigma^* - R_1$ is regular
- (d) R_1^* is not regular

[GATE-1990]

124. Let $r = 1(1 + 0)^*$, $s = 11^* 0$ and $t = 1^* 0$ be three regular expressions. Which one of the following is true?

- (a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$
- (b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$
- (c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$
- (d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$
- (e) None of the above

[GATE-1991]

125. Which one of the following is the strongest correct statement about a finite language over some finite alphabet Σ ?

- (a) It could be undecidable
- (b) It is Turing-machine recognizable
- (c) It is a context-sensitive language
- (d) It is a regular language
- (e) None of the above

[GATE-1991]

126. Which of the following regular expression identities are true?

- (a) $(r^*)^* = r^*$
- (b) $(r^* s^*) = (r + s)^*$
- (c) $(r + s)^* = r^* + s^*$
- (d) $r^* s^* = r^* + s^*$

[GATE-1992]

127. In which of the cases stated below is the following statement true?

"For every non deterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language."

- (a) M_1 is non deterministic finite automation
- (b) M_1 is non deterministic PDA
- (c) M_1 is a non deterministic Turing machine
- (d) For no machine M_1 is the above statement true

[GATE-1992]

128. Consider a DFA over $\Sigma = \{a\}$ accepting all strings which have number of a's divisible by 2 or number of a's divisible by 4. What is the minimum number of states that the DFA will have?

- (a) 1
- (b) 2
- (c) 4
- (d) 8

129. In some programming languages, an identifier is permitted to be a letter followed by a number of letters or digits. If L and D denote the sets of letters and digits respectively, which of the following expressions defines an identifier?

- (a) $(L \cup D)^+$
- (b) $L(L \cup D)^*$
- (c) $(L.D)^*$
- (d) $L.(L.D)^*$

[GATE-1995]

[GATE-1995]

[GATE-1995]

132. A finite state machine with the following state table has a single input x and a single output z .

	Present state	Next state
A	x = 1	x = 0
B	D, 0	B, 0
C	B, 1	C, 1
D	B, 0	D, 1
	B, 1	C, 0

If the initial state is unknown, then the shortest input sequence to reach the final state C is

[GATE-1995]

133. Let $\Sigma = \{0, 1\}$, $L = \Sigma^*$ and $R = \{0^n 1^n \mid n > 0\}$ then the languages $L \cup R$ and R are respectively

 - (a) Regular, Regular
 - (b) Not Regular, Regular
 - (c) Regular, Not Regular
 - (d) Not Regular, Not Regular

[GATE-1995]

[GATE-1996]

135. Let $L \subseteq \Sigma^*$ where $\Sigma = \{a, b\}$. Which of the following is true?

 - (a) $L = \{x \mid x \text{ has an equal number of } a\text{'s and } b\text{'s}\}$ is regular
 - (b) $L = \{a^n b^n \mid n \geq 1\}$ is regular
 - (c) $L = \{x \mid x \text{ has more } a\text{'s than } b\text{'s}\}$ is regular
 - (d) $L = \{a^m b^n \mid m \geq 1, n \geq 1\}$ is regular

[GATE-1996]

136. If L_1 and L_2 are context free languages and R is a regular set, one of the language below is not necessarily a context free language. Which one?

(a) $L_1 L_2$ (b) $L_1 \cap L_2$
(c) $L_1 \cap R$ (d) $L_1 \cup L_2$

[GATE-1996]

137. Define a context free language $L \subseteq \{0, 1\}^*$
 $\text{init}(L) = \{u \mid uv \in L \text{ for some } v \in \{0, 1\}^*\}$ (in other words, $\text{init}(L)$ is the set of prefixes of L)
Let $L = \{w \mid w \text{ is non empty and has equal number of } 0\text{'s and } 1\text{'s}$
Then $\text{init}(L)$ is

 - (a) the set of all binary strings with unequal number of 0's and 1's
 - (b) the set of all binary string including the null string
 - (c) the set of all binary strings with exactly one more 0's than the number of 1's or one more 1 than the number of 0's
 - (d) None of the above

[GATE-1996]

- 138.** Which one of the following regular expressions over {0, 1} denotes the set of all strings not containing 100 as a substring?

- (a) $0^*(1+0)^*$ (b) 0^*1010^*
 (c) 0^*1^*01 (d) $0^*(10+1)^*$

[GATE-1997]

139. Which of the following set can be recognized by a Deterministic Finite-state Automation?
 (a) The numbers 1, 2, 4, 8, ... 2^n , ... written in binary
 (b) The numbers 1, 2, 4, ..., 2^n , ... written in unary
 (c) The set of binary string in which the number of zeros is the same as the number of ones
 (d) The set {1, 101, 11011, 1110111, ...}

[GATE-1998]

140. Regarding the power of recognition of languages, which of the following statements is false?
 (a) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata
 (b) Non-deterministic push-down automata are equivalent to deterministic push-down automata
 (c) Non-deterministic Turing machines are equivalent to deterministic push-down automata
 (d) Non-deterministic Turing machines are equivalent to deterministic Turing machine
 (e) Multi-tape TMs are equivalent to single-tape TMs

[GATE-1998]

141. The string 1101 does not belong to the set represented by
 (a) $110^*(0+1)$
 (b) $1(0+1)^*101$
 (c) $(10)^*(01)^*(00+11)^*$
 (d) $(00+(11)^*0)^*$

[GATE-1998]

142. Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite state automaton accepting L is

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- (a) 2 (b) 5
 (c) 8 (d) 3

[GATE-1998]

143. Which of the following statements is false?
 (a) Every finite subset of a non-regular set is regular
 (b) Every subset of a regular set is regular
 (c) Every finite subset of a regular set is regular
 (d) The intersection of two regular set is regular

[GATE-1998]

144. Consider the regular expression $(0+1)(0+1)\dots n$ times. The minimum state finite automata that recognizes the language represented by this regular expression contains
 (a) n states
 (b) $n+1$ states
 (c) $n+2$ states
 (d) None of these

[GATE-1999]

145. Let S and T be languages over $\Sigma = \{a, b\}$ represented by the regular expressions $(a+b^*)^*$ and $(a+b)^*$, respectively. Which of the following is true?
 (a) $S \subset T$ (b) $T \subset S$
 (c) $S = T$ (d) $S \cap T = \emptyset$

[GATE-2000]

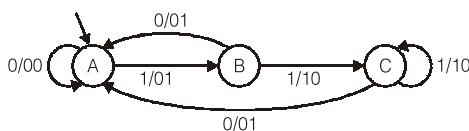
146. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is
 (a) N^2 (b) 2^N
 (c) $2N$ (d) $N!$

[GATE-2001]

147. Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?
 (a) 8 (b) 14
 (c) 15 (d) 48

[GATE-2001]

148. The Finite state machine described by the following state diagram with A as starting state, where an arc label is $x \mid y$ and x stands for 1-bit input and y stands for 2-bit output.



- (a) Outputs the sum of the present and the previous bits of the input
- (b) Outputs a "01" whenever the input sequence contain "11"
- (c) Outputs a "00" whenever the input sequence contains "10"
- (d) None of the above

[GATE-2002]

149. The smallest finite automaton which accepts the language $\{x \mid \text{length of } x \text{ is divisible by 3}\}$ has
- (a) 2 states
 - (b) 3 states
 - (c) 4 states
 - (d) 5 states

[GATE-2002]

150. Which one of the following is a regular language?
- (a) $\{ww \mid w \in \{0, 1\}^*\}$
 - (b) $\{ww^R \mid w \in \{0, 1\}^*\}$
 - (c) $\{w(w^R)^* \mid w \in \{0, 1\}^*\}$
 - (d) $\{w^n w^m \mid 0 \leq n \leq m, w \in \{0, 1\}\}$

[JNU]

151. For a deterministic finite automata, which one of the following statements is false?
- (a) $\delta^*(q, xw) = \delta^*(\delta^*(q, x), w)$
 - (b) $\delta^*(q, xw) = \delta^*(q, wx)$
 - (c) $\delta^*(q, \lambda) = q$
 - (d) If $\delta^*(q, y) = \delta^*(q, x)$, then $\delta^*(q, xz) = \delta^*(q, yz)$

[JNU]

152. For a non-deterministic finite accepter M = $(Q, \Sigma, \delta, q_0, F)$, the transition function δ is defined as
- (a) $\delta : Q \times \Sigma \rightarrow Q$
 - (b) $\delta : Q \times \Sigma \rightarrow 2^Q$
 - (c) $\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$
 - (d) None of the above

[JNU]

153. The logic of pumping lemma is a good example of
- the pigeonhole principle
 - the divide and conquer technique
 - recursion
 - iteration

[JNU]

154. ϕ^* (Kleene's closure of ϕ , where ϕ is the empty language over Σ) is equivalent to
- ϕ
 - $\{\lambda\}$
 - Σ
 - None of these

[JNU]

155. Application of finite automata cannot be found in
- storage purpose
 - spelling checker
 - lexical analysis
 - string matching

[JNU]

156. Let L be the language $\{\lambda, 0, 10\}$ over $\{0, 1\}$. Determine the set $L \cup \bar{L}$
- ϕ
 - Same as the set L
 - $\{0, 1\}^*$
 - None of the above

[JNU]

157. If S is the number of states in NDFA, then equivalent DFA can have maximum of
- S states
 - $S - 1$ states
 - 2^S states
 - $2^S - 1$ states

[JNU]

158. If x is a string, then x^R denotes the reversal of x . If x and y are strings, then $(xy)^R$ is equal to
- xy^R
 - $y^R x$
 - $x^R y^R$
 - $y^R x^R$

[JNU]

159. Which one of the following regular expressions is false?
- $(r^*)^* = r^*$
 - $r_1^* (r_1 + r_2)^* = (r_1 + r_2)^*$
 - $(r_1 + r_2)^* = (r_1^* r_2^*)^*$
 - None of the above

[JNU]

160. Which one of the following is a property of Moore machine?
- Output of Moore machine depends on present state and input
 - Output of Moore machine depends on a state only
 - Output of Moore machine depends on output only
 - None of the above

[JNU]

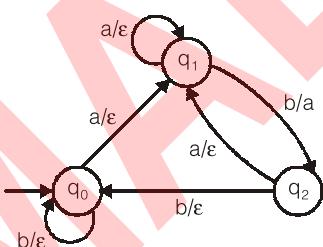
161. Which of the following pairs is not equivalent?
- $1(0\ 1)^*$ and $(1\ 0)^* 1$
 - $x(xx)^*$ and $(xx)^* x$
 - $(ab)^*$ and $a^* b^*$
 - x^+ and $x^* x^+$

[JNU]

162. Let $k \geq 2$, let L be the set of strings in $(0, 1)^*$ such that $x \in L$ if the number of 0's in x is divisible by k and the number of 1's in x is odd. The minimum number of states in a Deterministic Finite Automata (DFA) that recognizes L is
- $k + 2$
 - $2k$
 - $k \log k$
 - 2^k

[JNU]

163. Consider the following finite state transducer where the label on an edge $x | t$ denotes if the input is x , follow the arrow and emit t



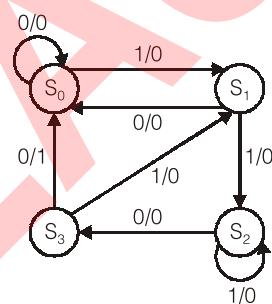
- For the input, aabbbaaabbbbababb the output is
- aaaa
 - aaaaaaaaaa
 - abababab
 - abbbabbbbabbabb

[JNU]

164. Let f be a homomorphism, $f(a) = 0$, $f(b) = 1$ and $f(c) = \epsilon$ (empty symbol). Consider a language $L = \{0^n 1^n \mid n > 0\}$, then which one of the following languages is $f^{-1}(L)$?
- $\{a^n b^n \mid n > 0\}$
 - $\{b^n a^n \mid n > 0\}$
 - $\{a^n c^m b^n \mid n \geq 0, m \geq 0\}$
 - $\{a^n b^n c^m \mid n \geq 0, m > 0\}$
 - None of these

[JNU]

165. Consider an output-producing deterministic finite state automaton (DFA) of the kind indicated below, in which it is assumed that every state is a final state.



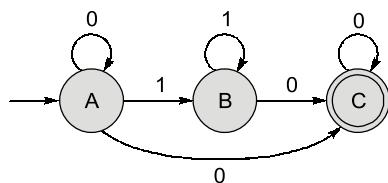
Assume that the input is at least four bits long. Which of the following is (are) true?

- The last bit of the output depends on the start state.
 - If the input ends with '1100', then the output must end with '1'.
 - The output cannot end with '1' unless the input ends with '1100'.
- 1 only
 - 2 only
 - 2 and 3
 - 1, 2 and 3

[JNU]

166. Let L_1 be a regular language over $\Sigma = \{a, b, c\}$ and $L_2 = \{a^n b^n c^n\}$. Concatenation of L_1 and L_2 is _____ (i.e., $L_1 L_2$)
- Regular language
 - Non-regular language
 - Context sensitive language
 - None of these

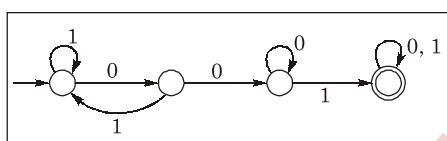
167. What is the language accepted by the following NFA?



