Assignment_02

MAT216_14_15

Spring 2024

Remarks: Release date: 23.02.2024

Submission date: 4.03.2024 (Monday, Class time)

Total Marks: 40 (will be converted to 20)

1. [10]

Let V be a subset of R^4 consisting of vectors that are perpendicular to vectors $a, b,$ and c where $a = <$ $1, 0, 1, 0 >, b = < 1, 1, 0, 0 >, c =< 0, 1, -1, 0 >,$

Namely, $V = \{x \in R^4 | a^T x = 0, b^T x = 0,$ and $C^T x = 0\}$

a. Prove that V is a subspace of R^4

b. Find a basis for V

c. Determine the Dimension of V

2. [10]

Let $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and consider the following subset V of the 2-dimensional vector space R^2, Na

$\{x \in R^2 | Ax = 5x\}$

a) Prove that the subset V is a subspace of R^2

b) Find a basis for V and determine the dimension of V

3. Find the basis for the row and column spaces of the following matrix: [10]

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

4. Let $V = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R}\right\}$ and $W = \{A \in V : A^2 = A\}$. Is W a subspace of V ? [5]

5. **Vector Space:** A set $\boldsymbol{V}$ equipped with two binary operations addition and scalar multiplication is called a vector space over the field F, if V satisfies the following 10 axioms, [5]

(i) $u + v \in V\ for\ all\ u, v \in V$

(ii) $u + v = v + u$

(iii) $u + (v + w) = (u + v) + w$

(iv) There exists a $0 \in V\ s.t.\ u + 0 = u\ for\ all\ u \in V$

(v) There exists a $-u \in V\ for\ all\ u \in V, such\ that\ u + (-u) = 0.$

(vi) $ku \in V\ for\ all\ u \in V\ and\ k \in F$

(vii) $a(u + v) = au + av\ for\ all\ a \in F\ and\ u, v \in V$

(viii) $(a + b)u = au + bu\ for\ all\ a, b \in F\ and\ u \in V.$

(ix) $(ab)u = a(bu) for\ all\ a, b \in F\ and\ u \in V$

(x) $1u = u, where\ 1 \in F\ and\ for\ all\ u \in V.$

(a) Suppose $u = (u_1, u_2)$ the multiplication of cu is defined to produce $(cu_{1,}0)$ instead of (cu_1, cu_2). With usual addition in $\mathbb{R}^2$, which of the eight conditions are not satisfied.?