Department of Computer Science and Engineering (CSE)
BRAC University

Fall 2023

CSE250 - Circuits and Electronics

# FIRST ORDER CIRCUITS

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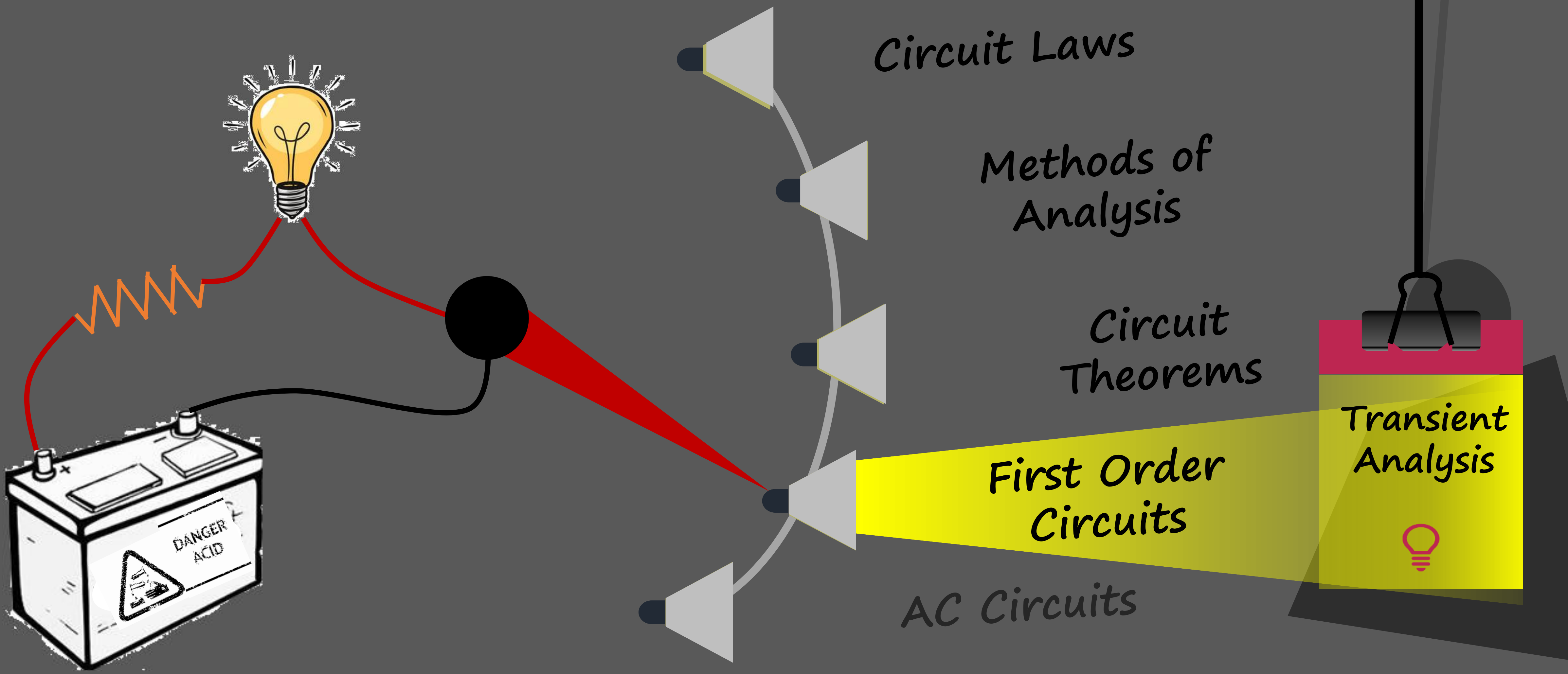
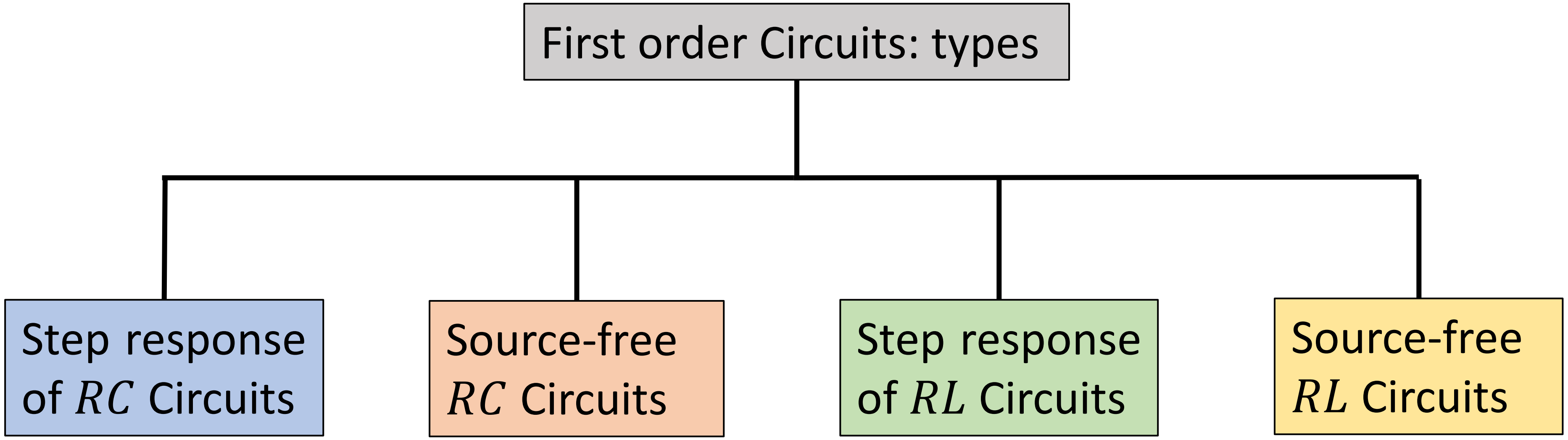
# Course Outline: broad themes



# First Order Circuits

- A first-order circuit is characterized by a first-order differential equation.
- We shall examine two types of differential circuits: circuit comprising resistors and capacitors ($RC$ circuit) and circuit comprising resistors and inductors ($RL$ circuit).
- Two ways to excite the circuits: $(i)$ by initial conditions of storage elements (source free circuits) and $(ii)$ by independent sources (DC for this course).



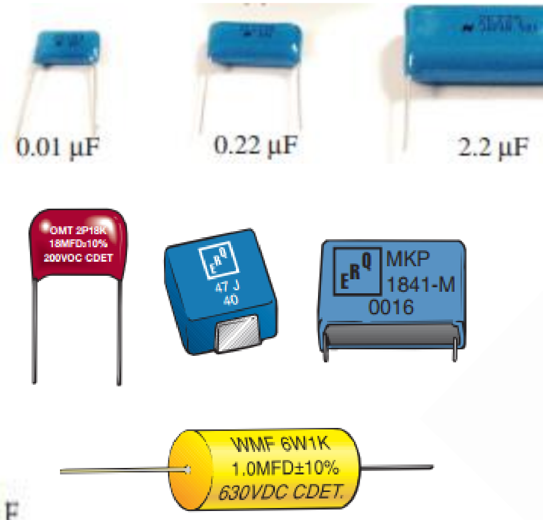
# Circuit Elements

- **Active element**
  - An *active element* is capable of generating energy.
  - In other words, an element is said to be active if it can add some gain (in terms of voltage or current) to a circuit.
  - Active elements can absorb energy if they are forced to do so by other active elements.
  - Examples: *Voltage/current sources, generators, transistors, operational amplifiers.*
- **Passive element**
  - *Passive elements* cannot supply energy. They can only consume/dissipate/store energy.
  - Examples: *Resistors, capacitors, inductors, transformers.*
  - Transformers change the voltage or current levels, but the power is unchanged. This is why transformers are passive element.

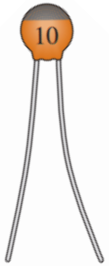
# Capacitors

- A capacitor is a passive circuit element designed to store energy in its electric field.
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.


Electrolytic

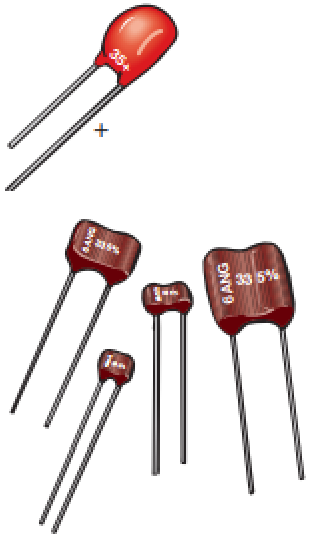



Polyester Film

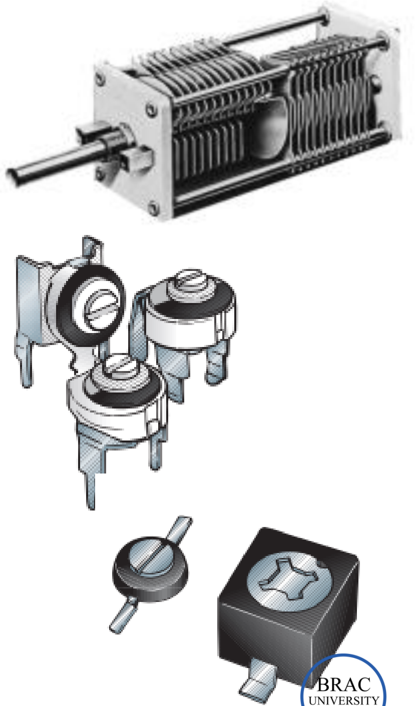



Ceramic

Mica Capacitors



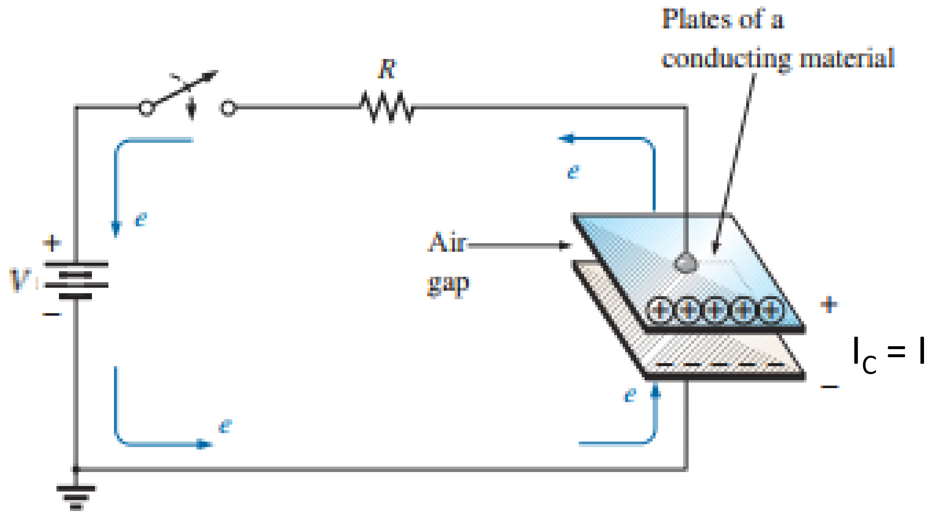
Dipped Capacitors

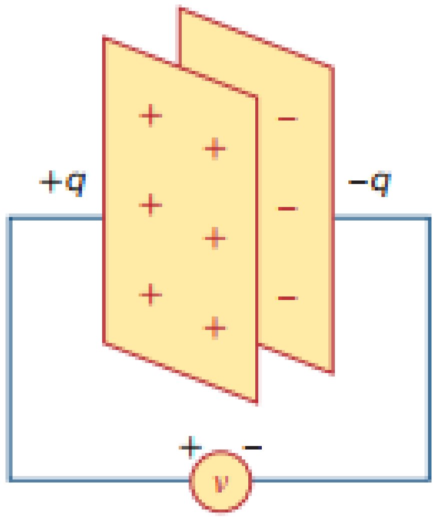


Variable Capacitors

# Parallel Plate Capacitor

- Most widely used configuration is the two conducting surfaces (aluminium mainly) separated by a dielectric (air, ceramic, paper, or mica).
- The switch is open initially (no net charge).
- Closing the switch causes electrons to flow from and to the upper and lower plates respectively as shown by the arrows.
- Electron flow continues until the potential difference between the plates equals the applied potential.
- The final result is a net positive charge on the top plate and a negative charge on the bottom plate.



# Capacitance

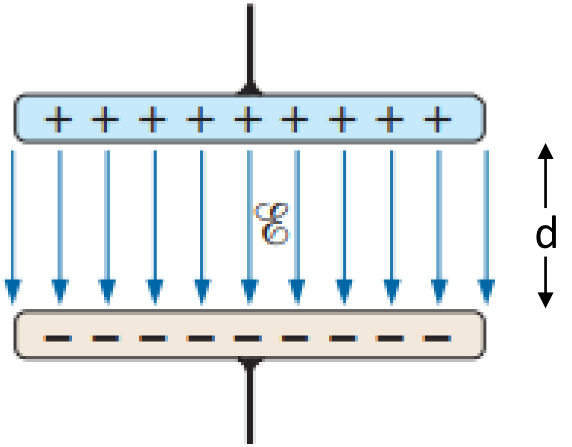
- *Capacitance* is a measure of a capacitor's ability to store charge.
- Increasing $V$ increases $E$ as $E \propto \frac{V}{d}$ as long as $d$ is constant. An increase in $E$ field causes increased charge separation i.e. increases $q$.
- So, $q \propto V$

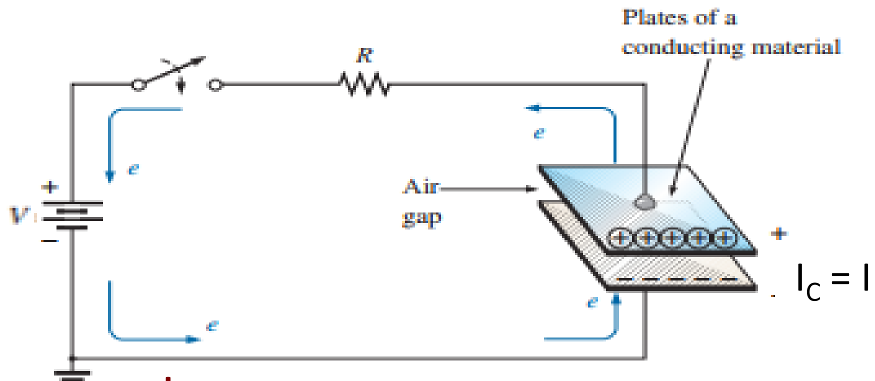
$\Rightarrow$ $q = CV$ [$C$ is a proportionality constant $\equiv$ Capacitance]

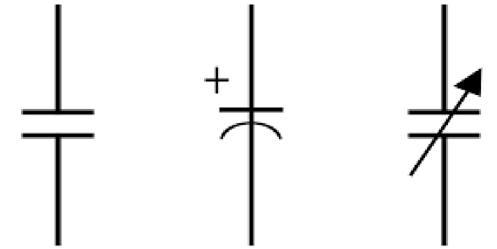
$\Rightarrow$ $C = \frac{q}{V}$ $[F\ (Farad),\ mF,\ \mu F]$

$\Rightarrow$ For a particular capacitor $\uparrow V, \uparrow q$ but $\frac{q}{V} =$ const. So, $C$ does not depend on $q$ or $v$. It depends on the physical dimension of the capacitor.

$\Rightarrow$ For the parallel plate capacitor, $C = \frac{\varepsilon A}{d}$



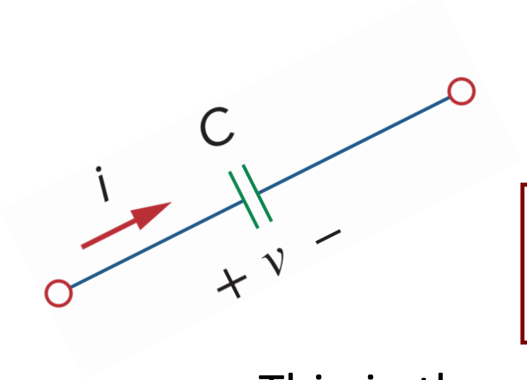
# I-V relation of a Capacitor

- From the definition of the capacitance,

$$C = \frac{q}{v}$$

$$\Rightarrow \quad q = Cv$$

- Differentiating with respect to time,




$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$$\boxed{i = C\frac{dv}{dt}}$$

- This is the characteristic equation of a capacitor.

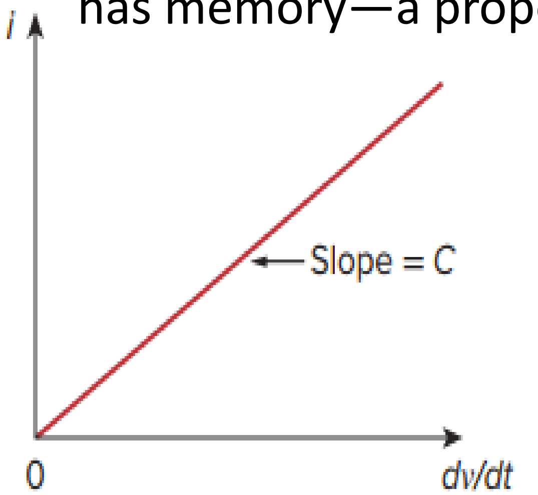
- Integrating with respect to time,

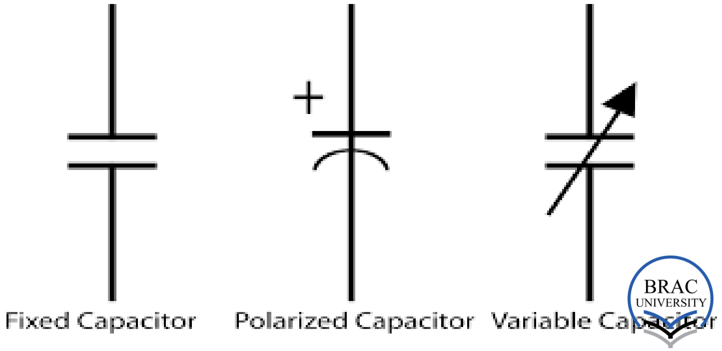
$$v(t) = \frac{1}{C}\int i(t)\,dt$$

- If the voltage of the capacitor at any time $t_0$ is $v(t_0) = q(t_0)/C$, then,

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(i_0)$$

- It shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.



# Energy & Power of a Capacitor

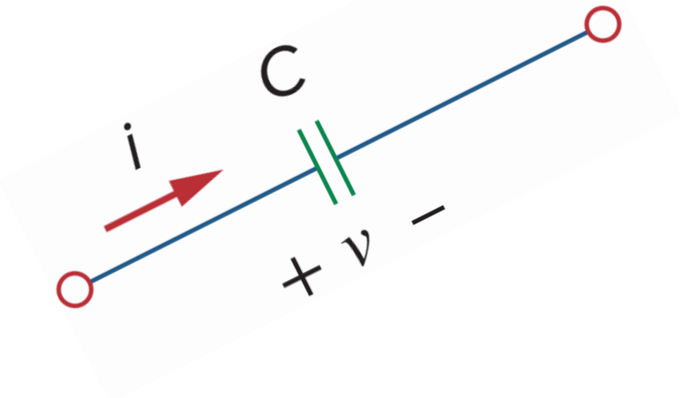
- The instantaneous power delivered to a capacitor according to the passive sign convention is,

$$p = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

- The energy stored in the capacitor is therefore

$$w(t) = \int_{-\infty}^{t} p(t)\, dt = \int_{-\infty}^{t} Cv(t)\frac{dv(t)}{dt}\, dt = \int_{v(-\infty)}^{v(t)} Cv(t)\; dv$$

$$\Rightarrow \quad w(t) = \frac{1}{2}Cv^2\Big|_{v(-\infty)=V_0}^{v(t)=V}$$




- If the voltage across the capacitor was initially (at $t = -\infty$) $V_0$, then,

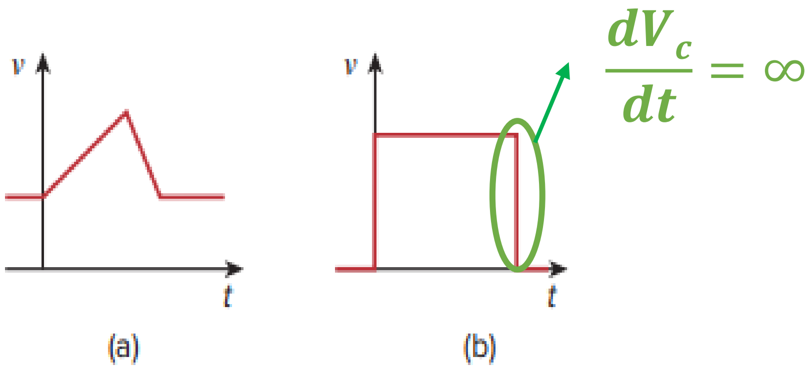
$$\Rightarrow \quad w(t) = \frac{1}{2}CV^2 - \frac{1}{2}CV_0^2$$

- In general, at any time $t$, if the voltage across a capacitor is $V$, then the stored energy can be found as,

$$w(t) = \frac{1}{2}Cv(t)^2 = \frac{1}{2}CV^2$$

# Capacitor: important properties

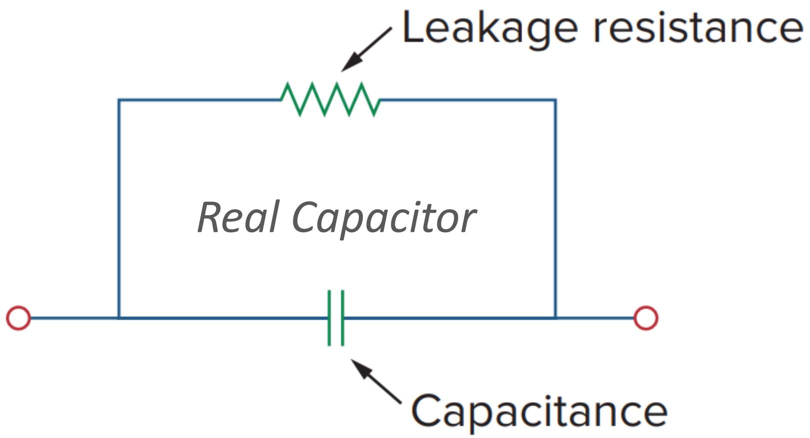
1. A capacitor is an <u>open circuit</u> to dc. At dc, $i_C = C\frac{dV_{c\,dc}}{dt} = 0$ [Open circuit]
2. The <u>voltage</u> on a capacitor cannot change abruptly.



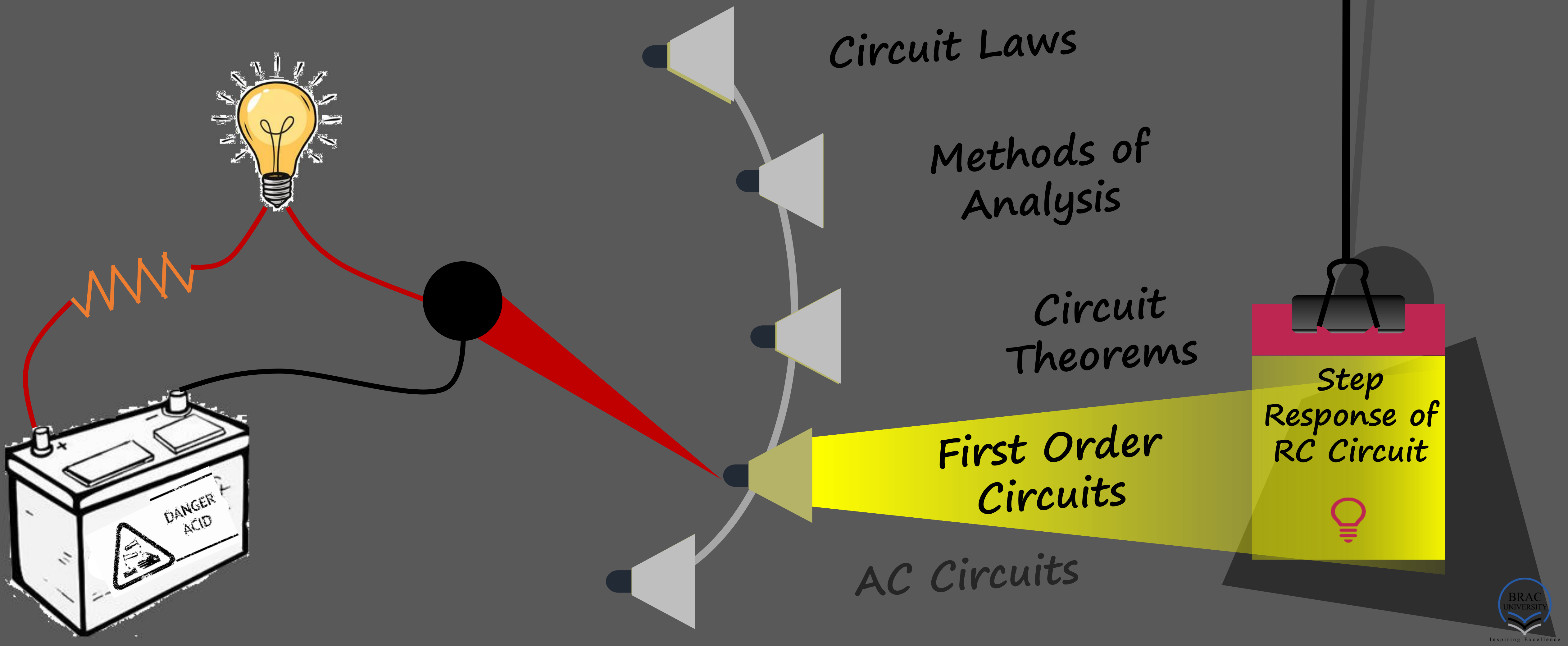

*Voltage change across a capacitor (a) allowed and (b) not allowed*

3. An ideal capacitor does not dissipate energy.

4. A real, nonideal capacitor has a parallel-model leakage resistance.

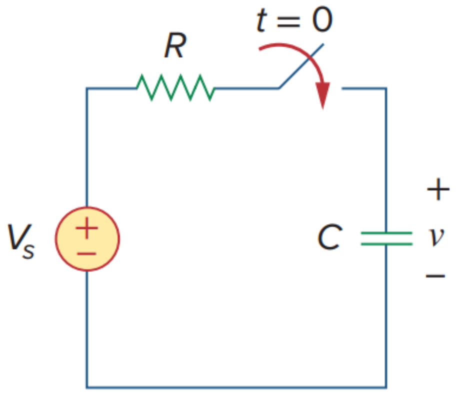


# Course Outline: broad themes



# Step Response of a RC circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the capacitor voltage.




$\Rightarrow$ Since the voltage of a capacitor cannot change instantaneously,

$$\Rightarrow v(0^-) = v(0^+) = V_0$$

$\Rightarrow$ Using KCL (for $t > 0$),

$$\Rightarrow C\frac{dv}{dt} + \frac{v - V_s}{R} = 0$$

$$\Rightarrow \frac{dv}{dt} = -\frac{v - V_s}{RC}$$

$$\Rightarrow \frac{dv}{v - V_s} = -\frac{1}{RC}dt$$

Integrating both sides,

$$\Rightarrow [\ln(v - V_s)]_{V_0}^{v(t)} = -\left[\frac{t}{RC}\right]_0^t$$

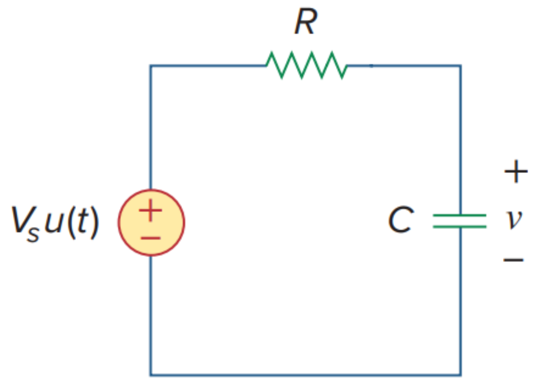
$$\Rightarrow \ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

$$\Rightarrow ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

$$\Rightarrow \frac{v - V_s}{V_0 - V_s} = e^{-t/RC}$$

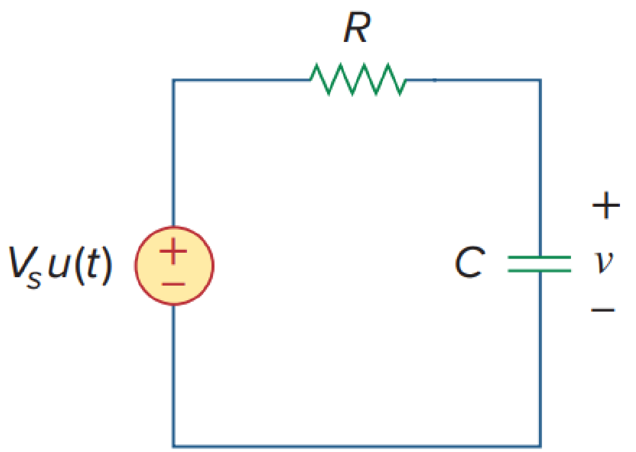
$$\Rightarrow v - V_s = (V_0 - V_s)e^{-t/RC}$$

$$\Rightarrow v(t) = V_s + (V_0 - V_s)e^{-t/RC}$$



# Time Constant (charging) for RC circuit

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$



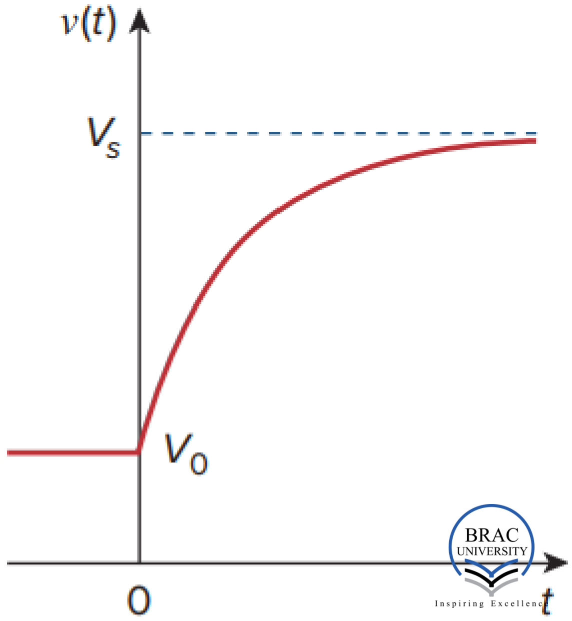

- This is known as the complete response (or total response) of the RC circuit to a sudden application of a dc voltage source. It is assumed that the capacitor was initially charged to $V_0$.

$$\Rightarrow \quad v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

$\Rightarrow$ where $\tau = RC$ is the *time constant* (unit in sec).

