DSC510: Introduction to Data Science and Analytics
Lab 11: Introduction to Timeseries

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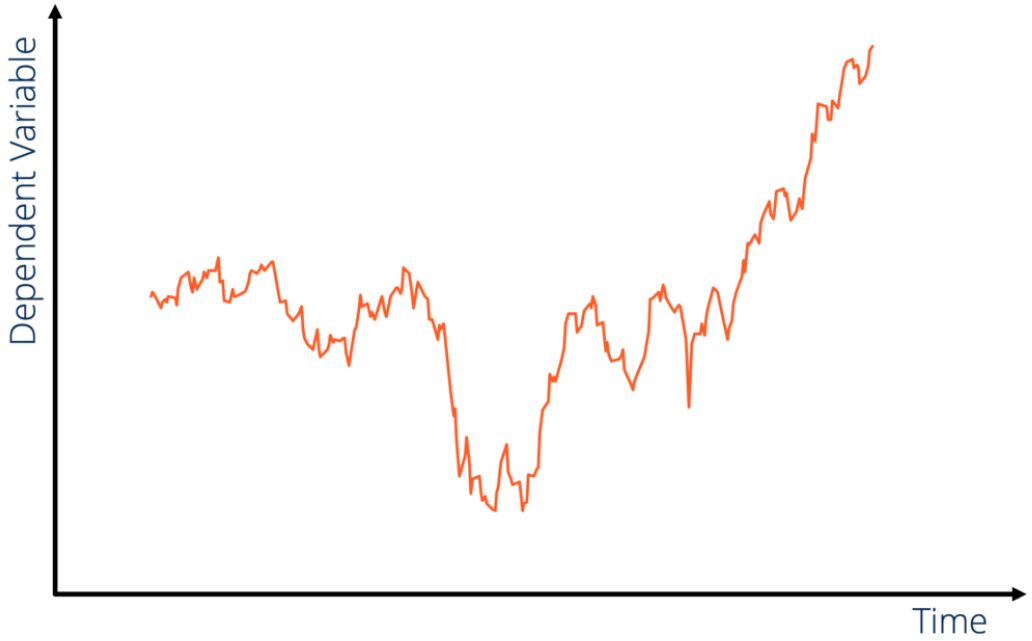
** More detailed information on timeseries can be found here (L14, L15) **

Introduction to time series

- A collection of observations for a time-dependent variable at various time intervals is known as time series

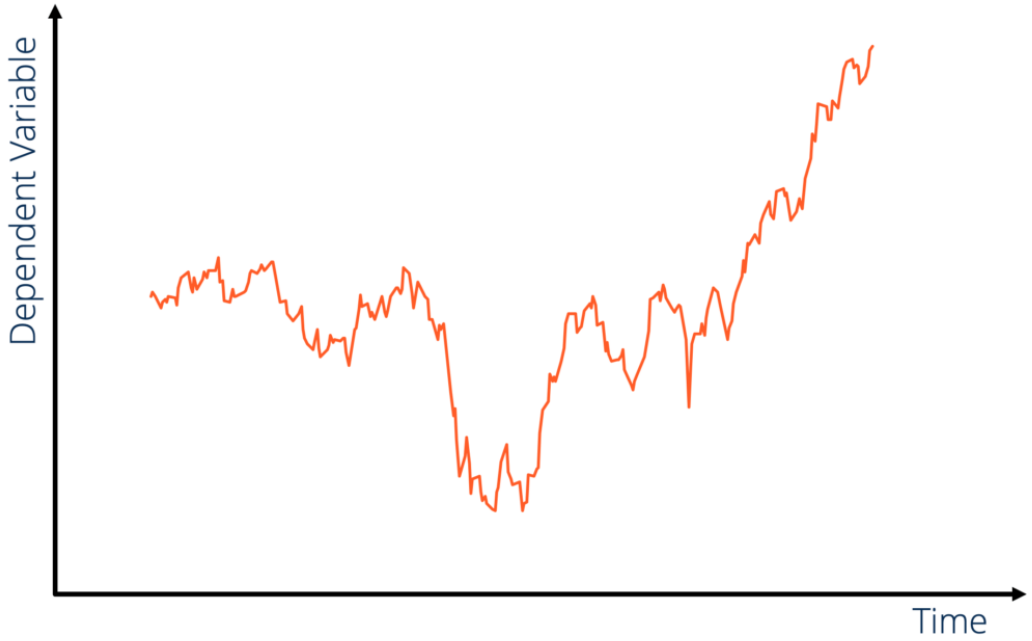


2019-01-01 00:00:00	49
2019-01-01 00:01:00	58
2019-01-01 00:02:00	48
2019-01-01 00:03:00	96
2019-01-01 00:04:00	42
2019-01-01 00:05:00	8
2019-01-01 00:06:00	20
2019-01-01 00:07:00	96
2019-01-01 00:08:00	48
2019-01-01 00:09:00	78

- Time series data is omnipresent in our lives as it can be generated in pretty much any domain
  - Sensors monitoring weather conditions, PV inverters measuring generated solar energy, stock price variations in the stock market

Introduction to time series

- A collection of observations for a time-dependent variable at various time intervals is known as time series



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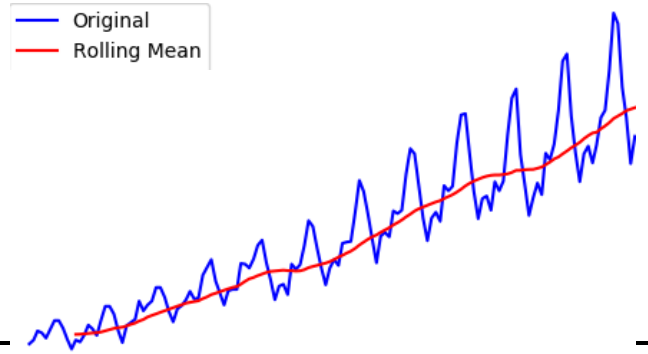
- Pandas DataFrames organize time series data by using date/time based indexing (row labels)

Introduction to time series

- Many time series are uniformly (equally) spaced at a specific frequency
  - For example: hourly weather measurements, daily counts of web site visits, or monthly sales totals
- Time series can also be irregularly (unequally) spaced and sporadic (bursts)
  - For example: timestamped data in a computer system's event log or a history of ambulance emergency calls
- Pandas time series tools apply well to either type of time series

Resampling, Shifting, and Windowing

- Important time series operations available by Pandas DataFrames:
  - Resampling: converting the time series from one frequency to another
    - e.g. convert hourly data (one sample per hour) to daily data (one sample per day) or vice versa
  - Time-shifting: going forward and backward over time
    - Can be performed by shifting the datetime-based index
  - Moving window functions (rolling statistics)
    - e.g. calculate moving (rolling) mean values taking into account the last ten samples
    - used to smooth data and capture the trend (uptrend, downtrend)



Dataset for practice

- Load Google stock price dataset as DataFrame:

```
import pandas as pd
goog = pd.read_csv("google_stocks_2005_2021.csv", index_col="Date",
                   parse_dates=True)
print(type(goog.index)) # <class 'pandas.core.indexes.datetimes.DatetimeIndex'>
goog.head()
```

If True, Pandas tries to parse the index to examine if can be represented as an array of datetimes. If not set, dates will be loaded as strings and timeseries operations cannot be applied.

Many TS operations require date-oriented index

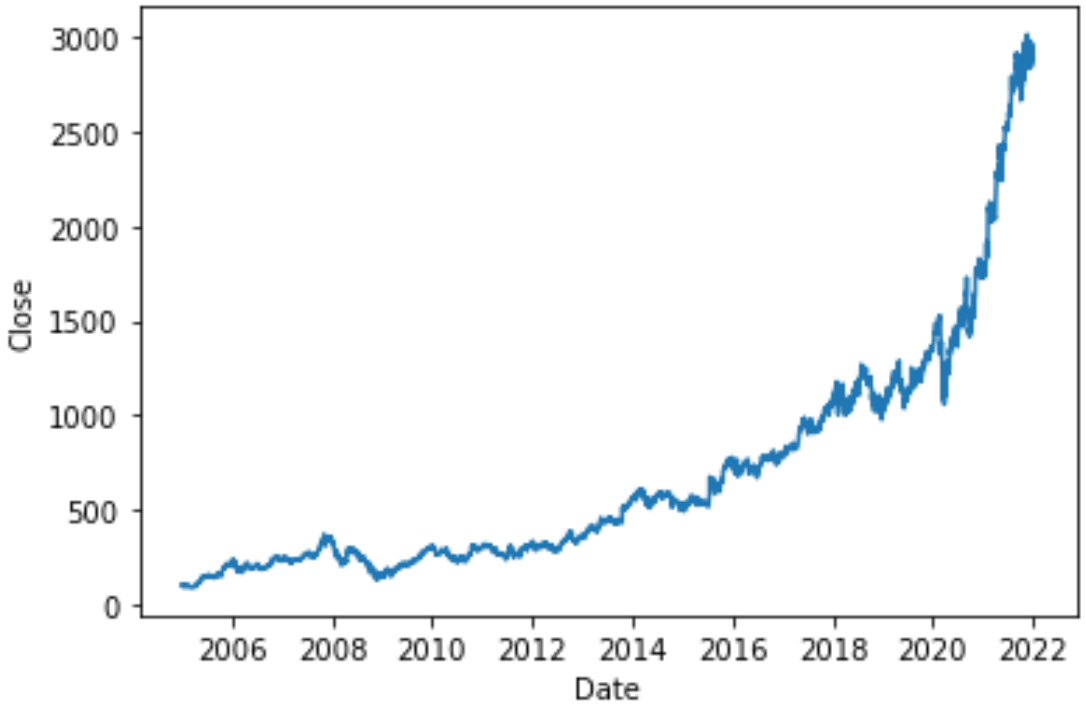
Date	High	Low	Open	Close	Volume	Adj Close
2005-01-03	101.439781	97.365051	98.331429	100.976517	31807176.0	100.976517
2005-01-04	101.086105	96.378746	100.323959	96.886841	27614921.0	96.886841
2005-01-05	98.082359	95.756081	96.363808	96.393692	16534946.0	96.393692
2005-01-06	97.584229	93.509506	97.175758	93.922951	20852067.0	93.922951
2005-01-07	96.762314	94.037521	94.964050	96.563057	19398238.0	96.563057

Only business days are included (no weekends or public holidays) in dataset

Plotting time series

- For simplicity, we'll use just the closing price
- We can visualize this using the Seaborn `lineplot()` function

```
import matplotlib.pyplot as plt
import seaborn as sns
fig, ax = plt.subplots()
sns.lineplot(data=goog, x=goog.index, y='Close')
```



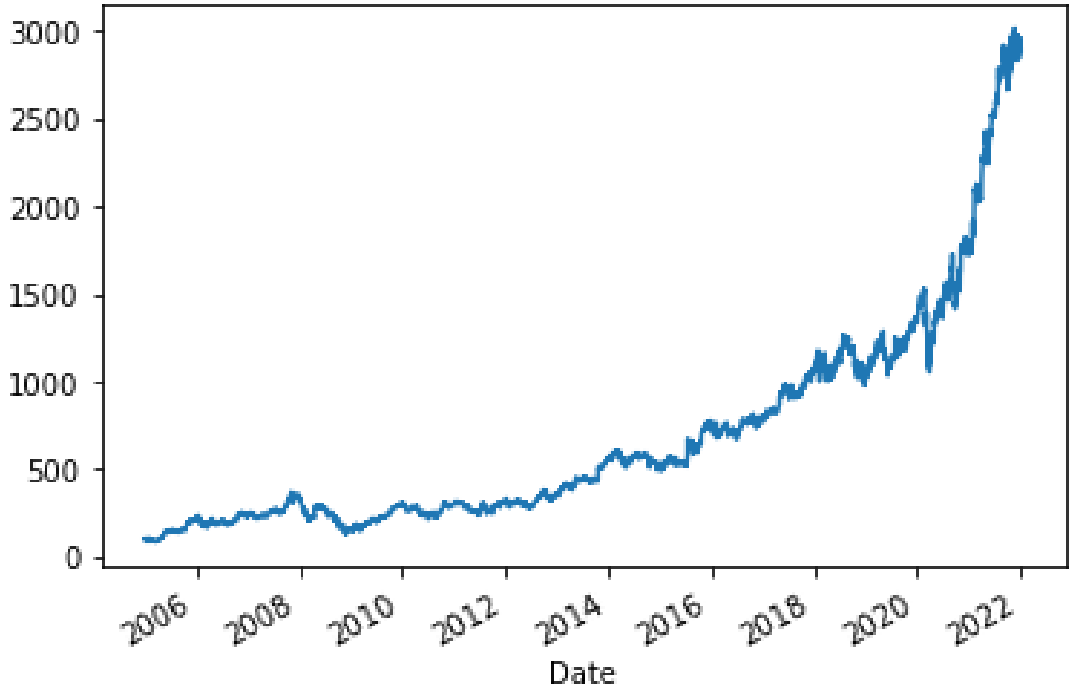
Plotting time series (alternative)

- Alternatively, we can extract the Close column as a Series object

```
goog = goog['Close']
```

- Plot the series using the `.plot()` function on the Series object

```
import matplotlib.pyplot as plt
fig, ax = plt.subplots()
goog.plot()
```



Resampling data

- Resampling is the process of changing the frequency of time-dependent (timeseries) features

- Two types of resampling are:
  - **Down-sampling**: When you reduce the frequency of data (lower granularity) by aggregating over a longer time period
    - for example, converting hourly data to daily data by averaging or summing the values
  - **Up-sampling**: When you increase the frequency of data (higher granularity) by interpolating or filling in additional data points
    - for instance, converting daily data to hourly data by estimating values between days

