PHY-112
Principles of Physics-II
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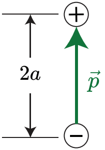
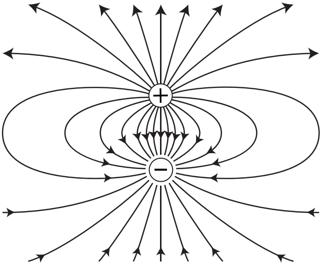
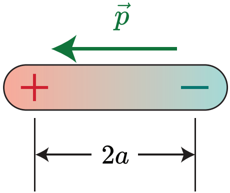
Spring-24 | Class-4

A system consisting of two equal and opposite point charges, typically denoted as $q = +ne$ and $q = -ne$, separated by a small distance $d = 2a$.



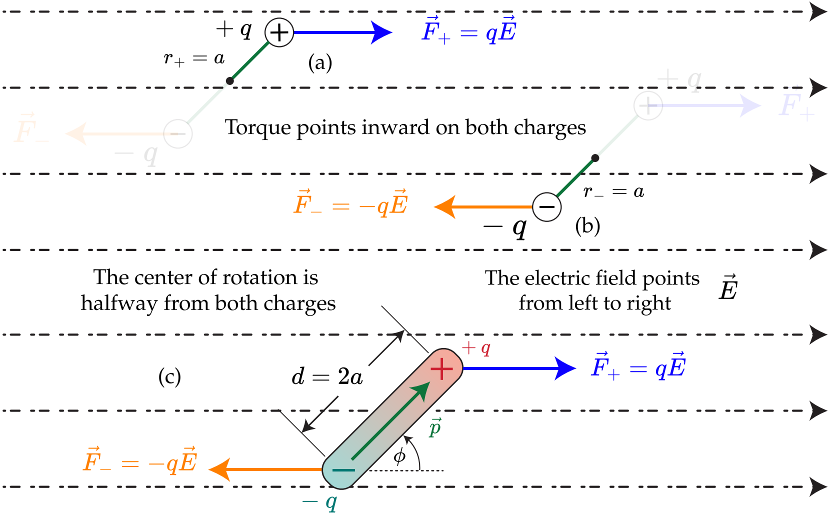


The direction of $\vec{p}$ indicates the orientation of the dipole
The magnitude of $\vec{p}$ measures the strength of the dipole



$$\vec{p} = q \cdot 2\vec{a}.$$

Forces on Electric Dipole in a Uniform $\vec{E}$-field



Forces on Electric Dipole in a Uniform $\vec{E}$-field

- The positive charge experiences a Coulomb force $\vec{F}_+ = +Q\vec{E}$ that points along $\vec{E}$
- The negative charge of the dipole also feels an equal but opposite force $\vec{F}_- = -Q\vec{E}$ that points opposite to $\vec{E}$
- The net force on the dipole thus $\vec{F}_{\text{net}} = \vec{F}_+ + \vec{F}_- = 0$
- This does not mean the dipole is motionless

Torque on Electric Dipole in a Uniform $\vec{E}$-field

- The torque for the positive charge would be
$$\tau_+ = \vec{r}_+ \times \vec{F}_+ = a\hat{d} \times Q\vec{E} = (Qa)\hat{d} \times \vec{E} = \frac{1}{2}\left(\vec{p} \times \vec{E}\right)$$
- The torque for the negative charge would be
$$\tau_- = \vec{r}_- \times \vec{F}_- = \left(-L\hat{d}\right) \times \left(-Q\vec{E}\right) = (Qa)\hat{d} \times \vec{E} = \frac{1}{2}\left(\vec{p} \times \vec{E}\right)$$
- The total torque of the dipole system
$$\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_- = \frac{1}{2}\left(\vec{p} \times \vec{E}\right) + \frac{1}{2}\left(\vec{p} \times \vec{E}\right) = \vec{p} \times \vec{E}$$

Testing Concepts (1)

- Q: The permanent electric dipole moment of a particular molecule is 1.1×10^{-30} C m. What is the MAXIMUM possible torque on the molecule in a 8.0×10^{8} N C^{-1} field?

