How to Guide: I-V Characteristics

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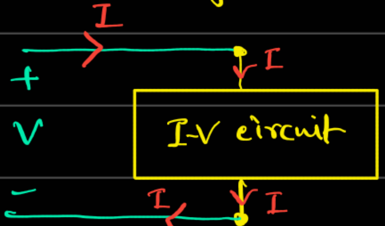
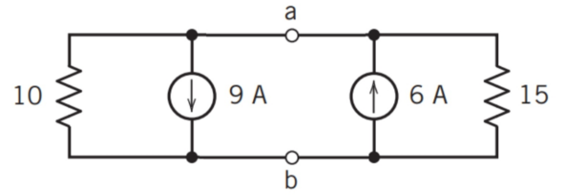
- Please read through all the details.
- Both methods are useful. If asked for a graph first method is a bit easier. If asked for equation better use Method 2.
- Both methods are actually same thing. If you are having trouble working with x volt and y current, use 1V or 1A to get a sense of how you should solve it.
- You may use any circuit solving techniques such as: KCL, KVL, Nodal/Mesh Analysis, Source transformation, superposition etc.
- Keep Passive Sign Convention in mind. That is:

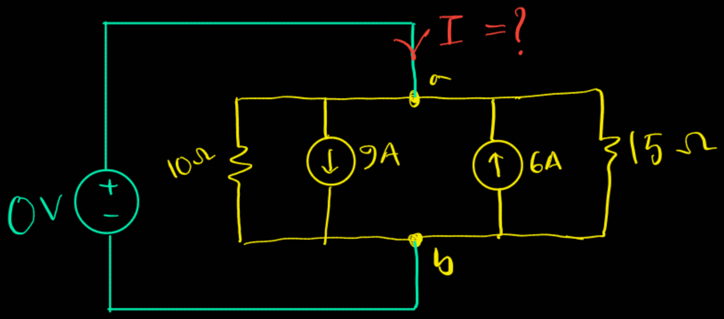


Current should always enter through the (+ve) terminal and leave via (-ve).

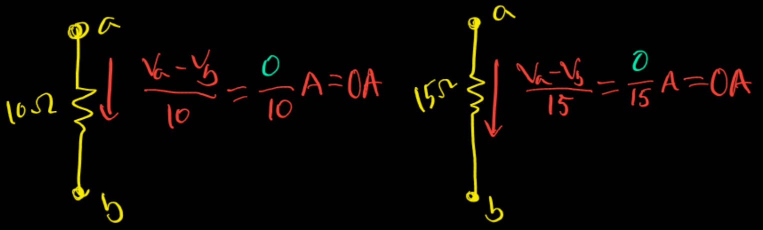
Problem 1

- Derive the $I - V$ characteristics of the following circuit with respect to the terminals $a - b$.





$$V_a - V_b = 0V$$



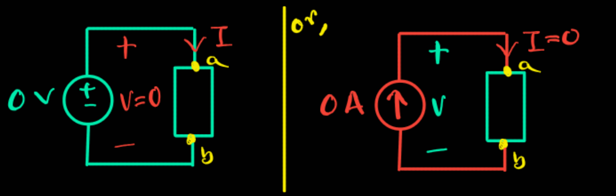
$$\frac{V_a - V_b}{10} = \frac{0}{10} A = 0A \qquad \frac{V_a - V_b}{15} = \frac{0}{15} A = 0A$$

Method 1: Two data points

① Apply some voltage across the terminals using a 0V voltage source. (You can use any voltage you like.)

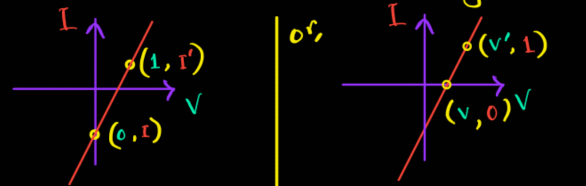
⊛ Use 0A current source if voltage source can't be applied (e.g. another voltage src in parallel)

② Find current I supplied by the voltage source. (Notice the direction of current. Must follow passive sign convention for the circuit given in question)

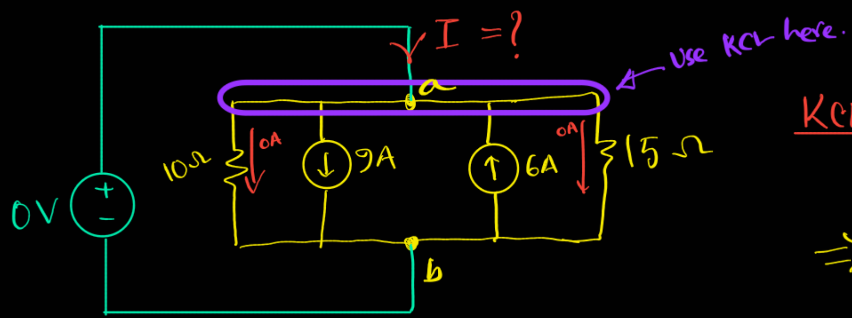


③ Repeat for 1V, find new current I'.

④ Plot the two data points in graph and connect them via straight line.



⑤ Use $y = mx + c$ to get equation.



KCL @ Node a:

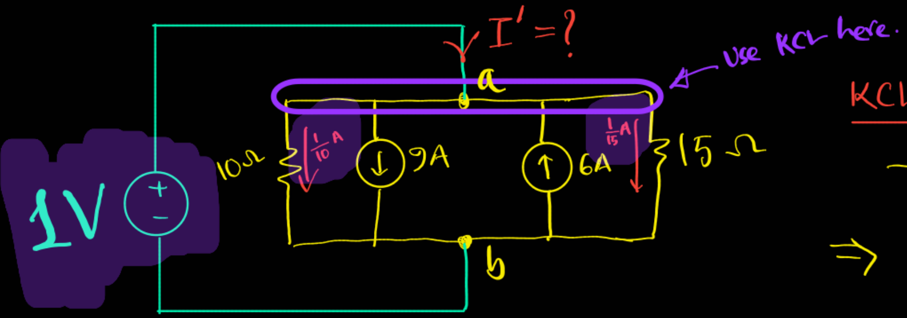
$$-I + 0 + 9 - 6 + 0 = 0$$

$$\Rightarrow I = 9 - 6$$

$$\therefore I = 3A$$

$\therefore$ one data point $\Rightarrow (0, 3)$

Similarly, for 1V,



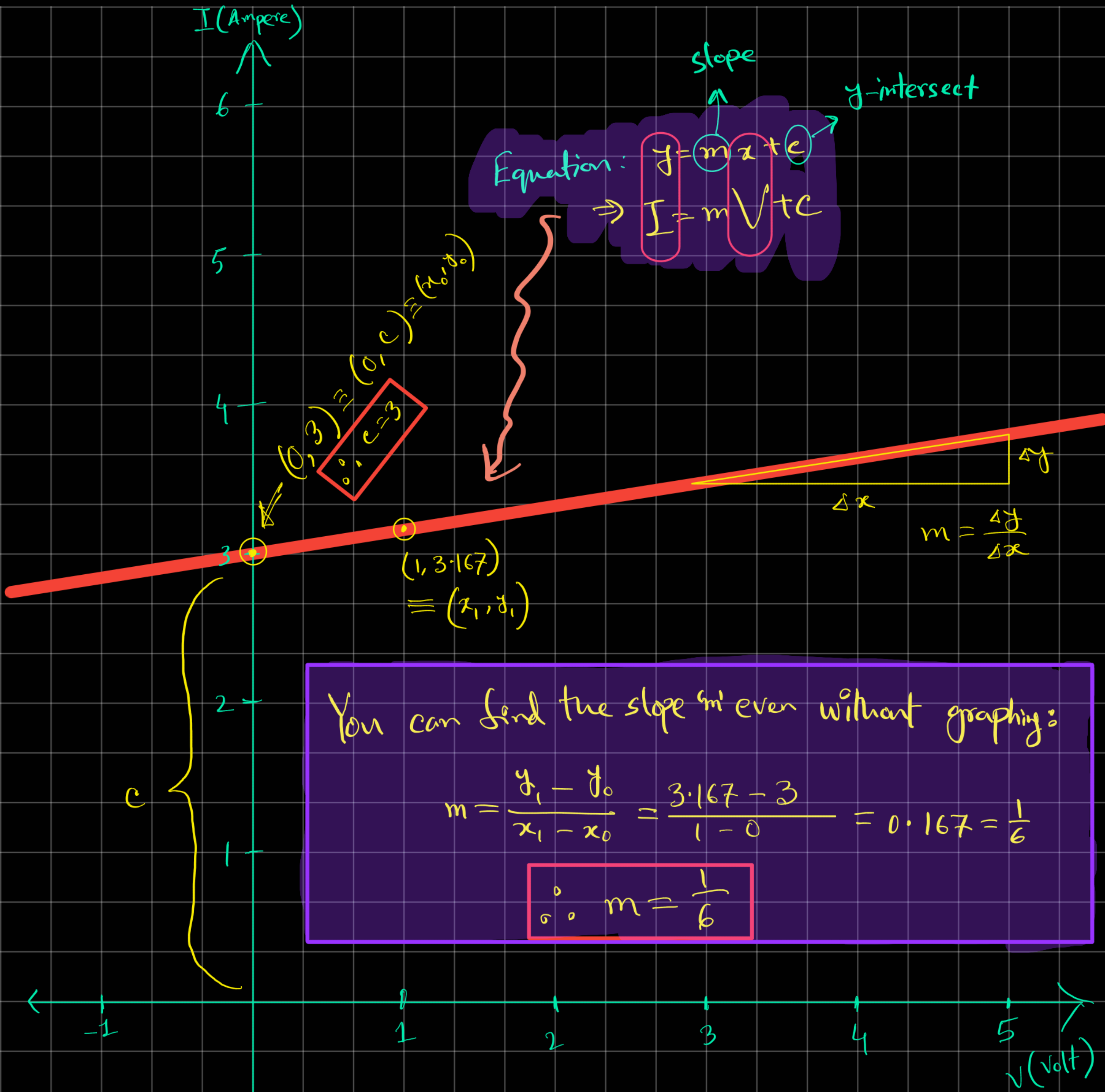
KCL @ Node a:

$$-I' + \frac{1}{10} + 9 - 6 + \frac{1}{15} = 0$$

$$\Rightarrow I' = \frac{1}{10} + 9 - 6 + \frac{1}{15}$$

$$\therefore I' = 3.167 A$$

$\therefore$ second data point $\Rightarrow (1, 3.167)$



Equation: $y = mx + c$

$\Rightarrow I = mV + c$

$m = \dfrac{\Delta y}{\Delta x}$

You can find the slope 'm' even without graphing:

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{3.167 - 3}{1 - 0} = 0.167 = \frac{1}{6}$$

$$\boxed{\therefore\ m = \frac{1}{6}}$$

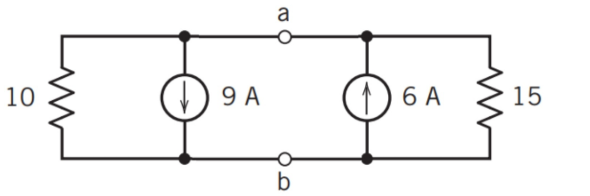
$$y = mx + c$$

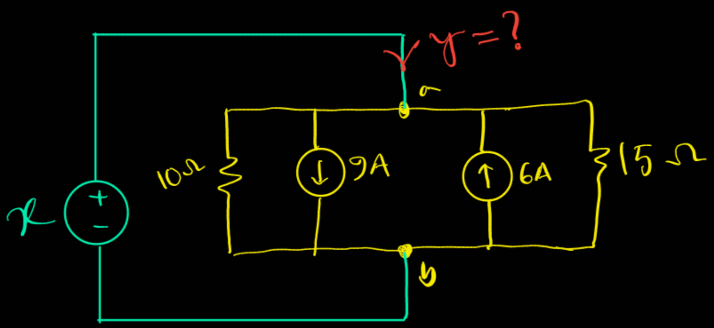
$$\Rightarrow y = \frac{1}{6}x + 3$$

$$\Rightarrow I = \frac{V}{6} + 3$$

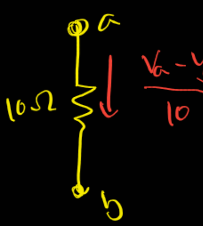
Problem 1

- Derive the $I - V$ characteristics of the following circuit with respect to the terminals $a - b$.