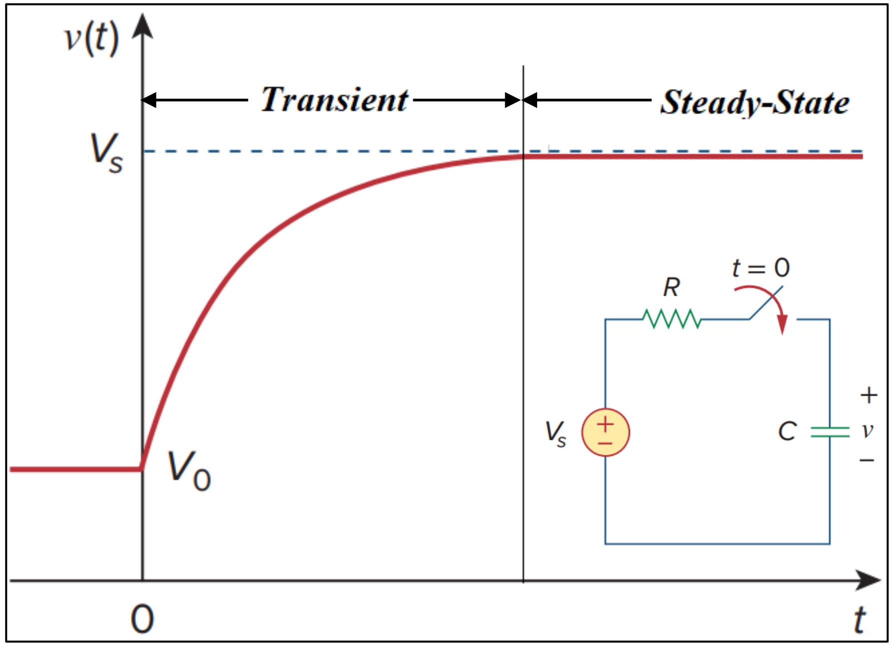
- Notice that, we write $\tau = RC$ for the circuit consisting of only a resistor $R$ in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor $R$ is actually the Thevenin Resistance $R_{Th}$. Therefore,

$$\boxed{\tau = R_{Th}C}$$



# Transient and Steady-State Response

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$




- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,

$$v(t) = v_{ss} + v_t, \quad \text{where,}$$

$$v_{ss} = V_s \quad \& \quad v_t = (V_0 - V_s)e^{-\frac{t}{\tau}}$$

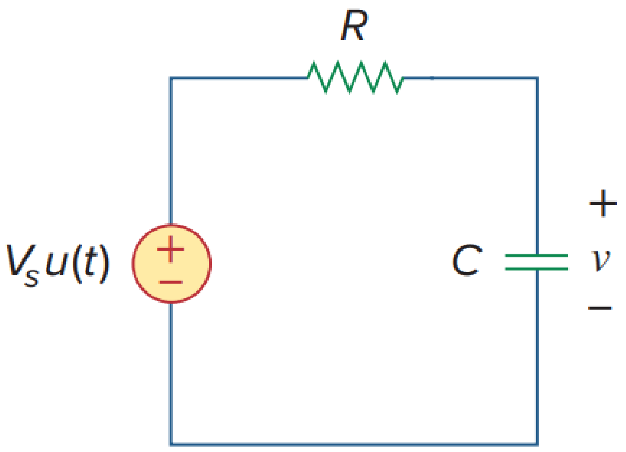
- The *transient response* $(v_t)$ is the circuit's temporary response that will die out with time.
- The *steady-state response* $(v_{ss})$ is the behaviour of the circuit a long time after an external excitation is applied.
- The complete response can be written as,

$$v(t) = V_{final} + [V_{initial} - V_{final}]e^{-\frac{t}{\tau}}$$

$$or, \quad v(t) = V(\infty) + [V(0) - V(\infty)]e^{-\frac{t}{\tau}}$$

# Definition of $\tau$ (charging)

$$v(t) = V_{final} + [V_{initial} - V_{final}]e^{-\frac{t}{\tau}}$$



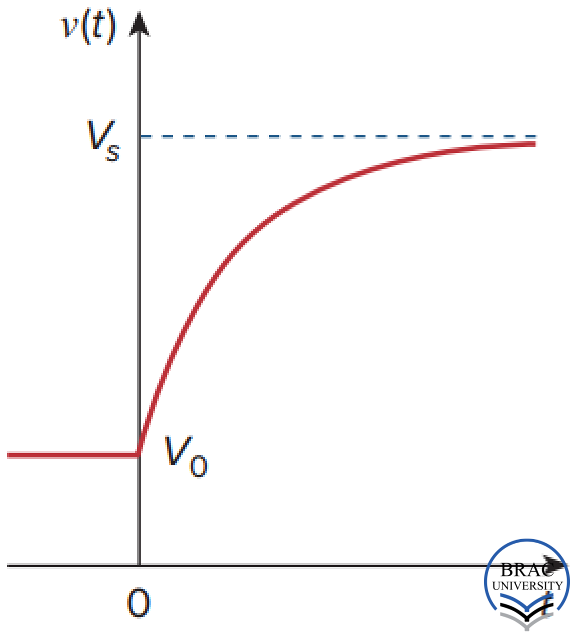

- At $t = \tau$,

$$v(t) = V_{final} + [V_{initial} - V_{final}]e^{-1}$$

$$\Rightarrow \quad v(t) = V_{final}\left(1 - {}^{1}\!/_{e}\right) + V_{initial}\left({}^{1}\!/_{e}\right)$$

$$\Rightarrow \quad v(t) = V_{final}\left(1 - {}^{1}\!/_{e}\right) - V_{initial}\left(1 - {}^{1}\!/_{e}\right) + V_{initial}$$

$$\Rightarrow \quad v(t) = V_{initial} + [V_{final} - V_{initial}]\left(1 - {}^{1}\!/_{e}\right)$$




- We can define the time constant in this way,

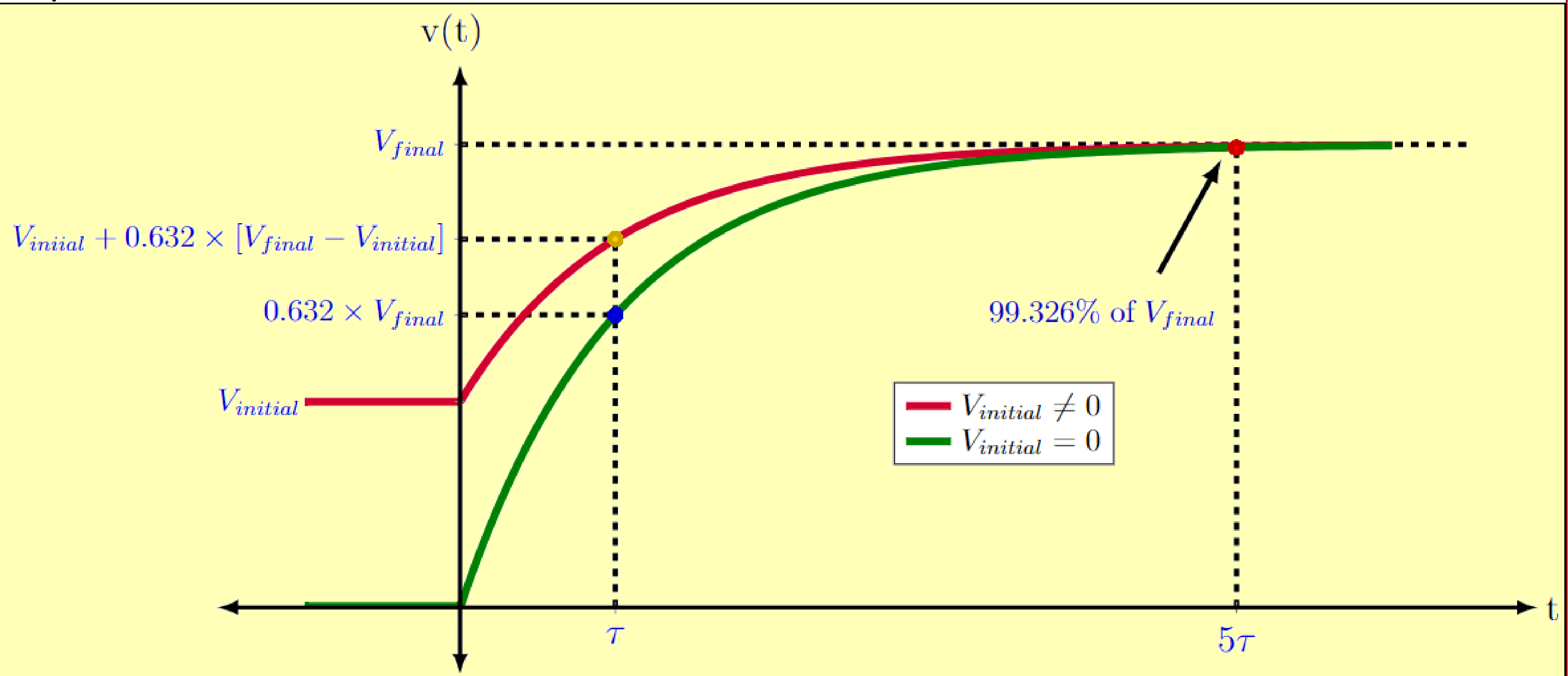
- The *charging time constant* is the time required for the response to reach to a factor of $(1 - 1/e)$ or 63.2% towards $V_{final}$ from an initial response $V_{initial}$.

# Time Constant ($\tau$): graphically

At $t = \tau$, $\quad v(t) = V_{initial} + [V_{final} - V_{initial}](1 - {}^{1}/_{e})$

$$\Rightarrow \quad v(t) = 63.2\% \times V_{final} \quad \text{when } V_{initial} = 0$$

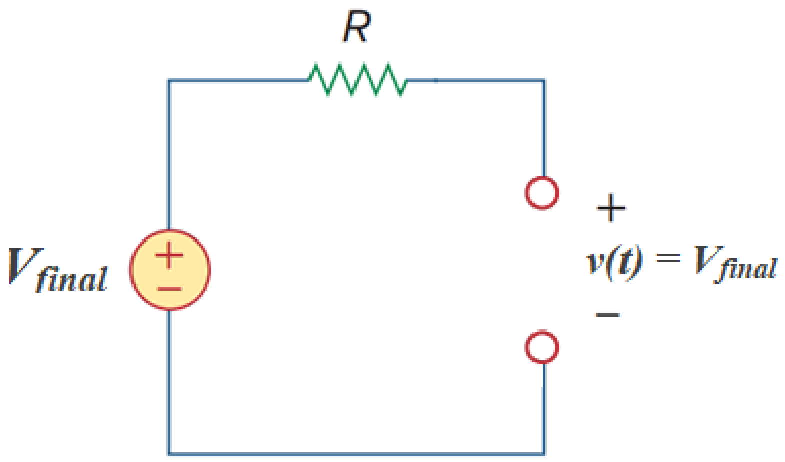
$$\Rightarrow \quad v(t) = V_{initial} + 63.2\% \times [V_{final} - V_{initial}] \quad \text{when } V_{initial} \neq 0$$



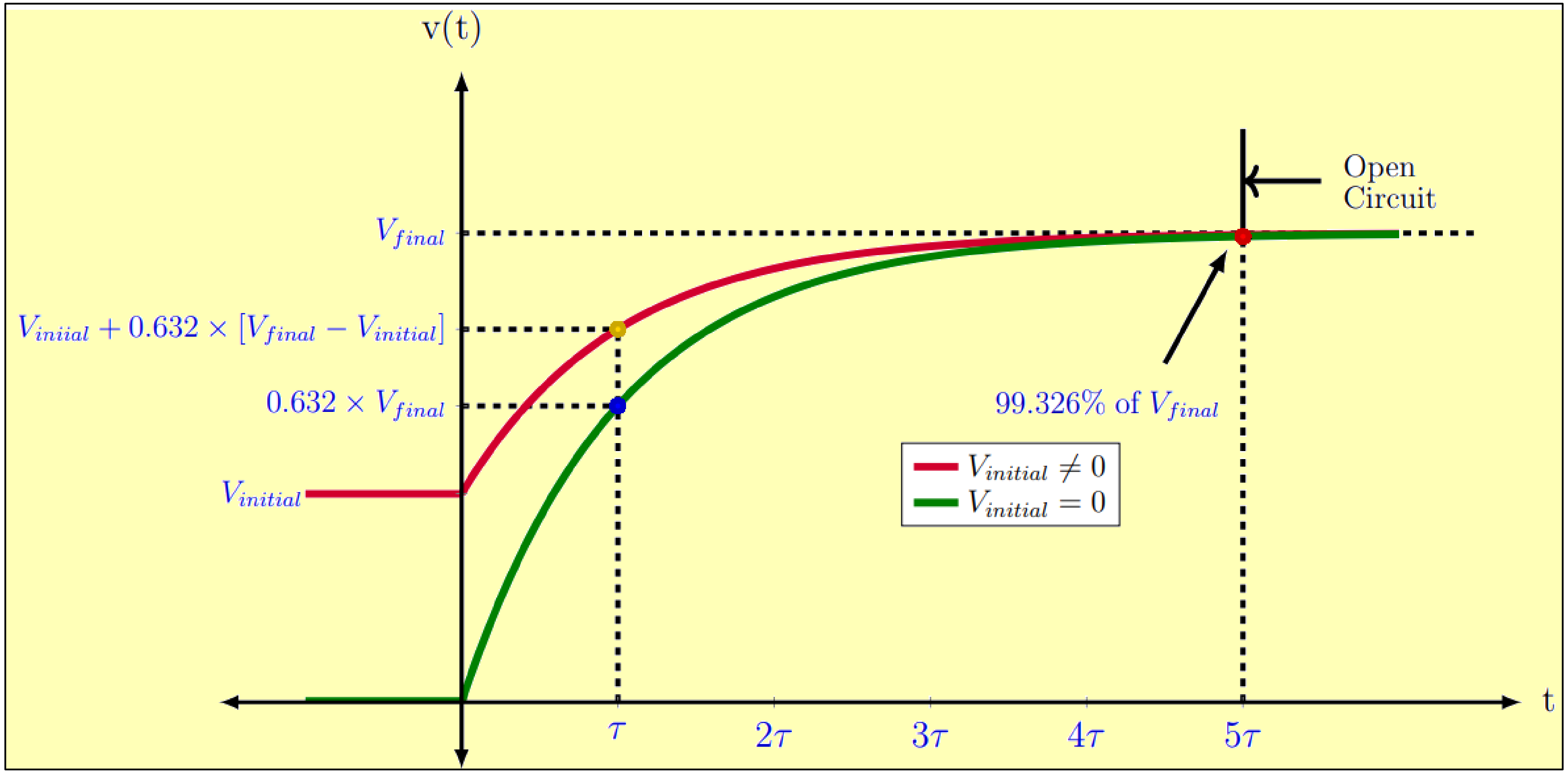

- As $\tau$ only depends on $R_{Th}$ and $C$ ($\tau = R_{Th}C$), for a given circuit, that is, for a fixed $R_{Th}$ and $C$, the time needed for the capacitor voltage to rise to the final value ($V_{final}$) is the same whether or not the capacitor is initially charged ($V_{initial}$ zero or nonzero).

# Significance of τ (charging): 5τ Time

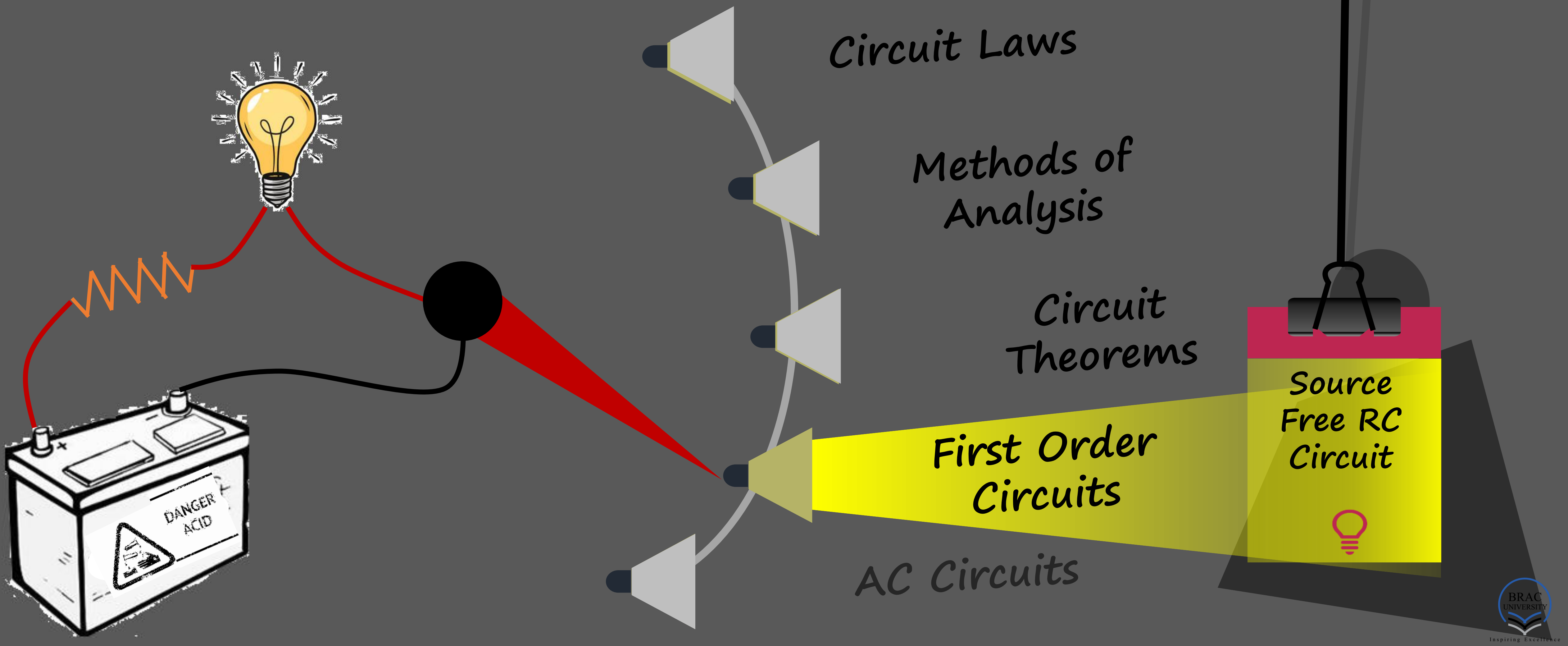
- As can be seen from the following plot, the capacitor voltage reaches the final voltage approximately after 5 times the Time Constant (τ). The capacitor is fully charged and acts as open circuit from 5τ time onward. So, when designing circuits, the charging time of a capacitor under the application of a certain dc supply can be set by choosing $R_{Th}$.




at or after $t = 5\tau$



# Course Outline: broad themes



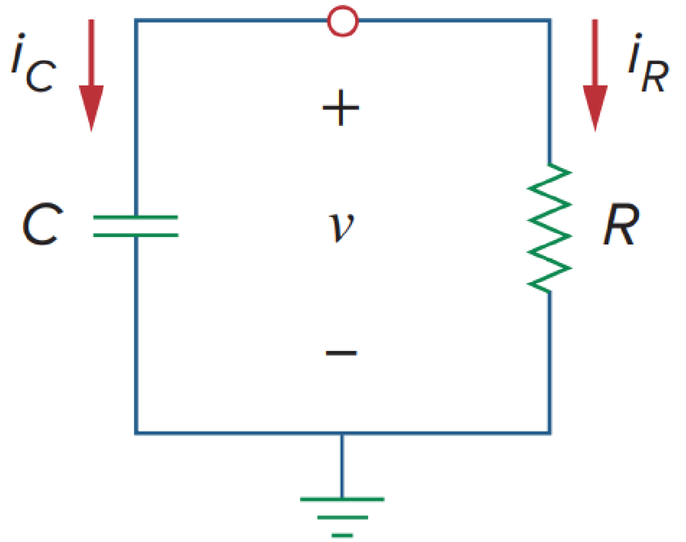
# Source-Free RC circuit

- A *source-free RC circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

⇒ Assume that a capacitor is charged to $V_0$ and then it is connected to a resistor as shown. The capacitor starts to discharge the stored energy to the resistor.

⇒ Initially stored charge, $w(0) = \frac{1}{2}CV_0^2$

⇒ From the figure using KCL, $i_C + i_R = 0$




$\Rightarrow \ C\frac{dv}{dt} + \frac{v}{R} = 0$

$\Rightarrow \ \frac{dv}{dt} + \frac{v}{RC} = 0$

$\Rightarrow \ \frac{dv}{v} = -\frac{1}{RC}dt$

Integrating both sides,

$\Rightarrow \ lnv = -\frac{t}{RC} + lnA$

$\Rightarrow \ ln\frac{v}{A} = -\frac{t}{RC}$

$\Rightarrow \ v = Ae^{-\frac{t}{RC}}$

At $t = 0, v(0) = A = V_0$. So, $\boxed{v(t) = V_0 e^{-\frac{t}{RC}}}$

# Time Constant (discharging) for RC circuit

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

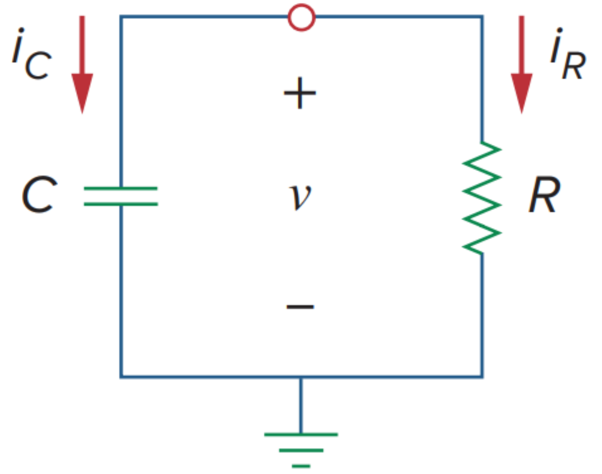
- This shows that the voltage response of the $RC$ circuit is an exponential decay of the initial voltage. It is called the *natural response* of the circuit.

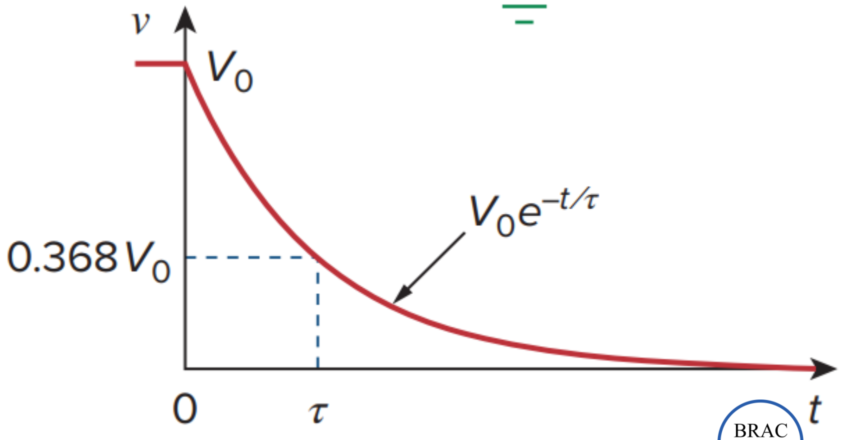
$$\Rightarrow \quad v(t) = V_0 e^{-\frac{t}{\tau}}$$

- where $\tau \ = \ RC$ is the time constant (unit in sec).

- Notice that, we write $\tau = RC$ for the circuit consisting of only a resistor $R$ in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor $R$ is actually the Thevenin Resistance $R_{Th}$. Therefore,

$$\boxed{\tau = R_{Th} C}$$



# Definition of $\tau$ (discharging)

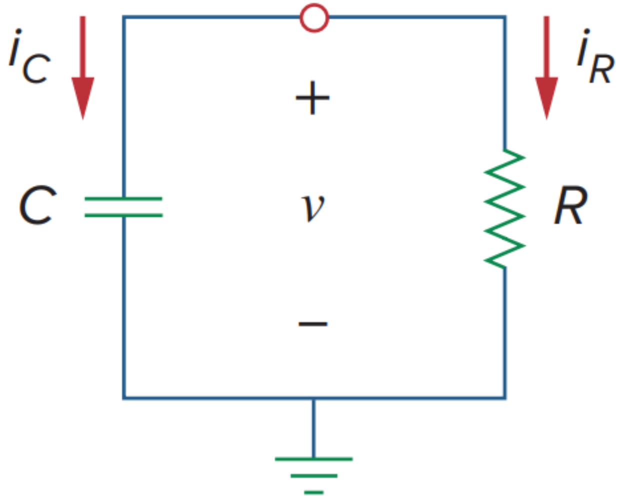
$$v(t) = V_0 e^{-\frac{t}{\tau}}$$

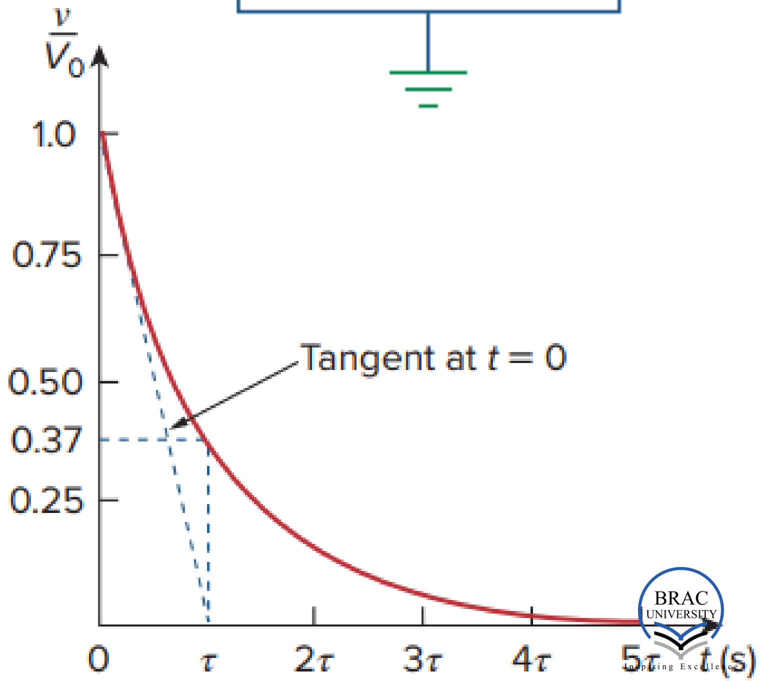
- At $t = \tau$,

$$v(t) = V_0 e^{-1}$$

$$\Rightarrow \quad v(t) = 0.368 \times V_0$$

- We can define the discharging time constant in this way,
- The *discharging time constant* is the time required for the response to fall to a factor of $^1/_e$ or 36.8% from an initial response $V_{initial}$ or $V_0$.
- Recall that the *charging time constant* is the time required for the response to reach to a factor of $(1 - 1\,/\,e)$ or 63.2% towards $V_{final}$ from an initial response $V_{initial}$.