  $\longrightarrow$ Take $\phi = 90^\circ$

- Q: Find the MINIMUM torque

  $\longrightarrow$ Take $\phi = 0^\circ$

Potential Energy stored by an Electric Dipole

$$
\begin{aligned}
U_{\text{dipole}} &= -\int \tau d\phi \\
&= -\int_{90^\circ}^{\phi} \; pE \sin\phi \; d\phi \\
&= \Big[pE \cos\phi \Big]_{90^\circ}^{\phi} \\
&= \vec{p} \cdot \vec{E}
\end{aligned}
$$

Testing Concepts (2)

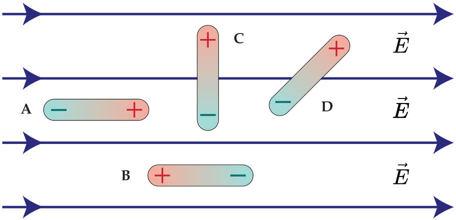
- Q: The permanent electric dipole moment of a particular molecule is 1.1×10^{-30} C m. What is the stored energy on the molecule (when placed parallel) in a 8.0×10^{8} N C^{-1}?

  $\longrightarrow$ Take $\phi = 0^\circ$

- Q: What is the stored energy on the molecule (when placed perpendicular)

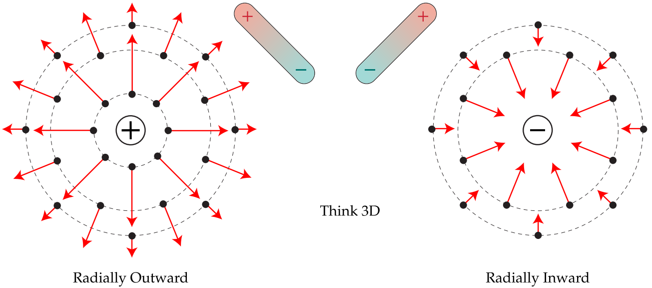
  $\longrightarrow$ Take $\phi = 90^\circ$

Testing Concepts (3)



- Q: Rank the Net Forces experienced by the dipoles in descending order.
- Q: Rank the Net Torques experienced by the dipoles in descending order.
- Q: Rank the Potential Energies stored by the dipoles in descending order.

Electric Dipoles in a Non-Uniform $\vec{E}$-field



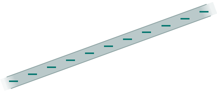
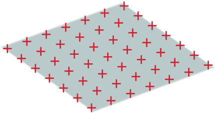
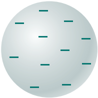
- Step-1: Orient $\vec{p}$ to $\vec{E}$
- Step-2: Apply Force $\vec{F}_E$ (push/pull) accordingly

$\vec{E}$-fields due to Continuous Charge Distributions

One Problem Solving Strategy to Rule them All

- Start with $\vec{E} = \left(\frac{Cq}{r^2}\right)\hat{r}$
- Superpose them: $\vec{E} = \sum_i^N \left(\frac{Cq_i}{r_i^2}\right)\hat{r}_i$ (Discrete)
- Start with $d\vec{E} = \left(\frac{Cdq}{r_{dq}^2}\right)\hat{r}_{dq}$
- Integrate them: $\vec{E} = \int \left(\frac{Cdq}{r_{dq}^2}\right)\hat{r}_{dq}$ (Continuous)

4 Key $\vec{E}$ Field Sources: Continuous Distributions

Line Charge	Surface Charge	Volume Charge
		
$dq = \lambda dl$	$dq = \sigma da$	$dq = \rho dV$
$\vec{E} = \left(C \int \frac{\lambda dl}{r^2} \right) \hat{r}$	$\vec{E} = \left(C \int \frac{\sigma da}{r^2} \right) \hat{r}$	$\vec{E} = \left(C \int \frac{\rho dV}{r^2} \right) \hat{r}$
$\hat{r}$ → away is + ; → toward if -	$\hat{r}$ → away is + ; → toward if -	$\hat{r}$ → away is + ; → toward if -

Gauss's Law for Electrostatics: The 1st Maxwell's Equation

Gauss's Law for Electrostatics: Why do We need it?

- It is more fundamental than Coulomb's law
- 1^{st} of Maxwell's equations
- Relates Electric Charges to Electric Fields and vice versa
- Easier to use than superposing many $\vec{E}s$
- Can measure $\vec{E}$ both inside and outside of sources.
- Suitable for Symmetric $\vec{E}$-Field models

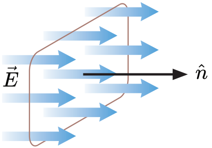
Gauss's Law for Electrostatics: What do You need?

