Final

1(a)

(i) $\bar{V} = 8\angle -70^\circ$

$\bar{I} = 4\angle -70^\circ$

$\bar{V}$ and $\bar{I}$ are in phase

(ii) equivalent impedance $= 2\,\Omega$

(iii) $j\omega L + \frac{1}{j\omega C} = 0$

$\Rightarrow j\omega L + \frac{-j}{\omega C} = 0$

$\Rightarrow \omega L - \frac{1}{\omega C} = 0$

$\Rightarrow \omega L = \frac{1}{\omega C}$

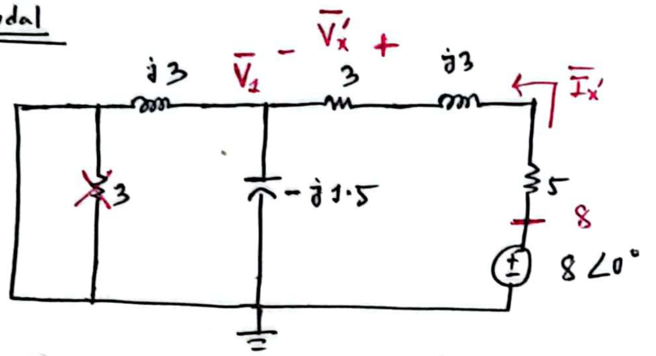
$\Rightarrow \omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{0.1\times 0.1}}$

$= 10$ rad/s

(b)

Only $8\cos(6t)$ V active

Nodal



$\frac{1}{2}$ H $\Leftrightarrow j\omega L = j\times 6\times \frac{1}{2} = j3\,\Omega$

$\frac{1}{9}$ F $\Leftrightarrow \frac{1}{j\omega C} = \frac{1}{j\times 6\times \frac{1}{9}} = -j1.5\,\Omega$

$$\frac{\bar{V}_1}{-j1.5} + \frac{\bar{V}_1}{j3} + \frac{\bar{V}_1 - 8}{3+5+j3} = 0$$

$$\Rightarrow \bar{V}_1\left(\frac{1}{-j1.5} + \frac{1}{j3} + \frac{1}{8+j3}\right) = \frac{8}{8+j3}$$

$$\Rightarrow \bar{V}_1\left(\frac{8}{73} + j\frac{64}{219}\right) = \frac{64}{73} - \frac{24}{73}j$$

$$\Rightarrow \bar{V}_1 = -j3 = 3\angle -90^\circ$$

$\bar{V}_x' = 3\bar{I}_x'$

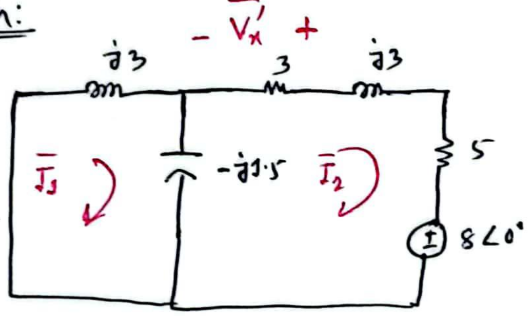
$= 3\times \frac{8 - \bar{V}_1}{8+j3}$

$= 3\times \frac{8-(-j3)}{8+j3}$

$= 3\angle 0^\circ$

$V_x'(t) = 3\cos(6t)$ V

Mesh:



$j3\bar{I}_1 - j1.5(\bar{I}_1 - \bar{I}_2) = 0$

$\Rightarrow j1.5\bar{I}_1 + j1.5\bar{I}_2 = 0$

$\Rightarrow \bar{I}_1 + \bar{I}_2 = 0 \quad \ldots(1)$

$(3+5+j3)\bar{I}_2 + 8 - j1.5(\bar{I}_2 - \bar{I}_1) = 0$

$\Rightarrow j1.5\bar{I}_1 + (8+j1.5)\bar{I}_2 = -8 \quad \ldots(2)$

$$\begin{bmatrix} 1 & 1 \\ j1.5 & 8+j1.5 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \end{bmatrix}$$

