예제 0.1.1

Slope $= \frac{-2}{-3}$, $y = \frac{2}{3}(x-1)+1$

$y = \frac{2}{3}x + \frac{1}{3}$

예제 0.1.2

$y = 2x^2 - 3x + 1$

$= 2\left(x^2 - \frac{3}{2}x + \frac{1}{2}\right)$

$= 2\left(x - \frac{3}{4}\right)^2 + 1 - \frac{9}{8}$

$= 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$

$= (x-1)(2x-1)$

x-Intercept : $(1,0)$ / $\left(\frac{1}{2}, 0\right)$

y-Intercept : $(0,1)$

꼭지점 : $\left(\frac{3}{4}, -\frac{1}{8}\right)$

예제 0.2.1

(1) $y = 5x^2 + 7x - 6$, $y' = 10x + 7$

(2) $f(x) = (3x^2-1)^2(2x^3+5) = (3x^2-1)^{2\prime}(2x^3+5) + (3x^2-1)^2(2x^3+5)'$

$= 2(\cancel{3x^2-1}) \cdot \cancel{6x}(2x^3+5) + (3x^2-1)^{\cancel{2}} \cdot \cancel{6x^2}$

$= 6x(3x^2-1)(4x^3+10 + 3x^3 - x)$

$= 6x(3x^2-1)(7x^3 - x + 10)$

(3) $v(x) = \frac{2x^2+1}{x-3}$, $\frac{dv}{dx} = \frac{4x(x-3) - (2x^2+1) \cdot 1}{(x-3)^2}$

$= \frac{2x^2 - 12x - 1}{(x-3)^2}$

예제 0.2.2

(1) $y = (x^2-2)^{10}$, $y' = 10(x^2-2)^9 \cdot 2x$

$= 20x(x^2-2)^9$

(2) $f(x) = \dfrac{x}{(2x+1)^3}$. $f'(x) = \dfrac{(2x+1)^3 - x(3(2x+1)^2 \cdot 2)}{(2x+1)^6}$

$$= \frac{(2x+1)^3 - 6x(2x+1)^2}{(2x+1)^6}$$

$$= \frac{(2x+1)^2\{2x+1-6x\}}{(2x+1)^6}$$

$$= \frac{-4x+1}{(2x+1)^4}$$

예제 0.2.3

(1) $f(x) = x^3+x^2-3$, $f'(x) = 3x^2+2x$, $f''(x) = 6x+2$

(2) $y = x^3 - \dfrac{1}{x^2} = x^3 - x^{-2}$, $f'(x) = 3x^2+2x^{-3}$, $f''(x) = 6x-6x^{-4}$

예제 0.3.1.

(1) $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \dfrac{1}{1+\frac{1}{2}} x^{1+\frac{1}{2}} = \dfrac{2}{3}x^{\frac{3}{2}}+C$

(2) $\int (2x^2+x-1)\,dx = \dfrac{2}{3}x^3+\dfrac{1}{2}x^2-x+C$

예제 0.4.1

$$\int_0^1 (2x-1)^8 dx$$

$2x-1 = t$. $\Rightarrow 2\frac{dx}{dt} = 1 \Rightarrow dx = \frac{1}{2}dt$

$$\int_{-1}^{1} t^8 \frac{1}{2} dt = \left[\frac{1}{2}\cdot\frac{1}{9}\cdot t^9\right]_{-1}^{1} = \frac{1}{18} - \left(-\frac{1}{18}\right) = \frac{1}{9}$$

예제 0.4.2

$$\int x\sqrt{2x+1}\, dx$$

$2x+1 = t \Rightarrow 2\frac{dx}{dt} = 1 \Rightarrow dx = \frac{1}{2}dt$

$$\int x\sqrt{2x+1}\, dx = \int \frac{(t-1)}{2}\sqrt{t}\,\frac{1}{2}dt = \frac{1}{4}\int t^{\frac{3}{2}} - t^{\frac{1}{2}}\, dt$$

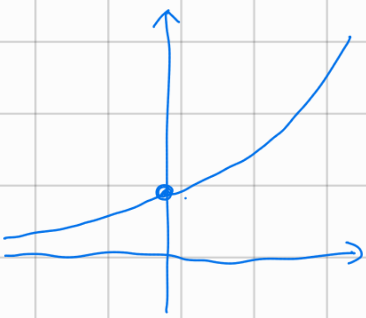
$$= \frac{1}{4}\left\{\left[\frac{2}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}}\right]\right\} + C$$