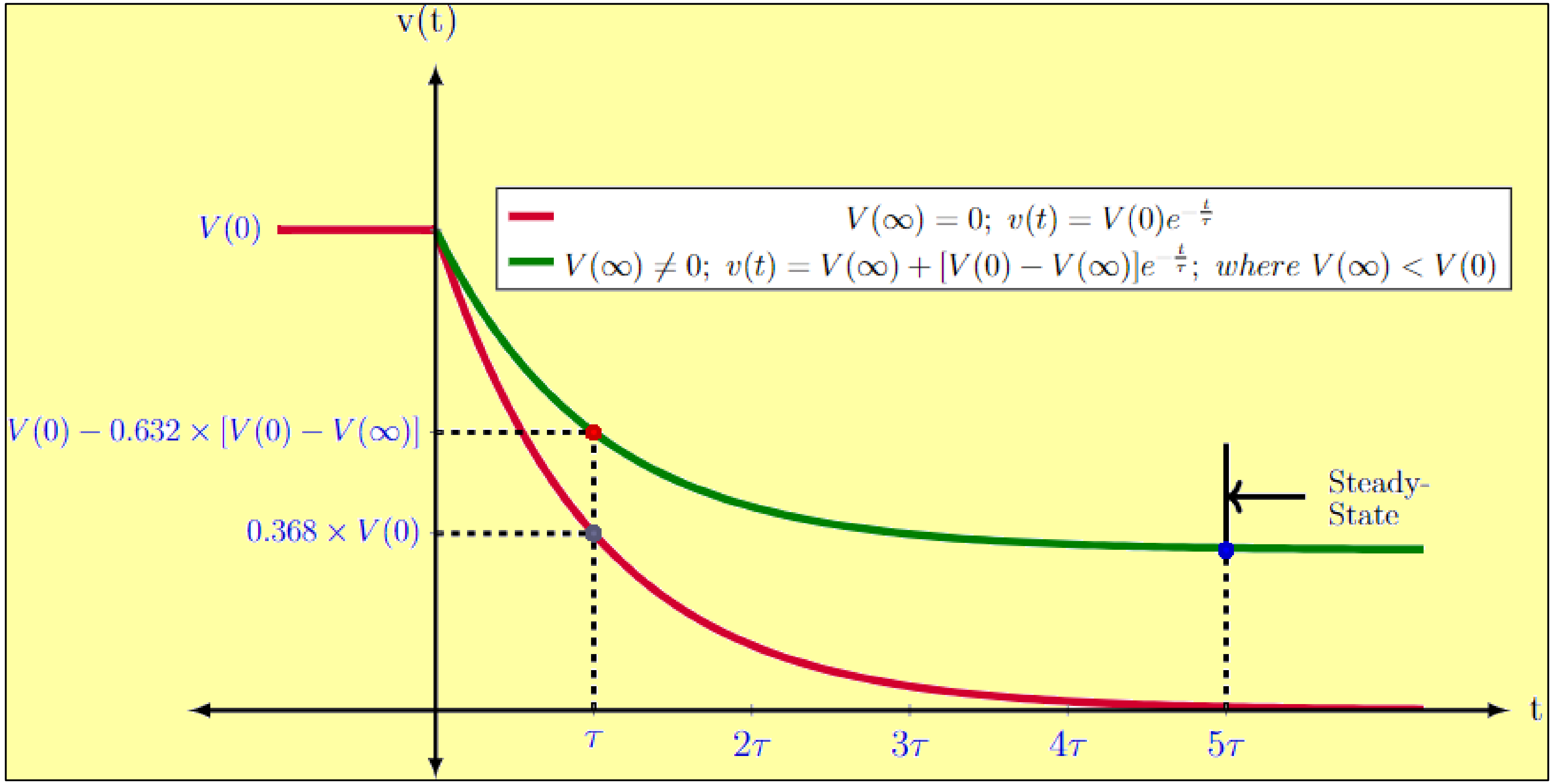


# Significance of $\tau$ (discharging): $5\tau$ Time

- As can be seen from the following plot, the capacitor voltage decreases to the final voltage approximately after $5$ times the Time Constant ($\tau$). In case where $V(\infty) = 0$, the capacitor is fully discharged from $5\tau$ time onward. So, when designing circuits, the discharging time of a capacitor can be set by choosing $R_{Th}$.

- In the case that a capacitor is subjected to a final voltage lower than its initial voltage, the discharging $\tau$ is the time required for the response to decay to 63.2% from $V(0)$ towards $V(\infty)$. See Problem 6



# Procedure

$$v(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

**Determine the initial voltage of the capacitor $V_{initial}$ or $V(0)$**

Consider only the active‡ portion of the circuit before switching. For example, if switching occurs at $t = 0$, consider the circuit for $t < 0$.

If the circuit includes any dc source (current or voltage), open the capacitor and determine the voltage at the open terminal. This is the $V(0)$. $V(0) = 0$ if there is no independent source in the circuit.

**Determine the final voltage of the capacitor $V_{final}$ or $V(\infty)$**

Now consider the active‡ portion of the circuit after switching. For example, for $t > 0$.

Repeat the step. This time, the voltage across the capacitor is $V(\infty)$. Circuits with $V(\infty) = 0$ are called source free.

**Determine the time constant ($\tau$)**

Again, only consider the active‡ portion after switching. For example, for $t > 0$.

Determine the Thevenin resistance ($R_{Th}$) as seen from the capacitor terminals

$\tau = R_{Th}C$

**Determine $v(t)$**

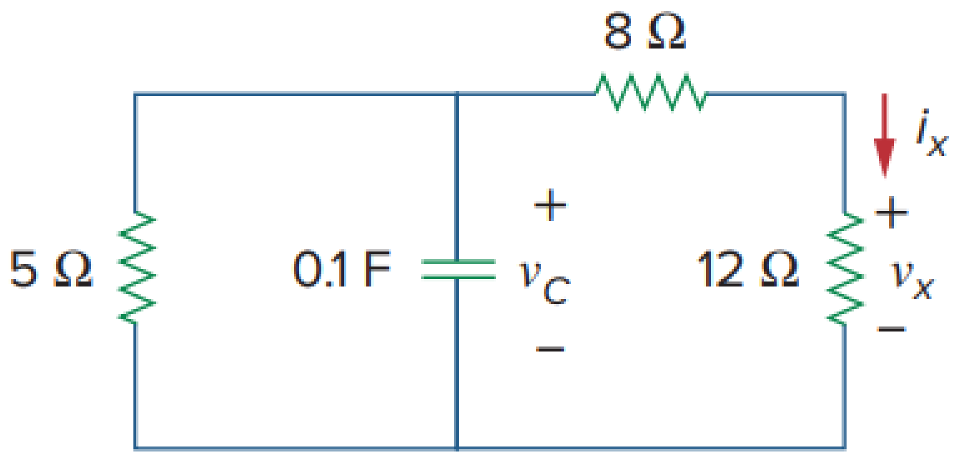
Plug in $V(0)$, $V(\infty)$, and $\tau$ into the equation for $v(t)$

Determine any other voltages or currents in the circuit using $v(t)$ and the circuit laws.

‡ active portion of the circuit excludes everything that has no influence on the capacitor

# Example 1

- Let $V_C(0) = 15\ V$ , Determine $v_C$, $v_x$, and $i_x$ for $t > 0$.




**Solution**

The equivalent resistance as seen from the capacitor terminal is,

$$R_{eq} = (8+12)\,||\,5 = 4\,\Omega$$

Time constant, $\tau = R_{eq}C = 4 \times 0.1 = 0.4\ s$

Thus, for a source-free RC circuit, $V(\infty) = 0$. So,

$$v_C(t) = V(0)e^{-\frac{t}{\tau}} = 15e^{-2.5t}\ (V)$$

The voltage $v_x$ can be found by simple voltage division.

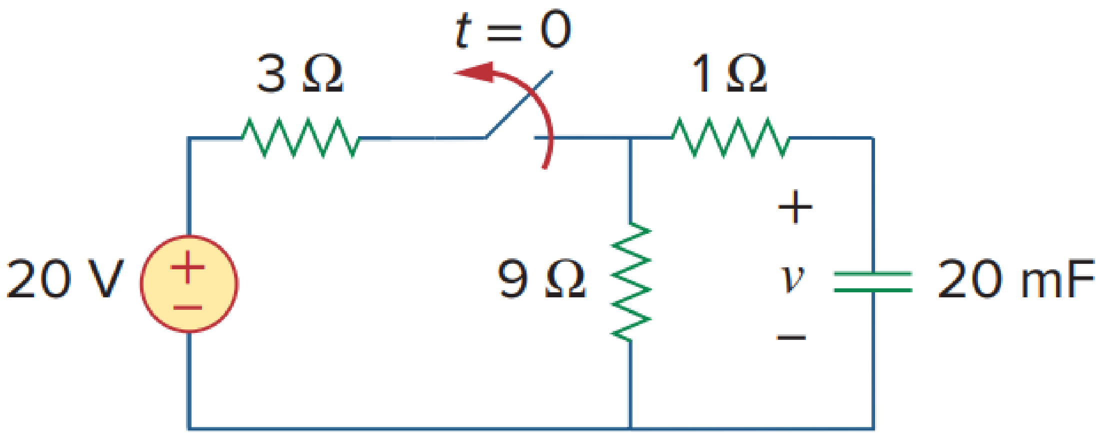
$$v_x(t) = \frac{12}{12+8} \times v_C(t) = 9e^{-2.5t}\ (V)$$

According to the Ohm's law,

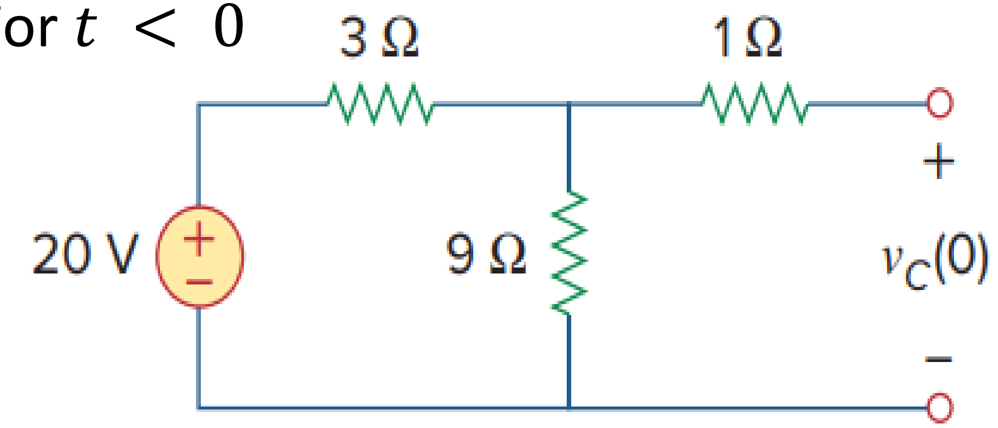
$$i_x = \frac{v_x}{12} = \frac{9e^{-2.5t}}{12} = 0.75e^{-2.5t}\ (\mathrm{A})$$

# Example 2

- The switch in the circuit has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t > 0$. Calculate the initial energy stored in the capacitor.




For $t < 0$




For $t < 0$, the switch is closed. With the capacitor open at dc, the circuit transforms into the one shown above.

No current flows through the $1\,\Omega$. So, the voltage across the $9\,\Omega$ is the $v_C(t)$ for $t < 0$,

$$v_C(t) = \frac{9}{9+3} \times 20 = 15\,V, \qquad t < 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0) = v_C(0^-) = 15\,V$$

# Example 2: $t > 0$

For $t > 0$, the switch is open. The circuit transforms into the one shown above. As there is no independent source in the circuit, $V(\infty) = 0$.
The Thevenin resistance as seen from the capacitor terminal,

$$R_{Th} = 1 + 9 = 10\,\Omega$$

The time constant is,

$$\tau = R_{Th}C = 10 \times 20 \times 10^{-3} = 0.2\ s$$

So, the voltage across the capacitor for t > 0 is,

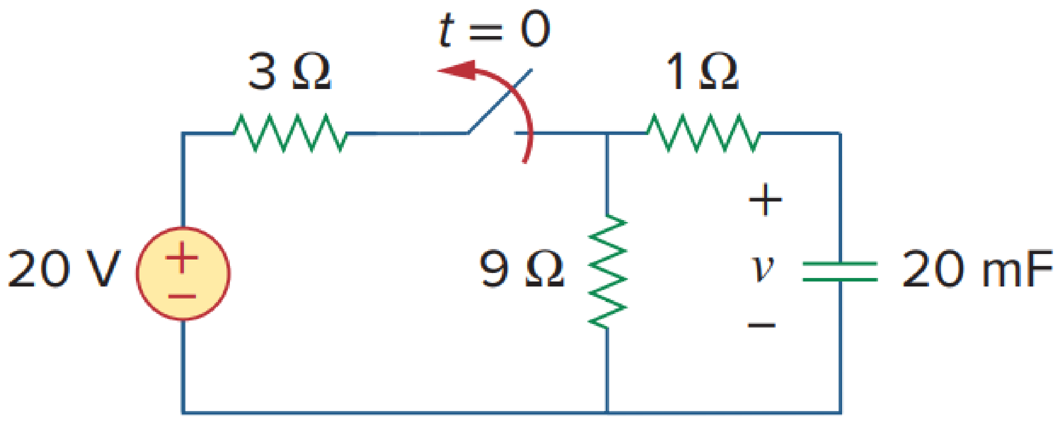
$$v_C(t) = V(0)e^{-\frac{t}{\tau}}$$

$$= 15e^{-5t}\ (V)$$

The initial energy stored in the capacitor is,

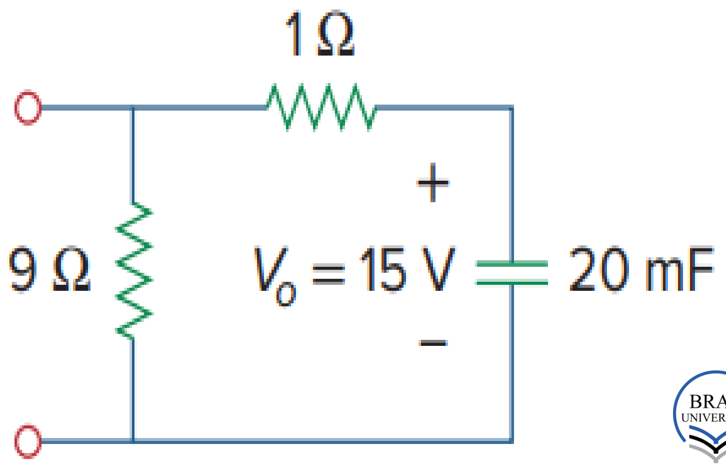
$$w_C(t) = \frac{1}{2}CV(0)^2$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\,J$$



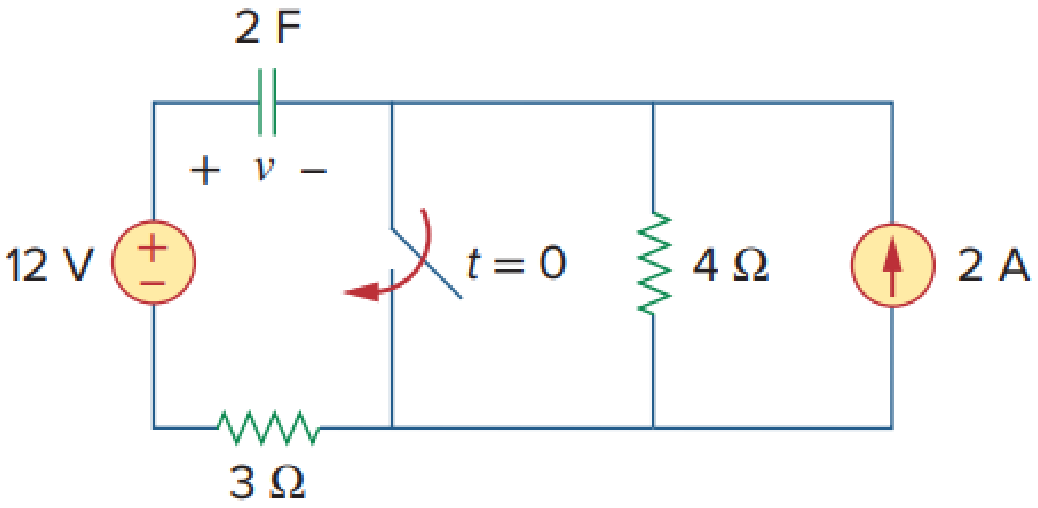

*Original circuit*

For $t > 0$

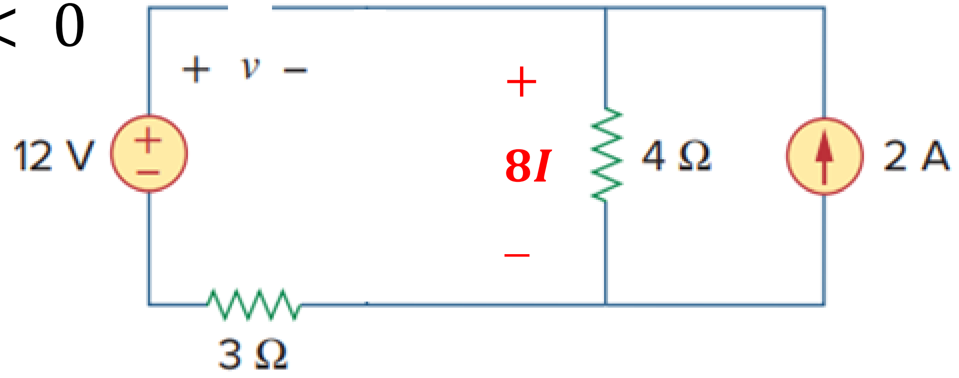


# Example 3

- Calculate the capacitor voltage $v(t)$ for $t < 0$ and for $t > 0$.




For $t < 0$




For $t < 0$, the switch is open. With the capacitor open at dc, the circuit transforms into the one shown above.

The $2\ A$ current from the current source will flow only through the $4\ \Omega$ resistance. The voltage drop across the $4\ \Omega$ resistance is, $4 \times 2 = 8\ V$.

There is no voltage drop across the $3\ \Omega$ ($i = 0$ at open circuit). So,

$$v(t) = 12 - 8 = 4\ V, \qquad t < 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v(0) = v(0^-) = 4\ V$$

# Example 3: $t > 0$

For $t > 0$, the switch is closed. With the capacitor again open at dc, the circuit transforms into the one shown above.

Again, there is no voltage drop across the $3\,\Omega$ ($i = 0$ at open circuit). So,

$$v(t) = 12\,V, \qquad t > 0$$

This is the steady-state voltage across the capacitor for $t > 0$.

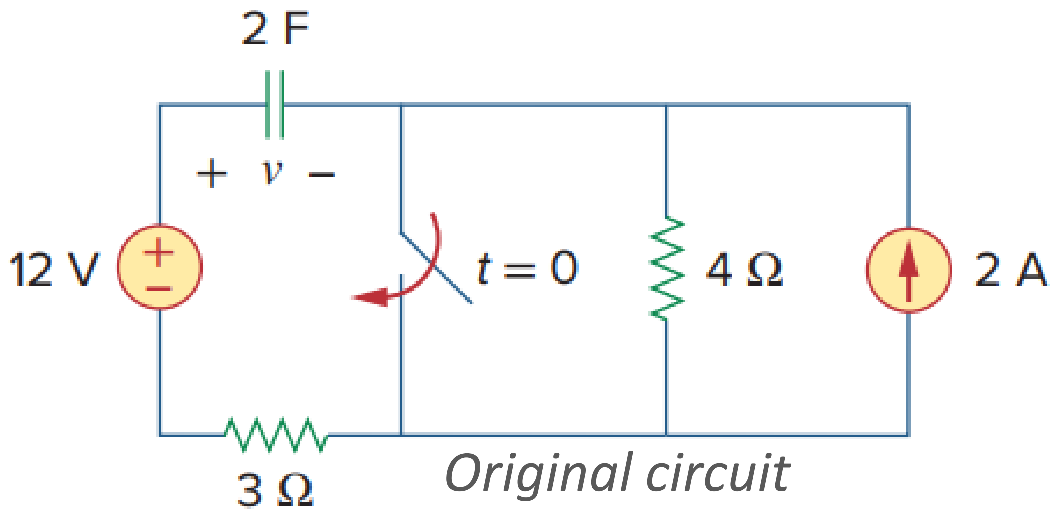
$$v(\infty) = 12\,V$$

The time constant is, $\tau = R_{Th}C = 3 \times 2 = 6\,s$

So,

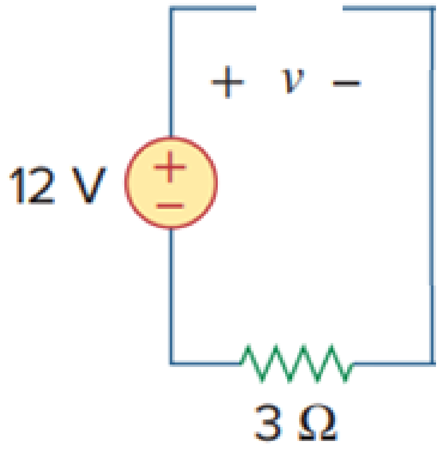
$$v(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 12 + [4 - 12]e^{-\frac{t}{6}} = 12 - 8e^{-\frac{t}{6}}$$

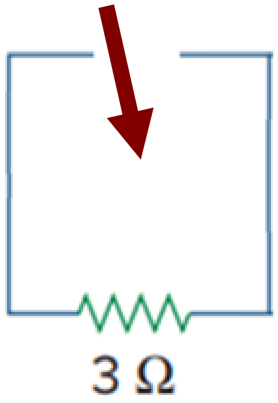



*Original circuit*

For $t > 0$

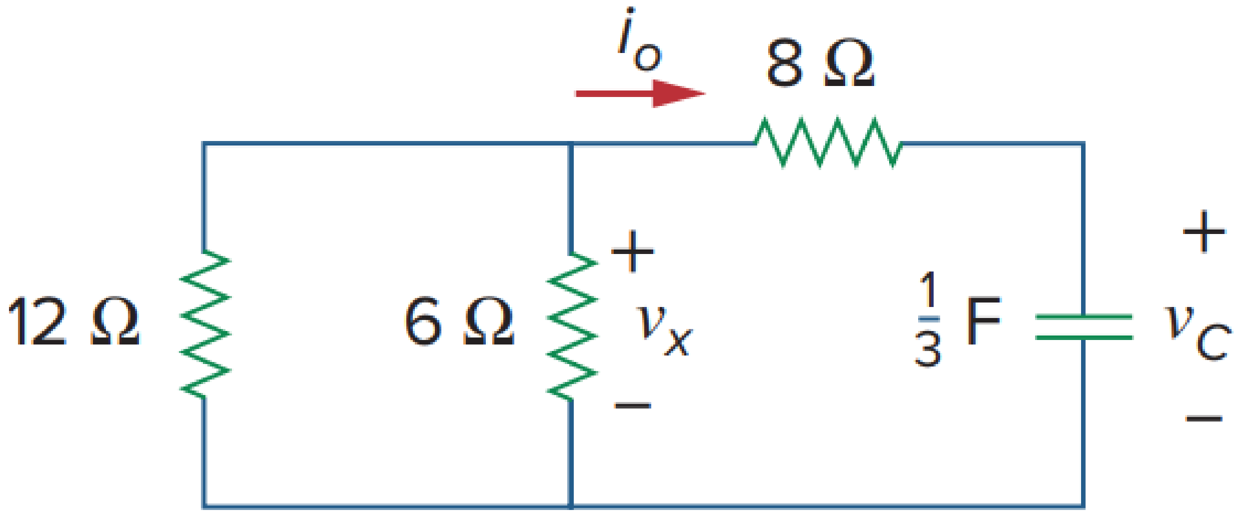



$R_{Th} = 3\,\Omega$



# Problem 1

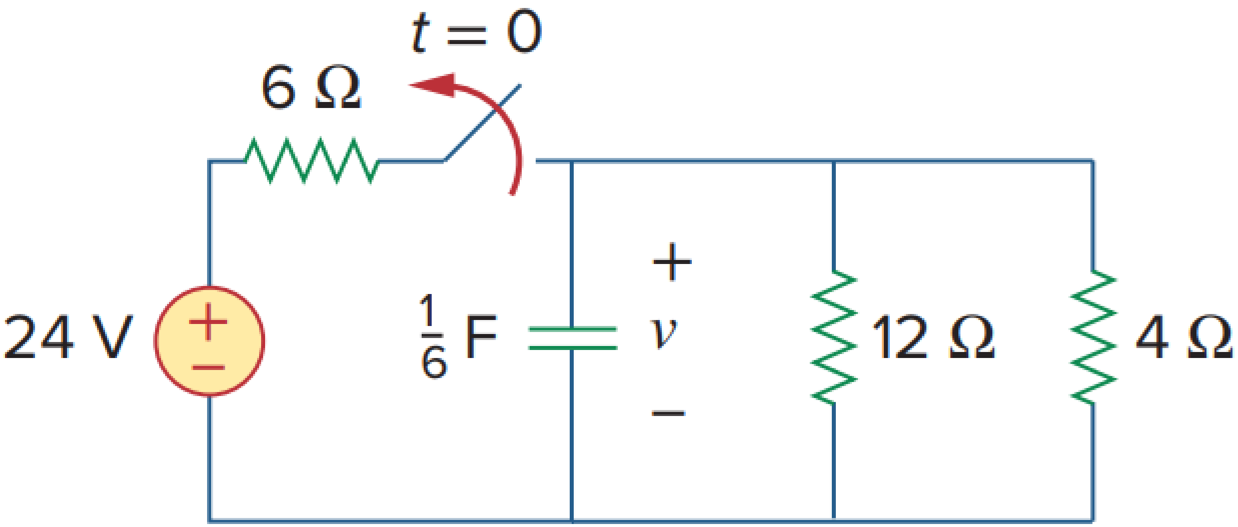
- Let $V_C\ (0) = 60\,V$, Find $v_C, v_x$, and $i_x$ for $t > 0$.




Ans: $\boldsymbol{v_C = 60e^{-0.25t}\ V;\ v_x = 20e^{-0.25t}\ V;\ i_x = -5e^{-0.25t}\ A}$

# Problem 2

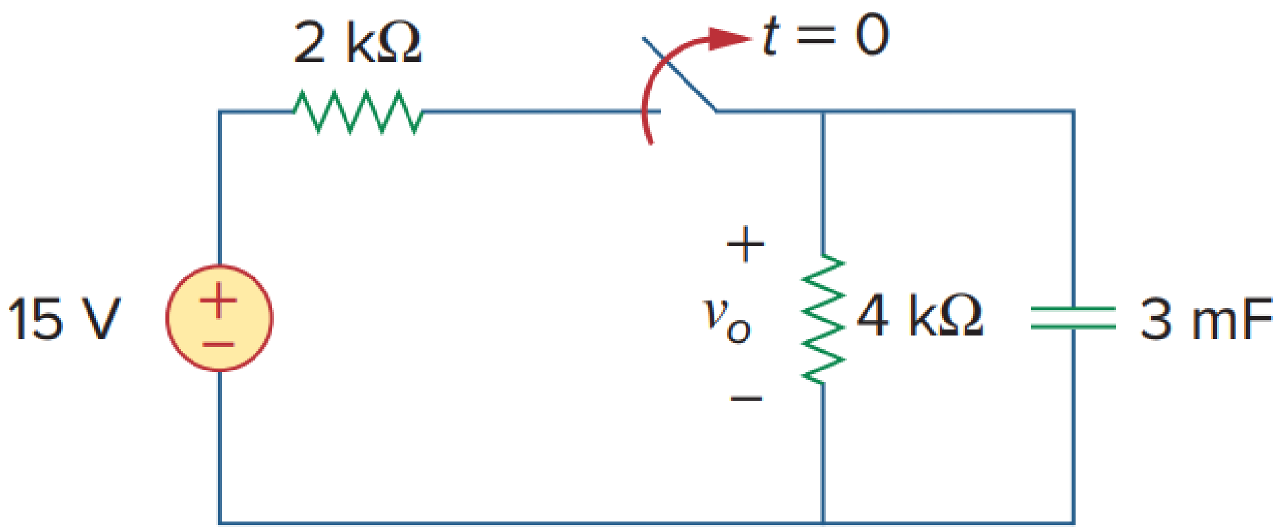
- The switch in the circuit has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t > 0$. Calculate the initial energy stored in the capacitor.




Ans: $\boldsymbol{v(t) = 8e^{-2t}\ V;\ w_C(0) = 5.333\ J}$

# Problem 3

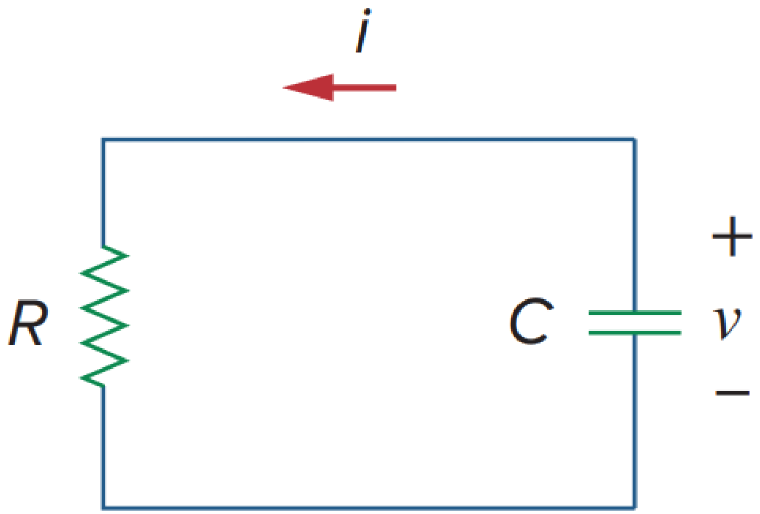
- The switch opens at $t = 0$. Find $v_0(t)$ for $t > 0$.




Ans: $\boldsymbol{v(t) = 10e^{-t/12}\ V}$

# Problem 4

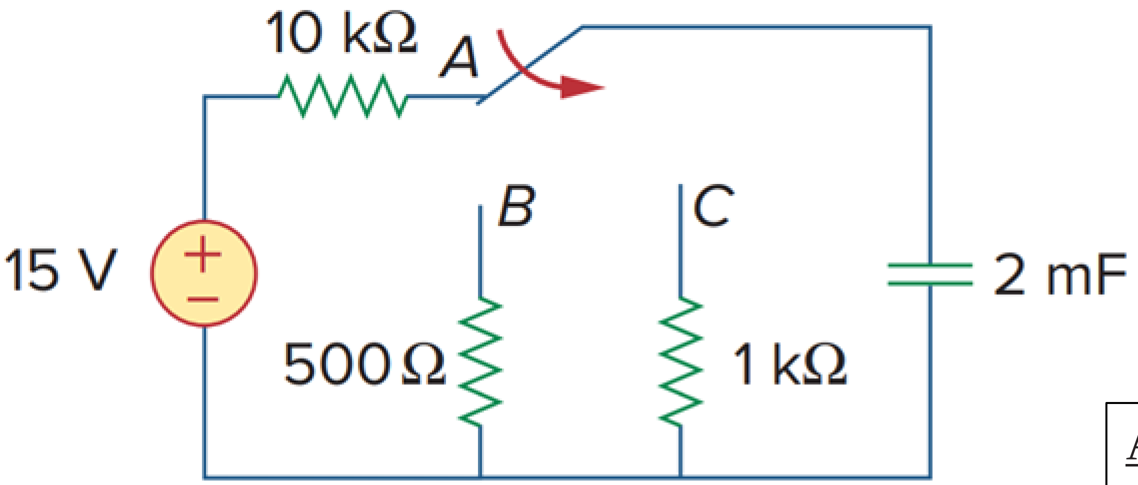
- For the circuit below, $v = 10e^{-4t}\,V$ and $i = 0.2e^{-4t}A$
  - (a) Find $R$ and $C$.
  - (b) Determine the time constant.
  - (c) Calculate the initial energy in the capacitor.
  - (d) Obtain the time it takes to dissipate $50\%$ of the initial energy.




