Optimal bound on the quantum Fisher Information

Based on few initial expectation values of the prove state.

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Outline

1. *Many* **inequalities** have been proposed to lower bound the quantum Fisher Information.

Bounds for qFI

$$\mathcal{F}[\varrho, J_z] \geq \frac{\langle J_x \rangle^2}{(\Delta J_y)^2}, \qquad \mathcal{F}[\varrho, J_y] \geq \beta^{-2} \frac{\langle J_x^2 + J_z^2 \rangle}{(\Delta J_z)^2 + \frac{1}{4}},$$

$$\mathcal{F}[\varrho, J_z] \geq \frac{4(\langle J_x^2 + J_y^2 \rangle)^2}{2\sqrt{(\Delta J_x^2)^2 \, (\Delta J_y^2)^2} + \langle J_x^2 \rangle - 2\langle J_y^2 \rangle (1 + \langle J_x^2 \rangle) + 6}$$

[**I.A.**, B. Lücke, J. Peise, C. Klempt & G. Toth, New J. Phys. **17**, 083027 (2015)]
[L. Pezzé & A. Smerzi, Phys. Rev. Lett. **102**, 100401 (2009)]
[Z. Zhang & L.-M. Duan, 2014 New J. Phys. **16** 103037 (2014)]

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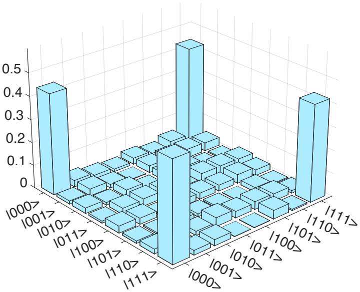
2. Typically, we only have *a couple of expectation values* to characterize the state.



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2. Typically, we only have *a couple of expectation values* to characterize the state.
3. The archetypical criteria that demonstrates *useful entanglement* on the state.
4. It is essential either to *verify them or find new ones* for different set of expectation values.

The non-trivial exercise of computing the qFI

1. Different forms of the qFI

$$\mathcal{F}[\varrho, J_z] = 2\sum_{\lambda,\gamma} \frac{(p_\lambda - p_\gamma)^2}{p_\lambda + p_\gamma} |\langle\lambda|J_z|\gamma\rangle|^2$$

$$\mathcal{F}[\varrho, J_z] = \min_{\{p_k, |\Psi_k\rangle\}} 4\sum_k p_k \left(\Delta J_z\right)^2_{|\Psi_k\rangle}$$

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3. In the general case, usually *lower bounded* by its *"classical"* counterparts.

Optimization: Legendre Transform

- For a convex function of the state, we construct a *thight lower bound* as follows,

$$g(\varrho) \geq \mathcal{B}\big(\{w_k := \langle W_k \rangle\}\big) = \sup_{\{r_k\}} \big(r \cdot w - \sup_{\varrho}[r \cdot \langle W \rangle - g(\varrho)]\big).$$

- When $g(\varrho)$ is deffined as infimum over the convex roof, the $2^{\rm nd}$ optimization simplified to pure states only,

$$\mathcal{B}(\{w_k\}) = \sup_{\{r_k\}} \big(r \cdot w - \sup_{|\psi\rangle}[r \cdot \langle W \rangle - g(|\psi\rangle)]\big).$$

[O. Gühne, M. Reimpell, and R.F. Werner, Phys. Rev. Lett. **98**, 110502 (2007)]

Optimization for the qFI

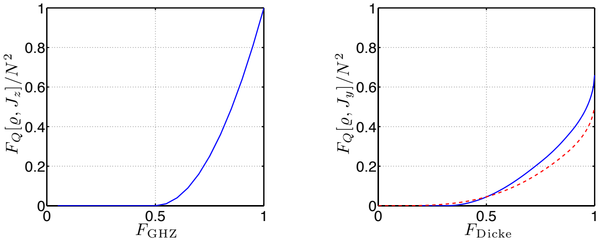
Different because of *simplicity of the qFI for pure states*.

$$\mathcal{F}(\{w_k\}) = \sup_{\{r_k\}} \left(r \cdot w - \sup_{\mu}[\lambda_{\max}(r \cdot W - 4(J_z - \mu)^2)]\right).$$

- Therefore, we have parametrised the optimization, which leads to a *more efficient finding* of the solution.

[**I.A.**, M. Kleinmann, O. Güne & G. Tóth, arXiv:1511.05203]

- We'll present 2 main cases, spin-squeezed states and unpolarized Dicke.
- Though, we apply our method to projectors with great success, we will focus on global $J_{\mathbf{n}}$ momentums.
- One of the cases using projector operators, *i.e.*, using the *fidelity* leads to *analytic soulution*!



Measuring $\langle J_z \rangle$ and $(\Delta J_x)^2$ for Spin Squeezed States

1. We use the following 3 operators $\{J_z, J_x, J_x^2\}$ to characterize the input state with their respective expectation values.
2. In the direction of $\langle J_x \rangle$ the worst case is when it takes the *value zero*.
3. Therefore, the optimisation can be accomplished only for 2 operators $\{J_z, J_x^2\}$ while it is mapped directly to $\langle J_z \rangle, (\Delta J_x)^2$.

- We numerically optimise the lower bound of qFI for a 4 particle system for all possible values of $\langle J_z \rangle$ and $(\Delta J_x)^2$.

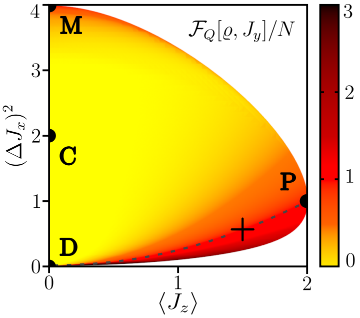
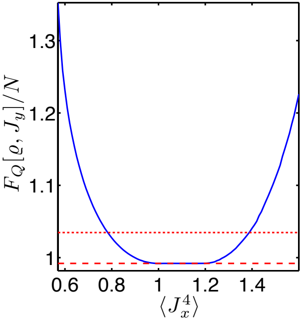


Figure: Below the dashed line, $\mathcal{F}$ surpasses the shot noise limit. The crossed point to be extended next. It shows an extremely similarity with $\mathcal{F} \geq \langle J_z \rangle^2 / (\Delta J_x)^2$, TODO

- For the crossed point, we now study what happens with the extra information of a 3$^{\mathrm{rd}}$ expectation value.



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