

Department of Computer Science and Engineering (CSE)
BRAC University

Fall 2023

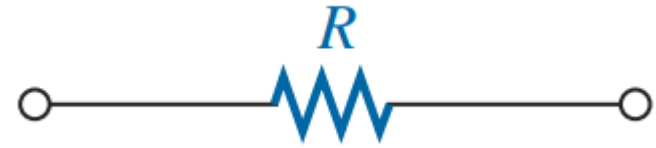
CSE250 - Circuits and Electronics

SERIES AND PARALLEL RESISTORS

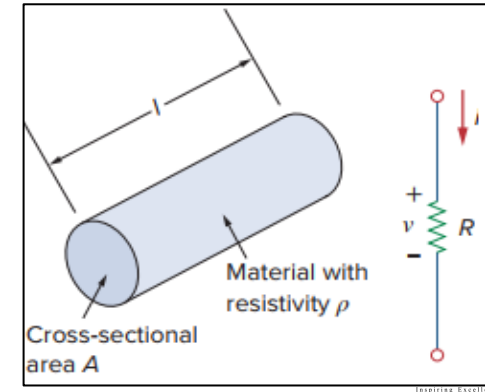


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Resistance



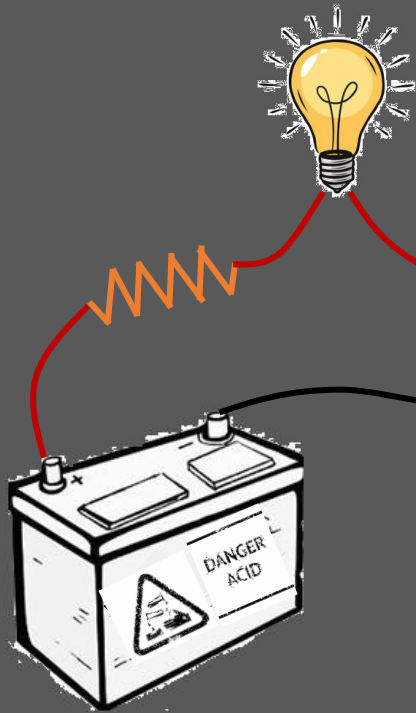
- What determines the level of current that results when a particular voltage is applied across a wire?
The answers lie in the fact that there is an opposition to the flow of charge in the system that depends on the components of the circuit. This opposition to the flow of charge through an electrical circuit, called resistance.
- This opposition, due primarily to collisions and friction between the free electrons and other electrons, ions, and atoms in the path of motion, converts the supplied electrical energy into heat that raises the temperature of the electrical component and surrounding medium.
- So, *resistance* is a physical property of materials that refers to the ability to resist current.
- The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ . Mathematically, $R = \rho \frac{\ell}{A}$, where ρ is known as the resistivity of the material in ohm-meters ($\Omega - m$).
- The measuring unit for resistance is *ohm* (Ω)



Conductance

- By finding the reciprocal of the resistance of a material, we have a measure of how well the material conducts electricity. The quantity is called conductance, has the symbol G , and is measured in *siemens* (S) or *mhos* (\mathcal{U})
- So, the *conductance* is a measure of how well an element will conduct electric current.
- $G = \frac{1}{R}$ [$1 \mathcal{U} = 1 A/V = 1 \text{ Siemen } (S)$]
- $G = \frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma A}{L}$, where $\sigma = \frac{1}{\rho}$ is a material-specific parameter called *conductivity*, measured in *siemens per meter* ($S m^{-1}$)

Course Outline: broad themes



Circuit Laws

Ohm's
Law



Methods of
Analysis

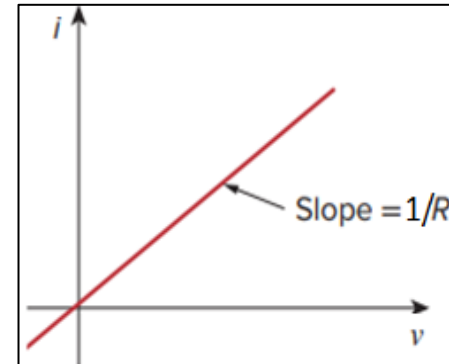
Circuit
Theorems

First Order
Circuits

AC Circuits

Ohm's Law

- One of the basic equations for any physical system is, $Effect = \frac{Cause}{Opposition}$. Every conversion of energy from one form to another can be related to this equation.
- In electric circuits, the effect we are trying to establish is the flow of charge, or **current**. The potential difference, or **voltage**, between two points is the cause (“pressure”), and the opposition is the **resistance** encountered. Substituting the terms,
- $Current = \frac{Voltage}{Resistance} \Rightarrow I = \frac{V}{R}$
- **Ohm's law** states that the voltage across a resistor is directly proportional to the current flowing through the resistor.
- That is, $v \propto i$ or $v = Ri$. Ohm defined the constant of proportionality for a resistor to be the resistance, R , measured in **ohm** (Ω). [$1 \Omega = 1 \text{ V/A}$]
- $p = vi = i^2 R = \frac{v^2}{R}$ (always +ve for passive elements like resistor)

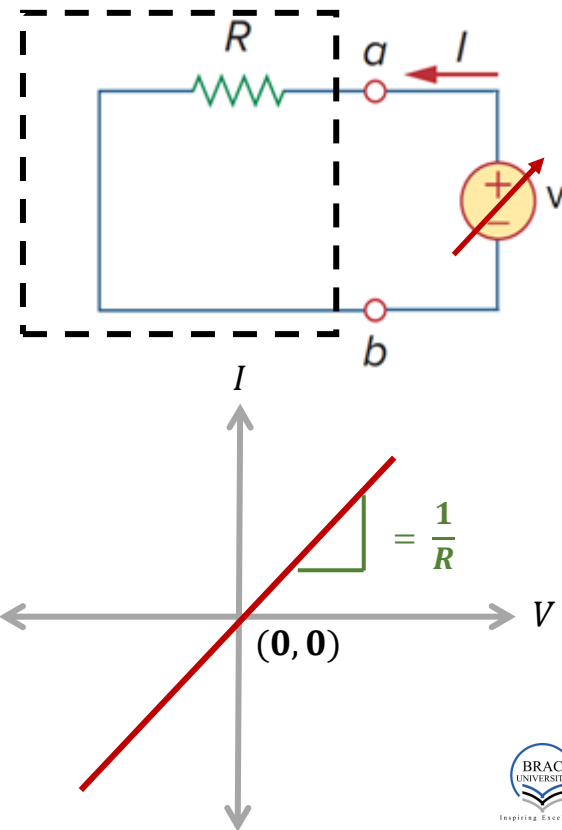


I-V characteristics of a Resistor

- The current voltage relationship or *I – V characteristics* of a resistor follows Ohm's law.
- Let's say we have a resistor (R) between terminals a and b . To determine the $I – V$ characteristics, if applying a voltage V between $a – b$ makes the resistor to draw a current I , we can write,

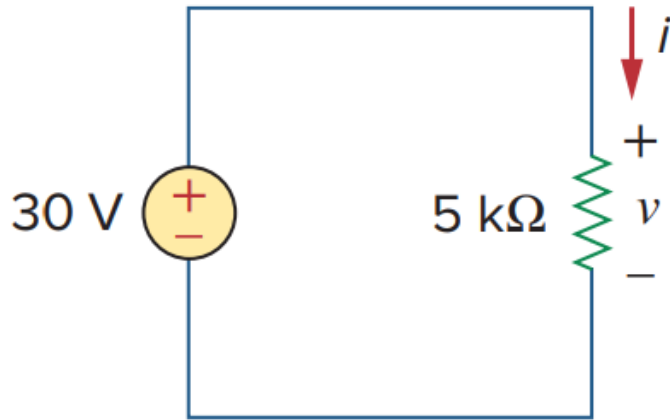
$$V = IR$$
$$\Rightarrow I = \frac{1}{R}V$$

- The equation results in a linear I vs V plot that intersects the axes at the origin.
- The slope of the line depends on the value of the resistance. The greater the resistance, the less current flows through it, and the curve deviates from the current axis and flattens.



Example 1

- In the circuit shown below, calculate the current i , the conductance G , and the power p .