Ans: $\boldsymbol{R = 50\,\Omega;\ C = 5\,mF;\ \tau = 0.25\,s;\ w_{C(0)} = 0.25\,J;\ t = 86\,ms}$

# Problem 5

- Assume that the switch has been in position A for a long time and is moved to position B at $t = 0$. Then at $t = 1s$, the switch moves from $B$ to $C$. Find $I_C(t)$ for $t > 0$.



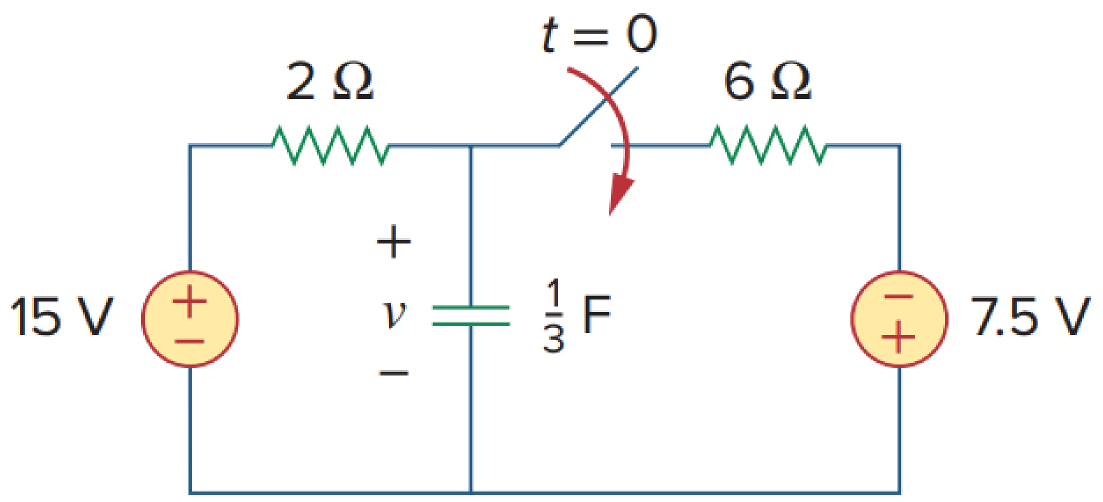

Ans: $\boldsymbol{v(t) = 15e^{-t}\,V\ for\ 0 < t < 1\ sec;}$

$\boldsymbol{v(t) = 5.518e^{-(t-1)/2}\,V\ for\ 1 < t < \infty\ sec;}$

# Problem 6

- Find $v(t)$ for $t > 0$ in the circuit shown below. Assume the switch has been open for a long time and is closed at $t = 0$. Calculate $v(t)$ at $t = 0.5s$.

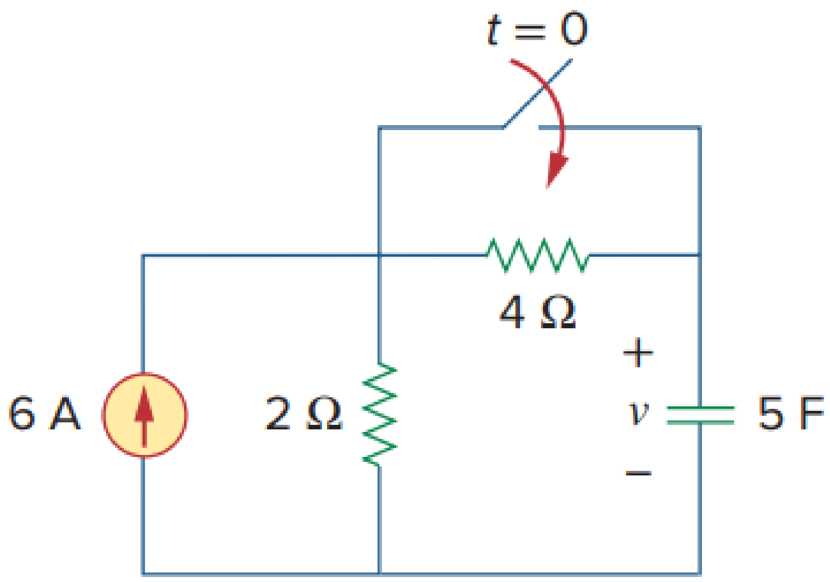
.




Ans: $\boldsymbol{v_c(t) = 9.375 + 5.625e^{-2t}\ V\ for\ t > 0;\ v_c(0.5) = 11.444\ V}$

# Problem 7

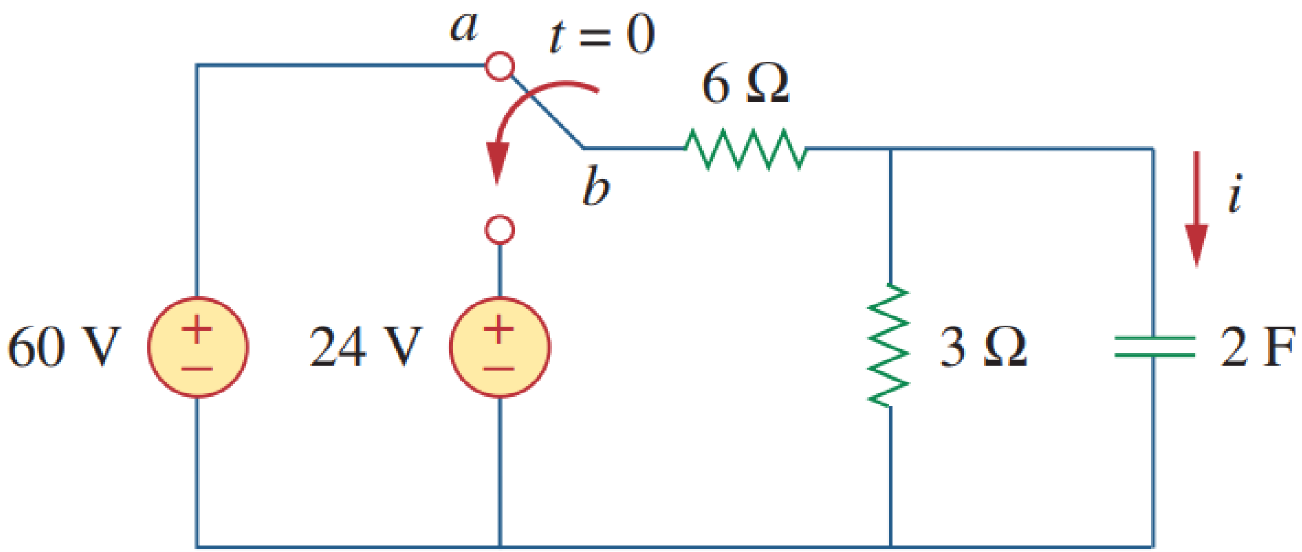
- Calculate the capacitor voltage for $t < 0$ and for $t > 0$.




Ans: $\boldsymbol{v(t) = 12\,V\ for\ t < 0;\ v(t) = 12\,V\ for\ t > 0}$

# Problem 8

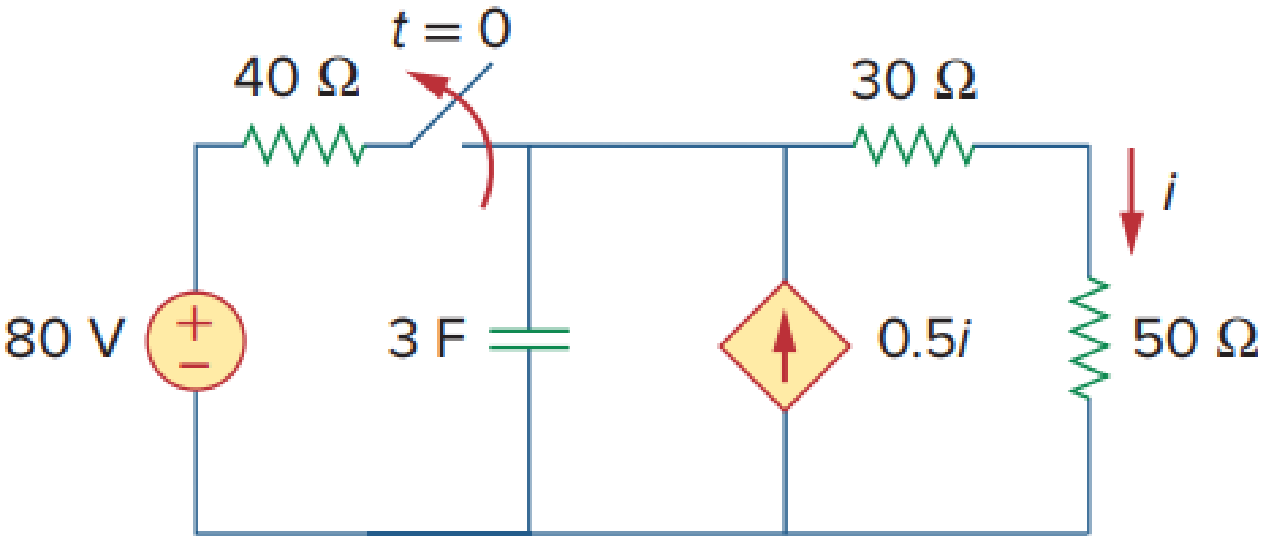
- The switch has been in position $a$ for a long time. At t = 0 it moves to position $b$. Calculate $i(t)$ for all $t > 0$.




Ans: $\boldsymbol{i(t) = -6e^{-0.25t}\ A\ for\ t > 0}$

# Problem 9

- Consider the circuit shown below. Find $i(t)$ for $t < 0$ and $t > 0$.




Ans: $\boldsymbol{i(t) = 0.8\,A\ for\ t < 0;\ i(t) = 0.8e^{-t/480}\,A\ for\ t > 0}$

# Problem 11

- The figure below shows the voltage response of an RC circuit to a sudden DC voltage applied through an equivalent resistance of $4\ k\Omega$.

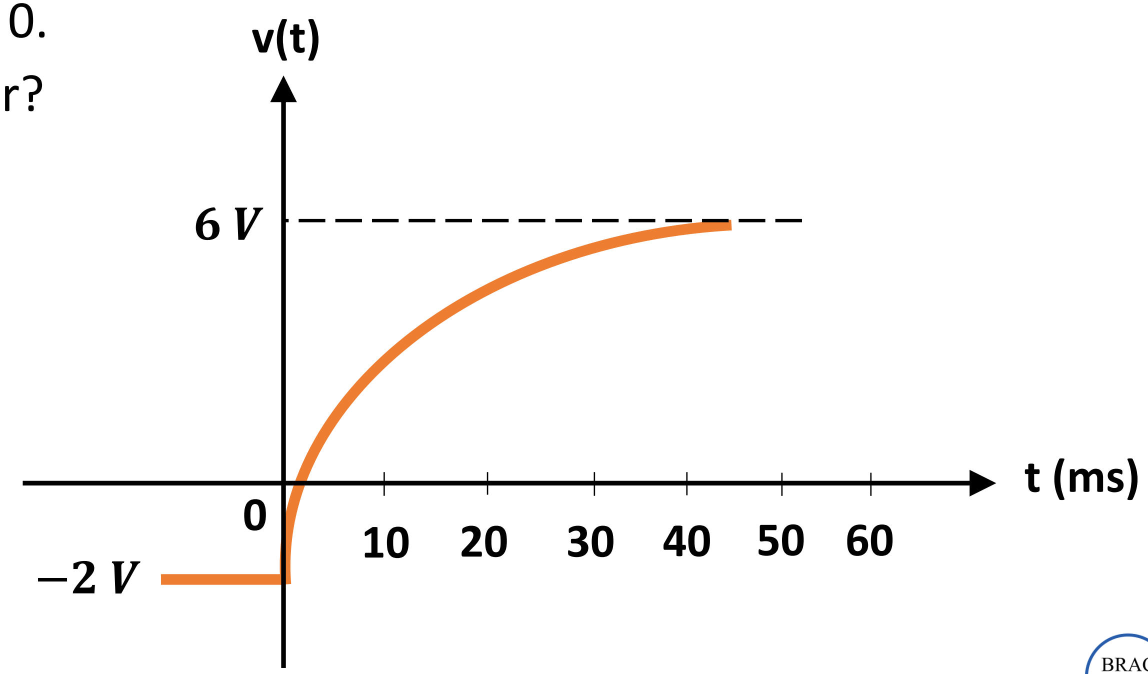
I. Define time constant.

II. Determine the approximate time constant from the figure.

III. Find the mathematical expression of v(t) for t > 0.

IV. What is the initial energy stored in the capacitor?

V. Draw the circuit diagram.

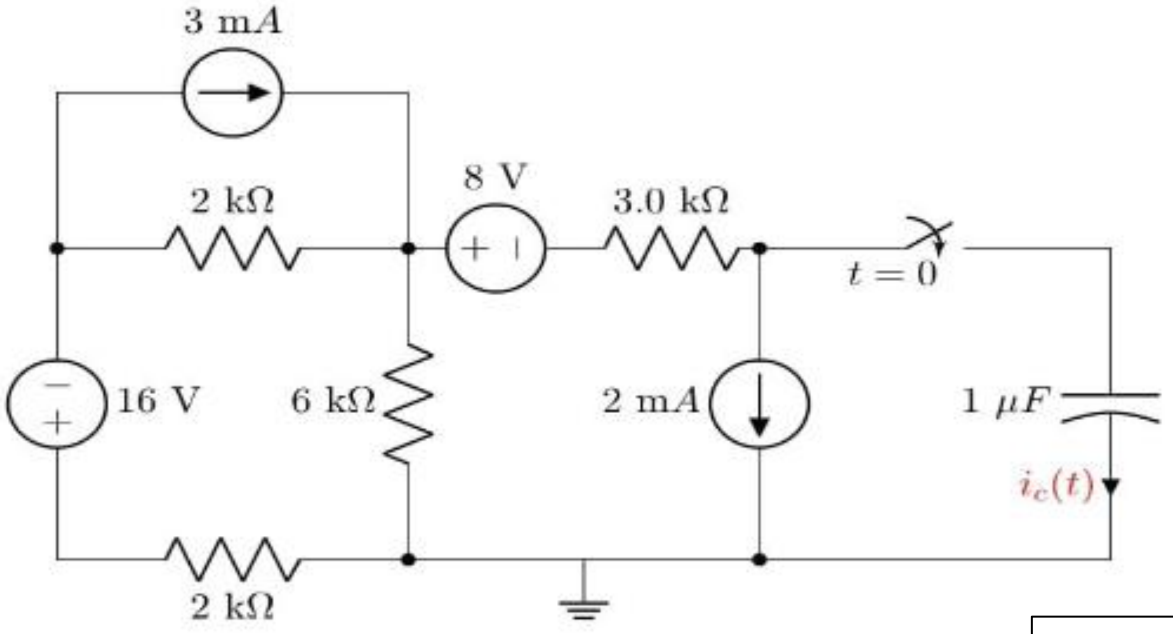



Ans: $(ii)\ \tau = \mathbf{9\ ms}$; $(iii)\ \boldsymbol{v(t) = 6 - 8e^{-1000t/9}\ I\ for\ t > 0}$; $(iv)\ \boldsymbol{w = 4.5 \times 10^{-6}\ J}$

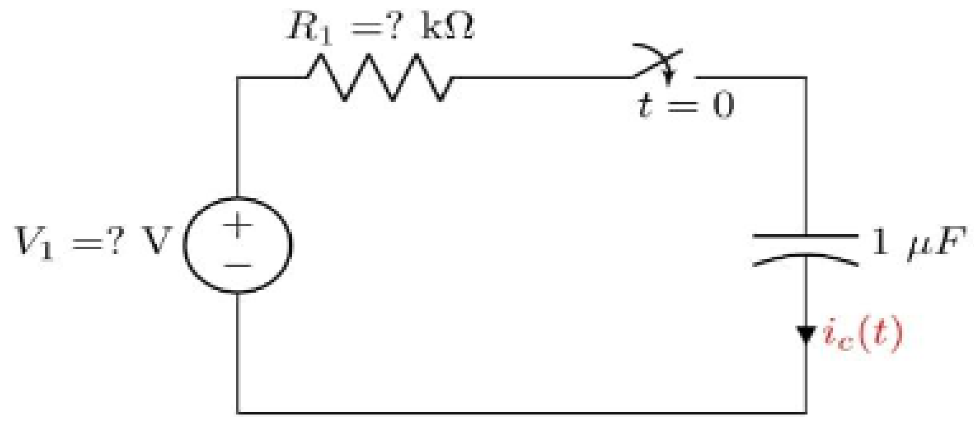
# Problem 12

I. Simplify the circuit 1 below so that it takes the form of the circuit 2. Determine the values of $V_1$ and $R_1$.

II. Perform transient analysis to determine $i_c(t)$ through the capacitor for $t > 0$.




*Circuit 1*

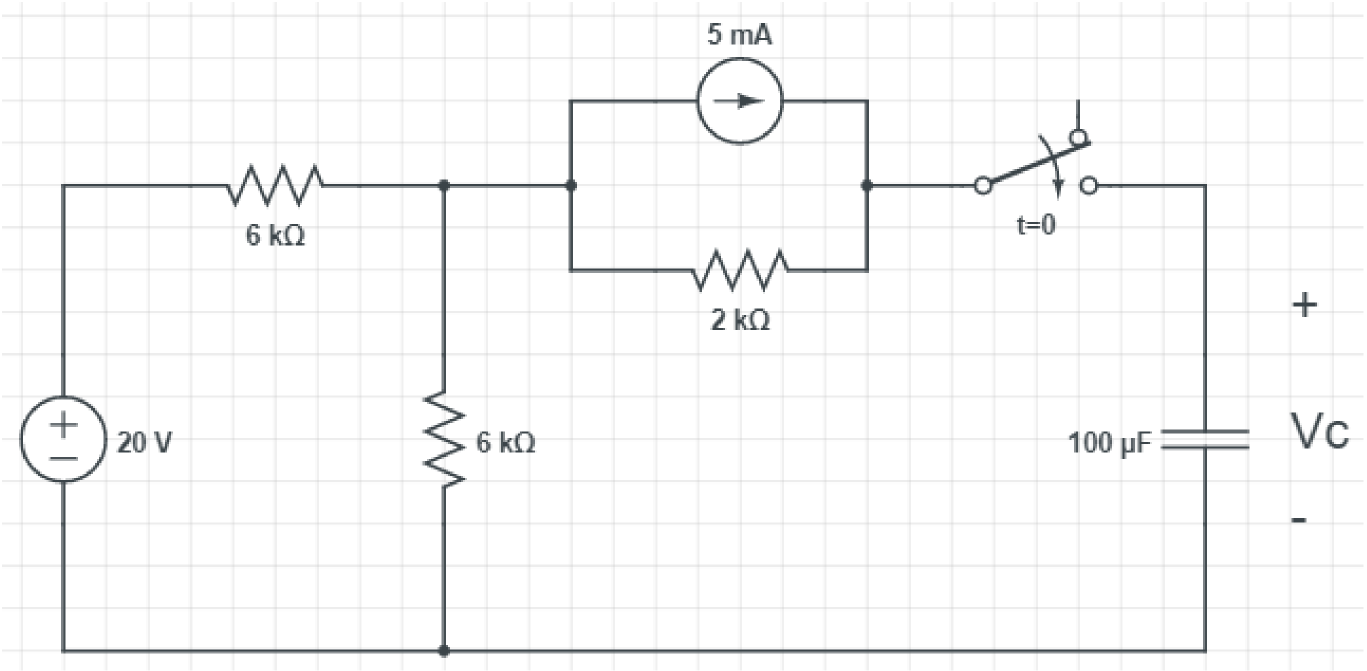



*Circuit 2*

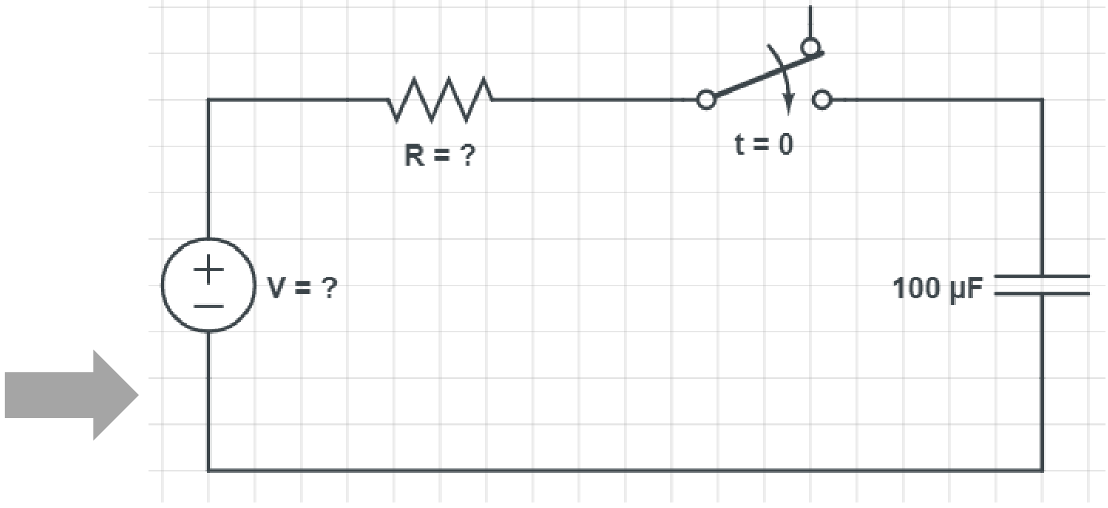
<u>Ans</u>: $\mathbf{V_1 = -24.8\ V}$; $\boldsymbol{R_1} = \mathbf{5.4\ k\Omega}$; $\boldsymbol{i_c(t) = -4.6e^{-1000t/5.4}\ A}$

# Problem 13

- Simplify the Circuit 1 below so that it takes the form of the Circuit 2. Determine the values of $I$ and $R$.
- Perform transient analysis to determine $V_C(t)$ across the capacitor for $t > 0$.




Circuit 1

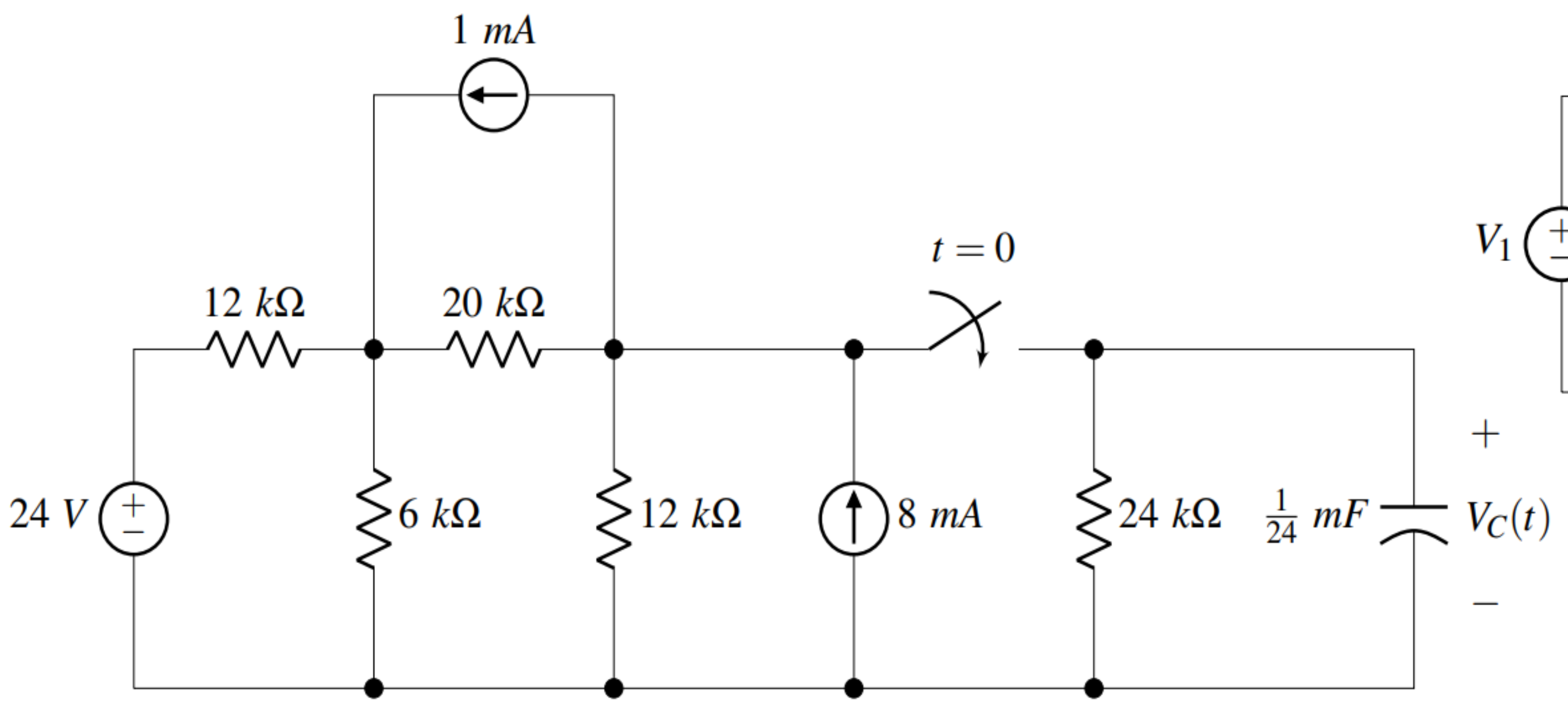



Circuit 2

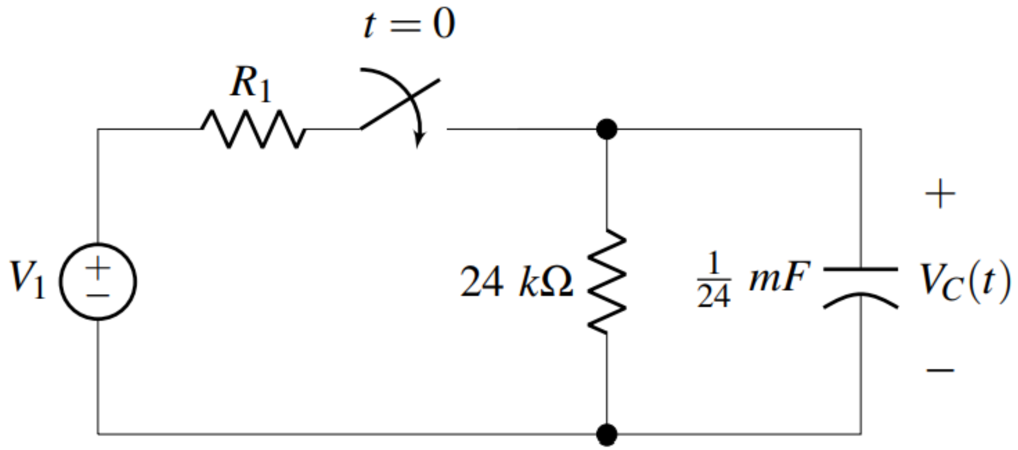
<u>Ans</u>: $\mathbf{V = 20\ V; R = 5\ k\Omega;} \boldsymbol{V_C(t) = 20(1 - e^{-2t})\ V}$

# Problem 14

- Simplify the Circuit 1 below so that it takes the form of the Circuit 2. Determine the values of $V_1$ and $R_1$. Perform transient analysis to determine $V_C(t)$ across the capacitor for $t > 0$.