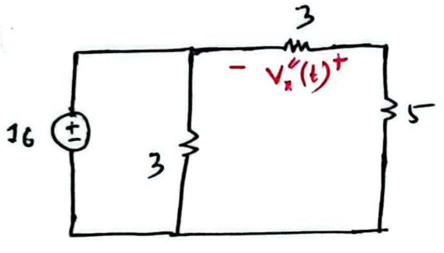
$$\Delta = \begin{vmatrix} 1 & 1 \\ j1.5 & 8+j1.5 \end{vmatrix} = 1(8+j1.5) - j1.5 = 8$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ j1.5 & -8 \end{vmatrix} = -8-0 = -8$$

$$\bar{I}_2 = \frac{\Delta_2}{\Delta} = \frac{-8}{8} = -1 = 1\angle 180^\circ$$

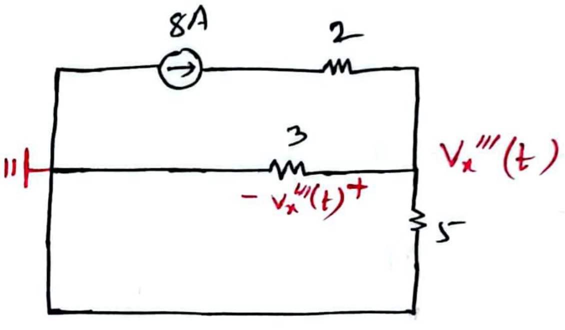
$$\bar{V}_x' = -3\bar{I}_2 = 3\angle 0^\circ \qquad V_x'(t) = 3\cos(6t) \text{ V}$$

Only 16V active



$$V_x''(t) = -\frac{3}{3+5} \times 16 = -6\text{ V}$$

Only 8A active



$$\frac{V_x'''(t)}{5} + \frac{V_x'''(t)}{3} = 8$$

$$\Rightarrow V_x'''(t) \times \frac{8}{15} = 8$$

$$\Rightarrow V_x'''(t) = 15 \text{ V}$$

$$\therefore V_x(t) = V_x'(t) + V_x''(t) + V_x'''(t)$$

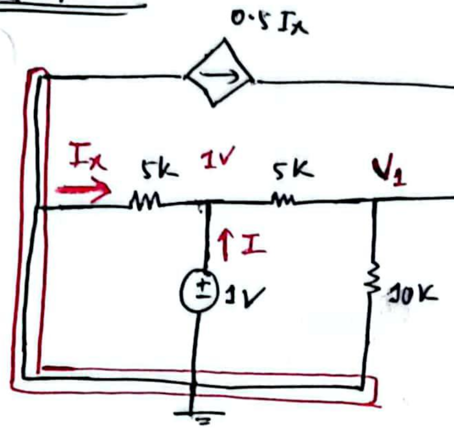
$$= 3\cos(6t) - 6 + 15$$

$$= 3\cos(6t) + 9$$

(Ans)

2
(a)

R_{TH}/R_N:



$$I_x = \frac{0-1}{5} = -\frac{1}{5} \text{ mA}$$

Node-1 KCL:

$$\frac{V_1}{10} + \frac{V_1 - 1}{5} = 0.5 I_x$$

$$\Rightarrow V_1\left(\frac{1}{10}+\frac{1}{5}\right) = \frac{1}{5} + 0.5 \times \left(-\frac{1}{5}\right)$$

$$\Rightarrow 0.3 V_1 = \frac{1}{10}$$

$$\Rightarrow V_1 = \frac{1}{3} \text{ V}$$

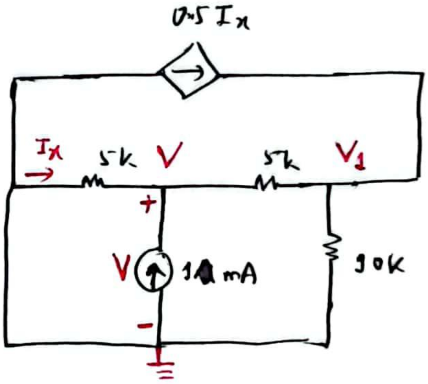
$$I + I_x = \frac{1 - V_1}{5}$$

$$\Rightarrow I = \frac{1-\frac{1}{3}}{5} - \left(-\frac{1}{5}\right) = \frac{1}{3} \text{ mA}$$