Solution

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is,

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 0.006 \text{ A} = 6 \text{ mA}$$

The conductance is,

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.0002 \text{ S} = 0.2 \text{ mS}$$

The power can be calculated in various ways

$$p = vi = 30 \times (6 \times 10^{-3}) = 0.18 \text{ W} = 180 \text{ mW}$$

Or,

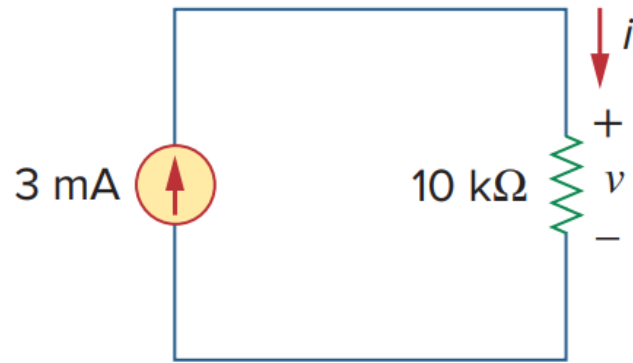
$$p = i^2 R = (6 \times 10^{-3})^2 \times (5 \times 10^3) = 180 \text{ mW}$$

Or,

$$p = \frac{v^2}{R} = \frac{30^2}{5 \times 10^3} = 180 \text{ mW}$$

Problem 1

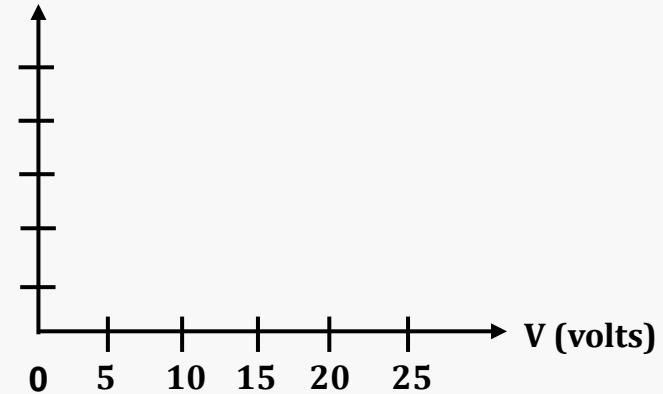
- i. For the circuit shown below, calculate the voltage v , the conductance G , and the power p .



Ans: (i) $v = 30 \text{ V}$; $G = 100 \mu\text{S}$; $p = 90 \text{ mW}$

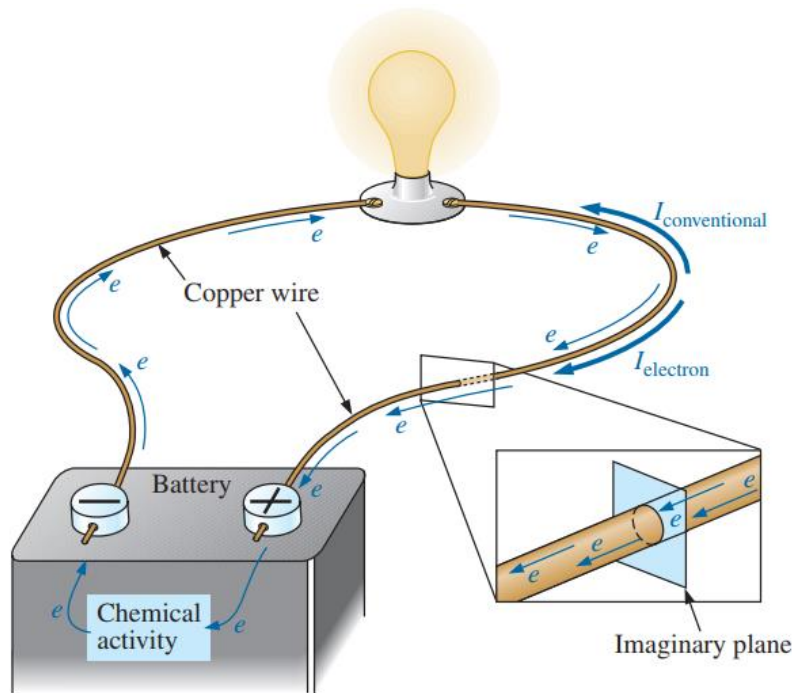
- ii. Draw the $I - V$ characteristics of a $10 \text{ k}\Omega$ resistor using the following template. Label the axes appropriately.

I (amperes)



Problem 2

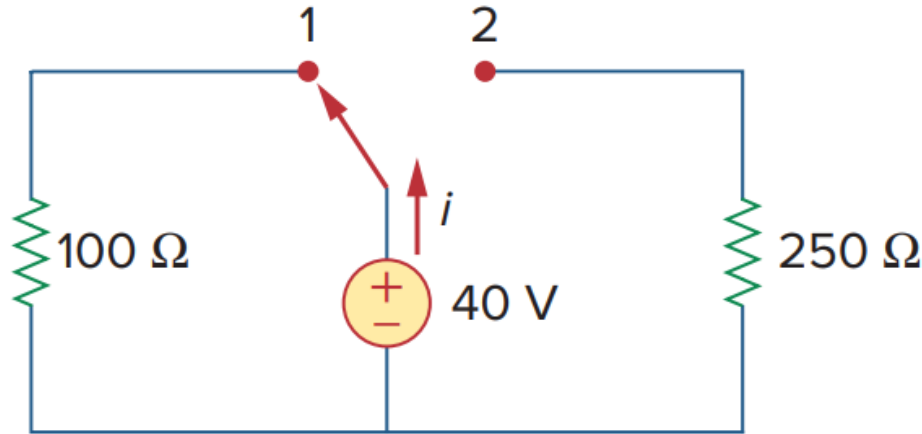
- Find the hot resistance of a light bulb rated 60 W, 120 V.



$$\overline{\text{Ans: } R = 240 \, \Omega}$$

Problem 3

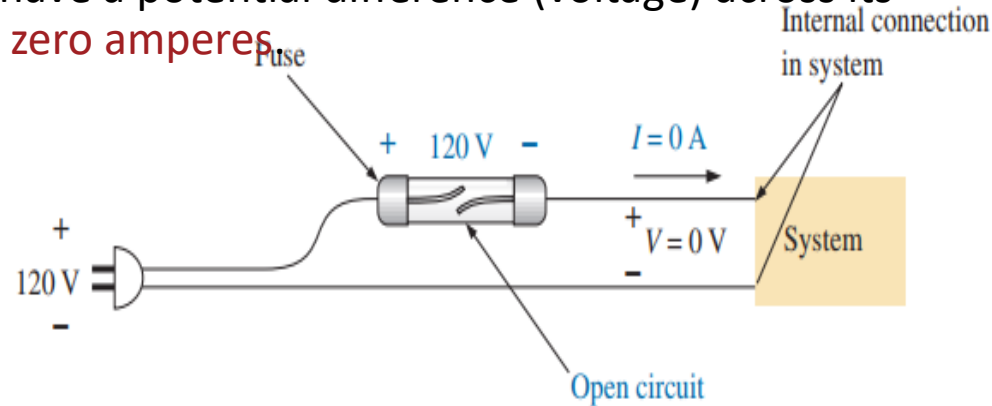
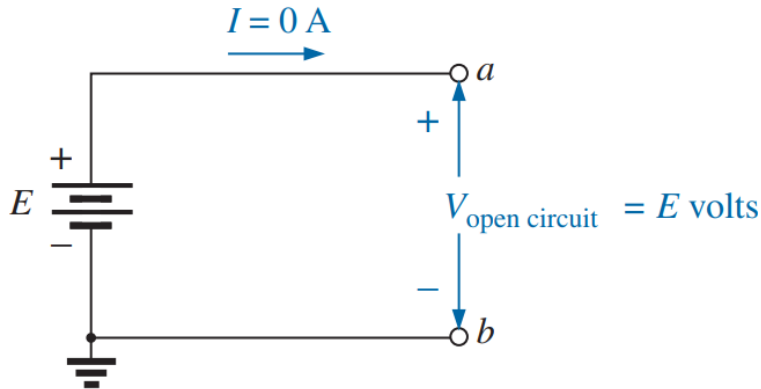
- (a) Calculate current i when the switch is in position 1.
(b) Find the current when the switch is in position 2.



Ans: (a) $i = 0.4\text{ A}$; (b) $i = 0.16\text{ A}$

Open circuit

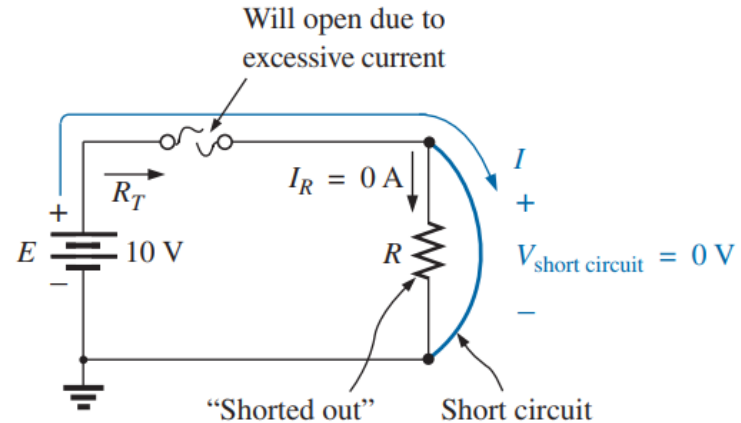
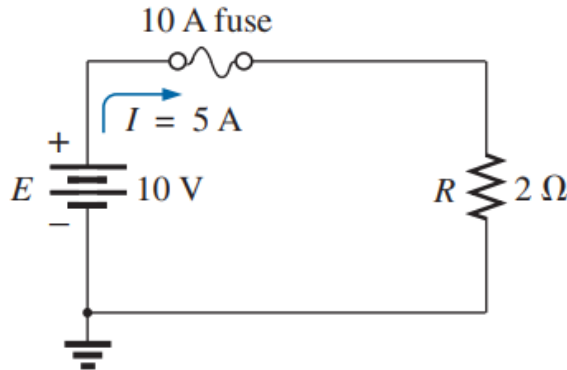
- An *open circuit* is two isolated terminals not connected by an element of any kind. It is the limiting case of a resistor where the resistance approaches infinite.
- Any element with $R \rightarrow \infty$ is an open circuit. $i = 0 = \lim_{R \rightarrow \infty} \frac{v}{R}$
- Indicating that, an open circuit can have a potential difference (voltage) across its terminals, but the **current is always zero amperes**.