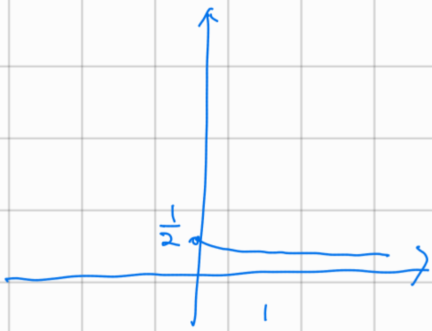
$$= \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + C.$$

예제 0.5.1

(1) $y = e^x$ (2) $y = \frac{1}{2}e^{-\frac{x-1}{2}}$ (3) $y = 1 - e^{-\frac{(x-3)}{2}}$

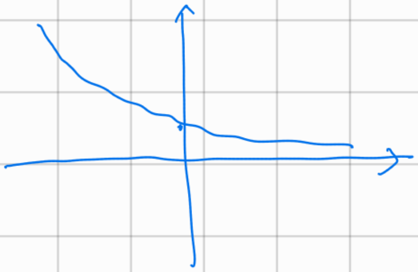
$y = -e^{-\frac{1}{2}x}$

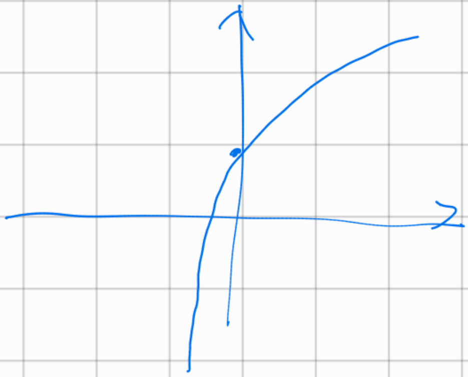




(4) $y = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

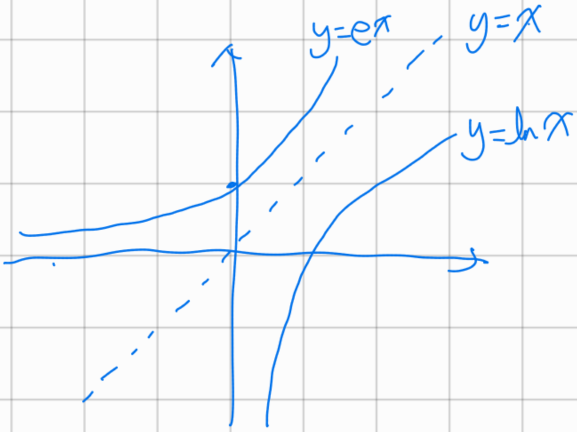
(5) $y = 1 + 2\ln(x+1)$





예제 0.5.2

$y=e^x$, $y=\ln x$, $y=x$



예제 0.5.3

(1) $y=e^{-3x}$

$y' = e^{-3x} \cdot -3 = -3e^{-3x}$

(2) $y=e^{-x^2}$

$y' = e^{-x^2}(-2x) = -2xe^{x^2}$

(3) $y=3^{x^2}$

$y' = 3^{x^2}\ln 3 \, 2x = 2\ln 3 \, x \, 3^{x^2}$

(4) $y=\ln[(x^2+2)(x-3)]$

$y' = \frac{1}{(x^2+2)(x-3)}\left(2x(x-3) + x^2+2\right)$

$= \frac{1}{(x^2+2)(x-3)}\left(3x^2-6x+2\right)$

예제 0.5.4

(1) $y = x\ln x$

$$y' = \ln x + x\frac{1}{x} = 1 + \ln x$$

(2) $y = \dfrac{\ln 2x}{x}$

$$y' = \frac{\frac{1}{2x}\cdot 2\cdot x - \ln 2x \cdot 1}{x^2} = \frac{1-\ln 2x}{x^2}$$

예제 0.5.5

(1) $\displaystyle\int_0^{\ln 5} e^x\,dx = \left[e^x\right]_0^{\ln 5} = 5 - 1 = 4.$

(2) $\displaystyle\int (2e^{-x} + 3^{x+1})\,dx$

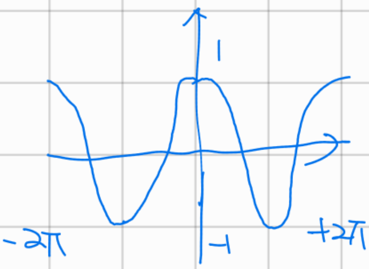
$*\ a^x = e^{\ln a^x} = e^{x\ln a},\ (a^x)' = e^{x\ln a}\cdot\ln a = a^x\ln a$

$$= -2e^{-x} + \frac{1}{\ln 3}3^{x+1} + C$$

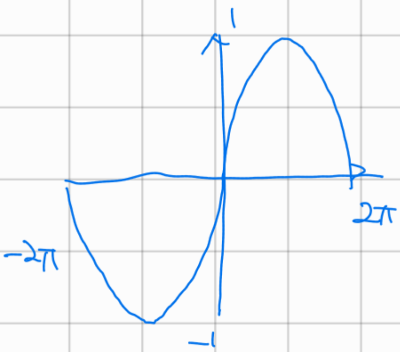
(3) $\displaystyle\int_0^{\ln 3} e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_0^{\ln 3} = \frac{1}{2}(9-1) = 4.$