- (a) $(0 + 1)^*0$
(b) $0^* 1^* 0^*$
(c) $0^* 1^* 0^* 0$
(d) $(0 + 1)^+$

168. Find the number of states in minimized DFA that accepts a language over $\Sigma = \{a, b\}$ where each string has exactly three a's and atleast two b's.
(a) 12
(b) 13
(c) 14
(d) 15

169. Consider the following deterministic finite state automaton M.

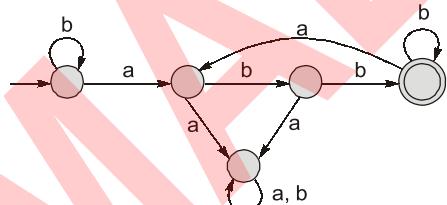


Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is

- (a) 1
(b) 5
(c) 7
(d) 8

[GATE-2003]

170. Consider the machine M:



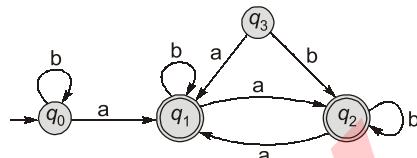
The language recognized by M is

- (a) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by exactly two } b's\}$
(b) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by at least two } b's\}$
(c) $\{w \in \{a, b\}^* \mid w \text{ contains the substring 'abb'}$
(d) $\{w \in \{a, b\}^* \mid w \text{ does not contain 'aa' as a substring}\}$

[GATE-2005]

Common Data for Q.171 & Q.172

Consider the following Finite State Automation



171. The language accepted by this automaton is given by the regular expression

- (a) $b^* ab^* ab^* ab^*$
(b) $(a+b)^*$
(c) $b^* a (a+b)^*$
(d) $b^* ab^* ab^*$

[GATE-2007]

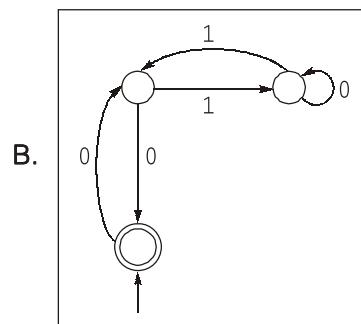
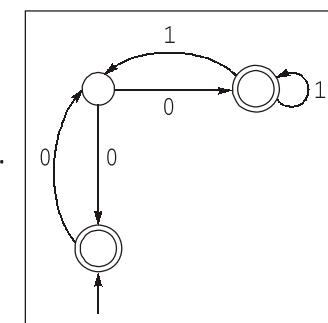
172. The minimum state automaton equivalent to the above FSA has the following number of states

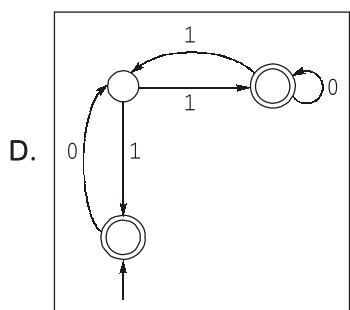
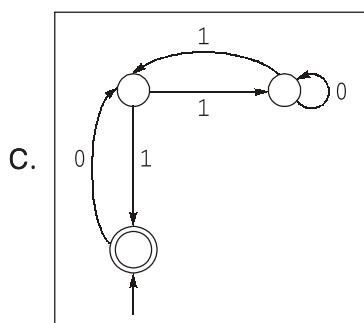
- (a) 1
(b) 2
(c) 3
(d) 4

[GATE-2007]

173. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I





List-II

1. $\epsilon + 0(01^* 1 + 00)^* 01^*$
2. $\epsilon + 0(10^* 1 + 00)^* 0$
3. $\epsilon + 0(10^* 1 + 10)^* 1$
4. $\epsilon + 0(10^* 1 + 10)^* 10^*$

Codes:

A	B	C	D
(a) 2	1	3	4
(b) 1	3	2	4
(c) 1	2	3	4
(d) 3	2	1	4

[GATE-2008]

174. Let $L = \{\omega \in (0+1)^* \mid \omega \text{ has even number of } 1s\}$, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L ?

- (a) $(0^* 1 0^*)^*$
- (b) $0^*(10^* 10^*)^*$
- (c) $0^*(10^* 1)^* 0^*$
- (d) $0^* 1 (10^* 1)^* 10^*$

[GATE-2010]

175. Definition of a language L with alphabet $\{a\}$ is given as follows:

$L = \{a^{nk} \mid k > 0, \text{ and } n \text{ is a positive integer constant}\}$.

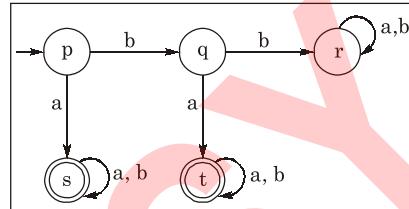
What is the minimum number of states needed in a DFA to recognize L ?

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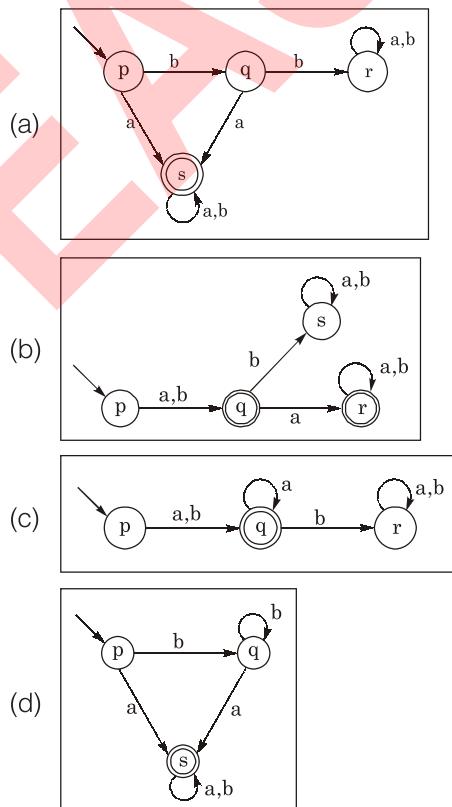
- (a) $k + 1$
- (b) $n + 1$
- (c) 2^{n+1}
- (d) $2^k + 1$

[GATE-2011]

176. A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below:



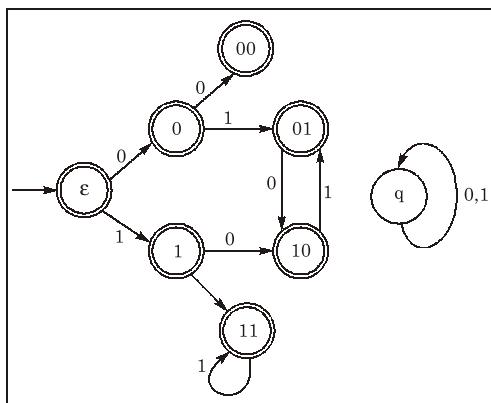
Which of the following finite state machines is a valid minimal DFA which accepts the same language as D ?



[GATE-2011]

177. Consider the set of strings on $\{0, 1\}$ in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially complete DFA that accepts this language is shown below.

The missing arcs in the DFA are



	00	01	10	11	q
00	1	0			
01				1	
10	0				
11		0			

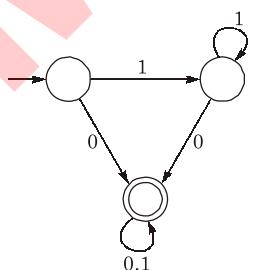
	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

	00	01	10	11	q
00		1			0
01				1	
10	0				
11		0			

[GATE-2012]

178. Consider the DFA A is given below:



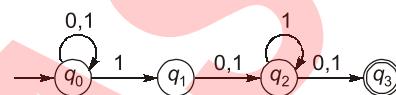
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Which of the following are False?

1. Complement of $L(A)$ is context-free.
 2. $L(A) = L((11^*0 + 0)(0 + 1)^*0^*1^*)$
 3. For the language accepted by A, A is the minimal DFA.
 4. A accepts all strings over $\{0, 1\}$ of length at least 2.
- (a) 1 and 3 only (b) 2 and 4 only
(c) 2 and 3 only (d) 3 and 4 only

[GATE-2013]

179. Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?

- (a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q_1\}$
(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_3\}$

[GATE-2014 (Set-1)]

180. If $L_1 = \{a^n | n \geq 0\}$ and $L_2 = \{b^n | n \geq 0\}$, consider

- I. $L_1 \cdot L_2$ is a regular language
II. $L_1 \cdot L_2 = \{a^n b^n | n \geq 0\}$

Which one of the following is CORRECT?

- (a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

[GATE-2014 (Set-2)]

181. Consider alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0 , X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0 , X_1 and X_2 are related as follows:

$$\begin{aligned} X_0 &= 1X_1 \\ X_1 &= 0X_1 + 1X_2 \\ X_2 &= 0X_1 + \{\lambda\} \end{aligned}$$

Which one of the following choices precisely represents the strings in X_0 ?

- (a) $10(0^* + (10)^*)1$
(b) $10(0^* + (10)^*)^*1$
(c) $1(0 + 10)^*1$
(d) $10(0 + 10)^*1 + 110(0 + 10)^*1$

[GATE-2015 (Set-2)]

182. Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive 1s?*
- $(0+1)^*0011(0+1)^* + (0+1)^*1100(0+1)^*$
 - $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(0+1)^*$
 - $(0+1)^*00(0+1)^* + (0+1)^*11(0+1)^*$
 - $00(0+1)^*11 + 11(0+1)^*00$

[GATE-2016 (Set-1)]

183. Consider the following two statements:
- If all states of an NFA are accepting states then the language accepted by the NFA is Σ^* .
 - There exists a regular language A such that for all languages B , $A \cap B$ is regular.

Which one of the following is **CORRECT**?

- Only I is true
- Only II is true
- Both I and II are true
- Both I and II are false

[GATE-2016 (Set-2)]

184. Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N . Which one of the following is necessarily true?
- $k \geq 2^n$
 - $k \geq n$
 - $k \leq n^2$
 - $k \leq 2^n$

[GATE-2018]

185. For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L ?
- 9
 - 24
 - 3
 - 5

[GATE-2019]

186. If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is NOT regular?
- $L \cdot L^R = \{xy \mid x \in L, y^R \in L\}$
 - Suffix (L) = $\{y \in \Sigma^* \mid \exists x \in \Sigma^* \text{ such that } xy \in L\}$

- Prefix (L) = $\{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$
- $\{ww^R \mid w \in L\}$

[GATE-2019]

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2
3
6
9

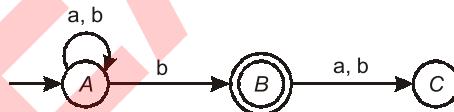
Numerical Answer Type Questions

187. Consider the following regular language on $\Sigma = \{a\}$.

$$L = \{a^{m^n} \mid n \geq 1, m > n\}$$

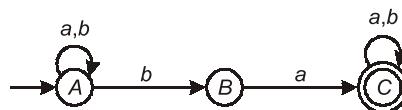
How many states required to construct equivalent minimal DFA that accepts L ?

188. Consider the following NFA that accepts a regular language L over $\Sigma = \{a, b\}$.



Find the number of states in the minimized DFA that accepts a complement of language L .

189. Consider the following NFA.



How many final states required in the equivalent DFA obtained by subset-construction algorithm?

190. Find the minimum number of states in DFA that accepts a language where each string starts with 'a' and ends with 'ab' over $\Sigma = \{a, b\}$.

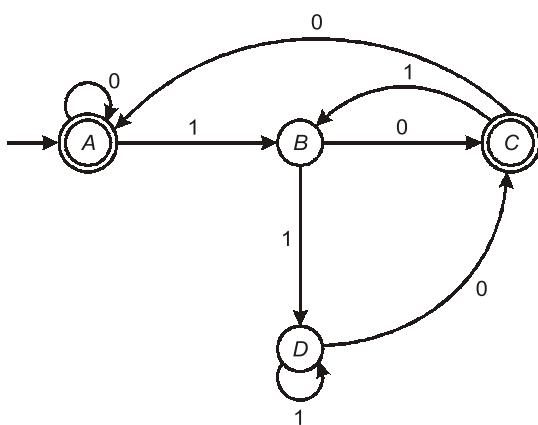
191. Let $L = \{(a^P)^* \mid P \text{ is a prime number}\}$ and $\Sigma = \{a\}$. How many minimum number of states in DFA that accepts a language L ?

192. How many minimum number of states required to construct a DFA that accepts a language represented by the regular expression $(a^* + a^*b^*)$?

193. Let $L_1 = 0^*1^*$, $L_2 = 1^*0^*$, $L_3 = (0+1)^*$ and $L_4 = 0^*1^*0^*$. Then find the number of strings in the following language L.

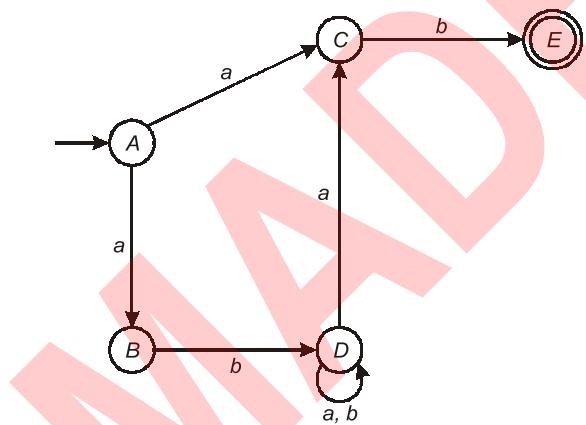
$$L = (L_1 \cap L_2) - (L_3 \cap L_4)$$

194. Consider the following DFA that accepts some regular language L over {0, 1}.



How many states in equivalent minimized DFA that accepts the language L?

195. Consider the following non-deterministic finite automata over $\Sigma = \{a, b\}$.



How many states in the minimized DFA which is equivalent to the above NFA?

196. Let $L = \{aba\}$. Prefix and suffix operations over the language L is used to perform the following.

$$X = (\text{prefix}(L) \cap \text{suffix}(L))/L$$

Quotient operation (/) is also used in the above. How many strings are there in the language X?

197. How many final states required in minimized DFA that accepts the language represented by the following regular expression R?

$$R = a^+b^+ + b^+a^+$$

198. How many equivalence classes are exist for the following regular expression R?

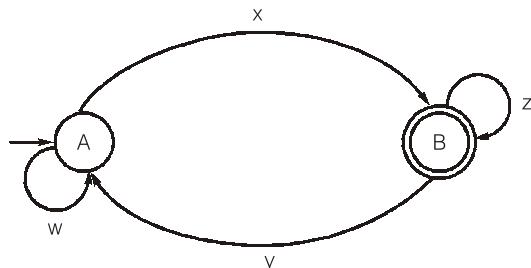
$$R = (a + b)^* b(a + b + \epsilon)$$

199. Let $L_1 = (0 + 1)^*$, $L_2 = (01^*0 + 1^*)$ and $L_3 = (01^+0 + 1^+ + \epsilon)$
If $L = L_1 \cap L_2 \cap L_3$ then find number of strings in L which do not contain 11.

200. Let $L = \{w \mid w \in (a + b)^* \text{ and } w \text{ does not end in } ba\}$. Find the number of states needed for minimal DFA to accept L

201. Let $L = \{\text{madeeasy}\}$ over $\Sigma = \{m, a, d, e, s, y\}$, $L_1 = \text{prefix}(L)$ and $L_2 = L_1/\Sigma^*$. Find the number of strings in L_2 . Assume L_1 and L_2 do not include empty string.

202. Consider the following state diagram:



Assume $w, x, y, z \in \{0, 1\}$.

