Graph	$\chi(G)$	Matching Number	Line Covering Number	Vertix Covering Number	Independent set number	$\lambda(G)$	$K(G)$	Circuit Rank $(E-V+1)$	Total no of Edges
K_n	n	$\lfloor \frac{n}{2} \rfloor$	$\lceil \frac{n}{2} \rceil$	$n-1$	1	$n-1$	$n-1$	$\frac{n(n-1)}{2}-n+1$	$\frac{n(n-1)}{2}$
$K_{m,n}$	2	$\min(m,n)$	$\max(m,n)$	$\min(m,n)$	$\max(m,n)$	$\max\{m,n\}$	$\min\{m,n\}$	$mn-(m+n)+1$	mn
$C_n (n \geq 3)$	$n-2\lfloor \frac{n}{2} \rfloor+2$	$\lfloor \frac{n}{2} \rfloor$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lfloor \frac{n}{2} \rfloor$	2	2	1	n
$W_n (n \geq 4)$	$n-2\lceil \frac{n}{2} \rceil+4$	$\lfloor \frac{n}{2} \rfloor$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n+1}{2} \rceil$	$\lfloor \frac{n-1}{2} \rfloor$	3	3	$n-1$	$2(n-1)$
Star graph	2	1	$n-1$	1	$n-1$	1	1	0	$n-1$

Discrete Mathematics

Graph Theory Short Notes

Made Easy GATE, IES

Basics

i) Graph, Nullgraph, Trivial, Paralleledges, Multigraph, Simple graph, Regular, Complete, Cyclegraph, wheel graph, Bipartite, Complete Bipartite

ii) $\sum_{i=1}^{n} deg(v_i) = 2|E|$

iii) $\delta(G) \le \frac{2|E|}{|V|} \le \Delta(G)$

iv) $|E(G)| + |E(\bar{G})| = |E(K_n)|$

v) Havel-Hakimi Result

i) S $t_1, t_2 \cdots t_s, d_1, d_2 \cdots d_n$

ii) $t_1-1, t_2-1, \ldots t_s-1, d_1, d_2 \cdots d_n$

Sequence (i) is graphic iff Sequence (ii) is graphic

vi) Planar graph

→ $\sum_{i=1}^{n} deg(R_i) = 2|E|$

Where $R_i = i^{th}$ Region

→ Eulers Formula

$|V| + |R| = |E| + 2$

(This is for Connected planar)

Suppose if graph G is planar with 'K' Connected components then

$|V| + |R| = |E| + (K+1)$

vii) Kurtowskis Theorem

A graph 'G' is not planar iff 'G' Contains a Subgraph homeomorphic to $K_{3,3}$ (or) K_5

viii) Isomorphism

$G \cong G'$ if there is a function $f: V(G) \to V(G')$ such that

i) f is a bijection and

ii) for each pair of vertices u and v of G, $\{u,v\} \in E(G)$ iff $\{f(u), f(v)\} \in E(G')$

Note 1: Two simple graphs are Isomorphic iff their Complements are Isomorphic with each other

Note 2: If $G \cong G'$ then following Conditions must hold

i) $|V(G)| = |V(G')|$

ii) $|E(G)| = |E(G')|$

iii) degree seq in G = deg seq in G'

iv) Circuit length in G = Circuit length in G'

Chromatic Number

i) $\chi(G)$

ii) Four color Theorem
Every planar graph is four colorable

iii) Welch-Powel's algorithm

Matching

→ $deg(v) \le 1$ $\forall v \in G$ in M

→ maximal, maximum

→ perfect matching: $deg(v) = 1$ $\forall v \in G$, in M

→ No of perfect matching in $K_{2n} = \frac{2n!}{n!\,2^n}$

→ $K_{m,n}$ has perfect matching iff $m = n$

→ No of perfect matching in $C_n = 2$
in $W_n = n-1$

→ In $K_{m,n}$ perfect matching exist iff $m = n$

Line Covering: $C \subseteq E$, $deg(v) \ge 1$ $\forall v \in G$ in G
then 'C' is called line Covering

→ A line Covering exists iff G has no isolated vertix

→ A line covering of 'n' vertices graph has atleast $\lceil \frac{n}{2} \rceil$ edges

*** $\boxed{\text{Matching Number + Line Covering Number = Total no of vertices}}$

Vertex Covering:- $K \subseteq V$, if every edge of 'G' incident with a vertix in 'K', then 'K' is vertix covering

Independent set:- $S \subseteq V$, if no two vertices of 'S' are adjacent then 'S' is independent set.

$\boxed{\text{Vertix Covering Number + Independent Set No = Total no of vertices}}$

Connectivity

If there are 'K' Connected Components

$(n-K) \le E(G) \le \frac{(n-K)(n-K+1)}{2}$

• Cut vertix, Cut edge, properties

Hamiltonian, Eulerian

i) properties

ii) Dirac's Theorem

iii) Ore's Theorem.

Discrete Maths Short notes of Graph Theory

• Made Easy

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(please send mail if anything found wrong)