In the event of an excessive current flow, a fuse opens to protect appliances.

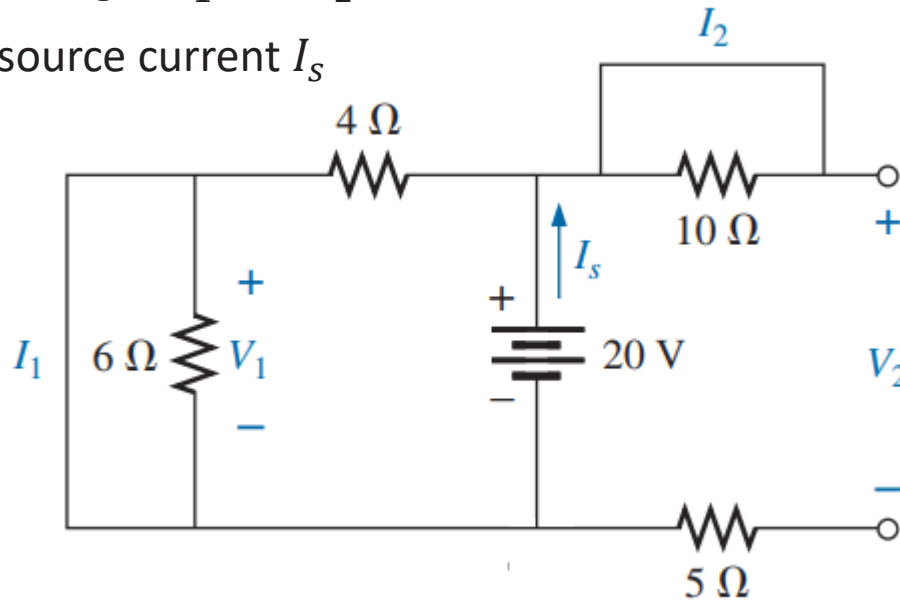
Short circuit

- A *short circuit* is a very low resistance, direct connection between two terminals of a network. It is the limiting case of a resistor where the resistance approaches zero.
- Any element with $R = 0$ is a short circuit. $v = 0 = \lim_{R \rightarrow 0} iR$
- Indicating that, a short circuit can carry a current of a level determined by the external circuit, but the **potential difference (voltage) across its terminals is always zero volts.**



Problem 4

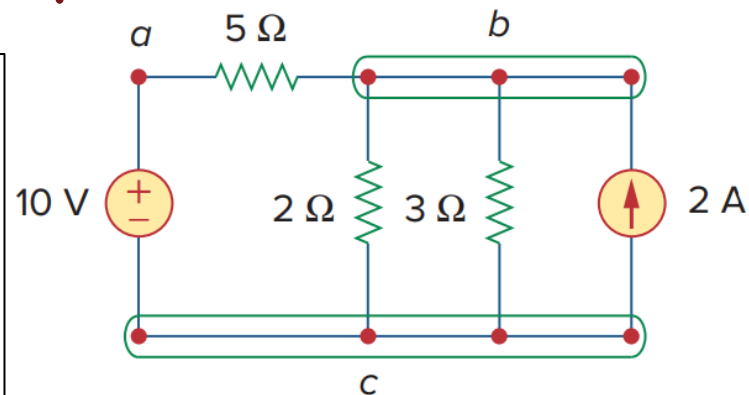
- Determine the short circuit currents I_1 and I_2 .
- The voltages V_1 and V_2 .
- The source current I_s



Ans:
a. 5 A, 0 A
b. 0 V, 20 V
c. 5 A

Nodes, Branches, Loops, & Mesh

- A **branch** represents a single element such as a voltage source or a resistor. In other words, a branch represents a two-terminal element.
- A **node** is the point of connection between two or more branches.
- A **loop** is a path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- A loop is said to be **independent** if it contains at least one branch which is not part of any other **independent** loops.
- A **mesh** is a loop which does not contain any other loops within it.



👉 5 branches: 10 V source, 2 Ω, 3 Ω, and 5 Ω resistors, 2 A current source

👉 3 nodes (n): a, b, c

👉 3 independent loops (l)

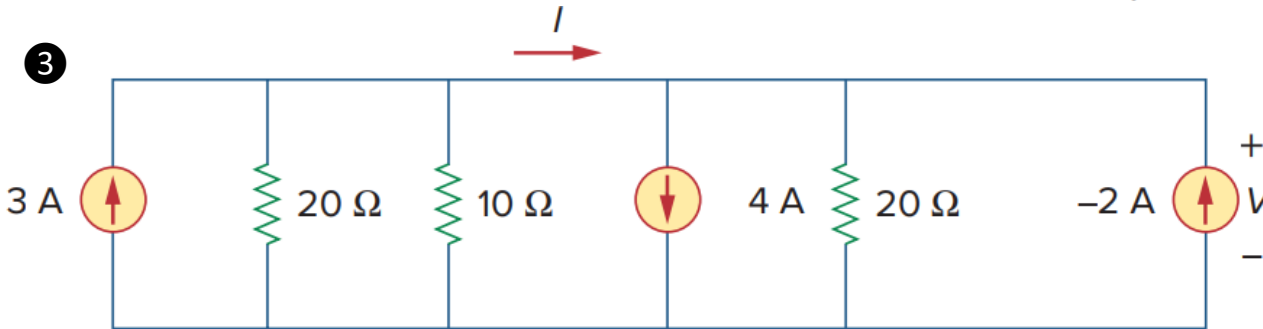
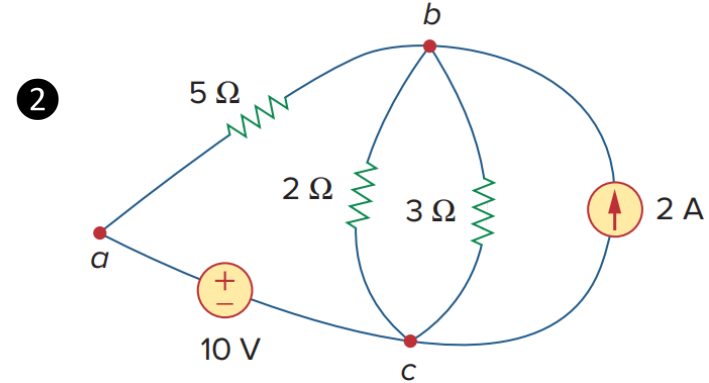
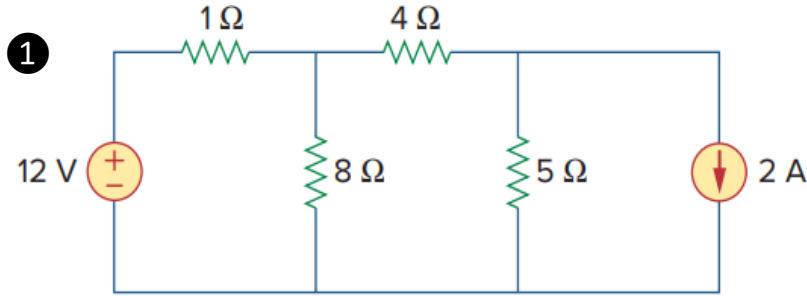
👉 3 dependent loops (l_T)

👉 3 meshes

Problem 5

Ans: $l = 6; n = 4; l_T = 3 + 3; m = 3$
 $2. b = 5; n = 3; l_T = 3 + 3; m = 3$
 $3. b = 6; n = 2; l_T = 5 + 10; m = 5$

