III

Algorithms

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1 Loop Analysis, Asymptotic Notations, Recursive Algorithm and Methods to Solve Recurrence Relations

Multiple Choice Questions

Q.1 What does the following algorithm approximate? (Assume $m > 1, \in > 0$).

```
x = m;
y = 1;
while (x – y > ∈)
{
    x = (x + y) / 2;
    y = m / x;
}
print (x);
```

(a) $\log m$ (b) m^2
(c) $m^{1/2}$ (d) $m^{1/3}$

[GATE-2004]

Q.2 Consider the following C-program fragment in which i, j and n are integer variables.

```
for (i = n, j = 0; i > 0; i / = 2, j + = i);
```

Let Val (j) denote the value stored in the variable j after termination of the for loop. Which one of the following is true?

(a) val (j) = $\Theta(\log n)$ (b) val (j) = $\Theta(\sqrt{n})$
(c) val (j) = $\Theta(n)$ (d) val (j) = $\Theta(n \log n)$

[GATE-2006]

Q.3 Consider the following C code segment:

```
int isPrime (n)
{   int i, n;
    for (i = 2; i < = sqrt (n); i ++)
    {   if (n mod i == 0)
        {
        printf ("Not Prime")
        return 0;
        }
    }
    return 1;
}
```

Let $T(n)$ denote the number of times the for loop is executed by the program on input n. Which of the following is TRUE?

(a) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(\sqrt{n})$
(b) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(1)$
(c) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$
(d) None of the above

[GATE-2007]

Q.4 Consider the following function:

```
int unknown (int n)
{
  int i, j, k = 0;
  for (i = n/2; i <= n; i++)
    for (j = 2; j <= n; j = j*2)
      k = k + n/2;
  return (k);
}
```

The return value of the function is

(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$
(c) $\Theta(n^3)$ (d) $\Theta(n^3 \log n)$

[GATE-2013]

Q.5 Consider the following C function:

```
int fun1 (int n)
{   int i, j, k, p, q = 0;
    for (i = 1; i < n; ++i)
    {   p = 0;
        for (j = n; j > 1; j = j/2)
        ++p;
        for (k = 1; k < p; k = k*2)
        ++q;
    } return q;
}
```

Which one of the following most closely approximates the return value of the function fun1?

(a) n^3 (b) $n(\log n)^2$
(c) $n \log n$ (d) $n \log (\log n)$

[GATE-2015]

Q.6 Consider the following C function:

```
int fun {int n)
{       int i, j;
        for (i = 1; i <= n; i++)
        {
                for (j = 1; j < n; j + = i)
                {
                        printf{"%d %d", i, j);
                }
        }
}
```

Time complexity of fun in terms of Θ notation is

(a) $\Theta(n\sqrt{n})$ (b) $\Theta(n^2)$
(c) $\Theta(n\log n)$ (d) $\Theta(n^2 \log n)$

[GATE-2017]

Q.7 Consider the following functions:

$f(n) = 3n^{\sqrt{n}}$
$g(n) = 2^{\sqrt{n}\log_2 n}$
$h(n) = n!$

Which of the following is true?

(a) $h(n)$ is $O(f(n))$ (b) $h(n)$ is $O(g(n))$
(c) $g(n)$ is not $O(f(n))$ (d) $f(n)$ is $O(g(n))$

[GATE-2000]

Q.8 Let $f(n) = n^2 \log n$ and $g(n) = n(\log n)^{10}$ be two positive functions of n. Which of the following statements is correct?

(a) $f(n) = O(g(n)$ and $g(n) \neq O(f(n))$
(b) $g(n) = O(f(n)$ and $f(n) \neq O(g(n))$
(c) $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$
(d) $f(n) = O(g(n))$ and $g(n) = O(f(n))$

[GATE-2001]

Q.9 If $g = O(f)$ then find true statement from the following.

(a) $f = O(g)$ (b) $g = \Theta(f)$
(c) $f + g = \Theta(g)$ (d) $f + g = \Theta(f)$

Q.10 Consider the following three claim:

1. $(n + k)^m = \Theta(n^m)$ where k and m are constants
2. $2^{n+1} = O(2^n)$
3. $2^{2n+1} = O(2^n)$

Which of these claim are correct?

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

[GATE-2003]

Q.11 Arrange the following functions in increasing asymptotic order:

A. $n^{1/3}$ B. e^n
C. $n^{7/4}$ D. $n \log^9 n$
E. 1.0000001^n

(a) A, D, C, E, B (b) D, A, C, E, B
(c) A, C, D, E, B (d) A, C, D, B, E

[GATE-2008]

Q.12 Two alternative packages A and B are available for processing a database having 10^k records. Package A requires 0.0001 n^2 time units and package B requires $10n\log_{10}n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

(a) 12 (b) 10
(c) 6 (d) 5

[GATE-2010]

Q.13 Let $W(n)$ and $A(n)$ denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?

(a) $A(n) = \Omega(W(n))$ (b) $A(n) = \Theta(W(n))$
(c) $A(n) = O(W(n))$ (d) $A(n) = o(W(n))$

[GATE-2012]

Q.14 Let $f(n) = n$ and $g(n) = n^{(1+\sin n)}$, where n is a positive integer. Which of the following statements is/are correct?

I. $f(n) = O(g(n))$
II. $f(n) = \Omega(g(n))$

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

[GATE-2015]

Q.15 Write the following statements True or False

(a) If $f(n) = O(g(n))$ then $g(n) = O(f(n))$

(b) $f(n) + O(f(n)) = \Theta(f(n))$

(c) $f(n) + o(f(n)) = \Theta(f(n))$

(d) $(\log n)!$ and $(\log\log n)!$ are polynomially bounded

Q.16 Consider the following functions from positive integers to real numbers:

$$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$$

Tire CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

(a) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(b) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(c) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(d) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

[GATE-2017]

Q.17 Consider following function f

```
void f(int n)
{
    if (n ≤ 0) return;
        else
    {
        print (n);
        f(n – 2);
        print (n);
        f(n – 1);
    }
}
```

Let $R(n)$ be recurrence relation which computes sum of values printed by $f(n)$. Then $R(n)$ is

(a) $R(n - 1) + R(n - 2)$

(b) $R(n - 1) + R(n - 2) + n$

(c) $R(n - 1) + R(n - 2) + 2n$

(d) None of these

Common Data for Q.18 and Q.19

Consider the following C function:

```
double foo (int n)
{
        int i;
        double sum;
        if (n = = 0) return 1.0;
        else
        {
                sum = 0.0;
                for (i = 0; i < n ; i +)
                sum + = foo (i);
                return sum;
        }
}
```

Q.18 The space complexity of the above function is

(a) $O(1)$ (b) $O(n)$

(c) $O(n!)$ (d) $O(n^n)$

[GATE-2005]

Q.19 Suppose we modify the above function foo() and store the values of foo(i), $0 < = i < n$, as and when they are computed. With this modification, the time complexity for function foo() is significantly reduced. The space complexity of the modified function would be:

(a) $O(1)$ (b) $O(n)$

(c) $O(n^2)$ (d) $O(n!)$

[GATE-2005]

Common Data Questions Q.20 and Q.21

Consider the following C functions:

```
int f1(int n)
{
  if(n = = 0 || n = = 1) return n;
  else
     return (2 * f1(n – 1) + 3 * f1(n – 2));
}
int f2( int n)
{
  int i;
  int X[N], Y[N], Z[N];
  X[1] = 1; Y[1] = 2; Z[1] = 3;
  for(i = 2; i < = n; i++)
  {
```

```
        X[i] = Y[i – 1] + Z[i – 2];
        Y[i] = 2 * X[i];
        Z[i] = 3 * X[i];
    }
    return X[n];
}
```

Q.20 The running time of $f1(n)$ and $f2(n)$ are

(a) $\Theta(n)$ and $\Theta(n)$ (b) $\Theta(2^n)$ and $O(n)$
(c) $\Theta(n)$ and $\Theta(2^n)$ (d) $\Theta(2^n)$ and $\Theta(2^n)$

[GATE-2008]

Q.21 $f1(8)$ return the values

(a) 1661 (b) 59
(c) 1640 (d) 1640

[GATE-2008]

Q.22 In the following C function, let $n \geq m$.

```
int gcd (n, m)
{   if (n%m = = 0} return m;
    n = n%m;
    return gcd (m, n);
}
```

How many recursive calls are made by this function?

(a) $\Theta(\log_2 n)$ (b) $\Omega(n)$
(c) $\Theta(\log_2 \log_2 n)$ (d) $\Theta(\sqrt{n})$

[GATE-2007]

Q.23 Let $T(n)$ be the function defined by

$$T(n) = \begin{cases} T(1) \text{ for } n = 1 \\ 7T(n/2) + 18n^2 \text{ for } n \geq 2 \end{cases}$$

Now which of the following statement is true?

(a) $T(n) = \Theta(\log_2 n)$ (b) $T(n)\Theta(n^{\log_2 7})$
(c) $T(n) = O(n^{\log_2 n})$ (d) None of these

Q.24 If $T_1 = O(1)$, match **List-I** with **List-II** select the correct answer using the codes given below the Lists:

List-I	List-II
A. $T_n = T_{n-1} + n$	1. $T_n = O(n)$
B. $T_n = T_{n/2} + n$	2. $T_n = O(n^2)$
C. $T_n = T_{n/2} + n \log n$	3. $T_n = O(n \log n)$
D. $T_n = T_{n-1} + \log n$	4. $T_n = O(n^3)$

Codes:

	A	B	C	D
(a)	2	1	3	3
(b)	3	1	4	2
(c)	2	3	4	1
(d)	3	1	2	4

[GATE-1999]

Q.25 The recurrence equation:

$T(1) = 1$
$T(n) = 2T(n-1) + n, n \geq 2$

evaluates to

(a) $2^{n+1} - n - 2$ (b) $2^n - n$
(c) $2^{n+1} - 2n - 2$ (d) $2^n + n$

[GATE-2004]

Q.26 Consider the following program:

```
int Bar (int n)
{
    if (n < 2) return;
    else
    {
        int sum = 0;
        int i, j;
        for (i = 1; i < = 4; i++) Bar (n/2);
        for (i = 1; i < = n; j++)
        {
            for (j = 1; j < = i; j++)
            {
                sum = sum + 1;
            }
        }
    }
}
```

Now consider the following statements:

S_1 : The time complexity of Bar(n) is $\theta(n^2 \log(n))$.
S_2 : The time complexity of Bar(n) is $\Omega(n^2 \log(n^2))$.
S_3 : The time complexity of Bar(n) is $O(n^3 \log(n^2))$.

The number of correct assertions are _______.