If above state diagram is possible to make a DFA with deterministic inputs w, x, y, z then find number of possible DFA's. [Assume all given states are fixed]

203. Let L be the language over {a, b} and it contain all strings with following rules.

1. Every string starts with "a"
2. Every string contains "ab"
3. Every string ends with "b"

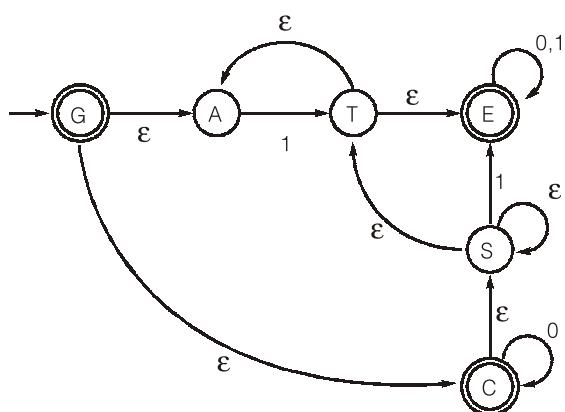
Find the number of states required in the minimized DFA that accepts L.

204. Let L be a language and $L = \{w \mid w \text{ has exactly one 'b', } w \in \{a, b\}^*, |w| \text{ is even}\}$.

Find the minimum number of states required for DFA that accepts L _____.

Common Data for Q.205 & Q.206

Consider the following ϵ -NFA



205. Find the cardinality of epsilon closure of state 'S' [ϵ -closure (q) is the set which contain all the states those are reachable from state q without reading any input symbol]

206. Find the number of states required to construct an equivalent minimized DFA that accepts the same language as the given ϵ -NFA.

207. Consider a regular language R over $\Sigma = \{a, b\}$ which is equivalent to the regular expression $(ab + b)^*$. How many equivalence classes of Σ^* are present for the language R ?

208. Let $L_1 = a^*b^*$ and $L_2 = \{ab\}$. $L_3 = \text{Prefix}(L_1^* \cap L_2)$, where $\text{prefix}(L) = \{u \mid uv \in L \text{ for any } v\}$. Find the number of strings in L_3 ?

209. How many number of states are there in the minimized DFA that accepts the following language L .

$$L = \{a^{3(n+1)} \mid n \geq 0\}$$

210. Find the number of states in DFA which accepts a language such that each block of 4 consecutive symbols of every string contain at least two a's for $\Sigma = \{a, b\}$. [If string length is less than 4 then it must be accepted]

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Common Data for Q.211 & Q.212

Consider the following regular expression R .

$$R = a^*b^* + b^*a^*$$

211. How many final states exist in the minimized DFA that accepts a language equivalent to R .

212. How many equivalence classes of Σ^* to represent a language which is equivalent to R .

213. The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is _____.
 $a^*b^*(ba)^*a^*$

[GATE-2014 (Set-3)]

214. The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1aw_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is _____.
[GATE-2017]

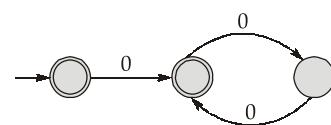
215. Given a language L , define L^i as follows:

$$L^0 = \{\epsilon\}$$

$$L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is _____.
[GATE-2018]



Multiple Select Questions

216. Let L be the language over the alphabet $\{1, 2, 3, (,)\}$ generated by the following grammar (with start symbol S and non-terminals {A, B, C}):

$$S \rightarrow ABC$$

$$A \rightarrow ($$

$$B \rightarrow 1B \mid 2B \mid 3B$$

$$B \rightarrow 1 \mid 2 \mid 3$$

$$C \rightarrow)$$

Then which of the following is false?

- (a) L is finite
- (b) L contains only strings of even length
- (c) L is regular
- (d) L is context free

217. Which of the following are true?

- (a) A DFA with n states must accept atleast one string of length greater than n .
- (b) A DFA that has n states and accepts an infinite language must accept at least one string x such that $n \leq |x| \leq 2n - 1$, where $|x|$ denotes the length of x .
- (c) If a language L is accepted by an NFA with n states then there is a DFA with no more than 2^n states accepting L .
- (d) None of these

218. Which of the following statements are true?

- (a) The intersection of a context free language with a regular language is context free.
- (b) The intersection of two context free languages is context free.
- (c) The intersection of a contexts free language and the complement of a regular language is context free.
- (d) The intersection of a regular language and the complement of a regular language is regular.

219. Consider the languages L_1 and L_2 :

$$L_1 = \{a^m b^n c^p \mid (m = n \text{ or } n = p) \text{ and } m + n + p \geq 20\}$$

$$L_2 = \{a^m b^n c^p \mid (m = n \text{ or } n = p) \text{ and } m + n + p \leq 20\}$$

Which of the following is true?

- (a) L_2 is finite
- (b) Both L_1 and L_2 are finite
- (c) L_1 is regular
- (d) L_2 is regular

220. Let L_1 and L_2 be languages over an alphabet Σ such that $L_1 \subseteq L_2$. Which of the following are false?

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- (a) If L_2 is regular then L_1 must also be regular.
- (b) If L_1 is regular then L_2 must also be regular.
- (c) Either both L_1 and L_2 are regular or both are not regular.
- (d) If L_2 is regular then L_1 need not be regular.

221. Let r, s and t be regular expression. Which of the following identities are incorrect?

- (a) $(rs + r)^*r = r(sr + r)^*$
- (b) $r(s + t) = rs + t$
- (c) $(r + s)^* = r^*s^*$
- (d) $(r^*s)^* = (rs)^*$

222. Let $L \subseteq \{0, 1\}^*$. Which of the following are false?

- (a) If L is regular, all subsets of L are regular.
- (b) If all proper subsets of L are regular then L is regular.
- (c) If all finite subsets of L are regular then L is regular.
- (d) If a proper subset of L is not regular then L is not regular.

223. Which of the following is correct?

- (a) The Kleene Closure (L^*) of L where $L = \{0^{m^2} : m \geq 3\}$ is regular
- (b) For all languages $L \subseteq \{0, 1\}^*$ if L^* is regular then L need not to be regular.
- (c) Intersection of infinitely many regular languages must be regular.
- (d) If L is regular and M is not regular then $L.M$ is necessarily not regular.

224. Which of the following regular expressions represent(s) the set of all binary numbers that are divisible by three? Assume that the string e is divisible by three.

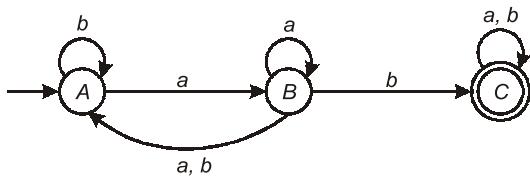
- (a) $(0^*(101^*0)^*1)^*$
- (b) $(0+1(01^*0)^*1)^*$
- (c) $(0+11+10(1+00)^*01)^*$
- (d) $(0+11+11(1+00)^*00)^*$

[GATE : 2021 (Set-2)]



Try Yourself

- T1.** Identify the language accepted by the following NFA over $\Sigma = \{a, b\}$



- (a) All strings which do not contain two consecutive a's.
- (b) All strings which contain two consecutive a's
- (c) All strings which contain "ab" as substring
- (d) None of these

[Ans: (c)]

- T2.** Consider the following two regular expressions R_1 and R_2 .

$$R_1 = (a + b)^*a(a + b)(a + b)^*$$

$$R_2 = a^*bbab$$

Which of the following relation is correct?

- (a) $R_1 = R_2$
- (b) $R_1 \subseteq R_2$
- (c) $R_1 \supseteq R_2$
- (d) None of these

[Ans: (c)]

- T3.** Consider the regular expression $R = (a + b)^*a(ab)^* + \epsilon$. Which of the following are possible as subset of R ?

- (i) $(aa)^*$
- (ii) $(ba)^*$
- (iii) $(aa)^*(ba)^*$
- (a) (i) and (ii) only
- (b) (ii) and (iii) only
- (c) (i), (ii) and (iii)
- (d) None of these

[Ans: (c)]

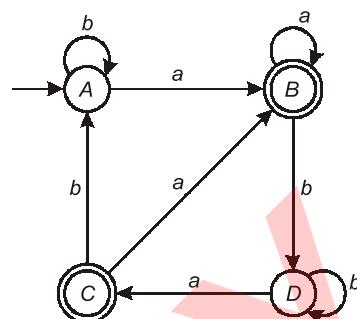
- T4.** Let $A = \{(a^*b^*)\}$ and $B = \{bb, ba, bbb\}$. Then A/B represents the following language when / is quotient operation.

- (a) \emptyset
- (b) $\{b^*\}$
- (c) $\{a^*b^*\}$
- (d) None of these

[Ans: (c)]

- T5.** Identify the language accepted by the following deterministic finite automata over the input alphabet $\Sigma = \{a, b\}$.

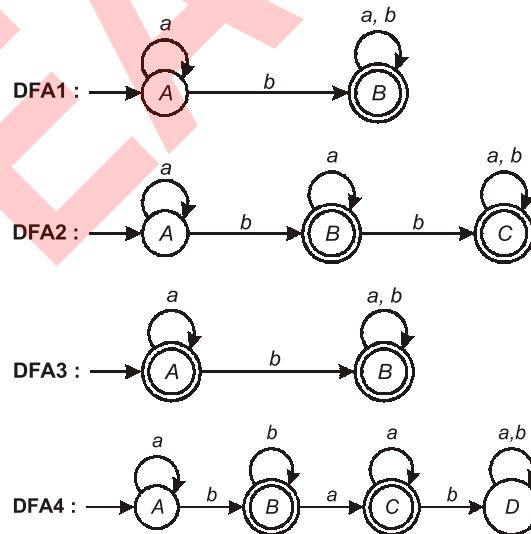
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- (a) All strings of a's and b's
- (b) All strings which are ending with a
- (c) All strings which do not end with b
- (d) All strings which contain 'a' as the substring

[Ans: (b)]

- T6.** Consider the following DFA's.



Which of the above DFA's are equivalent?

- (a) DFA1 and DFA2
- (b) DFA2 and DFA3
- (c) DFA3 and DFA4
- (d) None of these

[Ans: (a)]

- T7.** Consider the following regular expression (RE)

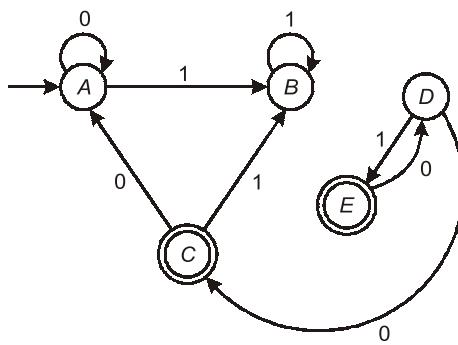
$$RE = (a + b)^*(a + b + \epsilon)a$$

Which of the following is equivalent to the above RE?

- (a) $(a^* + b^*)^*(aa + ba)$
- (b) $(\epsilon + a + b^*)^*a$
- (c) $(a + b)^*(a + b + \epsilon)a$
- (d) None of these

[Ans: (b)]

T8. Consider the following DFA.



If above DFA accepts the language denoted by the regular expression $(0 + 1)^*10(0 + 1)$, find the missing transitions.

- (a) $\delta(B, 0) = A, \delta(E, 1) = C$
- (b) $\delta(B, 0) = B, \delta(E, 1) = B$
- (c) $\delta(B, 0) = D, \delta(E, 1) = B$
- (d) $\delta(B, 0) = A, \delta(E, 1) = B$

[Ans: (c)]

T9. Which of the following is a regular language?

- (a) $L = \{a^{n^n} \mid n \geq 1\}$
- (b) $L = \{a^{m^n} \mid n \geq 1, m = n^2\}$
- (c) $L = \{a^{m^n} \mid n \geq 1, m > n\}$
- (d) None of these

[Ans: (c)]

T10. Which of the following is non-regular language?

- (a) $L = \{wxwy \mid x, y, w \in (a + b)^+\}$
- (b) $L = \{xwyw \mid x, y, w \in (a + b)^+\}$
- (c) $L = \{wxyw \mid x, y, w \in (a + b)^+\}$
- (d) All of the above

[Ans: (c)]

T11. Assume R_1, R_2 and R_3 are three regular expressions.

Given, $R_1 + R_2R_3 = (R_1 + R_2)(R_1 + R_3)$ for any R_2 and R_3 .

Which of the following could be correct condition which always satisfies the above equation.

- (i) $R_1 = R_2$
- (ii) $R_1 = R_3$
- (iii) $R_1 = \emptyset$

- (a) Only (i) and (ii) are correct
- (b) Only (i) and (iii) are correct
- (c) Only (ii) and (iii) are correct
- (d) Only (iii) is correct

[Ans: (d)]

T12. Let L be any formal language. If L^* is regular language then what is L ?

- (a) L is regular
- (b) L is non-regular
- (c) L is CFL
- (d) None of these

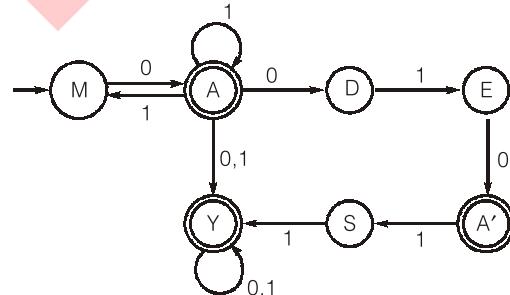
[Ans: (d)]

T13. Find the number of states in DFA that recognizes the language where all strings that do not contain the substring bab, for $\Sigma = \{a, b\}$

- (a) 3
- (b) 4
- (c) 5
- (d) 6

[Ans: (b)]

T14. Find the language accepted by the following NFA.



- (a) All strings of 0's and 1's
- (b) All strings starting with '1'
- (c) All strings starting with '0'
- (d) None of these

[Ans: (c)]

T15. Let L be the language formed by tossing a coin. Each string in L has n -length and the sequence in each string depends on the result of ' n ' tosses. Each toss can result in either Head (H) or Tail (T). Find the number of strings in L over the input alphabet $\Sigma = \{T, H\}$

- (a) n
- (b) n^2
- (c) 2^n
- (d) n^n

[Ans: (c)]

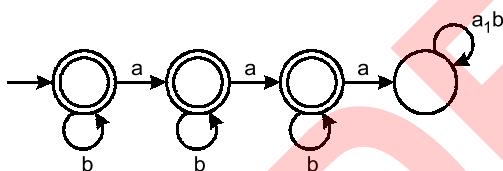
- T25.** Consider a regular expression $R = (a + \epsilon)(bb^*a)^*$. What is the language generated by R over $\Sigma = \{a, b\}$
- Set of all strings that do not contain aa .
 - Set of all strings that do not contain two or more consecutive a 's.
 - Set of all strings that do not end with b and do not contain two or more consecutive a 's.
 - None of these

[Ans: (c)]

- T26.** Let $L = \{pq \mid p, q \in \{0, 1\}^*\text{ and } \text{dec}(p) \times \text{dec}(q) = 10\text{ (ten)}\}$, where $\text{dec}(x)$ is decimal equivalent of binary number x . Find the language L by assuming that leading zero's are allowed for the strings of L . [01 = 001 = 0001]
- Regular
 - CFL but not regular
 - Recursive but not CFL
 - None of these

[Ans: (a)]

- T27.** Consider the following DFA:



Identify the language accepted by the DFA?