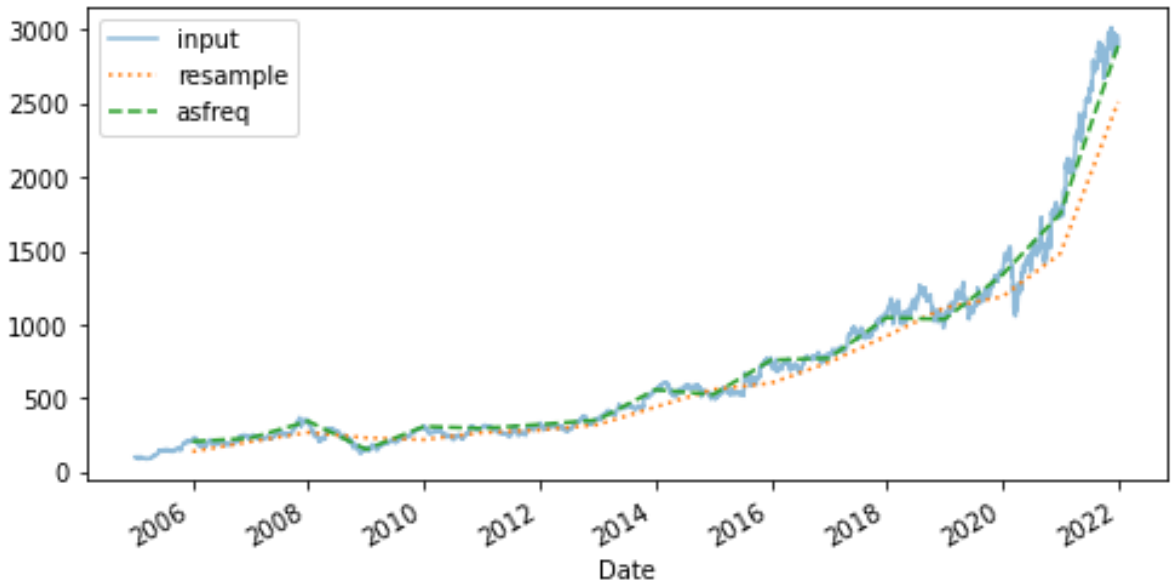
Resampling data

- We can resample a time series at a higher or lower frequency:
  - to inspect how dataset behaves differently under different resolutions or frequency
    - For example, you have a feature measured on a daily basis and you want to make predictions on a monthly basis => you need to downsample it to a monthly level
  - to join datasets with initially different resolutions and obtain a larger dataset
  - there is an extremely high number of observations that needs to be reduced to speedup both EDA and ML algorithms execution time
    - Need for down-sampling
- Resample functions: `resample()`, `asfreq()`
  - `resample()` is fundamentally a data aggregation, while `asfreq()` is fundamentally a data selection
  - The dataset to be resampled must have a datetime-like index

Down-sampling

- Down-sample (lower freq) Google closing price using both functions
- Example: we resample the data at the end of business year:

```
fig, ax = plt.subplots(figsize=(8,4))
goog.plot(alpha=0.5, style='-')
goog.resample('BYE').mean().plot(style=':')
goog.asfreq('BYE').plot(style='--');
plt.legend(['input', 'resample', 'asfreq'], loc='upper left')
```



Notice the difference: at each point, resample() reports the average of the previous year, while asfreq() reports (selects) the value at the end of the year.

resample() is like a groupby function and should be used in conjunction with other aggregation functions such as sum, median, etc.

Frequencies and offsets

- We can use frequency codes to specify any desired frequency spacing. The following table summarizes the main codes available:

Code	Description	Code	Description
D	Calendar day	B	Business day
W	Weekly		
ME	Month end	BME	Business month end
QE	Quarter end	BQE	Business quarter end
YE	Year end	BYE	Business year end
h	Hours	bh	Business hours
min	Minutes		
s	Seconds		
ms	Milliseconds		
us	Microseconds		
ns	nanoseconds		

The monthly, quarterly, and annual frequencies are all marked at the end of the specified period. By adding an S suffix to any of these, they instead will be marked at the beginning:

MS	Month start	BMS	Business month start
QS	Quarter start	BQS	Business quarter start
YS	Year start	BYS	Business year start

https://pandas.pydata.org/pandas-docs/stable/user_guide/timeseries.html#offset-aliases

Up-sampling

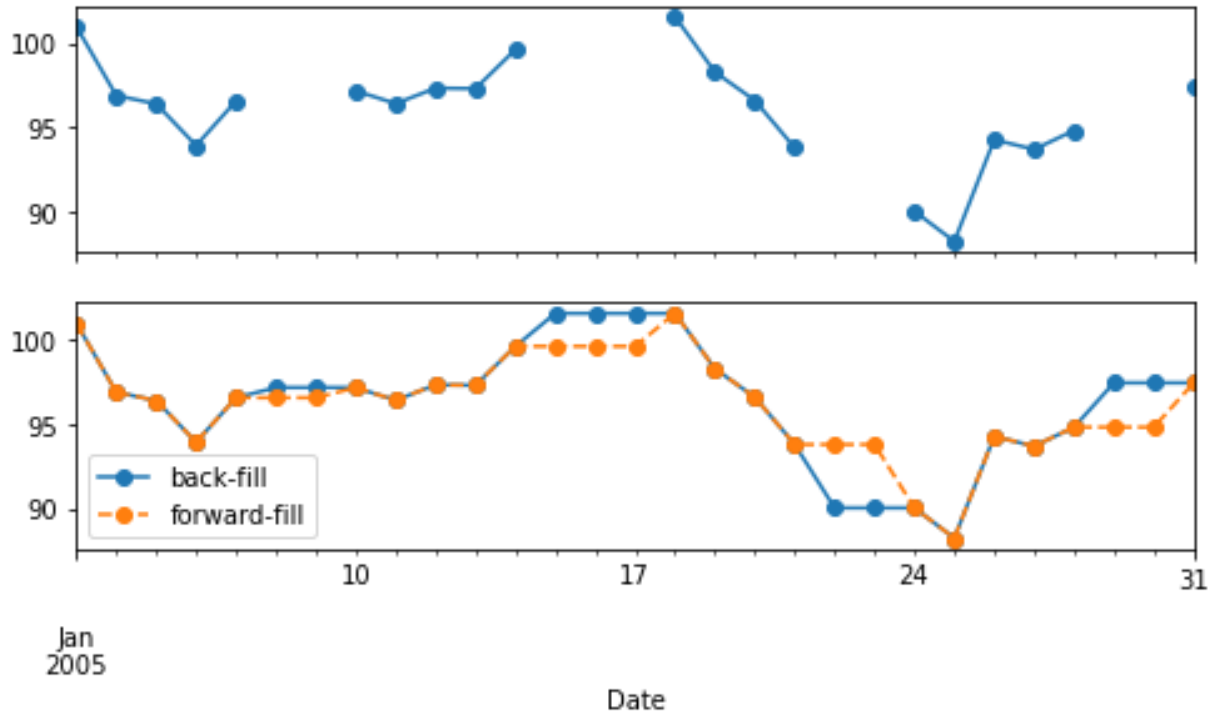
- For up-sampling (high freq) , `resample()` and `asfreq()` are largely equivalent
  - By default, both functions insert the new higher-frequency timestamps into the index, and for each of these new moments time-based variable contains NaN
  - Example: If original df of temperature has timestamps at 12:00 and 14:00, but you resample to hourly:

index	temperature
12:00	(existing data)
13:00	(**new** → **NaN**)
14:00	(existing data)

- `asfreq()` accepts a method argument to specify how NaN values are imputed:
  - pad / ffill (forward fill): propagate last valid observation forward to next valid
  - bfill (back fill): use NEXT valid observation to fill
- Example: in Google dataset, we will resample at a daily frequency so as days related to weekends and other public holidays (that are missing) to be created

Up-sampling (asfreq)

```
fig, axs = plt.subplots(2, 1, sharex=True, figsize=(8,4))
data = goog.iloc[:20]
# Up-sampling without filling (default action)
data.asfreq('D').plot(ax=axs[0], marker='o')

data.asfreq('D', method='bfill').plot(ax=axs[1], style='-o')
data.asfreq('D', method='ffill').plot(ax=axs[1], style='--o')
axs[1].legend(["back-fill", "forward-fill"])
```

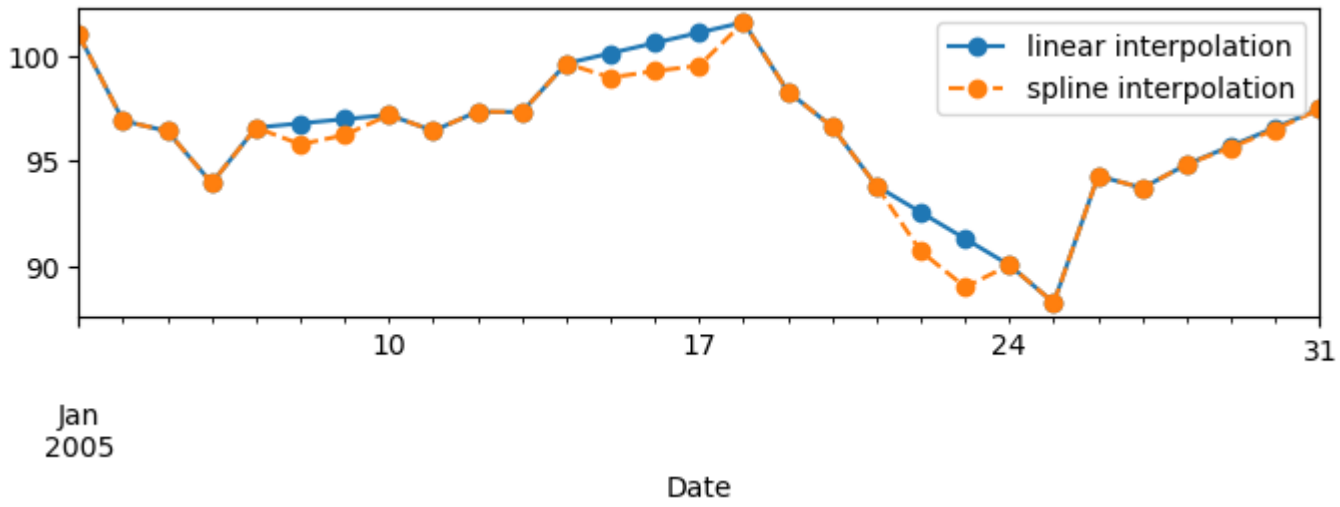


Up-sampling without filling (default action) => weekends and public holidays are present in the dataset but have NA (no value) as close price

** Note: SimpleImputer from Lab4 could be used to fill gaps with statistical-based parameters such as mean, median.

Up-sampling (resample)

```
fig, axs = plt.subplots(2, 1, sharex=True, figsize=(8,4))
data = goog.iloc[:20]

data.resample('D').interpolate(method='linear').plot(style='-o')
data.resample('D').interpolate(method='spline',order=2).plot(style='--o')
axs[1].legend(["linear interpolation", "spline interpolation"])
```



Up-sampling with linear and spline interpolations

Linear interpolation is a method to fill in missing values in a dataset by **drawing a straight line** between known data points and estimating the missing values along that line.
Spline interpolation is a method to fill in missing values in a dataset by **fitting a smooth curve** through the known data points and estimating the missing values along that curve.

Time shifts

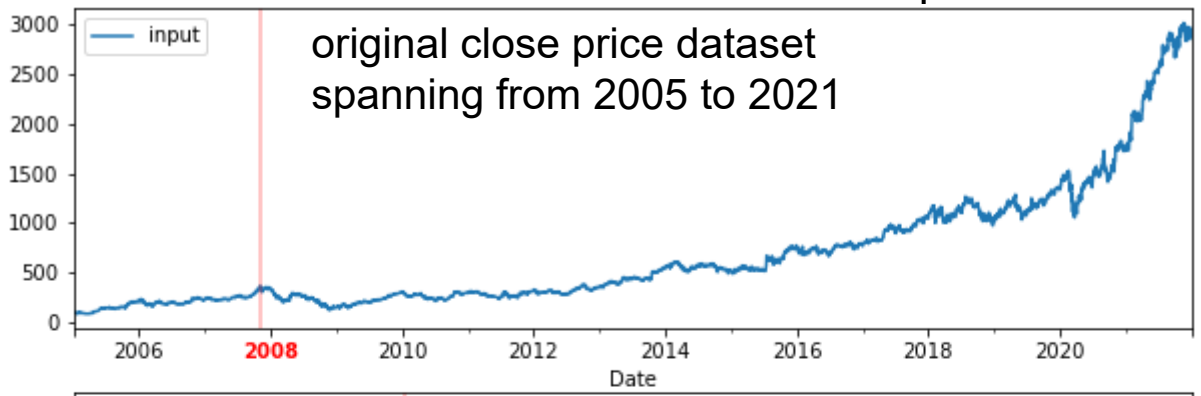
- Common time series-specific operation: shifting of data in time

- Time shift function: `shift()`
  - Shifts index by the desired number of `periods` with an optional time `freq`
  - When `freq` is **not passed**, data values will be shifted **without re-aligning** the index
  - When `freq` is passed, both data values and index will be shifted using the `periods` and the `freq`

