Optimal bound on the quantum Fisher Information

Based on few initial expectation values of the prove state.

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Outline

1. *Many* **inequalities** have been proposed to lower bound the quantum Fisher Information.

Bounds for qFI

$$\mathcal{F}[\varrho, J_z] \geq \frac{\langle J_x \rangle^2}{(\Delta J_y)^2}, \qquad \mathcal{F}[\varrho, J_y] \geq \beta^{-2} \frac{\langle J_x^2 + J_z^2 \rangle}{(\Delta J_z)^2 + \frac{1}{4}},$$

$$\mathcal{F}[\varrho, J_z] \geq \frac{4(\langle J_x^2 + J_y^2 \rangle)^2}{2\sqrt{(\Delta J_x^2)^2 (\Delta J_y^2)^2} + \langle J_x^2 \rangle - 2\langle J_y^2 \rangle (1 + \langle J_x^2 \rangle) + 6}$$

[**I.A.**, B. Lücke, J. Peise, C. Klempt & G. Toth, New J. Phys. **17**, 083027 (2015)]
[L. Pezzé & A. Smerzi, Phys. Rev. Lett. **102**, 100401 (2009)]
[Z. Zhang & L.-M. Duan, 2014 New J. Phys. **16** 103037 (2014)]

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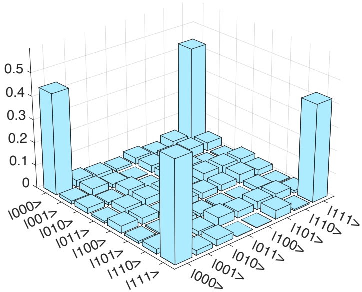
2. Typically, we only have *a couple of expectation values* to characterize the state.



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2. Typically, we only have *a couple of expectation values* to characterize the state.
3. The archetypical criteria that demonstrates *useful entanglement* on the state.
4. It is essential either to *verify them or find new ones* for different set of expectation values.

The non-trivial exercise of computing the qFI

1. Different forms of the qFI

$$\mathcal{F}[\varrho, J_z] = 2\sum_{\lambda,\gamma} \frac{(p_\lambda - p_\gamma)^2}{p_\lambda + p_\gamma} |\langle\lambda|J_z|\gamma\rangle|^2$$

$$\mathcal{F}[\varrho, J_z] = \min_{\{p_k,|\Psi_k\rangle\}} 4\sum_k p_k \left(\Delta J_z\right)^2_{|\Psi_k\rangle}$$

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3. In the general case, usually *lower bounded* by its *"classical"* counterparts.

Optimization: Legendre Transform

- For a convex function of the state, we construct a *thight lower bound* as follows,

$$g(\varrho) \geq \mathcal{B}\big(\{w_k := \langle W_k \rangle\}\big) = \sup_{\{r_k\}} \big(r \cdot w - \sup_{\varrho} [r \cdot \langle W \rangle - g(\varrho)]\big).$$

- When $g(\varrho)$ is deffined as infimum over the convex roof, the 2^{nd} optimization simplified to pure states only,

$$\mathcal{B}(\{w_k\}) = \sup_{\{r_k\}} \big(r \cdot w - \sup_{|\psi\rangle} [r \cdot \langle W \rangle - g(|\psi\rangle)]\big).$$

[O. Gühne, M. Reimpell, and R.F. Werner, Phys. Rev. Lett. **98**, 110502 (2007)]

Optimization for the qFI

Different because of *simplicity of the qFI for pure states*.

$$\mathcal{F}(\{w_k\}) = \sup_{\{r_k\}} \Big(r \cdot w - \sup_{\mu} [\lambda_{\max}(r \cdot W - 4(J_z - \mu)^2)] \Big).$$

- Therefore, we have parametrised the optimization, which leads to a *more efficient finding* of the solution.

[**I.A.**, M. Kleinmann, O. Güne & G. Tóth, arXiv:1511.05203]

Measuring $\langle J_z \rangle$ and $(\Delta J_x)^2$ for Spin Squeezed States

1. We use the following 3 operators $\{J_z, J_x, J_x^2\}$ to characterize the input state with their respective expectation values.
2. In the direction of $\langle J_x \rangle$ the worst case is it take the value zero
3. Therefore the optimisation can be done only for 2 operators $\{J_z, J_x^2\}$ and it can be mapep directly to $\langle J_z \rangle, (\Delta J_x)^2$.

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