Propositional Logic

Workbook Solutions.

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Q No: 01 : Ans (c)

A declarative sentence which we can assign truth value either TRUE (or) False is proposition

1+1 = 4 (FALSE) is its truth value

Q No: 02
Ans (C), (D)

Q No: 03
b, c, d

a) $\left(\underbrace{2+2=4}_{T} \wedge \underbrace{x^2 \text{ is negative for a rea } x}_{F}\right) = F$

b) $\left(\underbrace{(2+2=5)}_{F} \rightarrow \underbrace{2x \text{ is even}}_{T}\right) = T$

c) $\underbrace{(x \text{ is divisible by } 2)}_{P}$ or $(x \underbrace{\text{ is not divisible by } 2}_{\sim P})$

$\therefore P \vee \sim P \cong T$

d) $\underbrace{(2x \text{ divisible by } 2)}_{T} \longleftrightarrow \underbrace{(3x \text{ is not divisible by } 2)}_{T}$

$\longleftrightarrow$

$\therefore T \longleftrightarrow T = T$

Q No: 04
Ans (b)

x is even iff x is divisible by 2

$\cong$ (x is even $\rightarrow$ x is divisible by 2) and (x is divisible by 2 $\rightarrow$ x is even)

$\cong$ (x is not even $\vee$ x is divisible by 2) and (x is not divisible by 2 $\vee$ x is even)

$[\because P \rightarrow q \cong \sim P \vee q]$

Now apply negation to the above statement then

we get (x is even $\wedge$ x is not divisible by 2) $\vee$ (x is divisible by 2 $\wedge$ x is not even)

QNo:05
Ans(d)

a) $p \leftrightarrow q \cong q \leftrightarrow p$ True

b) c) True by Contrapositive

$p \leftrightarrow q \cong \sim p \leftrightarrow \sim q$

$\equiv \sim q \leftrightarrow \sim p$

QNo:06
Ans (c)

Refer Discrete mathematics Theory book page no: 06.

$(\vee, \wedge)$ is not functionally Complete.

QNo:07
Ans (c)

Refer Theory book Page no: 06

QNo:08
Ans (b)

Refer Theory book page no: 06

QNo:09
Ans (a)

Refer Theory book page no:- 07

NAND is Commutative but not associative

QNo:10
Ans(d)

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\cong (\sim p \vee q) \wedge (\sim q \vee p)$$

$$\cong [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$$

$$\cong [(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \vee [(\sim p \wedge p) \vee (q \wedge p)]$$

$$\cong [p'.q' \vee F \vee F \vee q.p]$$

$$\cong p'.q' + q.p$$

$(\because \vee \cong + \quad \wedge \cong \cdot)$

$(\sim p \cong p' \quad \sim q \cong q')$

QNo: 11

Ans(a)

$p \to q \cong \sim p \vee q$

QNo:12

Ans (a)

$[p \uparrow q \cong \sim(p \wedge q)]$

a) $p \uparrow p \cong \sim p$ True — here $\uparrow$ is NAND

b) $(p \downarrow q) \downarrow (p \downarrow q) \equiv p \vee q$ True — here $\downarrow$ is NOR

c) $(p \uparrow p) \uparrow (q \uparrow q) \cong p \vee q$ True

d) $(p \uparrow q) \uparrow (p \uparrow q) \cong p \wedge q$ True

hence option (a) $p \uparrow q \cong \sim p$ is False.

QNo:13

Ans (c)

Given $p \to (q \leftrightarrow r)$

$\cong \sim p \vee [(q \to r) \wedge (r \to q)]$

$\cong \sim p \vee [(\sim q \vee r) \wedge (\sim r \vee q)]$

QNo:14

Ans (b)

$\sim, \wedge, \vee, \to, \leftrightarrow$ here $\sim$ has highest precedence

$\leftrightarrow$ has lowest precedence

QNo:15

Ans(b)

Duality law: Two formulas A and A' are said to be duals of each other, if either can be obtained from the other by replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$, 0 by 1 and 1 by 0

Ex:1 $(P \wedge Q) \vee T$ has dual is $(P \vee Q) \wedge F$

Ex 2 Dual of $A + 1 = 1$ is $A \cdot 0 = 0$

Q NO: 16 Ans (c)

dual of $P \vee T \equiv T$ is $P \wedge F \equiv F$

Q NO: 17 Ans (b)

dual of $\sim(\sim P) \equiv P$ is itself

Q NO: 18 Ans (d)