$$R_{TH} = \frac{1}{I} = 3 \text{ k}\Omega$$

Alternate:

$$I_x = \frac{0-V}{5} = -\frac{V}{5} \text{ mA}$$



Node-V

$$\frac{V}{5} + \frac{V - V_1}{5} = 1$$

$$\Rightarrow \frac{2V}{5} - \frac{V_1}{5} = 1 \quad \ldots(1)$$

Node-V_1

$$\frac{V_1}{10} + \frac{V_1 - V}{5} = 0.5 \times \left(-\frac{V}{5}\right)$$

$$\Rightarrow -\frac{V}{10} + \frac{3V_1}{10} = 0 \quad \ldots(2)$$

Solving eqn (1) and (2)

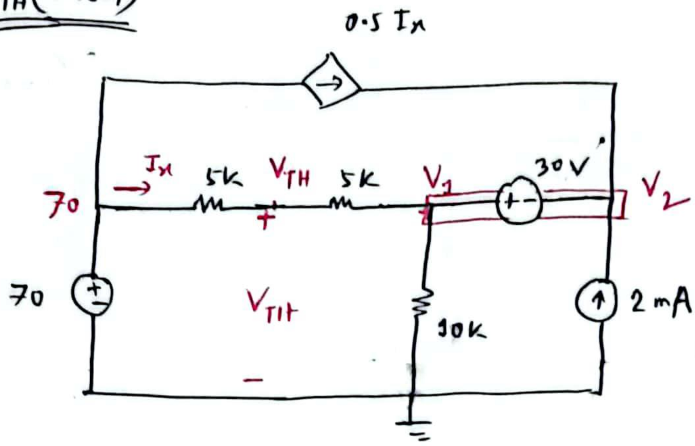
$V = 3V$, $V_1 = 1$ V

$$R_{TH} = R_N = \frac{V}{1 \text{ mA}} = \frac{3}{1} \text{ k}\Omega = 3 \text{ k}\Omega$$

(Ans)

(b)

V_{TH} (Nodal)



$$I_x = \frac{70 - V_{TH}}{5}$$

Supernode

KVL: $V_1 - V_2 = 30 \quad \cdots (1)$

KCL:

$$0.5 \times \frac{70 - V_{TH}}{5} + 2 = \frac{V_1}{10} + \frac{V_1 - V_{TH}}{5}$$

$$\Rightarrow \frac{3V_1}{10} - \frac{V_{TH}}{10} = 9 \quad \cdots (2)$$

Node-3 KCL

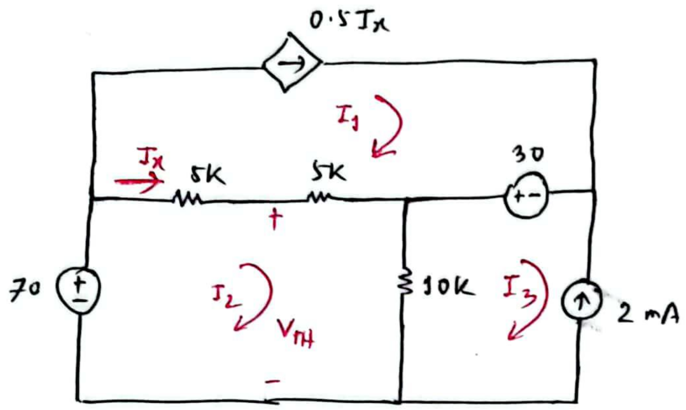
$$\frac{V_{TH} - 70}{5} + \frac{V_{TH} - V_1}{5} = 0$$

$$\Rightarrow -\frac{V_1}{5} + \frac{2V_{TH}}{5} = 14 \quad \cdots (3)$$

Solving eqn (1), (2), (3)

$V_1 = 50, \quad V_2 = 20, \quad V_{TH} = 60V$

V_{TH} (mesh)