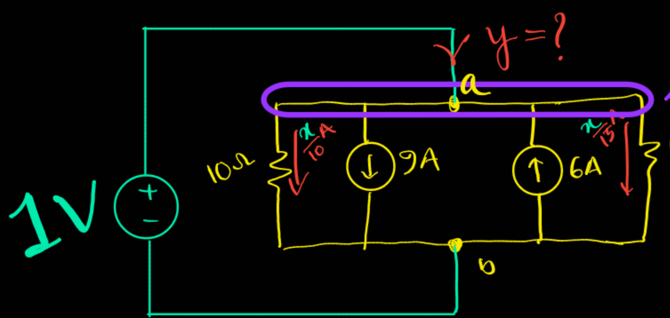


$V_a - V_b = x$



$\frac{V_a - V_b}{10} = \frac{x}{10}$ A

$\frac{V_a - V_b}{15} = \frac{x}{15}$ A



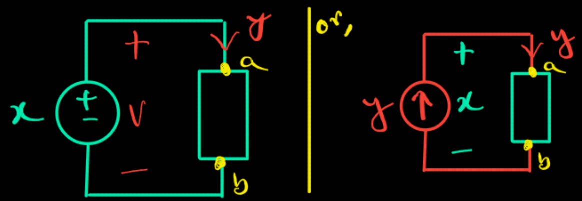
Use KCL here.

Method 2: x Volt − y current.

① Apply x voltage across the terminals using a voltage source of x volt. (This just like the 1V case as before, except you replace 1V with x everywhere)

⊗ Use current source if voltage source can't be applied (e.g. another voltage src in parallel)

② Find the current y supplied by the voltage source to the circuit.



③ Replace y with I and x with V for final answer.

KCL @ Node a:

$$-y + \frac{x}{10} + 9 - 6 + \frac{x}{15} = 0$$

$$\Rightarrow y = \frac{x}{10} + 9 - 6 + \frac{x}{15}$$

$$\Rightarrow y = x\left(\frac{1}{10} + \frac{1}{15}\right) + 3$$

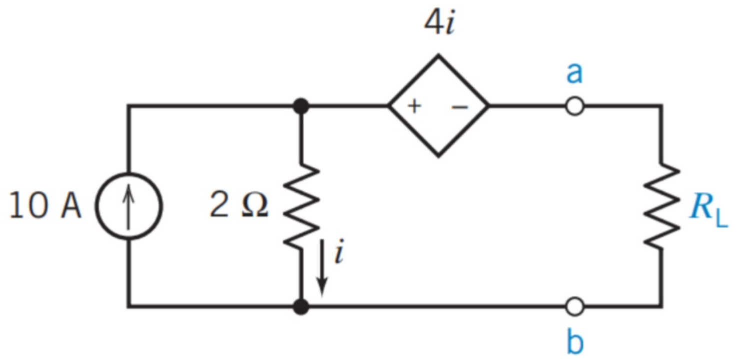
$$\therefore \boxed{y = \frac{x}{6} + 3}$$

You directly get the equation !!

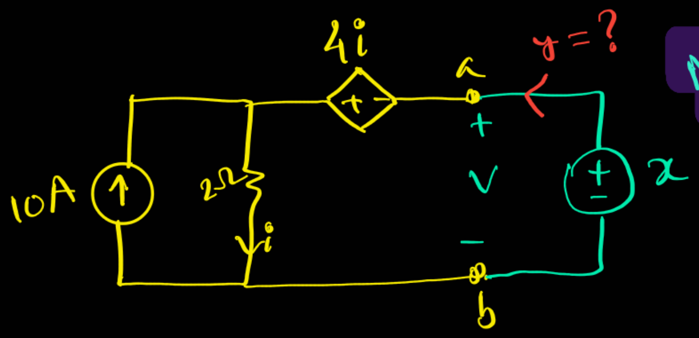
$$I = \frac{V}{6} + 3$$

Problem 2

- Derive the $I - V$ characteristics of the portion left to the terminals $a - b$.

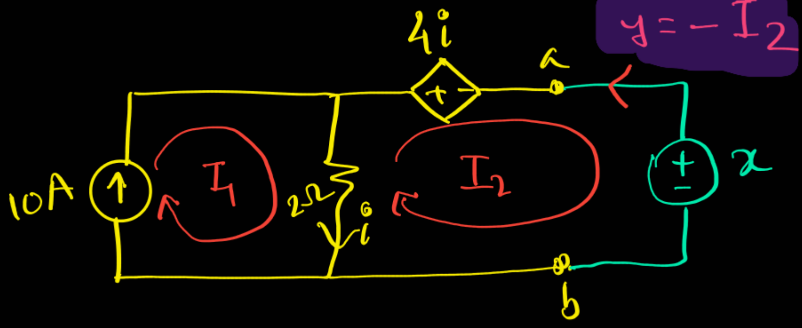


Ans: $I = -\frac{1}{2}V - 10$



Notice the direction of current y. Follows passive sign convention for the I-V circuit.

Using Mesh analysis:



$I_1 = 10$

KVL@2: $I_2(2) - 2I_1 + 4i + x = 0$

$\Rightarrow 2I_2 - 2 \times I_1 + 4(I_1 - I_2) + x = 0$

$\Rightarrow 2(-y) - 2 \times 10 + 4(10 - (-y)) + x = 0$

Already a relation between x & y.

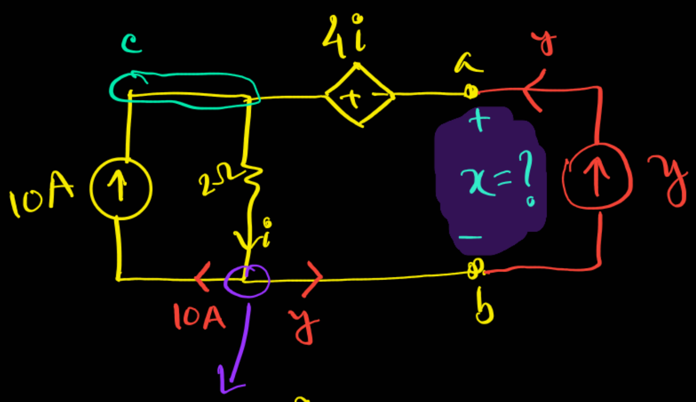
$\Rightarrow -2y - 20 + 40 + 4y + x = 0$

$\Rightarrow x + 2y + 20 = 0$

$\Rightarrow y = -\frac{x}{2} - 10$

$I = -\frac{V}{2} - 10$

There is actually an easier way to solve Problem 2. You don't even need Mesh Analysis. Read this first:



KCL: $i = 10 + y$

$$x = V_a - V_b$$
$$= V_a - V_c + V_c - V_b$$
$$= -(V_c - V_a) + (V_c - V_b)$$
$$= -(4i) + 2 \times i$$
$$= -2i$$