(a) 0 (b) 1
(c) 2 (d) 3

Q.27 The recurrence relation capturing the optimal execution time of the Towers of Hanoi problem with n discs is

(a) $T(n) = 2T(n-2) + 2$
(b) $T(n) = 2T(n-1) + n$
(c) $T(n) = 2T(n/2) + 1$
(d) $T(n) = 2T(n-1) + 1$

[GATE-2012]

Q.28 What is the time complexity of the following recursive function:

```
int DoSomething ( int n)
{   if (n < = 2)
    return 1;
    else
    return (DoSomething (floor (sqrt (n))) + n);
}
```

(a) $\Theta(n^2)$ (b) $\Theta(n \log_2 n)$
(c) $\Theta(\log_2 n)$ (d) $\Theta(\log_2 \log_2 n)$

[GATE-2007]

Q.29 When $n = 2^{2k}$ for some $k \geq 0$, the recurrence relation $T(n) = \sqrt{2}\,T(n/2) + \sqrt{n}$, $T(1) = 1$ evaluates to:

(a) $\sqrt{n}(\log n + 1)$ (b) $\sqrt{n}\log n$
(c) $\sqrt{n}\log\sqrt{n}$ (d) $n\log\sqrt{n}$

[GATE-2008]

Q.30 Which one of the following correctly determines the solution of the recurrence relation with $T(1) = 1$?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

(a) $\Theta(n)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^2)$ (d) $\Theta(\log n)$

[GATE-2014]

Q.31 Find the running time of the following algorithm.
procedure $A(n)$
If $n < = 2$ return (1);
else return $\left(A\left(\left\lceil\sqrt{n}\right\rceil\right)\right)$;

(a) $O(n)$ (b) $O(\log n)$
(c) $O(\log \log n)$ (d) $O(1)$

[GATE-2002]

Q.32 If $T(n) = 3T(n/2) + n$, if $n > 1$. $T(1) = 1$. Then $T(n) = ?$

(a) $\Theta(n)$ (b) $\Theta(n^{\log_2 3})$
(c) $\Theta(n^{3/2})$ (d) $\Theta(n^{\log_2 3} \log_2 n)$

[DRDO-2008]

Q.33 Let $S_1 = \sum_{r=0}^{\log n - 1} \frac{nr}{2^r}$ and $S_2 = \sum_{r=0}^{\log n - 1} r2^r$. Which of the following is true?

(a) $S_1 = \Theta(n \log n)$, $S_2 = \Theta(n \log n)$
(b) $S_1 = \Theta(n)$, $S_2 = \Theta(n \log n)$
(c) $S_1 = \Theta(n \log n)$, $S_2 = \Theta(n)$
(d) $S_1 = \Theta(n)$, $S_2 = \Theta(n)$

[DRDO-2008]

Q.34 We have the following recurrence relation:

$$T(n) = \begin{cases} 1 & n \leq 5 \\ T(n/5) + T(3n/4) + n & n > 5 \end{cases}$$

Then which of the following statement is TRUE?

(a) $T(n) \in \Theta(n^2)$ (b) $T(n) \in \Omega(\sqrt{n})$
(c) $T(n) \in \Theta(n)$ (d) $T(n) \in \Theta(n \log n)$

[DRDO-2009]

Q.35 If $T_1 = O(1)$, give the correct matching for the following pairs:

(m) $T_n = T_{n-1} + n$ (u) $T_n = O(n)$
(n) $T_n = T_{\frac{n}{2}} + n$ (v) $T_n = O(n \log n)$
(o) $T_n = T_{n/2} + n \log n$ (w) $T_n = O(n^2)$
(p) $T_n = T_{n-1} + \log n$ (x) $T_n = O(\log^2 n)$

(a) m – w, n – v, o – u, p – x
(b) m – w, n – u, o – x, p – v
(c) m – v, n – w, o – x, p – u
(d) m – w, n – u, o – v, p – x

[GATE-1999]

Q.36 Let $T(n)$ be a function defined by the recurrence $T(n) = 2T(n/2) + \sqrt{n}$ for $n \geq 2$ and $T(1) = 1$. Which of the following statements is TRUE?

(a) $T(n) = \Theta(\log n)$ (b) $T(n) = \Theta(\sqrt{n})$
(c) $T(n) = \Theta(n)$ (d) $T(n) = \Theta(n \log n)$

[GATE-2005]

Q.37 Consider the following is true:

$$T(n) = 2T(\lceil\sqrt{n}\rceil) + 1,\ T(1) = 1$$

Which one of the following is true?

(a) $T(n) = \Theta(\log\log n)$ (b) $T(n) = \Theta(\log n)$

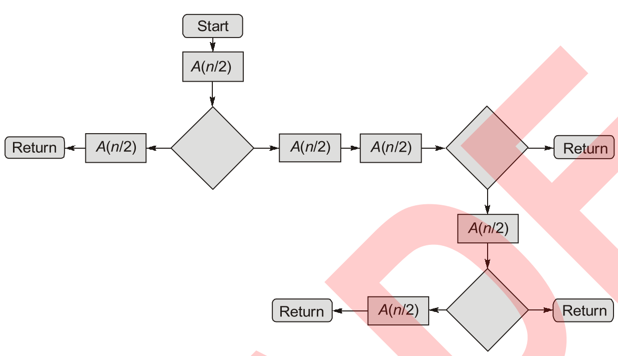
(c) $T(n) = \Theta(\sqrt{n})$ (d) $T(n) = \Theta(n)$

[GATE-2006]

Numerical Answer Type Questions

Q.38 The given diagram shows the flow chart for a recursive function $A(n)$. Assume that all statements, except for the recursive calls, have $O(1)$ time complexity. If the worst case time complexity of this function is $O(n^{\alpha})$, then the least possible value (accurate up to two decimal positions) of α is ______.

Flow chart for Recursive Function $A(n)$



[GATE-2016]

Multiple Select Questions

Q.39 Consider the given recursive algo

```
Algo rec(n)
{
    if (n ≤ 1)
        return(2)
    else
    {
        x = 2 * rec(n – 1)
        call f(x)
    }
}
f(x)
{
    for (i = 1; i ≤ x; i = i * 2)
        printf("MADE EASY");
}
```

Which of the following statements are true?

(a) Time complexity of rec(n) is $\theta(2^n)$.

(b) Space complexity of rec(n) is $\theta(n)$.

(c) Time complexity of rec(n) is $\theta(n^2)$.

(d) Time complexity of rec(n) is $\theta(n)$.

Q.40 Consider the following recursive algo:

```
Algo rec(n)
{
    if (n ≤ 2)
        return(1)
    else
        return(2 * rec(sqrt(n)) + 2 * rec(sqrt(n))
        + log2 n)
}
```

Which of the following statements is/are true?

(a) Time complexity of rec(n) is $\theta(\log n)$.

(b) Value returns by rec(n) is $\theta(\log^2 n)$.

(c) Number of fun calls of rec(n) is $\theta(\log n)$.

(d) Space complexity of rec(n) is $\theta(\log n)$.

Q.41 Consider given

$f_1 = \log n!$

$f_2 = (\log n)!$

$f_3 = (\log n)^{\log n}$

$f_4 = (\log n)^{\log\log n}$

$f_5 = (\log\log n)^{\log n}$

(a) $f_4\ f_1\ f_2\ f_5\ f_3$ (b) $f_4\ f_5\ f_1\ f_2\ f_3$

(c) $f_4\ f_1\ f_5\ f_2\ f_3$ (d) $f_1\ f_2\ f_4\ f_5\ f_3$

Q.42 Let $n = m!$ Which of the following is not true?

(a) $m = \theta(\log n/\log\log n)$

(b) $m = \Omega(\log n/\log\log n)$ but not $m = O(\log n/\log\log n)$

(c) $m = \theta(\log^2 n)$

(d) $m = \Omega(\log^2 n)$ but not $m = O(\log^2 n)$

Q.43 Which of the following are asymptotically correct for $T(n)$?

(a) $T_{n-1} + n = O(n^2)$

(b) $T_{n-1} + n = O(n \log n)$

(c) $T\left(\frac{n}{2}\right) + n \log n = O(n \log n)$

(d) $T\left(\frac{n}{2}\right) + n \log n = \theta(n^2)$

Try Yourself

T1. Let $f(n) = \Omega(n)$, $g(n) = O(n)$ and $h(n) = \Theta(n)$. Then $[f(n) \cdot g(n)] + h(n)$ is _______

(a) $\Omega(n)$ (b) $O(n)$

(c) $\Theta(n)$ (d) None of these

[Ans: (a)]

T2. $f_1 = 10^n$, $f_2 = n^{1/3}$, $f_3 = n^n$, $f_4 = \log n$, $f_5 = 2^{\log n}$. Arrange all these five functions in increasing order.

[Ans: $f_4 < f_2 < f_5 < f_1 < f_3$]

T3. Consider the following function:

```
find (int n)
{    if (n < 2) then return;
     else
     {    sum = 0;
          for (i = 1; i ≤ 4; i++) find(n/2);
          for (i = 1; i ≤ n*n; i++)
          sum = sum +1;
     }
}
```

Assume that the division operation takes constant time and "sum" is global variable. What is the time complexity of "find (n)"?

(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$

(c) $\Theta(n^3)$ (d) None of these

[Ans: (b)]

T4. Consider the following sorting algorithm.

```
Sorting (A, low, high)
{    if (low = high) return;
     if (low +1 = high)
     {    if (A[low] > A[high])
          Swap (A[low], A[high]);
          return;
     }
```

$$t_1 = \text{low} + \left(\frac{\text{high} - \text{low} + 1}{3}\right);$$

$$t_2 = \text{low} + 2 \cdot \left(\frac{\text{high} - \text{low} + 1}{3}\right);$$

```
     Sorting (A, low, t2);
     Sorting (A, t1, high);
     Sorting (A, low, t2);
}
```

What is the running time of Sorting(A, 1, n) function.

(a) $\Theta(n^{1.7})$ (b) $\Theta(n^{2.7})$

(c) $\Theta(n^{3.7})$ (d) $\Theta(n^{0.7})$

[Ans: (b)]

2 Divide and Conquer

Q.1 An array A[1 .... n] contains all the integers from 0 to n, except one element. How much time it will take to determine the missing integer?
(a) $O(n)$ (b) $O(\log n)$
(c) $O(n^2)$ (d) None of these

Q.2 For merging two sorted lists of sizes m and n into a sorted list of size $m + n$, we required comparisons of
(a) $O(m)$ (b) $O(n)$
(c) $O(m + n)$ (d) $O(\log m + \log n)$
[GATE-1995]

Common Data for Q.3 & Q.4
In a permutation a_1 ... a_n of n distinct integers, an inversion is a pair (a_i, a_j) such that $i < j$ and $a_i > a_j$.

