

# DSC510: Introduction to Data Science and Analytics

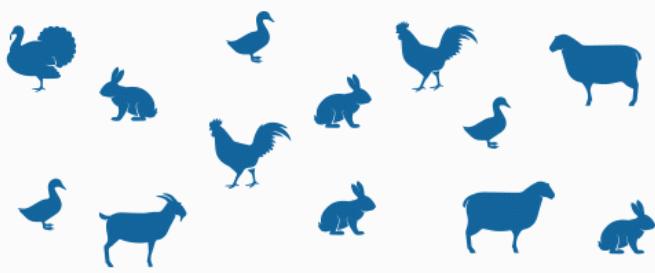
## Lab 8: Classification



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Classification

Clustering



## Recap from last lab

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- In the previous two labs, we explored several regression techniques, including linear regression, polynomial regression, Support Vector Regression (SVR), and ensemble methods such as Random Forest.
- We discussed cross-validation, a technique used to reduce overfitting by splitting the training data into multiple folds for training and validation.
- We introduced GridSearchCV, a method for systematically exploring a grid of hyperparameter combinations for an estimator or a pipeline (including transformations and a regressor).
- In this lab, our focus shifts to classification problems – exploring classification techniques and learning how to evaluate their performance.

# Classification

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- Supervised Machine Learning (ML) process of predicting the class or category of data based on a set of predefined classes
- Classification algorithms are trained on datasets containing labeled data
- Labeled data is data observations that have already been classified
  - For example, in a dataset of emails, each email might be labeled as "spam" or "not spam".
  - These labels (target variable) then provide a clear guide for a classification machine learning algorithm to learn from.



# Types of Classification

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- **Binary Classification**
  - process of classification in which **input data observations are being classified into one of two discrete classes**
  - For example, a medical dataset which classifies patients into those that have a specific disease versus those that do not (e.g. COVID positive vs COVID negative)
- **Multi-class Classification**
  - Process of classification in which **input data observations are being classified into one of three or more classes**
  - For example, medical profiling dataset that classifies patients into those with kidney, liver, lung, or stomach infection symptoms



# Classification algorithms

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- Popular algorithms used for both binary and multi-class classification:
    - Logistic Regression
    - Support Vector Classifier (SVC)
    - Multinomial Logistic Regression
    - k-Nearest Neighbors (kNN)
    - Decision Tree Classifier
    - Gaussian Naive Bayes Classifier
    - Stochastic Gradient Descent Classifier
    - Linear Discriminant Analysis
    - Ensemble algorithms (Random Forest, Gradient Boosting, XGBoost)
-



# Is rescaling/unskewing needed?

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- Feature scaling is recommended prior training classification algorithms that use the notion of (Euclidean) distance between data points to determine their similarity (whether they belong to the same class or category) such as:
  - k-Nearest Neighbors (kNN)
    - Normalization (MinMax scaler) is usually more effective than standardization for KNN because all features are mapped to the same range of values (e.g. between 0 and 1)
  - Support Vector Classifier (SVC)
    - Standardization (Standard or Robust scaler) is usually more effective when the rbf kernel is used in SVC because rbf assumes that features are centered around zero
  - Stochastic Gradient Descent Classifier (SGDClassifier)
    - Standardization (Standard or Robust scaler) benefits SGDClassifier because it allows faster, more stable convergence, preventing biases caused by feature scales



# Is rescaling/unskewing needed?

- Unskewing techniques (e.g. BoxCox, Yeo-Johnson, Sqrt, Log) are generally recommended on highly skewed features. In a related study<sup>1</sup>, the use of BoxCox transformation has been shown to increase the accuracy of various classifiers (Linear Classifier, KNN, SVC, Bayesian)
- Rescaling/unskewing of target variable (that includes class/category values) does not make any sense in classification problems
- There is no way to know in advance if feature rescaling or unskewing will provide better prediction results. You can always start by fitting (training) your model on (a) raw, (b) normalized, (c) standardized and (d) unskewed data and then comparing the prediction performance of each model