$\therefore\ x = -2(10 + y)$

$\Rightarrow -\dfrac{x}{2} = 10 + y$

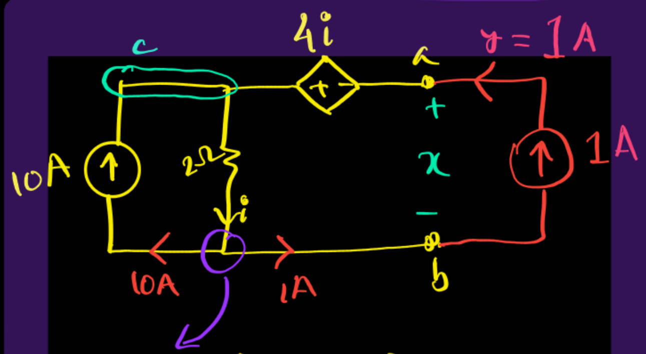
$\therefore\ y = -\dfrac{x}{2} - 10$

$\therefore\ \boxed{I = -\dfrac{V}{2} - 10}$

Rough

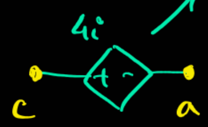
use 1A instead of y if you can't figure out what to do.

Here is the example:

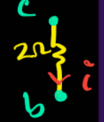


KCL: $i_o = (10 + 1)A = 11A$

$$x = V_a - V_b = V_a - V_c + V_c - V_b$$
$$= -(V_c - V_a) + (V_c - V_b)$$
$$= -(4i) + 2 \times i$$



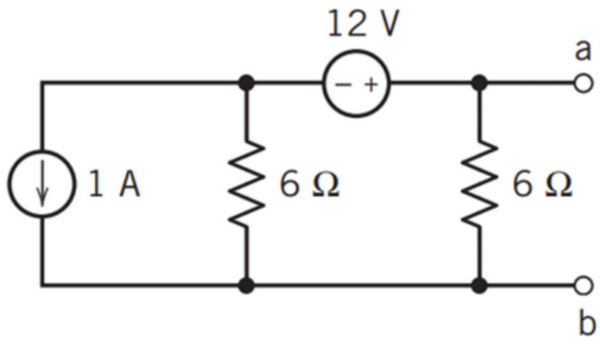
Ohm's Law for 2Ω



$$= -2i$$
$$= -2 \times 11\ V$$
$$= -22\ V$$

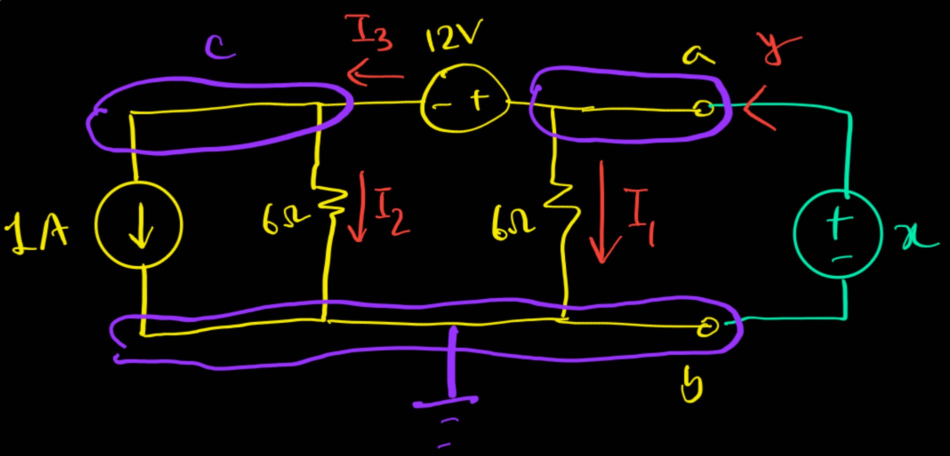
Problem 3

- From the following circuit, derive the current–voltage characteristics equation between the terminals $a - b$.



Ans: $I = \frac{1}{3}V - 1$

Try to solve this yourself by applying 1V first.
Then replace 1V with x. (I'm doing it with just x)
Do this for all problems discussed later as well.



$V_a - V_b = x \Rightarrow V_a = x$

$V_a - V_c = 12 \Rightarrow x - V_c = 12 \Rightarrow V_c = x - 12$

$I_1 = \dfrac{V_a - V_b}{6} = \dfrac{x}{6}$, $I_2 = \dfrac{V_c - V_b}{6} = \dfrac{(x-12) - 0}{6} = \dfrac{x}{6} - 2$

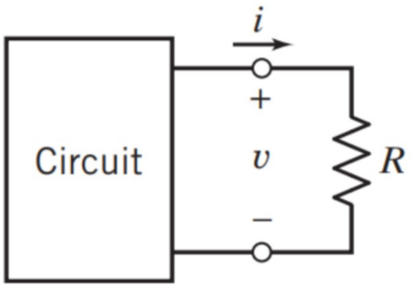
KCL @ c: $I_3 = 1 + I_2$

KCL @ a: $y = I_1 + I_3 = I_1 + 1 + I_2 = \dfrac{x}{6} + 1 + \dfrac{x}{6} - 2$

$\therefore y = \dfrac{x}{3} - 1 \Rightarrow \boxed{I = \dfrac{V}{3} - 1}$

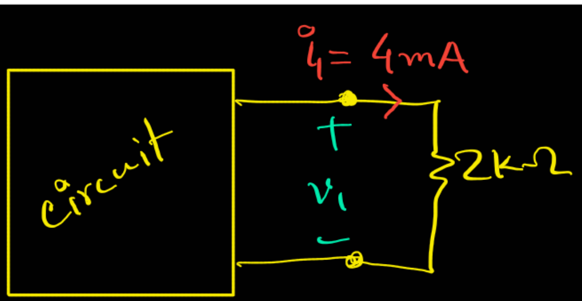
Problem 4

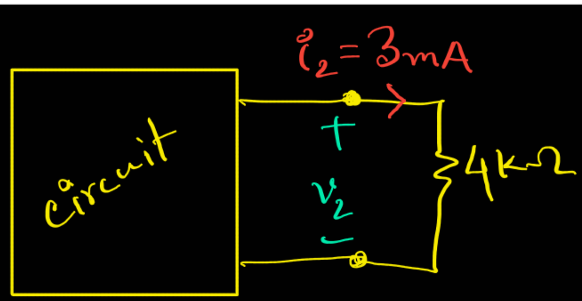
- A resistor, R, was connected to a circuit box as shown below. The current i was measured. The resistance was changed, and the current was measured again. The results are shown in the table.
  - i. Plot the relationship between i and v.
  - ii. Draw a circuit diagram with minimum number of circuit elements that can give rise to the same $i - v$ curve derived in i.