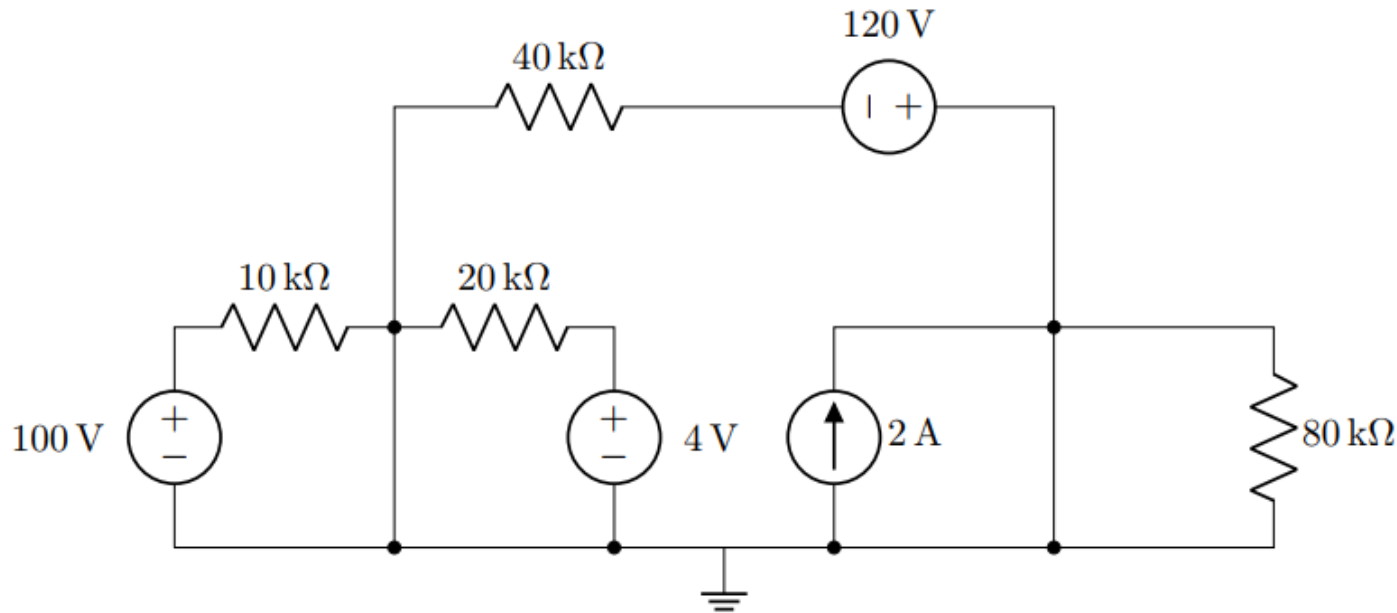
- Determine the number of **branches**, **nodes**, **loops**, and **meshes** in the following circuits.



The three can be related as, $l = b - n + 1$, where l is the number of independent loops

Problem 6

- Determine the number of **nodes** and **meshes** in the following circuit.

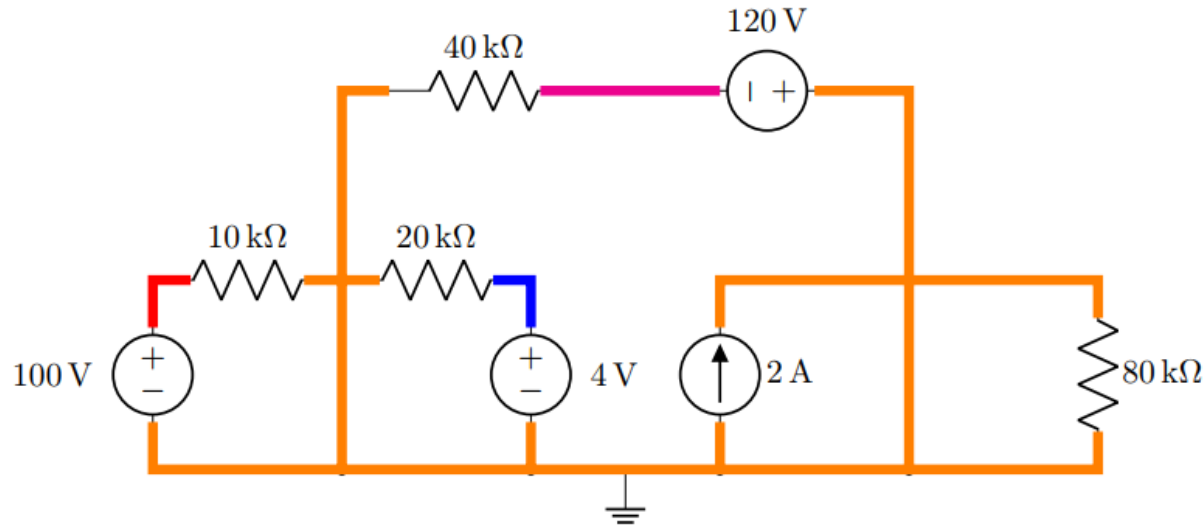


Ans: Try yourself

Problem 6: Solution

- Question: Determine the number of **nodes** in the following circuit.

Solution:

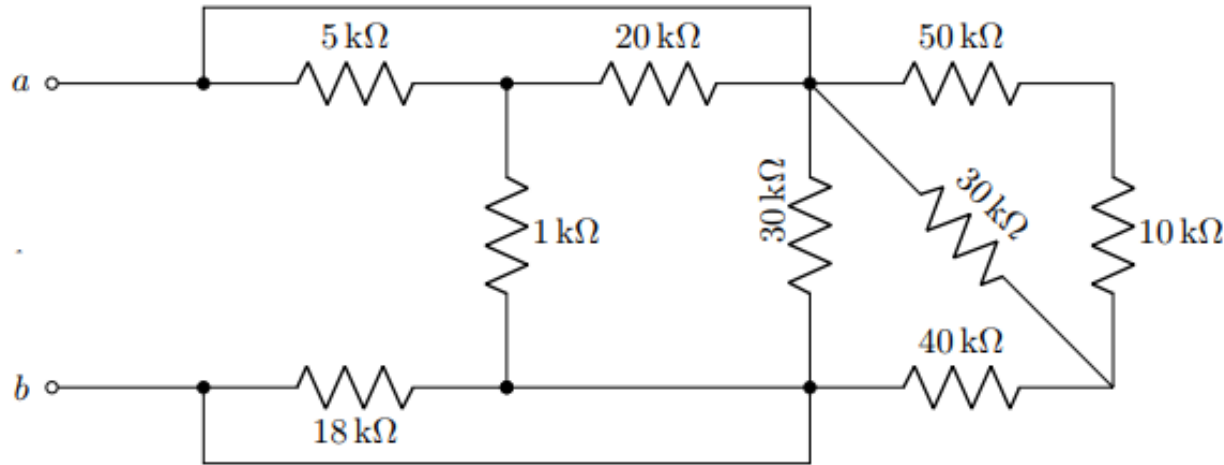


—	Node 1
—	Node 2
—	Node 3
—	Node 4

Ans: **3 mesh**

Problem 7

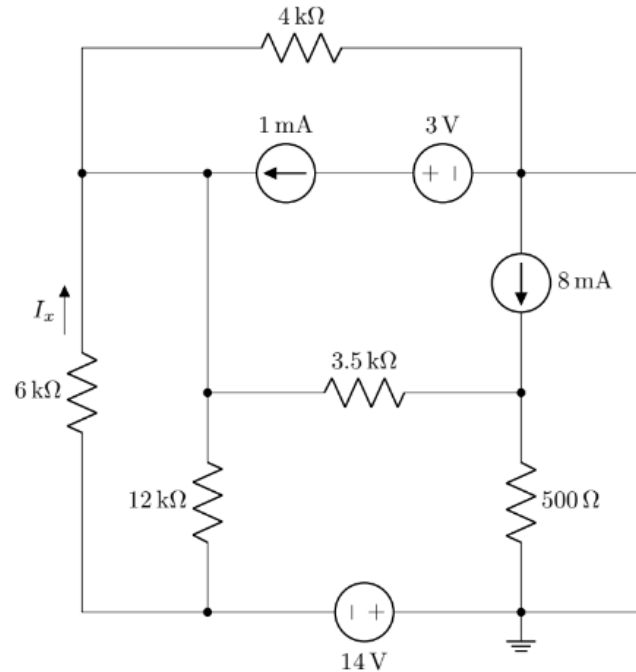
- Determine the number of **nodes** in the following circuit.



Ans: Try yourself

Problem 8

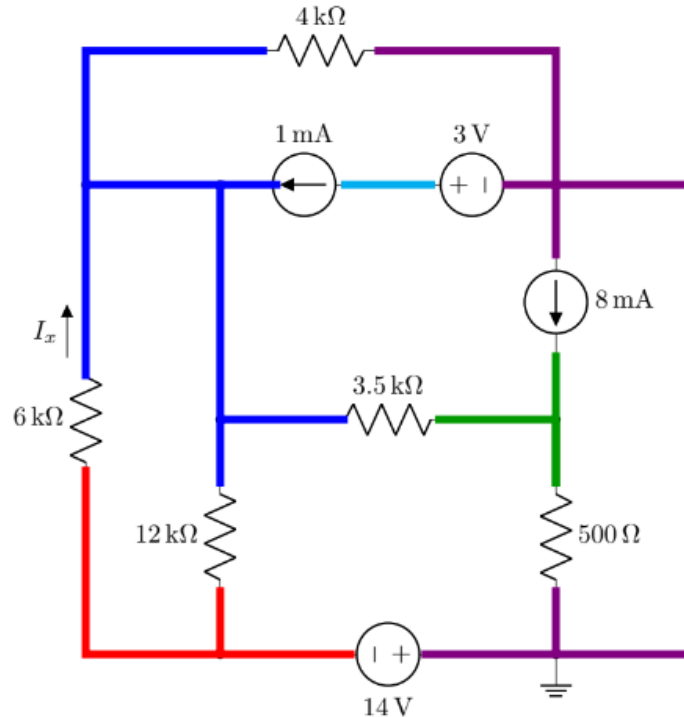
- Determine the number of **nodes** in the following circuit.