$I_x = I_2 - I_1$

Mesh-1

$I_1 = 0.5 I_x = 0.5(I_2 - I_1)$

$\Rightarrow 1.5 I_1 - 0.5 I_2 = 0 \quad \cdots (1)$

Mesh-3

$I_3 = -2 \text{ mA} \quad \cdots (2)$

Mesh-2: $70 = 5(I_2 - I_1) + 5(I_2 - I_1) + 10(I_2 - I_3)$

$\Rightarrow -10 I_1 + 20 I_2 - 10 I_3 = 70 \quad \cdots (3)$

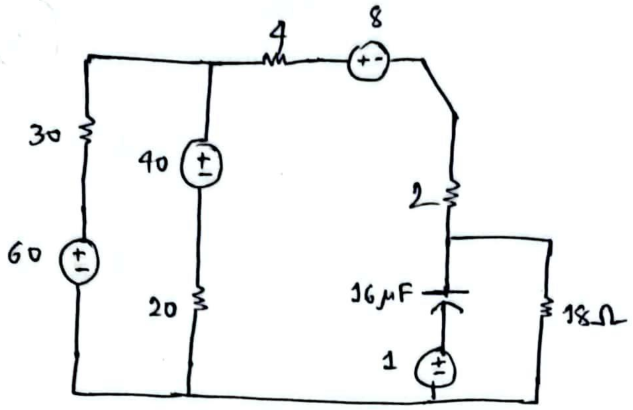
Solving eqn (1), (2), (3) we get $I_1 = 1 \text{ mA}, \quad I_2 = 3 \text{ mA}, \quad I_3 = -2 \text{ mA}$

$I_x = I_2 - I_1 = 3 - 1 = 2 \text{ mA}$

$70 = 5 I_x + V_{TH} \quad \Rightarrow V_{TH} = 70 - 5I_x = 70 - 5 \times 2 = 60 \text{ V}$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{60^2}{4 \times 3 \times 10^3} = 0.3 \text{ W}$$

$t<0$:



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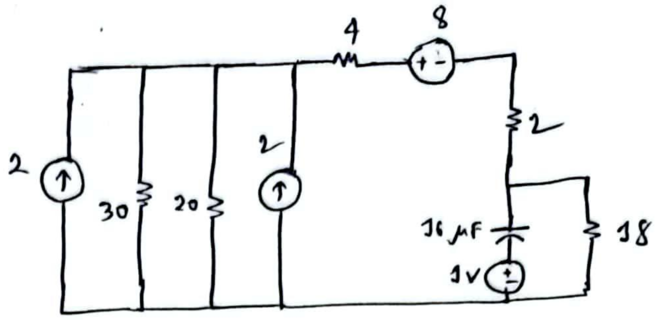

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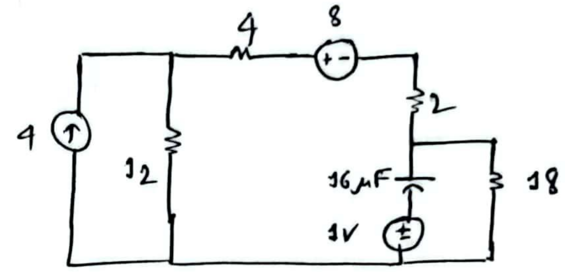