예제 0.6.1

(1) $y = \cos x$



(2) $y = \sin x$



(3) $y = \cos(4x + \pi)$

$4(x + \frac{\pi}{4})$

(4) $y = 0.5 + 0.5\sin\left(2x - \frac{\pi}{3}\right)$

skip.

예제 0.6.2

(1) $\cos\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos t = \frac{\sqrt{3}}{2}$, $t = \pm\frac{\pi}{6} + 2\pi k \ (k \in \mathbb{Z})$

$2x - \frac{\pi}{3} = \pm\frac{\pi}{6} + 2\pi k$

$x = \pi k + \frac{\pi}{2}$ or $\pi k + \frac{\pi}{6}$

$\therefore x = -\frac{\pi}{2}, \frac{\pi}{2}, -\frac{5}{6}\pi, \frac{\pi}{6}$

(2), (3) skip.

(4) $\sin\left(3x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$

$\sin t = \frac{\sqrt{3}}{2}$

$t = \frac{\pi}{3} + 2k\pi$ or $\frac{2}{3}\pi + 2k\pi$

a. $3x - \frac{\pi}{2} = \frac{\pi}{3} + 2k\pi$

$3x = \frac{5}{6}\pi + 2k\pi$

$x = \frac{5}{18}\pi + \frac{2}{3}k\pi$

$= \frac{(5-12k)\pi}{18}$

b. $3x - \frac{\pi}{2} = \frac{2}{3}\pi + 2k\pi$

$3x = \frac{7}{6}\pi + 2k\pi$

$x = \frac{(7+12k)\pi}{18}$

$-24+7$

$-\frac{5}{18}\pi$

$\frac{17}{18}\pi, \frac{5}{18}\pi, -\frac{7}{18}\pi$

$-\frac{17}{18}\pi, -\frac{5}{18}\pi, \frac{7}{18}\pi.$

예제 0.6.3

(1) $y = x\cos 2x$

$y' = \cos 2x + x\cdot(-\sin 2x \cdot 2) = \cos 2x - 2x\sin 2x.$

(2) $y = e^{-2x}\cdot \sin x$

$y' = -2\cdot e^{-2x}\sin x + e^{-2x}\cos x.$

(3) $y = \sin(3x+2)$

$y' = \cos(3x+2)\cdot 3 = 3\cos(3x+2)$

(4) $y = \dfrac{\cos x}{x}$

$y' = \left(\dfrac{1}{x}\cdot\cos x\right)' = -\dfrac{1}{x^2}\cos x + \dfrac{1}{x}\,{-\sin x}$

$= \dfrac{-x\sin x - \cos x}{x^2}$

예제 0.6.4

(1) $\displaystyle\int_0^{\frac{\pi}{2}} (\sin 2x + \cos 3x)\,dx$

$= \left[-\dfrac{1}{2}\cos 2x + \dfrac{1}{3}\sin 3x\right]_0^{\frac{\pi}{2}} = \left(\dfrac{1}{2} - \dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right) = 1 - \dfrac{1}{3} = \dfrac{2}{3}.$

(2) $\displaystyle\int \sin^2\left(\dfrac{x}{2}\right)dx$

Note. $\cos\alpha = \cos\dfrac{\alpha}{2}\cdot\cos\dfrac{\alpha}{2} - \sin\dfrac{\alpha}{2}\sin\dfrac{\alpha}{2}$

$\cos\alpha = \cos^2\dfrac{\alpha}{2} - \sin^2\dfrac{\alpha}{2}$

$= \left(1 - \sin^2\dfrac{\alpha}{2}\right) - \sin^2\dfrac{\alpha}{2}$

$\Rightarrow$ $\cos\alpha = 1 - 2\sin^2\dfrac{\alpha}{2}$

$\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$

$\displaystyle\int \dfrac{1-\cos x}{2}\,dx = \dfrac{1}{2}x - \dfrac{1}{2}\sin x + C.$