Circuit 1



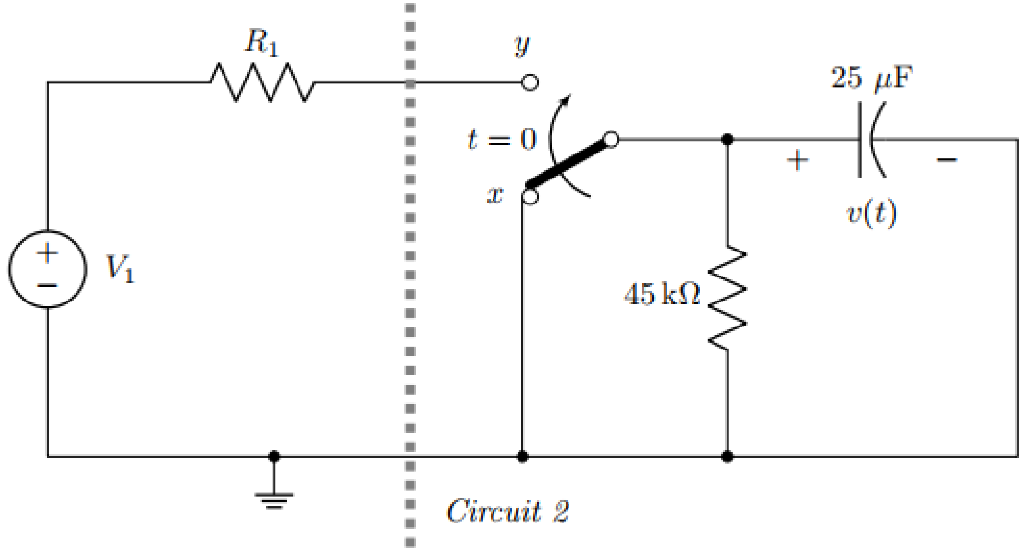

Circuit 2

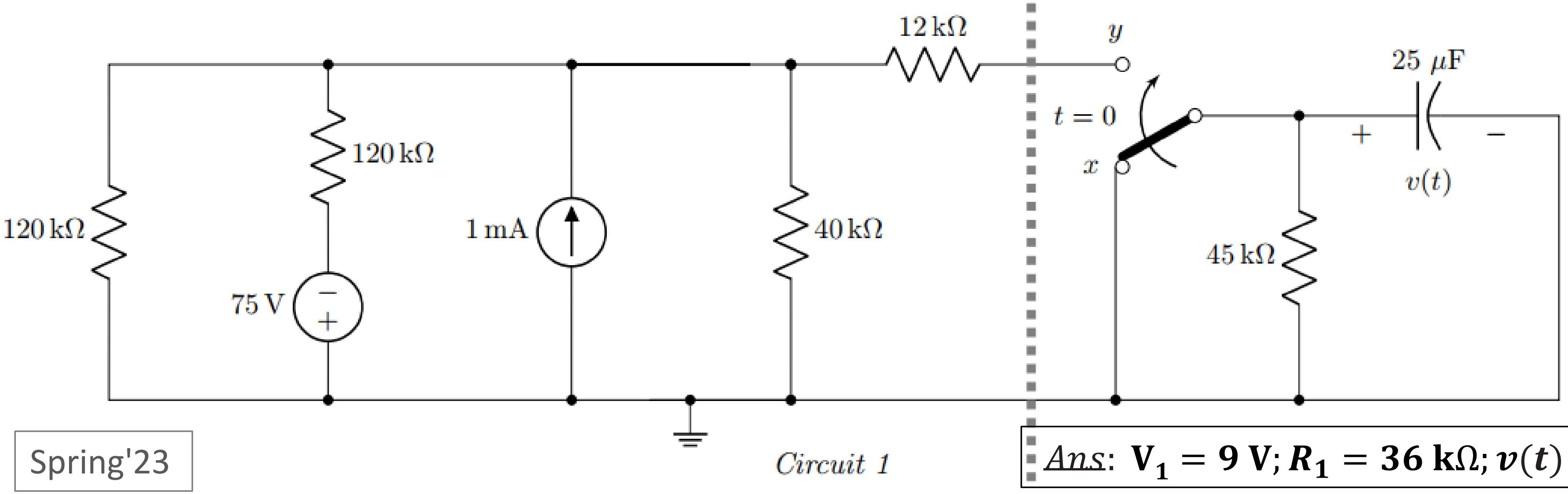
Fall'22

*Ans*: $\mathbf{V_1 = 60\ V}$; $\boldsymbol{R_1 = 8\ \mathrm{k\Omega}}$; $\boldsymbol{V_C(t) = 45(1 - e^{-4t})\ V}$

# Problem 15

- Reduce the left portion with respect to the dashed grey line of Circuit 1 so that it takes the form of Circuit 2 as shown. Write down the values of $V_1$ and $R_1$.
- Now, analyse the Transient Behaviour of the circuit assuming that the switch moves from position $x$ to position $y$ at $t = 0$. Determine $v(t)$ for $t > 0$.



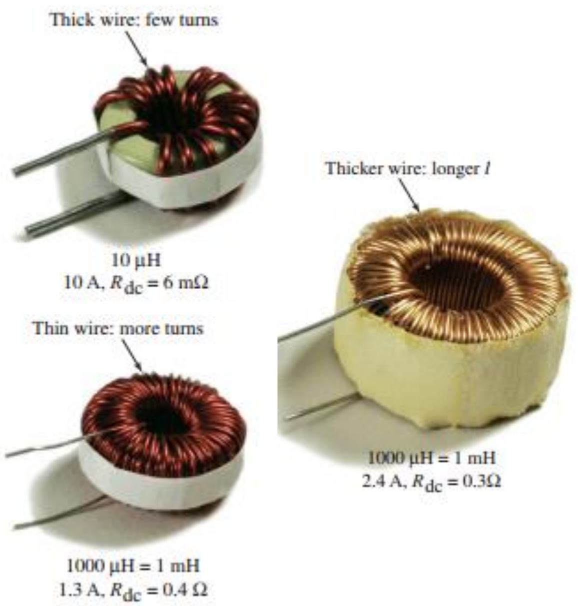




Spring'23

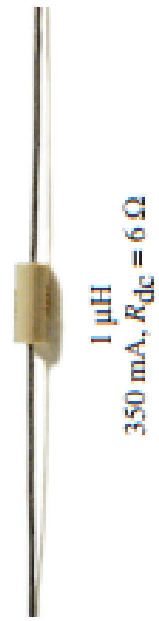
<u>Ans</u>: $\mathbf{V_1 = 9\ V; R_1 = 36\ k\Omega; v(t) = 5(1 - e^{-2t})\ V}$

# Inductors

- An inductor is a passive circuit element designed to store energy in its magnetic field.
- Inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.

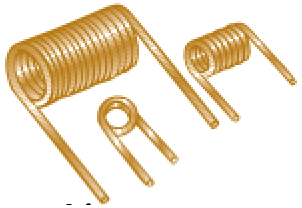


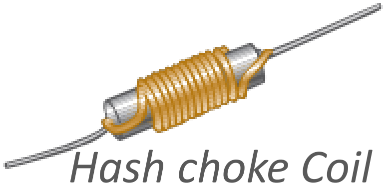

Toroid



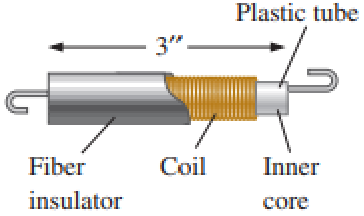

Plastic core


Ferrite core

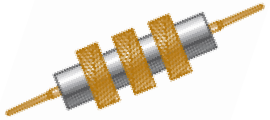




Hash choke Coil

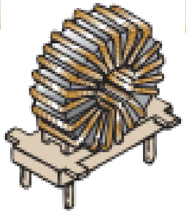



Delay line Coil

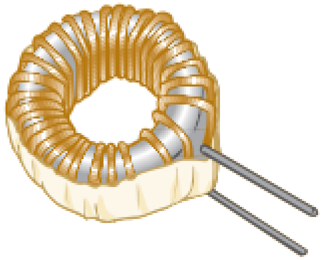


RF chokes

Molded Coil

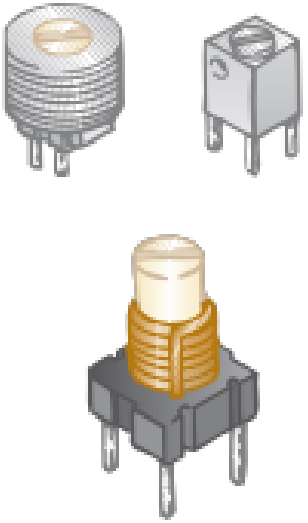


Common-mode choke Coil



Toroid Coil

Surface mount

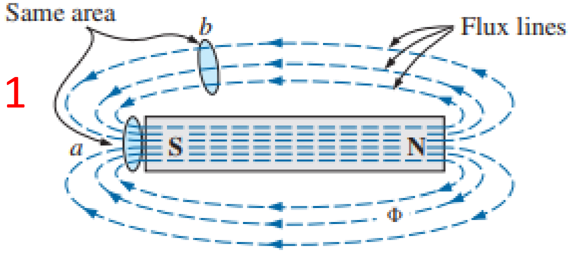


Variable inductors

# Magnetic Field

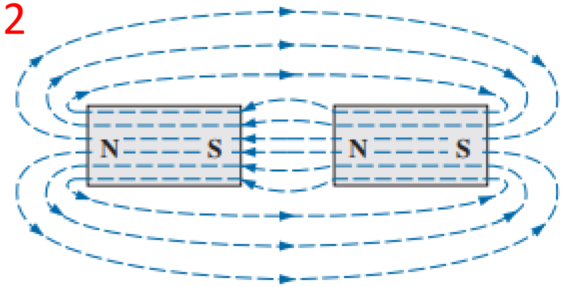
- A magnetic field exists in the region surrounding a permanent magnet, which can be represented by magnetic flux lines (imaginary) similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops
- In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass deflects if brought near a current-carrying conductor. This was the first demonstration that electricity and magnetism were related.

1



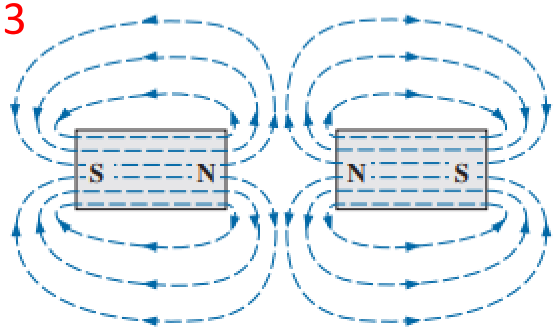

*Flux distribution for a permanent magnet.*

2



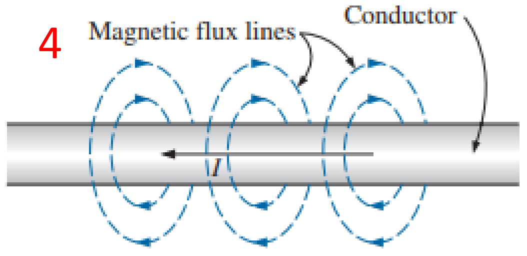

*Flux distribution for two adjacent, opposite poles.*

3



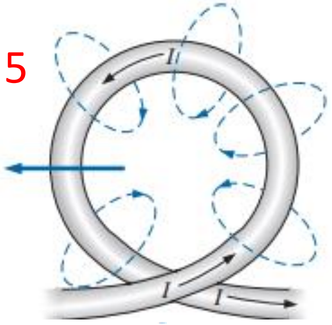

*Flux distribution for two adjacent, like poles.*

4



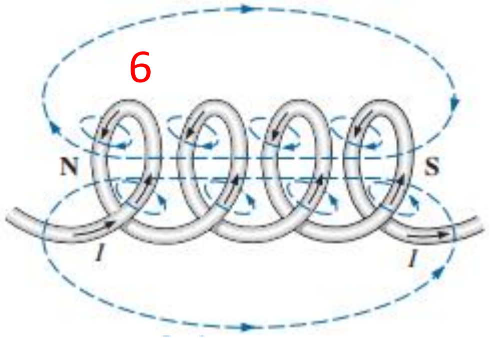

*Magnetic flux lines around a current-carrying conductor.*

5



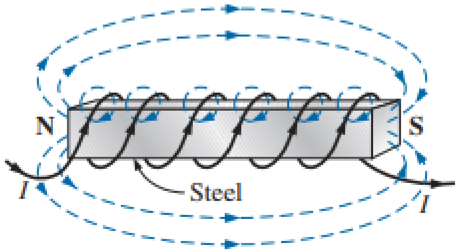

*Flux distribution of a single-turn coil.*

6



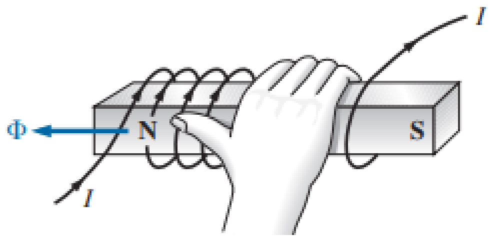

*Flux distribution of a current- carrying coil.*

7

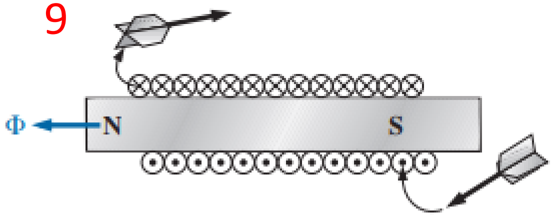



*Electromagnet.*

8



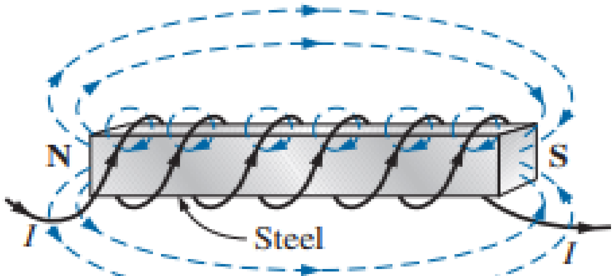

9




Flux distribution is quite similar to that of permanent magnet

# Magnetic Flux Density

- *Magnetic flux* ($\phi$) is measured in $webers$ $(Wb)$ as derived from the surname of Wilhelm Eduard Weber.
- The number of flux lines per unit area is called *flux density* $(B)$. Measured in *tesla* $(T)$.

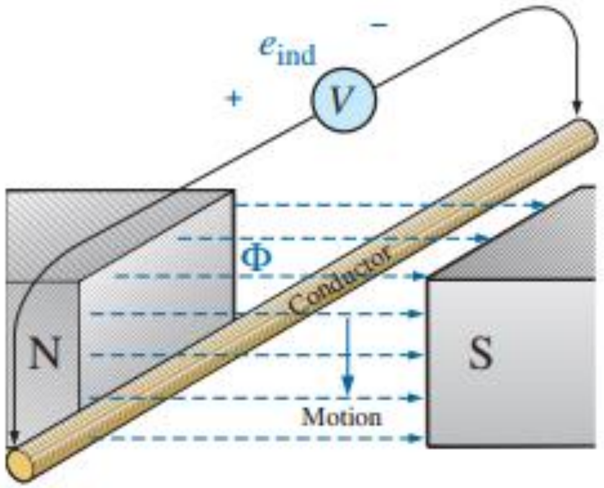


*Electromagnet.*

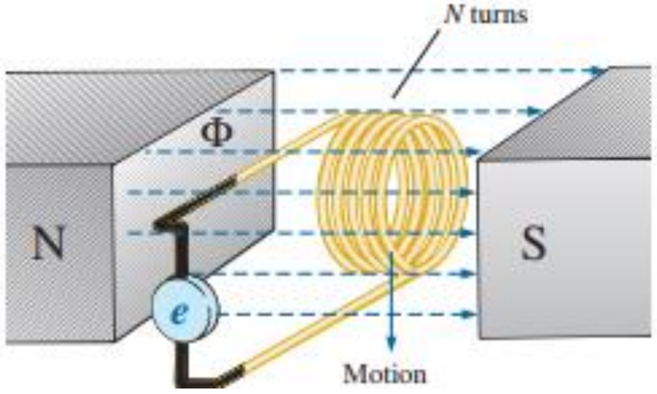
- In equation form,
- $B = \dfrac{\phi}{A} \quad (Wb/m^2 \equiv T \equiv 10^4\ Gauss)$
- $B$ of an electromagnet is directly proportional to the number of turns of, and current through, coil. Increasing either one (or both) results in increasing magnetic field.
- Another factor that affects the magnetic field strength is the type of core used. Ferromagnetic materials such as iron, cobalt, nickel, steel, alloys provide higher magnetic flux, high permeability ($\mu$).

# Faraday's Law of Electromagnetic Induction

- *Faraday's law* states that, a conductor exposed to a **changing magnetic flux** will develop an induced voltage across it.
- It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field: The only requirement is that the flux linking (passing through) the coil changes with time.
- In the form of equation,
- $e = N\frac{d\phi}{dt} \quad (volts, V)$
- This important phenomenon can now be applied to an inductor



Generating an induced voltage by moving a conductor through a magnetic field.



Demonstrating Faraday's law.

# Inductance

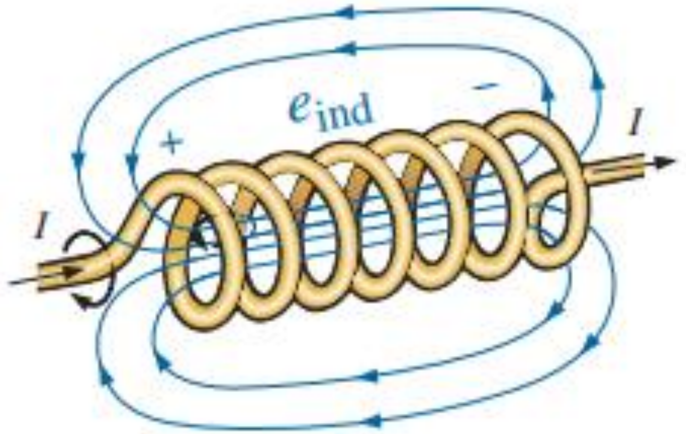
- We found that the magnetic flux linking the coil of $N$ turns with a current $I$ has the distribution shown in the figure.
- If the current $(I)$ through the coil increases in magnitude, the flux $(\phi)$ linking the coil also increases.
- So, $\phi \propto I$

$\Rightarrow$ $\phi = LI$ [$L$ is a proportionality constant ≡ Self-inductance]

- If the loop has $N$ number of turns,

$\Rightarrow$ $N\phi = LI$

$\Rightarrow$ $L = \frac{N\phi}{I}$ $(Henry,\ H,\ mH,\ \mu H)$; $\left[L = \frac{N^2\mu A}{l}\text{ for a solenoid}\right]$




*Demonstrating the effect of Lenz's law.*

- For a particular inductor, $\uparrow I, \uparrow \phi$ but $\frac{\phi}{I} = const.$ So, $L$ does not depend on $\phi$ or $I$. It depends on the physical dimension (length, # of turns, area, material) of the inductor.

# Lenz's Law of Electromagnetic Induction

- It is very important to note in the figure that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil.
- The quicker the change in current through the coil, the greater is the opposing induced voltage to squelch the attempt of the current to increase in magnitude
- Lenz's law says that an induced effect is always such as to oppose the cause that produced it.

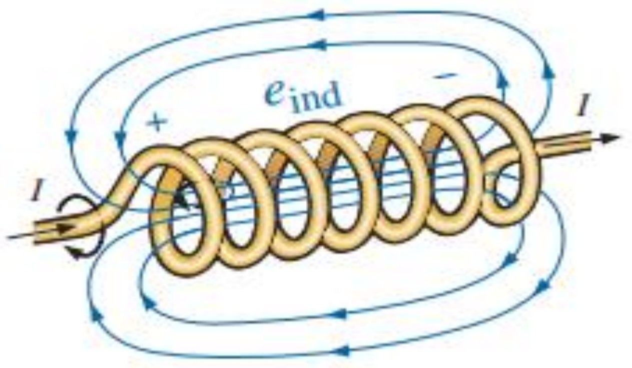
$$L = \frac{N^2 \mu A}{l}$$

$\Rightarrow \quad L = \dfrac{N\phi}{I}$

$\Rightarrow$ Differentiating with respect to $t$, $\dfrac{d\phi}{dt} = \dfrac{L}{N}\dfrac{dI}{dt}$

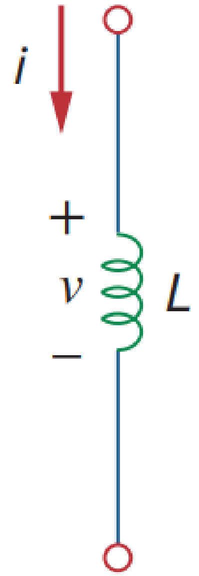
$\Rightarrow$ Substituting into Faraday's law, $e = N\dfrac{d\phi}{dt} = N\dfrac{L}{N}\dfrac{dI}{dt}$

$\Rightarrow \quad e = L\dfrac{dI}{dt}$




*Demonstrating the effect of Lenz's law.*

# I-V relation of an Inductor




- In network analysis, $e = L\frac{dI}{dt}$ is expressed as,

$$\boxed{v = L\frac{di}{dt}}$$

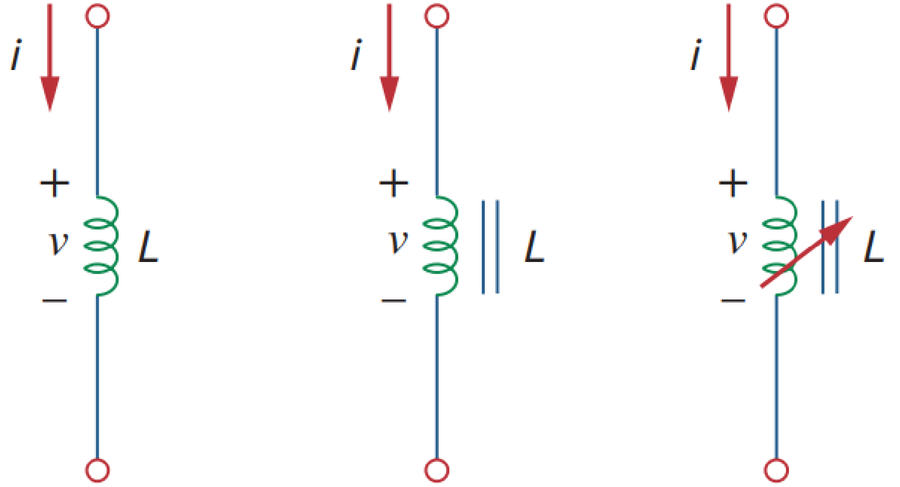
- This is the characteristic equation of an inductor.
- The current can be found as,

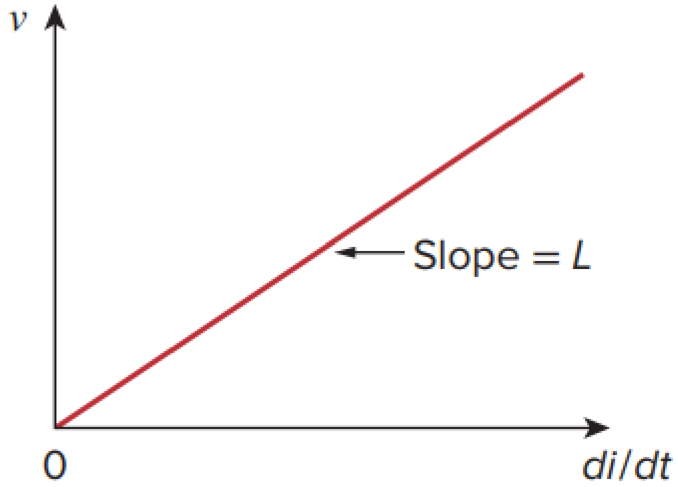
$$\Rightarrow \quad di = \frac{1}{L} v\, dt$$

- Integrating both sides,

$$i(t) = \frac{1}{L}\int_{\infty}^{t} v(t)\, dt$$

$$\Rightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\, dt + i(t_0)$$



# Energy stored & Power of an Inductor

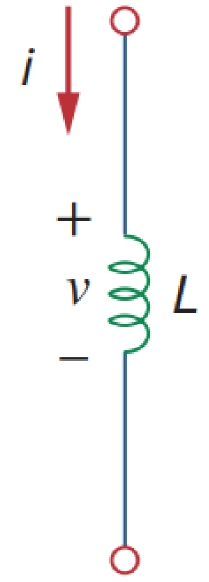
- The instantaneous power delivered to an inductor according to the passive sign convention is,

$$p = v(t)i(t) = Li(t)\frac{di(t)}{dt}$$

- The energy stored in the inductor is therefore

$$w(t) = \int_{-\infty}^{t} p(t)\,dt = \int_{-\infty}^{t} Li(t)\frac{di(t)}{dt}\,dt = \int_{v(-\infty)}^{v(t)} Li(t)\;di$$

$$\Rightarrow \quad w(t) = \frac{1}{2}Li^2\Big|_{i(-\infty)=I_0}^{i(t)=I}$$




- If the current through the inductor was initially (at $t = -\infty$) $I_0$, then,

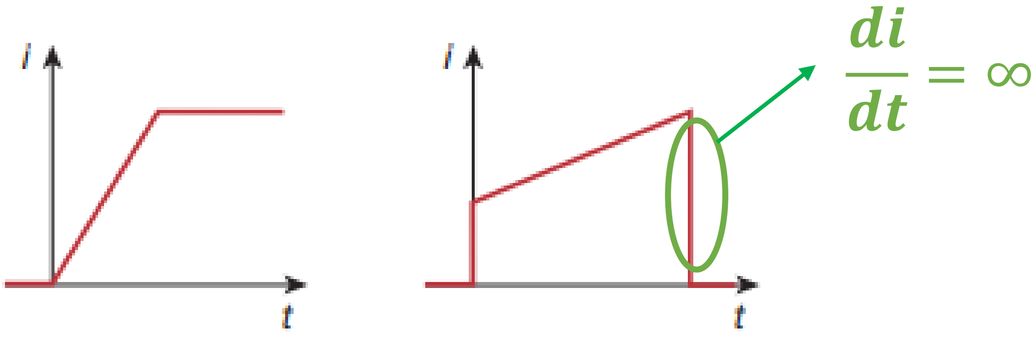
$$\Rightarrow \quad w(t) = \frac{1}{2}LI^2 - \frac{1}{2}LI_0^2$$

- In general, at any time $t$, if the current through an inductor is $I$, then the stored energy can be found as,

$$w(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}LI^2$$

# Inductor: important properties

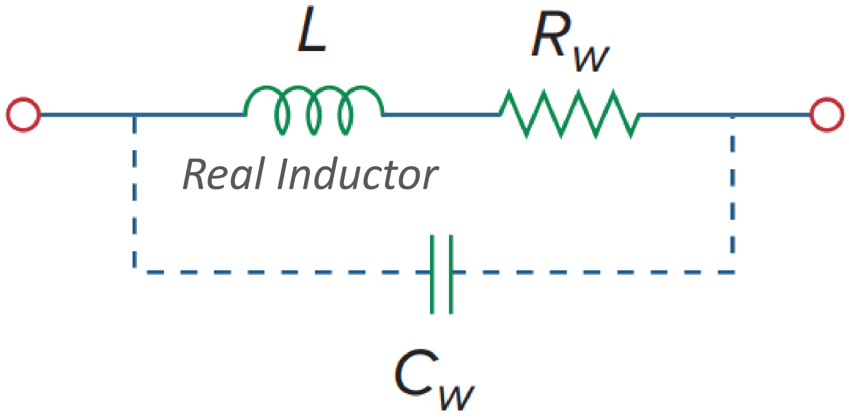
1. An inductor is a short circuit to dc. At dc, $v_L = L\frac{di_{L,dc}}{dt} = 0$ [Short circuit]
2. The current through an inductor cannot change abruptly.