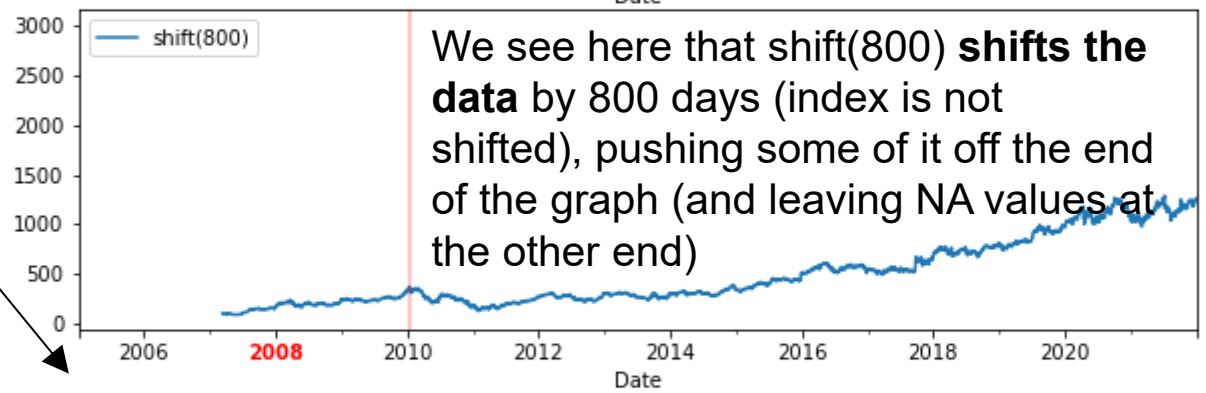
- Example: use function with / without `freq` to shift dataset by 800 days

Time shifts

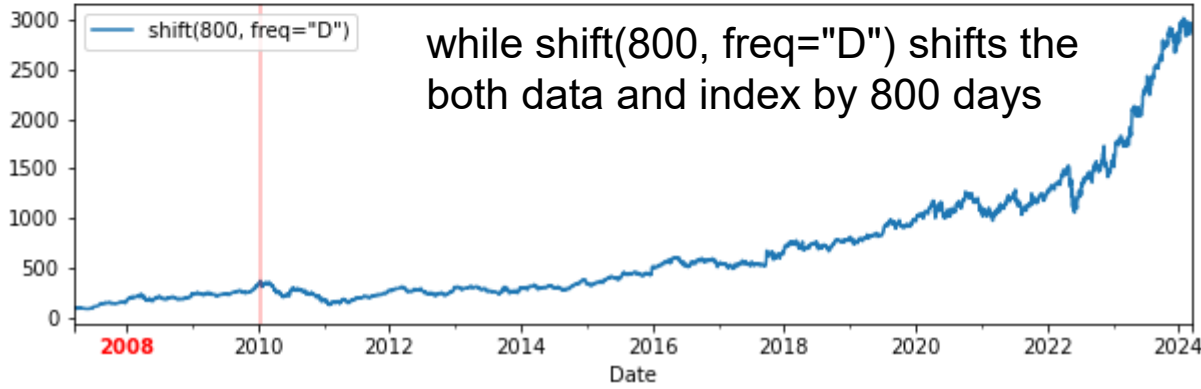
```
fig, axs = plt.subplots(3, 1, figsize=(10, 10), sharey=True)

# fill-in weekends
goog_filled = goog.asfreq('D', method='ffill')

goog_filled.plot(ax=axs[0])
goog_filled.shift(periods=800).plot(ax=axs[1])
goog_filled.shift(periods=800, freq='D').plot(ax=axs[2])

# legends and annotations
local_max = pd.to_datetime('2007-11-05')
offset = pd.Timedelta(800, 'D')

axs[0].legend(['input'], loc='upper left')
axs[0].get_xticklabels()[2].set(weight='heavy', color='red')
axs[0].axvline(local_max, alpha=0.3, color='red')

axs[1].legend(['shift(800)'], loc='upper left')
axs[1].get_xticklabels()[2].set(weight='heavy', color='red')
axs[1].axvline(local_max + offset, alpha=0.3, color='red')

axs[2].legend(['shift(800, freq="D")'], loc='upper left')
axs[2].get_xticklabels()[1].set(weight='heavy', color='red')
axs[2].axvline(local_max + offset, alpha=0.3, color='red')
```



original close price dataset spanning from 2005 to 2021



We see here that shift(800) **shifts the data** by 800 days (index is not shifted), pushing some of it off the end of the graph (and leaving NA values at the other end)



while shift(800, freq="D") shifts the both data and index by 800 days

Used in many tasks e.g. for detrending or deseasonalizing (here) or for creating lagged features (here)

Moving window functions (rolling statistics)

- Rolling statistics are a core time-series operation that compute values over a moving window.
- Pandas provides the `rolling()` function, which creates a rolling "view," similar in spirit to a groupby object – key parameters:
  - window:
    - Number of observations (or time-based span) used to compute each rolling statistic.
  - center:
    - False (default): assign the computed value to the right edge of the window
    - True: assign the computed value to the center of the window
- This rolling view supports many aggregation methods out of the box (e.g., sum, mean, median, std, …) that can be applied on the values within the window

Moving window functions (rolling statistics)

window=3

```
df['Rolling_Volume_Sum'] = df['Volume'].rolling(3, center = ).sum()
```

False

True

evaluate a rolling sum of volume values over a windows of size 3

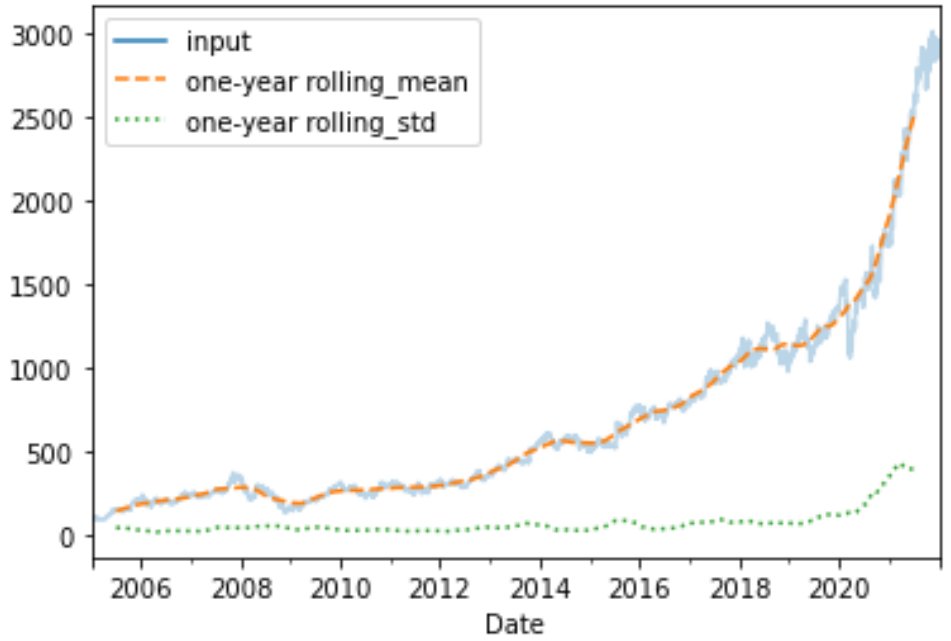
	Volume	Rolling_Volume_Sum
0	3641	NaN
1	3675	NaN
2	3371	10687.0
3	2519	9565.0
4	3260	9150.0
5	2922	8701.0

	Volume	Rolling_Volume_Sum
0	3641	NaN
1	3675	10687.0
2	3371	9565.0
3	2519	9150.0
4	3260	8701.0
5	2922	NaN

Rolling statistics

- One-year centered rolling mean and standard deviation of the Google closing stock prices

```
rolling = goog.rolling(365, center=True)
data = pd.DataFrame({'input': goog,
                     'one-year rolling_mean': rolling.mean(),
                     'one-year rolling_std': rolling.std()})
ax = data.plot(style=['-', '--', ':'])
ax.lines[0].set_alpha(0.3)
```



Timeseries Analysis & Forecasting

- **Time Series Analysis**
  - Uses mathematical and statistical methods to describe and understand an observed time series.
  - Aims to identify patterns, structures, and trends in the data.
  - Focuses on uncovering the underlying causes (“why”) behind the observed behavior.
- **Time Series Forecasting**
  - Involves fitting models to historical time series data.
  - Uses these models to predict future values of the series.
  - Goal is not explanation but accurate future estimation.

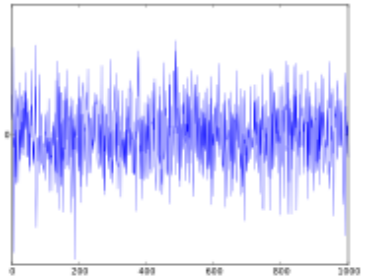
Timeseries Forecasting Concerns

- Time series forecasting often requires pre-processing (e.g. cleaning) and even transformation (e.g. frequency scaling)
- Cleaning
  - Missing data*: Perhaps there are gaps or missing data that need to be interpolated or imputed
  - Outliers: Perhaps there are extreme outlier values that need to be identified and handled
- Transformation
  - Frequency scaling*: Perhaps data is provided at a frequency that is too high to model or is unevenly spaced through time requiring resampling for use in some models

(*) Handled using `asfreq()` or `resample()`

Stationarity in timeseries

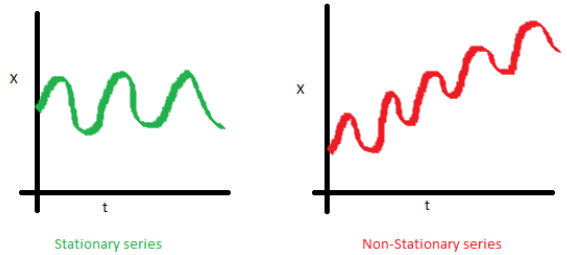
- Many time series techniques assume that the underlying data-generating process is stationary
- A TS is said to be stationary if its statistical properties such as mean, variance remain constant over time
  - The Poisson process and white noise are perfect stationary signals



- In other words, if TS is stationary and has a particular behavior over time, the TS will follow the same behavior in the future
- Most TS forecasting models work on the assumption that the TS is stationary → look at examples at the end of that lab

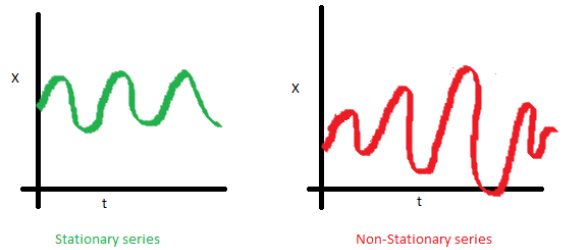
Stationarity in timeseries

- Stationarity is defined using very strict criterion
- In theory, a TS is assumed to be stationary if it has constant statistical properties over time:
  - constant mean



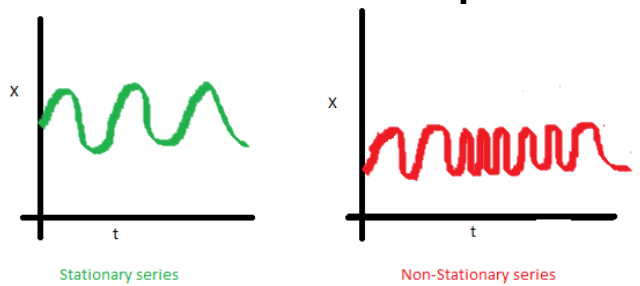
Increasing trend

  - constant variance



Seasonal variations

  - an autocovariance that does not depend on time.

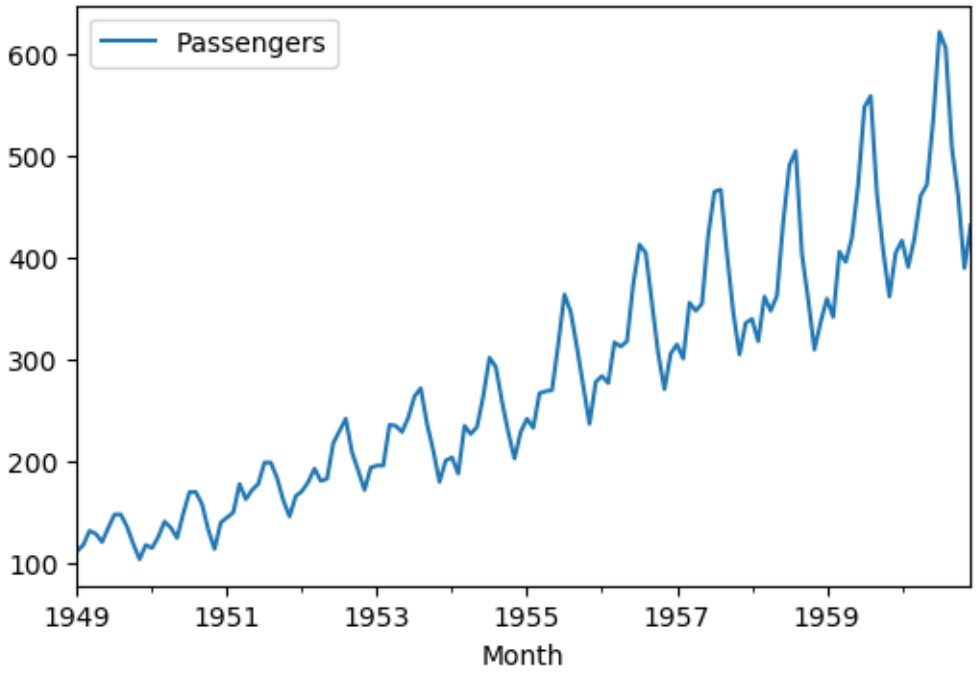


There is a statistical dependence of samples taken at different time points

Time series example

- Dataset: Airline Passengers
- Describes the total number of airline passengers (in thousands)
- 144 monthly observations from 1949 to 1960

```
"Month","Passengers"
"1949-01",112
"1949-02",118
"1949-03",132
"1949-04",129
```




```
import pandas as pd
import matplotlib.pyplot as plt
df = pd.read_csv('airline-passengers.csv',
index_col='Month', parse_dates=True,
infer_datetime_format=True)
# plot original timeseries
df.plot()
plt.show()
```