Ans: Try yourself

Problem 8: solution

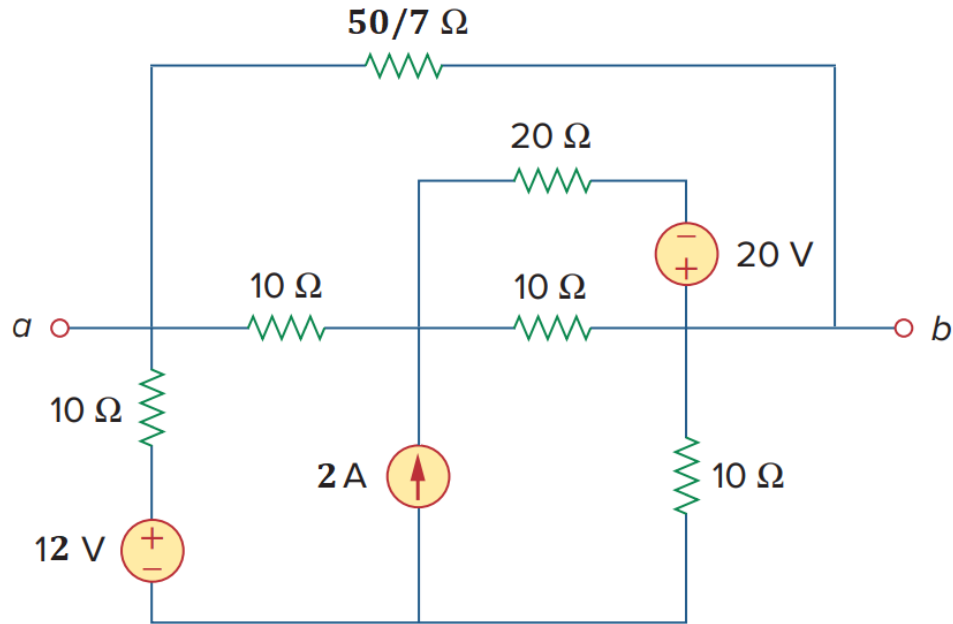
- Question: Determine the number of **nodes** and **meshes** in the following circuit.



Ans: 5 nodes; 5 meshes

Problem 9

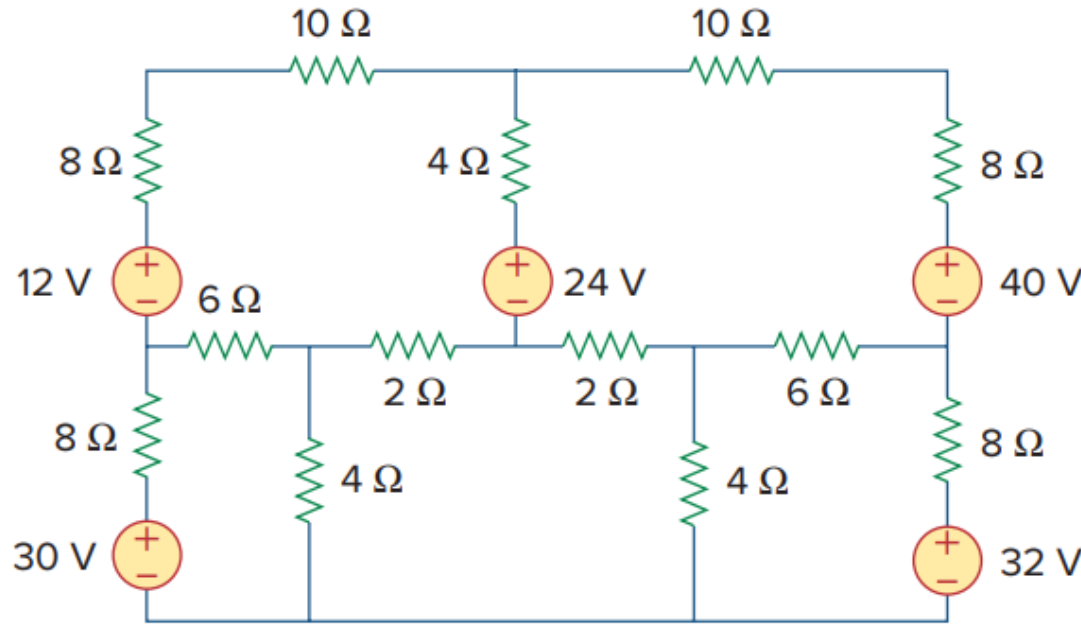
- Determine the number of **nodes** in the following circuit.



Ans: Try yourself

Problem 10

- How many **nodes** and **meshes** are there in the following circuit.

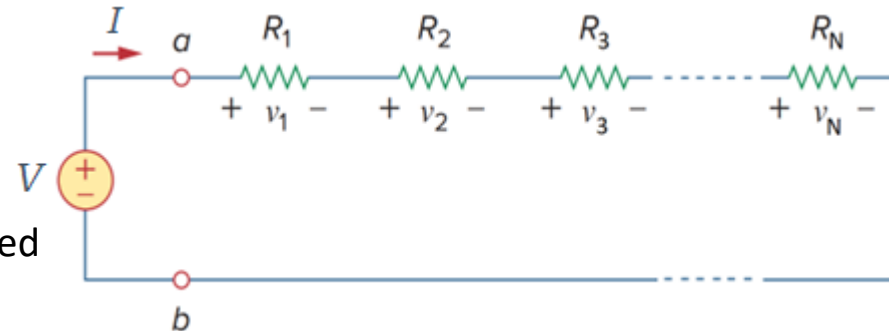


Ans: 14 nodes, 5 meshes

Circuit Configurations

- Circuit elements can be connected to each other in two main ways: series connection and parallel connection.
- In *series configuration*, components are connected end-to-end in a line. The same current flows through all the components. If one component breaks down, the whole circuit will burn out. *So, if same current flows through two circuit elements having a common node, they are said to be in series.*
- In *parallel configuration*, all components are connected across each other leads forming exactly two sets of electrically common points. There are many paths for current flow, but only one voltage across all components. *So, if multiple circuit elements are connected between the same two nodes, they are said to be in parallel.*
- Another configuration occurs when the circuit components are not connected in series or parallel but rather in a 'Y' or ' Δ ' configuration. *Wye-Delta transformation* is required to simplify such configuration.
- The majority of electric circuits use all configurations simultaneously.

Series resistors



- Consider N number of resistors connected in series with a power supply of v volts.
- If the current flowing through the series circuit is i , then the voltage drops across the resistors can be written as,

- $v_1 = IR_1, v_2 = IR_2, v_3 = IR_3, \dots, v_N = IR_N$

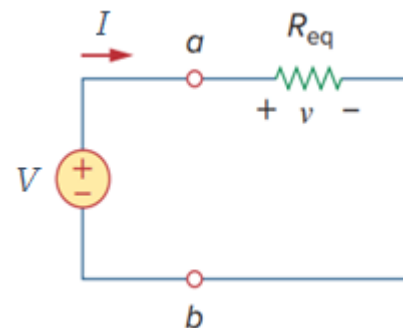
- According to the KVL, $v = v_1 + v_2 + v_3 + \dots + v_N$

$$\Rightarrow V = I(R_1 + R_2 + R_3 + \dots + R_N)$$

$$\Rightarrow I = \frac{V}{R_1 + R_2 + R_3 + \dots + R_N}$$

- It can be written as, $v = V = IR_{eq}$, implying that the series resistors can be replaced by an equivalent resistor R_{eq} ; that is,

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N$$



Graphically

- We can also graphically derive the condition for the two circuits to be equivalent to each other.
- From the previous slide, the current voltage relationship for N number of resistors connected in series is,

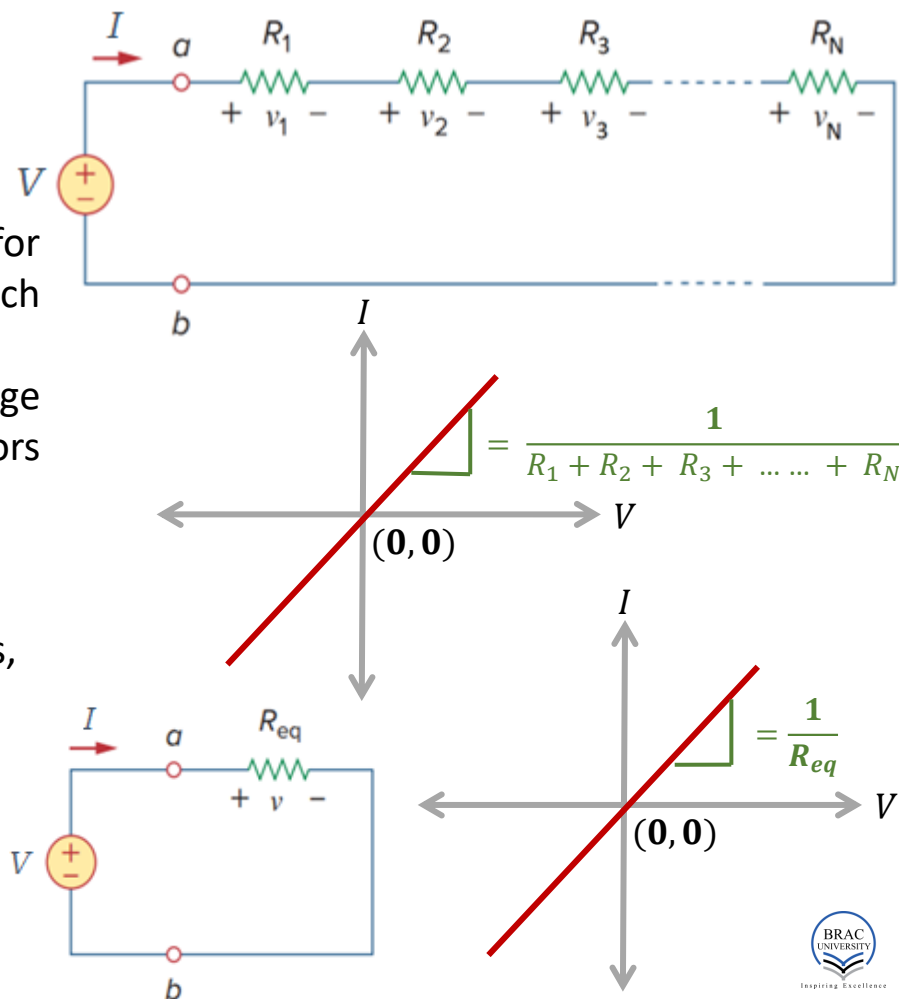
$$I = \frac{1}{R_1 + R_2 + R_3 + \dots + R_N} V$$