(3) $\int_0^{\frac{2}{3}\pi} \sin\left(2x-\frac{\pi}{3}\right) dx = \left[-\frac{1}{2}\cos\left(2x-\frac{\pi}{3}\right)\right]_0^{\frac{2}{3}\pi}$

$= \left(\frac{1}{2}\right) - \left(-\frac{1}{4}\right) = \frac{3}{4}.$

(4) $\int_0^{\frac{\pi}{2}} \sin x \cdot \cos x \, dx.$

Note. $\sin\alpha = \sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2} + \cos\frac{\alpha}{2}\cdot\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} = \frac{\sin\alpha}{2}$

$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2} dx = \left[-\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}} = \left(\frac{1}{4}\right) - \left(-\frac{1}{4}\right) = \frac{1}{2}.$

(5) $\int_0^{\frac{\pi}{2}} \sin 3x \cdot \cos 2x \, dx$

Note. $\sin(3x+2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$

$\sin(3x-2x) = \sin 3x \cdot \cos 2x - \cos 3x \cdot \sin 2x$

$\sin 5x + \sin x = 2 \cdot \sin 3x \cdot \cos 2x.$

$\int_0^{\frac{\pi}{2}} \sin 3x \cos 2x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(\sin 5x + \sin x)\, dx = \left[-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x\right]_0^{\frac{\pi}{2}}$

$= \frac{3}{5}.$

(6) $\int_{-\pi}^{\pi} \sin^2 x \cdot \sin 3x \, dx.$

Note. $\cos 2x = \cos^2 x - \sin^2 x$

$= (1-\sin^2 x) - \sin^2 x$ $\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$\int \sin^2 x \cdot \sin 3x \, dx = \int \left(\frac{1-\cos 2x}{2}\right) \sin 3x \, dx$

$$\int \left(\frac{1-\cos 2x}{2}\right) \sin 3x \, dx = \int \frac{1}{2}\sin 3x - \frac{1}{2}\cos 2x \sin 3x \, dx.$$

Note. $\sin 5x = \sin 3x \cdot \cos 2x + \cos 3x \cdot \sin 2x$

$\sin x = \sin 3x \cos 2x - \cos 3x \cdot \sin 2x$

$$\sin 3x \cdot \cos 2x = \frac{1}{2}(\sin 5x + \sin x)$$

$$\int \frac{1}{2}\sin 3x - \frac{1}{4}\sin 5x - \frac{1}{4}\sin x \, dx = \left[-\frac{1}{6}\cos 3x + \frac{1}{20}\cos 5x + \frac{1}{4}\cos x\right]_{-\pi}^{\pi}$$

$$\left(\frac{1}{6} - \frac{1}{20} - \frac{1}{4}\right) - \left(\frac{1}{6} - \frac{1}{20} - \frac{1}{4}\right) = 0.$$

(7) $\int_{-\pi}^{\pi} \sin 3x \cdot \cos 4x \, dx$

Note. $\sin 7x = \sin 3x \cos 4x + \cos 3x \cdot \sin 4x$

$\sin -x = \sin 3x \cos 4x - \cos 3x \cdot \sin 4x$

$$\frac{\sin 7x - \sin x}{2} = \sin 3x \cdot \cos 4x.$$

$$\int_{-\pi}^{\pi} \sin 3x \cos 4x \, dx = \int_{-\pi}^{\pi} \frac{1}{2}\sin 7x - \frac{1}{2}\sin x \, dx = \left[-\frac{1}{14}\cos 7x + \frac{1}{2}\cos x\right]_{-\pi}^{\pi}$$

$$= 0$$

(8) $\int_{-\pi}^{\pi} \cos^2 2x \, dx. = \int_{-\pi}^{\pi} \frac{1+\cos 4x}{2} \, dx = \left[\frac{1}{2}x + \frac{1}{8}\sin 4x\right]_{-\pi}^{\pi} = \left(\frac{\pi}{2}\right) - \left(\frac{-\pi}{2}\right)$