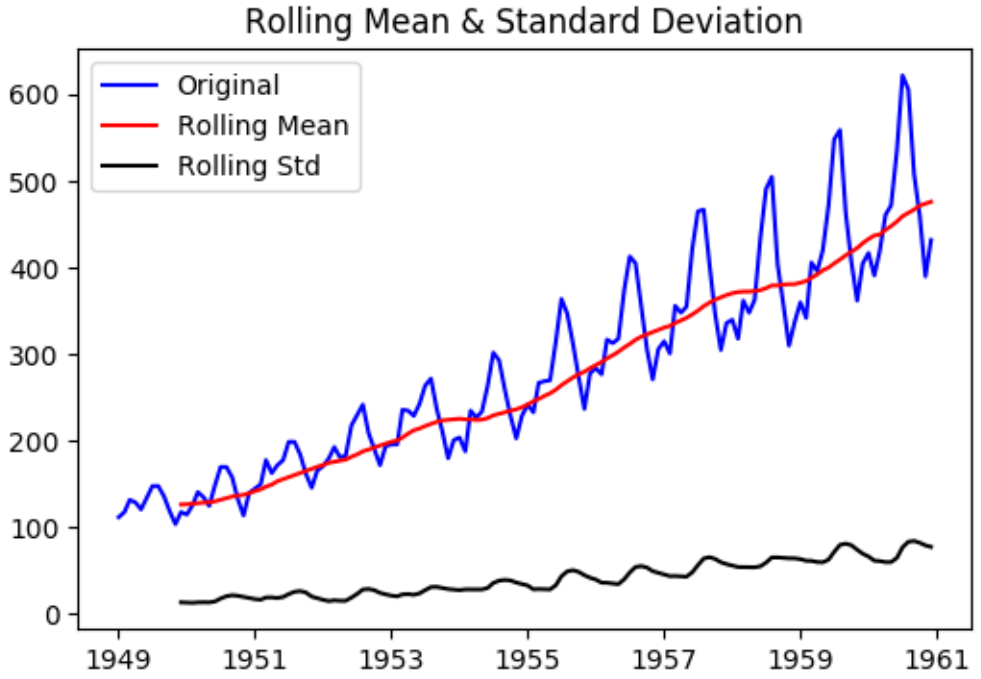
Observations:

- overall increasing trend
- some seasonal variations → people tend to travel more in a particular time period e.g. during summertime or in the Christmas period

Check stationarity

- **Plot Rolling Statistics – a visual technique**
  - Plot the moving average & moving variance and see if they vary with time
  - Moving average/variance: At any time t, take the average/variance of the last year, i.e. last 12 months
- **Dickey-Fuller Test – a statistical test**
  - Null hypothesis is that the TS is non-stationary
  - Use a **Test Statistic** and some **Critical Values** for difference confidence levels. If the 'Test Statistic' is less than the 'Critical Value', we can reject the null hypothesis and say that the series is stationary
  - See here for details

Check stationarity



Apply rolling function on a window of size 12 Observations

- Mean is clearly increasing with time (uptrend behavior) ➔ not a stationary timeseries
- Variation in standard deviation is small
- Test statistic is way more than the critical values ➔ null hypothesis is not rejected ➔ time series is not stationary

Results of **Dickey-Fuller Test:**

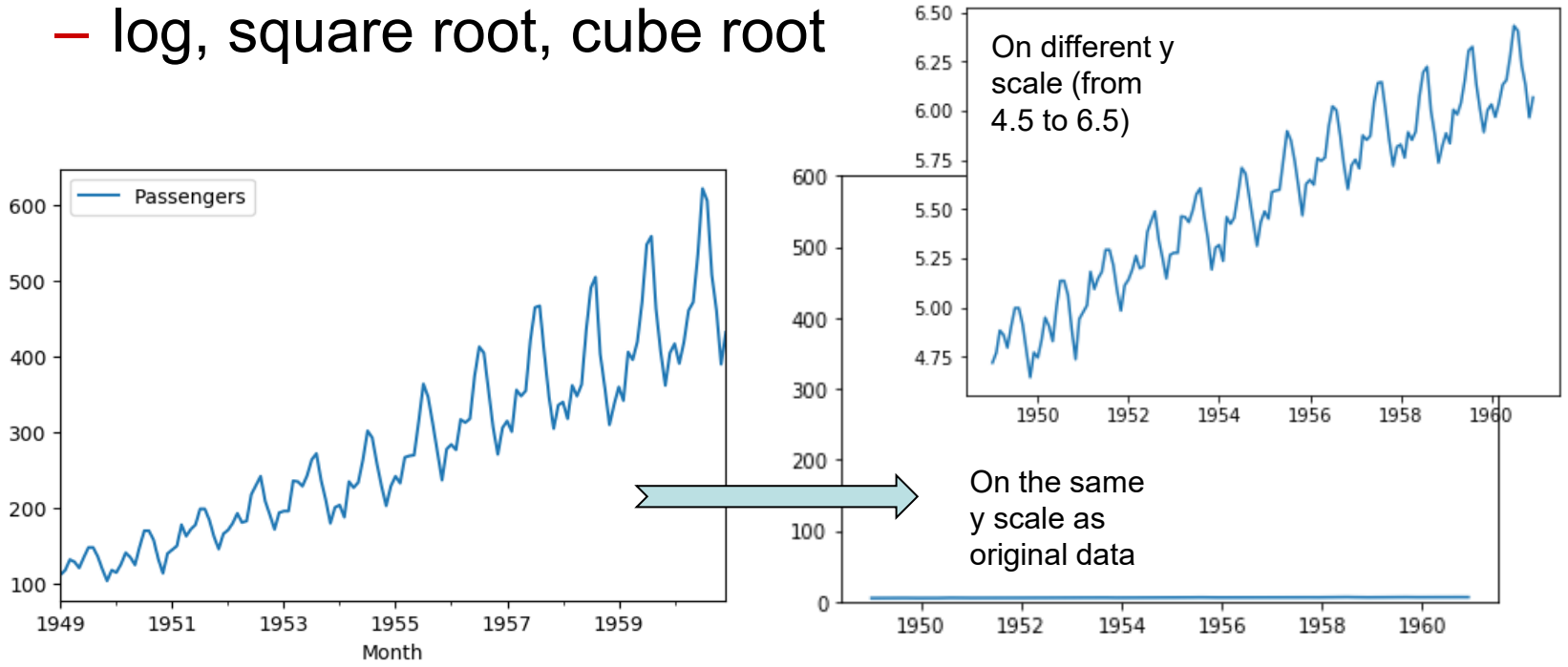
Test Statistic	0.815369
p-value	0.991880
#Lags Used	13.000000
Number of Observations Used	130.000000
Critical Value (1%)	-3.481682
Critical Value (5%)	-2.884042
Critical Value (10%)	-2.578770

How to make a TS Stationary?

- Impossible to make a series perfectly stationary
- 2 major reasons behind non-stationarity of a TS:
  - **Trend** – long term increase or decrease in data → observe mean over time
    - e.g., in airline passengers we observed an **increasing trend**: on average, the number of passengers is growing over time
  - **Seasonality** – systematic (repeated), calendar related movements → observe variations at specific time-frames
    - e.g., in airline passengers we observed people have a tendency to travel more in a particular time period e.g. during summertime / Christmas
- Next steps:
  - model or estimate trend and seasonality components and remove them to convert a non-stationary TS to a stationary TS
  - implement forecasting techniques on stationary TS and predict TS values
  - convert the predicted values into the original scale by re-applying trend and seasonality components back

Estimating and Eliminating Trend

- Reduce trend by transformation
- Suppress increasing trend => apply transformation that penalizes higher values more than small values
  - log, square root, cube root




```
ts_log = np.log(ts)
plt.plot(ts_log)
```

```
"Month","Passengers"
"1949-01",4.718499
"1949-02",4.770685
"1949-03",4.882802
"1949-04",4.859812
```

Passenger values are minimized; however, the curve maintains the seasonality variations in a smaller scale

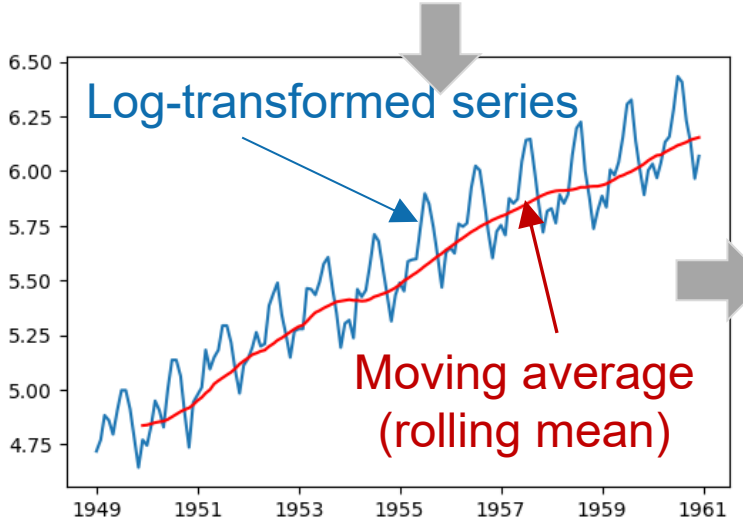
Estimating and Eliminating Trend

- Common techniques to estimate or model trend :
  - **Aggregation** – taking average for a time period like monthly/weekly averages
  - **Smoothing** – taking rolling (moving) averages
  - **Polynomial Fitting** – fit a polynomial regression model
    - Fit a polynomial curve (e.g., quadratic or cubic) to the time series to capture long-term trends: $Yt=\beta 0+\beta 1t+\beta 2t^2+\cdots+\beta pt^p+\epsilon t$

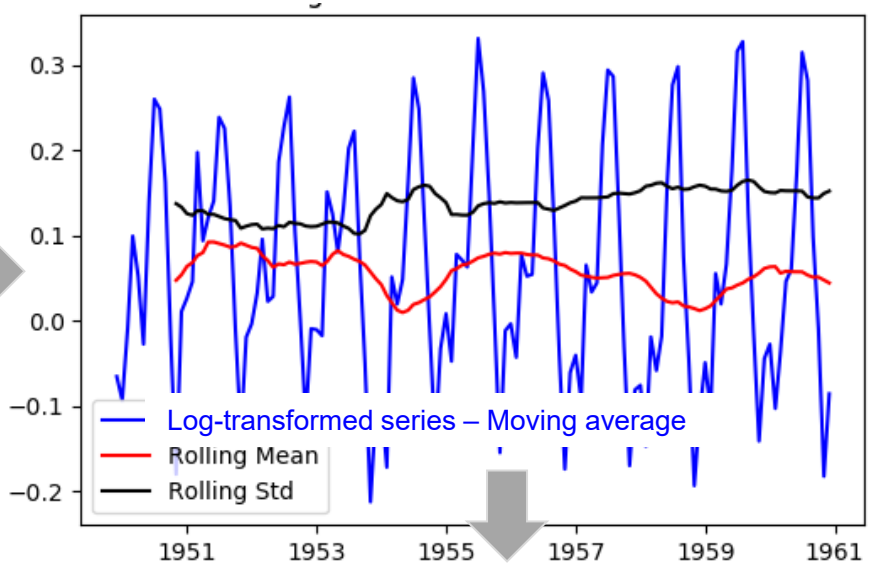
- After estimating trend, we need to remove it from the series to make it stationary

Estimating and Eliminating Trend

- **Smoothing**
  - Evaluate moving (rolling) average of the past ‘k’ consecutive values depending on the frequency of time series (e.g. over the past 1 year, i.e. last 12 values) and subtract it from the original series & re-run stationarity test




```
moving_avg = tslog.rolling(12).mean()
plt.plot(ts_log)
plt.plot(moving_avg, color='red')
```

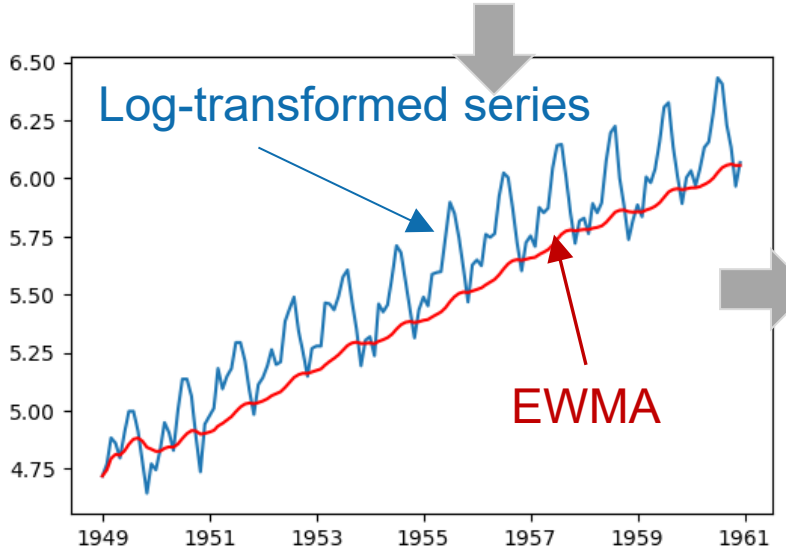


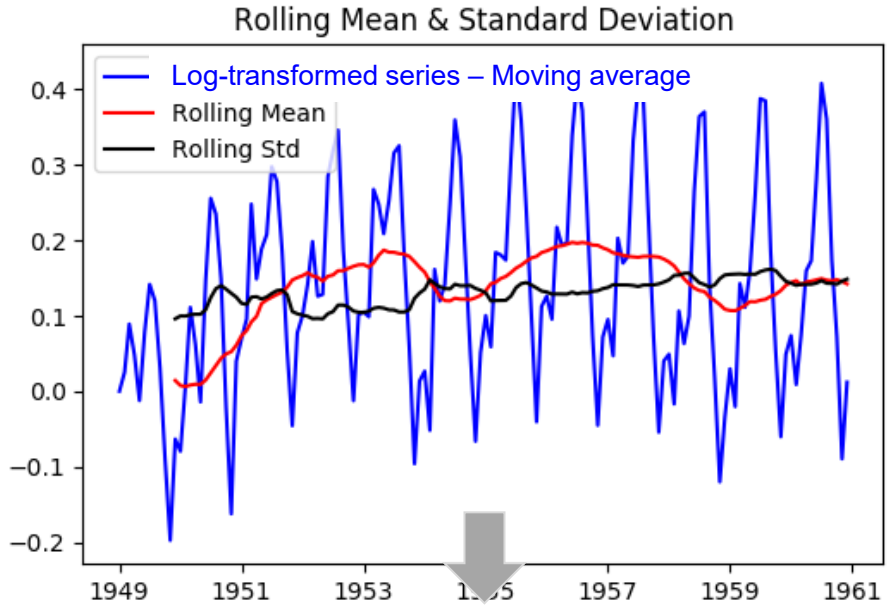

```
Results of Dickey-Fuller Test:
Test Statistic                  -3.162908
p-value                          0.022235
Critical Value (5%)             -2.886151
Critical Value (1%)             -3.486535
Critical Value (10%)            -2.579896
```

Test statistic is **smaller than the 5% critical values** so we can say with 95% confidence that this is a stationary series.