- And the $I - V$ relationship for a resistor R_{eq} is,

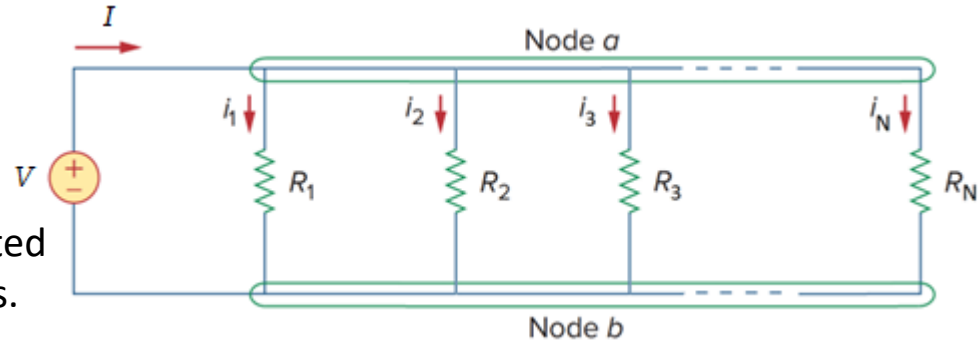
$$I = \frac{1}{R_{eq}} V$$

- The two circuits will be equivalent to each other if the two $I - V$ lines are identical. It requires the slopes to be equal. Thus,

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N$$



Parallel resistors



- Consider N number of resistors connected in parallel with a power supply of V volts.

- Therefore, they have the same voltage V across them. So,

$$V = i_1 R_1 = i_2 R_2 = i_3 R_3 = \dots = i_N R_N$$

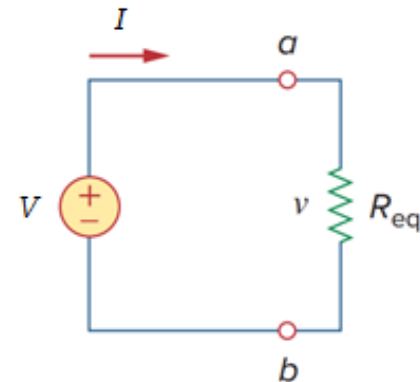
- According to the KCL, $I = i_1 + i_2 + i_3 + \dots + i_N$

$$\Rightarrow I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \dots + \frac{V}{R_N}$$

$$\Rightarrow I = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N} \right)$$

- It can be written as, $I = \frac{V}{R_{eq}}$, implying that the parallel resistors can be replaced by an equivalent resistor R_{eq} ; that is,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$



Graphically

- From the previous slide, the current voltage relationship for N number of resistors connected in parallel is,

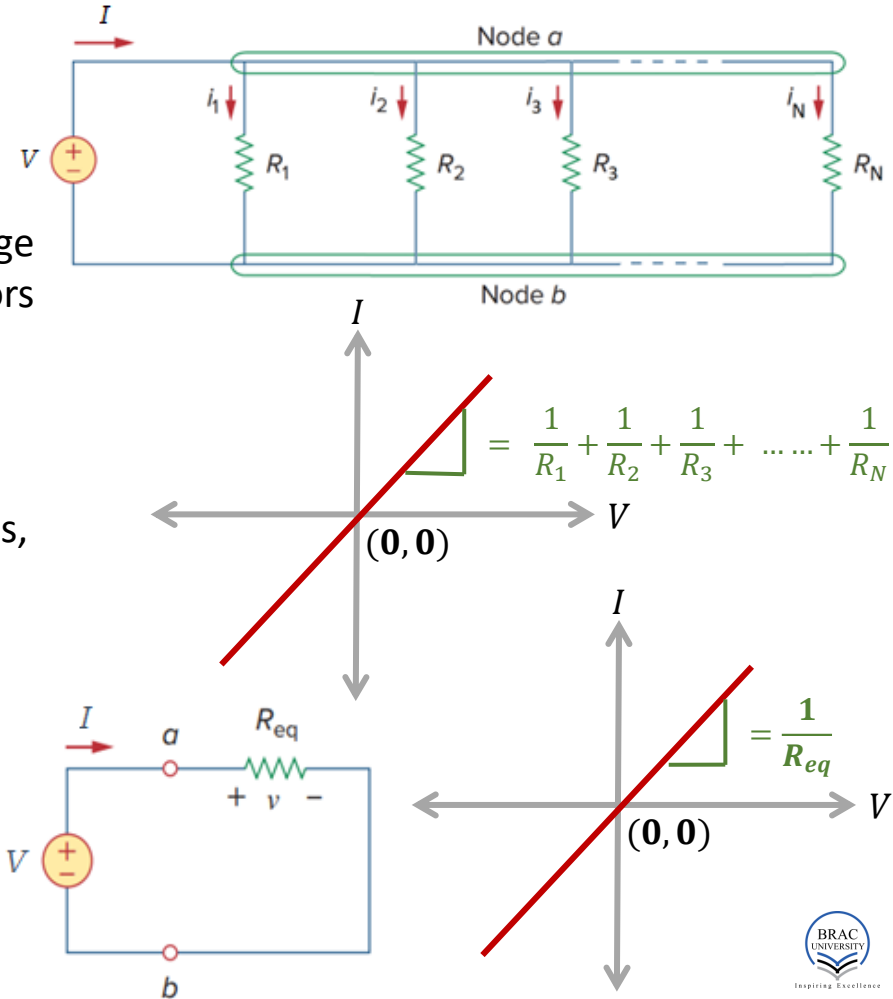
$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N} \right) V$$

- And the $I - V$ relationship for a resistor R_{eq} is,

$$I = \frac{1}{R_{eq}} V$$

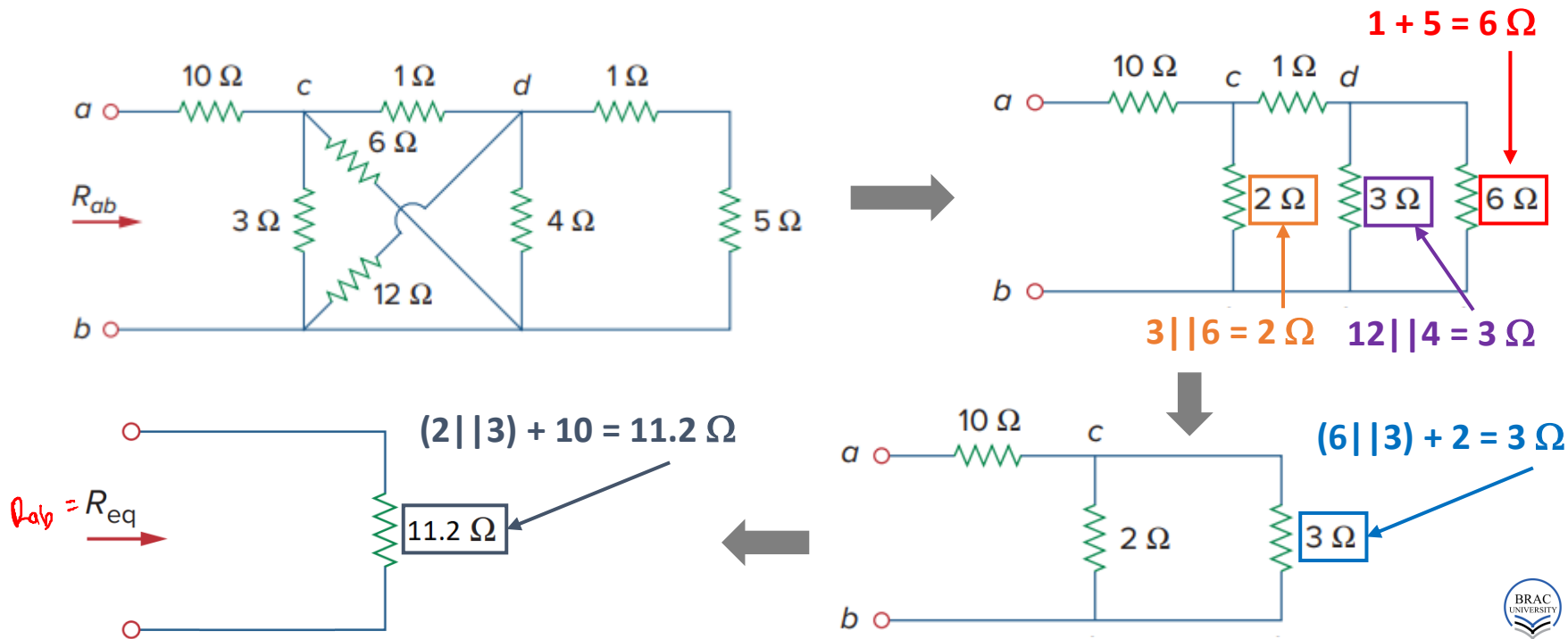
- The two circuits will be equivalent to each other if the two $I - V$ lines are identical. It requires the slopes to be equal. Thus,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$