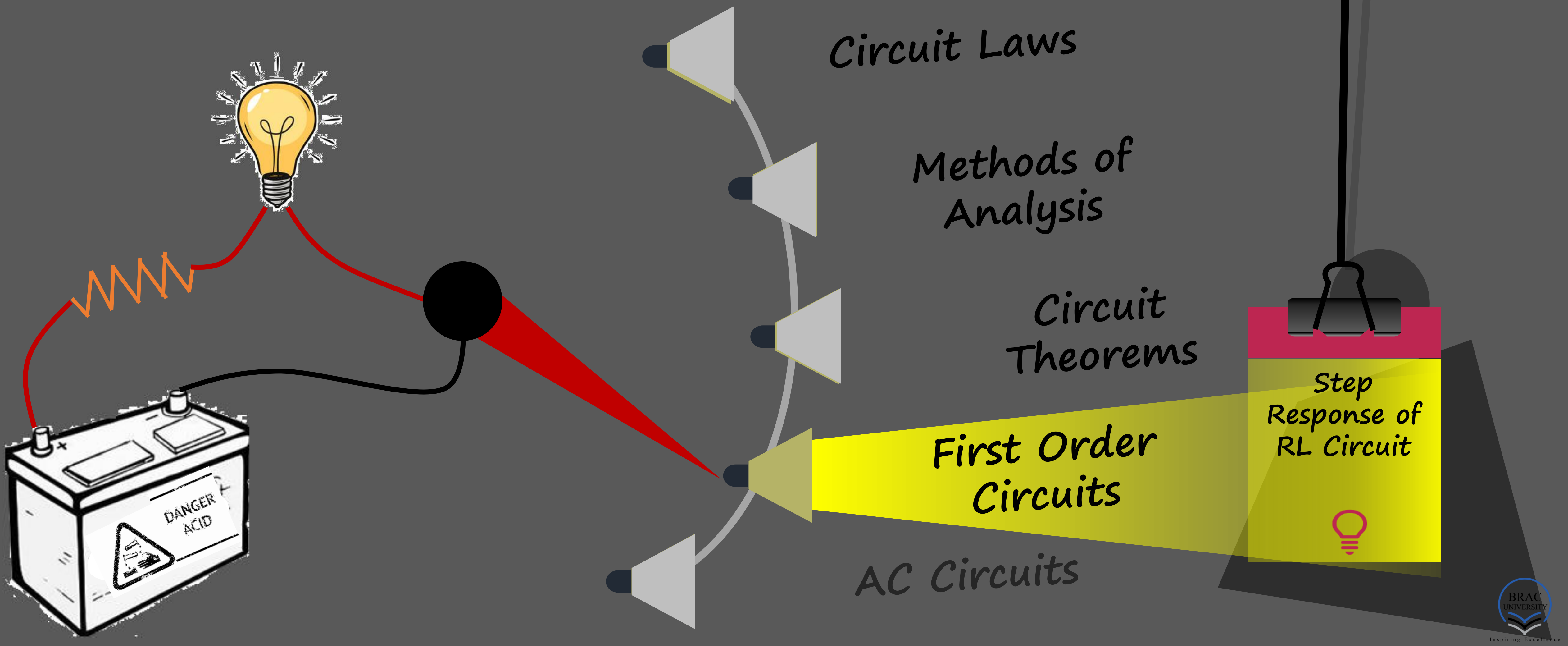

*Current through an inductor (a) allowed and (b) not allowed*

3. An ideal inductor does not dissipate energy.

4. A real, nonideal inductor has a series winding resistance and a capacitive coupling between the conducting coils.

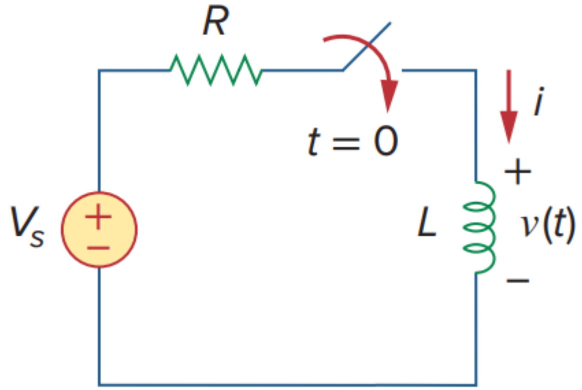


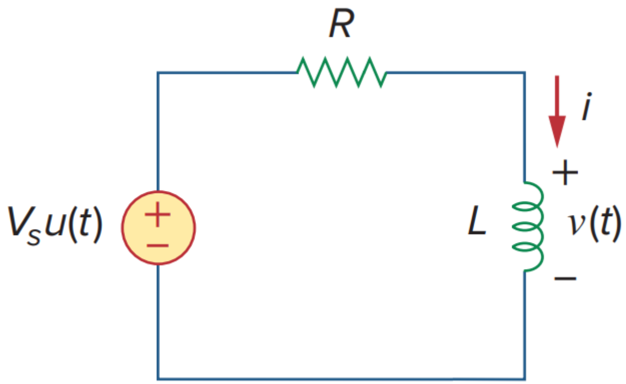
# Course Outline: broad themes



# Step Response of a RL circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the inductor current.




⇒ Since the current through an inductor cannot change instantaneously,

$$\Rightarrow \quad i(0^-) = i(0^+) = I_0$$

⇒ Using KVL (for $t > 0$),

$$\Rightarrow \quad L\frac{di}{dt} + iR = V_s$$

$$\Rightarrow \quad \frac{di}{dt} + \frac{R}{L}i = \frac{V_s}{L}$$

Multiplying both sides by $e^{\frac{R}{L}t}$,

$$\Rightarrow \quad e^{\frac{R}{L}t}\frac{di}{dt} + e^{\frac{R}{L}t}\frac{R}{L}i = \frac{V_s}{L}e^{\frac{R}{L}t}$$

$$\Rightarrow \quad \frac{d}{dt}\left[e^{\frac{R}{L}t}.i\right] = \frac{V_s}{L}e^{\frac{R}{L}t}$$

Integrating both sides with respect to $t$,

$$\Rightarrow \quad e^{\frac{R}{L}t}.i = \frac{V_s}{R}e^{\frac{R}{L}t} + C$$

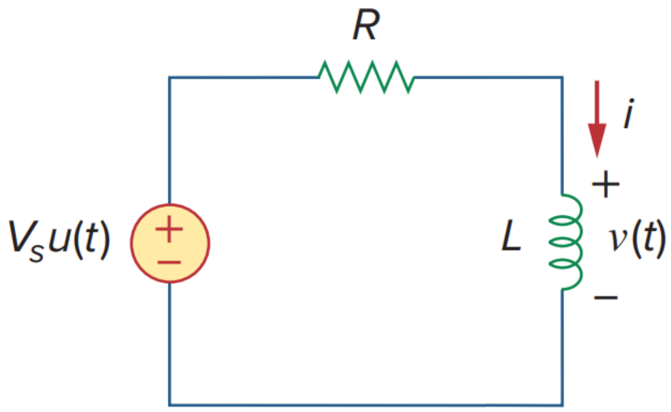
$$\Rightarrow \quad i = \frac{V_s}{R} + Ce^{-\frac{R}{L}t}$$

At $t = 0,\ i = I_0$. $\quad So,\ C = i - \frac{V_s}{R}$

Substituting, $i = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}$

# Time Constant (charging) for RL circuit

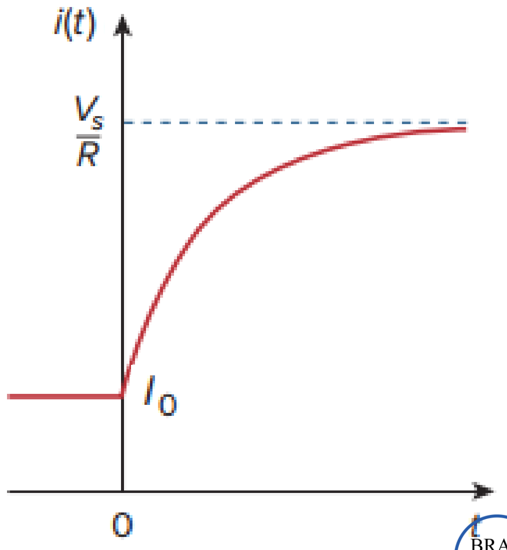
$$i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$




- This is known as the complete response (or total response) of the RL circuit to a sudden application of a dc voltage source. It is assumed that the inductor was initially charged to $I_0$.

$$\Rightarrow \quad i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$

$\Rightarrow$ where $\tau = {}^{L}/_{R}$ is the *time constant* (unit in sec).

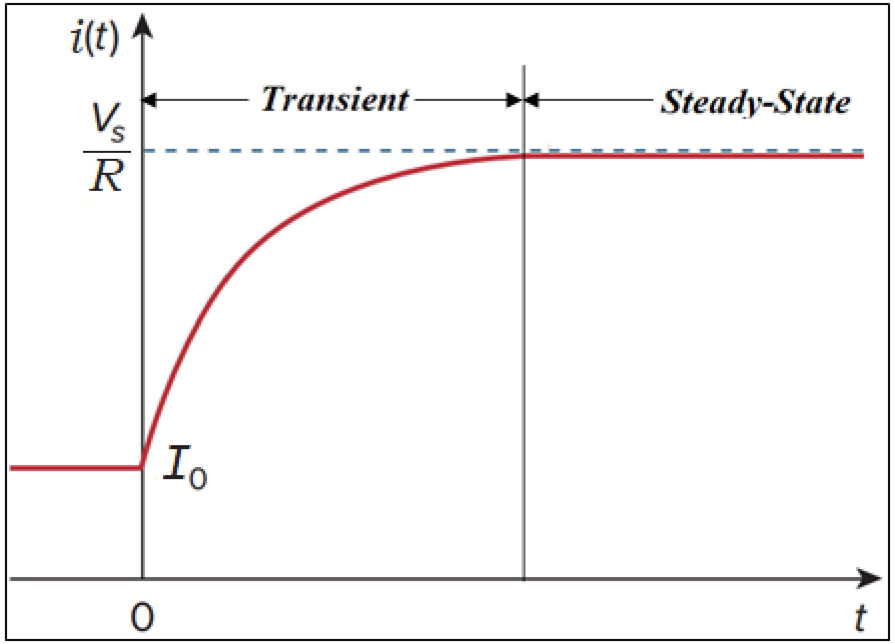



- Notice that, we write $\tau = {}^{L}/_{R}$ for the circuit consisting of only a resistor $R$ in series with the inductor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor $R$ is actually the Thevenin Resistance $R_{Th}$. Therefore,

$$\boxed{\tau = \frac{L}{R_{Th}}}$$

# Transient and Steady-State Response

$$i(t) = \begin{cases} I_0, & t < 0 \\ \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$




- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,

$$i(t) = i_{ss} + i_t, \quad \text{where,}$$
$$i_{ss} = \frac{V_s}{R} \quad \& \quad i_t = \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$$

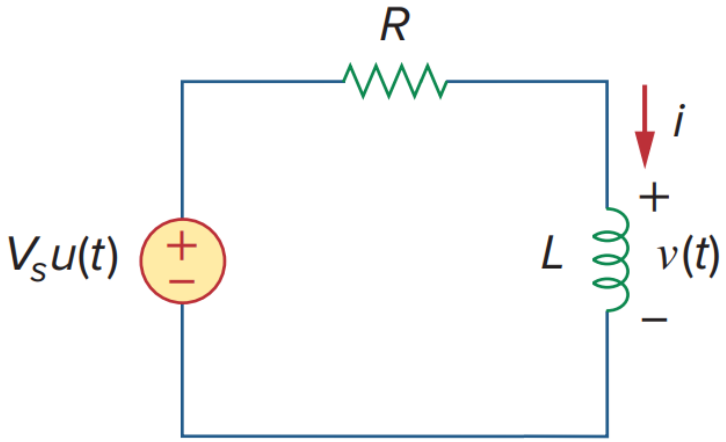
- The *transient response* $(i_t)$ is the circuit's temporary response that will die out with time.
- The *steady-state response* $(i_{ss})$ is the behaviour of the circuit a long time after an external excitation is applied.

- The complete response can be written as,

$$i(t) = I_{final} + [I_{initial} - I_{final}]e^{-\frac{t}{\tau}}$$

$$or, \quad i(t) = I(\infty) + [I(0) - I(\infty)]e^{-\frac{t}{\tau}}$$

# Definition of $\tau$ (charging)




$$i(t) = I_{final} + \left[I_{initial} - I_{final}\right]e^{-\frac{t}{\tau}}$$

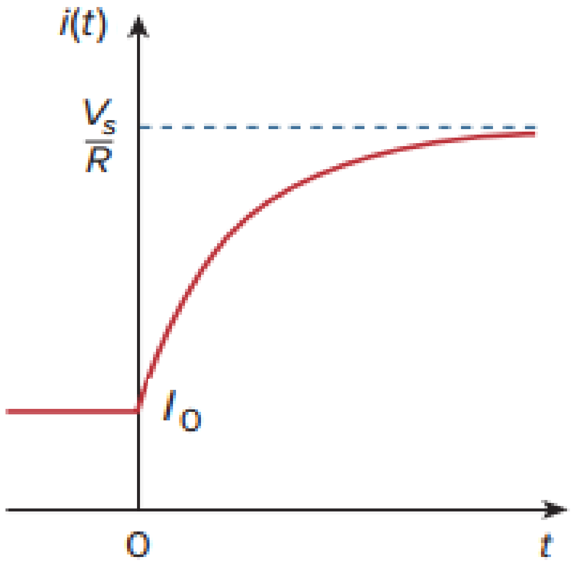
- At $t = \tau$,

$$i(t) = I_{final} + \left[I_{initial} - I_{final}\right]e^{-1}$$

$$\Rightarrow \quad i(t) = I_{final}\left(1 - {}^{1}\!/_{e}\right) + I_{initial}\left({}^{1}\!/_{e}\right)$$

$$\Rightarrow \quad i(t) = I_{final}\left(1 - {}^{1}\!/_{e}\right) - I_{initial}\left(1 - {}^{1}\!/_{e}\right) + I_{initial}$$

$$\Rightarrow \quad i(t) = I_{initial} + \left[I_{final} - I_{initial}\right]\left(1 - {}^{1}\!/_{e}\right)$$



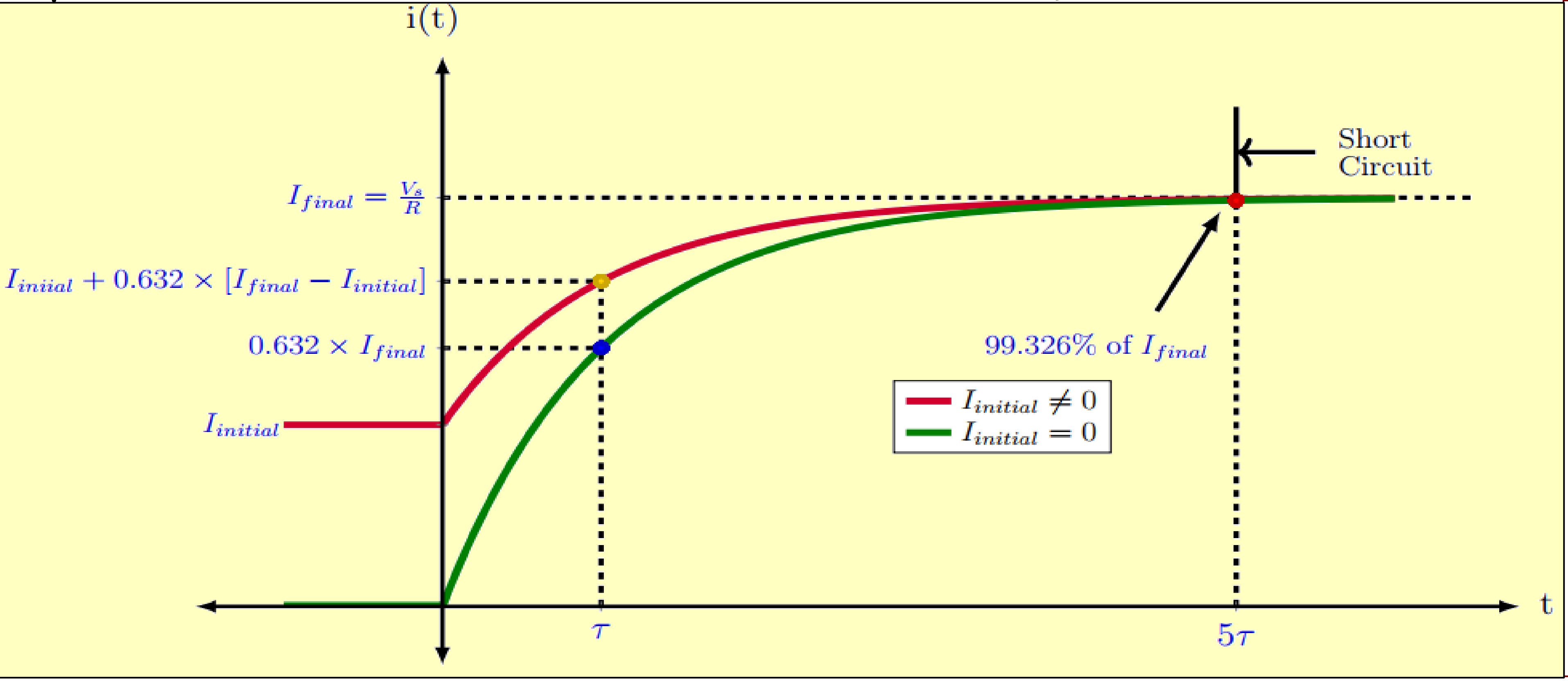

- We can define the time constant in this way,
- The *charging time constant* is the time required for the response to reach to a factor of $(1 - 1 / e)$ or 63.2% towards $I_{final}$ from an initial response $I_{initial}$.

# Time Constant ($\tau$): graphically

$$\text{At } t = \tau, \quad i(t) = I_{initial} + [I_{final} - I_{initial}](1 - {}^{1}/_{e})$$

$$\Rightarrow \quad i(t) = 63.2\% \times I_{final} \quad \text{when } I_{initial} = 0$$

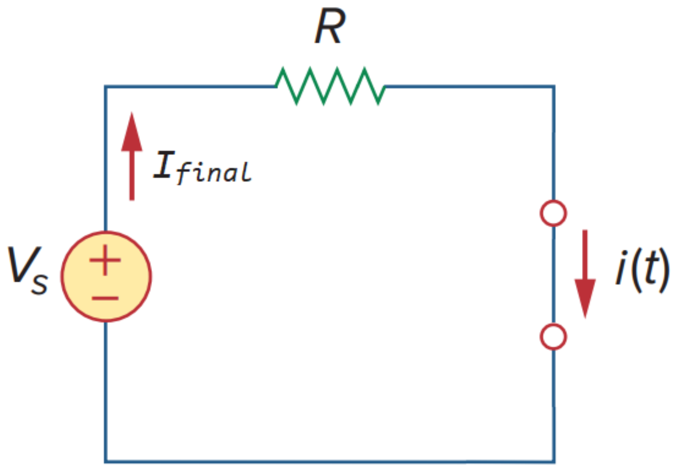
$$\Rightarrow \quad i(t) = I_{initial} + 63.2\% \times [I_{final} - I_{initial}] \quad \text{when } I_{initial} \neq 0$$



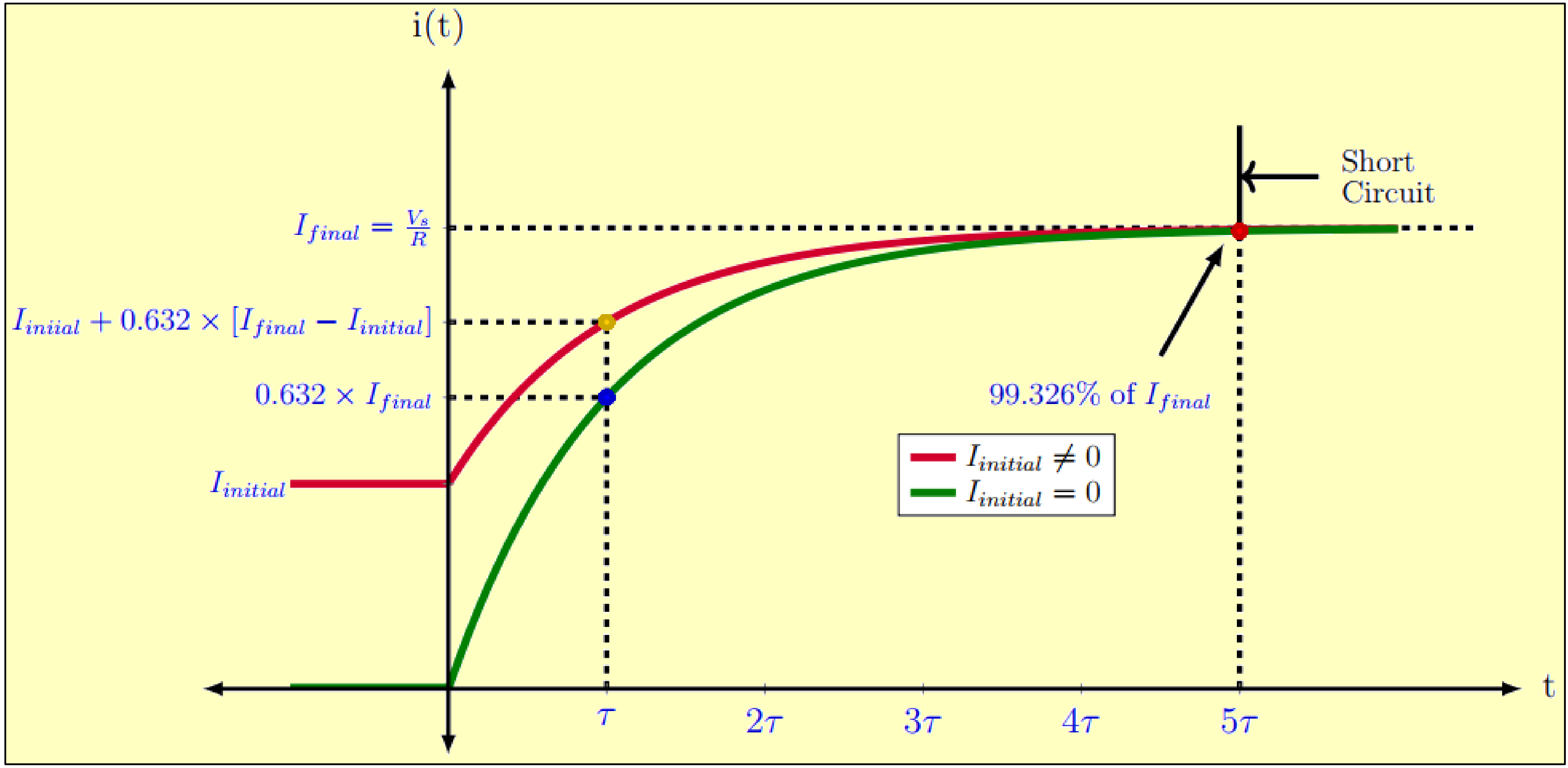

- As $\tau$ only depends on $R_{Th}$ and $L$ ($\tau = {}^{L}/_{R_{Th}}$), for a given circuit, that is, for a fixed $R_{Th}$ and $L$, the time needed for the inductor current to rise to the final value ($I_{final}$) is the same whether or not the capacitor is initially charged ($I_{initial}$ zero or nonzero).

# Significance of τ (charging): 5τ Time

- As can be seen from the following plot, the inductor voltage reaches the final voltage approximately after 5 times the Time Constant (τ). The inductor is fully charged and acts as short circuit from 5τ time onward. So, when designing circuits, the charging time of an inductor under the application of a certain dc supply can be set by choosing $R_{Th}$.



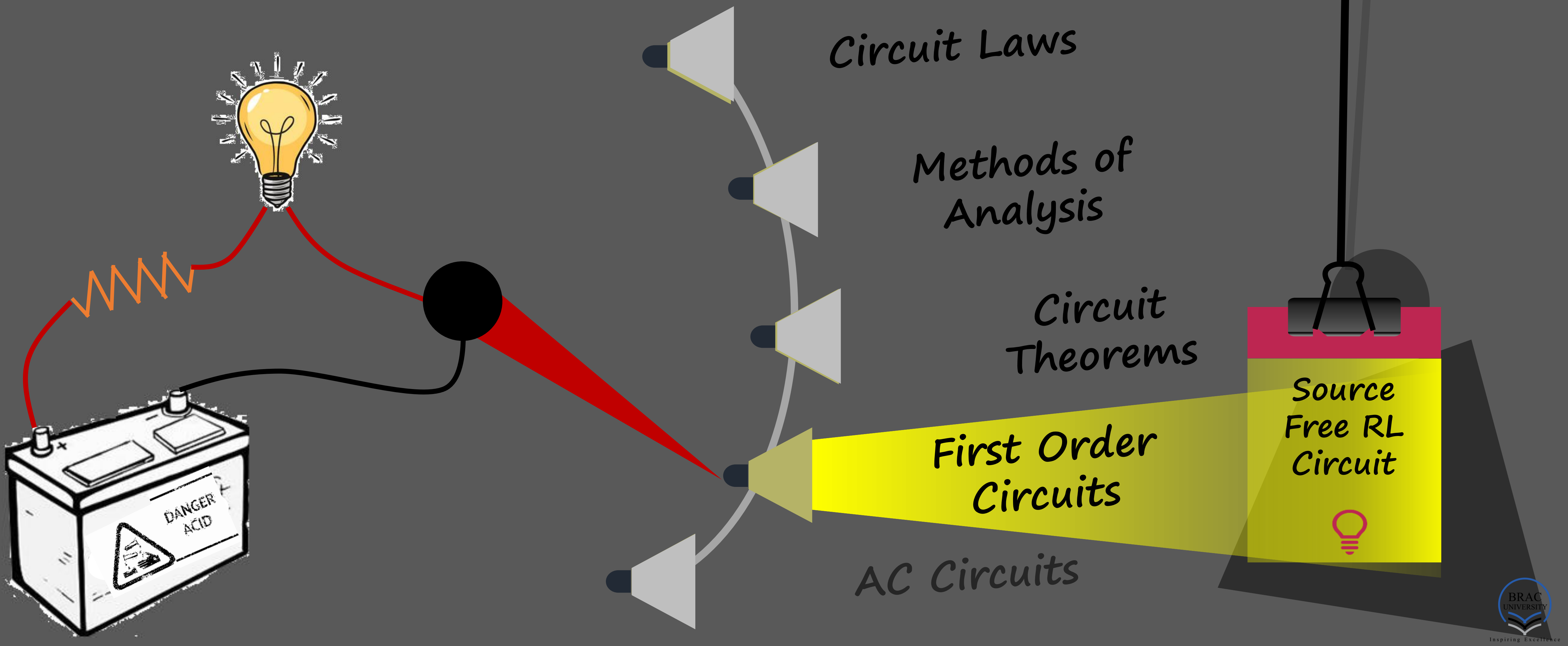

at or after $t = 5\tau$



# Summary: Capacitor vs. Inductor

| | Capacitor | Inductor |
|---|---|---|
| Capacitance/ Inductance | $C = \frac{q}{v}$ | $L = \frac{N\varphi}{I}$ |
| $I - V$ Characteristics | $I = C\frac{dV}{dt}$ | $V = L\frac{dI}{dt}$ |
| Behavior | *open circuit at dc* | *short circuit at dc* |
| Energy | $W = \frac{1}{2}CV^2$ | $W = \frac{1}{2}LI^2$ |
| Step response | $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}$ | $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$ |
| Time constant | $\tau = R_{Th}C$ | $\tau = \frac{L}{R_{Th}}$ |

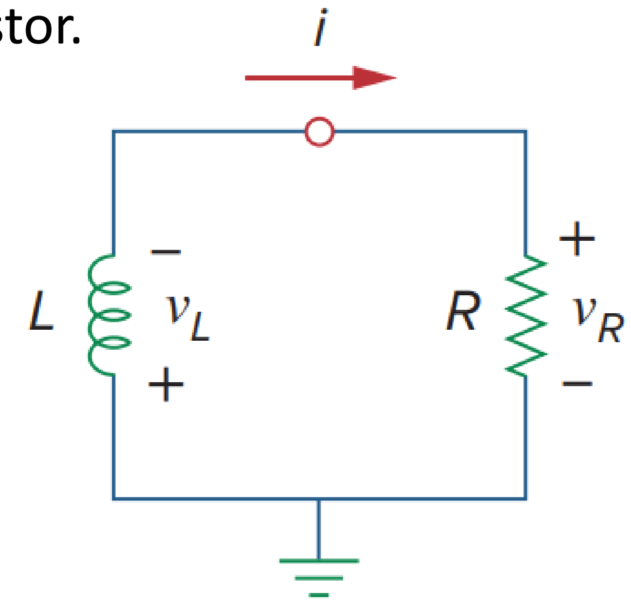
# Course Outline: broad themes



# Source-Free RL circuit

- A *source-free RL circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

$\Rightarrow$ Assume that an inductor is charged to $I_0$ and then it is connected to a resistor as shown. The inductor starts to discharge the stored energy to the resistor.




$\Rightarrow$ Initially stored charge, $w(0) = \frac{1}{2}LI_0^2$

$\Rightarrow$ From the figure using KVL, $v_L + v_R = 0$

$$\Rightarrow \mathrm{L}\frac{di}{dt} + Ri = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides,

$$\Rightarrow \ ln i = -\frac{R}{L}t + lnA$$

$$\Rightarrow \ ln\frac{i}{A} = -\frac{R}{L}t$$

$$\Rightarrow \ i = Ae^{-\frac{R}{L}t}$$

At t = 0, $i(0) = \mathrm{A} = I_0$ So, $\boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$

# Time Constant (discharging) for RL circuit

$$i(t) = I_0 e^{-\frac{R}{L}t}$$

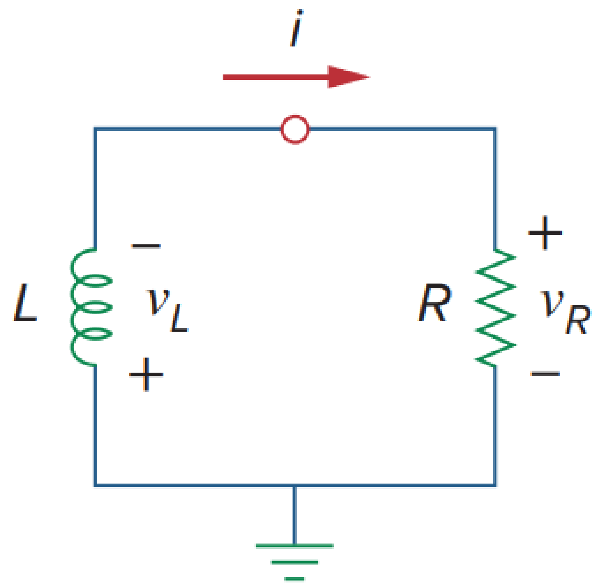
- This shows that the voltage response of the $RL$ circuit is an exponential decay of the initial voltage. It is called the *natural response* of the circuit.

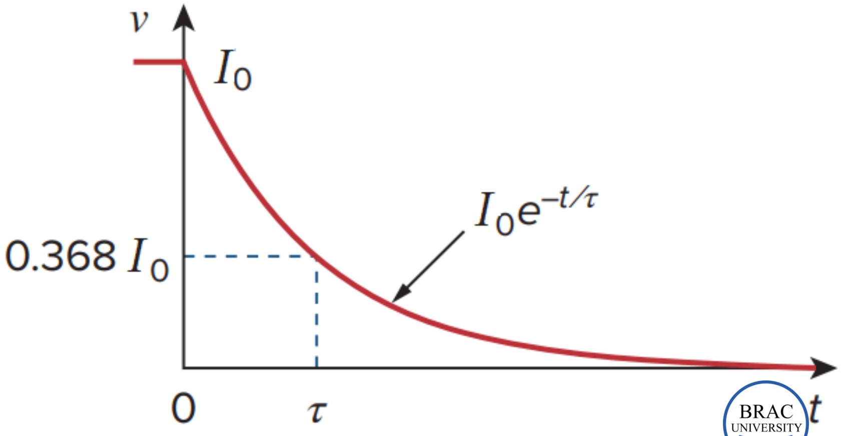
$$\Rightarrow \quad i(t) = I_0 e^{-\frac{t}{\tau}}$$

- where $\tau = {}^{L}/_{R}$ is the time constant (unit in sec).

