S8 Assignment #4

1/

(i) At junction b, $I = I_1 + I_2$.

In the left loop, abefa path, apply KVL,

$$-(4\Omega)I_1 - (3\Omega)I + 12V = 0$$

$$\Rightarrow -(4\Omega)I_1 - (3\Omega)I_1 - (3\Omega)I_2 + 12V = 0$$

$$\Rightarrow (7\Omega)I_1 + (3\Omega)I_2 = 12V \quad \text{——— } (1)$$

In the right loop, bcdde path, apply KVL,

$$-(2\Omega)I_2 - 5V + (4\Omega)I_1 = 0$$

$$\Rightarrow (4\Omega)I_1 - (2\Omega)I_2 = 5V \quad \text{——— } (2)$$

$$2\times(1) + 3\times(2) \Rightarrow (14\Omega)I_1 + (12\Omega)I_1 = 24V + 15V$$

$$\Rightarrow (26\Omega)I_1 = 39V$$

$$\Rightarrow I_1 = \frac{39V}{26\Omega} = \frac{3}{2}A.$$

Substitute I_1 into (2) to find I_2.

$$(4\Omega)\times\frac{3}{2}A - (2\Omega)I_2 = 5V$$

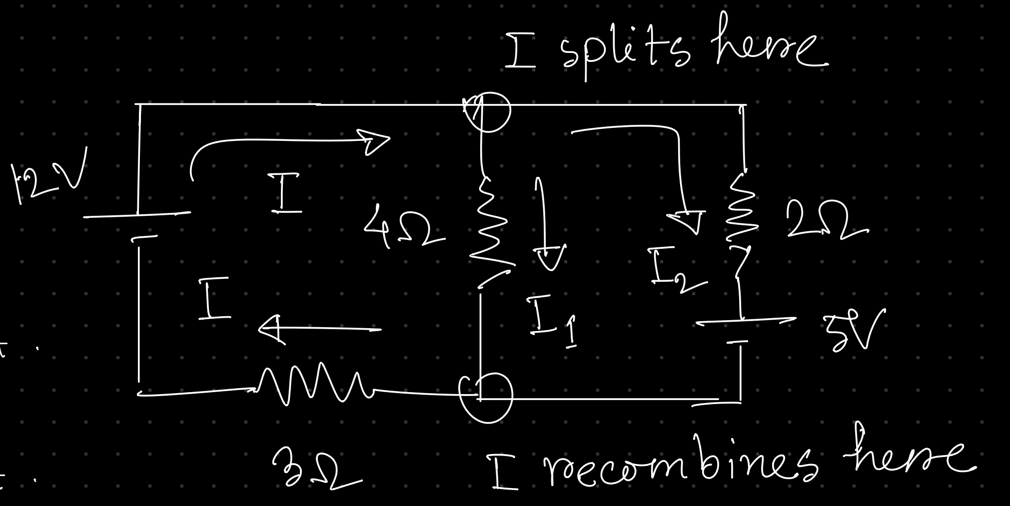
$$\Rightarrow (2\Omega)I_2 = -5V + 6V$$

$$\Rightarrow I_2 = \frac{1}{2}A.$$

$$I = \frac{3}{2}A + \frac{1}{2}A$$

$$= 2A$$

Currents through:



4Ω resistor : $\frac{3}{2}$ A.

3Ω resistor : 2 A.

2Ω resistor : $\frac{1}{2}$ A.

(ii) Power dissipation through:

4Ω resistor : $\left(\frac{3}{2}\right)^2 \times 4$ W $= 9$W

3Ω resistor : $(2)^2 \times 3$ W $= 12$W

2Ω resistor : $\left(\frac{1}{2}\right)^2 \times 2$W $= 0.5$W.

Note to TA: Conventions must be different for each student. Only the simplest application of KVL/KCL is to be graded. Energy within each loop cannot be violated.

2) We need to apply Biot-Savart Law here,

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

Given, $\vec{v} = -2\times10^{7}\, ms^{-1}\, \hat{i}$.

At $(0,0)\times10^{-2}$ m location,

$$\vec{r} = (0-0)\times10^{-2}\,m\,\hat{i} + (0-1)\times10^{-2}\,m\,\hat{j}$$

$$= (0\,\hat{i} - 10^{-2}\hat{j})\,m$$

$$r = \sqrt{(0\,m)^2 + (-10^{-2}m)^2}$$

$$= 10^{-2}\,m$$

$$\hat{r} = \frac{\vec{r}}{r} = -\hat{j}$$

Here,

$$\vec{v}\times\hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\times10^{7} & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(+2\times10^{7})\,ms^{-1}$$

$$= (+2\times10^{7}\,\hat{k})\,ms^{-1}$$

Plug this into the Biot-Savart Law,

$$\vec{B} = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{(-1.602 \times 10^{-19} C)(+2\times 10^{7}\,\hat{k})\,ms^{-1}}{(10^{-2} m)^{2}}$$

$$\vec{B} = (-3.204 \times 10^{-15}\,\hat{k})\,T.$$

Similar treatment to $(0,0)\times 10^{-2} m$ and $(2,0)\times 10^{-2} m$ gives you the following values:

At $(0,1)\times 10^{-2} m$ point

$\vec{r} = (0\hat{i} + 0\hat{j})\,m$

$r = 0 m$

$\hat{r} = 0\hat{i} + 0\hat{j}$

$\vec{v} \times \hat{r} = 0\ ms^{-1}$

$\vec{B} =$ Undefined

At $(2,0)\times 10^{-2} m$ point

$\vec{r} = (2\times 10^{-2}\hat{i} - 10^{-2}\hat{j})\,m$

$r = 2.24 \times 10^{-2} m$

$\hat{r} = \frac{25}{28}\hat{i} - \frac{25}{56}\hat{j}$

$\vec{v} \times \hat{r} = (+8\times 10^{6}\,\hat{k})\,ms^{-1}$

$\vec{B} = (+2.554 \times 10^{-16}\,\hat{k})\,T$

Note to TA: Students who followed only the magnitude part of cross-product approach may be graded correct so long their directions are properly chosen.

3/ First, one needs to explain how to set the cyclotron motion problem. A diagram here is a must here.

$$\text{Magnetic Force} = \text{Centripetal force}$$

$$qvB = m\frac{v^2}{r_c}$$

Note:
This is an impossible value, but given our miniscule B-field strength and cyclotron radius, we are accepting this on the basis of correct solution procedure.

(i) $$v = \frac{qBr_c}{m}$$ $r_c = 20\times10^{-2}\,m.$

$$= 5.27\times10^{9}\,ms^{-1}.$$

(ii) Period of the cyclotron motion, $T = \frac{2\pi r_c}{v}$

$$\Rightarrow T = 2.385\times10^{-10}\,s.$$

(iii) Work done by any uniform circular motion is zero. Since $\vec{F}$ and displacements are always at 90° with each other.

(iv) Kinetic energy upon exiting the cyclotron,

$$K = \frac{1}{2} m r_c^2 \omega_c^2$$

$$= \frac{1}{2} m r_c^2 \left(\frac{2\pi}{T}\right)^2$$

$$= 2 m r_c^2 \pi^2 \times \frac{1}{T^2}.$$

$$= 1.26 \times 10^{-11}\ \text{J}.$$