$$= \pi.$$

Note $\cos 2x = \cos x \cdot \cos x - \sin x \cdot \sin x$

$1 = \cos x \cos x + \sin x \sin x$

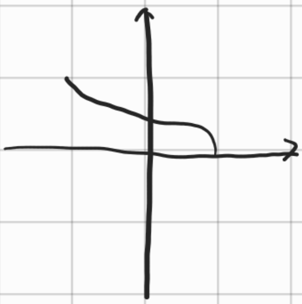
$$\frac{1+\cos 2x}{2} = \cos^2 x.$$

예제 0.7.1

(a) $y=\cos^{-1}x$

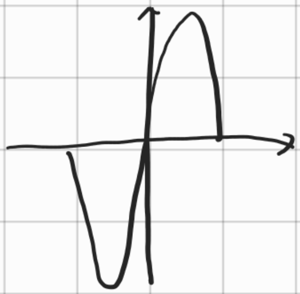
$y=\cos x$

$y=\cos^{-1}x$

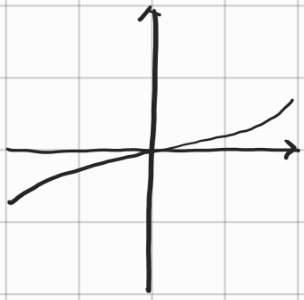


(b) $y=\sin^{-1}x$

$y=\sin x$

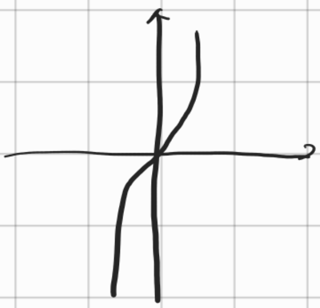


$y=\sin^{-1}x$

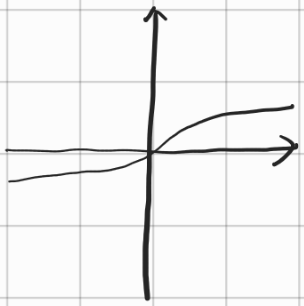


(c) $y=\tan^{-1}x$

$y=\tan x$



$y=\tan^{-1}x$



(d) Skip!

예제 0.7.2

(1) $y=\tan^{-1}\sqrt{x}$

$$y'=\frac{1}{1+x}\cdot\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2(1+x)\sqrt{x}}$$

(2) $y=\sin^{-1}x^3$

$$y'=\frac{1}{\sqrt{1-x^6}}3x^2$$

(3) $$\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}dx=\Big[\sin^{-1}x\Big]_{-\frac{1}{2}}^{\frac{1}{2}}=\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)=\frac{\pi}{3}.$$

(4) $$\int_0^1\frac{1}{1+x^2}dx=\Big[\tan^{-1}x\Big]_0^1=\frac{\pi}{4}.$$

예제 0.8.1.

(1) $y = \cosh^2(2x+1)$

$$y = \frac{\cosh(4x+2)+1}{2}$$

$$y' = \frac{1}{2}\sinh(4x+2)\cdot 4 = 2\sinh(4x+2).$$

(2) $\int \sinh 2x \, dx$

$$= \frac{1}{2}\cosh 2x + C$$

(3) $\int \tanh\frac{x}{3} dx$

$$t := \frac{x}{3}, \quad \frac{dt}{dx} = \frac{1}{3}$$

$$\int \frac{\sinh t}{\cosh t} 3 \cdot dt = 3 \cdot \ln|\cosh t| + C. = 3 \cdot \ln|\cosh\frac{x}{3}| + C.$$

예제 0.9.2

(1) $z = \sqrt{x^2+y^2}$

$$\frac{\partial z}{\partial x} = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}, \quad \frac{\partial z}{\partial y} = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+y^2}} 2y = \frac{y}{\sqrt{x^2+y^2}}$$

(2) $z = \frac{y}{x}$

$$\frac{\partial z}{\partial x} = y \cdot -\frac{1}{x^2} = -\frac{y}{x^2}, \quad \frac{\partial z}{\partial y} = \frac{1}{x}$$

(3) $z = \cos(xy)$

$$\frac{\partial z}{\partial x} = -y \cdot \sin(xy), \quad \frac{\partial z}{\partial y} = -x \cdot \sin(xy)$$

예제 0.9.3

(1) $z = \frac{y}{x}$

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2}, \quad \frac{\partial z}{\partial y} = \frac{1}{x} \Rightarrow dz = -\frac{y}{x^2}dx + \frac{1}{x}dy$$

(2) $z = r\cos\theta$

$$\frac{\partial z}{\partial r} = \cos\theta, \quad \frac{\partial z}{\partial \theta} = -r\sin\theta \Rightarrow dz = \cos\theta \, dr - r\sin\theta \, d\theta$$

예제 0.10.2

$z = x^2 + 2xy$, $x = \cos\theta$, $y = \sin\theta$.

$$\frac{dz}{d\theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= (2x+2y)(-\sin\theta) + 2x\cos\theta$$

$$(2x+2y)(-\sin\theta) + 2x\cos\theta \Big|_{\theta=\pi} = 2.$$

예제 0.10.3

$z = u^3 + v^3$, $u = x+y$, $v = x-y$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = 3u^2 \cdot 1 + 3v^2 \cdot 1 = 3(x+y)^2 + 3(x-y)^2$$