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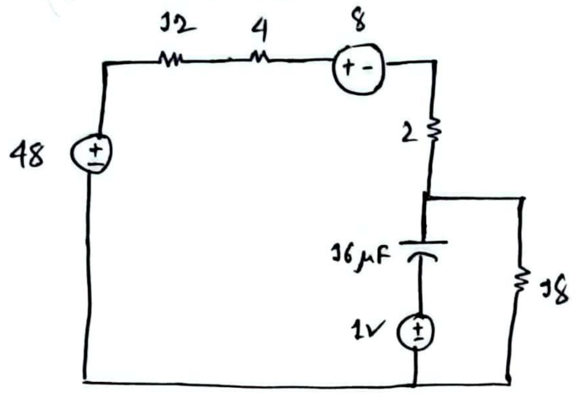
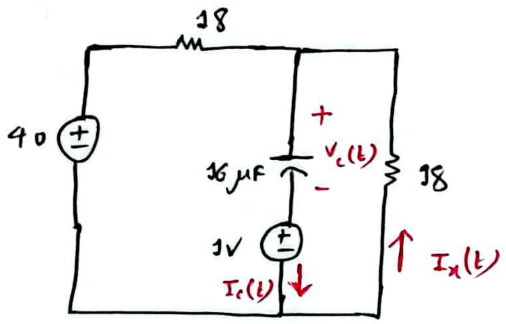
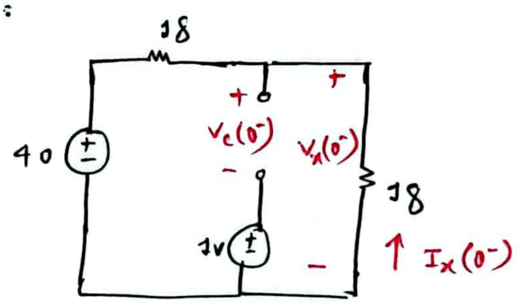


⇓



⇒

$t=0^-$:

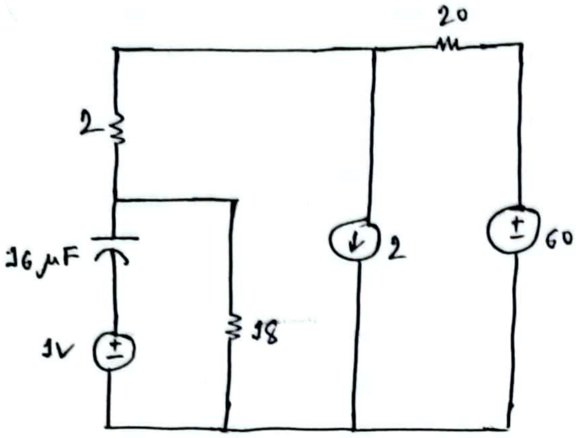


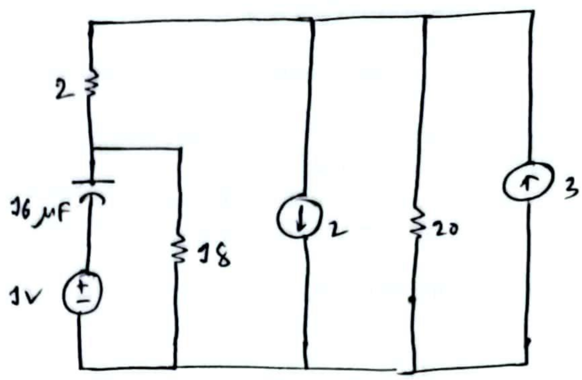
$$V_x(0^-) = \frac{18}{18+18} \times 40 = 20\ V$$

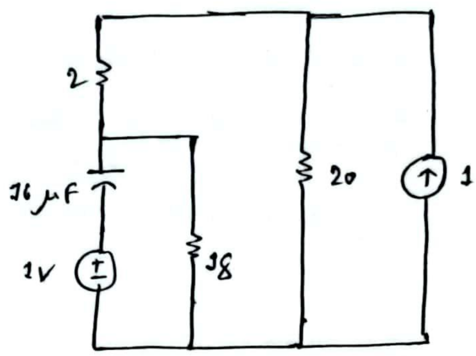
Applying KVL, $V_x(0^-) = V_c(0^-) + 1 \Rightarrow V_c(0^-) = 19\ V$