Estimating and Eliminating Trend

- Evaluate ‘weighted moving average’ where more recent values within the windows are given a higher weight
  - Use exponentially weighted moving average (EWMA) where weights are assigned to all the previous values with a decay factor





Test statistic is **smaller than the 1% critical value => TS is stationary with 99% confidence => better than previous case**

```
Results of Dickey-Fuller Test:
Test Statistic                  -3.601262
Critical Value (5%)             -2.884042
Critical Value (1%)             -3.481682
Critical Value (10%)            -2.578770
```

Eliminating Trend and Seasonality

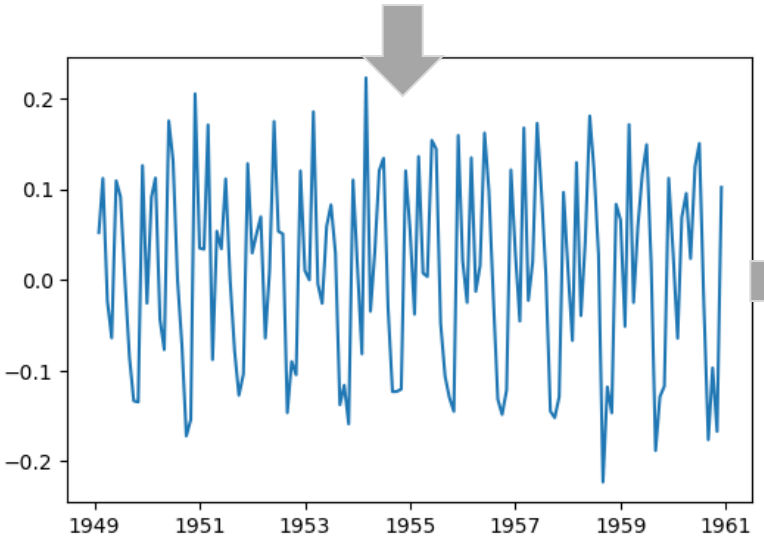
- Simple trend reduction techniques do not work in cases with high seasonality
- Common methods dealing with both trend and seasonality:
  - **Differencing** – creates a new TS by taking the differences between observations
  - **Decomposition** – models both trend and seasonality components and removes them from the model

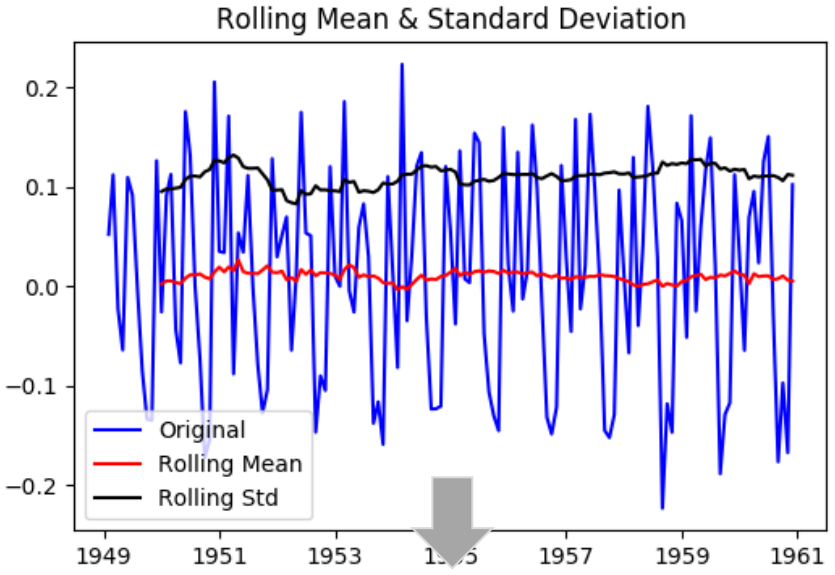
Eliminating Trend and Seasonality

- **First-order differencing**: takes the difference of the observation at a particular instant with that at the prev instant
- First order differencing can be done in Pandas as:

```
"Month","Passengers"
"1949-01",Nan
"1949-02",0.052186
"1949-03",0.112117
"1949-04",-0.02299
```

```
ts_log_diff = ts_log - ts_log.shift()
plt.plot(ts_log_diff)
```





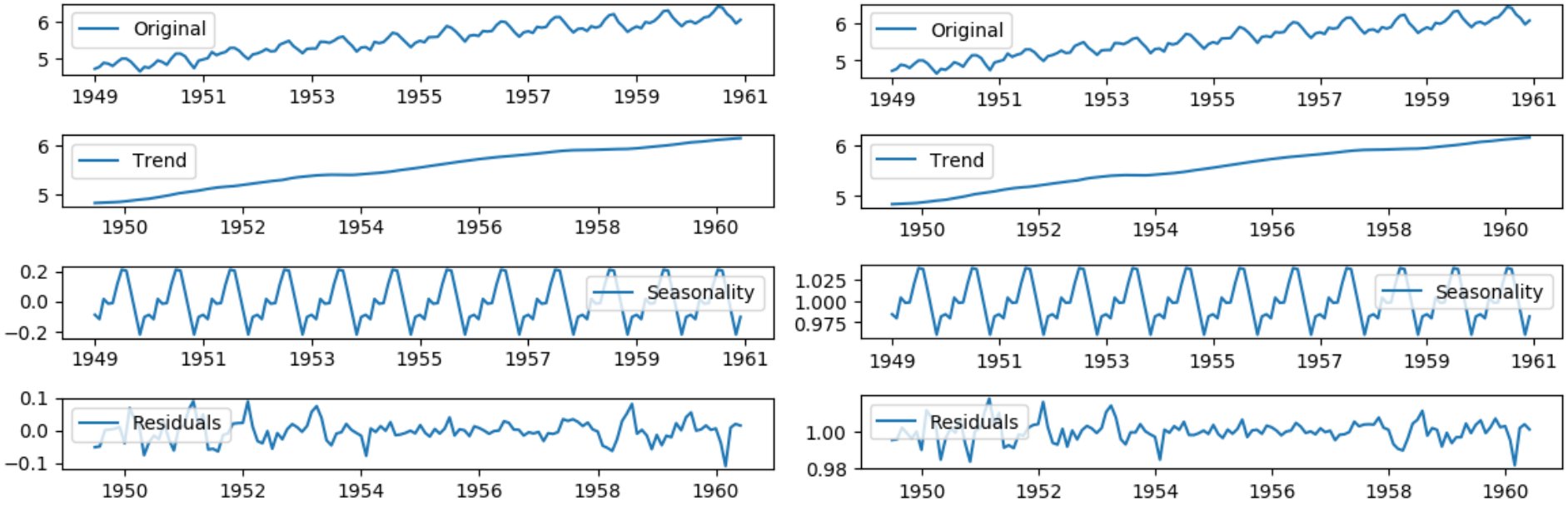

```
Results of Dickey-Fuller Test:
Test Statistic                  -2.717131
Critical Value (1%)             -3.482501
Critical Value (10%)            -2.578960
Critical Value (5%)             -2.884398
```

Test statistic is **less than the 10% critical value**, thus the TS is stationary with 90% confidence

Eliminating Trend and Seasonality

- **Decomposing**: given a TS, find the constituent components (trend, seasonality, residual or random or noise)
- Components can be combined in some way to provide the observed time series e.g.
  - Additive model:
    - `Y(t) = trend + seasonality + residual`
  - Multiplicative model:
    - `Y(t) = trend * seasonality * residual`
- There are methods to automatically decompose TS
- Python: seasonal_decompose()
  - Install statsmodels library: conda install -c conda-forge statsmodels
  - specify whether the model is additive or multiplicative

Eliminating Trend and Seasonality



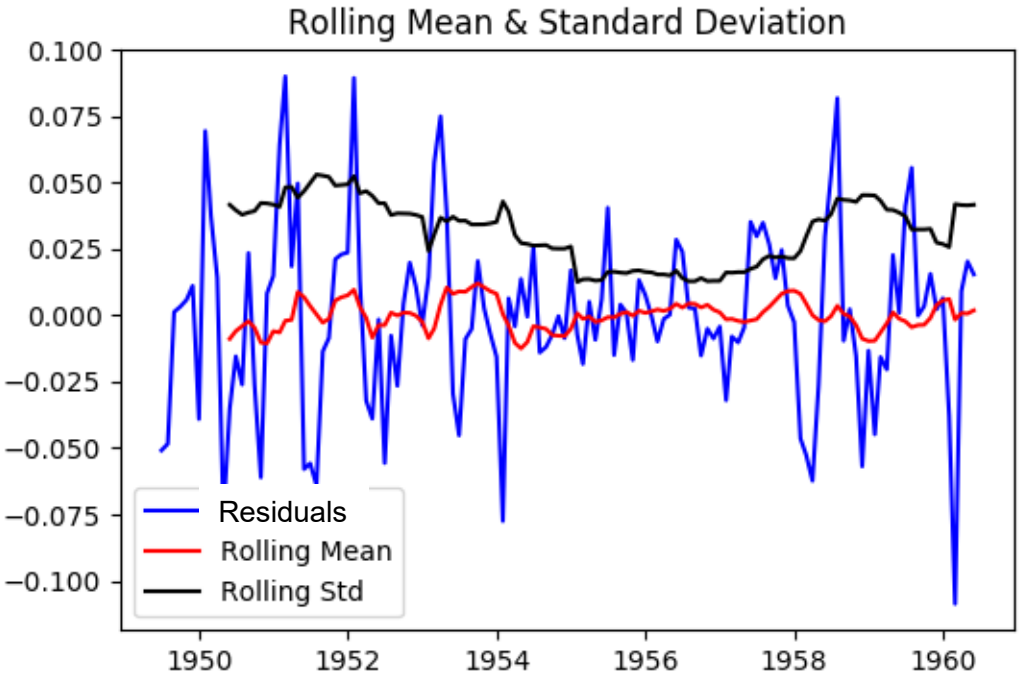
Additive Model

Multiplicative Model

- Trend, seasonality are separated out from data
- Residual or error is the random component of a TS which will be tested for stationarity

Eliminating Trend and Seasonality

- Test stationarity of residuals




```
Results of Dickey-Fuller Test:
Test Statistic                  -6.332387e+00
Critical Value (1%)             -3.485122e+00
Critical Value (10%)            -2.579569e+00
Critical Value (5%)             -2.885538e+00
```

- The Dickey-Fuller test statistic is significantly **lower than the 1% critical value** => this TS is very close to stationary
- You can try advanced decomposition techniques as well which can generate better results

Forecasting a Timeseries dataset

- After trend/seasonality elimination, we need to build a forecasting model to predict # of airline passengers
  - Train model using as train data:
    - If dataset contains differences (if differencing is used) → predict differences
    - If dataset contains the residuals (if decomposition is used) → predict residuals
  - Add trend and seasonality back into predicted values (differences or residuals) to evaluate the predicted # of airline passengers

- Forecasting methods
  - Use statistical methods like Holt-Winters, ARIMA or
  - Convert TS forecasting problem to supervised learning problem (generate features) and solve it using ML algorithms

Statistical forecasting methods

- Holt-Winters Methods
- Autoregression (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), and Seasonal Autoregressive Integrated Moving-Average (SARIMA)

Supervised learning problem

- TS forecasting problem can be restructured as a supervised learning problem, by generating features => gain access to the suite of standard linear & nonlinear ML predictive algorithms discussed in previous labs

- Feature generation options:
  1. Transform datetime index to a set of new features: date, hour, day of week, quarter, year, month, day of year, day of month, week of year

  2. Use the observed entity (time-dependent variable) to create new features: e.g. use previous (shifted) instants of the observed entity as features.