R	i
2 kΩ	4 mA
4 kΩ	3 mA

Ans: $\boldsymbol{i} = -\frac{1}{4}\boldsymbol{v} + \boldsymbol{6}$





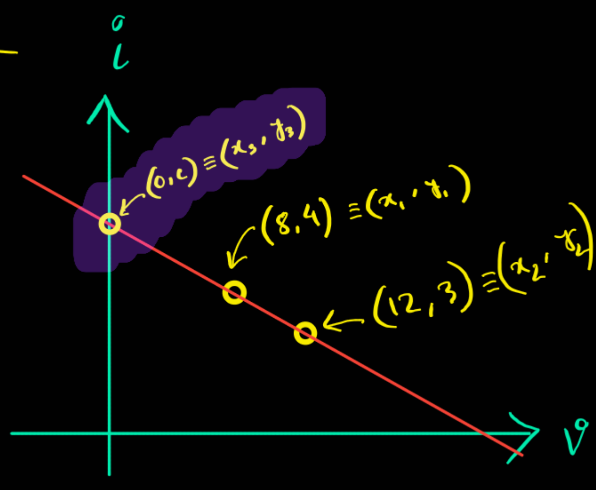
$v_1 = 4mA \times 2k\Omega$
$= 8V$

$v_2 = 3mA \times 4k\Omega$
$= 12V$

Data Point 1: (v_1, i_1)
$= (8, 4)$

Data Point 2: (v_2, i_2)
$= (12, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{12 - 8} = \frac{-1}{4}$$



⊛ But how do we find 'c' ?

Notice that (x_1, y_1), (x_2, y_2) and $(0, c)$ are all points on the line.

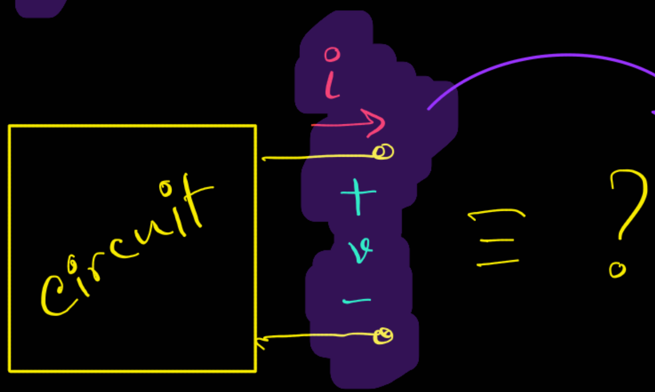
Since (x_1, y_1) and $(x_3, y_3) \equiv (0, c)$ are on the same line, slope 'm' should be same as before $\left(-\frac{1}{4}\right)$.

$$m = \frac{y_3 - y_1}{x_3 - x_1} = \frac{c-4}{0-8} = \frac{c-4}{-8}$$

But, $m = \frac{-1}{4}$. So, $\frac{c-4}{-8} = \frac{-1}{4} \Rightarrow c = 4 + \frac{8}{4} = 6$

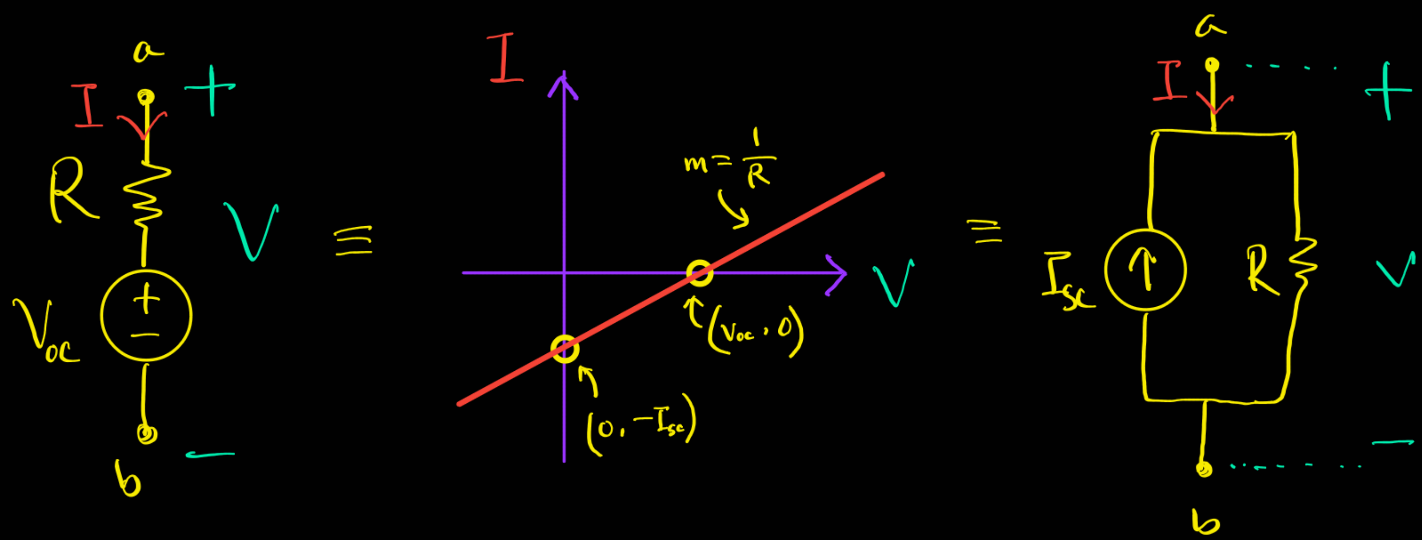
$$\therefore y = mx + c \Rightarrow y = -\frac{1}{4}x + 6 \Rightarrow \boxed{i = -\frac{v}{4} + 6}$$

(*) But how to find a proper circuit?

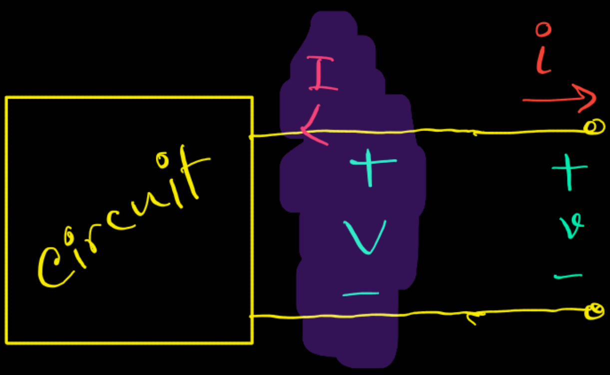


Notice how v and i are not following passive sign convention. This will be important.