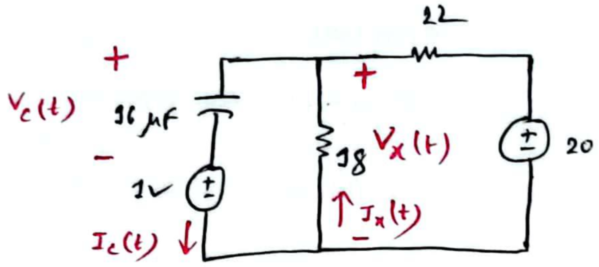
$\cancel{V_c(t) = 19 + (0-19)e^{-}}$ $I_c(0) = 0\ A$

t>0









After long time

$$V_x(\infty) = \frac{18}{18+22} \times 20 = 9\ V$$

$$V_c(\infty) = 9 - 1 = 8\ V$$

$$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]\, e^{-t/\tau}$$

$$= 8 + (19-8)\, e^{-\frac{t}{1.584\times10^{-4}}}$$

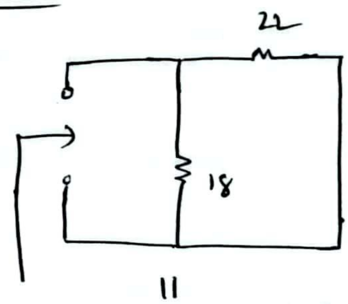
$$= 8 + 11\, e^{-6313.13t}\ V$$

$$i_c(t) = C\,\frac{dV_c(t)}{dt}$$

$$= 16\times10^{-6} \times 11 \times (-6313.13)\, e^{-6313.13t}$$

$$= -1.11\, e^{-6313.13t}$$

R_{TH}:



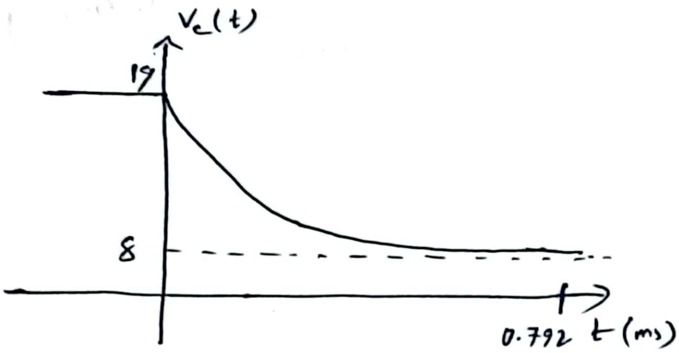
$R_{TH} = 22 \parallel 18 = 9.9\,\Omega$

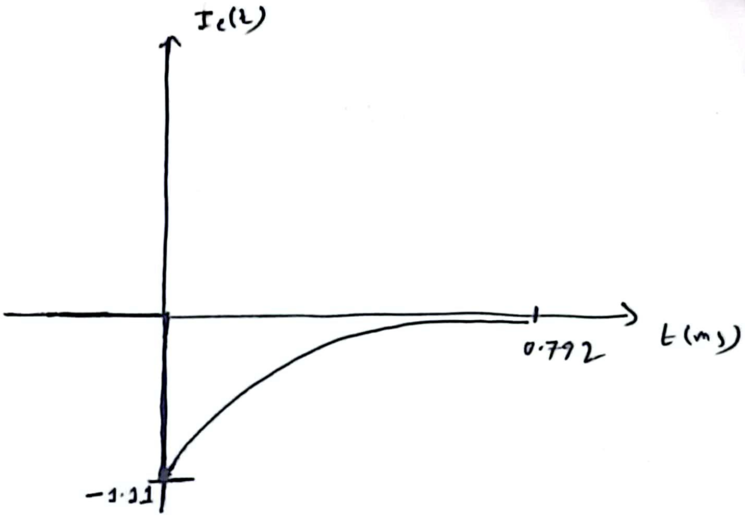
$\tau = R_{TH} C = 16\times10^{-6} \times 9.9$

$= 1.584\times10^{-4}\ s$

$= 0.1584\ ms$

$5\tau = 0.792\ ms$





(c)

$$V_x(t) = V_c(t) + 1$$

$$\Rightarrow V_x(t) = 9 + 11\, e^{-6313.13\, t} \quad V$$

$$I_x(t) = -\frac{V_x(t)}{18} = -0.5 - 0.61\, e^{-6313.13 t} \quad A$$

$$I_x(0.1) = -0.5 - 0.61\, e^{-6313.13 \times 0.1} = -0.5 \quad A$$