Optimal bound on the quantum Fisher Information

Based on few initial expectation values of the prove state.

Iagoba Apellaniz [1], Matthias Kleinmann [1], Otfried Ghüne [2], & Géza Tóth [1,3,4]

iagoba.apellaniz@gmail.com

[1]Department of Theoretical Physics, University of the Basque Country, Spain
[2]Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Germany
[3]IKERBASQUE, Basque Foundation for Science, Spain
[4]Wigner Research Centre for Physics, Hungarian Academy of Sciences, Hungary

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Outline

1. *Many* **inequalities** have been proposed to lower bound the quantum Fisher Information.

Bounds for qFI

$$\mathcal{F}[\varrho, J_z] \geq \frac{\langle J_x \rangle^2}{(\Delta J_y)^2}, \qquad \mathcal{F}[\varrho, J_y] \geq \beta^{-2} \frac{\langle J_x^2 + J_z^2 \rangle}{(\Delta J_z)^2 + \frac{1}{4}},$$

$$\mathcal{F}[\varrho, J_z] \geq \frac{4(\langle J_x^2 + J_y^2 \rangle)^2}{2\sqrt{(\Delta J_x^2)^2 \left(\Delta J_y^2\right)^2} + \langle J_x^2 \rangle - 2\langle J_y^2 \rangle (1 + \langle J_x^2 \rangle) + 6}$$

[**I.A.**, B. Lücke, J. Peise, C. Klempt & G. Toth, New J. Phys. **17**, 083027 (2015)]
[L. Pezzé & A. Smerzi, Phys. Rev. Lett. **102**, 100401 (2009)]
[Z. Zhang & L.-M. Duan, 2014 New J. Phys. **16** 103037 (2014)]

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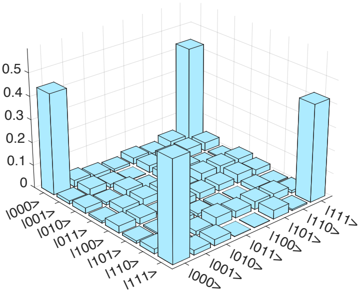
2. Typically, we only have *a couple of expectation values* to characterize the state.



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2. Typically, we only have *a couple of expectation values* to characterize the state.
3. The archetypical criteria that demonstrates *useful entanglement* on the state.
4. It is essential either to *verify them or find new ones* for different set of expectation values.

1. Introduction and Motivation
2. QFI based on expectation values: Are they optimal?
   - Optimization problem
3. Case study
   - Spin squeezed states
   - Unpolarized Dicke states
4. Conclusion and outlook

The non trivial exercise of computing the qFI

1. Different forms of the qFI

$$\mathcal{F}[\varrho, J_z] = 2\sum_{\lambda,\gamma} \frac{(p_\lambda - p_\gamma)^2}{p_\lambda + p_\gamma} |\langle\lambda|J_z|\gamma\rangle|^2$$

$$\mathcal{F}[\varrho, J_z] = \min_{\{p_k, |\Psi_k\rangle\}} 4\sum_k p_k \left(\Delta J_z\right)^2_{|\Psi_k\rangle}$$

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3. In the general case, usually *lower bounded* by its *"classical"* counterparts.

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