We need to memorize a few I-V circuit:



Now, Since our given circuit doesn't follow passive sign convention, Let's fix it. I am denoting the proper direction with capital letters.



Follows passive sign convention.

$$V = v \,, \quad I = -i$$

We know from part (i) of the question,

$$i = \frac{-v}{4} + 6 \Rightarrow -I = \frac{-V}{4} + 6$$

$$\Rightarrow I = \frac{V}{4} - 6$$

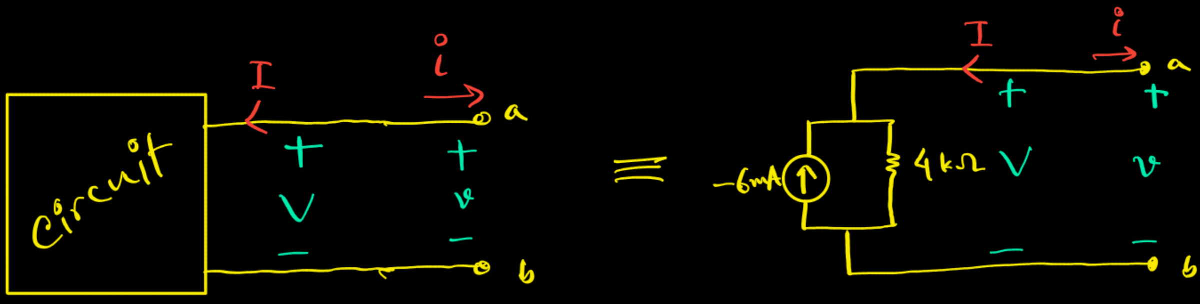
Now the slope is positive as expected.

$$\therefore m = \frac{1}{4} = \frac{1}{R} \Rightarrow \boxed{R = 4\ k\Omega}$$

Taking, $V = 0$, we get, $I = \frac{0}{4} + 6 = 6$

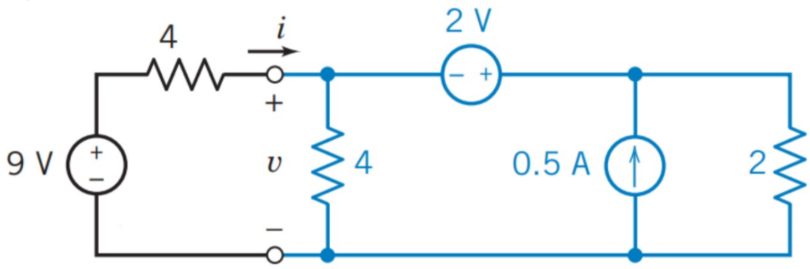
So, it goes through $(0, 6) \equiv (0, -I_{sc})$ data point.

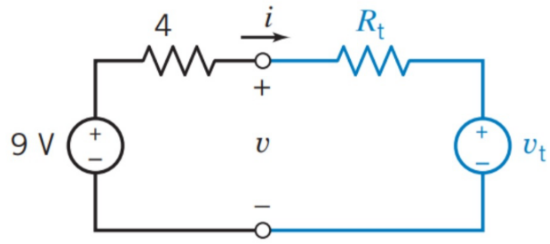
$$\therefore 6 = -I_{sc} \Rightarrow \boxed{I_{sc} = -6\,mA}$$



Problem 5

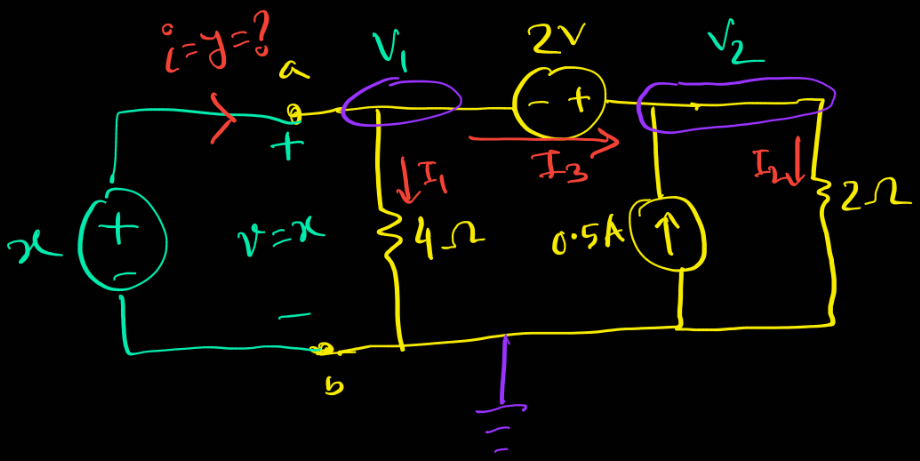
- Determine the values of R_t and v_t, if the following two circuits are equivalent to each other.





Ans: $v_t = -2/3\,V,\ R_t = 4/3\ \Omega$

Since the black part is same on both circuits, we only need to find the equivalent of the blue part of the circuit.



$$V_1 - 0 = x \Rightarrow V_1 = x$$

$$V_2 - V_1 = 2 \Rightarrow V_2 = V_1 + 2 = x + 2$$

$$I_1 = \frac{V_1 - 0}{4} = \frac{x}{4} \quad , \quad I_2 = \frac{V_2 - 0}{2} = \frac{x+2}{2} = \frac{x}{2} + 1$$

KCL @ 2: $\quad -I_3 - 0.5 + I_2 = 0 \Rightarrow I_3 = I_2 - 0.5 = \frac{x}{2} + 1 - 0.5$

KCL @ 1: $\quad y = I_1 + I_3 = \frac{x}{4} + \frac{x}{2} + 1 - 0.5 = \frac{3x}{4} + 0.5$

$$\therefore \boxed{I = \frac{3V}{4} + 0.5}$$

Comparing with $y = mx + c$,

$$m = \frac{3}{4} = \frac{1}{R} \Rightarrow \boxed{R = \frac{4}{3}\ \Omega}$$

In order to find v_t we set $V = v_t$ and $I = 0$, because the I-V of voltage src + series resistor goes through $(v_t, 0)$ data point.