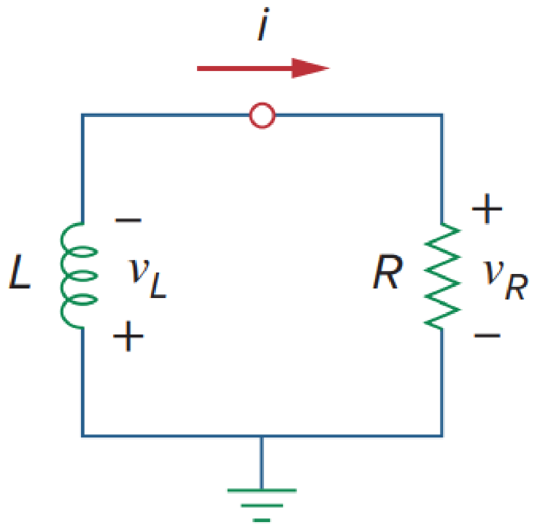
- Notice that, we write $\tau = {}^{L}/_{R}$ for the circuit consisting of only a resistor $R$ in series with the capacitor. As we know, all the linear two terminal circuits can be reduced to this form by Thevenin's Theorem, so the resistor $R$ is actually the Thevenin Resistance $R_{Th}$. Therefore,

$$\boxed{\tau = {}^{L}/_{R_{Th}}}$$



# Definition of $\tau$ (discharging)




$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

- At $t = \tau$,

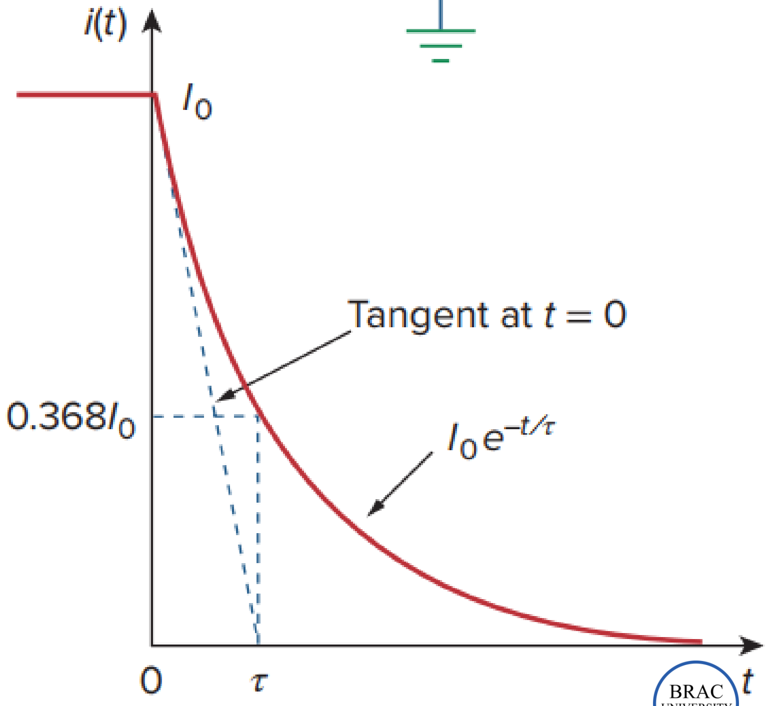
$$i(t) = I_0 e^{-1}$$

$$\Rightarrow \quad i(t) = 0.368 \times I_0$$

- We can define the discharging time constant in this way,

- The *discharging time constant* is the time required for the response to fall to a factor of $^1/_e$ or 36.8% from an initial response $I_{initial}$ or $I_0$.

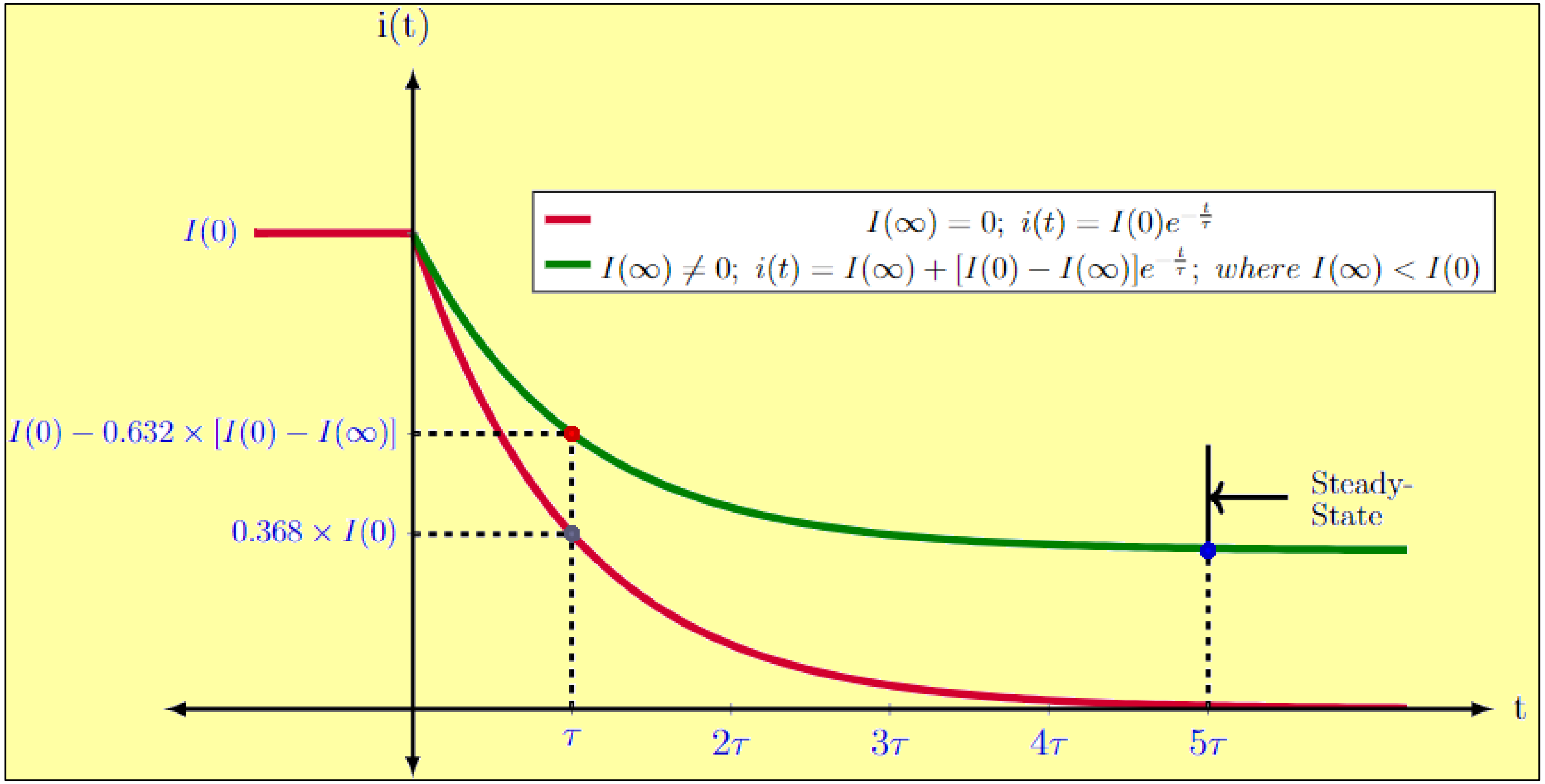
- Recall that the *charging time constant* is the time required for the response to reach to a factor of $(1 - 1/e)$ or 63.2% towards $I_{final}$ from an initial response $I_{initial}$.



# Significance of $\tau$ (discharging): $5\tau$ Time

- As can be seen from the following plot, the inductor current decreases to the final value approximately after $5$ times the Time Constant ($\tau$). In case where $I(\infty) = 0$, the inductor is fully discharged after $5\tau$ time. So, when designing circuits, the discharging time of a capacitor can be set by choosing $R_{Th}$.

- In the case that a capacitor is subjected to a final voltage lower than its initial voltage, the discharging $\tau$ is the time required for the response to decay to 63.2% from $I(0)$ towards $I(\infty)$. See Example 6.



# Procedure

$$i(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

<u>Determine the initial current of the inductor $I_{initial}$ or $I(0)$</u>

Consider only the active‡ portion of the circuit before switching. For example, if switching occurs at $t = 0$, consider the circuit for $t < 0$.

If the circuit includes any dc source (current or voltage), short the capacitor and determine the voltage at the open terminal. This is the $I(0)$. $I(0) = 0$ if there is no independent source in the circuit.

<u>Determine the final voltage of the inductor $I_{final}$ or $I(\infty)$</u>

Now consider the active‡ portion of the circuit after switching. For example, for $t > 0$.

Repeat the step. This time, the voltage across the capacitor is $I(\infty)$.

<u>Determine the time constant ($\tau$)</u>

Again, only consider the active‡ portion after switching. For example, for $t > 0$.

Determine the Thevenin resistance ($R_{Th}$) as seen from the capacitor terminals

$\tau = L/R_{Th}$

<u>Determine $i(t)$</u>

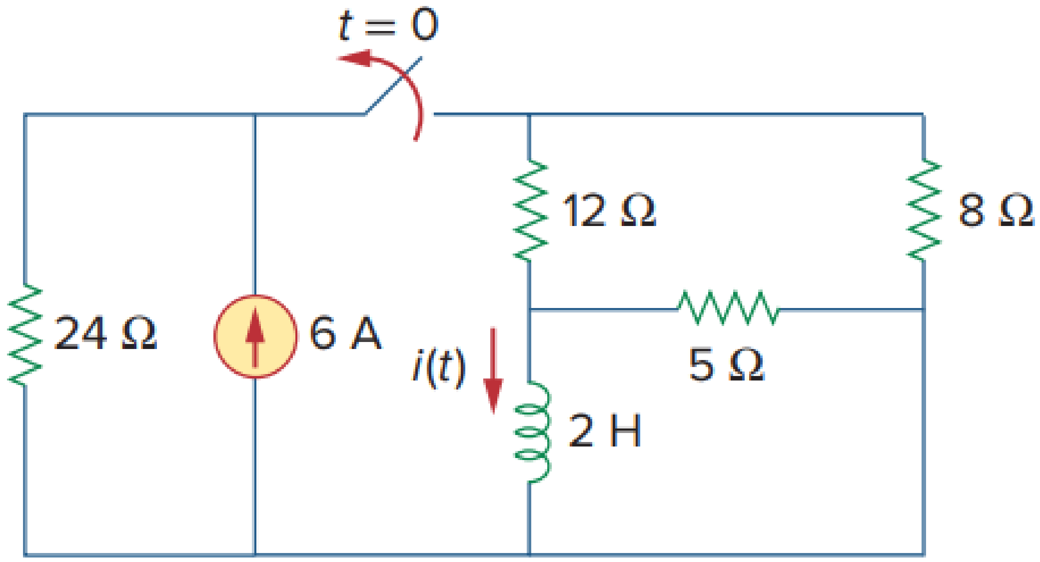
Plug in $I(0)$, $I(\infty)$, and $\tau$ into the equation for $I(t)$

Determine any other voltages or currents in the circuit using $i(t)$ and the circuit laws.

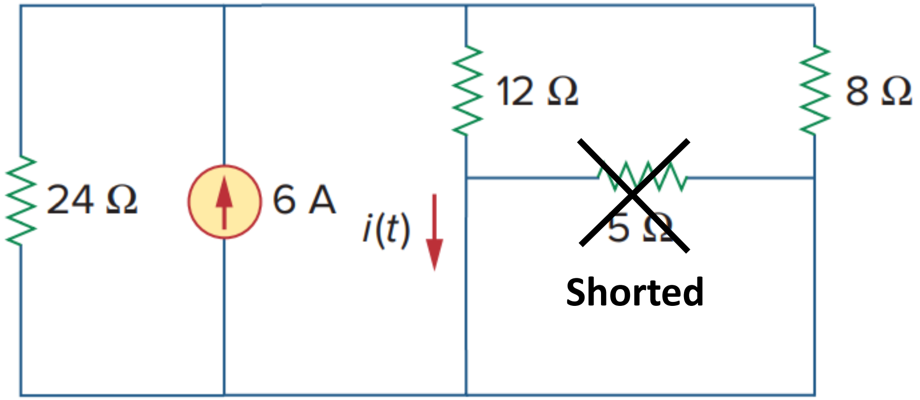
‡ active portion of the circuit excludes everything that has no influence on the inductor

# Example 4

- Find $i(t)$ for $t > 0$. Find the initial energy stored in the inductor.




For $t < 0$




For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

The $5\,\Omega$ resistance has zero potential difference, hence, shorted out. The current through the $12\,\Omega$ can be found using current division rule.

$$i(t) = \frac{12^{-1}}{12^{-1} + 24^{-1} + 8^{-1}} \times 6 = 2\,A, \qquad t < 0$$

Since the current though the inductor cannot change instantaneously,

$$i(0) = i(0^-) = 2\,A$$

# Example 4 (contd …2)

For $t > 0$, the switch is open. transforms into the one shown above. As there is no independent source in the circuit, $I(\infty) = 0$. The Thevenin resistance as seen from the inductor terminal,

$$R_{Th} = (12 + 8)\,||\,5 = 4\,\Omega$$

The time constant is,

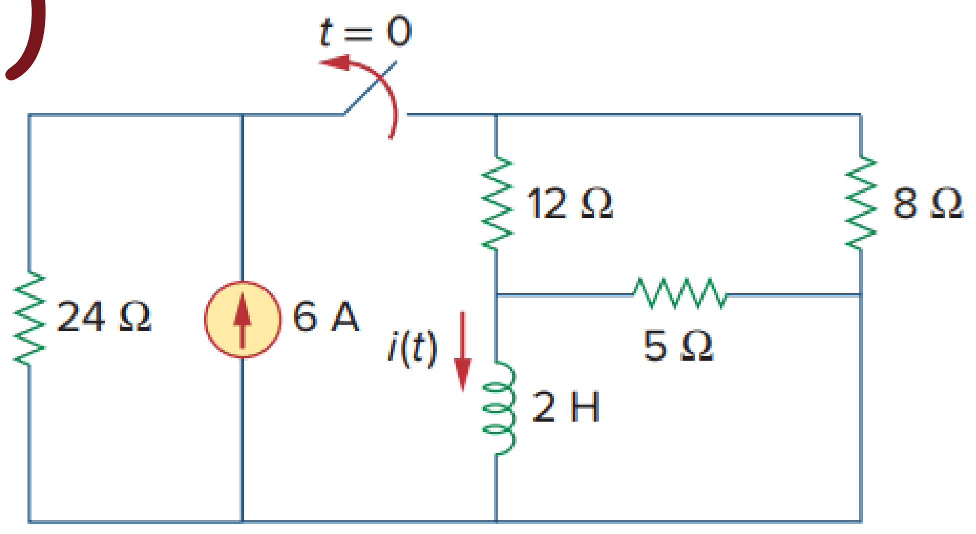
$$\tau = \frac{L}{R_{Th}} = \frac{2}{4} = 0.5\ s$$

So, the current through the inductor for t > 0 is,

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$
$$= 2e^{-2t}\ (A)$$

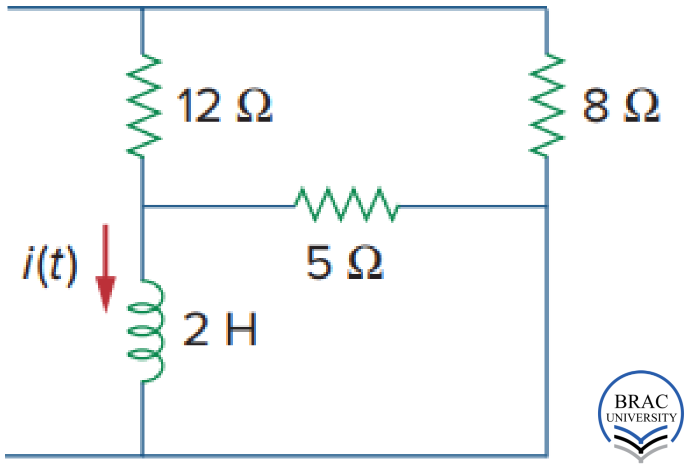
The initial energy stored in the inductor is, $w_L(0) = \frac{1}{2}LI(0)^2$

$$\Rightarrow\quad \frac{1}{2} \times 2 \times 2^2 = 4\ J$$



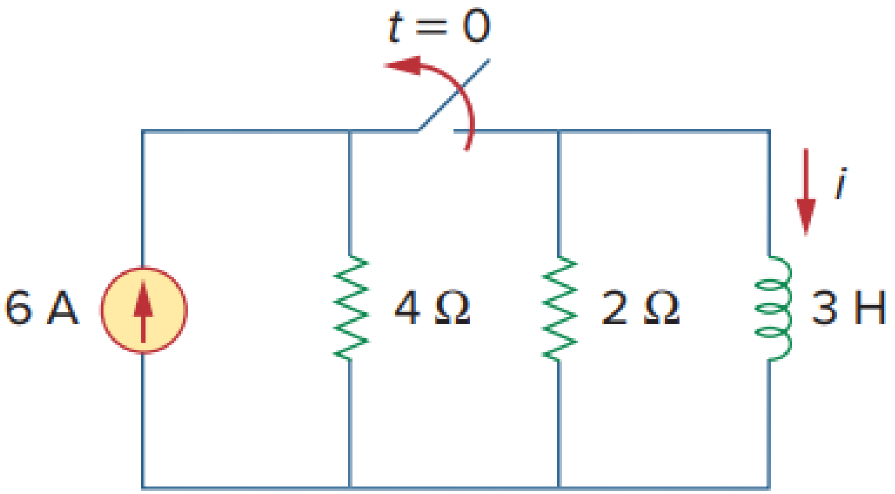

*Original circuit*

For $t > 0$

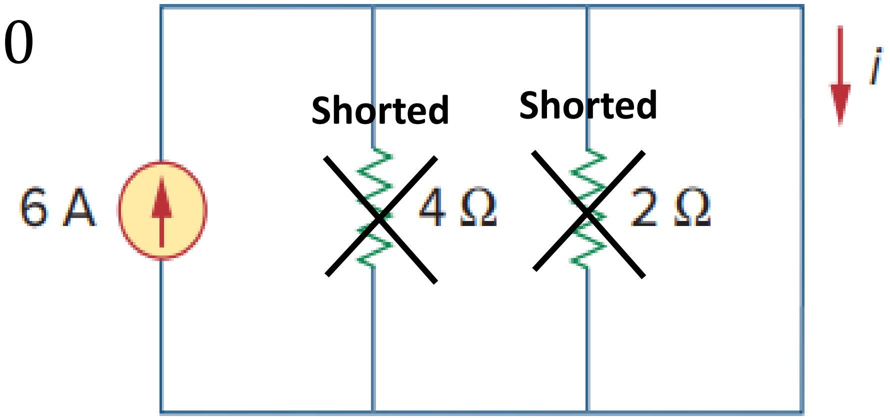


# Example 5

- Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$.




For $t < 0$




For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

As the potentials across each of the $4\,\Omega$ and $2\,\Omega$ resistances are equal due to the short circuit at the inductor, no current will flow through them.

The current $6\ A$ will flow only through the short circuit at the inductor.

$$i(t) = 6\ A, \qquad \mathrm{t} < 0$$

Since the current through the inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6\ A$$

# Example 5: $t > 0$

For $t > 0$, the switch is open. The circuit transforms into the one shown above. As there is no dc source, inductor is not short circuited and $I(\infty) = 0$.

The time constant is,

$$\tau = \frac{L}{R_{Th}} = \frac{3}{2} s$$

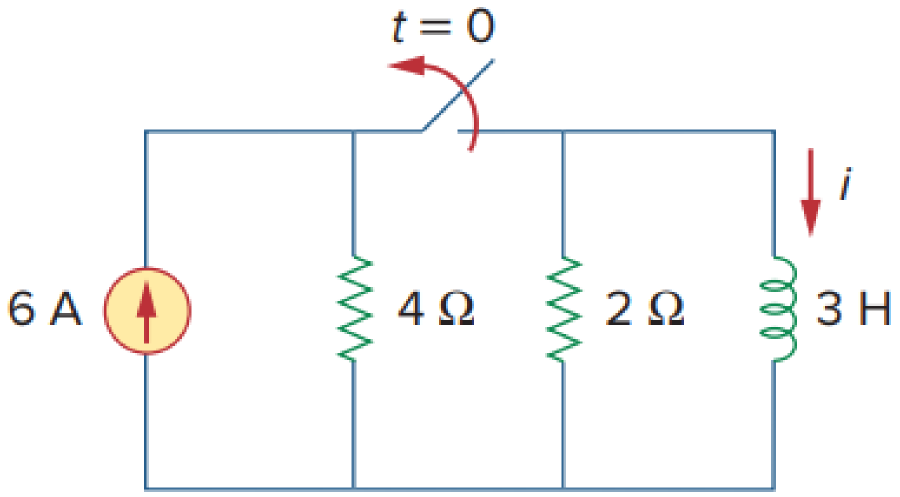
The current through the inductor for t > 0 is thus,

$$i(t) = i(0)e^{-\frac{t}{\tau}} = 6e^{-\frac{2t}{3}}\,(A), \qquad t > 0$$

The voltage across the inductor for $t > 0$ is,

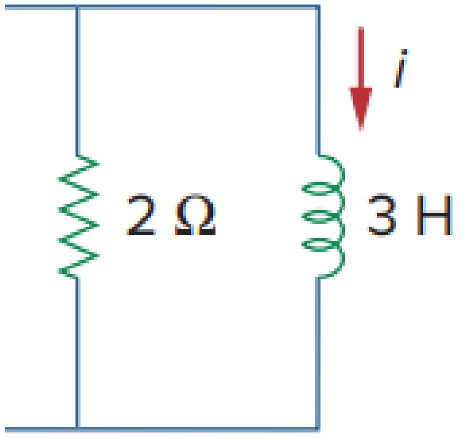
$$v_L = L\frac{di}{dt} = 3\frac{d}{dt}\left(6e^{-\frac{2t}{3}}\right) = 3 \times 6 \times \left(-\frac{2}{3}\right)e^{-\frac{2t}{3}}$$

$$= -12e^{-\frac{2t}{3}}, \qquad t > 0$$



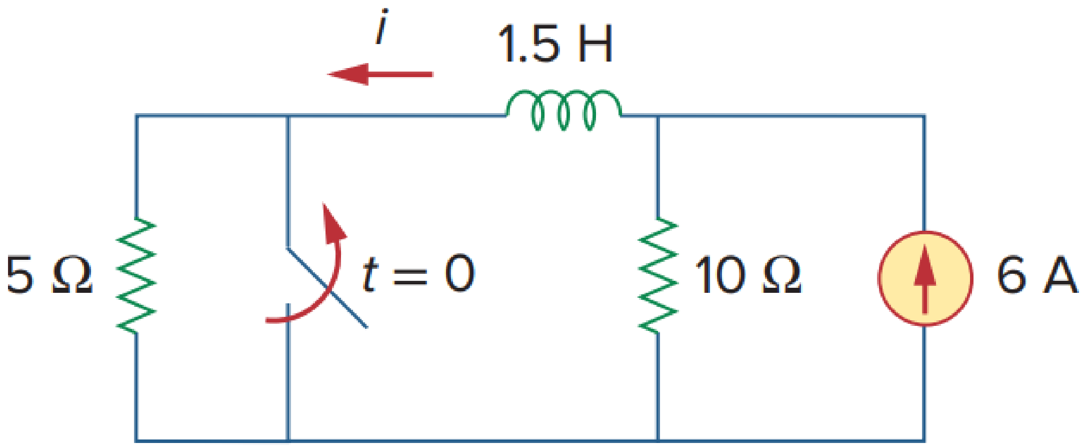

*Original circuit*

For $t > 0$

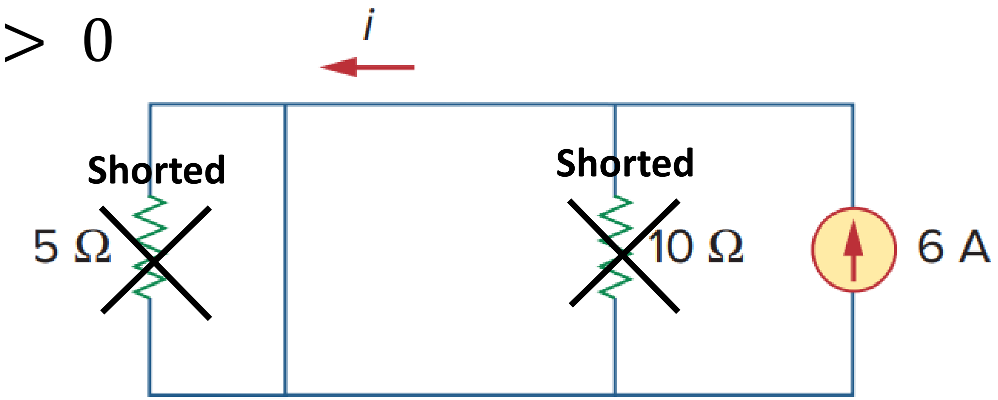


# Example 6

- The switch has been closed for a long time. It opens at $t = 0$. Find $i(t)$ for $t > 0$.




For $t > 0$




For $t < 0$, the switch is closed. With the inductor shorted at dc, the circuit transforms into the one shown above.

As the potentials across $5\ \Omega$ resistor are equal due to the short circuit at the switch, no current will flow through it.

The $10\ \Omega$ resistor is also shorted out due the short circuits at the inductor and switch. Consequently the $6\ A$ current will flow through the short circuit. So,

$$i(t) = 6\ A, \qquad t < 0$$

Since the current through the inductor cannot change instantaneously,

$$i(0) = i(0^{-}) = 6\ A$$

# Example 6 : $t > 0$

For $t > 0$, the switch is open. With the inductor shorted at dc, the circuit transforms into the one shown above.

The current, $i(t)$ through the $5\,\Omega$ resistor can be found using voltage division as,

$$i(t) = \frac{10}{10+5} \times 6 = 4\,A, \qquad t > 0$$

This is the inductor's steady-state current until the circuit is modified. So,

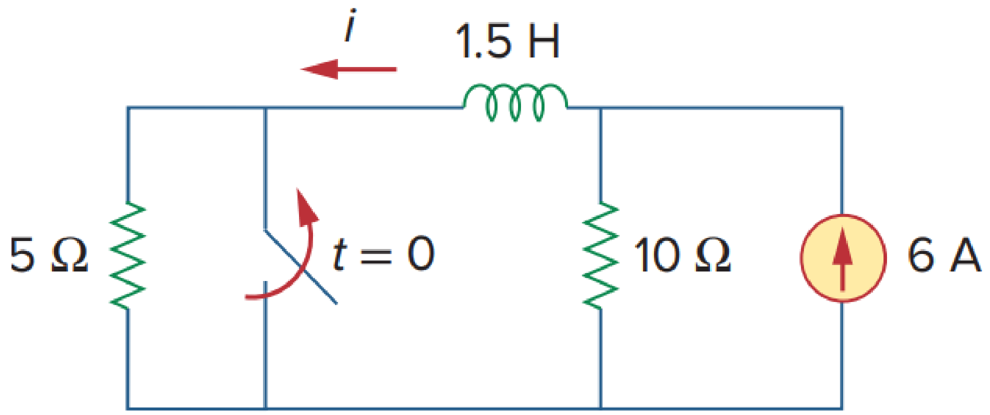
$$i(\infty) = 4\,A$$

The time constant is, $\tau = \frac{L}{R_{Th}} = \frac{1.5}{5+10} = 0.1\,s$

So,

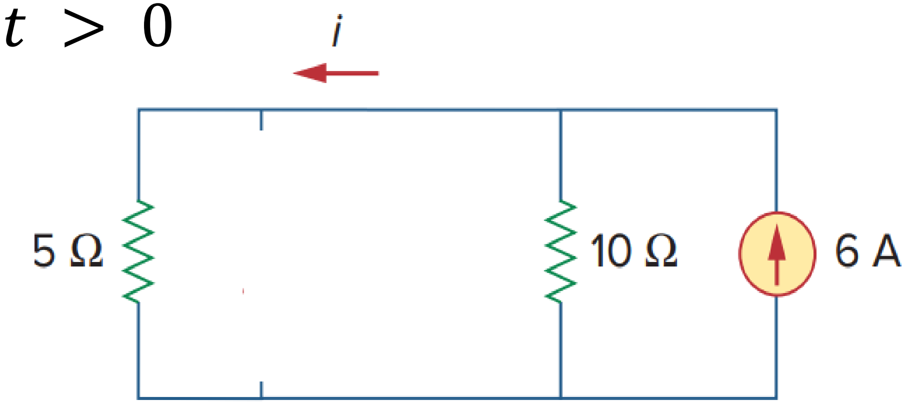
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 4 + [6-4]e^{-t/0.1} = 4 + 2e^{-10t}\ (A), \quad t > 0$$

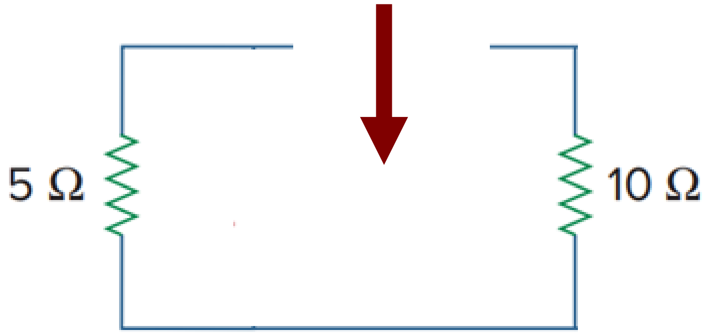



*Original circuit*

For $t > 0$




$$R_{Th} = 5 + 10 = 15\,\Omega$$



# Problem 16

- Find $i_0, v_0$, and $i$ for all time, assuming that the switch was open for a long time.