By using properties of dual

a) $A \Leftrightarrow B$ then $A^* \Leftrightarrow B^*$

b) $(A^*)^* = A$

c) $\sim A(P_1, P_2, \cdots P_n) \Leftrightarrow A^*(\sim P_1, \sim P_2, \cdots \sim P_n)$

By using demorgan laws.

but $A = \sim A^*$ is False

Q NO: 19 Ans (b)

Given that $A(P, Q, R) = \sim P \wedge \sim(Q \vee R)$

$$A^*(\sim P, \sim Q, \sim R) = \sim(\sim P) \vee \sim(\sim Q \wedge \sim R)$$
$$= P \vee \sim(\sim(Q \vee R))$$
$$= P \vee (Q \vee R)$$

Q NO: 20 Ans (a)

: Refer class notes

Q NO: 21 Ans (c)

Given $a \cong F$ and $b \leftrightarrow c$ is a Tautology

$$\left[\begin{array}{l} b \leftrightarrow c \cong T \\ (b \to c) \wedge (c \to b) \cong T \\ \therefore\ b = T,\ c = T \text{ or} \\ \quad b = F,\ c = F \end{array}\right]$$

Now $a \vee (b \wedge c)$

$\cong$ ~~$(a \vee b)$~~ .

$\cong (a \vee b) \wedge (a \vee c)$

$\cong (\sim a \to b) \wedge (\sim a \to c)$

$\equiv (F \to b) \wedge (F \to c)$ $(\because a \equiv F)$

$= T \wedge T$

$= T$

QNo: 22
Ans (a)

If p is a false then $p \wedge q \rightarrow r$

Case (i) If 'q' is also F then $p \wedge q \rightarrow r \cong F \rightarrow r = T$

Case (ii) If q is True then $p \wedge q \rightarrow r \cong F \rightarrow r = T$

QNo: 23
Ans (c)

Given $p \rightarrow q$ is True

p	q	$p \rightarrow q$
T	T	T
F	T	T
F	F	T

we know that $p \rightarrow q \cong \sim p \vee q$ True

$p \rightarrow q \cong \sim q \rightarrow \sim p$ Contrapositive

$p \rightarrow q \equiv \sim (p \wedge \sim q)$

$\equiv \sim p \vee q$ (using Demorgan)

QNo: 24
Ans (d)

$p \rightarrow q \cong \sim p \vee q$

QNo: 25
Ans (c)

If $p \rightarrow q$ is True and $q \rightarrow p$ is also True then $p \leftrightarrow q$ is True

$\therefore$ $p \leftrightarrow q$ is True only when both p and q are T
(or) both p and q are F

and $p \oplus q \equiv F$ (If both 'p', 'q' have same Truth values)

QNo: 26
Ans (b)

$(p \leftrightarrow q) \wedge (p \oplus q)$ is Contradiction.

p	q	$p \leftrightarrow q$	$p \oplus q$	$(p \leftrightarrow q) \wedge (p \oplus q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	T	F	F

QNo: 27

Ans (b)

$p \to q$ is false when $p(T)$ and $q(F)$

a) $q \to p$ is satisfiable $\left(\because \underset{(F)}{q} \to \underset{(T)}{p} \cong T\right)$

b) $q \to \sim p$ is unsatifiable $\left(\because \underset{F}{q} \to \underset{F}{\sim p} \cong T\right)$ is wrong.

c) $q \to (p \vee r)$ is satisfiable ($\because$ $q(F)$ so $F \to ?$ is TRUE)

d) $q \to (p \vee r)$ is tautology

($\because$ $q(T)$, $p(T)$ so $q \to (p \vee r)$ is T)

QNo: 28

Ans (c)

F_1: $p \vee (p \wedge q) \cong (p \vee p) \wedge (p \vee q) \cong p \wedge (p \vee q) \equiv F_2$

F_2: $p \wedge (p \vee q)$

dual of $F_1 = F_2$

F_1 and F_2 are not tautologies.

QNo: 29

Ans (d)

If $\underbrace{\text{Today is monday}}_{p}$, $\underbrace{\text{tomorrow is Tuesday}}_{q}$

$p \to q$ has ~~inverse~~ Converse: $q \to p$

[If Tomorrow is Tuesday then Today is monday

(or)

Today is Monday is sufficient for Tomorrow is ~~monda~~ Tuesday.]