So, $0 = \frac{3v_t}{4} + 0.5$

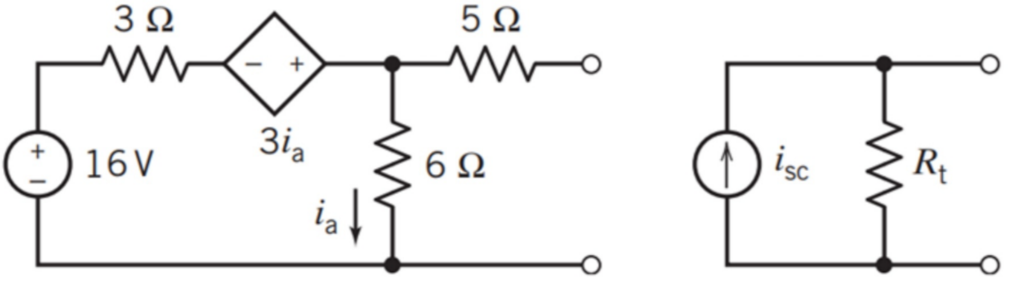
$$\Rightarrow \frac{3v_t}{4} = -0.5$$

$$\Rightarrow v_t = -0.5 \times \frac{4}{3} = -\frac{2}{3}\ \text{V}$$

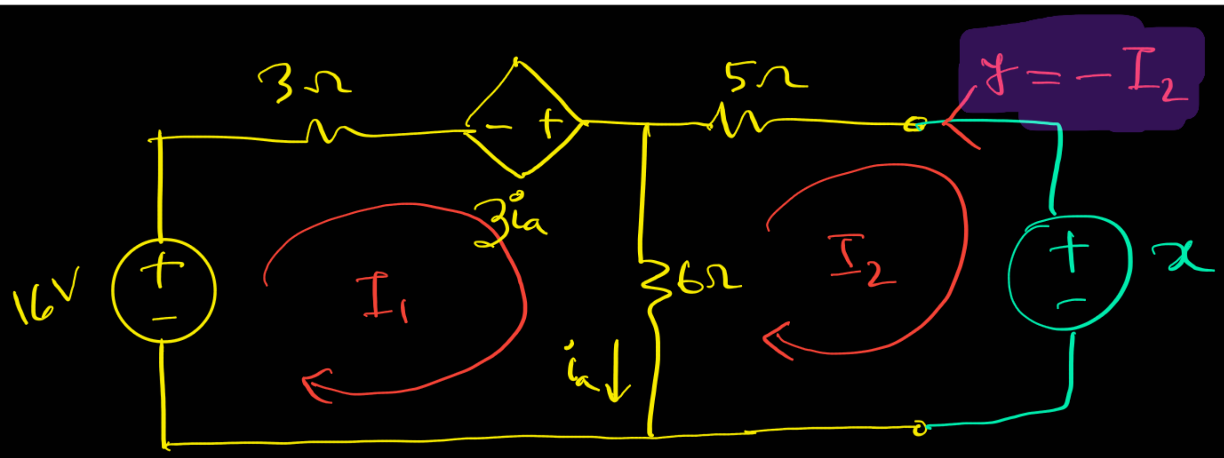
$$\therefore \boxed{v_t = -2/3\ \text{V}}$$

Problem 6

- Determine the values of R_t and i_{sc}, if the following two circuits are equivalent to each other.



Ans: $i_{sc} = 2\ A,\ R_t = 8\ \Omega$



KVL@1: $I_1(3+6) - 6I_2 - 16 - 3i_a = 0$

$\Rightarrow 9I_1 - 6I_2 - 16 - 3(I_1 - I_2) = 0$

$\Rightarrow 6I_1 - 3I_2 - 16 = 0$

$\Rightarrow 6I_1 + 3y - 16 = 0$ — (I)

KVL@2: $I_2(5+6) - 6I_1 + x = 0$

$\Rightarrow 11I_2 - 6I_1 + x = 0$

$\Rightarrow -11y - 6I_1 + x = 0$ — (II)

(I)+(II) $\Rightarrow \cancel{6I_1} + 3y - 16 - 11y - \cancel{6I_1} + x = 0$

$\Rightarrow 8y = x - 16 \Rightarrow y = \frac{x-16}{8} \Rightarrow \boxed{y = \frac{x}{8} - 2}$

Comparing with $y = mx + c$,

$$m = \frac{1}{8} = \frac{1}{R_t} \Rightarrow \boxed{R_t = 8\ \Omega}$$

In order to find i_{sc} we set $V=0$ and $I = -i_{sc}$, because the I-V of current src + parallel resistor goes through $(0, -i_{sc})$ data point.

So, $$y = \frac{x}{8} - 2$$

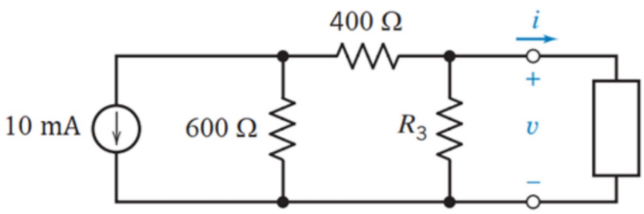
$$\Rightarrow -i_{sc} = \frac{0}{8} - 2$$

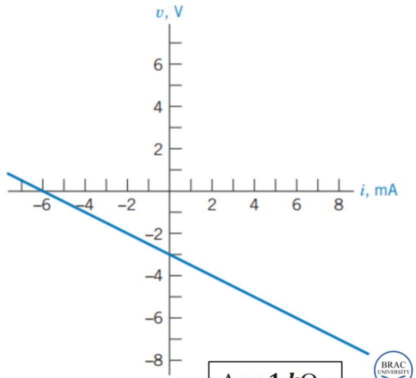
$$\Rightarrow -i_{sc} = -2\ A$$

$$\therefore \boxed{i_{sc} = 2\ A}$$

Problem 7

- If the voltage v vs. current i has the following relationship expressed graphically, determine the value of R_3.