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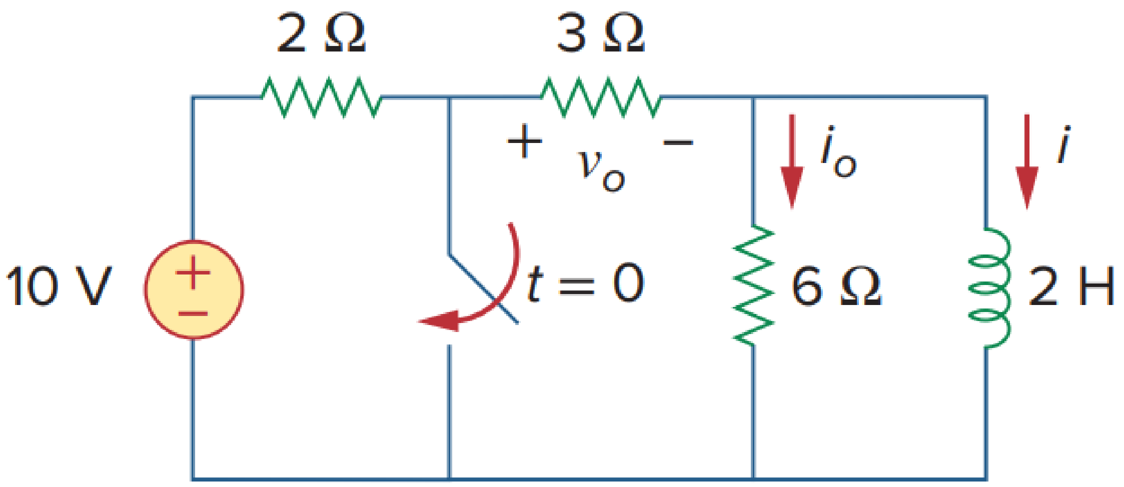
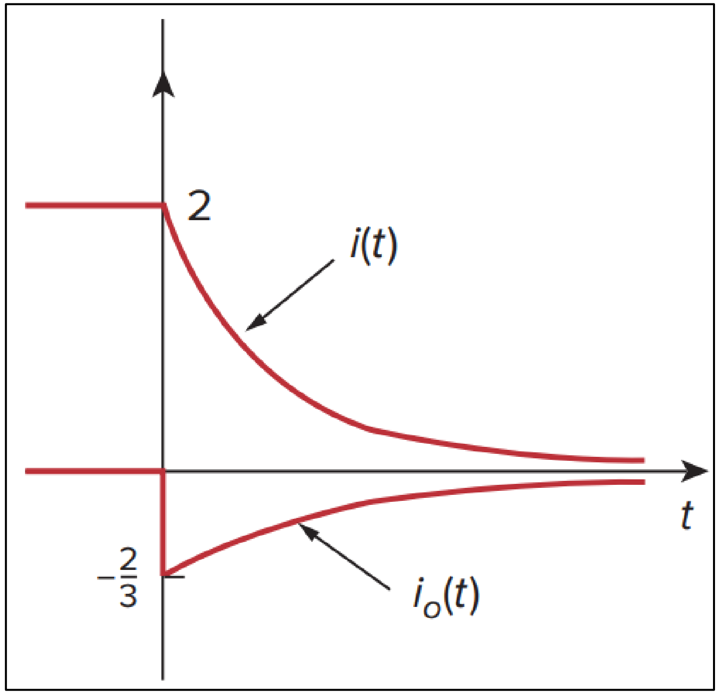


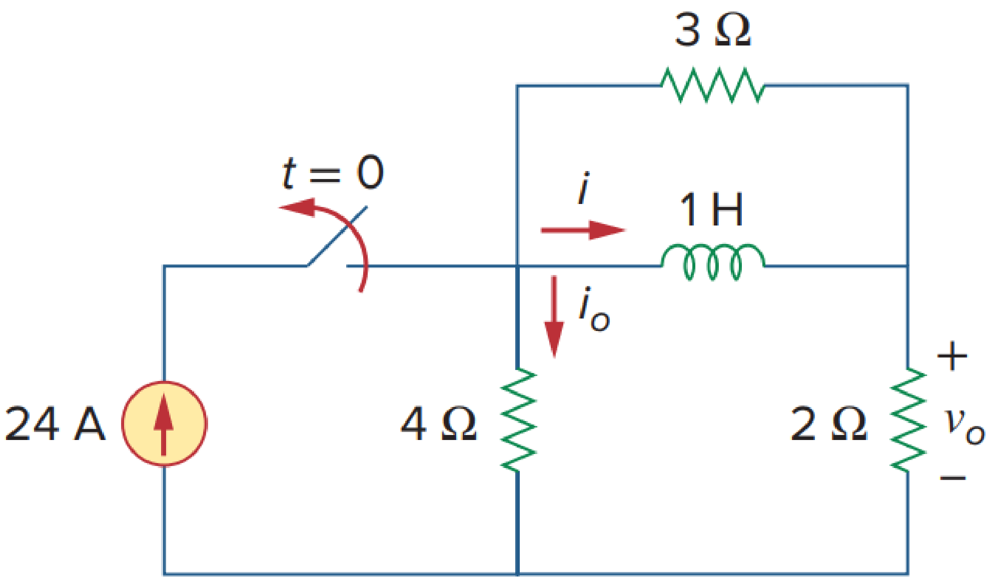

Ans:

$$i_o(t) = \begin{cases} 0\ \text{A}, & t < 0 \\ -\dfrac{2}{3}e^{-t}\ \text{A}, & t > 0 \end{cases}, \quad v_o(t) = \begin{cases} 6\ \text{V}, & t < 0 \\ 4e^{-t}\ \text{V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\ \text{A}, & t < 0 \\ 2e^{-t}\ \text{A}, & t \geq 0 \end{cases}$$

# Problem 17

- Determine $i, i_0$, and $v_0$ for all $t$ in the circuit shown below. Assume that the switch was closed for a long time.



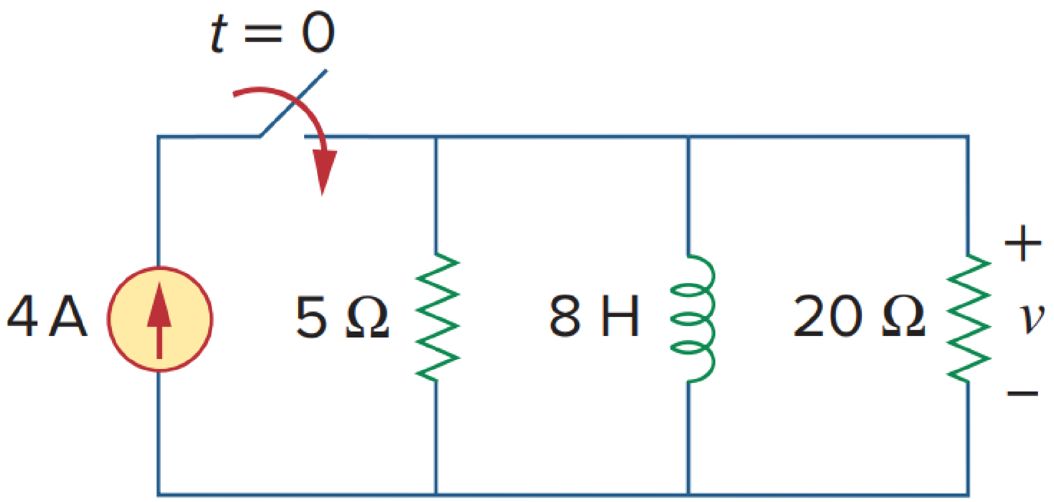

Ans:

$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \geq 0 \end{cases}, \qquad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

# Problem 18

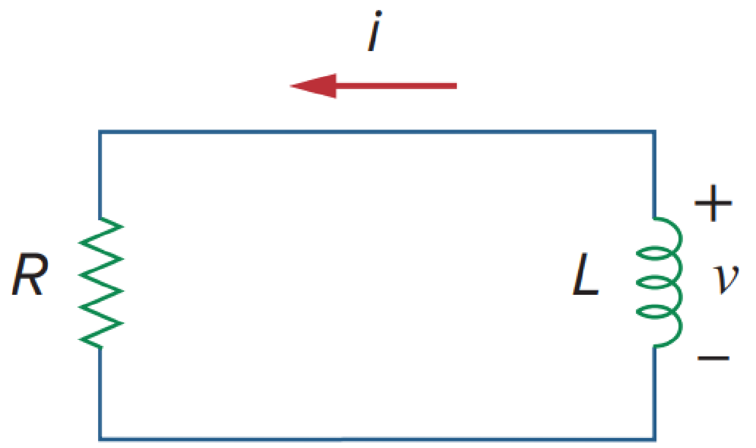
- Find $v(t)$ for $t > 0$.




Ans: $\boldsymbol{v(t) = (80e^{-t/2} - 60)\ V\ for\ t > 0}$

# Problem 19

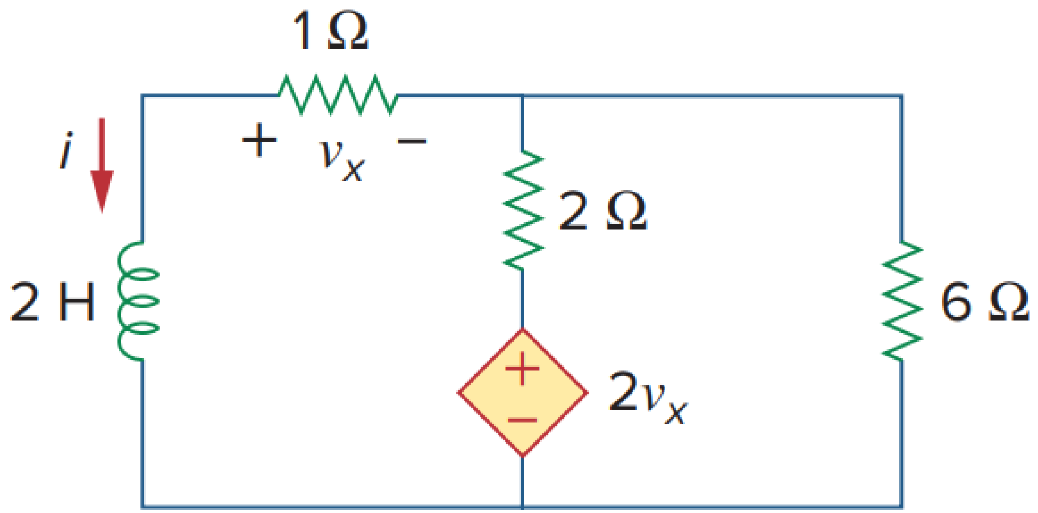
- For the circuit below, $v = 90e^{-50t}$ V and $i = 30e^{-50t}$ A for $t > 0$
  (a) Find $L$ and $R$.
  (b) Determine the time constant.
  (c) Calculate the initial energy in the inductor.
  (d) What fraction of the initial energy is dissipated in 10 $ms$.




Ans: $\boldsymbol{R = 3\ \Omega;\ L = 60\ mH;\ \tau = 0.02\ s;\ w_L(0) = 27\ J;\ \% = 94.7}$

# Problem 20

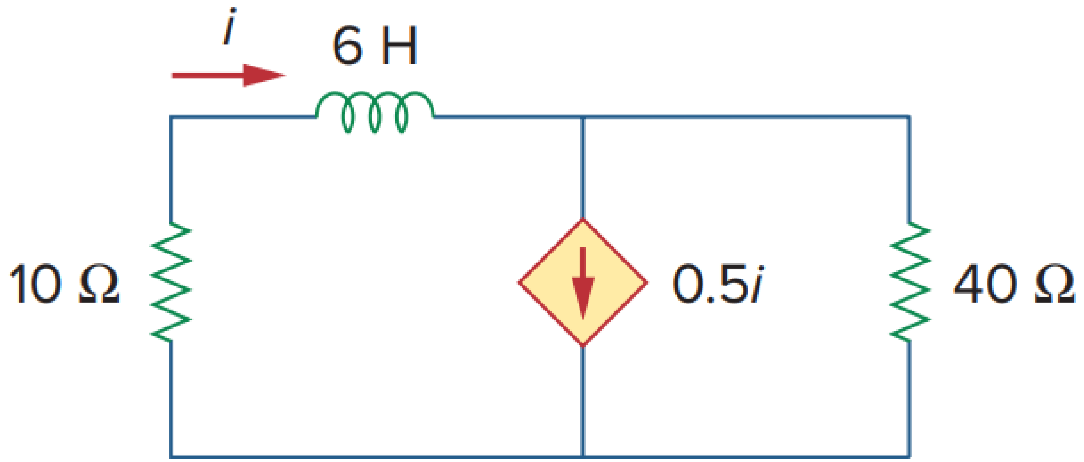
- Find $i$ and $v_x$ if $i(0) = 7\,A$.




Ans: $\boldsymbol{i(t) = 7e^{-2t}A;\ v_x(t) = -7e^{-50t}\ V}$

# Problem 21

- Find $i(t)$ if $i(0) = 5\,A$.




Ans: $\boldsymbol{i(t) = 5e^{-5t}\,A}$

# Problem 22

- Obtain the inductor current $i(t)$ for $t > 0$.

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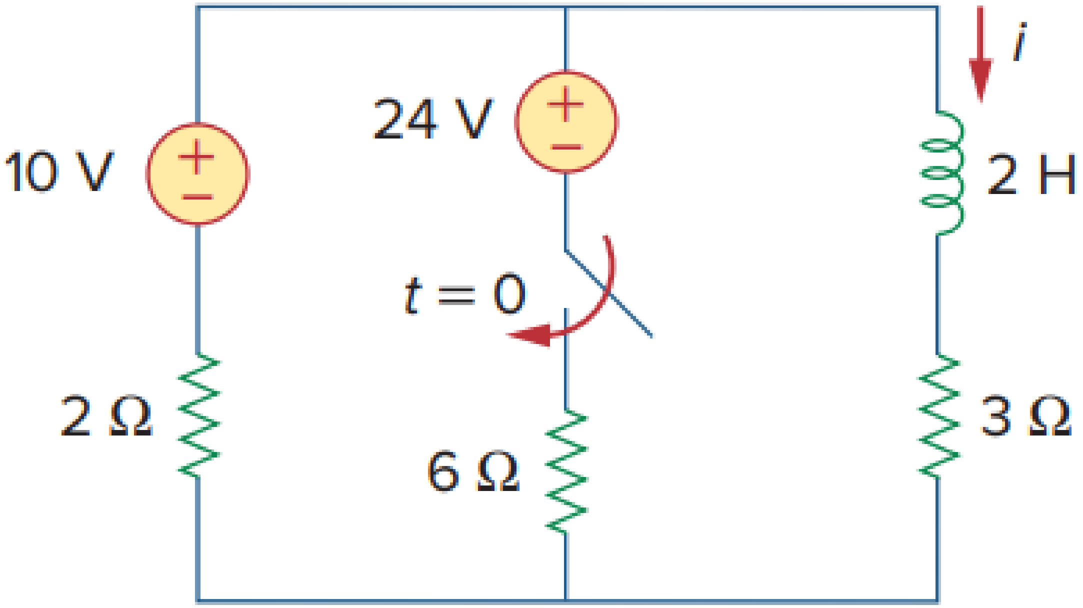
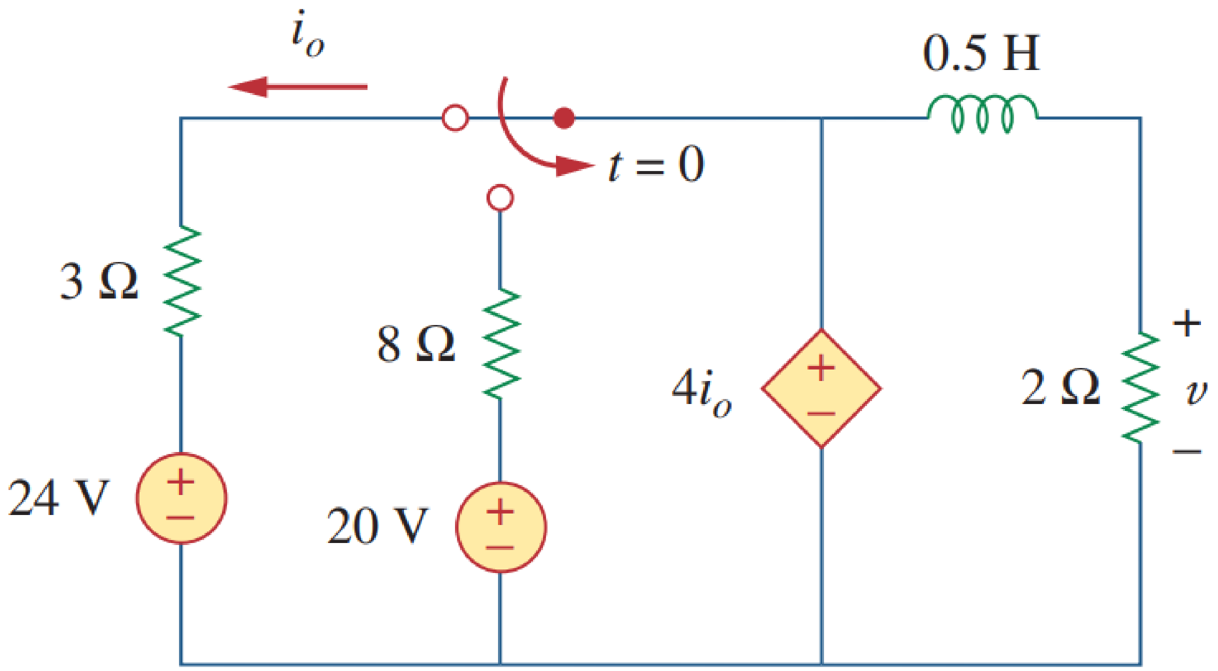



Ans: $\boldsymbol{i(t) = 3 - e^{-9t/4}\ A\ for\ t > 0}$

# Problem 23

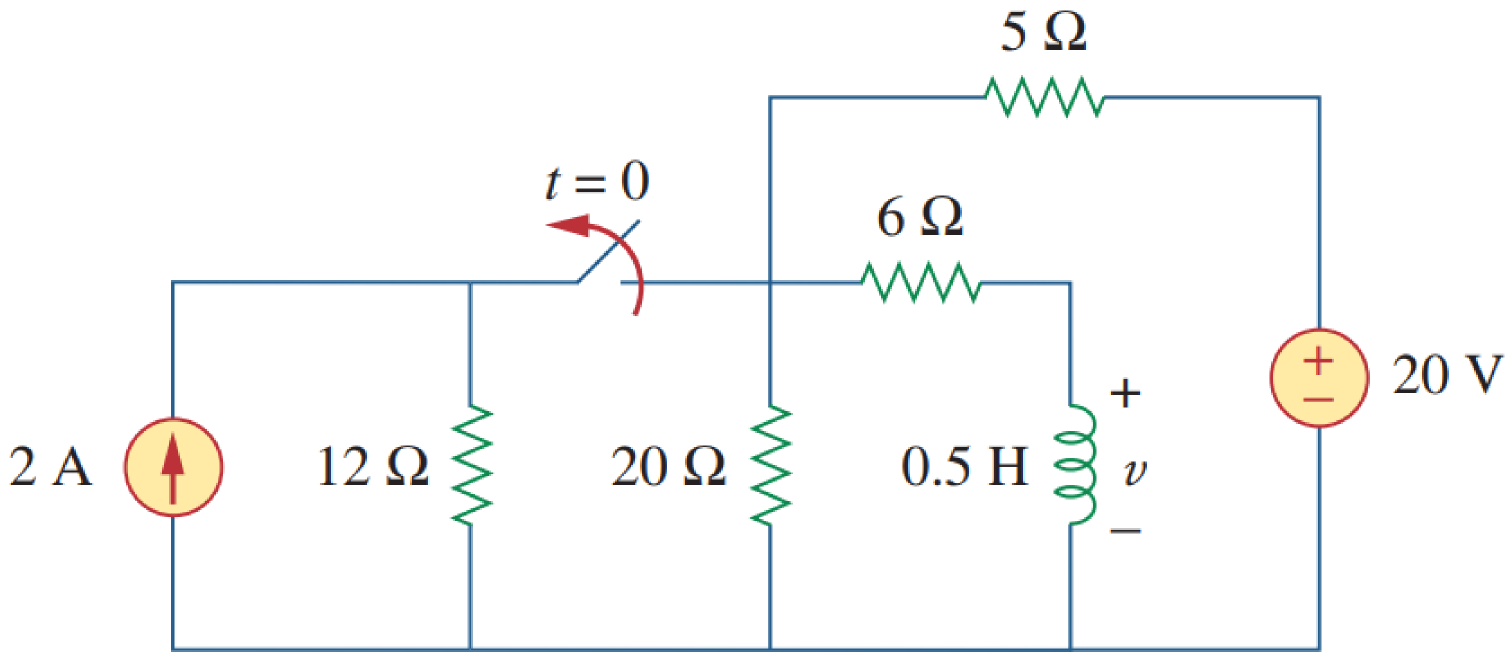
- Find $v(t)$ for both $t < 0$ and $t > 0$.




Ans: $\boldsymbol{v(t) = 96\,V\ for\ t < 0;\ v(t) = 96e^{-4t}\,V\ for\ t > 0}$

# Problem 24

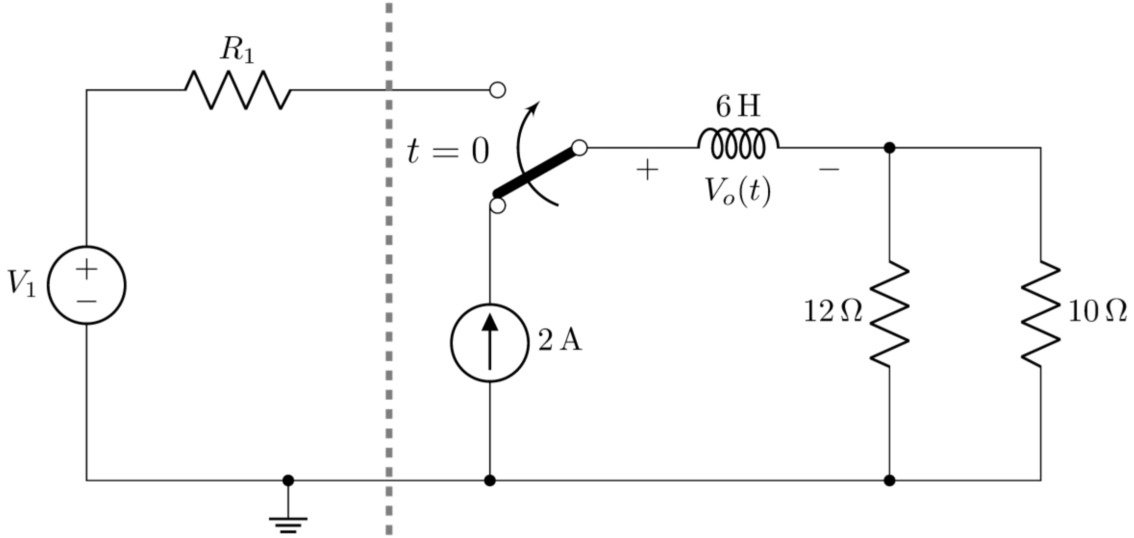
- Find $v(t)$ for $t > 0$.



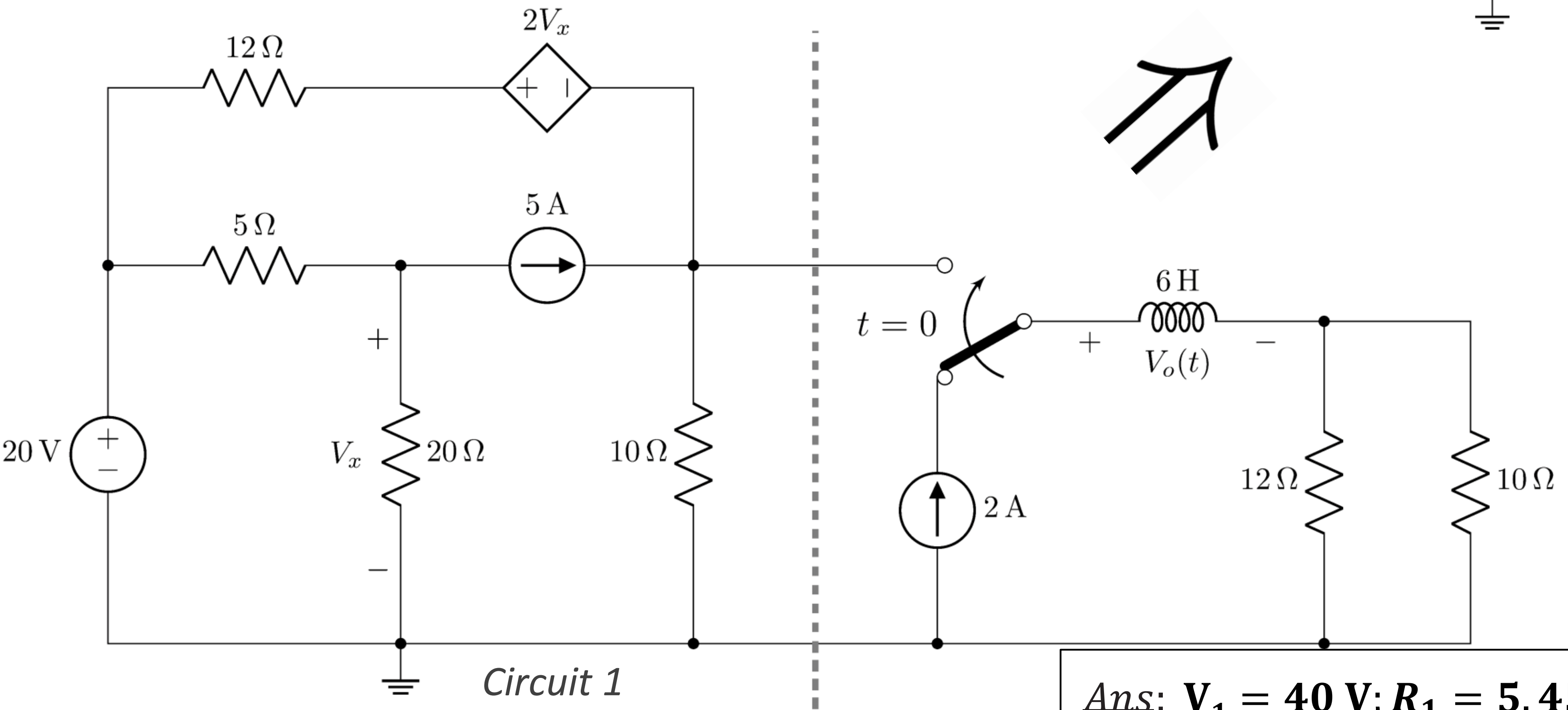

Ans: $\boldsymbol{v(t) = -4e^{-20t}\ V\ for\ t > 0}$

# Problem 25

- Reduce Circuit 1 so that it takes the form of Circuit 2. Determine $V_1$, and $R_1$.
- Find $\mathrm{V_o}(t)$ for $t > 0$.




Circuit 2

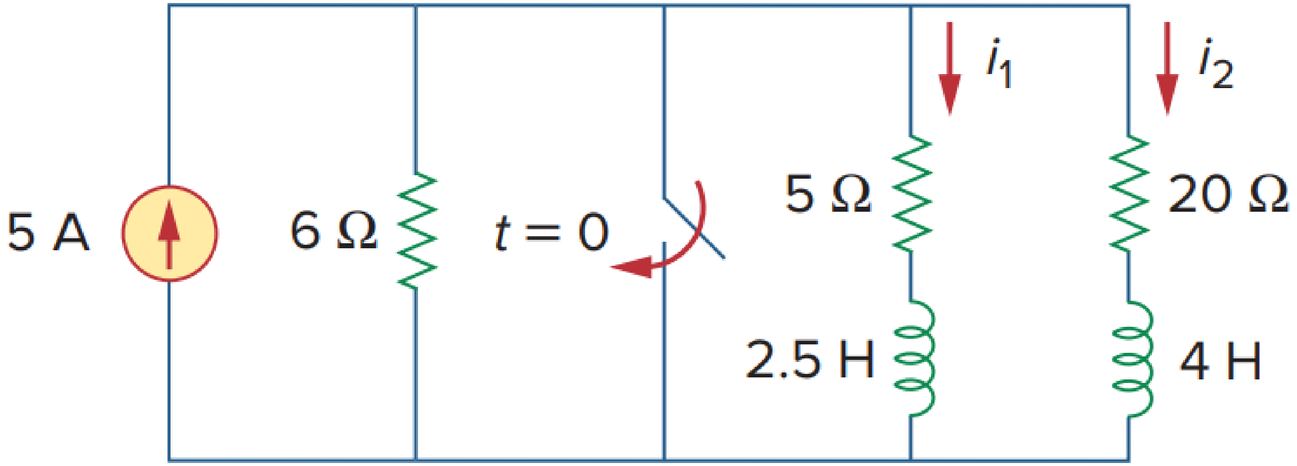



Circuit 1

<u>Ans</u>: $\mathbf{V_1 = 40\ V};\ \boldsymbol{R_1 = 5.45\ \Omega};\ \boldsymbol{V_o(t) = 18.22 e^{-t/0.55}\ V}$

# Problem 26

- Find $i_1(t)$ and $i_2(t)$ for $t > 0$.




Ans: $\boldsymbol{i_1(t) = 2.4e^{-2t} A\ for\ t > 0;\ i_2(t) = 0.6e^{-5t} A\ for\ t > 0}$

# Thank you for your attention