Q.3 If all permutations are equally likely, what is the expected number of inversions in a randomly chosen permutation of 1...n?
(a) $n(n-1)/2$ (b) $n(n-1)/4$
(c) $n(n+1)/4$ (d) $2n[\log_2 n]$
[GATE-2003]

Q.4 What would be the worst case time complexity of the insertion sort algorithm, if the inputs are restricted to permutations of 1...n with at most n inversions?
(a) $\Theta(n^2)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^{1.5})$ (d) $\Theta(n)$
[GATE-2003]

Q.5 Let F_k denote the k^{th} Fibonacci number with $F_k = F_{k-1} + F_{k-2}$ for $k \geq 2$, $F_0 = F_1 = 1$.
Consider the following variation of an merge sort, which assumes that the number of elements in its list argument L is a Fibonacci number F_k.
Algorithm FibMergeSort(L)
L is a list of items from a totally ordered set, whose length is a Fibonacci number F_k.
{
If L contains only 1 element, then return L;
else
{
divide L into L_1 (the first F_{k-1} items) and L_2 (the remaining F_{k-2} items)
sorted L_1 := FibMergeSorte(L_1)
sorted L_2 := FibMergeSort (L_2)
sorted L := Merge(sortedL_1, sortedL_2)
return sortedL;
}
}
Assuming that the "divide" step in FibMergeSort takes constant time (no comparisons) and Merge behaves similar to the merge in the normal merge sort. Identify which of the following expressions most closely matches the total number of comparisons performed by FibMergeSort when initially given a list of F_k elements?
(a) $O(k \log k)$ (b) $O(K^2)$
(c) $O(k F_k)$ (d) $O(F_k \log k)$

Q.6 If one uses straight two-way merge sort algorithm to sort the following elements in ascending order:
20, 47, 15, 8, 9, 4, 40, 30, 12, 17
Then the order of these elements after second pass of the algorithm is
(a) 8, 9, 15, 20, 47, 4, 12, 17, 30, 40
(b) 8, 15, 20, 47, 4, 9, 30, 40, 12, 17
(c) 15, 20, 47, 4, 8, 9, 12, 30, 40, 17
(d) 4, 8, 9, 15, 20, 47, 12, 17, 30, 40
[GATE-1999]

Q.7 We are given a sequence of n-numbers $a_1, a_2, a_3 \ldots a_n$, we will assume that all the number are distinct. We say two indices $i < j$ form an inversion if $a_i > a_j$.
How much time it will take to find total number of inversions in the given array?
(a) $O(n^2)$ (b) $O(n \log n)$
(c) $O(n)$ (d) None of these

Q.8 Suppose you have k-sorted arrays, each with n-elements and you want to combine those k-sorted arrays into a single sorted array of kn elements. How much time it will take?
(a) $O(kn)$ (b) $O(kn \log k)$
(c) $O(k^2)$ (d) None of these

Q.9 Suppose there are 4 sorted lists of $n/4$ elements each. If we merge these lists into a single sorted list of n elements, for the $n = 400$ number of key comparisons in the worst case using an efficient algorithm is ________.

Q.10 Assume that a mergesort algorithm in the worst case takes 30 seconds for an input of size 64. Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes?

Q.11 Which of the following is divide and conquer application?
(a) Heapsort (b) Insertion sort
(c) Bubble sort (d) Merge sort
[DRDO-2008]

Q.12 The worst case time complexity of Quicksort for n elements when the median is selected as the pivot is:
(a) $O(n)$ (b) $O(n^2)$
(c) $O(n \log n)$ (d) $O(n \log n)$
[DRDO-2008]

Q.13 Randomized quicksort is an extension of quicksort where the pivot is chosen randomly. What is the worst case complexity of sorting n numbers using randomized quicksort?
(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(n!)$
[GATE-2001]

Q.14 Given two arrays of numbers $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i, j) such that $a_i + a_{i+1} + \ldots + a_j = b_i + b_{i+1} + \ldots + b_j$, or report that there is no such span,
(a) Takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted
(b) Takes $O(n^3)$ and $\Omega(n^{2.5})$ time in the key comparison model
(c) Takes $\Theta(n)$ time and space
(d) Takes $O(\sqrt{n})$ time only if the sum of the $2n$ elements is an even number
[GATE-2006]

Q.15 Suppose you are provided with the following function declaration in the C programming language.

int partition (int *a*[ ], int *n*);

The function treats the first element of a[ ] as a pivot, and rearranges the array so that all elements less than or equal to the pivot is in the left part of the array, and all elements greater than the pivot is in the right part. In addition, it moves the pivot so that the pivot is the last element of the left part. The return value is the number of elements in the left part.
The following partially given function in the C programming language is used to find the k^{th} smallest element in an array *a*[ ] of size n using the partition function. We assume $k \le n$.

```
int kth_smallest (int a[ ], int n, int k)
{
    int left_end = partition (a, n);
    if (left_end + 1==k)
    {
        return a [left_end];
    }
    if (left_end+1 > k)
    {
        return kth_smallest (______);
    }
    else
    {
```

return k^{th}_smallest (______);
}
}

The missing argument lists are respectively

(a) (a, left_end, k) and (a +left_end+1, n–left_end–1, k–left_end–1)
(b) (a, left_end, k) and (a, n–left_end–1, k–left_end–1)
(c) (a+left_end+1, n–left_end–1, k–left_end–1) and(a, left_end, k)
(d) (a, n–left_end–1, k–left_end–1) and (a, left_end, k)

[GATE-2015]

Q.16 You are given an infinite array A in which the first n-cells contains integers in sorted order and the rest of the cells are filled with ∞. If you are not given the value of n, find time complexity of an algorithm that takes an integer X as input and find the position of element X in the given array A.

(a) $O(n)$ (b) $O(\log n)$
(c) $O(n^2)$ (d) None of these

Q.17 The minimum number of comparisons required to sort 5 elements is

(a) 4 (b) 5
(c) 6 (d) 7

[DRDO-2008]

Q.18 An element in an array X is called a leader if it is greater than all elements to the right of it in X. The best algorithm to find all leaders in an array.

(a) Solves it in linear time using a left to right pass of the array
(b) Solves in linear time using a right to left pass
(c) Solves it is using divide and conquer in time $\Theta(n \log n)$
(d) Solves it in time $\Theta(n^2)$

[GATE-2006]

Q.19 The number of comparisons required to find maximum and minimum in the given array of n-elements using divide and conquer is ______

(a) $\left\lfloor \frac{3n}{2} \right\rfloor$ (b) $\left\lceil \frac{3n}{2} \right\rceil$
(c) $\left\lfloor \frac{3n}{2} \right\rfloor + 2$ (d) $\left\lceil \frac{3n}{2} \right\rceil - 2$

Q.20 An unordered list contains n distinct elements. The number of comparisons to find an element in this list that is neither 2^{nd} maximum nor 2^{nd} minimum is

(a) $\Theta(n \log n)$ (b) $\Theta(n)$
(c) $\Theta(\log n)$ (d) $\Theta(1)$

Q.21 Assume an array A [1, ..., n] has n-elements, and every element of an array is positive and less than or equal to n. An element is said to be "majority element", if it is occurred in more than $\frac{n}{2}$ positions of an array. What is the time complexity to check whether the majority element exist or not in the given array? [Best answer]

(a) $O(\log n)$ (b) $O(n)$
(c) $O(n \log n)$ (d) $O(n^2)$

Q.22 Binary search can be carried out on a set of ordered data items stored in a

(a) Array (b) Stack
(c) Queue (d) List

[DRDO-2008]

Q.23 Given 2-sorted arrays each of n-elements and distinct. How much time it will take to find middle element of the union sorted array?

(a) $O(1)$ (b) $O(\log n)$
(c) $O(n)$ (d) None of these

Q.24 Which of the following statements is true?

I. As the number of entries in a hash table increases, the number of collisions increases.
II. Recursive programs are efficient
III. The worst case complexity for Quicksort is $O(n^2)$
IV. Binary search using a linear linked list is efficient.

(a) I and II (b) II and III
(c) I and IV (d) I and III

Q.25 Let s be a sorted array of n integers. Let $t(n)$ denote the time taken for the most efficient algorithm to determined if there are two elements with sum less than 1000 in s. Which of the following statements is true?

(a) $t(n)$ is 0(1)
(b) $n \leq t(n) \leq n \log_2 n$
(c) $n \log_2 n \leq t(n) < \binom{n}{2}$
(d) $t(n) = \binom{n}{2}$

[GATE-2000]

Q.26 The cube root of a natural number n is defined as the larger natural number m such that $m^3 \leq n$. The complexity of computing the cube root of n(n is represented in binary notation) is

(a) $O(n)$ but not $O(n^{0.5})$
(b) $O(n^{0.5})$ but not $O((\log n)^k)$ for any constant $k > 0$
(c) $O((\log n)^k)$ for some constant $k > 0$, but not $O((\log\log n)^m)$ for any constant $m > 0$
(d) $O((\log\log n)^k)$ for some constant $k > 0.5$, but not $O((\log\log n)^{0.5})$

[GATE-2003]

Q.27 Consider a list of recursive algorithms and a list of recurrence relations as shown below. Each recurrence relation corresponds to exactly one algorithm and is used to derive the time complexity of the algorithm.

List-I (Recursive Algorithm)

P. Binary search
Q. Merge sort
R. Quick sort
S. Tower of Hanoi

List-II (Recurrence Relation)

I. $T(n) = T(n-k) + T(k) + cn$
II. $T(n) = 2T(n-1) + 1$
III. $T(n) = 2T(n/2) + cn$
IV. $T(n) = T(n/2) + 1$

Which of the following is the correct match between the algorithms and their recurrence relations?

Codes:

	P	Q	R	S
(a)	II	III	IV	I
(b)	IV	III	I	II
(c)	III	II	IV	I
(d)	IV	II	I	III

[GATE-2004]

Linked Answers Q.28 and Q.29

Consider the following C program that attempts to locate an element x in an array Y[ ] using binary search. The program is erroneous.

```
1.  f(int Y[10] , int x) {
2.      int i, j, k;
3.      i = 0; j = 9;
4.      do {
5.          k = (i + j) / 2;
6.          if (Y[k] < x) i = k; else j = k;
7.      } while (( Y[k] ! = x) && (i < j));
8.      if (Y[k] = = x)
            printf("x is in the array") ;
9.      else
            printf("x is not in the array");
10. }
```

Q.28 On which of the following contents of Y and x does the program fail?

(a) Y is [1 2 3 4 5 6 7 8 9 10] and $x < 10$
(b) Y is [1 3 5 7 9 11 13 15 17 19] and $x < 1$
(c) Y is [2 2 2 2 2 2 2 2 2 2] and $x > 2$
(d) Y is [2 4 6 8 10 12 14 16 18 20] and $2 < x < 20$ and x is even

[GATE-2008]

Q.29 The correction needed in the program to make it work properly is

(a) change line 6 to : if (Y[k] < x) $i = k + 1$; else $j = k - 1$;
(b) change line 6 to : if (Y[k] < x) $i = k - 1$; else $j = k + 1$;
(c) change line 6 to : if (Y[k] < x) $i = k$; else $j = k$;
(d) change line 7 to : while ((Y[k] = = x) && ($i < j$));

[GATE-2008]

Q.30 Given a sorted array of n-elements where other than one element x every other element repeat two times. Then how much time will it take to find position of x.

Multiple Select Questions

Q.31 Consider the following statements with respect to quick sort and select the false options given below:

(a) While performing sorting at any iteration only 1 element can be present at its correct position.

(b) At first iteration, none of the element changes its position if given input is already sorted.

(c) After i^{th} iteration applied on the set of elements left after $(i - 1)$ iterations if any element is found to be at its correct position, then that element must have been the pivot for the i^{th} iteration.

(d) None of these

Q.32 Which of the following statements are correct?

(a) An insertion algorithm consists of N – 1 passes when an array of N elements is given.

(b) If the input is pre-sorted, the running time is O(N).

(c) Insertion sort is stable and it sorts Out-place.

(d) When the input array is already sorted then Merge sort is best suited.

Try Yourself

T1. Merging k-sorted lists each of size n/k into one sorted list of n-elements using heap sort will take how much time?

[Ans: $O(n \log k)$]

T2. Consider a modification to merge sort in which m/k sublists each of length k are sorted using insertion sort and then merged using standard merge procedure. Then find total time complexity of modified merge sort.