- All strings not contain aaa
- All strings not starting with aaa .
- All strings with no more than two a 's
- None of these

[Ans: (c)]

- T28.** Let L_1 and L_2 be languages over Σ and assume that $L_1 \cap L_2 = \emptyset$. If L_1 is finite language and $L_1 \cup L_2$ is regular then L_2 is _____?
- Regular language and finite
 - Regular language and infinite
 - Need not be regular
 - None of these

[Ans: (d)]

- T29.** Find the number of states in minimized DFA for the regular expression $a^+b^+ + b^+a^+$
- 4
 - 5
 - 6
 - None of these

[Ans: (c)]

- T30.** Consider the following configuration of a DFA. $DFA = (Q, \Sigma, \delta, q_0, A)$ where $Q = \{q_0, q_1\}$, q_0 is initial state, A is set of final states.

Given, $A = \{q_1\}$ and $\delta(q_i, x) = q_{1-i}$, $x \in \Sigma, i = 0, 1$. Identify the language accepted by the above DFA?

- Set of all strings over a given alphabet Σ .
- Set of all strings of odd length over Σ .
- Set of all strings of even length over Σ .
- None of these

[Ans: (b)]

- T31.** Identify the non regular language from the following.

- $\{x \mid x \in (0+1)^*, |x| \text{ is odd, First symbol of } x \text{ is } 1\}$
- $\{x \mid x \in (0+1)^*, |x| \text{ is odd, Middle symbol of } x \text{ is } 1\}$
- $\{x \mid x \in (0+1)^*, |x| \text{ is odd, Last symbol of } x \text{ is } 1\}$
- None of these

[Ans: (b)]

- T32.** Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

- $(0+1)^*0011(0+1)^* + (0+1)^*1100(0+1)^*$
- $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(0+1)^*$
- $(0+1)^*00(0+1)^* + (0+1)^*11(0+1)^*$
- $00(0+1)^*11 + 11(0+1)^*00$

[GATE-2016, Ans: (b)]



2

Context Free Languages and Pushdown Automata



Multiple Choice Questions

1. Which of the following conversions is not possible?
 - (a) Regular grammar to Context free grammar.
 - (b) NDFA to DFA
 - (c) Non deterministic PDA to Deterministic PDA
 - (d) Non deterministic Turning machine to Deterministic Turning machine

2. The class of "Context Free Language" is not closed under
 - (a) Intersection
 - (b) Union
 - (c) Concatenation
 - (d) Kleene closure

3. The number of auxiliary memory required for a "Pushdown Machine" (PDM) to behave like a "Finite State Machine" (FSM) is
 - (a) 0
 - (b) 1
 - (c) 2
 - (d) 3

4. The language L is recognizable by a finite automaton. The language $L' = \{w \mid w \text{ is the prefix of } v \text{ where } v \in L\}$ then L' is a
 - (a) Context Free Language
 - (b) Context Sensitive Language
 - (c) Regular Language
 - (d) None of the above

5. The language $L = \{a^n b^m c^n b^m \mid n \geq 1, m \geq 1\}$ is
 - (a) is context free
 - (b) is not context free
 - (c) abstract problem of checking number of formal and actual parameters
 - (d) both (b) and (c)

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6. The language $L = \{0^n 1^n 2^n \mid n > 0\}$ is a
 - (a) Context free language
 - (b) Context sensitive language
 - (c) Regular language
 - (d) Recursive Enumerable language

7. Let $L_1 = \{a^n b^n a^m \mid n, m \in \mathbb{N}\}$
 $L_2 = \{a^n b^m a^m \mid n, m \in \mathbb{N}\}$
 $L_3 = \{a^n b^n a^n \mid n \in \mathbb{N}\}$
 where \mathbb{N} is a Natural Number
 Which is false?
 - (a) $L_3 = L_1 \cap L_2$
 - (b) $L_3 \subset L_1$
 - (c) L_1 and L_2 are context free languages but L_3 is not a context free language
 - (d) L_1 and L_2 are not context free languages but L_3 is a context free language

8. The set $L = \{0^{n^2} \mid n \text{ is Natural Number}\}$ is a
 - (a) Regular Language
 - (b) Context Sensitive Language
 - (c) Context Free Language
 - (d) None of the above

9. The Grammar $G = (\{S\}, \{0, 1\}, P, S)$ where $P = \{S \rightarrow SS, S \rightarrow 0S1, S \rightarrow 1S0, S \rightarrow \epsilon\}$ would generate
 - (a) Regular Language
 - (b) Context Free Language
 - (c) Context Sensitive but not CF Language
 - (d) None of the above

10. Any string of terminals that can be generated by the following CFG

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aX \mid bX \mid a \\ Y &\rightarrow Ya \mid Yb \mid a \end{aligned}$$

- (a) Has at least two a's
 (b) Should end in a 'a'
 (c) Has at least one b
 (d) Has no consecutive a's or b's
11. Give the production Grammar for the language $L = \{a^i b^j \mid i > j \geq 1\}$
 (a) $\{S \rightarrow A, A \rightarrow aA, A \rightarrow aB, B \rightarrow ab\}$
 (b) $\{S \rightarrow aS, S \rightarrow aB, B \rightarrow ab, A \rightarrow aaB, B \rightarrow b\}$
 (c) $\{S \rightarrow AB, A \rightarrow aA \mid a, B \rightarrow aBb \mid ab\}$
 (d) None of the above
12. The production grammar in $\{S \rightarrow aSbb, S \rightarrow abb\}$ is
 (a) Type - 0 Grammar (b) Type - 1 Grammar
 (c) Type - 2 Grammar (d) Type - 3 Grammar
13. Consider the production Grammar $S \rightarrow AA$, $A \rightarrow aa$, $A \rightarrow bb$. Describe the language specified by the production Grammar
 (a) $L = \{aaab, baba, bbaa, bbbb\}$
 (b) $L = \{aaaa, abab, bbaa, aaab\}$
 (c) $L = \{aaaa, aabb, bbaa, bbbb\}$
 (d) $L = \{abab, abaa, aaab, baaa\}$
14. The following CFG $S \rightarrow aS \mid bS \mid a \mid b$ is not equivalent to the regular expression
 (a) $(a^* + b)^*$ (b) $(a + b)^+$
 (c) $(a + b)(a + b)^*$ (d) $(a + b)^*(a + b)$
15. Give a production Grammar that satisfies the language $L = \{a^i b^{2i} \mid i \geq 1\}$
 (a) $S \rightarrow aSb, S \rightarrow b$
 (b) $S \rightarrow aA, S \rightarrow b, A \rightarrow b$
 (c) $S \rightarrow aSbb, S \rightarrow abb$
 (d) None of the above
16. The language generated by $S \rightarrow aS \mid aSb \mid \epsilon$ is
 (a) $L(G) = \{g \mid \text{each prefix of } g \text{ has same numbers of a's and b's}\}$
 (b) $L(G) = \{g \mid \text{each prefix of } g \text{ has at least as many a's as b's}\}$
 (c) $L(G) = \{g \mid \text{each prefix of } g \text{ has unequal number of a's and b's}\}$
 (d) $L(G) = \{g \mid \text{each prefix of } g \text{ has at least as many a's as b's and a's never follow the b's}\}$
17. Consider the grammar:
 $S \rightarrow PQ \mid SQ \mid PS$
 $P \rightarrow x$
 $Q \rightarrow y$
- To get a string of n terminals, the number of productions to be used is
 (a) n^2 (b) $n + 1$
 (c) $2n$ (d) $2n - 1$
18. Choose the correct statements
 (a) $A = \{a^n b^n \mid n = 0, 1, 2, 3 \dots\}$ is a regular language
 (b) the set B , of all strings of equal number of a's and b's defines a regular language
 (c) $L(a^*b^*) \cap B$ gives the set A
 (d) None of the above
19. Consider the left recursive grammar
 $S \rightarrow Aa \mid b$
 $A \rightarrow Ac \mid Sd \mid \epsilon$
- Which of the following grammar's are equivalent to the grammar given above when the left recursion is removed?
 (a) $S \rightarrow Aa \mid b$ (b) $S \rightarrow Aa \mid b$
 $A \rightarrow bdA' \mid A'$ $A \rightarrow ad \mid bd \mid cA$
 $A' \rightarrow cA' \mid adA' \mid \epsilon$
 (c) $S \rightarrow bA'$ (d) $S \rightarrow Aa \mid b$
 $A' \rightarrow c \mid da$ $A \rightarrow AC \mid Aad \mid bd$
20. Let L be a language recognizable by a finite automaton. The language REVERSE (L) = { w such that w is the reverse of v where $v \in L$ } is a
 (a) context free language
 (b) context sensitive language
 (c) regular language
 (d) None of the above
21. The context free grammar defining the language a^*b^* is
 (a) $S \rightarrow Sb \mid a$
 (b) $S \rightarrow XY$
 $X \rightarrow aX \mid \lambda$
 $Y \rightarrow bY \mid \lambda$
 (c) $S \rightarrow SS \mid baa \mid abb$
 $S \rightarrow A$
 (d) $S \rightarrow SS \mid baa \mid abb \mid \lambda$

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22. $L = \{a^n b^n a^n \mid n = 1, 2, 3, \dots\}$ is an example of a language that is
- context free
 - not context free
 - not context free but whose complement is CF
 - context free but whose complement is not CF
23. A PDM behaves like an FSM when the number of auxiliary memory it has, is
- 0
 - 1
 - 2
 - None of these
24. A PDM behaves like a TM when the number of auxiliary memory it has, is
- 0
 - 1 or more
 - 2 or more
 - None of these
25. A CFG is said to be in Chomsky Normal (CNF), if all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$. Let G be a CFG in CNF. To derive a string of terminals of length x , the number of productions to be used is
- $2x - 1$
 - $2x$
 - $2x + 1$
 - 2^x
26. Which of the following is accepted by an NDPDM, but not by a DPDM?
- All strings in which a given symbol is present atleast twice
 - Even palindromes (i.e., palindromes made up of even number of symbols)
 - Strings ending with a particular letter
 - None of the above
27. Which of the following is not necessarily a palindrome on $\Sigma = \{0, 1\}$
- ϵ
 - 0
 - $\{aa \mid a \in \{0, 1\}\}$
 - $\{0x \mid x \text{ is a palindrome}\}$
 - $\{axa \mid a \in \{0, 1\}, x \text{ is a palindrome}\}$
28. Which of the following is true?
- All unambiguous grammars are regular
 - Every LL(K) and LR(K) grammar is unambiguous and a regular language can never be inherently ambiguous
29. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:
- List-I**
- $S \rightarrow aS \mid \epsilon$
 - $S \rightarrow aS \mid bS \mid \epsilon$
 - $S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$
 - $S \rightarrow aSb \mid SS \mid \epsilon$
 - $S \rightarrow aSb \mid bSa \mid SS \mid \epsilon$
 - $S \rightarrow a \mid b \mid SS \mid \epsilon$
 - $S \rightarrow aa \mid bb \mid SS \mid \epsilon$
 - $S \rightarrow aSa \mid bSb \mid a \mid b$
 - $S \rightarrow aSa \mid bSb \mid \epsilon$
- List-II**
- $(a + b)^*$
 - Palindromes
 - a^*
 - $L = \{w \mid w \in \{a, b\}^* \text{ and } n_a(v) \geq n_b(v), \text{ where } v \text{ is any prefix of } w\}$ (properly balanced parenthesis)
 - $(aa + bb)^*$
 - $L = \{w \mid w \in \{a, b\}^* \text{ and } n_a(w) = n_b(w)\}$ (balanced parenthesis)
 - Even palindromes
 - Odd palindromes
- Codes:**
- | A | B | C | D | E | F | G | H | I |
|-------|---|---|---|---|---|---|---|---|
| (a) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1 |
| (b) 4 | 1 | 2 | 5 | 7 | 3 | 6 | 8 | 1 |
| (c) 3 | 1 | 2 | 4 | 6 | 1 | 5 | 8 | 7 |
| (d) 3 | 1 | 2 | 4 | 6 | 1 | 5 | 7 | 8 |
30. To prove that $L(M) = L(r)$ it is sufficient to prove
- That every string generated by r is accepted by M
 - That every string accepted by M is generated by r
 - Both (a) and (b)
 - None of above

31. Which is false?
- A CFG can generate a regular language
 - A regular grammar can generate a CFL
 - A regular grammar can generate any CFL
 - A regular grammar is always a CFG
32. CFL's are closed under
- Intersection
 - Complementation
 - Set difference
 - Regular difference
33. CFL's are closed under
- min
 - max
 - half
 - alt
 - None of these
34. CFL's are not closed under
- init
 - L/a
 - cycle
 - set difference
35. CFL's are not closed under
- Kleene Star
 - Union
 - Intersection
 - Product
36. Consider the following languages:
- $$L_1 = \{a^p b^q c^r d^t ; p + q = r + t\}$$
- $$L_2 = \{a^p b^q c^r ; p < q < r\}$$
- $$L_3 = \{a^p b^q c^r ; q = p + r\}$$
- Only L_1 is CFL
 - L_1 and L_3 are CFL
 - L_1, L_2 and L_3 are CFL
 - None are CFL
37. Consider the following languages:
- $$L_1 = \{a^p b^q c^r ; p = 3(q + r)\}$$
- $$L_2 = \{a^p b^q c^r ; p < q \text{ or } q < r\}$$
- $$L_3 = \{ww^R ; w \in \{a, b\}^*\}$$
- $$L_4 = \{wcw^R ; w \in \{a, b\}^*\}$$
- $$L_5 = \{a^n b^{2n}\} \cup \{a^n b^{3n}\} \cup \{a^n b^{4n}\}$$
- L_1 and L_4 are DCFL's while L_2 and L_3 and L_5 are CFL's but not DCFL's
 - L_1 and L_2 are DCFL's while L_3, L_4 and L_5 are CFL's but not DCFL's
 - L_4 is DCFL and rest are CFL's but not DCFL's
 - All 5 are DCFL's
38. Which of these grammars generate the same language?