# Classification model evaluation metrics

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- True Positive (TP): When you predict an observation belongs to a class and it actually does belong to that class
    - **Correctly** (true) predicted positive class
    - A passenger who is classified as COVID positive and is actually positive
  - True Negative (TN): When you predict an observation does not belong to a class and it actually does not belong to that class
    - **Correctly** (true) predicted negative class
    - A passenger who is classified as not COVID positive (negative) and is actually not COVID positive (negative)
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# Classification model evaluation metrics

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- False Positive (FP): When you predict an observation belongs to a class and it actually does not belong to that class
    - **Incorrectly** (false) predicted positive class
    - A passenger who is classified as COVID positive and is actually not COVID positive (negative)
  - False Negative (FN): When you predict an observation does not belong to a class and it actually does belong to that class
    - **Incorrectly** (false) predicted negative class
    - A passenger who is classified as not COVID positive (negative) and is actually COVID positive
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# Confusion matrix

- A confusion matrix is a table that is often used to describe the performance of a classification model (or "classifier")

|           |          | ACTUAL   |          |
|-----------|----------|----------|----------|
|           |          | Positive | Negative |
| PREDICTED | Positive | TP       | FP       |
|           | Negative | FN       | TN       |

Annotations:

- Correctly Predicted COVID +ve passenger as +ve (points to TP)
- Incorrectly Predicted COVID -ve passenger as +ve (points to FP)
- Incorrectly predicted COVID +ve Passenger as -ve (points to FN)
- Correctly predicted COVID -ve passenger as -ve (points to TN)



# Accuracy

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- Accuracy is one metric which gives the fraction of predictions our model got right
- $$Accuracy = \frac{Number\ of\ correct\ predictions}{Total\ number\ of\ predictions} = \frac{TP+TN}{TP+FP+TN+FN}$$
- Ranges from 0 to 1



# Is accuracy a good metric?

- Now, let's consider 50,000 passengers travel per day on an average. Out of which, 10 are actually COVID positive.
- One of the easiest strategies to increase accuracy is to classify every passenger as COVID negative. So, our confusion matrix looks like:

| ACTUAL    |          |            |                              |
|-----------|----------|------------|------------------------------|
|           |          | Positive   | Negative                     |
| PREDICTED | Positive | TP<br>= 0  | FP<br>= 0                    |
|           | Negative | FN<br>= 10 | TN<br>$50,000 - 10 = 49,990$ |

$$\text{Accuracy} = \frac{49,990}{50,000} = 0.9998 \text{ or } 99.98\%$$

- We achieve more accuracy than we have ever seen in any model, but this does not solve our purpose which is:
- We need to identify COVID positive passengers!



# Is accuracy a good metric?

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- Not labeling 10 of actually positive passengers entering the country will result in increasing the spread in the community
- Accuracy in this context is a terrible measure because it's easy to get extremely good accuracy but that's not what we are interested in
- But is accuracy always a poor measure? When the **data is balanced**, **accuracy is a good measure** of evaluating a model. On the other hand if **data is imbalanced** (as in our case), then **accuracy is not a correct measure** of evaluation
  - What is data imbalance: number of samples between classes is uneven



# Recall (Sensitivity or True Positive rate)

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- Recall gives the fraction you correctly identified as positive out of all actual positives – a **measure** of a **classifier's completeness**
  - how “sensitive” the classifier is to detecting positive cases
- $$\text{Recall} = \frac{\text{Number of correct positives}}{\text{All positives}} = \frac{TP}{TP+FN}$$
  - Out of all positive passengers what fraction you identified correctly
  - Going back to our previous strategy of labeling every passenger as COVID negative that will give recall of zero:  $\text{Recall} = 0/10 = 0$
  - So, in this context, Recall is a good measure. It says that the terrible strategy of identifying every passenger as COVID negative leads to zero recall
  - We want to maximize the recall  $\rightarrow 1$
  - Is Recall alone good enough to evaluate the performance of a classification model?