예제 0.11.1

$x\sin y + y\cos x = 0$

$$\frac{dx}{dy} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} = -\frac{x\cos y + \cos x}{\sin y - y\sin x} \qquad \frac{dy}{dx} = -\frac{\sin y - y\sin x}{x\cos y + \cos x}.$$

예제 0.11.2

$x^2 + y^2 - 1 = 0$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{2x}{2y} = -\frac{x}{y}.$$

예제 0.11.3

$x^3 + y^3 + z^3 + 3xyz = 0$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{3x^2 + 3yz}{3z^2 + 3xy} = -\frac{x^2 + yz}{z^2 + xy} , \quad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{3y^2 + 3xz}{3z^2 + 3xy} = -\frac{y^2 + xz}{z^2 + xy}$$

예제 0.12.1

$f(x) = 2x^3 + 3x^2 - 12x + 7$

$f'(x) = 6x^2 + 6x - 12$

$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$

증감표.

$\cdots$	-2	$\cdots$	1	$\cdots$
$+$		$-$		$+$
$\nearrow$		$\searrow$		$\nearrow$

극대 : $f(-2) = 27$

극소 : $f(1) = 0$.

예제 0.12.2

$f(x) = \sqrt[3]{x^2}$

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

미분 불가.

$\downarrow$

$\cdots$	0	$\cdots$
$-$	?	$+$
$\searrow$		$\nearrow$

극소 : $f(0) = 0$.

예제 0.12.3

$f(x) = xe^{-x}$, $[0, 2]$ $\Rightarrow$ 연속이므로 $f(0)$, $f(2)$, 극값 중 min/max 존재

$f(0) = 0$, $f(2) = 2e^{-2}$

$f'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$

$\cdots$	1	$\cdots$
$+$		$-$
$\nearrow$		$\searrow$

$f(1) = e^{-1} = \frac{e}{e^{-2}}$

min : 0

max : e^{-1}

예제 0.13.1.

$f(x,y) = x^2 + y^2 - 4x - 8y + 10$

$\frac{\partial f}{\partial x} = 2x - 4 \quad \frac{\partial f}{\partial y} = 2y - 8$, $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0 \Rightarrow x=2, y=4$.

$\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = 2 \quad \frac{\partial^2 f}{\partial x \partial y} = 0$

$D = 2 \times 2 - 0^2 > 0$, $\frac{\partial^2 f}{\partial x^2} > 0 \Rightarrow$ 극소.

극소 : $f(2,4) = -10$.

예제 0.13.2

$f(x,y) = y^2 - x^2$

$\frac{\partial f}{\partial x} = -2x \quad \frac{\partial f}{\partial y} = 2y$

$\frac{\partial^2 f}{\partial x^2} = -2 \quad \frac{\partial^2 f}{\partial y^2} = 2 \quad \frac{\partial f}{\partial y \partial x} = 0 \quad D = -4 - 0^2 < 0$.

$\Rightarrow$ 안장점.

예제 0.13.3

$f(x,y) = x^2 + 2y^2$, $x + 2y - 6 = 0$.

$L = x^2 + 2y^2 + \lambda(x + 2y - 6) = 0$

$\frac{\partial L}{\partial x} = 2x + \lambda \Rightarrow \lambda = -2x$

$\frac{\partial L}{\partial y} = 4y + 2\lambda \Rightarrow -4x + 4y = 0$

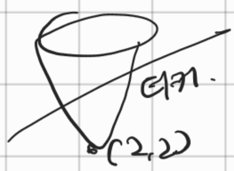
$\frac{\partial L}{\partial \lambda} = x + 2y - 6 \Rightarrow 2x + 4y = 12$

$\rightarrow 6x = 12 \Rightarrow x = 2, y = 2, \lambda = -4$

극값 (극소/대는 몰라) : $f(2,2) = 12$

아무거나 구속 만족 $(0,3)$

$f(0,3) = 18$



대개. $\therefore$ 최소 : $f(2,2) = 12$.

