

Please use PENCIL and erase mistakes. Closed books and notes, except for the 'cheat sheet' I posted on my Web site. Write your name on it (and NOTHING else), and turn in the cheat sheet with your exam. No electronic devices of any kind. For arithmetic answers, show your work and please put a box around your answer so I can tell it from your scratch work. I will not give full credit without showing work. Turn your cell phone OFF and put it away where you can't easily get to it. If your phone makes any noise whatever, I will take 5 points off your exam score. 100 total points.

1. What are the two primary pieces of information that the base of a number system gives you?
Illustrate both parts of your answer using base 12. (4 points)

1. It tells you about the set of digits and its cardinality.

For base 12: $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B\}$

$$|S| = 12$$

2. It tells you the place values for any number expressed in that base, which are always integer powers of '6'.

For base 12: 4A5 has place values $12^2 \ 12^1 \ 12^0$

2. Convert the following numbers from the indicated bases to decimal. (2 points each)

a. $\begin{array}{r} 210 \\ - 2547 \\ \hline 137 \end{array}$ dec = $(2 \times 7^2) + (5 \times 7^1) + (4 \times 7^0)$
 $= (2 \times 49) + (35) + 4 = 98 + 35 + 4$

$$\begin{array}{r} 98 \\ 35 \\ \hline 137 \end{array}$$

b. $\begin{array}{r} 218 \\ - 1A2_{12} \\ \hline 266 \end{array}$ dec = $(1 \times 12^2) + (10 \times 12^1) + (2 \times 12^0)$
 $= 144 + 120 + 2$

$$\begin{array}{r} 144 \\ 120 \\ \hline 266 \end{array}$$

c. $\begin{array}{r} 210 \\ - 2516 \\ \hline 103 \end{array}$ dec = $(2 \times 6^2) + (5 \times 6^1) + (1 \times 6^0)$
 $= 72 + 30 + 1$

$$\begin{array}{r} 72 \\ 30 \\ \hline 103 \end{array}$$

d. $\begin{array}{r} 210 \\ - 1234 \\ \hline 27 \end{array}$ dec = $(1 \times 4^2) + (2 \times 4^1) + (3 \times 4^0)$
 $= 16 + 8 + 3$

$$\begin{array}{r} 16 \\ 8 \\ 3 \\ \hline 27 \end{array}$$

e. $\begin{array}{r} 10 \\ A8_{15} \end{array}$ dec = $(10 \times 15^1) + (8 \times 15^0) =$

$\begin{array}{r} 150 \\ + 8 \\ \hline 158 \end{array}$ = 158

- 8
3. Using the repeated division method, convert the following decimal numbers to unsigned 8-bit binary.
(2 points each)

4

a.	146	0	1001 0010
	73	1	
	36	0	
	18	0	
	9	1	
	4	0	
	2	0	
	1	1	
	0		
b.	69	1	0100 0101
	34	0	
	17	1	
	8	0	
	4	0	
	2	0	
	1	1	
	0		zero on front

4. Using the double-and-add method, convert the following 8-bit unsigned binary numbers to decimal.
(2 points each)

a. 01011011
1, 2, 5, 11, 22, 45, 91

4
b. 11011010
1, 3, 6, 13, 27, 54, 109, 218

- 12
5. Using the place and digit values, convert the following 8-bit unsigned binary numbers to decimal. (2 points each)

a. 11110110

$$\begin{array}{r}
 128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1 \\
 \hline
 1 \cdot \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \\
 246 \checkmark \\
 128 + 64 + 32 + 16 + 6 = 128 + 118 \\
 \hline
 246
 \end{array}$$

b. 00111010

$$\begin{array}{r}
 128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1 \\
 \hline
 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \\
 32 + 16 + 10 = 58 \checkmark
 \end{array}$$