[Ans: $mk + m \log (m/k)$]

T3. Given array of n-distinct elements. What is the worst case running time to find i^{th} smallest element $(1 \le i \le n)$ from those n elements? (Select the best answer by assuming n is larger than 50)

(a) $O(\log n)$
(b) $O(n)$
(c) $O(n\log n)$
(d) $O(n^2)$

[Ans: (b)]

■■■■

3 Binary Trees, Binary Heaps and Greedy Algorithms

Q.1 The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is

(a) $2^h - 1$ (b) $2^{h-1} - 1$
(c) $2^{h+1} - 1$ (d) 2^{h+1}

[GATE-2007]

Q.2 The maximum number of binary trees that can be formed with three unlabeled nodes is

(a) 1 (b) 5
(c) 4 (d) 3

[GATE-2007]

Q.3 A program takes as input a balanced binary search tree with n leaf nodes and computes the value of a function $g(x)$ for each node x. If the cost of computing $g(x)$ is min (number of leaf-nodes in leaf-subtree of x, number of leaf-nodes in right-subtree of x) then the worst case time complexity of the program is

(a) $\Theta(n)$ (b) $O(n \log n)$
(c) $O(n)^2$ (d) $O(n^2 \log n)$

[GATE-2004]

Q.4 In a complete k-ary, every internal node has exactly k children. The number of leaves in such a tree with n internal nodes is

(a) $n k$ (b) $(n - 1) k + 1$
(c) $n(k - 1) + 1$ (d) $n (k - 1)$

[GATE-2005]

Q.5 Suppose there are $\log n$ sorted lists of $n/\log n$ elements each. The time complexity of producing a sorted list of all these elements is:
(**Hint:** Use a heap data structure)

(a) $O(n \log \log n)$ (b) $\Theta(n \log n)$
(c) $\Omega(n \log n)$ (d) $\Omega(n^{3/2})$

[GATE-2005]

Q.6 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, ..., v_n\}$ of n vertices?

(a) $\frac{n(n-1)}{2}$ (b) 2^n
(c) $n!$ (d) $2^{n(n-1)/2}$

[GATE-2001]

Q.7 The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is

(a) $\frac{n}{2}$ (b) $\frac{(n-1)}{3}$
(c) $\frac{(n-1)}{2}$ (d) $\frac{(2n+1)}{3}$

[GATE-2002]

Q.8 Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of $a + 10b$ is _________.

[GATE-2014]

Q.9 Adjacency list is preferred over adjacency matrix when the graph is

(a) Planar (b) Dense
(c) Clique (d) None of these

[DRDO-2008]

Q.10 In a simple connected undirected graph with n nodes (where $n \geq 2$) the maximum number of nodes with distinct degrees is

(a) $n-1$ (b) $n-2$
(c) $n-3$ (d) 2

[DRDO-2008]

Q.11 A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If $L = 41$, and $I = 10$, what is the value of n

(a) 3 (b) 4
(c) 5 (d) 6

[GATE-2007]

Q.12 Suppose we have a balanced binary search tree T holding n-numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T.

If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is ________.

[GATE-2014]

Q.13 Consider any array representation of an n element binary heap where the elements are sorted from index 1 to index n of the array. For the element sorted at index i of the array ($i \le n$), the index of the parent is

(a) $i-1$ (b) $\left\lfloor \frac{i}{2} \right\rfloor$
(c) $\left\lceil \frac{i}{2} \right\rceil$ (d) $\frac{(i+1)}{2}$

[GATE-2001]

Q.14 In a heap with n elements with the smallest element at the root, the 7th smallest element can be found in time

(a) $\Theta(n \log n)$ (b) $\Theta(n)$
(c) $\Theta(\log n)$ (d) $\Theta(1)$

[GATE-2003]

Q.15 The minimum element in a max-heap represented by an array can be computed in time

(a) $\Theta(n \log n)$ (b) $O(n)$
(c) $\Theta(n^2)$ (d) $O(1)$

[DRDO-2009]

Q.16 A weight-balanced tree is a binary tree in which for each node, the number of nodes in the left sub tree is at least half and at most twice the number of nodes in the right sub tree. The maximum possible height (number of nodes on the path from the root to the furthest leaf) of such a tree on n nodes is best described by which of the following?

(a) $\log_2 n$ (b) $\log_{4/3} n$
(c) $\log_3 n$ (d) $\log_{3/2} n$

[GATE-2002]

Q.17 A binary search tree is used to locate the number 43. Which of the following probe sequences are possible and which are not?

(a) 61 52 14 17 40 43
(b) 2 3 50 40 60 43
(c) 10 65 31 48 37 43
(d) 81 61 52 14 41 43
(e) 17 77 27 66 18 43

[GATE-1996]

Q.18 Let $T(n)$ be the number of different binary search trees on n distinct elements. Then

$$T(n) = \sum_{k=1}^{n} T(k-1)T(x)\text{, where } x \text{ is}$$

(a) $n-k+1$ (b) $n-k$
(c) $n-k-1$ (d) $n-k-2$

[GATE-2003]

Q.19 Which one of the following arrays satisfied max-heap property?

(a) 16, 10, 12, 8, 3, 5 (b) 16, 8, 5, 10, 12, 3
(c) 16, 12, 8, 3, 5, 10 (d) 10, 16, 12, 8, 5, 3

[DRDO-2008]

Common Data for Q.20 & Q.21

A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, $a[0]$, nodes in the next level, from left to right, is stored from $a[1]$ to $a[3]$. The nodes from the second level of the tree from left to right are stored from $a[4]$ location onward. An item x can be inserted into a 3-ary heap containing n items by placing x in the location $a[n]$ and pushing it up the tree to satisfy the heap property.

Q.20 Which one of the following is a valid sequence of elements in an array representing 3-ary max heap?
(a) 1, 3, 5, 6, 8, 9
(b) 9, 6, 3, 1, 8, 5
(c) 9, 3, 6, 8, 5, 1
(d) 9, 5, 6, 8, 3, 1
[GATE-2006]

Q.21 Suppose the elements 7, 2, 10, and 4 are insetrted, in that order, into the valid 3-ary max heap found in the question, Q. 30. Which one of the following is the sequence of items in the array representing the resultant heap?
(a) 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
(b) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
(c) 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
(d) 10, 8, 6, 9, 7, 2, 3, 4, 1, 5
[GATE-2006]

Q.22 The minimum number of interchanges needed to convert the array
89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70 into a heap with maximum element at the root is
(a) 0
(b) 1
(c) 2
(d) 3
[GATE-1996]

Q.23 The number of elements that can be stored in $\Theta(\log n)$ time using heap sort is
(a) $\Theta(1)$
(b) $\Theta(\sqrt{\log n})$
(c) $\Theta\left(\frac{\log n}{\log\log n}\right)$
(d) $\Theta(\log n)$
[GATE-2013]

Q.24 An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted. For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best total asymptotic complexity considering all the operations?
(a) Unsorted array
(b) Min-heap
(c) Sorted array
(d) Sorted doubly linked list

Q.25 Construct the Max Heap assuming the following set of integers were inserted into it in given order
20, 32, 1, 3, 4, 5, 6, 7, 10, 23, 45
Postorder traversal of the resultant max heap was stored in a array A with an index variable i inorder (Starting from 0). Similarly level order traversal was stored in the array B using index variable j in order (Starting from 0). For particular element, respective i and j location values from A and B were obtained and $|i-j|$ is calculated. What could be the maximum possible value for $|i-j|$?

Q.26 A data structure is required for storing a set of integers such that each of the following operations can be done in (log n) time, where n is the number of elements in the set.
1. Deletion of the smallest element.
2. Insertion of an element if it is not already present in the set.

Which of the following data structures can be used for this purpose?
(a) A heap can be used but not a balanced binary search tree
(b) A balanced binary search tree can be used but not a heap
(c) Both balanced binary search tree and heap can be used
(d) Neither balanced binary search tree nor heap can be used
[ISRO-2009]

Q.27 Postorder traversal of a binary search tree is given as follows 35, 40, 55, 60, 50, 100. Then the given tree is:
(a) Minheap tree
(b) Maxheap true
(c) Strict binary tree
(d) None of these

Q.28 The worst case running time complexity to search for an element in a balanced binary search tree with $(2n)!$ elements?

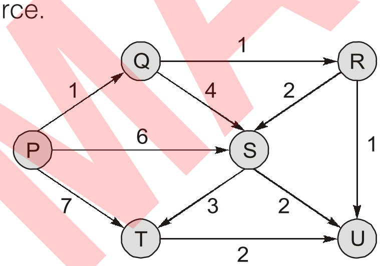
(a) $O(n^n)$ (b) $O(n \log n)$
(c) $O(n)$ (d) $O(n^2)$

Q.29 Consider the process of inserting an element into a Max Heap, where the Max Heap is represented by an array. Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of comparisons performed is
(a) $\theta(\log_2 n)$ (b) $\theta(\log_2 \log_2 n)$
(c) $\theta(n)$ (d) $\theta(n \log_2 n)$
[GATE-2007]

Q.30 We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is
(a) $\Theta(\log n)$ (b) $\Theta(n)$
(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$
[GATE-2008]

Q.31 Which one of the following statements are correct regarding Bellman-Ford shortest path algorithm?
P : Always finds a negative edge weight cycle if one exists.
Q : Find whether any negative edge weight cycle reachable from the source.
(a) P only (b) Q only
(c) Both P and Q (d) Neither P nor Q

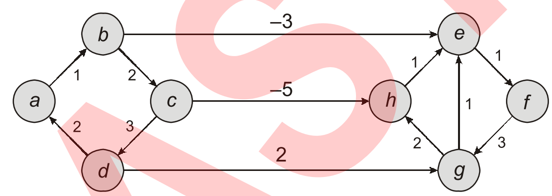
Q.32 Suppose we run Dijkstra's single source shortest-path algorithm on the following edge-weighted directed graph with vertex P as the source.



In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?
(a) P, Q, R, S, T, U (b) P, Q, R, U, S, T
(c) P, Q, R, U, T, S (d) P, Q, T, R, U, S
[GATE-2004]

Q.33 Let G (V, E) an undirected graph with positive edge weights. Dijkstra's single source-shortest path algorithm can be implemented using the sorted linked list data structure and adjacency list. What will the time complexity?
(a) $O(E\,|V|)$
(b) $O(|V|^3)$
(c) $O(|V| \log |V|)$
(d) $O((|E| + |V|) \log |V|)$

Q.34



Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the corrects shortest path distance to
(a) Only vertex a
(b) Only vertices a, e, f, g, h
(c) Only vertices a, b, c, d
(d) All the vertices
[GATE-2008]

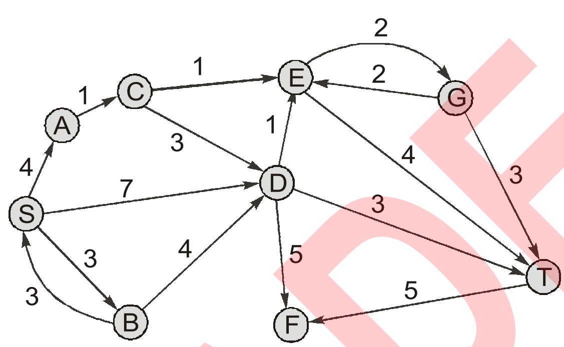
Q.35 Which one of the following statements are correct regarding Bellman-Ford shortest path algorithm?
P: Always finds a negative edge weight cycle if one exists.
Q: Find whether any negative edge weight cycle reachable from the source.
(a) P only (b) Q only
(c) Both P and Q (d) Neither P nor Q

Q.36 Consider the following statements:
I. For every weighted graph and any two vertices s and t, Bellman-ford algorithm starting at s will always return a shortest path to t.
II. At the termination of the Bellman-Ford algorithm, even if graph has negative weight cycle, a correct shortest path is found for a vertex for which shortest path is well-defined.