Similarly $p \to q$ has inverse is $\sim p \to \sim q$

Note :- $a \to b$ Can also be called as

a only if b

a is sufficient Condition for b

b is necessary Condition for a

b if a

b follows from a

b provided a

b is Consequent of a

QNO: 30

Ans (c)

By using Converse of $P \rightarrow q$ is $q \rightarrow P$

QNO: 31

Ans (d)

"All $\overline{\text{Triangles}}^{T(x)}$ are $\overline{\text{Polygons}}^{P(x)}$"

$\forall x\,(T(x) \rightarrow P(x))$

Now $\sim [\forall x\,(T(x) \rightarrow P(x))]$

$\exists x\,[\sim (T(x) \rightarrow P(x))]$

$\exists x\,[\sim (\sim T(x) \vee P(x))]$

$\exists x\,[T(x) \wedge \sim P(x)]$

There are some Triangles which are not polygon.

QNO: 32

Ans (b)

$\sim [\forall x\, \forall y\;\; x+y=10]$

$\cong \;\; \exists x \exists y \;\; x+y \neq 0$

QNO: 33

Ans (a)

$\sim [\,x$ is prime iff x is not Composite$]$

$= \sim [\,P(x) \leftrightarrow C(x)]$ where $P(x) = x$ is prime
$C(x) = x$ is Composite

$= \sim [[P(x) \rightarrow C(x)] \wedge [C(x) \rightarrow P(x)]]$

$= [\sim [\sim P(x) \vee C(x)] \vee [\sim (\sim C(x) \vee P(x))]]$

$= [P(x) \wedge \sim C(x)] \vee [C(x) \wedge \sim P(x)]$

$\therefore$ x is prime and x is not Composite or x is not prime and x is Composite

QNo: 34
a, c, d

$P \to Q \equiv \sim P \vee Q$ and

$P \to Q \equiv \sim Q \to \sim P$ [Contrapositive]

$\therefore \; a = c = d$

QNo: 35
Ans(b)

If $F_1 \wedge F_2$ is not satisfiable (i.e Contradiction)
then $F_1 \wedge F_2 \longrightarrow F_3$ and
$F_1 \wedge F_2 \longrightarrow \sim F_3$ are Tautologies

QNo: 36
Ans(d)

AND, OR, EX-OR are associative and Commutative
NAND is Commutative but not Associative

QNo: 37
Ans(a)

The following are well known Results

1. $\forall x\,(P(x) \wedge Q(x)) \longleftrightarrow \forall x\,P(x) \wedge \forall x\,Q(x)$

2. $\exists x\,(P(x) \vee Q(x)) \longleftrightarrow \exists x\,P(x) \vee \exists x\,Q(x)$

3. $\forall x\,P(x) \vee \forall x\,Q(x) \longrightarrow \forall x\,(P(x) \vee Q(x))$

4. $\exists x\,(P(x) \wedge Q(x)) \longrightarrow \exists x\,P(x) \wedge \exists x\,Q(x)$

(I advise to ~~[illegible]~~ remember above '4' Rules by using examples which we Covered in the class)

QNo: 38
Ans(b)

Whenever $a \wedge b$ is True then $b \vee c$ also True

$\therefore \; (a \wedge b) \to (b \vee c)$ is Tautology

(please Refer Classnotes approach)

QNo: 39
Ans(b)

$$P \wedge (\sim P \vee q) \equiv (P \wedge \sim P) \vee (P \wedge q)$$
$$\equiv F \vee (P \wedge q)$$
$$\equiv P \wedge q$$

QNo: 40

Ans (d)

Given $\sim p \to q$ is True

i.e $p \vee q$ is True [$\because$ we know that $p \to q \equiv \sim p \vee q$
$\sim p \to q \cong p \vee q$]

$\Rightarrow$ either 'p' is T (or) q is T or both (T)

i.e {T T, T F, F T} are possible values

Now $\sim p \vee (p \to q) \equiv \sim p \vee (\sim p \vee q)$

$\equiv (\sim p \vee \sim p) \vee q$

$\cong \sim p \vee q$

Now if we use above possible values of p, q and if we Try to find $\sim p \vee q$ then we get following

p	q	$\sim p \vee q$
T	T	T
T	F	F
F	T	T

Some times True and some times False so we can not determined.

Note :- We can not use the word multiple-valued in proposition, because proposition is either True (or) False but not both.