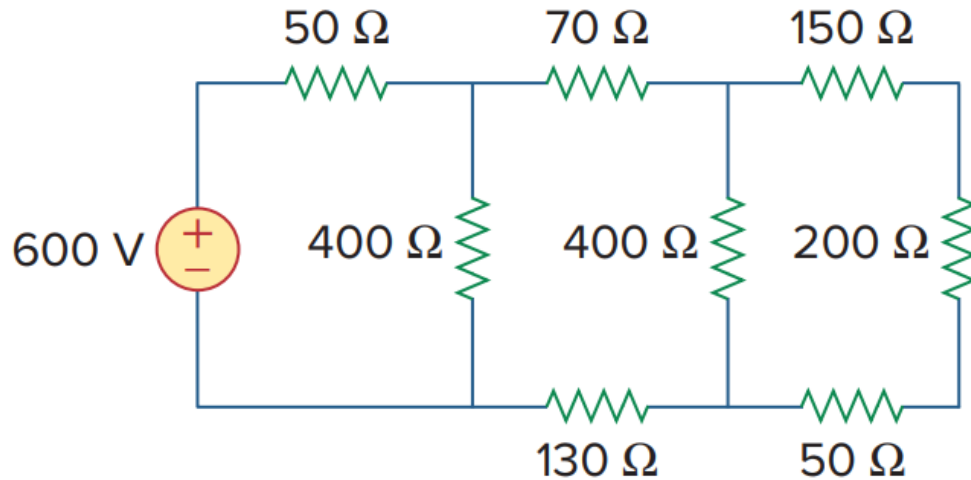
Example 2

- Using series/parallel resistance combination, find R_{ab} for the circuit shown below.



Problem 11

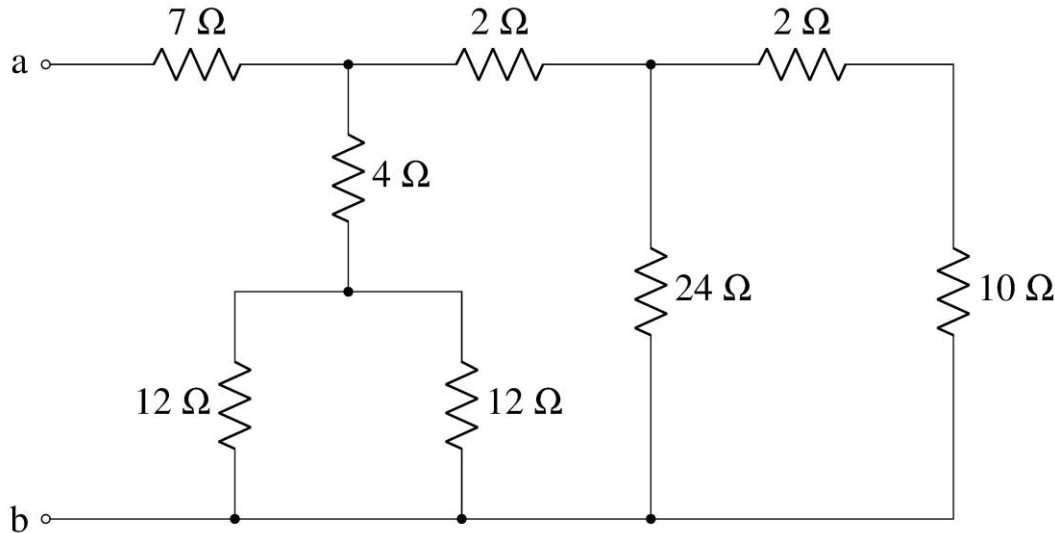
- Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit below.



$$\underline{\text{Ans: } R_{eq} = 250 \, \Omega}$$

Problem 12

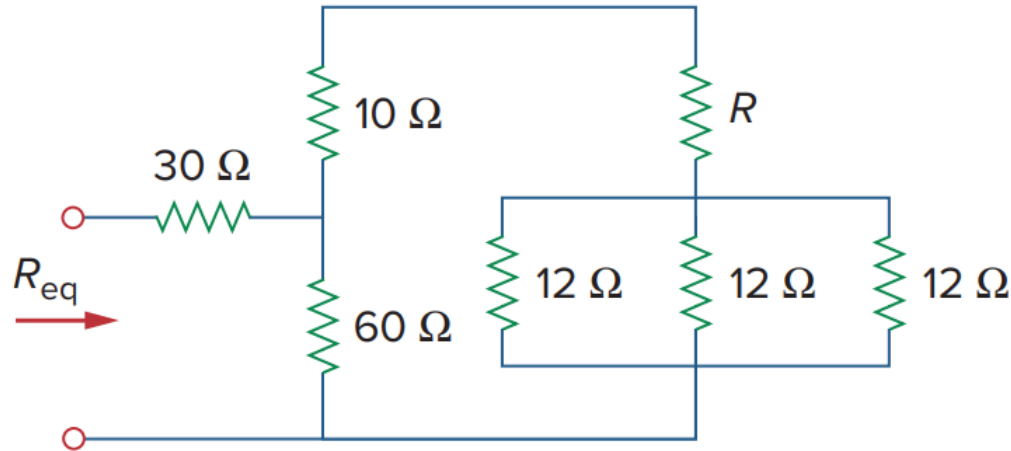
- Find the equivalent resistance between terminals a and b .



Ans: $R_{ab} = 12\ \Omega$

Problem 13

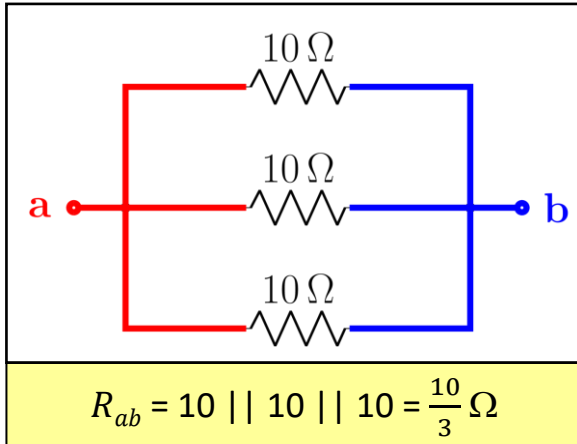
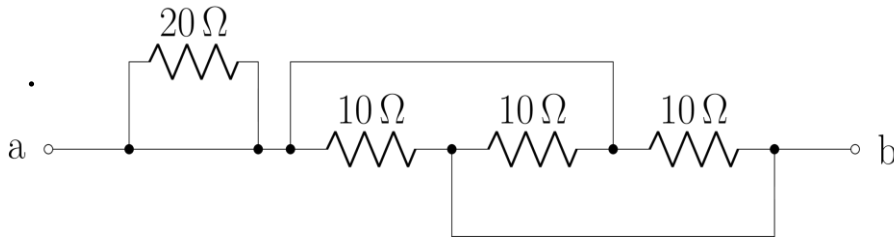
- If $R_{eq} = 50 \Omega$ in the circuit, find R .