Which of the above statements are true?
(a) Only I (b) Only II
(c) Both I and II (d) None of these

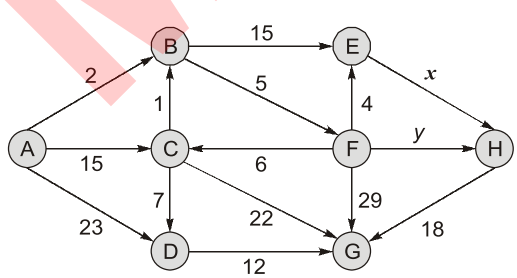
Q.37 If graph contains negative weight edges then which of the following is correct when we run Dijkstra's algorithm?
(a) It may not terminate
(b) It terminates but may produce incorrect results
(c) It never terminates due to cycles in graph
(d) None of these

Q.38 Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



(a) SDT (b) SVDT
(c) SACDT (d) SACET
[GATE-2012]

Q.39 Suppose that you are running Dijkstra's algorithm on the edge-weighted digraph below, starting from vertex A.

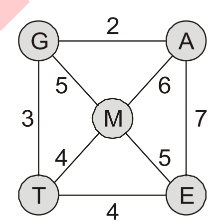


The table gives 'Distance' and 'Parent' entry of each vertex after vertex E has been deleted from the priority queue and relaxed.

Vertex	Distance	Parent
A	0	NULL
B	2	A
C	13	F
D	23	A
E	11	F
F	7	B
G	36	F
H	19	E

What could be the possible value of expression $x + y$?

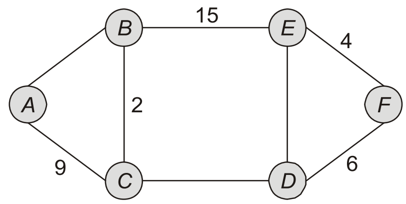
Q.40 Assume Dijkstra's algorithm is used to find the shortest paths from node 'G' in the following graph.



Find the number of edges which are not included in any of the shortest paths from node G.

Q.41 What is the upper bound on the number of edge disjoint spanning trees in a complete graph of n vertices?
(a) n (b) $n - 1$
(c) $\left[\frac{n}{2}\right]$ (d) $\left[\frac{n}{3}\right]$
[DRDO-2009]

Q.42 The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: {(A, C), (B, C), (B, E), (E, F), (D, F)}. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is ________.



Q.43 Let G be an undirected connected graph with distinct edge weight. Let e_{max} be the edge with maximum weight and e_{min} with minimum weight. Which of the following statements is false?

(a) Every minimum spanning tree of G must contain e_{min}

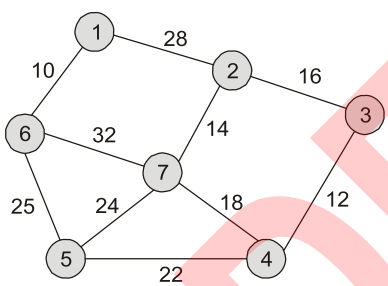
(b) If e_{max} is in a minimum spanning tree, then its removal must disconnect G

(c) No minimum spanning tree contains e_{max}

(d) G has a unique minimum spanning tree

[GATE-2000]

Q.44 Consider the following graph where the numbers denotes the weight of the particular edge

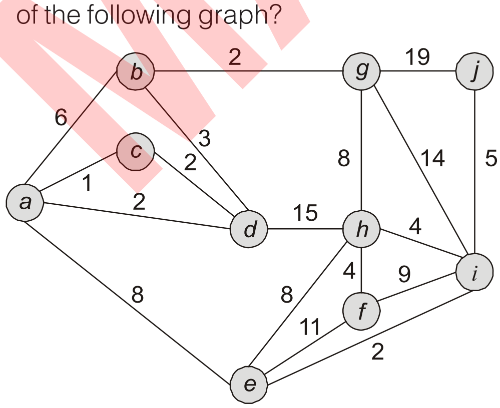


Now, calculate the minimum cost spanning tree of the above graph using either prim's or Kruskal's algorithm.

(a) 92 (b) 99

(c) 102 (d) 123

Q.45 What is the weight of a minimum spanning tree of the following graph?



(a) 29 (b) 31

(c) 38 (d) 41

[GATE-2003]

Common Data for Q.46 & Q.47

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

Q.46 What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(a) 7 (b) 8

(c) 9 (d) 10

[GATE-2010]

Q.47 What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(a) 7 (b) 8

(c) 9 (d) 10

[GATE-2010]

Q.48 Consider a weighted complete graph G on the vertex set $\{v_1, v_2, \ldots, v_n\}$ such that the weight of the edge (v_i, v_j) is $2\,|i-j|$. The weight of a minimum spanning tree of G is

(a) $n-1$ (b) $2n-2$

(c) $(n/2)$ (d) n^2

[GATE-2006]

Q.49 Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE?

(a) There is a minimum spanning tree containing e.

(b) If e is not in a minimum spanning tree T, then in the cycle formed by adding e to T, all edges have the same weight.

(c) Every minimum spanning tree has an edge of weight *w*

(d) *e* is present in every minimum spanning tree

[GATE-2007]

Q.50 Consider the following statements:

I. Let T be a minimum spanning tree of a graph G. Then for any two vertices *u* and *v* the path from *u* to *v* in T is the shortest path from *u* to *v* in the graph G.

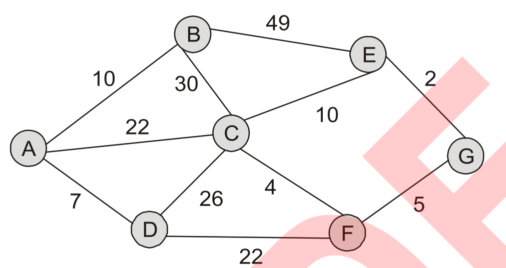
II. Suppose that average edge weight for a graph G is A_{avg}. Then the minimum spanning tree of G will have weight at most $(n - 1)$ A_{avg}. Where *n* is number of vertices in graph G.

Which of the above statements are true?

(a) Only I (b) Only II

(c) Both I and II (d) None of these

Q.51 Consider the undirected graph below:



Using Prim's algorithm to construct a minimum spanning tree starting with node A, which one of the following sequences of edges represents a possible order in which the edges would be added to construct the minimum spanning tree?

(a) (E,G), (C,F), (F,G), (A,D), (A,B), (A,C)

(b) (A,D), (A,B), (A,C), (C,F), (G,E), (F,G)

(c) (A,B), (A,D), (D,F), (F,G), (G,E), (F,C)

(d) (A,D), (A,B), (D,F), (F,C), (F,G), (G,E)

[GATE-2004]

Q.52 Let G be a weighted undirected graph and *e* be an edge with maximum weight in G. Suppose there is a minimum weight spanning tree in G containing the edge *e*. Which of the following statements is always TRUE?

(a) There exists a cutset in G having all edges of maximum weight.

(b) There exists a cycle in G having all edges of maximum weight.

(c) Edge *e* cannot be contained in a cycle.

(d) All edges in G have the same weight.

[GATE-2005]

Q.53 G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and *e* is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTS) of G is/are TRUE?

I. If *e* is the lightest edge of some cycle in G, then every MST of G includes *e*.

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes *e*.

(a) I only (b) II only

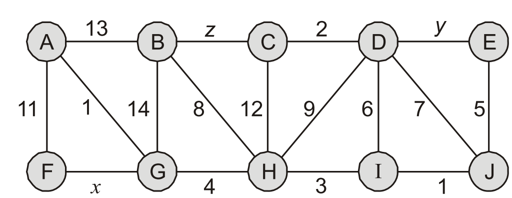
(c) Both I and II (d) Neither I nor II

[GATE-2016]

Q.54 Let *G* be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of *G* can have is _______.

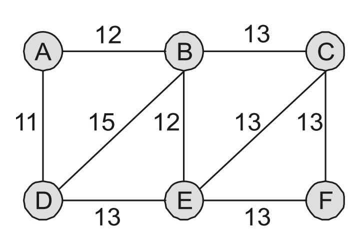
[GATE-2016]

Q.55 Suppose that minimum spanning tree of the following edge weighted graph contains the edges with weights x, y and z



What is the maximum value of $x + y + z$?

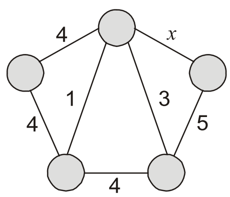
Q.56 Consider the following graph G.



Find the number of minimum cost spanning trees using Kruskal's algorithm or Prim's algorithm.

Q.57 Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes _________.

Q.58 Consider the following undirected graph G:



Choose a value of x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is _______.

[GATE-2018]

Q.59 Let G = (V, E) be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:

I. Minimum Spanning Tree of G is always unique.

II. Shortest path between any two vertices of G is always unique.

Which of the above statements is/are necessarily true?

(a) I only
(b) II only
(c) Both I nor II
(d) Neither I nor II

[GATE-2017]

Q.60 A file contains characters *a*, *e*, *i*, *o*, *u*, *s* and *t* with frequencies 10, 15, 12, 3, 4, 13 and 1 respectively. If we use Huffman coding for data compression then the average code length will be:

(a) $\frac{140}{58}$
(b) $\frac{146}{58}$
(c) $\frac{150}{58}$
(d) $\frac{174}{58}$

[DRDO-2009]

Linked Answer for Q.61 & Q.62

Consider the following message:

aabbbbabccdddccccbbdd

Q.61 Find the number of bits required for Huffman encoding of the above message.

(a) 30
(b) 38
(c) 42
(d) 46

Q.62 If Huffman tree coded as left child with '0' and right child with '1' from every node then what is the decoded message for 110100

(a) *abc*
(b) *bcd*
(c) *acb*
(d) *bda*

Common Data for Q.63 & Q.64

Suppose the letters *a*, *b*, *c*, *d*, *e* have probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ respectively.

Q.63 Which of the following is the Huffman code for the letters *a*, *b*, *c*, *d*, *e*?

(a) 0, 10, 110, 1110, 1111
(b) 11, 10, 011, 010, 001
(c) 11, 10, 01, 001, 0001
(d) 110, 100, 010, 000, 001

[GATE-2007]

Q.64 What is the average length of the correct answer to above question?

(a) 3
(b) 2.1875
(c) 2.25
(d) 1.781

[GATE-2007]

Q.65 Suppose P, Q, R, S, T are sorted sequences having lengths 20, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ________.

Q.66 The average number of key comparisons done on a successful sequential search in list of length *n* is

(a) $\log n$ (b) $\frac{n-1}{2}$

(c) $\frac{n}{2}$ (d) $\frac{n+1}{2}$

[GATE-1996]

Common Data for Q.67 & Q.68

We are given 9 tasks T_1, T_2 ... T_9. The execution of each task requires one unit of time. We can execute one task at a time. T_i has a profit P_i and a deadline d_i profit P_i is earned if the task is completed before the end of the d_i^{th} unit of time.

Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Profit	15	20	30	18	18	10	23	16	25
Deadline	7	2	5	3	4	5	2	7	3

Q.67 Are all tasks completed in the schedule that gives maximum profit?
(a) All tasks are completed
(b) T_1 and T_6 are left out
(c) T_1 and T_8 are left out
(d) T_4 and T_6 are left out

[GATE-2005]

Q.68 What is the maximum profit earned?
(a) 147 (b) 165
(c) 167 (d) 175

[GATE-2005]

Q.69 The following are the starting and ending times of activities A, B, C, D E, F, G, and H respectively in chronological order:

"a_s b_s c_s a_e d_s c_e e_s f_s b_e d_e g_s e_e f_e h_s g_e h_e"

Here, x_s denotes the starting time and x_e denotes the ending time of activity X. We need to schedule the activities in a set of rooms available to us. An activity can be scheduled in a room only if the room is reserved for the activity for its entire duration. What is the minimum number of rooms required?
(a) 3 (b) 4
(c) 5 (d) 6

[GATE-2003]

Q.70 Consider the Knapsack instance:
- Capacity of Knapsack is 15 and 7 objects
- Profits: $(P_1, P_2, ..., P_7)$ = (10,5,15,7,6,18,3)
- Weights: $(w_1, w_2, ..., w_7)$ = (2,3,5,7,1,4,1)
- Objects: $(x_1, x_2, ..., x_7)$

If Knapsack problem is solved using maximum profit per unit weight then find the object which is partially placed in the Knapsack.
(a) x_1 (b) x_2
(c) x_3 (d) x_4

Q.71 Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers i, j with $i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j. Suppose $T(k)$ denotes the smallest number of steps needed to move from k to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15.
Let y and z be such that $T(9) = 1 + \min(T(y), T(z))$. Then the value of the product yz is ______.

[GATE-2014]

Multiple Select Questions

Q.72 Consider the following statements. Which of the following is/are true?
(a) The time complexity to construct an AVL tree from an array which satisfies the min heap property is $O(n \log n)$.
(b) The time complexity to sort 'n' elements by first constructing a Binary Search Tree on those 'n' elements followed by an inorder traversal on the tree is $O(n \log n)$.
(c) The worst case time complexity to sort on array A[0 to n] where the elements at index A[$2i$] (at even array index) is in its correct position is $O(n \log n)$.
(d) None of these

Q.73 Consider the following statements and select the correct options.
(a) Given a min heap of height log$n$, it will take O(log$n$) time to delete the root element of the heap.
(b) Time complexity of fractional Knapsack using greedy algorithm is O(n^2).
(c) Selection sort works on greedy approach.
(d) The running time of radix sort is effectively independent of whether the input is already sorted or not.

Q.74 Consider the already built max-heap?

24, 20, 20, 7, 8, 9, 19, 1, 2

After root deletion operation, which of the following max-heaps are valid?
(a) 20, 20, 9, 7, 8, 2, 19, 1
(b) 20, 8, 20, 7, 2, 9, 19, 1
(c) 20, 20, 19, 7, 8, 9, 2, 1
(d) 20, 7, 20, 2, 8, 9, 19, 1

Q.75 Consider the following statement and select the incorrect options:
(a) There exist an algorithm that can sort any list in linear time, given only an appropriate comparison function.
(b) There exist an algorithm that can sort a list in linear time, assuming that each item's position is no more than 1000 away from its position in the final sorted list.
(c) There exist an algorithm that can find the largest number in a given unsorted array in logarithmic time.
(d) None of these

Q.76 Which of the following problems can be solved by a standard greedy algorithm?
(a) Finding a minimum spanning tree in a undirected graph with positive-integer edge weights.
(b) Finding a maximum clique in an undirected graph.
(c) Finding a maximum flow from a source node to sink node in a directed graph with positive-integer edge capacities.
(d) Solving Knapsack problem.

Q.77 Select statements which are false:
(a) Bellman's Ford gives correct result and can detect negative weight cycle.
(b) Bellman's Ford does not work properly on negative weighted cycle graph.
(c) Dijkstra's algorithm always give correct result when apply to directed weighted graph.
(d) Dijkstra's algorithm can detect negative weight cycle in a graph.

Q.78 Which of the following are correct regarding Huffman coding?
(a) Greedy algorithm is the best approach for solving the Huffman codes problem since it greedily searches for an optimal solution.
(b) The code length does not depend on the frequency of occurrence of characters.
(c) In Huffman encoding, data is always stored at the leaves of a tree in order to compute the codeword effectively.
(d) If the implementation of the priority queue is done using linked lists, the running time of Huffman algorithm is O(c^2).

Try Yourself

T1. Given an adjacency-list representation of directed graph G(V, E). Then how much time does it take to compute the out-degree of each vertex. **[Ans: O(E + V)]**

T2. If the graph G(V, E) is represented using adjacency matrix then to find universal sink (in-degree is V – 1 and out-degree is 0). **[Ans: O(V)]**

T3. An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i^{th} node. Assume that the heap is implemented in an array and i refers to the i^{th} index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

(a) $O(1)$
(b) $O(d)$ but not $O(1)$
(c) $O(2^d)$ but not $O(d)$
(d) $O(d2^d)$ but not $O(2d)$

[GATE-2016, Ans: (b)]

T4. A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _________.

[GATE-2016]

T5. Given two inputs BST and Min heap tree with n-nodes. To get the sorted order which is better and how much time?

[Ans: BST and O(n)]

T6. Consider the following "Max-heapify" algorithm. Array has size atleast n and $1 \le i \le n$. After applying the Max-heapify rooted at A[i], the result will be the subtree of A [1, ...n] rooted at A[i] is max heap. [Assume that except root A[i], all its children satisfies heap property]

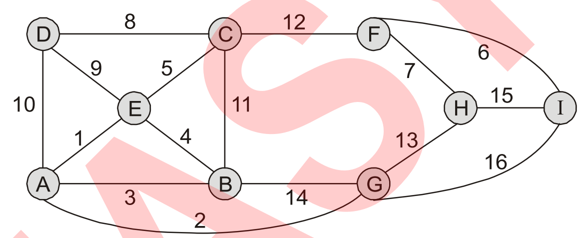
```
Max-heapify (int A[ ], int n, int i)
{     int p, m;
      p = i;
      while (X)
      {     if (Y && Z)
            m = 2p + 1;
            else m = 2p;
            if (A[p] < A[m])
            {
                  Swap (A[p], A [m]);
                  p = m;
            }
      else
return;
      }
}
```

Find missing statements at **X**, **Y** and **Z** respectively to apply the heapify for subtree rooted at A[i].

(a) $p \le n$, $(2p + 1) \ge n$, $A[2p + 1] > A[2p]$
(b) $2p \le n$, $(2p + 1) \le n$, $A[2p + 1] > A[2p]$
(c) $2p \le n$, $(2p + 1) \ge n$, $A[2p + 1] < A[2p]$
(d) $p \le n$, $(2p + 1) \le n$, $A[2p + 1] < A[2p]$

[Ans: (b)]

T7. Consider the weighted undirected graph below



Assume Prim's algorithm and kruskal's algorithm are executed on the above graph to find the minimum spanning tree. For a particular edge (e_i) which is included in minimum spanning tree and the position of an edge in minimum spanning tree is denoted by e_{p_i}. Where $1 \le e_{p_i} \le 8$ (where position defines the order in which edges are included in the MST). Then what is the maximum value of $\left|(e_{p_i})_{\text{prim's}} - (e_{p_i})_{\text{kruskal's}}\right|$?

[Ans: (7)]

T8. Complexity of Kruskal algorithm for finding the minimum cost spanning tree of an undirected graph contain n-vertices and m-edges, if the edges are already sorted.

[Ans: O(m)]

T9. Let T be a MST of G. Suppose that we decreased the weight of one of the edge present in G but not in T. Then how much time will take to construct MST for the modified graph G?

[Ans: O(V)]

■■■■

4 Sorting Algorithms, Graph Traversals and Dynamic Programming

Q.1 Consider a sequence A of length n which is sorted except for one item that appears out of order. Which of the following can sort the sequence in $O(n)$ time?
(a) Heapsort (b) Quicksort
(c) Merge sort (d) Insertion sort
[DRDO-2008]

Q.2 Consider the following algorithm for searching for a given number x in an unsorted array A[1..n] having n distinct values:
1. Choose an i uniformly at random from [1..n]
2. If A[i] = x then Stop else Goto 1;

Assuming that x is present A, what is the expected number of comparisons made by the algorithm before it terminates?
(a) n (b) $n-1$
(c) $2n$ (d) $n/2$
[GATE-2002]

Q.3 The usual $\Theta(n^2)$ implementation of insertion sort to sort an array uses linear search to identify the position where an element is to be inserted into the already sorted part of the array. If, instead, we use binary search to identify the position, the worst case running time will
(a) remain $\Theta(n^2)$
(b) become $\Theta(n(\log n)^2)$
(c) become $\Theta(n \log n)$
(d) become $\Theta(n)$
[GATE-2003]

Q.4 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that (i) each is sorted in ascending order, (ii) B has 5 and C has 3 elements, and (iii) the result of merging B and C gives A?
(a) 2 (b) 30
(c) 56 (d) 256
[GATE-2003]

Q.5 A sort method is said to be stable if the relative order of keys is the same after the sort as it was before the sort. In which of the following pairs both sorting algorithms are stable?
(a) Quick-sort and Insertion-sort
(b) Insertion-sort and Bubble-sort
(c) Quick-sort and Heap-sort
(d) Quick-sort and Bubble-sort
[DRDO-2009]

Q.6 Give the correct matching for the following pairs:

List-I		List-II	
A.	$O(\log n)$	P.	Selection
B.	$O(n)$	Q.	Insertion sort
C.	$O(n \log n)$	R.	Binary search
D.	$O(n^2)$	S.	Merge sort

Codes:
(a) A – R B – P C – Q D – S
(b) A – R B – P C – S D – Q
(c) A – P B – R C – S D – Q
(d) A – P B – S C – R D – Q
[GATE-1998]

Q.7 The tightest lower bound on the number of comparisons, in the worst case, for comparison-based sorting is
(a) $O(n)$ (b) $O(n^2)$
(c) $\Omega(n \log n)$ (d) $\Omega(n \log^2 n)$

Q.8 Assume that the algorithms considered here sort the input sequences in ascending order. If the input is already in ascending order, which of the following are TRUE?

I. Quicksort runs in $\Theta(n^2)$ time
II. Bubblesort runs in $\Theta(n^2)$ time
III. Mergesort runs in $\Theta(n)$ time
IV. Insertion sort runs in $\Theta(n)$ time
(a) I and II only (b) I and III only
(c) II and IV only (d) I and IV only

[GATE-2016]

Q.9 Using which algorithm an array of n-elements in the range $[1... n^3]$ will be sorted using $O(n)$ time?
(a) Merge sort (b) Quick sort
(c) Radix sort (d) Insertion sort

Q.10 If we use Radix Sort to sort n integers in the range $(n^{k/12}, n^k]$, for some $k > 0$ which is independent of n, the time taken would be
(a) $\Theta(n)$ (b) $\Theta(kn)$
(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

[GATE-2008]

Q.11 The most appropriate matching for the following pairs:
X : Depth first search **1** : Heap
Y : Breadth-first search **2** : Queue
Z : Sorting **3** : Stack
(a) X-1, Y-2, Z-3 (b) X-3, Y-1, Z-2
(c) X-3, Y-2, Z-1 (d) X-2, Y-3, Z-1

[GATE-2000]

Q.12 Match **List-I** with **List-II** select the correct answer using the codes given below the Lists:

List-I
A. All pairs shortest paths
B. Quick sort
C. Minimum weight spanning tree
D. Connected components

List-II
1. Greedy
2. Depth-first search
3. Dynamic programming
4. Divide and conquer

Codes:

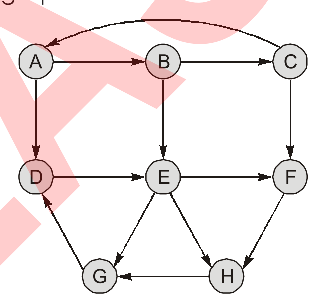
	A	B	C	D
(a)	2	4	1	3
(b)	3	4	1	2
(c)	3	4	2	1
(d)	4	1	2	3

[GATE-1997]

Q.13 Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing
(a) The shortest path between every pair of vertices.
(b) The shortest path from W to every vertex in the graph.
(c) The shortest paths from W to only those nodes that are leaves of T.
(d) The longest path in the graph.