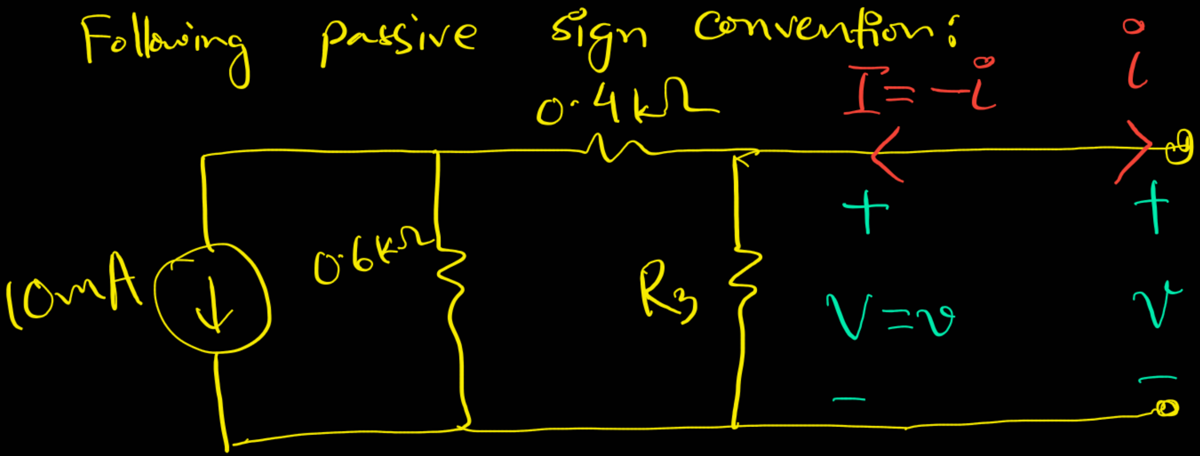
Ans: **1 kΩ**

Notice that i and v does not follow passive sign convention.

Also notice that the graph axes are flipped.

Here, i is in x-axis and v is in y-axis.

Following passive sign convention:



Choosing two data points from graph:

① $(-6, 0) \equiv (i_1, v_1) \equiv (-I_1, V_1) \Rightarrow V_1 = 0V,\ I_1 = 6\text{ mA}$

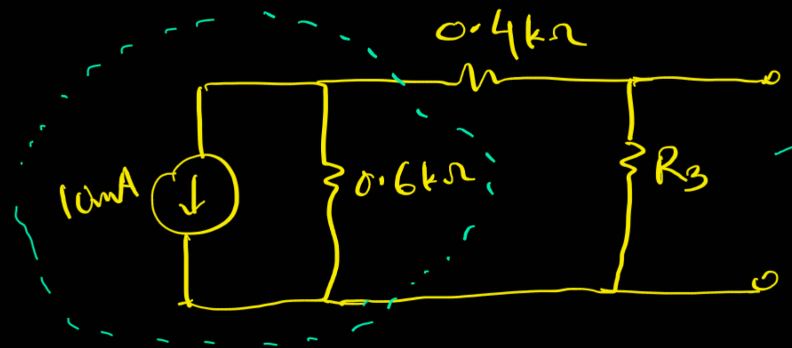
② $(0, -3) \equiv (i_2, v_2) \equiv (-I_2, V_2) \Rightarrow V_2 = -3V,\ I_2 = 0\text{mA}$

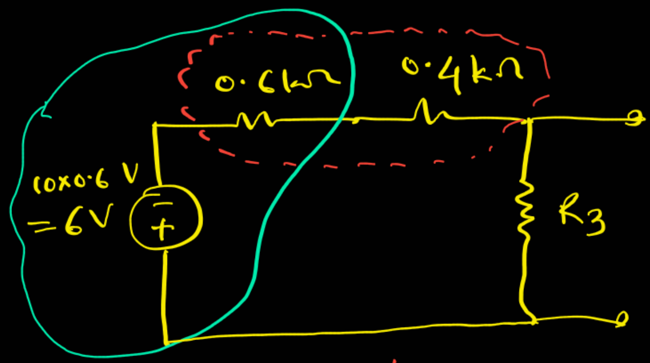
$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{I_2 - I_1}{V_2 - V_1} = \frac{0 - 6}{-3 - 0} = 2$$

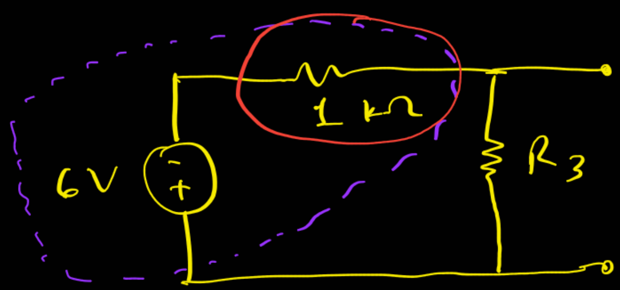
$$\therefore m = \frac{1}{R} = 2 \Rightarrow R = \frac{1}{2}\,k\Omega = 0.5\,k\Omega$$

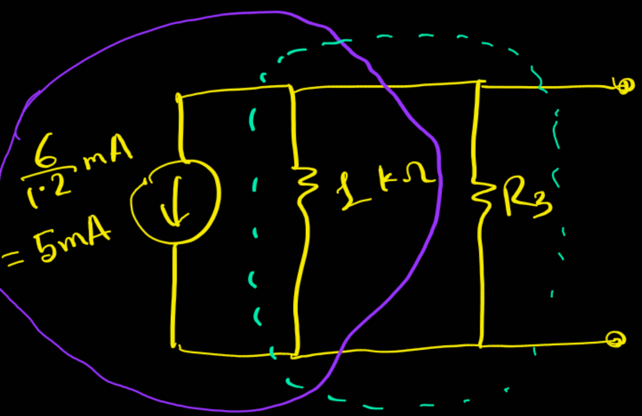
$$\therefore R = 0.5\,k\Omega$$

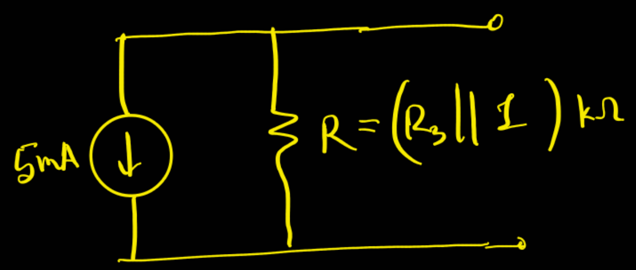
This is not R_3. It is the overall equivalent resistance. We can use source transformation to find the overall equivalent resistance.











Since, R is the parallel equivalent of R_3 and $1\ k\Omega$,

$$\frac{1}{R} = \frac{1}{R_3} + \frac{1}{1}$$

$$\Rightarrow \frac{1}{0.5} = \frac{1}{R_3} + \frac{1}{1}$$

$$\Rightarrow \frac{1}{R_3} = \frac{1}{0.5} - \frac{1}{1} = 2 - 1 = 1$$

$$\Rightarrow \boxed{R_3 = 1\ k\Omega}$$