1. Create features from the date time index

Original dataframe →

```
"Month","Passengers"
1949-01-01,112
1949-02-01,118
1949-03-01,132
1949-04-01,129
1949-05-01,121
```

New dataframe ↘

Month	Passengers	hour	dayofweek	quarter	month	year	dayofyear	dayofmonth	weekofyear
1949-01-01	112	0	5	1	1	1949	1	1	53
1949-02-01	118	0	1	1	2	1949	32	1	5
1949-03-01	132	0	1	1	3	1949	60	1	9
1949-04-01	129	0	4	2	4	1949	91	1	13
1949-05-01	121	0	6	2	5	1949	121	1	17

Y
target

X
Features created from different time components of the datetime-based index

1. Create features from the date time index

Original dataframe →

```
"Month","Passengers"
1949-01-01,4.718499
1949-02-01,4.770685
1949-03-01,4.882802
1949-04-01,4.859812
1949-05-01,4.795791
```

reduce trend with log transformation

New dataframe ↓

Month	Passengers	hour	dayofweek	quarter	month	year	dayofyear	dayofmonth	weekofyear
1949-01-01	4.718499	0	5	1	1	1949	1	1	53
1949-02-01	4.770685	0	1	1	2	1949	32	1	5
1949-03-01	4.882802	0	1	1	3	1949	60	1	9
1949-04-01	4.859812	0	4	2	4	1949	91	1	13
1949-05-01	4.795791	0	6	2	5	1949	121	1	17

Y
target

X
Features created from different time components of the datetime-based index

1. Create features from the date time index

Original dataframe →

```
"Month","Passengers"
1949-02-01,0.052186
1949-03-01,0.112117
1949-04-01,-0.022990
1949-05-01,-0.064022
1949-06-01,0.109484
```

reduce trend/ seasonality with differencing

New dataframe ↘

Month	Passengers	hour	dayofweek	quarter	month	year	dayofyear	dayofmonth	weekofyear
1949-02-01	0.052186	0	1	1	2	1949	32	1	5
1949-03-01	0.112117	0	1	1	3	1949	60	1	9
1949-04-01	-0.022990	0	4	2	4	1949	91	1	13
1949-05-01	-0.064022	0	6	2	5	1949	121	1	17
1949-06-01	0.109484	0	2	2	6	1949	152	1	22

Y
target

X
Features created from different time components of the datetime-based index

2. Create features using previous instants

Original dataframe

```
"Month","Passengers"
1949-01-01,112
1949-02-01,118
1949-03-01,132
1949-04-01,129
```

```
"Month","Passengers","shift1"
1949-01-01,112,Nan
1949-02-01,118,112
1949-03-01,132,118
1949-04-01,129,132
```

First row can be omitted

Y target X feature

New dataframe with window=1

number of passengers: target variable
previous instant of each observation: feature

```
"Month","Passengers","shift1","shift2"
1949-01-01,112,Nan,Nan
1949-02-01,118,112,Nan
1949-03-01,132,118,112
1949-04-01,129,132,118
```

First two rows can be omitted

Y target X features

Another new dataframe window=2

number of passengers: target variable
previous 2 instants of each observation: features

We can also reduce trend / seasonality by applying log-transformation and differencing on the target variable.

2. Create features using previous instants

- The use of prior time steps to predict the next time step is called the sliding window method
- In statistics and time series analysis, this is called a lag or lag method
- The number of previous time steps is called the window width or size of the lag

Train / test splitting

- Using train_test_split on time series data is not appropriate because it randomly splits the data, which breaks the temporal order
- Time series data has an inherent sequence, where past values influence future values. If you randomly split the data, you risk "leaking" information from the future into the training set, leading to unrealistic model performance during training and testing.
- Need for time-aware splitting methods that ensure that the test set consists of data points occurring after all the data points in the training set
  - This preserves the time order, preventing data leakage and allowing you to evaluate the model's performance as it would in real-world forecasting

Train / test splitting

1. Simple Train/Test Split
   - use a basic split by defining the training set as the data up to a certain date and the test set as the data after that date
   - works well for single evaluation

```
# OPTION 1: Assuming data is your DataFrame
 train_size = int(len(data) * 0.8) # Use 80% of data for training
train, test = data[:train_size], data[train_size:]

# OPTION 2: Or if your data has a datetime index and you want to split by a
specific date
train = data[data.index < '2023-01-01']
test = data[data.index >= '2023-01-01']
```

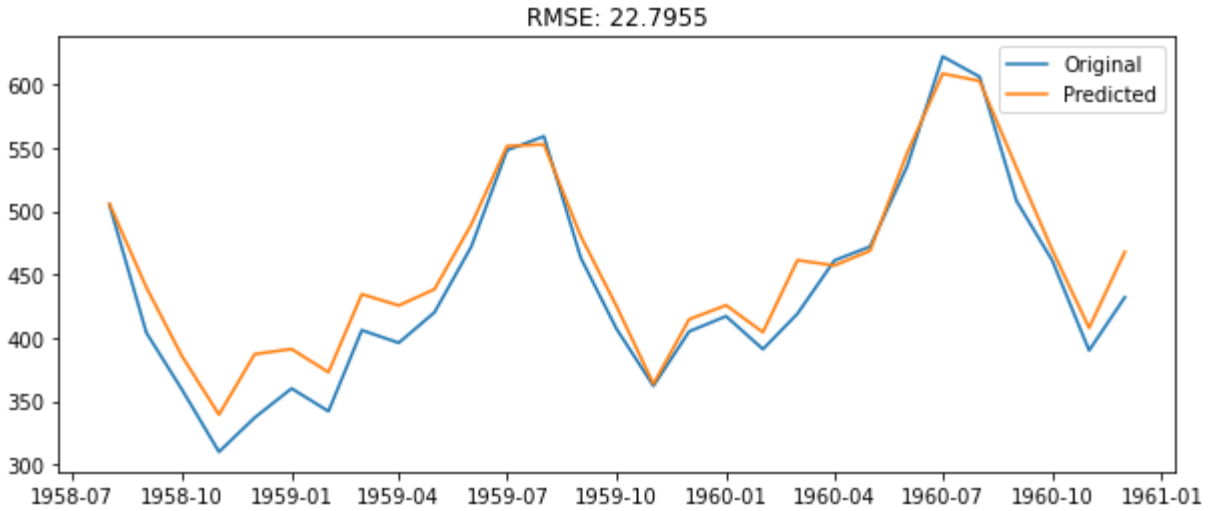
Train / test splitting

2. Using TimeSeriesSplit from sklearn for Cross-Validation
   - TimeSeriesSplit provides a way to perform cross-validation for time series by creating a series of train/test splits where the training set includes all previous data up to the test set. This is helpful for validating the model in a time series context.

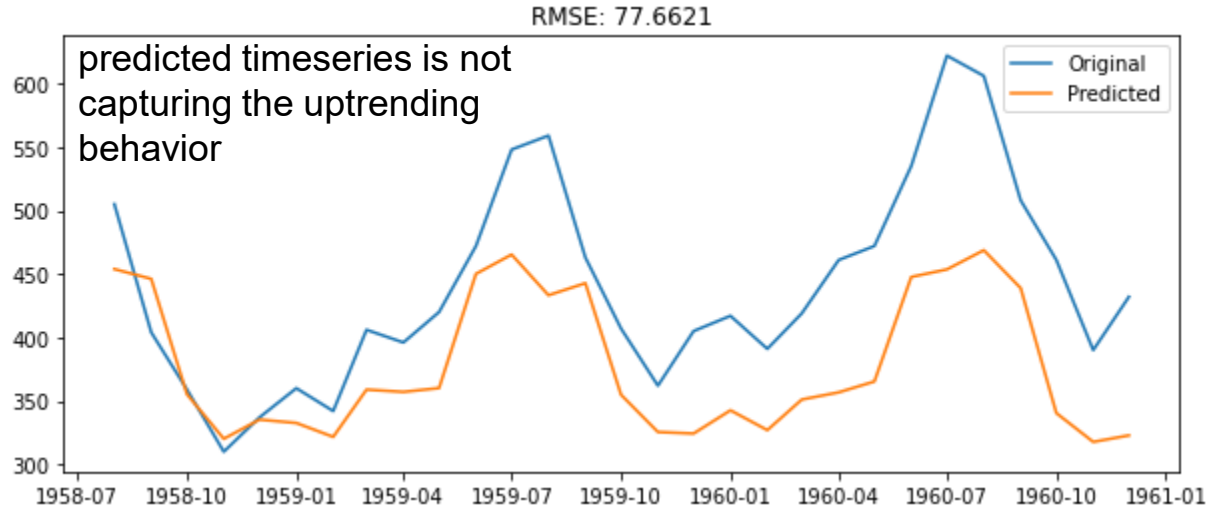
```
from sklearn.model_selection import TimeSeriesSplit

# Initialize time series splitter
tscv = TimeSeriesSplit(n_splits=5)

for train_index, test_index in tscv.split(data):
    train, test = data.iloc[train_index], data.iloc[test_index]
    # Train your model on `train`, evaluate on `test`
    # Store performance measure (MSE, RMSE, r2) value in a list
# print performance average
```

Some results…

- Features created from the date time index
- Simple train/test split: 80% (01/1949 – 06/1957) / 20% (07/1957 – 12/1960)
- Applied ML algorithm: RandomForest (100 estimators)



Log-transformation and differencing applied



NO Log-transformation and differencing applied

Thank you for your patience

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Good luck with your exams

APPENDIX A: Clustering / Classification of Time series

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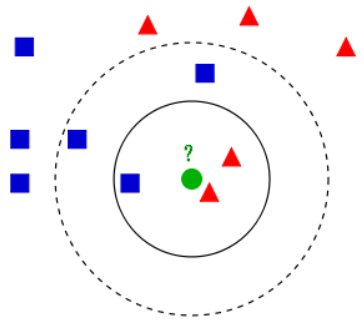
Clustering and classification

- Time series clustering is to partition time series data into groups based on similarity or distance, so that time series in the same cluster are similar.
- Time series classification is to build a classification model based on labelled time series and then use the model to predict the label of unlabelled time series.

How to use clustering & classification?

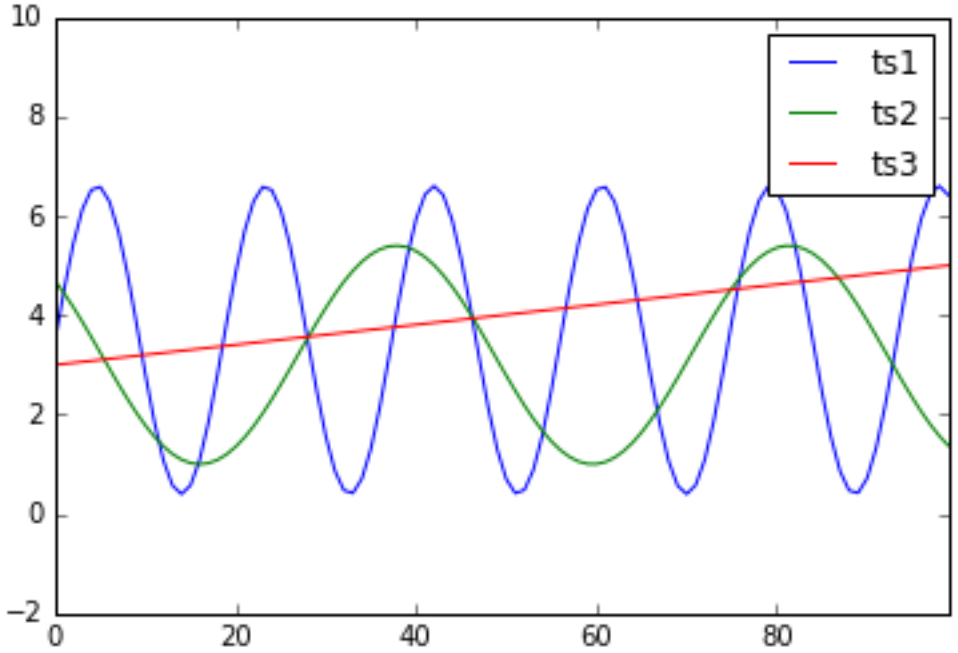
- One approach [extracting features]
  - Extract features from time series:
    - mean, maximum, minimum, other differential features
  - Use well-known algorithms with these features to make a prediction
    - Clustering: kMeans / Classification: Naive Bayes, SVMs, etc.
- Another approach [w/o features, use time series]
  - k-NN for classification
    - For a given time series test sample that you want to predict, find the most **similar (small distance) time series** samples in the training set and use its corresponding output as the prediction.



  - kMeans for clustering
    - points/centroids are time series; notion of **distance** needed.
  - Find a good similarity (distance) measure is not trivial

Similarity measures in time series

- Good similarity measure:
  - small changes in two time series result in small changes in their similarity
- How about Euclidean distance?
  - **true** for changes in the **y-axis** but **false** for changes in the **time axis** (i.e. compression and stretching)



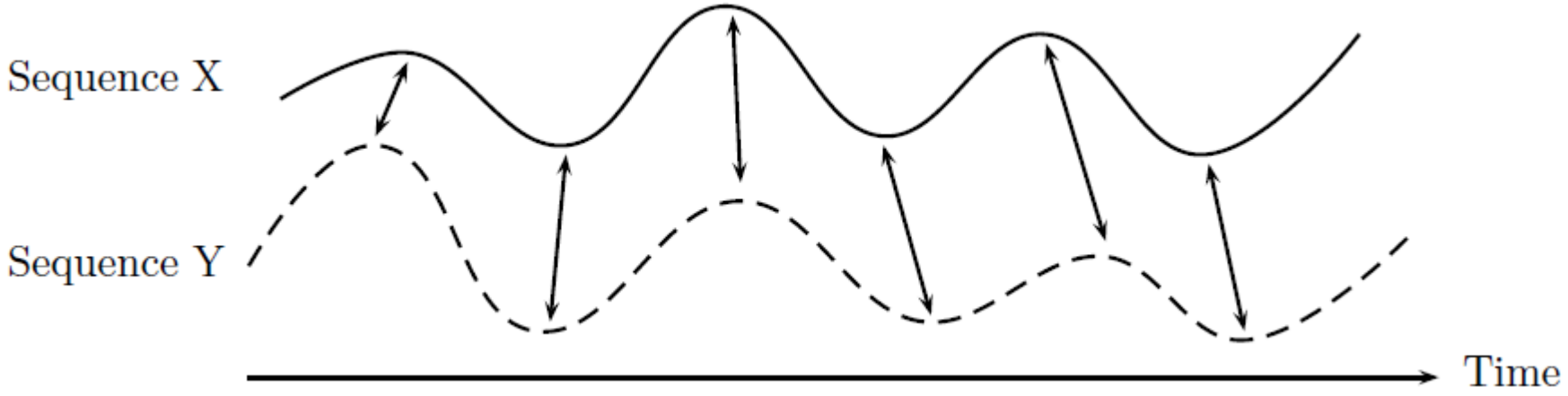

```
euclidean_distance(ts1,ts2)
= 26.959216038

euclidean_distance(ts1,ts3)
= 23.1892491903
```

According to Euclidean distance, ts1 is more similar to ts3 than to ts2 which contradicts our intuition!!!