[GATE-2014]

Q.14 Find the number of strong components in the following graph.



Q.15 Consider an undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let $d(r, u)$ and $d(r, v)$ be the lengths of the shortest paths from r to u and v respectively in G. If u is visited before v during the breadth-first traversal, which of the following statements is correct?
(a) $d(r, u) < d(r, v)$ (b) $d(r, u) > d(r, v)$
(c) $d(r, u) \le d(r, v)$ (d) None of these

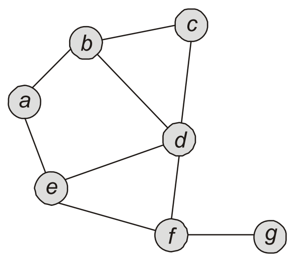
[GATE-2001]

Q.16 Let G be an undirected graph. Consider a depth-first traversal of G, and let T be the resulting depth-first search tree. Let u be a vertex in G and let v be the first new (unvisited) vertex visited after visiting u in the traversal. Which of the following statements is always true?
(a) $\{u, v\}$ must be an edge in G, and u is a descendant of v in T
(b) $\{u, v\}$ must be an edge in G, and v is a descendant of u in T
(c) If $\{u, v\}$ is not an edge in G then u is a leaf in T

(d) If {u, v} is not an edge in G then u and v must have the same parent in T

[GATE-2000]

Q.17 Consider the following graph:



A possible Depth First Search (DFS) sequence for the above graph is

(a) *d, e, b, a, c, f, g* (b) *b, a, e, c, d, f, g*
(c) *d, b, a, e, f, c, g* (d) *b, c, d, e, f, g, a*

[DRDO-2009]

Q.18 The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity.

(a) $\Theta(n)$ (b) $\Theta(m)$
(c) $\Theta(m+n)$ (d) $\Theta(mn)$

[GATE-2007]

Q.19 Which of the following statement is false?

(a) The depth of any DFS (Depth First Search) tree rooted at a vertex is at least as much as the depth of any BFS tree rooted at the same vertex.

(b) If all edges in a graph have distinct weight then the shortest path between two vertices is unique

(c) For a directed graph, the absence of back edges in a DFS tree means graph has no cycle

(d) BFS takes $O(V^2)$ time in a graph $G(V, E)$ if graph is represented with an adjacency matrix.

Q.20 Consider the tree arcs of a BFS traversal from a source node w in an weighted, connected, undirected graph with equal edge weights. The tree T formed by the tree arcs is a data structure for computing.

(a) the shortest path between every pair of vertices

(b) The shortest path from w to every vertex in the graph

(c) the shortest path from w to only those nodes that are leaves of T

(d) the longest path in the graph

Q.21 Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements:

I. No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D).

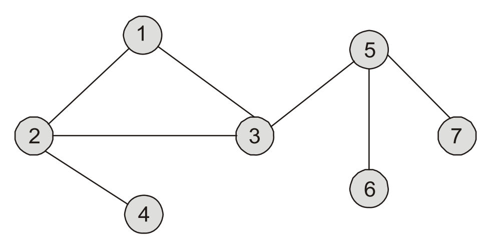
II. For every edge (u, v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

[GATE-2018]

Q.22 The number of articulation points of the following graph is



(a) 0 (b) 1
(c) 2 (d) 3

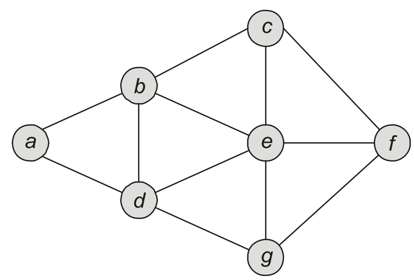
[GATE-1999]

Q.23 A depth-first search is performed on a directed acyclic graph. Let $d[u]$ denote the time at which vertex u is visited for the first time and $f[u]$ the time at which the DFS call to the vertex u terminates. Which of the following statements is always true for all edges (u, v) in the graph?

(a) $d[u] < d[v]$ (b) $d[u] < f[v]$
(c) $f[u] < f[v]$ (d) $f[u] > f[v]$

[GATE-2007]

Q.24 Consider the following sequence of nodes for the undirected graph given below:



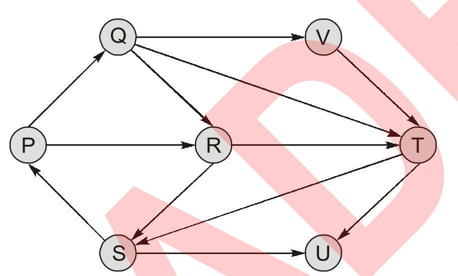
1. *a b e f d g c*   2. *a b e f c g d*
3. *a d g e b c f*   4. *a d b c g e f*

A Depth First Search (DFS) is started at node *a*. The nodes are listed in the order they are first visited. Which all of the above is (are) possible output(s)?

(a) 1 and 3 only (b) 2 and 3 only
(c) 2, 3 and 4 only (d) 1, 2 and 3 only

[GATE-2008]

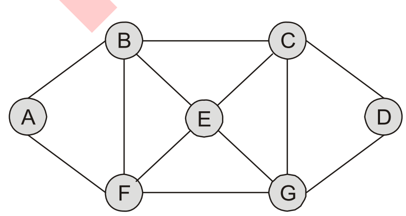
Q.25 Which of the following is the correct decomposition of the directed graph given below into its strongly connected components?



(a) {P, Q, R, S}, {T}, {U}, {V}
(b) {P,Q, R, S, T, V}, {U}
(c) {P, Q, S, T, V}, {R}, {U}
(d) {P, Q, R, S, T, U, V}

[GATE-2006]

Q.26 Consider the following graph:



The maximum and minimum size or queue and stack required when performing BFS and DFS respectively on the above graph is ________.

Q.27 Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex *t* at a distance four from the root. If *t* is the n^{th} vertex in this BFS traversal, then the maximum possible value of *n* is ________.

[GATE-2016]

Common Data for Q.28 & Q.29

A sub-sequence of a given sequence is just the given sequence with some elements (possibly none or all) left out. We are given two sequences X[*m*] and Y[*n*] of lengths *m* and *n*, respectively, with indexes of X and Y starting from 0.

Q.28 We wish to find the length of the longest common sub-sequence (LCS) of X[*m*] and Y[*n*] as $l(m, n)$, where an incomplete recursive definition for the function $l(i, j)$ to compute the length of the LCS of X[*m*] and Y[*n*] is given below:

$$l(i, j) = 0, \text{ if either } i = 0 \text{ or } j = 0$$
$$= \text{expr1, if } i, j > 0 \text{ and } X[i-1] = Y[j-1]$$
$$= \text{expr2, if } i, j > 0 \text{ and } X[i-1] \neq Y[j-1]$$

Which one of the following options is correct?

(a) expr1 $\equiv l(i-1, j) + 1$
(b) expr1 $\equiv l(i, j-1)$
(c) expr2 $\equiv \max(l(i-1, j), l(i, j-1))$
(d) expr2 $\equiv \max(l(i-1, j-1), l(i, j))$

[GATE-2009]

Q.29 The values of $l(i, j)$ could be obtained by dynamic programming based on the correct recursive definition of $l(i, j)$ of the form given above, using an array L[M, N], where M = m + 1 and N = n + 1, such that L[i, j] = $l(i, j)$.

Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of $l(i, j)$?

(a) All elements of L should be initialized to 0 for the values of $l(i, j)$ to be properly computed
(b) The values of $l(i, j)$ may be computed in a row major order or column major order of L[M, N]

(c) The values of $l(i, j)$ cannot be computed in either row major order or column major order of L[M, N]

(d) L[p, q] needs to be computed before L[r, s] if either $p < r$ or $q < s$

[GATE-2009]

Q.30 The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + \ldots a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$ and Y the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_{n-1}$.
Then X is equal to

(a) max(Y, a_0 + Y)
(b) max(Y, a_0 + Y/2)
(c) max(Y, a_0 + 2Y)
(d) a_0 + Y/2

[GATE-2010]

Q.31 Let $P_0 = 5$, $P_1 = 6$, $P_2 = 7$, $P_3 = 1$, $P_4 = 10$, $P_5 = 2$ and for $1 \le i \le 5$. Let X_i be a matrix with P_{i-1} rows and P_i columns, and $X = X_1.X_2.X_3.X_4.X_5$. Which of the following is optimum parenthesization for computing X?

(a) $((X_1(X_2 \cdot X_3))\ X_4)X_5$
(b) $(X_1(X_2 \cdot X_3))\ (X_4 \cdot X_5)$
(c) $(X_1X_2)\ ((X_3X_4) \cdot X_5)$
(d) $((X_1 \cdot X_2)\ (X_3 \cdot X_4))\ X_5$

Q.32 The correct matching for the following pairs is

List-I
A. All pairs shortest paths
B. Quick Sort
C. Minimum weight spanning tree
D. Connected Components

List-II
1. Greedy
2. Depth-First search
3. Dynamic Programming
4. Divide and Conquer

Codes:
(a) A – 2 B – 4 C – 1 D – 3
(b) A – 3 B – 4 C – 1 D – 2
(c) A – 3 B – 4 C – 2 D – 1
(d) A – 4 B – 1 C – 2 D – 3

[GATE : 1997]

Common Data for Q.33 & Q.34

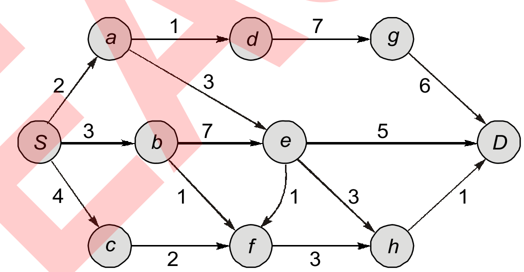
Matrix multiplication is associative and matrix chain multiplication uses following matrices:

A_1 is 10 × 100,
A_2 is 100 × 5,
A_3 is 5 × 50, and
A_4 is 50 × 1

Q.33 Find the number of orderings that are possible to compute $A_1\ A_2\ A_3\ A_4$.

Q.34 Find the maximum number of multiplications required to compute $A_1\ A_2\ A_3\ A_4$.

Q.35 Consider the following graph G.



What is minimum distance from S to D?

Q.36 Which one of the following statements is false?