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- $G_1 : S \rightarrow aaSb | b$
 $G_2 : S \rightarrow ASb | b, A \rightarrow Ba, B \rightarrow a$
 $G_3 : S \rightarrow ASb | b, A \rightarrow Aa | a$
 (a) G_1 and G_2 only (b) G_1 and G_3 only
 (c) G_2 and G_3 only (d) G_1, G_2 and G_3
39. Consider the following grammars;
- $$G_1: S \rightarrow aSb | SS | \epsilon$$
- $$G_2: S \rightarrow aSb | bSa | SS | \epsilon$$
- $$G_3: S \rightarrow aSa | bSb | \epsilon$$
- $$G_4: S \rightarrow aSa | bSb | \epsilon | a | b$$
- $L(G_2) \subseteq L(G_1)$ and $L(G_4) \subseteq L(G_3)$
 - $L(G_1) \subseteq L(G_3)$ and $L(G_2) \subseteq L(G_4)$
 - $L(G_1) \subseteq L(G_2)$ and $L(G_3) \subseteq L(G_4)$
 - $L(G_2) \subseteq L(G_3)$ and $L(G_1) \subseteq L(G_4)$
40. Which of the following is false?
- Given that L_1 is DCFL, L_2 is DCFL, L_3 is CFL, L_4 is CFL and L_5 is regular
- $(L_3 \cup L_4)^C$ is recursive
 - $L_1^C - L_5$ is DCFL
 - $(L_3^R \cup L_4^C) \cap L_5$ is CFL
 - $L_3^C \cap L_4^C$ is CFL
41. DCFL's are not closed under which operations
- Kleene closure, positive closure, union concatenation, homomorphism and intersection
 - inverse homomorphism and complement
 - regular intersection and regular difference
 - quotient, min and max
42. $L_1 = \{w \in \{a, b, c\}^* : n_a(w) = n_b(w) = n_c(w)\}$ and $L_2 = \{ww^T | w \in \{a, b\}^*\}$ are examples of languages which are
- not CFL's and whose complement is also not CFL's
 - not CFL's but whose complements are CFL's
 - are CFL's whose complements are also CFL's
 - are CFL's whose complements are not CFL's
43. $L = \{a^n b^n ; n \geq 0, n \neq 13\}$ is
- a DCFL
 - a CFL but not DCFL
 - not a CFL
 - a recursive set but not CFL

44. $\{a^n b^n : n \geq 0, n \text{ is not a multiple of } 3\}$
 (a) may or may not be a CFL
 (b) is a DCFL and hence a CFL
 (c) is a CFL but not a DCFL
 (d) is recursive but not CFL
45. $L_1 = \{x \in y^R\}$
 $L_2 = \{x \in y^R, x \neq y^R\}$
 $L_3 = \{x \in y^R, x^R \neq y\}$
 (a) L_1 is a CFL, but L_2 and L_3 are not
 (b) L_2 and L_3 are CFL, but L_1 is not
 (c) All three are CFL's
 (d) None of them is CFL
46. A CFG G is ambiguous. Which of the following is false?
 (a) at least one $w \in L(G)$ such that w has at least 2 distinct derivation trees
 (b) at least one $w \in L(G)$ such that w has at least 2 RMD's and at least 2 LMD's
 (c) at least one $w \in L(G)$ such that w has exactly 2 derivation trees
 (d) None of these
47. Let G be an arbitrary CFG. Which of the following is false?
 (a) There is an algorithm which can check if $L(G)$ is finite, infinite or empty
 (b) There is an algorithm to check for any $w \in \Sigma^*$, if $w \in L(G)$ or not
 (c) There is an algorithm which can check if $L(G)$ is regular
 (d) There is an algorithm which can check if $w \in L(G)$ such that $|w| \leq n$
 (e) There is an algorithm to check which productions in G are nullable
48. Which of following is false? For an arbitrary CFG, G, there exists an algorithm
 (a) to check if $L(G)$ contains infinite strings
 (b) to check which variables appear in some sentential form
 (c) to check which variables are nullable
 (d) to check which variables are useless
 (e) None of these

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49. $L_1 = \{wxw : w \in \{a, b\}^*, x \notin \{a, b\}\}$
 $L_2 = \{wxw^R : w \in \{a, b\}^*, x \notin \{a, b\}\}$
 $L_3 = \{w^R x w : w \in \{a, b\}^*, x \notin \{a, b\}\}$
 $L_4 = \{wxw : w, x \in \{a, b\}^*\}$
 (a) L_1 is CS, L_2 is DCFL, L_3 is not CFL, L_4 is regular
 (b) L_1 is CFL, L_2 and L_3 are DCFL's, L_4 is CS
 (c) L_1 is CS, L_2 and L_3 are DCFL's, L_4 is regular
 (d) L_1 and L_4 are CS, L_2 and L_3 are DCFL's
50. $L_1 = \{wxw^R : w, x \in \{a, b\}^+\}$ is
 (a) CFL but not DCFL
 (b) DCFL and hence a CFL
 (c) not CFL
 (d) regular
51. $L_2 = \{xww^R : w, x \in \{a, b\}^+\}$ is
 (a) a CFL but not DCFL
 (b) a DCFL and hence a CFL
 (c) not a CFL
 (d) regular
52. $L_3 = \{wxw^R : w \in \{a, b\}^+, x \notin \{a, b\}\}$ is
 (a) a CFL but not DCFL
 (b) a DCFL and hence a CFL
 (c) not a CFL
 (d) regular
53. If a parse tree (derivation tree) is made for a $w \in L(G)$, when G is in Chomsky-Normal form, then what would be its least height?
 (a) $\log_2 |w|$ (b) $\log_2 |w| - 1$
 (c) $\log_2 (|w| + 1)$ (d) $\lceil \log_2 |w| \rceil + 1$
54. If L_1 and L_2 are both not regular, then $L_1 \cup L_2$
 (a) can never be regular
 (b) can be regular
 (c) is surely regular
 (d) must be a CFL
55. Let L is a non-regular language. Then \bar{L}
 (a) can be regular
 (b) can never be regular
 (c) is regular
 (d) has to be context free

56. If L_1 is CFL and L_2 is CFL, then $L_1 \cap L_2$ has to be
 (a) a CFL
 (b) context sensitive
 (c) recursive
 (d) recursively enumerable
57. If L_1 is CFL and L_2 is CFL, then $L_1 \cap L_2^c$ has to be
 (a) CFL
 (b) context sensitive
 (c) recursive
 (d) recursively enumerable
58. Consider the following CFG's
 $G_1 : S \rightarrow xSS \mid \epsilon \mid y$
 $G_2 : S \rightarrow xSyS \mid \epsilon \mid y$
 Both G_1 and G_2 are ambiguous because
 (a) The string " xyy " belonging to both $L(G_1)$ and $L(G_2)$, has more than one parse tree
 (b) The string " xy " belonging to $L(G_1)$ and the string " xyy " belonging to $L(G_2)$, have more than one parse tree
 (c) The string " ϵ " belonging to both $L(G_1)$ and $L(G_2)$, has more than one parse tree
 (d) The string " x " belonging to $L(G_1)$ and the string " xyy " belonging to $L(G_2)$, have more than one parse trees
59. Consider the following two languages:
 $L_1 = \{a w a \mid a \in \{0, 1\}, w \in \{0, 1\}^*\}$
 $L_2 = \{w a w \mid a \in \{0, 1\}, w \in \{0, 1\}^*\}$
 Which of the following is true?
 (a) L_1 is regular but L_2 is not
 (b) L_2 is regular but L_1 is not
 (c) Both are regular
 (d) Both are not regular
60. $L_1 = L(a^* b^*)$
 $L_2 = \{a^n b^n; n \geq 0\}$
 $L_3 = \{w \mid n_a(w) = n_b(w)\}$
 Which of the following is true?
 (a) $L_2 \subseteq L_1$ and $L_3 \subseteq L_2$ and $L_1 \cap L_2 = L_3$
 (b) $L_1 \subseteq L_2$ and $L_2 \subseteq L_3$ and $L_1 \cap L_3 = L_2$
 (c) $L_3 \subseteq L_1$ and $L_2 \subseteq L_3$ and $L_2 \cap L_3 = L_1$
 (d) $L_2 \subseteq L_1$ and $L_2 \subseteq L_3$ and $L_1 \cap L_3 = L_2$

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61. $G_1 : S \rightarrow bSS \mid SS \mid \epsilon$
 In above grammar, the string "b" has how many parse tree
 (a) 0 (b) 1
 (c) 2 (d) infinite
62. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:
List-I
 A. $S \rightarrow aAa \mid bAb \mid a \mid b$
 $A \rightarrow aA \mid bA \mid \epsilon$
 B. $S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$
 C. $S \rightarrow aAa \mid bAb$
 $A \rightarrow a \mid b$
List-II
 1. $L = \{aaa, aba, bab, bbb\}$
 2. $L = \{w \mid w \in \{a, b\}^* \text{ and } w \text{ starts and ends with same letter}\}$
 3. $L = \{w \mid w \in \{a, b\}^*, w \text{ is a palindrome}\}$
Codes:

A	B	C
(a) 1	2	3
(b) 2	3	1
(c) 3	2	1
(d) 3	1	2

63. $S \rightarrow AC$
 $A \rightarrow aaAb \mid \epsilon$
 $C \rightarrow cC \mid \epsilon$
 The language generated by above grammar is
 (a) $\{a^{2n} b^n c^n; n \geq 0\}$
 (b) $\{a^{2n} b^n c^n; n \geq 1\}$
 (c) $\{a^{2n} b^n c^m; n \geq 0, m \geq 0\}$
 (d) $\{a^{2n} b^m c^p; n \geq 0, m \geq 0, p \geq 0\}$

64. $G : S \rightarrow (S) \mid SS \mid \epsilon$
 The above grammar
 (a) is an unambiguous grammar
 (b) generates the set of properly balanced parenthetical structures
 (c) generates the set of strings on $\Sigma = \{(,)\}$ such that the number of left and right parenthesis are equal
 (d) None of these

65. A context-free grammar is ambiguous if

 - (a) the grammar contains useless non-terminals
 - (b) it produces more than one parse tree for some sentence
 - (c) some production has two non-terminals side by side on the right-hand side
 - (d) None of the above

[GATE-1987]

66. FORTRAN is a

 - (a) Regular language
 - (b) Context-sensitive language
 - (c) Context-free language
 - (d) None of the above

[GATE-1987]

67. Context-free languages and regular languages are both closed under the operation(s) of

 - (a) Union
 - (b) Intersection
 - (c) Concatenation
 - (d) Complementation

[GATE-1989]

68. Which of the following problems are undecidable?

 - (a) Membership problem in context-free languages
 - (b) Whether a given context-free language is regular
 - (c) Whether a finite state automation halts on all inputs
 - (d) Membership problem for type 0 languages

[GATE-1989]

[GATE-1992]

70. Context-free languages are

 - (a) closed under union
 - (b) closed under complementation
 - (c) closed under intersection
 - (d) closed under Kleene closure

[GATE-1992]

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71. The grammar whose productions are

$$\begin{aligned} <\text{stmt}> &\rightarrow \text{if } <\text{id}> \text{ then } <\text{stmt}> \\ <\text{stmt}> &\rightarrow \text{if } <\text{id}> \text{ then } <\text{stmt}> \text{ else } <\text{stmt}> \\ <\text{stmt}> &\rightarrow <\text{id}> : = <\text{id}> \\ <\text{id}> &\rightarrow a | b | c | d | e | f \end{aligned}$$

is ambiguous because

 - (a) the sentence
if a then if b then c : = d has two parse trees
 - (b) the left most and right most derivations of the sentence
if a then if b then c : = d are not same
 - (c) the sentence
if a then if b then c : = d else c : = f has more than two parse trees
 - (d) the sentence
if a then if b then c : = d else c : = f has two parse trees

[GATE-1996]

72. In the following grammar:

$$X ::= (X \oplus Y) | Y$$
$$Y ::= (Z^* Y) | Z$$
$$Z ::= \text{id}$$

Which of the following is true?

 - (a) ' \oplus ' is left associative while ' $*$ ' is right associative
 - (b) Both ' \oplus ' and ' $*$ ' are left associative
 - (c) ' \oplus ' is right associative while ' $*$ ' is left associative
 - (d) None of the above

[GATE-1997]

73. Which of the following languages over $\{a, b, c\}$ is accepted by a deterministic pushdown automata?

 - (a) $\{wcw^R \mid w \in \{a, b\}^*\}$
 - (b) $\{ww^R \mid w \in \{a, b, c\}^*\}$
 - (c) $\{a^n b^n c^n \mid n \geq 0\}$
 - (d) $\{w \mid w \text{ is a palindrome over } \{a, b, c\}\}$

Note: w^R is the string obtained by reversing ' w '.

[GATE-1997]

74. Let L_D be the set of all languages accepted by a PDA by final state and L_E the set of all languages accepted by empty stack. Which of the following is true?

- (a) $L_D = L_E$ (b) $L_D \supset L_E$
 (c) $L_D \subset L_E$ (d) None of the above
[GATE-1999]

75. If L_1 is context free language and L_2 is a regular language which of the following is/are false?
 (a) $L_1 - L_2$ is not context free
 (b) $L_1 \cap L_2$ is context free
 (c) $\sim L_1$ is context free
 (d) $\sim L_2$ is regular

[GATE-1999]

76. A grammar that is both left and right recursive for a non-terminal, is
 (a) Ambiguous
 (b) Unambiguous
 (c) Information is not sufficient to decide whether it is ambiguous or unambiguous
 (d) None of the above

[GATE-1999]

77. Let L denotes the language generated by the grammar $S \rightarrow 0S0 \mid 00$. Which of the following is true?
 (a) $L = 0^+$
 (b) L is regular but not 0^*
 (c) L is not context free
 (d) L is context free but not regular

[GATE-2000]

78. Given the following expression grammar:

$$\begin{aligned} E &\rightarrow E * F \mid F + E \mid F \\ F &\rightarrow F - F \mid id \end{aligned}$$

Which of the following is true?

- (a) $*$ has higher precedence than $+$
 (b) $-$ has higher precedence than $*$
 (c) $+$ and $-$ have same precedence
 (d) $+$ has higher precedence than $*$

[GATE-2000]

79. Consider the following two statements:

- $S_1 : \{0^{2n} \mid n \geq 1\}$ is a regular language
 $S_2 : \{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is a regular language

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Which of the following statements is correct?