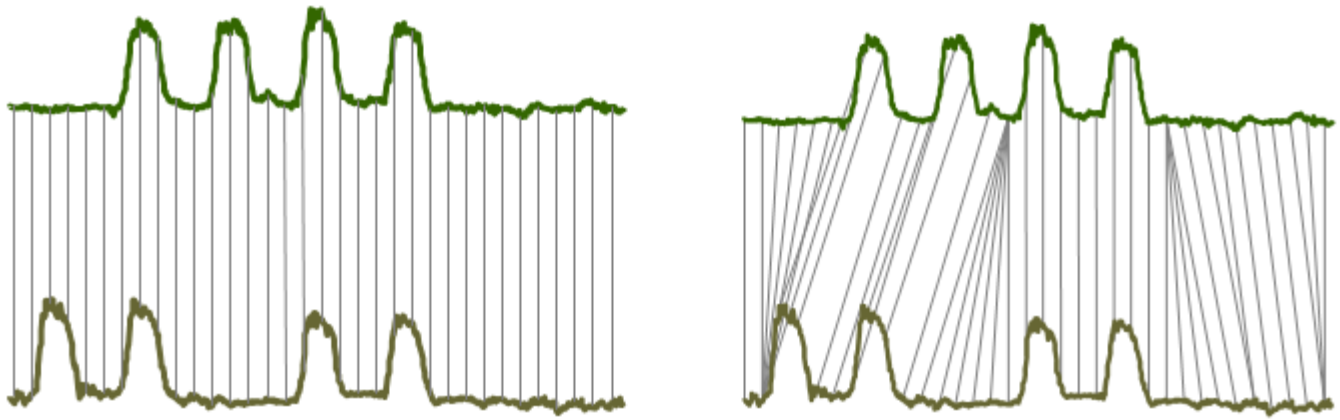
Dynamic Time Warping (DTW)

- Well-known technique to find optimal alignment between two given (time-dependent) sequences under certain restrictions
  - Allows acceleration-deceleration of sequences along the time dimension
  - Aligned points indicated by arrows in figure below



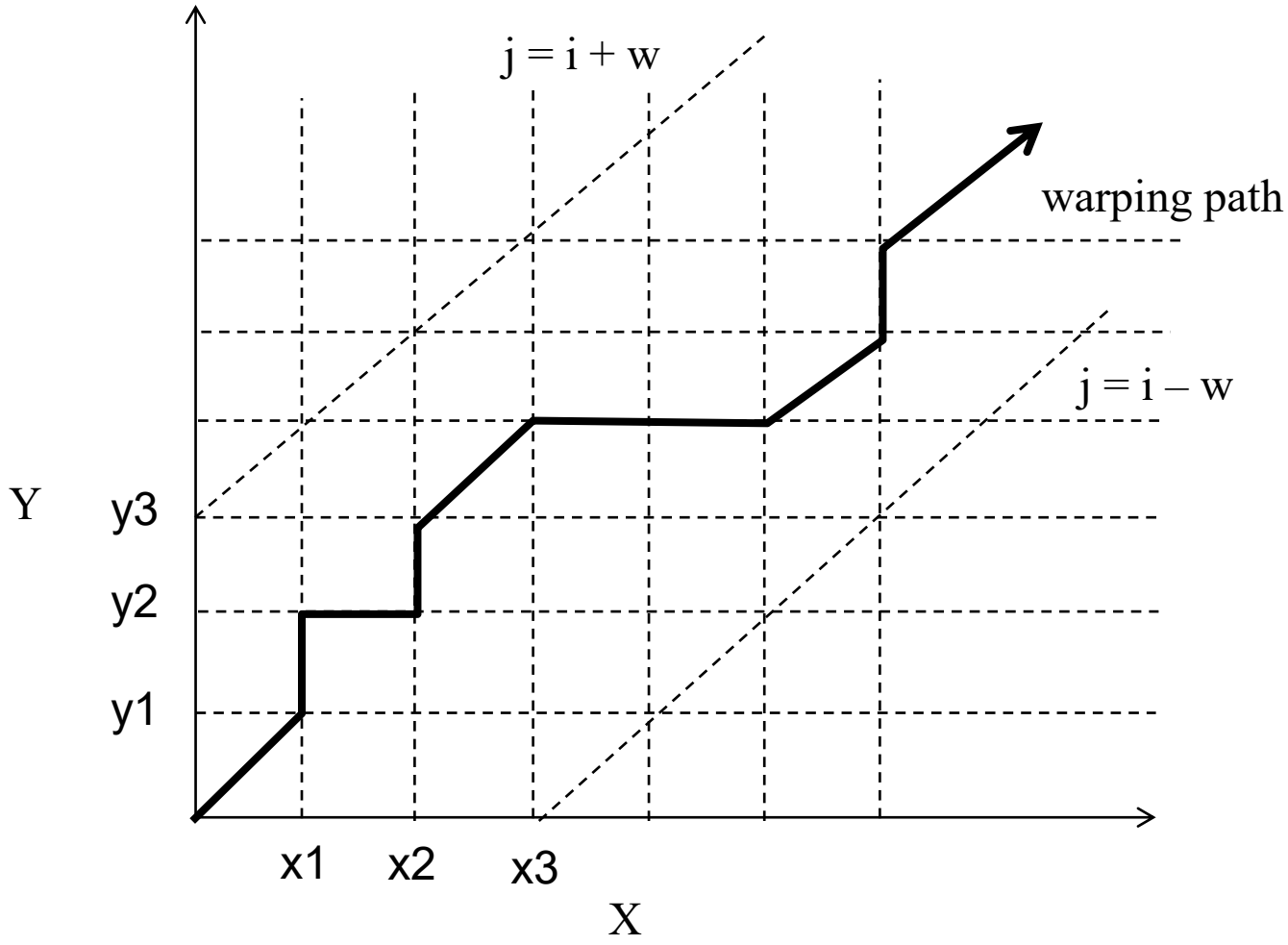
Dynamic Time Warping (DTW)

- Basic idea
  - Consider X = $x_1, x_2, \ldots, x_n$, and Y = $y_1, y_2, \ldots, y_n$
  - We are allowed to extend each sequence by repeating elements
  - Euclidean (or Manhattan) distance now calculated between the extended sequences X' and Y'



Euclidean distance vs DTW

Dynamic Time Warping (DTW)



Restrictions on Warping Paths

- Monotonicity
  - Path should not go down or to the left
- Continuity
  - No elements may be skipped in a sequence
- Speed up evaluations:
  1. Warping Window: | i – j | <= w
  2. Use *LB Keogh* lower bound of dynamic time warping
  - [*LB Keogh* lower bound method is linear whereas dynamic time warping is quadratic O(nm) in complexity which make it very advantageous for searching over large sets of time series]

Formulation

- Let D(i, j) refer to the dynamic time warping distance between the subsequences

$x_1, x_2, \ldots, x_i$

$y_1, y_2, \ldots, y_j$

$$D(i, j) = dist(x_i, y_j) + \min \{ D(i - 1, j), D(i - 1, j - 1), D(i, j - 1) \}$$

- $dist(x_i, y_j)$ can be Euclidean or Manhattan distance
- Faster solution: use Dynamic Programming instead of recursion

Classification

- k Nearest Neighbors (kNN)
  - Empirically, best results when k=1, & DTW with euclidean distance

```
neighbors = []
for every time series i in test set,
    distances = []
    for every time series j in training set
        distances.append(j, DTW(i,j))
    distances.sort(key = index(1))
    neighbors_i = []
    for m = 1..k
        neighbors_i.append(distances[m])
    neighbors.append(neighbors_i)
return neighbors
```

Clustering

- kMeans
  - choose centroids (e.g. randomly)
  - assign sequences to clusters (based on proximity to centroids)
    - Distance: DTW with euclidean
  - recalculate centroids of clusters iteratively

Hands on

- Download timeseries.zip
  - Lab11_DTW.py, dwt_functions.py
  - dtw_train.csv, dtw_test.csv
- dtw_functions.py
  - recursiveDTW, recursiveDTW_window, LB_Keogh, knn, kmeans
- Experiment with functions
  - convert knn from 1-nn to k-nn

APPENDIX B:
Holt-Winters and ARIMA methods

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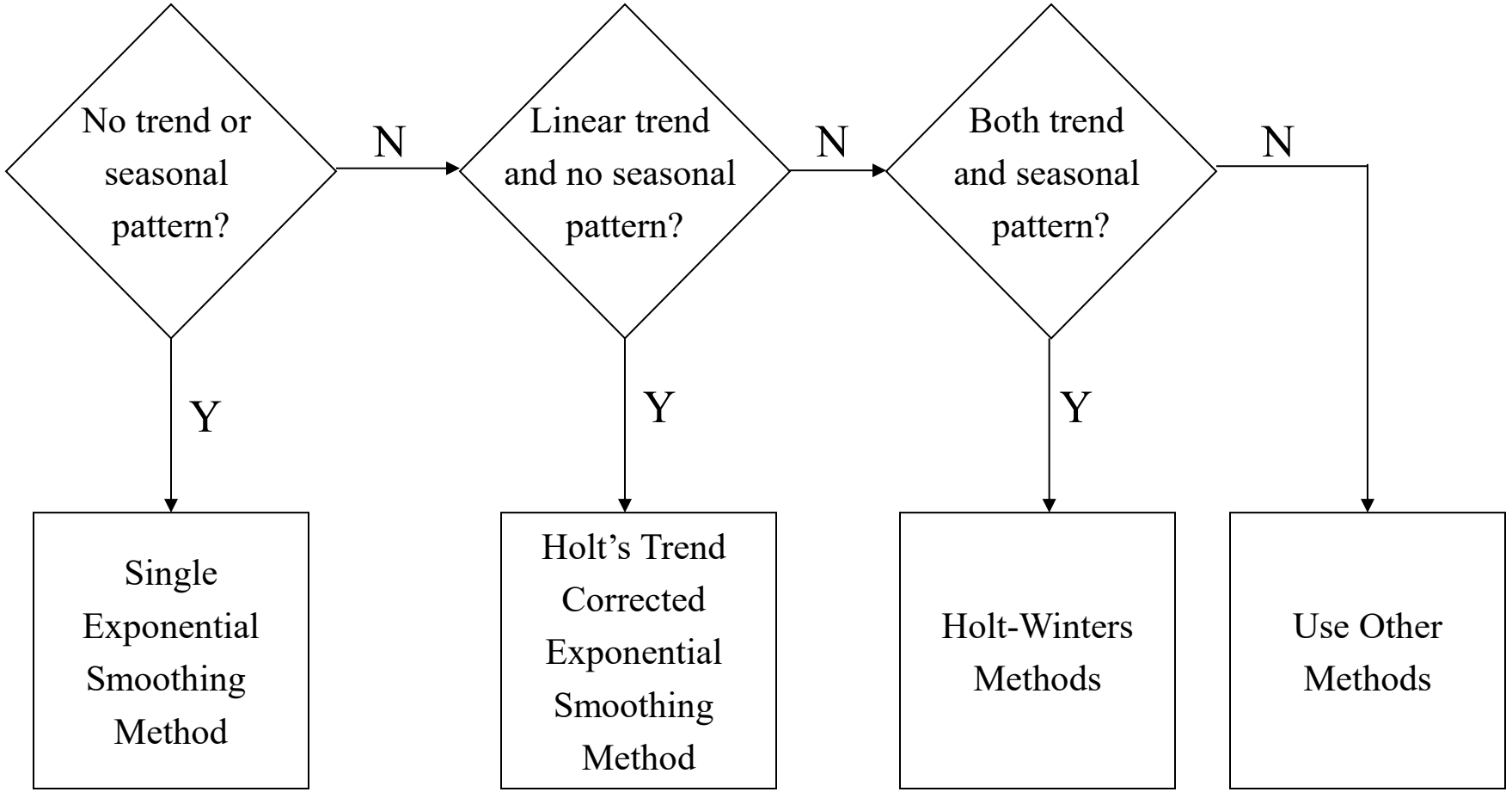
Holt-Winters Methods

- Univariate method: involves one input variable
- Forecasting using exponential smoothing
  - Larger weights to recent observations; exponential decrease of weights
    - Effective when variables of the TS changing slowly
- Used when data exhibits both seasonality and (linear) trend
  - Multiplicative model
    - Used for TS with increasing (multiplicative) seasonal variations
  - Additive model
    - for TS with a constant (additive) seasonal variations
- More information can be found here