(a) optimal binary search tree construction can be performed efficiently using dynamic programming
(b) breadth-first search cannot be used to find connected components of a graph
(c) given the prefix and postfix walks over a binary tree, the binary tree cannot be uniquely constructed
(d) depth-first search can be used to find connected components of a graph

[GATE-1994]

Q.37 Match **List-I** with **List-II** select the correct answer using the codes given below the Lists:

List-I
A. Strassen's matrix multiplication
B. Kruskal's minimum spanning tree
C. Bi-connected components algorithm
D. Floyd's shortest path

List-II
1. Greedy method
2. Dynamic programming
3. Divide and Conquer
4. Depth first search

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

Q.38 Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$.
If $p = 10$, $q = 100$, $r = 20$, $s = 5$ and $t = 80$, then the minimum number of scalar multiplications needed is
(a) 248000 (b) 44000
(c) 19000 (d) 25000
[GATE-2011]

Multiple Select Questions

Q.39 Which of the following is/are correct regarding dynamic algorithm?
(a) Time complexity of matrix chain multiplication is $O(n^3)$.
(b) Time complexity of travelling salesman problem is $O(n^n)$.
(c) Time complexity of 0/1 Knapsack algorithm is $O(mn)$.
(d) Time complexity of Fibonacci series is $O(n)$.

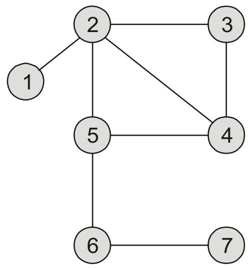
Q.40 Consider two vertices '*a*' and '*b*' that are simultaneously on the FIFO queue at same point during the execution of breadth first search from '*s*' in an undirected graph. Which of the following statements is/are correct?
(a) The number of edges on the shortest path between '*s*' and '*a*' is atmost one more than the number of edges on the shortest path between '*s*' and '*b*'.
(b) The number of edges on the shortest path between '*s*' and '*a*' is atleast one less than the number of edges on the shortest path between '*s*' and '*b*'.
(c) There is a path between '*a*' and '*b*'.
(d) None of these

Q.41 Consider the vertices '*a*' and '*b*' that are simultaneously on the function call stack at some point during the execution of DFS from vertices '*s*' in diagraph. Which of the following must be true?
(a) There exist directed path from *s* to '*a*' and directed path from *s* to '*b*'.
(b) There exist both a directed path from '*a*' to '*b*' and a directed path from '*b*' to '*a*'.
(c) If there is no directed path from '*a*' to '*b*' then there exist a directed path from *b* to '*a*'.
(d) None of these

Q.42 Consider a directed cyclic graph G with a property that it can be made acyclic by removing an edge from G. Assume depth first search is applied. Which of the following is/are incorrect?
(a) It will encounter no back edge.
(b) It will encounter exactly 1 back edge.
(c) It can encounter multiple number of back edges.
(d) DFS is not possible on G.

Q.43 Which of the following is/are correct?
(a) The depth of any DFS (Depth First Search) tree rooted at a vertex is at least as much as the depth of any BFS tree rooted at the same vertex.
(b) If all edges in a graph have distinct weight then the shortest path between 2 vertices is unique.
(c) For a directed graph, the absence of back edges in a DFS tree means graph has no cycle.
(d) BFS takes $O(V^2)$ time in a graph G(V, E) if graph is represented with an adjacency matrix.

Q.44 Consider the following graph G:



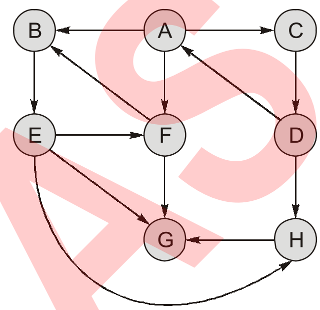
Which of the following is/are DFS sequence for the above graph G?
(a) 4, 5, 2, 1, 3, 6, 7 (b) 4, 5, 6, 7, 2, 1, 3
(c) 6, 5, 2, 1, 3, 4, 7 (d) 2, 3, 4, 1, 5, 6, 7

Q.45 Which of the following is correct regarding Dynamic programming?
(a) A problem that can be solved using dynamic programming possesses overlapping subproblems as well as optimal substructure properties.
(b) Overlapping subproblems is the property in which value of a subproblem is used several times.
(c) A greedy algorithm can be used to solve all the dynamic programming problems.
(d) Memorization is the technique in which previously calculated values are stored, so that, these values can be used to solve other subproblems.

Q.46 Which of the following is correct?
(a) When a top-down approach of dynamic programming is applied to a problem, it usually Decreases both, the time complexity and the space complexity.
(b) The fractional knapsack problem is not solved using dynamic programming.
(c) Longest common subsequence problems should be solved using Dynamic programming.
(d) Both Recursion and Dynamic Programming can be used to find nth Fibonacci numbers.

Q.47 Which of the following are correct with respect to Floyd Warshall's algorithm?
(a) Floyd Warshall's Algorithm is used for solving all pair shortest path problems.
(b) Floyd Warshall Algorithm can be applied in directed graphs.
(c) The running time of the Floyd Warshall Algorithm is $O(V^3)$.
(d) Floyd Warshall Algorithm can be used for finding Transitive closure.

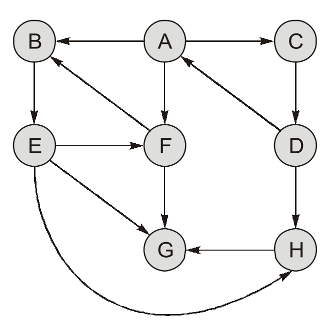
Q.48 Consider given graph, if DFS algo runs from vertex "A" and adjacency selection during DFS traversal is lexicographic order of vertex labels.



Which of the following statements is/are correct?
(a) Two back edges (b) Three cross edges
(c) Two cross edges (d) One forward edge

Q.49 If DFS traversal for directed graph:
X_s : Exploration begins
X_e : Exploration ends
Find correct order of starting and ending exploration of edge (u, v)?
(a) (u, v) back edge $V_s < U_s < U_e < V_e$
(b) (u, v) tree edge $U_s < V_s < V_e < U_e$
(c) (u, v) cross edge $V_s < V_e < U_s < U_e$
(d) (u, v) forward edge $U_s < V_s < V_e < U_e$

Q.50 Consider given graph, if BFS algo runs from vertex "A" and adjacency selection during BFS traversal is lexicographic order of vertex labels.

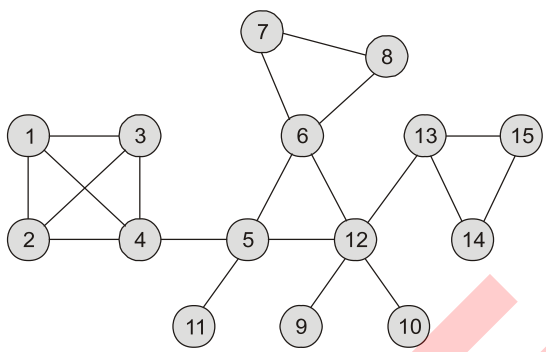


Which of the following statements is/are correct?

(a) One back edges (b) Five cross edges
(c) Four cross edges (d) One forward edge

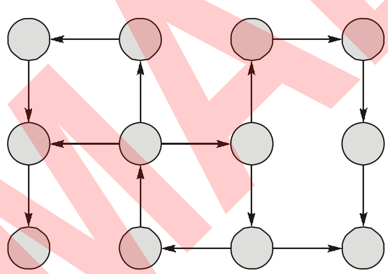
Q.51 Which of the correct statement?
(a) Forward edges not possible in BFS traversal of directed graph.
(b) Forward edges not possible in BFS traversal of un-directed graph.
(c) Forward edges not possible in DFS traversal of directed graph.
(d) Forward edges not possible in DFS traversal of un-directed graph.

Q.52 Consider given graph:



Which is/are correct statements?
(a) Total 9 bi-connected components in graph.
(b) Total 7 bi-connected components in graph.
(c) Total 5 cut vertices in graph.
(d) Total 4 cut vertices in graph.

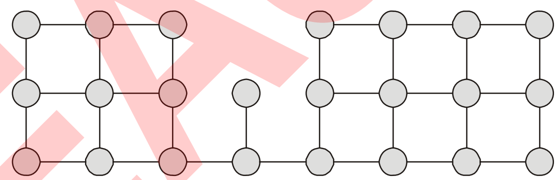
Q.53 Consider given graph:



(a) Number of strongly connected components are 9.
(b) Number of strongly connected components of single vertex are 8.
(c) Number of strongly connected components are 7.
(d) No back edges of DFS traversal of graph.

Q.54 Consider given claims
(a) If an edge (u, v) is contained in some minimal spanning tree, than it is a light edge crossing some cut of the graph.
(b) e be a maximum cost edge on some cycle of graph G with distinct edge costs, every MST of graph G excludes edge e.
(c) If (u, v) be a minimum edge in graph G then (u, v) belongs to some minimum spanning tree of graph G.
(d) If (u, v) be a minimum edge in graph G than (u, v) belongs to every minimal spanning tree of graph G.

Q.55 Consider the given graph:



Which of the following statements is/are true?
(a) Min stack entries required for DFS traversal of graph is 9.
(b) Max stack entries required for, DFS traversal of graph is 22.
(c) Max queue entries required BFS traversal of graph is 7.
(d) Max queue entries required BFS traversal of graph is 8.

Try Yourself

T1. The pseudo code for insertion sort is presented as a procedure called **Insertion-sort**, which takes array A[1...n] as parameter containing sequence of n elements to be sorted. Find the missing statements at X and Y respectively.

```
Insertion-sort (A)
{     for j ← 2 to length [A]
      {     key ← A[j]
            i = [ X ]
            While (i > 0 && A[i] > key)
```

```
        {    A[i + 1] ← A[i]
             i = [ Y ]
        }
        A[i + 1] ← key.
    }
}
```

(a)

X	Y
$j-1$	$i+1$

(b)

X	Y
$j+1$	$i+1$

(c)

X	Y
$j-1$	$i-1$

(d)

X	Y
$i-1$	$j-1$

[Ans: (c)]

T2. Given n integers in the range of 0 to K we want to preprocess the input in such a way that to any query about how many of the n-integers fall in the range $a...b$ is $O(1)$. Then what will the preprocess time?

[Ans: O(n)]

T3. Let G be a undirected graph on n-nodes. Any two of the following statements implies the 3rd. Is it True/False?

1. G is connected
2. G don't have cycle
3. G contain $n - 1$ edges

[Ans: True]

T4. Let G = (V, E) be a simple undirected graph, and s be a particular vertex in it called the source. For $x \in$ V, let $d(x)$ denote the shortest distance in G from s to x. A breadth first search (BFS) is performed starting at s. Let T be the resultant BFS tree. If (u, v) is an edge of G that is not in T, then which one of the following CANNOT be the value of $d(u) - d(v)$?

(a) –1 (b) 0
(c) 1 (d) 2 **[Ans: (d)]**

T5. The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is ________.
Note: The height of a tree with a single node is 0).

[GATE-2016, Ans: (64)]

T6. Given a sequence of n-real numbers $a_1, a_2, a_3 \ldots a_n$ then to find continuous subsequence $a_i, a_{i+1}, a_{i+2} \ldots a_j$. Such that its sum is maximum. How much time the above problem will take if you use dynamic program?

[Ans: O(n)]

T7. Let A_1, A_2, A_3 and A_4 be four matrices of dimensions 10×5, 5×20, 20×10, and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1 A_2 A_3 A_4$ using the basic matrix multiplication method is _______.

[GATE-2016, Ans: (1500)]

■■■■