

# Alogrithm exercise

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## 1 ex1

ex1.c in ex director

## 2 ex2

ex2.c in ex director

## 3 ex3

ex3.c in ex director

## 4 ex4

### Question:

Assume  $\epsilon, \delta$  is constant within  $(0, 1)$ , prove:

If  $I$  is the correct value of  $\int_0^1 f(x)dx$ , and  $h$  is the return value of alogrithm "HitorMiss", Then  $Prob[|h - I| < \epsilon] \geq 1 - \delta$  when  $n \geq I(1 - I)/\epsilon^2\delta$ .

### Prove:

Assume we hit the area  $n$  times and there are  $k$  points scatterd under the area  $f(x)$ . Apparently, the random  $X$  that the number of points scattered under  $f(x)$  is Binomial Distribution, that is  $Pr(X = k) = C_n^k p^k (1 - p)^{n-k}$  then,  $E(X) = np, Var(X) = np(1 - p)$ .

the  $h$  that HitOrMiss return is  $k/n$ , So  $h = X/n, E(h) = E(X/n) = p = k/n = I, Var(h) = Var(X/n) = \frac{p(1-p)}{n} = \delta^2$ .

According to Chebyshev's inequality  $Pr(|h - I| \leq \epsilon) \geq 1 - \delta^2/\epsilon^2$  And according to the cond:  $n \geq I(1 - I)/\epsilon^2\delta$ , So we have  $Pr(|h - I| \leq \epsilon) \geq 1 - \frac{p(1-p)}{\epsilon^2 n} \geq 1 - \frac{p(1-p)}{\epsilon^2} \frac{\epsilon^2 \delta}{I(1-I)} = 1 - \delta$ .

Q.E.D

## 5 ex5

ex5.py in ex director

## 6 ex6

问题： 写一Sherwood算法C 与算法A, B, D比较, 并给出实验结果。

答案：

程序ex6.c 及结果 见 ex 文件夹。

## 7 ex7

问题： 证明当放置第k+1个皇后时, 若有多多个位置是开放的, 则算法QueensLV选中其中一位置的概率相等。

证： 假设在放置k+1个皇后时, 有nb个位置可以放置, 并假设选中了第i个位置, 则其概率 $p(i) = \frac{1}{i} \times \frac{i}{i+1} \times \cdots \times \frac{nb-1}{nb} = \frac{1}{nb}$  即选中其中一个位置的概率均相等, 为 $\frac{1}{nb}$ 。

## 8 ex8

写一算法, 求n=12-20时最优的StepVegas值。

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**Algorithm 1** 求StepVegas最优值

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1: procedure OPTIMALSTEPVEGAS
2:   mintime  $\leftarrow \infty$ 
3:   k  $\leftarrow 0$ 
4:   for from n = 12 to 20 do
5:     for from k = 0 to 20 do
6:       time  $\leftarrow \text{QueensLv}(n, \text{success}, k)$ 
7:       if time < mintime then
8:         mintime  $\leftarrow \text{time}$ 
9:         stepVegas  $\leftarrow k$ 
10:    Print TheCountofQueens : n, OptimalStepVegasValueis :
        stepVegas
```

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