예제 0-13-4

$x^2+y^2=1$, $f(x,y)=x^2+2y^2$

Sol 1) Lagrange multiplier.

let. $L=x^2+2y^2+\lambda(x^2+y^2-1)$

$\frac{\partial L}{\partial x}=2x+2\lambda x$ $\quad \frac{\partial L}{\partial y}=4y+2\lambda y$ $\quad \frac{\partial L}{\partial \lambda}=x^2+y^2-1$

$$\frac{\partial L}{\partial x}=\frac{\partial L}{\partial y}=\frac{\partial L}{\partial \lambda}=0 \Rightarrow \begin{aligned} 2x+2\lambda x &= 0 \Rightarrow 2x(1+\lambda)=0 \\ 4y+2\lambda y &= 0 \Rightarrow 2y(2+\lambda)=0 \\ x^2+y^2-1 &= 0 \end{aligned}$$

i) $x=0$, $y=\pm1$, $\lambda=-2$

$f(0,1)=2$

$f(0,-1)=2$

ii) $x=\pm1$, $y=0$, $\lambda=-1$

$f(1,0)=1$

$f(-1,0)=1$

믿어도 되나 싶은 티스토리 블로그에 따르면; Lagrange Multiplier로 극값을 구했는데

- 2개 이상의 극값이 나옴 ⇒ 둘 중 큰게 max, 작은게 min.
- 1개 의 극값이 나옴 ⇒ constraint를 만족하는 아무거나 넣은 뒤에 그 값보다 [크면 : max / 작으면 : min

Sol 2) Quadratic Form.

$x^2+y^2=1$, $f(x,y) = x^2+2y^2$

$$f(x,y) = \underbrace{(x \quad y)}_{v_1^T} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{v_1}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2)$$

i) max . $\lambda = 2$

$Av_1 = \lambda v_1$

$(\lambda I - A)v_1 = 0$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v_1 = 0$

$v_1 = t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. $t \neq 0$

max : $f(x,y)\Big|_{\substack{x=0 \\ y=1}} = 2$

ii) min $\lambda = 1$

$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} v_1 = 0$

$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t$ $(t \neq 0)$

min: $f(x,y)\Big|_{\substack{x=1 \\ y=0}} = 1$.

예제 0.14.1

(1) $\int x\sin x\,dx$

$$\int x\sin x\,dx = x(-\cos x) + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

(2) $\int xe^{-2x}dx$

$$\int xe^{-2x}dx = x\left(-\frac{1}{2}\right)e^{-2x} + \int \frac{1}{2}e^{-2x}dx$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

(3) $\int e^x\cos x\,dx$

$$\int e^x\cos x\,dx = e^x\sin x - \int e^x\sin x\,dx$$

$$= e^x\sin x - \left(e^x(-\cos x) + \int e^x\cos x\,dx\right)$$

let, $\int e^x\cos x\,dx = T$

$$T = e^x\sin x + e^x\cos x - T$$

$$\therefore T = \int e^x\cos x\,dx = \frac{e^x(\sin x + \cos x)}{2} + C.$$

예제 0.14.2

$$\int \frac{2x+1}{x^2+3x+2}dx = \int \frac{A}{(x+1)} + \frac{B}{(x+2)}dx$$

$$Ax + 2A + Bx + B$$

$$= (A+B)x + (2A+B) = 2x+1, \Rightarrow A=-1, \; B=3$$

$$\int \frac{-1}{x+1}dx + \int \frac{3}{x+2}dx = -\ln|x+1| + 3\ln|x+2| + C$$

예제 0.14.3

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty x^{-\frac{1}{2}}\,dx = \lim_{t\to\infty}(2\sqrt{t}-2) = \infty.$$

그냥 위아래 끝 넣고 하면 이 결과 안나옴.

예제 0.14.4

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 x^{-\frac{1}{2}}\,dx = \lim_{t\to +0}(2-2\sqrt{t}) = 2.$$

예제 0.14.5

$$\int_0^\infty e^{-x}\,dx = \left[-e^{-x}\right]_0^\infty = \lim_{t\to\infty}(-e^{-t}+1) = 1.$$

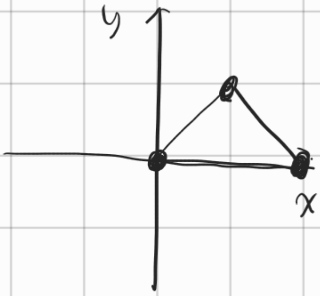
예제 0.15.1

$$\iint_R x+y\,dA = \int_0^1\int_1^2 x+y\,dy\,dx$$

$$= \int_0^1 \left[xy+\frac{1}{2}y^2\right]_1^2 dx = \int_0^1 x+\frac{3}{2}\,dx = \left[\frac{1}{2}x^2+\frac{3}{2}x\right]_0^1$$