- (a) Only S_1 is correct
 (b) Only S_2 is correct
 (c) Both S_1 and S_2 are correct
 (d) None of S_1 and S_2 is correct

[GATE-2001]

80. Which of the following statements is true?
 (a) If a language is context free it can always be accepted by a deterministic push-down automaton
 (b) The union of two context free languages is context free
 (c) The intersection of two context free languages is context free
 (d) The complement of a context free languages is context free

[GATE-2001]

81. The language accepted by a Pushdown Automaton in which the stack is limited to 10 items is best described as
 (a) Context free
 (b) Regular
 (c) Deterministic Context free
 (d) Recursive

[GATE-2002]

82. Which of the following suffices to convert an arbitrary CFG to an LL(1) grammar?
 (a) Removing left recursive alone
 (b) Factoring the grammar alone
 (c) Removing left recursion and factoring the grammar
 (d) None of the above

[GATE-2003]

83. Context free languages are closed under which of the following
 (a) left quotient (b) right quotient
 (c) prefix (d) suffix
 (e) All of the above

[GATE-2003]

84. Consider the following grammar C:

$$\begin{aligned} S &\rightarrow bS \mid aA \mid b \\ A &\rightarrow bA \mid aB \\ B &\rightarrow bB \mid aS \mid a \end{aligned}$$

Let $N_a(w)$ and $N_b(w)$ denote the number of a's and b's in a string w respectively. The language $L(G) \subseteq \{a, b\}^*$ generated by G is

- (a) $\{w \mid N_a(w) > 3N_b(w)\}$
- (b) $\{w \mid N_b(w) > 3N_a(w)\}$
- (c) $\{w \mid N_a(w) = 3k, k \in \{0, 1, 2, \dots\}\}$
- (d) $\{w \mid N_b(w) = 3k, k \in \{0, 1, 2, \dots\}\}$

[GATE-2004]

85. Let $L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$,
 $L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$, and
 $L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n, m \geq 0\}$.
 Which of these languages are NOT context free?
- (a) L_1 only
 - (b) L_3 only
 - (c) L_1 and L_2
 - (d) L_2 and L_3

[GATE-2006]

86. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

- A. Checking that identifiers are declared before their use
- B. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function
- C. Arithmetic expressions with matched pairs of parentheses
- D. Palindromes

List-II

1. $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
2. $X \rightarrow X b X \mid X c X \mid d X f \mid g$
3. $L = \{wcw \mid w \in (a \mid b)^*\}$
4. $X \rightarrow b X b \mid c X c \mid \epsilon$

Codes:

A	B	C	D
(a) 1	3	2	4
(b) 3	1	4	2
(c) 3	1	2	4
(d) 1	3	4	2

[GATE-2008]

87. Consider a CFG with the following productions.

$$\begin{aligned} S &\rightarrow AA \mid B \\ A &\rightarrow 0A \mid A0 \mid 1 \\ B &\rightarrow 0B00 \mid 1 \end{aligned}$$

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S is the start symbol, A and B are non-terminals and 0 and 1 are the terminals. The language generated by this grammar is

- (a) $\{0^n 10^{2n} \mid n \geq 1\}$
- (b) $\{0^i 10^j 10^k \mid i, j, k \geq 0\} \cup \{0^n 10^{2n} \mid n \geq 0\}$
- (c) $\{0^i 10^j \mid i, j \geq 0\} \cup \{0^n 10^{2n} \mid n \geq 1\}$
- (d) The set of all strings over $\{0, 1\}$ containing at least two 0's

[IT-2008]

Directions for Question 88 to 89:

A CFG G is given with the following productions where S is the start symbol, A is a non-terminal and a and b are terminals.

$$\begin{aligned} S &\rightarrow aS \mid A \\ A &\rightarrow aAb \mid bAa \mid \epsilon \end{aligned}$$

88. Which of the following strings is generated by the grammar above?

- (a) aabbaba
- (b) aabaaba
- (c) abababb
- (d) aabbaab

[IT-2008]

89. For the correct answer in above Question, how many steps are required to derive the string and how many parse trees are there?

- (a) 6 and 1
- (b) 6 and 2
- (c) 7 and 2
- (d) 4 and 2

[IT-2008]

90. Given the following state table of an FSM with two states A and B, one input and one output:

Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B = 0, what is the minimum length of an input string which will take the machine to the state A = 0, B = 1 with Output = 1?

- (c) $\{(ab)^n (cb^m)^n \mid m, n \geq 1\}$
- (d) $\{(ab)^n (cb^n)^m \mid m, n \geq 1\}$

[GATE-2017]

97. Consider the following languages:

- I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$
- II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$
- III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$
- IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

- (a) I and IV only
- (b) I and II only
- (c) II and III only
- (d) II and IV only

[GATE-2018]

98. Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?

- (a) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$
- (b) $\{w a^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$
- (c) $\{w w^R \mid w \in \{a, b\}^*\}$
- (d) $\{w a^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$

[GATE-2019]

99. Select from the given options the grammar rule after elimination of left recursion, equivalent to the following grammar rules with left recursion

- | | |
|---------------------------|---------------------------|
| $E \rightarrow E + T$ | |
| $E \rightarrow T$ | |
| (a) $E \rightarrow +TE'$ | (b) $E \rightarrow TE'$ |
| $E' \rightarrow TE'$ | $E' \rightarrow +TE'$ |
| $E' \rightarrow \epsilon$ | $E' \rightarrow \epsilon$ |
| (c) $E \rightarrow TE'$ | (b) $E \rightarrow +TE'$ |
| $E' \rightarrow +TE'$ | $E' \rightarrow TE'$ |

[JNU]

100. The productions $E \rightarrow E + E \mid E - E \mid E^* E \mid id$
- (a) generate an inherently ambiguous language
 - (b) generate an ambiguous language
 - (c) is an ambiguous grammar
 - (d) None of the above

[JNU]

101. Which of the following languages cannot be produced by a regular grammar?

- 1. $\{a^n b^n \mid n \geq 0\}$
- 2. $\{a^n b^k \mid k > n \geq 0\}$
- 3. $\{ww^R \mid w \in \{a, b\}\}$
- (a) Only 1
- (b) Only 1 and 2
- (c) Only 2 and 3
- (d) All 1, 2 and 3

[JNU]

102. A grammar G with the following production rules and $\Sigma = \{a, b\}$ is

$$S \rightarrow a \mid b$$

- (a) an LL(1) grammar
- (b) an LL(2) grammar
- (c) an LL(3) grammar
- (d) None of the above

[JNU]

103. A grammar G with the following production rules and $\Sigma = \{a, b\}$ is

$$S \rightarrow aA \mid aB$$

$$A \rightarrow b$$

$$B \rightarrow b$$

- (a) an LL(1) grammar
- (b) not an LL(2) grammar
- (c) an LR(0) grammar
- (d) None of the above

104. Identify from the following the string generated by the grammar rules,

$$\{S \rightarrow SS, S \rightarrow (S_1, S_1 \rightarrow S), S_1 \rightarrow \})\}$$

- (a) (())()
- (b) ((((())))))()
- (c) ()(()())
- (d) (()((())))

[JNU]

105. It is known that the language $L \subseteq \{a, b\}^*$ that consists of all strings that contain an equal number of a's and b's is context-free. Let M be the regular language $a^* b^*$. Which of the following is (are) true?

- 1. $L \cap M$ is a context-free language
- 2. $L \cap M$ is a regular language
- 3. $L \cap M = \{a^n b^m \mid n \text{ is a positive integer less than integer } m\}$
- (a) None
- (b) 1 only
- (c) 1 and 3
- (d) 2 and 3

[JNU]

106. If DFA denotes ‘deterministic finite automata’ and NDFA denotes ‘non-deterministic finite automata’, which of the following is false?
- For any language L, if L can be recognized by a DFA, then \bar{L} can be recognized by a DFA.
 - For any language L, if L can be recognized by an NDFA, then \bar{L} can be recognized by an NDFA.
 - For any language L, if L is context-free, then \bar{L} is context-free.
 - For any language L, if L is decidable, then \bar{L} is decidable.

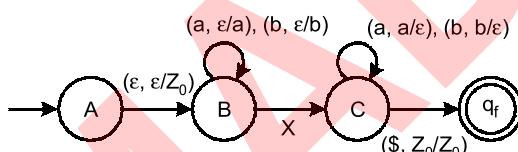
[JNU]

107. Identify the regular language:

- $F = \{ww \mid w \in \{0, 1\}^*\}$
- The complement of $\{0^n 1^n \mid n \geq 0\}$
- $F = \{w \in \{0, 1\}^* \mid w \text{ has an equal number of occurrences of } 01 \text{ and } 10 \text{ as substrings}\}$
- $F = \{w \in \{0, 1\}^* \mid w \text{ has equal number of } 0's \text{ and } 1's\}$

[JNU]

108. The following machine is designed with PDA acceptance by final state mechanism to accept the language L where all strings of L are odd length palindromes.



- What are the transitions at X to accept L?
- (a, a|ε), (b, b|b)
 - (a, a|ε), (b, b|ε)
 - (a, ε|ε), (b, ε|ε)
 - None of these

109. Assume PDA stack is limited to 2^{10} symbols. Now stack can contain maximum of 2^{10} symbols. The language accepted by such PDA is ____
- Regular language but not necessarily finite
 - DCFL but not regular
 - CFL but not DCFL
 - None of these

110. Let $L = \{wxw^R \mid w \in (a+b)^*, x \in (a+b)\}$. The complement of language L is ____
- Regular
 - DCFL but not regular
 - CFL but not DCFL
 - None of these

2
3
6
9

Numerical Answer Type Questions

111. Consider the following transition table from PDA.

δ	$\Sigma\epsilon$		
	a	b	ϵ
q_0	$(q_0, \epsilon/a)$	$(q_1, a/\epsilon)$	$(q_0, \epsilon/z_0)$
q_1	-	$(q_1, a/\epsilon)$	$(q_1, z_0/\epsilon)$

$PDA = (Q, \Sigma, \delta, \Gamma, q_0, z_0, \phi)$ where $Q = (q_0, q_1)$ and $\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \rightarrow Q \times \Gamma_\epsilon$.

[Example: In above table $\delta(q_0, a) = (q_0, \epsilon|a)$ is same as $\delta(q_0, a, \epsilon) = (q_0, a)$]

Consider the following languages.

- $$\begin{aligned}L_1 &= \{a^n b^n \mid m, n \geq 0\} \\L_2 &= \{a^n b^n \mid n \geq 1\} \\L_3 &= \{a^m b^n \mid m > n\} \\L_4 &= \{a^m b^n \mid m < n\}\end{aligned}$$

How many of the above languages L_1, L_2, L_3 and L_4 are subset of the language L where L is the language accepted by the given PDA?

112. Consider the following languages.

- $$\begin{aligned}L_1 &= \{a^i b^j c^k \mid i = j, j < k\} \\L_2 &= \{a^i b^j c^k \mid (i \leq j) \text{ or } (j \leq i), j = k\} \\L_3 &= \{a^m b^n c^n d^m \mid m \neq n\} \\L_4 &= \{a^i b^j c^k \mid \text{if } (i = j) \text{ then } k \text{ is even}\}\end{aligned}$$

How many of the above languages L_1, L_2, L_3 and L_4 are context free languages?

113. Consider the following CFG.

- $$\begin{aligned}S &\rightarrow aAbBC \mid abB \\A &\rightarrow bBd \mid \epsilon \\B &\rightarrow eBf \mid g \\C &\rightarrow f\end{aligned}$$

How many derivative steps required to derive the string “abegff” using the above grammar when start symbol ‘S’ is available?

114. Consider the following CFG:

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow BC \\ B &\rightarrow bAd \end{aligned}$$

What is the rank of a non-terminal ‘B’?

115. Consider the following CFG.

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$$

For the above CFG, find the total number of strings generated whose length is less than or equal to 10. [Exclude the empty string]

116. Consider the following CFG. Let G be the start symbol.

$$\begin{aligned} G &\rightarrow ATE \mid a \mid \epsilon \\ A &\rightarrow bT \mid b \mid a \mid \epsilon \\ T &\rightarrow aE \mid b \mid a \mid \epsilon \\ E &\rightarrow a \mid \epsilon \end{aligned}$$

Find the number of parse trees which can generate ‘a’.

117. What is the maximum possible string length that can be generated from the following CFG.

$$\begin{aligned} S &\rightarrow aAb \mid aBd \\ A &\rightarrow ab \mid Bd \mid e \\ B &\rightarrow abe \mid d \mid f \end{aligned}$$

118. Consider a language L which is generated by the following CFG.

$$\begin{aligned} S &\rightarrow aSa \mid B \\ B &\rightarrow \epsilon \mid bBa \end{aligned}$$

Find the number of strings in L such that the string length is less than 10.

119. Consider the following CFG G.

$$\begin{aligned} S &\rightarrow aA \mid bB \mid AB \mid \epsilon \\ A &\rightarrow aAb \mid \epsilon \\ B &\rightarrow bBa \mid \epsilon \end{aligned}$$

$L(G)$ is a language generated by G. If $L_1 = \{w \mid w \in L(G), |w| \leq 3\}$, how many strings are in L_1 ?

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Multiple Select Questions

120. Consider the following grammar G with terminals $\{[,]\}$ start symbol S and non-terminals $\{A, B, S\}$

$$\begin{aligned} S &\rightarrow AS \mid SS \mid AB \\ S &\rightarrow SB \\ A &\rightarrow [\\ B &\rightarrow] \end{aligned}$$

A language L is called prefix-closed if for every $x \in L$, every prefix of x is also in L. Which of the following true?

- (a) $L(G)$ is prefix closed
- (b) $L(G)$ is context free
- (c) $L(G)$ can be recognized by a deterministic push down automata
- (d) $L(G)$ is recursive

121. Let L be a given context free language over the alphabet $\{a, b\}$, construct L_1, L_2 as follows. Let $L_1 = L - \{xy \mid x, y \in \{a, b\}^*\}$ and $L_2 = L \cdot L$

Then

- (a) L_1 is regular
- (b) L_2 is regular
- (c) L_1 is CFL
- (d) L_2 is CFL

122. Consider the following languages over the alphabet $\{0, 1\}$

$$\begin{aligned} L_1 &= \{w.w \mid w \in \{0, 1\}^*\} \\ L_2 &= \{w.w^R \mid w \in \{0, 1\}^*\} \end{aligned}$$

Where w^R is the reverse of string w, then which of the following is true?

- (a) L_1 is CFL
- (b) L_1 is CSL but not CFL
- (c) L_2 is CFL
- (d) L_2 is CSL

123. Over the alphabet $\{0, 1\}$ consider the language $L = \{w \mid w \text{ does not contain the substring } 0011\}$. Which of the following is false about L?

- (a) L is not context free.
- (b) L is regular.

- (c) L is regular and it is context free.
 (d) L is context free but not recursively enumerable.