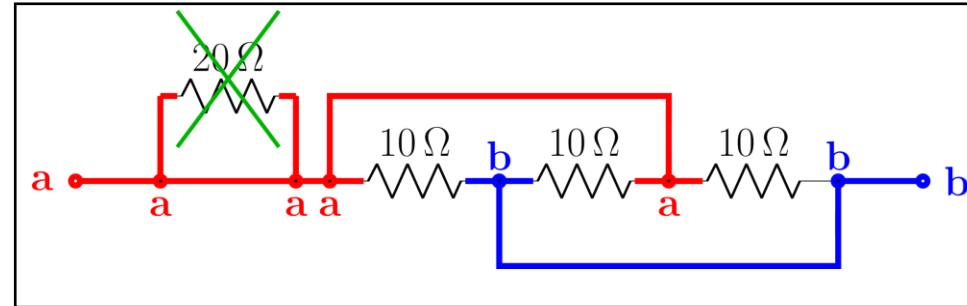
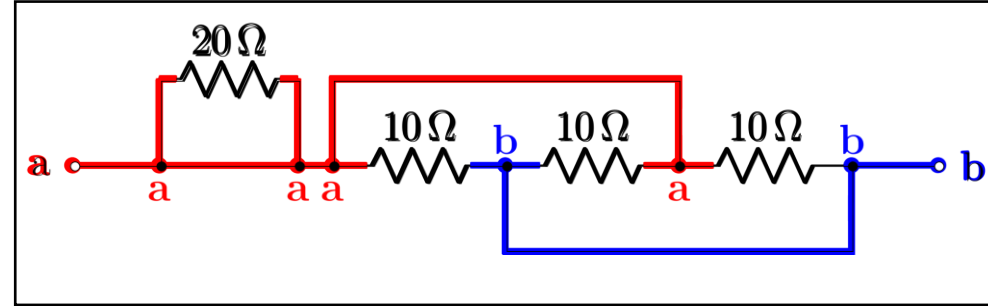
Ans: $R = 16 \Omega$

Example 3

- Find R_{ab}

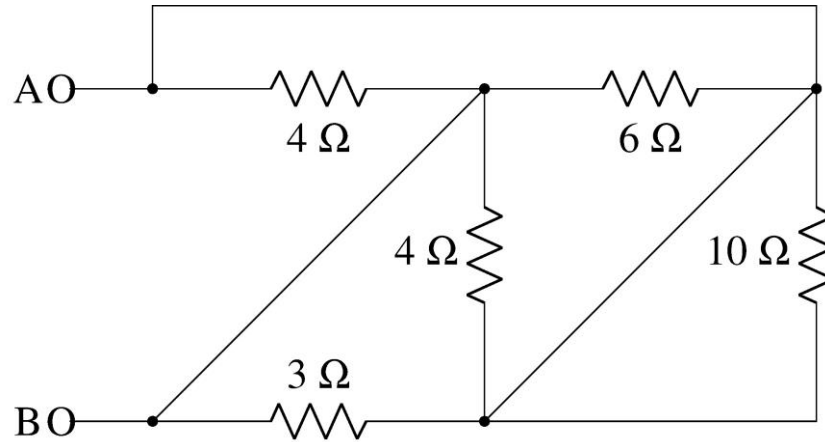


$$R_{ab} = 10 \parallel 10 \parallel 10 = \frac{10}{3} \Omega$$



Problem 14

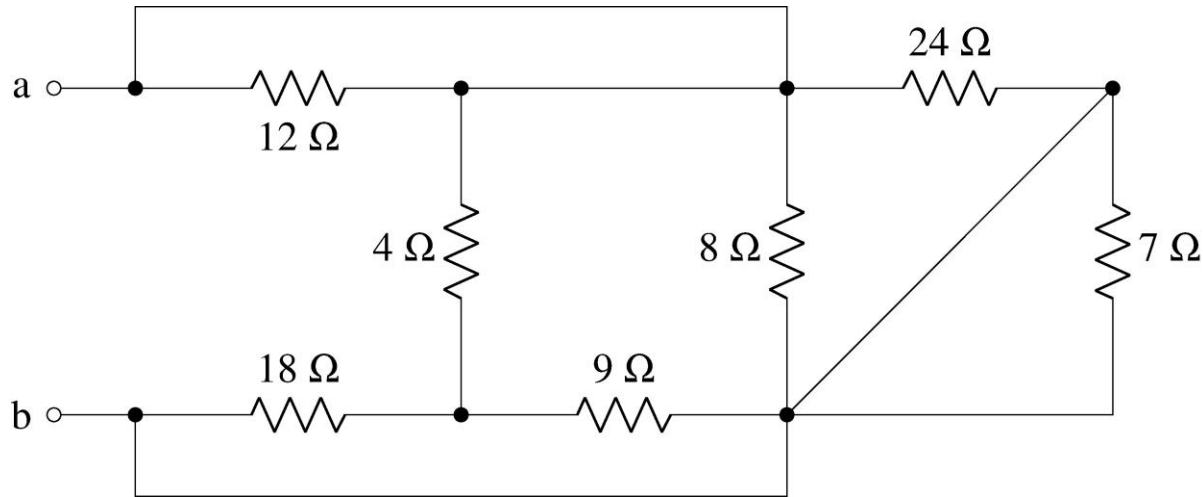
- Find the equivalent resistance between terminals A and B .



Ans: $R_{AB} = 1\ \Omega$

Problem 15

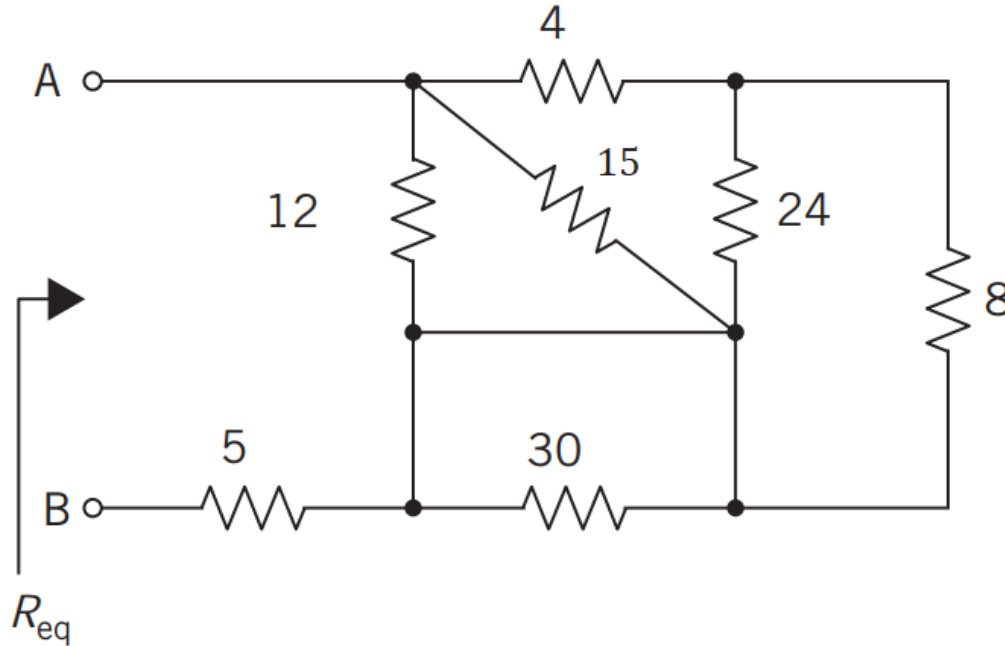
- Find the equivalent resistance between terminals a and b .



Ans: $R_{ab} = 3.75\ \Omega$

Problem 16

- Determine R_{eq} .

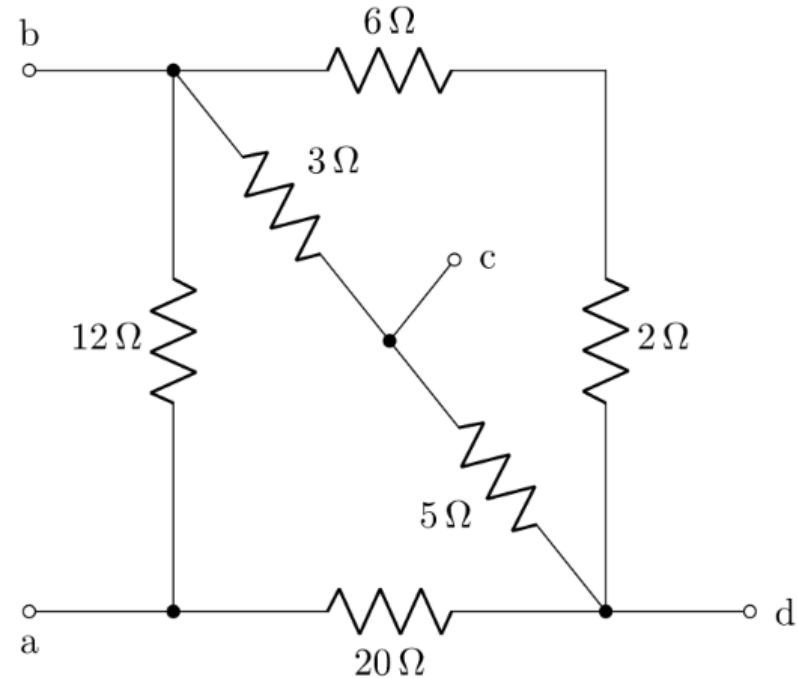


Ans: $R_{eq} = 9$

Problem 17

- Find the equivalent resistance between—

- I. a and b ,
- II. a and d ,
- III. b and d , and
- IV. b and c



$$\text{Ans: } R_{ab} = 8 \, \Omega; R_{ad} = \frac{80}{9} \, \Omega; R_{bd} = \frac{32}{9} \, \Omega; R_{bc} = \frac{19}{8} \, \Omega$$

Practice Problems

- Additional recommended practice problems: [here](#)
- Other suggested problems from the textbook: [here](#)

Thank you for your attention