$$= 2.$$

예제 0.15.2



$$\int_0^1\int_y^{2-y}(3x+2y)\,dx\,dy$$

$$= \int_0^1 \left[\frac{3}{2}x^2+2xy\right]_y^{2-y} dy$$

$$= \int_0^1 \frac{3}{2}(-4y+4)+2(2-2y)y\,dy$$

$$= \int_0^1 -6y+6+4y-4y^2\,dy = \int_0^1 -4y^2-2y+6\,dy$$

$$\left[-\frac{4}{3}y^3-y^2+6y\right]_0^1$$

$$= 6-1-\frac{4}{3}$$

$$= \frac{11}{3}$$

예제 0.15.3

$y=x, \quad y=x^2$

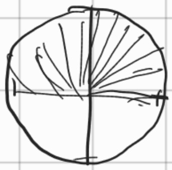
$$\int_0^1 \int_{x^2}^{x} 1 \cdot dy\, dx$$

$$= \int_0^1 x - x^2\, dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

예제 0.15.4

$$\iint_R (x^2+y^2)\, dA$$

$$\iint_R f(x,y)\, dx\, dy = \iint_Q f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$



$$= \iint_Q r^3\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 r^3\, dr\, d\theta = \frac{\pi}{8}.$$

예제 0.15.5.

$$\int_0^1 \int_0^2 \int_0^3 xyz\; dz\, dy\, dx$$

$$= \left[\frac{1}{2}xyz^2\right]_0^3 = \frac{9}{2}xy$$

$$\left[\frac{9}{4}xy^2\right]_0^2$$

$$\left[\frac{9}{2}x^2\right]_0^1 = \frac{9}{2}.$$

예제 0.15.6

$$\iiint_T f(x,y,z)\,dV = \iiint_Q f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\,\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} 1\cdot \rho^2\sin\phi\ d\rho\ d\phi\ d\theta$$

$$= \left[\frac{1}{3}\rho^3\sin\phi\right]_0^a$$

$$\left[-\frac{1}{3}a^2\cos\phi\right]_0^{\pi} = \frac{1}{3}a^2 - -\frac{1}{3}a^2 = \frac{2}{3}a^2$$

$$\left[\frac{2}{3}a^3\theta\right]_0^{2\pi} = \frac{4}{3}\pi a^3.$$

예제 0.17.1

$$|(2,-1,3)| = \sqrt{4+1+9} = \sqrt{14}.$$

예제 0.17.2

$$\mathbf{a}=(1,1,-2)\quad \mathbf{b}=(-3,2,1)$$

$$\mathbf{a}\cdot\mathbf{b} = -3+2-2 = -3$$

예제 0.18.1

$$\begin{pmatrix} 3 & 2 & -1 \\ 0 & 4 & 6 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 5 & 3 & 1 \\ 6 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 2 & 6 \\ 56 & 36 & 16 \end{pmatrix}$$

예제 0.18.2

$$\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 4 - -6 = 10.$$

$$\begin{vmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{vmatrix} = 6 \cdot \begin{vmatrix} 4 & 7 \\ 5 & 3 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} = \overline{6} \cdot -23 \ \overline{+3} \cdot 6$$

$$= +120.$$

예제 0.18.3

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = -\frac{1}{2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

예제 0.18.4

$-2 \ -3.$

$$A = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}, \quad \det(A) = 2 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= 2 + 10 = 12$$

$$\text{adj}\, A = \begin{pmatrix} 1 & 5 & -3 \\ -2 & 2 & 6 \\ +2 & -2 & +6 \end{pmatrix}^T \qquad A^{-1} = \frac{1}{12} \begin{pmatrix} 1 & -2 & +2 \\ 5 & 2 & -2 \\ -3 & 6 & +6 \end{pmatrix}.$$

예제 0.18.5

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 3 & 0 & 0 \end{matrix} = \hat{i}\,(0) + \hat{j}\,(-0) + \hat{k}\,(-3)$$

$$= (0, 0, -3).$$

예제 0.19.1

dir : $(1,1)$

$$\mathbb{r}(t) = t(1,1) + (1,0)$$

예제 0.19.2

$$\mathbb{r}(t) = t(1,1) + (1,0)$$

$$\frac{d\mathbb{r}}{dt} = \frac{d}{dt}(1+t,\, t) = (1,1)$$

예제 0.19.3

$$\mathbb{r}(t) = t(1,3) + (0,0) = t(1,3)$$

$$\frac{d\mathbb{r}}{dt} = (1,3)$$

$$\int_C F(\mathbb{r})\, d\mathbb{r} = \int_0^1 F(t,3t)\cdot(1,3)\, dt$$

내적이 되네!

$$= \int_0^1 (3t,\, t)\cdot(1,3)\, dt$$

$$= \int_0^1 6t\, dt = 3$$