QNo: 41

Refer class notes procedure

QNo: 42

Ans (a)

Given that $\underbrace{\text{I stay}}_{p}$ only if $\underbrace{\text{you go}}_{q}$

Converse of $p \to q$ is $q \to p$

We can Read $q \to p$ as : q only if p (or)
p if q

$\therefore$ I stay if you go

Refer class notes for $p \to q$

QNo: 43

Ans (a)

$a \leftrightarrow (b \vee \sim b)$

$\therefore$ $a \leftrightarrow$ True so a is True

and $b \leftrightarrow c$ holds i.e $b \equiv c$

Now $(a \wedge b) \rightarrow ((a \wedge c) \vee d)$

$b \rightarrow (c \vee d)$

$c \rightarrow (c \vee d)$ $(\because b \equiv c)$

$(\because a \equiv T)$ so $a \wedge b \equiv T \wedge b \equiv b$ and $a \wedge c \equiv T \wedge c = c$

This expression always True

QNo: 44

Ans (a)

F_1: $P \rightarrow \sim P$

F_2: $(P \rightarrow \sim P) \vee (\sim P \rightarrow P)$

Now F_1: $P \rightarrow \sim P$

: $\sim P \vee \sim P$ $(\because P \rightarrow Q \cong \sim P \vee Q)$

: T if P is F

F if P is T

$\therefore$ F_1 is Satisfiability

F_2: $(P \rightarrow \sim P) \vee (\sim P \rightarrow P)$

: $(\sim P \vee \sim P) \vee (P \vee P)$

: $\sim P \vee P$

: True for either P is T (or) F

$\therefore$ F_2 is Tautology so valid

QNo:45

Ans (a)

$(x \wedge \sim z) \rightarrow y$

QNo: 46

Ans (d)

$\forall x\,(\alpha \rightarrow \beta) \rightarrow ((\forall x)[\alpha] \rightarrow \forall x\,[\beta])$ is valid

QNo:47

Ans (b)

Derive clause $P \vee Q$ from clauses $P \vee R$, $Q \vee \sim R$

means that $(P \vee R) \wedge (Q \vee \sim R) \Rightarrow P \vee Q$

$\therefore$ (a) is True, Similarly (c) (d) also True (Refer class notes)

Since $x \Rightarrow y$ does not mean that $y \Rightarrow x$

So option (b) is FALSE

QNo: 48 : Refer class notes

Ans (c)

QNo: 49 Refer class notes

Ans (b)

QNo:50 Refer class notes

Ans (b)

QNo: 51
Ans(d)

By using Truth table approach
(OR)
You can solve this problem by using digital logic subject.

$A \odot B \cong \sim A \rightarrow \sim B \equiv \sim(\sim A) \vee \sim B$
$\equiv A \vee \sim B$
$\equiv$

A	B	$A \odot B$
True	True	True
True	False	True
False	True	False
False	False	True

QNo: 52
Ans(d)

Refer class notes

QNo: 53
Ans(a)

"For x which is an fsa there exist a 'y' which is a pda and which is equivalent to x"

QNo: 54
Ans(c)

Refer class notes approach.

QNo: 55
Ans(d)

Refer class notes for finding negation.

QNo: 56
Ans(b)

$\forall x \exists y \exists t \sim F(x, y, t)$

$= \sim\{\exists x \forall y \forall t \ F(x, y, t)\}$

= it is not true that (Some one can fool all people at all time)

= no one can fool everyone all the Time

Q No: 57 Refer class notes logic $

Ans (c)

Q No: 58 None of my friends are perfect

Ans (d)

i.e all of my friends are not perfect

$$\forall x \, (F(x) \rightarrow \sim P(x))$$

$$\forall x \, (\sim F(x) \vee \sim P(x))$$

$$\sim \exists x \, (F(x) \wedge P(x))$$

Q No: 59

Ans (d)

(a) All glitters are not gold

(b) All golds are glitter

(c) not all golds are glitters

d) Not all that glitter is gold

i.e There exist some glitters which is not gold

Q No: 60 : Apply Truth table approach.