- Electric Flux, Φ_E
- Gaussian Surfaces: Closed (Symmetric) surfaces
- Surface (Closed) Integral, $\oint$
- Divergence, $\vec{\nabla} \cdot \vec{E}$
- Ideas about Charge Distribution, λ, σ, ρ

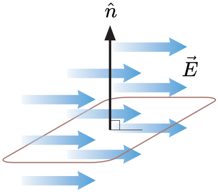
Q: What does it measure?

- Visually: the number of electric field lines passing through a given surface.
- Numerically: the surface integral of $\vec{E}$-fields
  - $\Phi_E = \int \vec{E} \cdot d\vec{A}$ (Non-Uniform)
  - $\Phi_E = \vec{E} \cdot \vec{A}$ (Uniform)
  - Unit: [N m^2 C^{-1}] or [V m]
  - It is a scalar

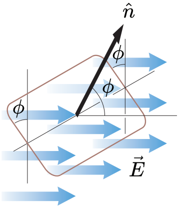
Maximum Flux



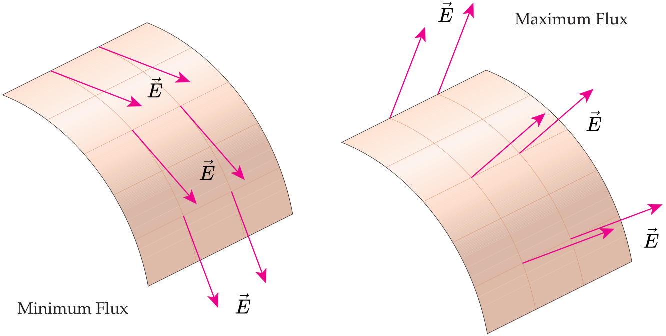
Minimum Flux



Arbitrary Flux



Electric Flux for Non-Uniform $\vec{E}$-field



It relates the behavior of the electric field to the distribution of electric charge. **One demands the presence of the other**.

Gauss's Law and $\vec{E}$-fields

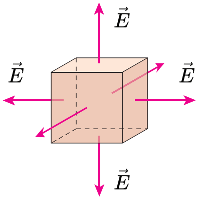
- The total Φ_E passing through a closed surface is proportional to the total electric charge Q_{enc} enclosed within that surface

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}. \qquad \text{(Integral Form)}$$

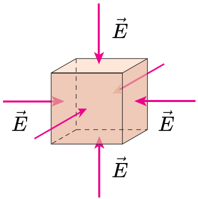
- The Divergence of $\vec{E}$-fields through a closed surface are directly proportional to the charge distribution within that surface

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{\text{enclosed}}}{\epsilon_0}. \qquad \text{(Differential Form)}$$

Q: Comment on the type of charges enclosed within the cube.



A



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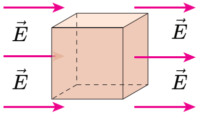

C

Gauss's Divergence Theorem

- Surface Integral (Vector Field) = Volume Integral (Divergence of the same Vector Field)
- The $\int$ count implies the dimension
- $$\iint_S \vec{V} \cdot d\vec{S} = \iiint_V (\vec{\nabla} \cdot \vec{V}) \cdot dV$$
- $$\int_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) \cdot dV$$

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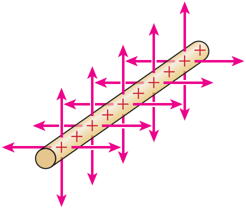
- $$\oint_S \vec{E} \cdot d\vec{a} = \oint_V (\vec{\nabla} \cdot \vec{E}) \cdot dV$$

Suitable Gaussian Surfaces for 4 Key $\vec{E}$ Field Sources

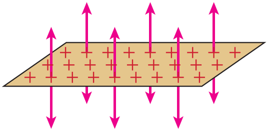
- Line Charge $\longrightarrow$ Current Wire $\longrightarrow$ Long Cylindrical
- Surface Charge $\longrightarrow$ Capacitors $\longrightarrow$ Wide Cylindrical
- Volume Charge $\longrightarrow$ Electrodes/Shell Charges $\longrightarrow$ Spherical

Suitable Gaussian Surfaces for 4 Key $\vec{E}$ Field Sources

Cylindrical symmetry



Planar symmetry



Spherical symmetry