MAT216

Assignment 1

Full Marks: 50

Due Date: 08.03.23 Spring 2024

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1. What is a vector space? Illustrate a 2D real vector space (IR^2), mention its conditions.

2. What is a vector subspace? Mention the conditions of it. Write down and illustrate the subspaces of IR^2 & IR^3

3. What is the column space? If,

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 2 & 2 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

   Then illustrate the column space of A or C(A).

4. What is null space? Describe it in a nutshell.

5. Illustrate row picture, matrix picture, and column picture of the following linear system.

$$2x + 3y = 5$$

$$7x - 2y = 5$$

6. Find the solution of the following linear system using gauss elimination. Also using the reduced row form.

$$\begin{aligned} x - 2y + z &= 8 \\ 2x + 2y + z &= 1 \\ 3x + 4y + 5z &= 10 \end{aligned}$$

7. Find the null space:

$$\begin{aligned} x + 3y + 5z &= 0 \\ 2x + 3y + 7z &= 0 \\ x + 4y + 6z &= 0 \end{aligned}$$

8. Illustrate the graphical representation of determinants of

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

9. Find the inverse of A using gauss-jordan elimination.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 7 & 0 & 5 \\ 5 & 2 & -3 \end{bmatrix}$$

10. Find the rank of A

$$A= \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 7 & -2 & 5 \\ 5 & 10 & 2 & 10 \end{bmatrix}$$

11. (a) Use Gauss-Jordan elimination (*equivalent to Reduced Row Echelon Form*) to solve the following system of linear equation.

$$\begin{array}{rrrrl} & -3x_2 & +2x_3 & +x_4 & = -1 \\ -x_1 & +x_2 & -2x_3 & +7x_4 & = -7 \\ x_1 & & +2x_3 & -10x_4 & = 10 \end{array}$$

(b) For what values of λ the following system of linear equation

$$\begin{array}{rrrl} x & +y & -z & = 1 \\ 2x & +3y & +\lambda z & = 3 \\ x & +\lambda y & +3z & = 2 \end{array}$$

has (i) a unique solution (ii) infinitely many solutions and (iii) no solution.

12. (a) Calculate the inverse of the matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ (i.e. A^{-1})by using $[A|I]$.

(b) Are the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ linearly independent?

[**Hints:** Check the solution of the system $c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0}$]

Vector Space: A set V equipped with two binary operations addition and scalar multiplication is called a vector space over the field $\mathbb{F}$, if V satisfies the following 10 axioms,

- $\mathbf{u} + \mathbf{v} \in V$ for all $\mathbf{u}, \mathbf{v} \in V$
- $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
- There exists a $\mathbf{0} \in V$ s.t. $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in V$
- There exists a $-\mathbf{u} \in V$ for all $\mathbf{u} \in V$, such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
- $k\mathbf{u} \in V$ for all $\mathbf{u} \in V$ and $k \in \mathbb{F}$
- $(ab)\mathbf{u} = a(b\mathbf{u})$ for all $a, b \in \mathbb{F}$ and $\mathbf{u} \in V$
- $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$ for all $a \in \mathbb{F}$ and $\mathbf{u}$,$\mathbf{v} \in V$
- $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{v}$ for all $a, b \in \mathbb{F}$ and $\mathbf{u} \in V$
- $1\mathbf{u} = \mathbf{u}$, where $1 \in \mathbb{F}$ and for all $\mathbf{u} \in V$.

13. (a) Check whether the set of vector $V = \{\vec{x} \mid I\vec{x} = \vec{0}\}$ is a vector space over the field of real number $\mathbb{R}$ and under the standard operation defined for addition and multiplication, where, I is the $n \times n$ **identity matrix**.

(b) Is the set of vectors $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \, y = -4x - z, z = 8x \right\}$ a subspace of the vector space $V = \mathbb{R}^3$?

Consider the following matrix,

$$A = \begin{bmatrix} 1 & -3 & 0 & 4 \\ -1 & 4 & -2 & 5 \\ -2 & 6 & 0 & -8 \end{bmatrix}$$

14. (a) Calculate the basis of the $Row(A)$. Also calculate the rank of A.

(b) Calculate the basis of the $Null(A)$. Also calculate the nullity of A.

15. (a) Consider the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$, where,

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ y - x + z \end{bmatrix}$$

Calculate the $dim(Img(T))$ and rank(T).

(b) Show that the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$ span $\mathbb{R}^3$.

16. Find the eigenvalues and corresponding eigenvectors of the matrix,

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$