- 124.** Which of the following problems are undecidable?
 (a) Given, a CFG G , find whether $L(G) = R$, where R is regular set.
 (b) Given a CFG G , find whether $L(G) = \{\}$.
 (c) Find whether the intersection of two CFL's is empty.
 (d) Find whether CFG G_1 and CFG G_2 generate the same language i.e. $L(G_1) = L(G_2)$.

- 125.** Consider the language $L \subseteq \{a, b, c\}^*$ defined as $L = \{a^p b^q c^r \mid p = q \text{ or } q = r \text{ or } r = p\}$

Which of the following is true about the type of language L ?

- (a) L is regular.
 (b) L is context free.
 (c) L is decidable.
 (d) The complement of L , defined as

$$\bar{L} = \{a, b, c\}^*/L \text{ is regular.}$$

- 126.** For a string w , we define w^R to be the reverse of w . For example, if $w = 01101$ then $w^R = 10110$.

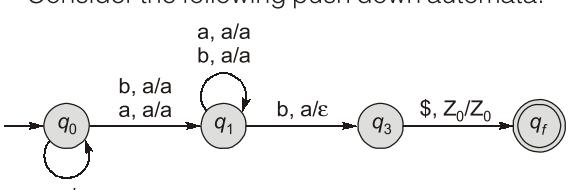
Which of the following languages is/are context-free?
 (a) $\{wxw^RxR \mid w, x \in \{0, 1\}^*\}$
 (b) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$
 (c) $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$
 (d) $\{wxw^Rw^R \mid w, x \in \{0, 1\}^*\}$

[GATE : 2021 (Set-2)]

- 127.** Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

- (a) $L_1 \cap \bar{L}_2$
 (b) $\bar{L}_1 \cup \bar{L}_2$
 (c) $L_1 \cup (L_2 \cup \bar{L}_2)$
 (d) $(L_1 \cap L_2) \cup (\bar{L}_1 \cap \bar{L}_2)$

[GATE : 2021 (Set-2)]

- 
Try Yourself
- T1.** Identify the language L which is not a CFL.
 (a) $L = \{a^m b^n c^k d^l \mid \text{if } (m = n) \text{ then } (k = l)\}$
 (b) $L = \{a^m b^n c^k d^l \mid \text{if } (n = k) \text{ then } (m = l)\}$
 (c) Both (a) and (b)
 (d) None of these
- [Ans: (d)]
- T2.** Consider the following languages L_1 and L_2
 $L_1 = \{a^m b^n \mid m, n \geq 0\}$
 $L_2 = \{a^m b^n \mid m = n\}$
- If $(L_1 \cap \bar{L}_2) = L$ then what is the language L ?
 (a) $L = \{a^n b^n\}$
 (b) $L = \{a^m b^n \mid m \neq n\}$
 (c) $L = (a + b)^* - \{a^n b^n\}$
 (d) $L = (a + b)^* - \{a^m b^n \mid m \neq n\}$
- [Ans: (b)]
- T3.** Which of the following CFG generates a regular language?
 (a) $S \rightarrow aAb$
 $A \rightarrow aB \mid \epsilon$
 $B \rightarrow Ab$
 (b) $S \rightarrow aABb$
 $A \rightarrow aA \mid \epsilon$
 $B \rightarrow bB \mid \epsilon$
 (c) Both (a) and (b)
 (d) Neither (a) nor (b)
- [Ans: (b)]
- T4.** Consider the following grammar:
 $S \rightarrow aSa \mid bSb \mid A$
 $A \rightarrow aBb$
 $B \rightarrow aB \mid bB \mid \epsilon$
- Identify the language generated by above CFG?
 (a) $L = \{ww^R \mid w \in (a + b)^*\}$
 (b) $L = \{xwabbw^R \mid w, x \in (a + b)^*\}$
 (c) $L = \{waxbw^R \mid w, x \in (a + b)^*\}$
 (d) None of these
- [Ans: (c)]
- T5.** Consider the following push down automata:
- 
- The language accepted by above PDA is _____.

- (a) Regular but infinite
- (b) DCFL but not regular
- (c) CFL but not DCFL
- (d) Finite language

[Ans: (a)]

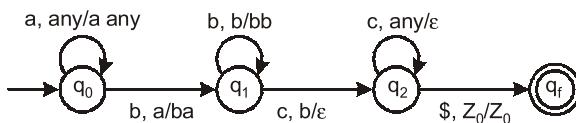
T6. Which of the following is CFL but not DCFL

- (a) $L = \{a^p \mid P \text{ is prime}\}$
- (b) $L = \{a^m b^n \mid (m < n) \text{ or } (m > n)\}$
- (c) $L = \{a^m b^n c^k \mid (m = n) \text{ or } (n = k)\}$
- (d) $L = \{a^m b^n c^k \mid m = n, n = k\}$

[Ans: (c)]

T7. Consider the following PDA:

$$PDA = (Q, \Sigma, \delta, \Gamma, q_0, Z_0, q_f)$$



Identify the language accepted by the above PDA?

- (a) $\{a^m b^n c^k \mid m = n = k, m, n, k \geq 1\}$
- (b) $\{a^m b^n c^k \mid m, n, k \geq 1, m = n + k\}$
- (c) $\{a^m b^n c^k \mid m, n, k \geq 1, m + n = k\}$
- (d) $\{a^m b^n c^k \mid m, n, k \geq 1\}$

[Ans: (c)]

T8. Let $L_1 = \{a^n b^m c^n \mid m, n \geq 0\}$ and $L_2 = \{a^n c^n \mid n \geq 0\}$. Both L_1 and L_2 are context free languages. If $L = (L_2 - L_1)$ then L is _____.

- (a) Finite language
- (b) Regular language
- (c) DCFL
- (d) Not DCFL

[Ans: (a)]

T9. DCFLs are not closed under _____.

- (a) Complement operation
- (b) Inverse homomorphism operation
- (c) Reversal operation
- (d) Prefix operation

[Ans: (c)]

T10. Find the grammar that generates inherently ambiguous context free language.

- | | |
|-----------------------------------|--|
| (a) $S \rightarrow AB \mid CD$ | (b) $S \rightarrow AB \mid CD$ |
| $A \rightarrow aAb \mid \epsilon$ | $A \rightarrow aAb \mid \epsilon$ |
| $B \rightarrow dB \mid \epsilon$ | $B \rightarrow bB \mid \epsilon$ |
| $C \rightarrow aC \mid \epsilon$ | $C \rightarrow aC \mid bC \mid \epsilon$ |
| $D \rightarrow bDd \mid \epsilon$ | $D \rightarrow bB \mid \epsilon$ |
| (c) Both (a) and (b) | (d) Neither (a) nor (b) |

[Ans: (a)]

T11. Find the language generated by the following grammar.

$$S \rightarrow aSa \mid aAa$$

$$A \rightarrow bA \mid b$$

$$(a) \{a^m b^n a^k \mid m, k > 0\}$$

$$(b) \{a^m b^n a^k \mid m = n, k > 0\}$$

$$(c) \{a^m b^n a^k \mid m = k, n > 0\}$$

$$(d) \{a^m b^n a^k \mid m, k, n > 0, m = k\}$$

[Ans: (d)]

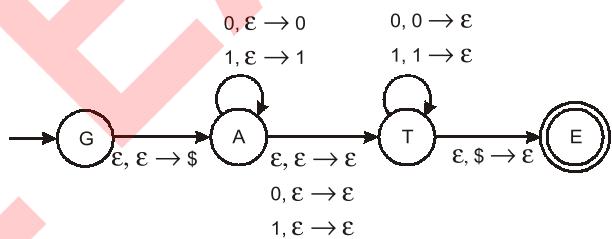
T12. Let $L = \{a^m b^n b^k d^\ell \mid \text{if}(n+k \text{ even}) \text{ then } m = \ell\}$.

Which of the following is true about L ?

- (a) L is CFL but not DCFL
- (b) L is regular but not CFL
- (c) L is DCFL but not regular
- (d) None of these

[Ans: (c)]

T13. Consider the following PDA:



The language accepted by above PDA is equivalent to the following grammar

- (a) $S \rightarrow 0S0 \mid 1S1 \mid \epsilon$
- (b) $S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1$
- (c) $S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$
- (d) None of these

[Ans: (c)]

T14. Consider the following CFG 'G':

$$S \rightarrow aA \mid bSS \mid SS$$

$$A \rightarrow aAb \mid bAa \mid AA \mid \epsilon$$

The language generated by G is _____.

- (a) Set of all strings with atleast one 'a'
- (b) Set of all strings with atleast two a's
- (c) Set of all strings with atleast one more 'a' than number of b's
- (d) None of these

[Ans: (c)]

- T15. Which of the following language is not a deterministic context free language?
- Set of all binary strings where every string not contain an equal number of zeros and one
 - Set of all binary strings where every string starts and ends with the same symbol, and have the same number of zeros as ones.
 - $\{10^n 1^n | n > 0\} \cup \{11^k 0^K 1^{2K} | k > 0\}$
 - None of these

[Ans: (d)]

- T16. Consider the following CFG with a start symbol S.

$$S \rightarrow AAaSb|\epsilon$$

$$A \rightarrow a|\epsilon$$

Which of the following language is generated by G?

- $\{a^m b^n | m \geq n \geq 0\}$
- $\{a^m b^n | 0 \leq n \leq m \leq 2n\}$
- $\{a^m b^n | 0 \leq n \leq m \leq 3n\}$
- None of these

[Ans: (c)]

- T17. Consider the following context-free grammars:

$$G_1 : S \rightarrow aS|B, B \rightarrow b|bB$$

$$G_2 : S \rightarrow aA|bB, A \rightarrow aA|B|\epsilon, B \rightarrow bB|\epsilon$$

Which one of the following pairs of languages is generated by G_1 and G_2 , respectively?

- $\{a^m b^n | m > 0 \text{ or } n > 0\}$ and $\{a^m b^n | m > 0 \text{ and } n > 0\}$
- $\{a^m b^n | m > 0 \text{ and } n > 0\}$ and $\{a^m b^n | m > 0 \text{ or } n \geq 0\}$
- $\{a^m b^n | m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n | m > 0 \text{ and } n > 0\}$
- $\{a^m b^n | m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n | m > 0 \text{ or } n > 0\}$

[GATE-2016, Ans: (d)]



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3

Recursive, Recursively Enumerable Languages and Turing Machines



Multiple Choice Questions

1. Recursively Enumerable Language are not closed under
 - (a) Complementation
 - (b) Union
 - (c) Intersection
 - (d) None of these
2. Choose the false statements.
 - (a) $L = \{a^n b^n a^n \mid n = 1, 2, 3 \dots\}$ is recursively enumerable
 - (b) Recursive languages are closed under union
 - (c) Every recursively enumerable language is recursive
 - (d) Recursive languages are closed under intersection
3. If there exists a language L , for which there exists a TM, T , that accepts every word in L and either rejects or loops for every word that is not in L , then L is said to be
 - (a) Recursive
 - (b) Recursively enumerable
 - (c) NP-HARD
 - (d) None of these
4. CSG can be recognized by a
 - (a) FSM
 - (b) DPDM
 - (c) NDPDM
 - (d) linearly bounded automata
5. Let A and B be countably infinite sets. Which of the following is false?
 - (a) Any subset of A or B is countably infinite
 - (b) $A \cup B$ and $A \times B$ are countably infinite
 - (c) The union of countably infinite collection of countably infinite sets is countably infinite

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6. Which of the following is countable?
 1. Number of hairs on a person's head.
 2. Points in a line.
 3. $\{2^I \mid I \text{ is the set of integers}\}$.
 4. The set of all languages on some alphabet Σ .
 5. The set of real numbers.
 6. Σ^* , where Σ is some alphabet.
 - (a) 1, 2 and 6
 - (b) 1, 3 and 6
 - (c) 2 and 5
 - (d) All of them
7. Which of the following is decidable?
 - (a) for some input if an arbitrary TM makes 5 moves
 - (b) whether an arbitrary TM halts within 5 steps
 - (c) whether an arbitrary TM prints some non blank character
 - (d) the set of codes for TM's that never make a left move
 - (e) All of the above
8. Which of the following is false?
 - (a) PC problem is decidable for a alphabet consisting of a single symbol
 - (b) Existence of MPC solution \Rightarrow Existence of PC solution
 - (c) Existence of PC solution \Rightarrow Existence of MPC solution
 - (d) Both MPC and PC problems are undecidable
9. Consider the following two languages:

$$L_d = \{(W_M, w) \mid w \notin L(M)\}$$

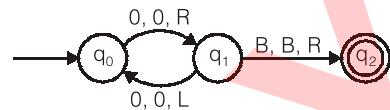
$$L_u = \{(W_M, w) \mid w \in L(M)\}$$

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- (a) $\{0^{2n}; n \geq 0\}$ (b) $\{0^{2n+1}; n \geq 0\}$
 (c) $\{0^n; n \geq 0\}$ (d) loops on all words

Linked Answer Q.16 & Q.17

16. The TM given below, hangs (loops) on which strings, if input alphabet is $\Sigma = \{0\}$?



- (a) $\{0^{2n+1}; n \geq 0\}$ (b) $\{0^{2n}; n \geq 1\}$
 (c) $\{0^n; n \geq 2\}$ (d) $\{0^n; n \geq 0\}$

17. What is the language accepted by above TM?

 - (a) {00}
 - (b) {0}
 - (c) \emptyset
 - (d) $\{0^n : n \geq 0\}$

- 18.** Which of the following is true?

 - (a) There are some regular languages for which no TM exists which accept it
 - (b) All languages accepted by TM's are infinite
 - (c) Languages which are not recursive are not accepted by any TM
 - (d) There are some languages which exist for which no TM exists which accept it

19. A TM which is allowed to move in one direction only and whose write capability is removed has same power as

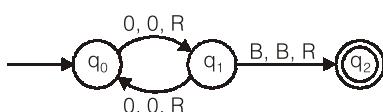
 - (a) a standard TM (b) an LBA
 - (c) a PDA (d) an FA

20. Which of the following is false?

 - (a) A regular language can be accepted by an LBA
 - (b) A DCFL can be accepted by an NPDA
 - (c) A recursive language can be accepted by a PDA
 - (d) Any recursive language can be accepted by some PDA

21. Recursive languages are

 - (a) a proper superset of context free languages
 - (b) always recognized by pushdown automata
 - (c) also called type 0 languages
 - (d) recognizable by Turing machines



22. It is undecidable whether
- an arbitrary Turing machine halts within 100 steps
 - a Turing machine prints a specific letter
 - a Turing machine computes the products of two numbers
 - None of the above

[GATE-1990]

23. Which of the following conversions is not possible (algorithmically)?
- Regular grammar to context-free grammar
 - Non-deterministic FSA to deterministic FSA
 - Non-deterministic PDA to deterministic PDA
 - Non-deterministic Turing machine to deterministic Turing machine

[GATE-1994]

24. Which of the following features cannot be captured by context-free grammars?
- Syntax of if-then-else statements
 - Syntax of recursive procedures
 - Whether a variable has been declared before its use
 - Variable names of arbitrary length

[GATE-1994]

25. Which of the following statements is false?
- The Halting problem of Turing machines is undecidable
 - Determining whether a context-free grammar is ambiguous is undecidable
 - Given two arbitrary context-free grammars G_1 and G_2 , it is undecidable whether $L(G_1) = L(G_2)$
 - Given two regular grammars G_1 and G_2 , it is undecidable whether $L(G_1) = L(G_2)$

[GATE-1996]

26. Which one of the following is not decidable?
- Given a Turing Machine M, a string s and an integer k, M accepts s within k steps
 - Equivalence of two given Turing machines
 - Language accepted by a given finite state machine is not empty
 - Language generated by a context free grammar is non empty

[GATE-1997]

27. Consider the following decision problems:

- Does a given finite state machine accept a given string.
- Does a given context free grammar generate an infinite number of strings.