- 4
6. Convert the following 32-bit binary numbers to hexadecimal. (2 points each)

a. 1001010111000110000111101110010
9 5 C 6 0 F 7 2

0x95C60F72

b. 11100001101010110011010011011000
E 1 A B 3 4 D 8

0xE1AB34D8

- 4
7. Convert the following 32-bit binary numbers to octal. (2 points each) why the ()?

a. 1001010111000110000111101110010
2 2 5 6 1 4 0 7 5 6 2

(22561407562)₈

b. 11100001101010110011010011011000
3 4 1 5 2 6 3 2 3 3 0

(34152632330)₈

12 8. Convert the following hexadecimal numbers to binary. Leave a gap between nibbles. (2 points each)

a. 0xD21A7FE8

1101 0010 0001 1010 0111 1111 1110 1000

b. 0xB03C6459

1011 0000 0011 1100 0110 0100 0101 1001

9. Add the following unsigned binary numbers. For each, tell me if overflow occurs. (2 points each)

$$\begin{array}{r} 111111 \\ 010111 \\ +00111011 \\ \hline 10011001 \end{array}$$

1001 1001

$$64 + 16 + 14 = 94$$

$$32 + 16 + 11 = 59$$

$$128 + 16 + 9 = 153$$

No overflow

$$\begin{array}{r} 111111 \\ 111011 \\ +00111100 \\ \hline 100101001 \end{array}$$

Because a 1 carried out at the MSB, there is overflow.

0010 1001

just "yes" worked too?

10. Subtract (by borrowing as needed) the following unsigned binary numbers. You may NOT convert using one's or two's complement. (2 points each)

$$\begin{array}{r} 00101101 \\ -00001010 \\ \hline 00011101 \end{array}$$

$$\begin{array}{r} 32 + 7 = 39 \\ 10 - 10 \\ \hline 29 \end{array}$$

$$16 + 13 = 29$$

0001 1101

$$\begin{array}{r} 01100100 \\ -01011111 \\ \hline 01001110 \end{array}$$

$$\begin{array}{r} 64 + 32 + 16 + 13 = 112 + 13 \\ 32 + 15 = 47 \\ \hline 111 = 125 \\ - 47 \\ \hline 78 \end{array}$$

$$64 + 14 = 78$$

0100 1110

14
 11. Subtract the following 8-bit binary numbers by converting using two's complement and adding. For each, tell me if overflow occurs (3 points each)

a. $\begin{array}{r} 01110010 \\ -00101111 \\ \hline \end{array}$

$$\begin{array}{r} 111' \\ 11010001 \\ + 01110010 \\ \hline 101000011 \end{array}$$

$$\begin{array}{r} 64 + 32 + 16 + 2 = 114 \\ 32 + 15 = 47 \end{array}$$

$$\begin{array}{r} 0101 \\ -47 \\ \hline 67 \end{array}$$

$$\boxed{0100_0011}$$

(NO) overflow.

b. $\begin{array}{r} 01001110 \\ -01111011 \\ \hline \end{array}$

$$\begin{array}{r} 1000'0101 \\ + 01001110 \\ \hline 11010011 \end{array}$$

$$\boxed{1101\ 0011}$$

$$64 + 14 = 78$$

$$64 + 32 + 16 + 11 = 112 + 11 = 123$$

$$\begin{array}{r} 011 \\ 0+23 \\ -78 \\ \hline 45 \end{array}$$

$$\begin{array}{r} -128 + 64 + 16 + 3 \\ -128 + 83 \end{array}$$

$$\begin{array}{r} 0128 \\ -83 \\ \hline 45 \end{array}$$

12. Convert the following 8-bit 2's complement binary numbers to decimal. (2 points each)

a. 10000000

$$\boxed{-128}$$

b. 11111111

$$\boxed{-1}$$

$$\begin{array}{r} -128 + 64 + 32 + 16 + 15 = -128 + 112 + 15 \\ -128 + 127 \end{array}$$