Ans (b)

Q No: 61

Ans (d)

g: mobile is good

c: mobile is cheap

P: Good mobile phones are not cheap

$$g \rightarrow \sim c \equiv \sim g \vee \sim c$$

Q: Cheap mobile phones are not good

$$c \rightarrow \sim g \equiv \sim c \vee \sim g$$

both P and Q are equivalent

$\therefore$ L: $P \rightarrow Q$

M: $Q \rightarrow P$

N: $P \equiv Q$

Q No: 62

Ans (d)

Not all rainy day are cold

$$\sim (\forall d\, (Rainy(d) \rightarrow Cold(d)))$$

$$= \sim (\forall d\, (\sim Rainy(d) \vee Cold(d)))$$

$$= \exists d\, (Rainy(d) \wedge \sim Cold(d))$$

Q No: 63

Ans (a)

Given truth table is $P \oplus q$

and $\oplus$ is both Commutative and associative

Q No: 64

Ans (c)

C: person is corrupt

K: person is kind

E: person is Elected

S_1: $C \rightarrow \sim E$

S_2: $K \rightarrow E$

S_3: $\sim E \rightarrow K$

So from S_1 and S_2: $(C \rightarrow \sim E) \wedge (\sim E \rightarrow \sim K) \cong \sim C \rightarrow \sim K$

we can conclude $C \rightarrow \sim K$ which is same as $K \rightarrow \sim C$

Q No: 65

Ans (C)

Since $P \rightarrow R \cong \sim P \vee R$ and quantifiers on both sides are same $\forall x \, \exists x$

$\therefore$ option (c) is tautology

Q No: 66

Ans (d)

$\forall x$ is only one way distribution over '$\vee$'

(d): $\forall x\, (P(x) \vee q(x)) \Rightarrow \forall x\, P(x) \vee \forall x\, q(x)$

L.H.S is True if

$P_1 \vee q_1$, True

$P_2 \vee q_2$ True

⋮

Let us Take $P_1(T)$, $q_1(F)$ and $P_2(F)$, $q_2(T)$

Then L.H.S True but R.H.S False

QNo:67

Ans (b)

I) $\forall x \exists y\, R(x,y) \rightarrow \exists y \exists x\, R(x,y)$ is True

Since $\exists y \exists x\, R(x,y) \equiv \exists x \exists y\, R(x,y)$

II) $\forall x \exists y\, R(x,y) \rightarrow \exists y (\forall x\, R(x,y))$ is false

III $\forall x \exists y\, R(x,y) \rightarrow \forall y \exists x\, R(x,y)$ is False

IV $\forall x \exists y\, R(x,y) \rightarrow \sim(\exists x \forall y \sim R(x,y))$ is True

Since $\sim(\exists x \forall y \sim R(x,y)) \equiv \forall x \exists y\, R(x,y)$

QNo:68

Ans d

Given $(p \rightarrow q) \rightarrow r \equiv F$

$(\sim(\sim p \vee q) \vee r) \equiv F$ $\quad (\because p \rightarrow q \equiv \sim p \vee q)$

$(p \wedge \sim q) \vee r \equiv F$

This is possible only if $p \wedge \sim q \equiv F$ (or) $r \equiv F$

Now $p \wedge \sim q \equiv F$ iff $p \equiv F$ (or) $\sim q \equiv F$

hence $(p \equiv F$ (or) $\sim q \equiv F)$ and $r \equiv F$

Now $(r \rightarrow p) \rightarrow q \equiv \sim(\sim r \vee p) \vee q$

$\equiv (r \wedge \sim p) \vee q$

Since $r \equiv F$ hence $r \wedge \sim p \equiv F$

$\therefore (r \rightarrow p) \rightarrow q \equiv (r \wedge \sim p) \vee q \equiv F \vee q \equiv q$

So wheneverer q is True, $(r \rightarrow p) \rightarrow q$ is True

QNO: 69

Ans (a)

p: It is raining

q: It is cold

r: It is pleasant

Answer: $(\sim p \wedge r) \wedge (\sim r \text{ only if } p \wedge q)$

$$= (\sim p \wedge r) \wedge (\sim r \rightarrow (p \wedge q))$$

QNO: 70

Ans (d)

$$\sim p \rightarrow \sim q \equiv p \vee \sim q = \sim q \vee p$$

I. $p \rightarrow q \equiv \sim p \vee q$

II. $q \rightarrow p = \sim q \vee p$

III. $\sim q \vee p$

IV. $\sim p \vee q$

$\therefore$

QNO: 71

Ans (b)

The predicate ϕ simply says that if z is a prime number in the set then there exist another prime number is the set which is larger.

Clearly ϕ is True in S_2 and S_3

Since in set of all integers as well as all positive integers there is a prime number greater than any given prime number

however in $S1: \{1, 2, 3, \ldots 100\}$ ϕ is false since for prime number $97 \in S1$ there exist no prime number in the set which is greater.

I wish you All the best

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