Which of the following statements is true?

- Both 1 and 2 are decidable
- Neither 1 nor 2 are decidable
- Only 1 is decidable
- Only 2 is decidable

[GATE-2000]

28. Consider the following languages:

- $$\begin{aligned}L_1 &= \{ww \mid w \in \{a, b\}^*\} \\L_2 &= \{ww^R \mid w \in \{a, b\}^*, w^R \text{ is the reverse of } w\} \\L_3 &= \{0^{2i} \mid i \text{ is an integer}\} \\L_4 &= \{0^{i^2} \mid i \text{ is an integer}\}\end{aligned}$$

Which of the languages are regular?

- Only L_1 and L_2
- Only L_2 , L_3 and L_4
- Only L_3 and L_4
- Only L_3

[GATE-2001]

29. Consider the following problem X:

Given a Turing machine M over the input alphabet Σ , any state q of M and a word $w \in \Sigma^*$, does the computation of M on w visit the state q ?

Which of the following statements about X is correct?

- X is decidable
- X is undecidable but partially decidable
- X is undecidable and not even partially decidable
- X is not a decision problem

[GATE-2001]

30. Which of the following is true?

- The complement of a recursive language is recursive
- The complement of a recursively enumerable language is recursively enumerable
- The complement of a recursive language is either recursive or recursively enumerable
- The complement of a context free language is context free

[GATE-2002]

31. The C language is
 (a) A context free language
 (b) A context sensitive language
 (c) A regular language
 (d) Parsable fully only by a TM

[GATE-2002]

32. A single tape Turing Machine M has two states q_0 and q_1 , of which q_0 is the starting state. The tape alphabet of M is $\{0, 1, B\}$ and its input alphabet is $\{0, 1\}$. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table:

	0	1	B
q_0	$q_1, 1, R$	$q_1, 1, R$	Halt
q_1	$q_1, 1, R$	$q_0, 1, L$	q_0, B, L

The table is interpreted as illustrated below. The entry $(q_1, 1, R)$ in row q_0 and column 1 signifies that if M is in state q_0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q_1 . Which of the following statements is true about M?
 (a) M does not halt on any string in $(0 + 1)^+$
 (b) M does not halt on any string in $(00 + 1)^*$
 (c) M halts on all string ending in a 0
 (d) M halts on all string ending in a 1

[GATE-2003]

33. Consider three decision problems P_1 , P_2 and P_3 . It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?
 (a) P_3 is decidable if P_1 is reducible to P_3
 (b) P_3 is undecidable if P_3 is reducible to P_2
 (c) P_3 is undecidable if P_2 is reducible to P_3
 (d) P_3 is decidable if P_3 is reducible to P_2 's complement

[GATE-2005]

34. Let L_1 be regular language, L_2 be a deterministic context-free language and L_3 a recursively enumerable, but not recursive, language. Which one of the following statements is false?

- (a) $L_1 \cap L_2$ is a deterministic CFL
 (b) $L_3 \cap L_1$ is recursive
 (c) $L_1 \cup L_2$ is context free
 (d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

[GATE-2006]

35. Let $\langle M \rangle$ be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let $L = \{\langle M \rangle \mid M \text{ is a Turing machine that accepts a string of length } 2014\}$. Then, L is
 (a) decidable and recursively enumerable
 (b) undecidable but recursively enumerable
 (c) undecidable and not recursively enumerable
 (d) decidable but not recursively enumerable

[GATE-2014 (Set-2)]

36. Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B . Which one of the following is FALSE?
 (a) If $A \leq_m B$ and B is recursive then A is recursive.
 (b) If $A \leq_m B$ and A is undecidable then B is undecidable.
 (c) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.
 (d) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

[GATE-2014 (Set-2)]

37. Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^* . Which one of the following is TRUE?
 (a) Both 2^{Σ^*} and Σ^* are countable
 (b) 2^{Σ^*} is countable and Σ^* is uncountable
 (c) 2^{Σ^*} is uncountable and Σ^* is countable
 (d) Both 2^{Σ^*} and Σ^* are uncountable

[GATE-2014 (Set-3)]

38. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that \bar{Y} reduces to W , and Z reduces to \bar{X}

(reduction means the standard many-one reduction). Which one of the following statements is TRUE?

- (a) W can be recursively enumerable and Z is recursive.
- (b) W can be recursive and Z is recursively enumerable.
- (c) W is not recursively enumerable and Z is recursive.
- (d) W is not recursively enumerable and Z is not recursive.

[GATE-2016 (Set-1)]

39. Let $L(R)$ be the language represented by regular expression R . Let $L(G)$ be the language generated by a context free grammar G . Let $L(M)$ be the language accepted by a Turing machine M .

Which of the following decision problems are undecidable?

- I. Given a regular expression R and a string w , is $w \in L(R)$?
 - II. Given a context-free grammar G , is $L(G) = \emptyset$?
 - III. Given a context-free grammar G , is $L(G) = \Sigma^*$ for some alphabet Σ ?
 - IV. Given a Turing machine M and a string w , is $w \in L(M)$?
- (a) I and IV only (b) II and III only
 (c) II, III and IV only (d) III and IV only

[GATE-2017 (Set-2)]

40. Consider the following problems $L(G)$ denotes the language generated by a grammar G . $L(M)$ denotes the language accepted by a machine M .

- I. For an unrestricted grammar G and a string w , whether $w \in L(G)$.
- II. Given a Turing Machine M , whether $L(M)$ is regular.
- III. Given two grammars G_1 and G_2 , whether $L(G_1) = L(G_2)$.
- IV. Given an NFA N , whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

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- (a) Only I and II are undecidable
- (b) Only III is undecidable
- (c) Only II and IV are undecidable
- (d) Only I, II and III are undecidable

[GATE-2018]

41. Identify from the following the INCORRECT statement
- (a) Finite automata have no storage and no computing capability
 - (b) Mealy machines produce outputs corresponding to any input based on λ , a mapping from set of states to output alphabet
 - (c) Non-deterministic Turing Machine M semi-decides a language L if and only if for all $w \in L$, M accepts w
 - (d) The class of recursive language is a strict subset of the class of recursively enumerable languages

[JNU]

42. The next move function δ of a Turing Machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ is a mapping
- (a) $\delta : Q \times \Sigma \rightarrow Q \times \Gamma$
 - (b) $\delta : Q \times \Gamma \rightarrow Q \times \Sigma \times \{L, R\}$
 - (c) $\delta : Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$
 - (d) $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

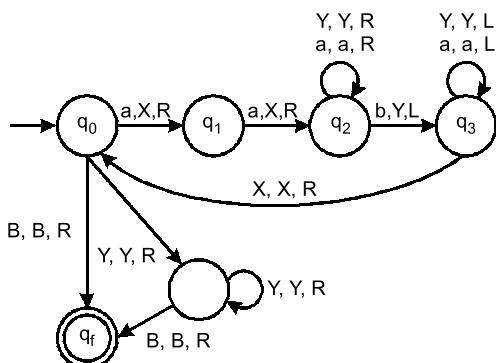
[JNU]

43. Which one of the following statements is false?
- (a) Any language $L(M)$, that is accepted by a Turing machine M is a recursively enumerable language
 - (b) The (pushdown) stack in a pushdown automata (PDA) is an auxiliary storage unit
 - (c) Let $L_1 = \{0^n 1^n \mid n \geq 0\}$ and $L_2 = \{(01 + 10)^*\}$. Then $L_1 \cap L_2$ is context-free language
 - (d) If L is a regular language, then L^R is not regular language but is a context-free language

[JNU]

44. Let L_1 be a recursive language and L_2 be a recursive enumerable language. Then $L_2 - L_1$ is
- (a) Recursive language
 - (b) Recursive enumerable language
 - (c) Non-recursive enumerable language
 - (d) None of these

45. Consider the following Turing machine.



Identify the language accepted by the above TM:

- (a) $\{a^m b^n \mid m > n\}$ (b) $\{a^m b^n \mid m = 2n\}$
 (c) $\{a^m b^n \mid n = 2m\}$ (d) $\{a^m b^n \mid m \geq n\}$

46. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that \bar{Y} reduces to W , and Z reduces to \bar{X} (reduction means the standard many-one reduction). Which one of the following statements is TRUE?
 (a) W can be recursively enumerable and Z is recursive.
 (b) W can be recursive and Z is recursively enumerable.
 (c) W is not recursively enumerable and Z is recursive.
 (d) W is not recursively enumerable and Z is not recursive.

[GATE-2016]

47. Let A and B be finite alphabets and let $\#$ be a symbol outside both A and B . Let f be a total function from A^* to B^* . We say f is computable if there exists a Turing machine M which given an input x in A^* , always halts with $f(x)$ on its tape. Let L_f denote the language $\{x \# f(x) \mid x \in A^*\}$. Which of the following statements is true:
 (a) f is computable if and only if L_f is recursive.
 (b) f is computable if and only if L_f is recursively enumerable.
 (c) If f is computable then L_f is recursive, but not conversely.

- (d) If f is computable then L_f is recursively enumerable, but not conversely.

[GATE-2017]

48. Consider the following sets:
 S_1 : Set of all recursively enumerable languages over the alphabet $\{0, 1\}$.
 S_2 : Set of all syntactically valid C programs.
 S_3 : Set of all languages over the alphabet $\{0, 1\}$.
 S_4 : Set of all non-regular languages over the alphabet $\{0, 1\}$.

Which of the above sets are uncountable?

- (a) S_1 and S_2 (b) S_3 and S_4
 (c) S_1 and S_4 (d) S_2 and S_3

[GATE-2019]



Multiple Select Questions

49. Which of the following is true?
 (a) A language recognized by a non-deterministic Turing machine can also be recognized by a deterministic Turing machine.
 (b) Complement of a context free language can be recognized by a Turing machine.
 (c) If a language and its complement are both recursively enumerable then it is recursive.
 (d) Complement of a non-recursive language can never be recognized by any Turing machine.
50. Which of the following statements are false?
 (a) The set of Turing machines which do not halt on empty input forms a recursively enumerable set.
 (b) The complement of a context free language is context free.
 (c) The complement of every recursively enumerable language is recursively enumerable.
 (d) Every Turing machine recognizable language is recursive.

51. Which of the following are true?
- All recursive sets are recursively enumerable.
 - The complement of every Not-RE language is Not-RE.
 - All finite sets are recursive.
 - The complement of every recursive set is recursive.



Try Yourself

- T1. Recursively enumerable language (problem) is _____.
- Computable
 - Turing recognizable
 - Turing enumerable
 - All of these

[Ans: (d)]

- T2. Consider the following unrestricted grammar.

$$\begin{aligned} S &\rightarrow A1B \\ A &\rightarrow AA \\ A1 &\rightarrow 11B \\ B &\rightarrow \epsilon \end{aligned}$$

What is the language generated by above grammar?

- $L = \{1^n \mid n > 0\}$
- $L = \{1^{2n} \mid n > 0\}$
- $L = \{1^{2^n} \mid n > 0\}$
- None of these

[Ans: (d)]

- T3. Which of the following language is a recursive language?

- $\{< M > \mid M \text{ is a TM and there exist an input whose length is less than } 100, \text{ on which } M \text{ halts}\}$
- $\{< M > \mid M \text{ is a TM and } L(M) = \{00, 11\}\}$
- $\{M_1, M_2, M_3 \mid L(M_1) = L(M_2) \cup L(M_3)\}$
- None of these

[Ans: (d)]

- T4. Which of the following is decidable. Assume given a Turing machine M , a state q , a symbol ' x ' and a string ' w '.

- Whether M ever reaches state q when started with input w from its initial state.
- Whether M ever writes the symbol ' x ' when started with an empty tape.
- Whether M ever moves its head to the left when started with input w .
- Whether the language accepted by M is finite.
- None of the above

[Ans: (e)]

- T5. Let $L = \{< M > \mid M \text{ accepts some string}\}$ where M is a Turing machine. Find the language L ?

- Turing recognizable
- Turing decidable
- Both (a) and (b)
- Neither (a) nor (b)

[Ans: (a)]

- T6. The languages generated by a grammar is

- Recursively enumerable languages
- Turing recognizable languages
- Partially decidable languages
- All of these

[Ans: (d)]

- T7. Which of the following is decidable?

- $L = \{< M > \mid \text{Turing machine 'M' accepts } L(M) \subseteq \{aa, bb\}\}$
- $L = \{< M > \mid \text{Turing machine 'M' accepts } L(M) = \{aa, bb\}\}$
- $L = \{< M, q > \mid \text{Turing machine 'M' visits state } q \text{ on some input}\}$
- $L = \{< M, q > \mid \text{Turing machine 'M' visits state } q \text{ on some input within } 10 \text{ steps}\}$

[Ans: (d)]

- T8. Let G be a CFG and for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w . Then find G ?

- CNF
- GNF
- Both (a) and (b)
- Neither (a) nor (b)

[Ans: (a)]

[Ans: (d)]

Linked Answer for T10 and T11

Let L_1 is reducible to L_2 and L_2 is reducible to L_3 .

- T10.** If L_3 is decidable then which of the following statement is correct?

 - (a) L_1 is decidable but L_2 is undecidable
 - (b) L_2 is decidable but L_1 is undecidable
 - (c) Both L_1 and L_2 are decidable
 - (d) None of these

[Ans: (c)]

- T11.** Which of the following can be valid using the above languages L_1 and L_2 ?

 - (a) $L_1 \cap L_2$ is decidable
 - (b) Complement of L_1 is undecidable
 - (c) Homomorphism of L_1 is decidable
 - (d) None of these

[Ans: (a)]

100

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