c. 10101011

$$\boxed{-85}$$

$$-128 + 32 + 11 = -128 + 43$$

$$\begin{array}{r} 0128 \\ -43 \\ \hline 85 \end{array}$$

d. 00101111

$$32 + 15 = 47$$

$$\boxed{47}$$

14 13. Sign extend the following hexadecimal numbers to 32 bits (write them in hexadecimal) (1 point each):

a. 0x6F 8 bits

0x0000006F

b. 0x8C 8 bits

0x FFFFFFF8C

14. Zero extend the following hexadecimal numbers to 32 bits (write them in hexadecimal) (1 point each):

a. 0xF3 8 bits

0x 000000F3

b. 0x2B 8 bits

0x 0000002B

15. Convert the following decimal values to the indicated bases (2 points each):

$$\text{a. } 73 = \frac{243}{5} = \frac{67}{11} = \frac{124}{7} \quad \boxed{133}$$

$\begin{array}{r} 73 \\ | \\ 14 \\ | \\ 2 \end{array}$
 $\begin{array}{r} 73 \\ | \\ 6 \\ | \\ 6 \end{array}$
 $\begin{array}{r} 73 \\ | \\ 9 \\ | \\ 1 \end{array}$

4, 3, 2, 1

$$49 + 14 + 10 = 73$$

10

$$\text{b. } 56 = \frac{62}{9} = \frac{2002}{3} = \frac{44}{13}$$

$\begin{array}{r} 56 \\ | \\ 6 \end{array}$
 $\begin{array}{r} 56 \\ | \\ 18 \\ | \\ 6 \\ | \\ 2 \end{array}$
 $\begin{array}{r} 56 \\ | \\ 4 \\ | \\ 4 \end{array}$

- 9 16. Assume that a 32-bit word held in \$t0 has the following fields (9 points):

31	30	- - - - -	21	20	- - -	15	14	- - -	10	9	- - -	4	3	- 1	0
A	B	10	C	6	D	5	E	6	F	3	G				

So, field A is one bit wide in bit 31 (the high-order bit), and field E is 6 bits wide in bits 9 – 4, etc. Each of the following can be done in two or three instructions.

- a. Write MIPS instructions to convert field B into an unsigned 32-bit value in register \$t0. (3 points)

3 sll \$t0, \$t0, 1.

3 srl \$t0, \$t0, 22.

- b. Write MIPS instructions to test the bit in field G and branch to LABEL if it is 1. (3 points)

3 andi \$t1, \$t0, 1.

3 bne \$t1, \$zero, LABEL

- c. Write MIPS instructions to convert field C into a two's-complement signed value in register \$t0. Assume field C is signed two's-complement already. (3 points)

3 sll \$t0, \$t0, 11.

3 sra \$t0, \$t0, 26

$$0x69 = 0110\ 1001$$

17. Write a complete MIPS program to put the following bit pattern into register \$t0: 0x69696969.

Start by putting a 1 in the low order bit of register \$t0. Using only bitwise logical operations on that 1 bit, and on bits derived from that 1 bit, build the value for register \$t1. (Fewer than 15 instructions) (15 points)

.text

.globl main

main:

ori \$t0, \$zero, 1

sll \$t1, \$t0, 1

or \$t2, \$t0, \$t1

sll \$t2, \$t2, 1

sll \$t1, \$t0, 3

or \$t1, \$t0, \$t1

sll \$t2, \$t2, 4

or \$t1, \$t1, \$t2

sll \$t2, \$t1, 8

or \$t1, \$t1, \$t2

sll \$t2, \$t1, 16

or \$t1, \$t1, \$t2

\$t0 = 0001

\$t1 = 0010

\$t2 = 0011

\$t2 = 0110

\$t1 = 1000

\$t1 = 1001

\$t2 = 0110 0000

\$t1 = 0110 1001

\$t2 = 0x6900

\$t1 = 0x6969

\$t2 = 0x69690000

\$t1 = 0x69696969

Syscall or/ code 10?