Data vs Methods



Multiplicative Holt-Winters Method

- $y_{t+h|h} = (l_t + h * b_t) * S_{t-m+h_m}$

  Predicted values on a horizon h points

  Level **Trend** **Seasonal** component

- Estimate of the level

$$l_t = a * \left(\frac{y_t}{S_{t-m}}\right) + (1-a) * (l_{t-1} + b_{t-1})$$

- Estimate of the trend

$$b_t = \beta * (l_t - l_{t-1}) + (1-\beta) * b_{t-1}$$

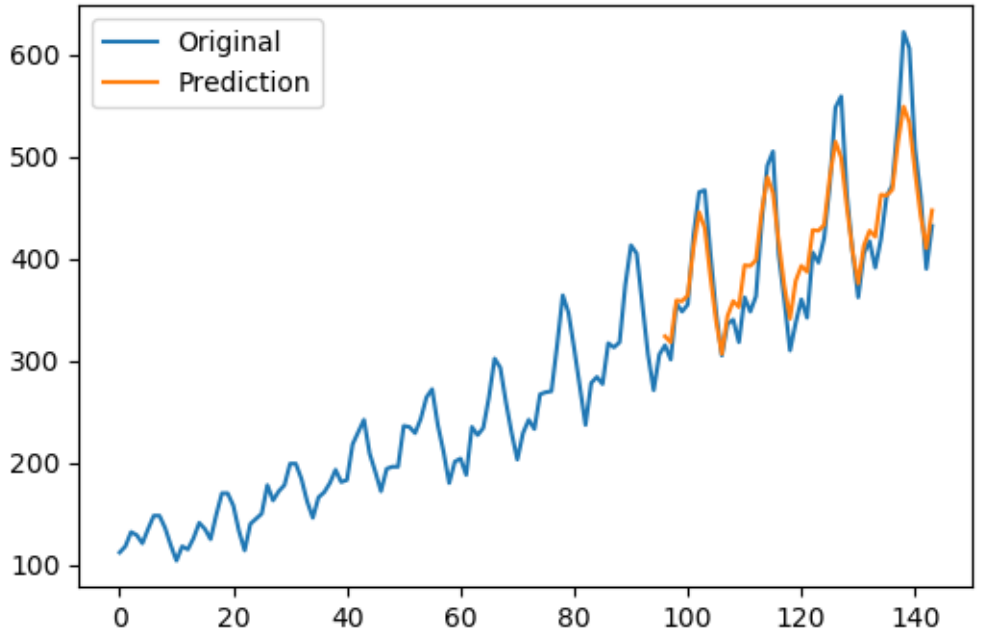
- Estimate of the seasonal factor

$$S_t = \gamma * \left(\frac{y_t}{l_t + b_t}\right) + (1-\gamma) * S_{t-m}$$

where α, β, and γ are smoothing constants between 0 and 1, m = number of seasons in a year (m = 12 for monthly data, and m = 4 for quarterly data)

Holt-Winters Forecasting

- $m = 12, \alpha = 0.26, \beta = 0.19, \gamma = 0.5$
- Predict the last 4 years ($h = 48$)



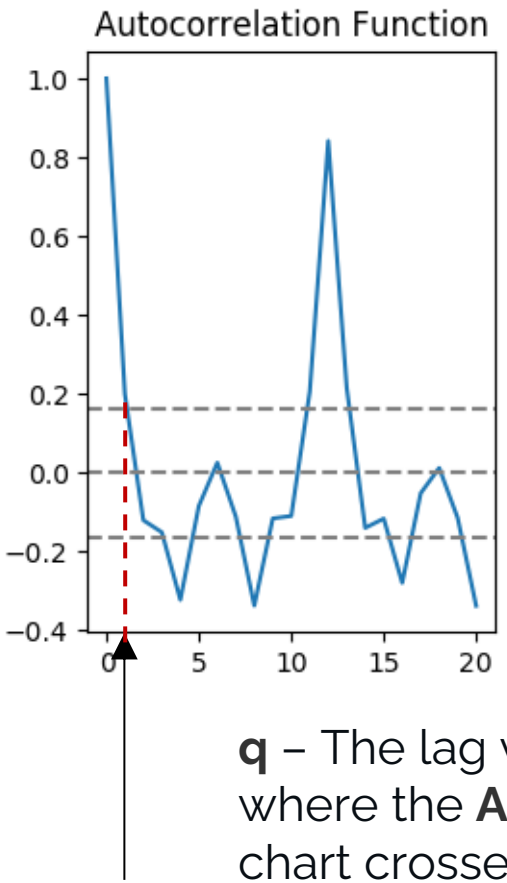
ARIMA statistical model

- Auto-Regressive Integrated Moving Averages
  - a linear equation, where predictors (features) depend on the parameters (p,d,q) of the ARIMA model:
    - **Number of AR (Auto-Regressive) terms (p):** AR terms are just lags of dependent variable. For instance if p is 5, the predictors for x(t) will be x(t-1)….x(t-5).
    - **Number of MA (Moving Average) terms (q):** MA terms are lagged forecast errors in prediction equation. For instance if q is 5, the predictors for x(t) will be e(t-1)….e(t-5) where e(i) is the difference between the moving average at i^{th} instant and actual value.
    - **Number of Differences (d):** These are the number of nonseasonal differences, i.e. in this case we took the first order difference. So either we can pass that variable and put d=0 or pass the original variable and put d=1. Both will generate same results.

ARIMA statistical model

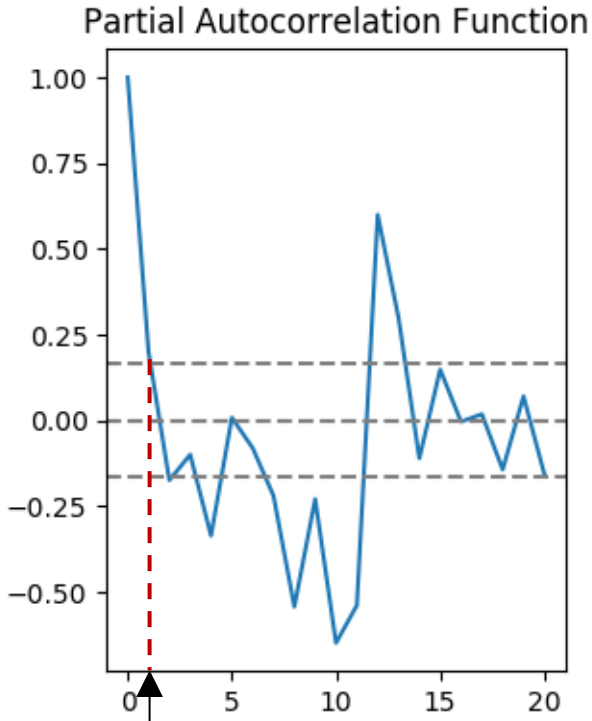
- Use 2 plots to determine the value of ‘p’ and ‘q’:
  - **Autocorrelation Function (ACF):** It is a measure of the correlation between the TS with a lagged version of itself. For instance at lag 5, ACF would compare series at time instant ‘t1’…’t2’ with series at instant ‘t1-5’…’t2-5’ (t1-5 and t2 being end points).
  - **Partial Autocorrelation Function (PACF):** This measures the correlation between the TS with a lagged version of itself but after eliminating the variations already explained by the intervening comparisons. E.g. at lag 5, it will check the correlation but remove the effects already explained by lags 1 to 4.

Choosing “best” q and p values



q=2

q – The lag value where the **ACF** chart crosses the upper confidence interval for the first time.

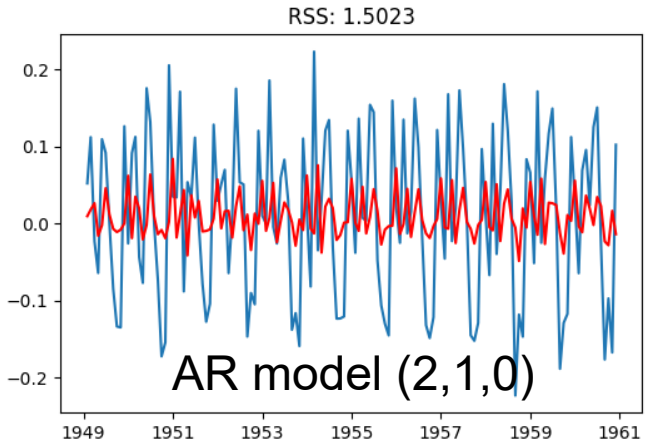


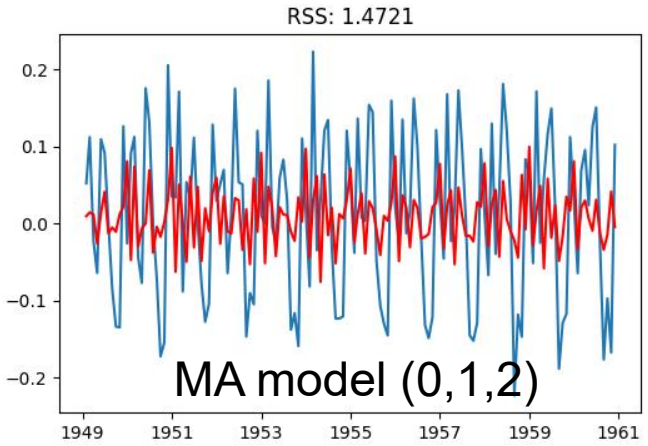
p=2

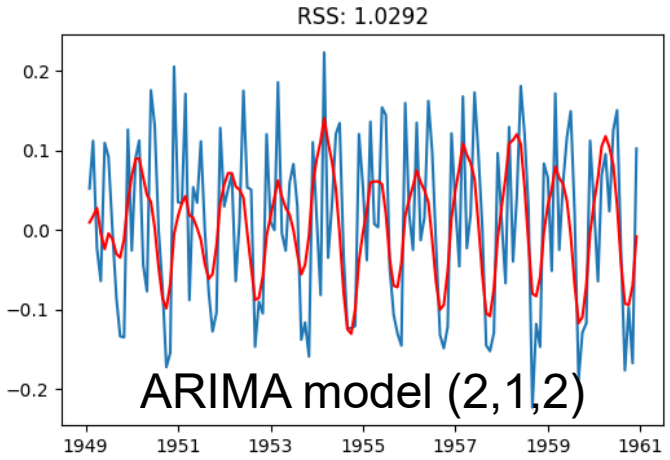
p – The lag value where the **PACF** chart crosses the upper confidence interval for the first time.

ARIMA models

- 3 different ARIMA models (AR, MA, ARIMA) considering individual as well as combined effects

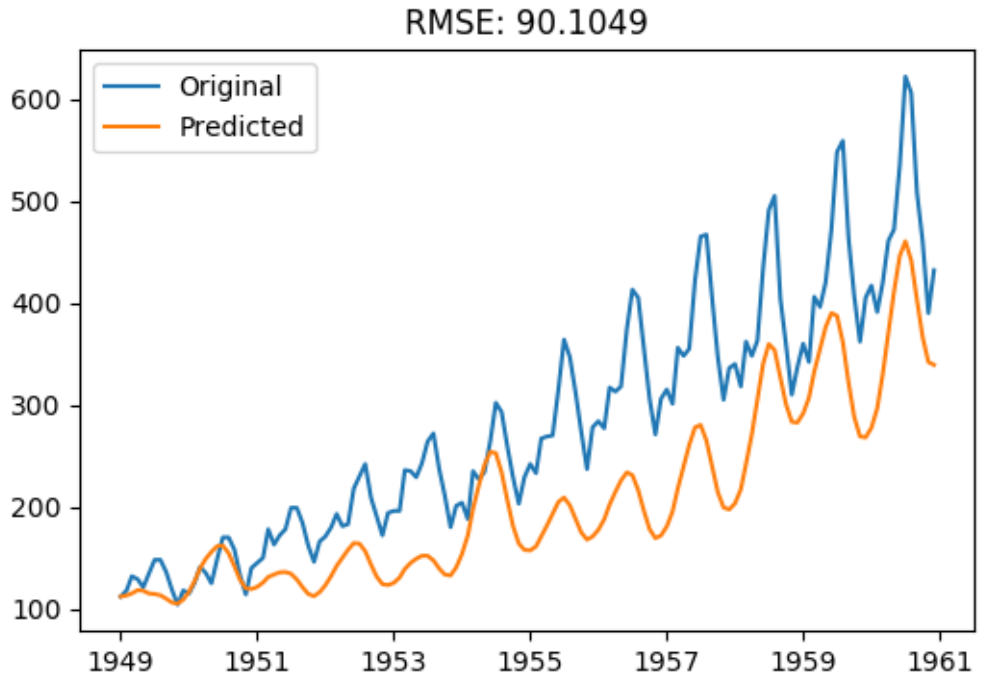






ARIMA results

- Original TS vs predicted TS



APPENDIX C: Exponential Weighted Moving Average (EWMA)

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EWMA Formula

$$EWMA_t = \alpha * r_t + (1 - \alpha) * EWMA_{t-1}$$

- **alpha** = The weight decided by the user
- **r** = Value of the series in the current period
- The EWMA is a recursive function, which means that the current observation is calculated using the previous observation. The EWMA's recursive property leads to the exponentially decaying weights as shown below:

$$EWMA_t = \alpha * r_t + (1 - \alpha) * (\alpha * r_{t-1} + (1 - \alpha) * EWMA_{t-2})$$

$$EWMA_t = \alpha * r_t + (1 - \alpha) * \left(\alpha * r_{t-1} + (1 - \alpha) * (\alpha * r_{t-2} + (1 - \alpha) * EWMA_{t-3})\right)$$

EWMA Formula

- The process continues until we reach the base term $EWMA_0$. The equation can be rearranged to show that the $EWMA_t$ is the weighted average of all the preceding observations, where the weight of the observation r_{t-k} is given by: $\alpha * (1 - a)^k$
- Since alpha is between 0 and 1, the weight becomes smaller as k becomes larger. In other words, as we go back further in history, the weight becomes smaller. The fact is illustrated in the chart below, which plots the weights of observation as k increases for different choices of the parameter alpha.

EWMA Weights