# Recall

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- To answer the previous question, consider another naive strategy of labeling every passenger as COVID positive
- The confusion matrix will look like:
- $Recall = \frac{TP}{TP+FN} = \frac{10}{10+0} = 1$
- Although recall is perfect, this strategy produces a very large number of FP
- Recall alone is not a sufficient performance measure in this scenario

|           |          | ACTUAL     |                              |
|-----------|----------|------------|------------------------------|
|           |          | Positive   | Negative                     |
| PREDICTED | Positive | TP<br>= 10 | FP<br>$50,000 - 10 = 49,990$ |
|           | Negative | FN<br>= 0  | TN<br>= 0                    |



# Precision

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- Fraction of correctly identified as positive out of all predicted as positives – a **measure** of a **classifier's exactness**
  - Refers to a model's ability to correctly interpret positive observations
- $Precision = \frac{\text{Number of correct positives}}{\text{All predicted positives}} = \frac{TP}{TP+FP}$
- Considering our second bad strategy of labeling every passenger as positive, the precision would be :
- $Precision = \frac{TP}{TP+FP} = \frac{10}{10+49990} = 0.0002$



# Recall vs Precision

- While this bad (2<sup>nd</sup>) strategy has a good recall value of 1 but it has a terrible precision value of 0.0002
- This clarifies that recall alone is not a good measure, we need to consider precision value as well
- Considering another (3<sup>rd</sup>) strategy of labeling only one passenger (correctly) as COVID positive whereas the rest as COVID negative. The confusion matrix in this case will be

$$Precision = \frac{TP}{TP + FP} = \frac{1}{1 + 0} = 1$$

$$Recall = \frac{TP}{TP + FN} = \frac{1}{1 + 9} = 0.1$$

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN} = \frac{49991}{50000} = 0.99984$$

|           |          | ACTUAL    |                             |
|-----------|----------|-----------|-----------------------------|
|           |          | Positive  | Negative                    |
| PREDICTED | Positive | TP<br>= 1 | FP<br>= 0                   |
|           | Negative | FN<br>= 9 | TN<br>$50,000 - 9 = 49,991$ |



# Recall vs Precision

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- In some cases, we want to maximize either recall or precision at the cost of the other
  - As in this case of labeling passengers, we really **want to get the predictions right for COVID positive passengers** because it is really expensive to not predict the passenger right as allowing COVID positive person to proceed will result in increasing the spread. So, here, we are **more interested in recall**.
- Unfortunately, sometimes it's difficult to have it both ways: often, increasing precision reduces recall and vice versa. This is called precision/recall tradeoff.

$$Precision = \frac{TP}{TP + FP}$$

$$Recall = \frac{TP}{TP + FN}$$

High scores for both show that the classifier is returning accurate results (high precision), as well as returning a majority of all positive results (high recall). **To maximize both, we need to minimize the number of false labels.**



# F-1 score

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- Often convenient to combine precision and recall into a single metric
- F1 score is the harmonic mean of the model's precision and recall
- $$F1 \text{ score} = \frac{2 * Precision * Recall}{Precision + Recall} = \frac{TP}{TP + \frac{1}{2}(FP + FN)}$$
- Why harmonic mean and not simple average?
  - Not sensitive to extremely large values, unlike simple averages
  - Example: a model with a precision of 1, and recall of 0 gives a simple average as 0.5 and an F-1 score of 0
  - If one of the parameters is low, the second one no longer matters in the F-1 score.
  - F-1 score favors classifiers that have similar precision and recall
  - F-1 score is a better measure to use if you are seeking a balance between Precision and Recall



# F-1 score

- Previous formula can be only used in a binary classification problem
- In a multi-class classification problem, we obtain **one F1-score per class** (instead of a single overall F1-score)
  - instead of having multiple per-class F1-scores, it would be better to average them to obtain a single number to describe overall performance
    - **Macro average:** mean of all the per-class F1 scores. This method treats all classes equally regardless of the number of samples in each class. Not recommended for unbalanced datasets.
    - **Weighted average:** weighted mean of all the per-class F1 scores. Each class F1-score is multiplied by the percentage of samples belonging to this class (e.g. majority class is given higher weight). **Recommended for unbalanced multi-class datasets.**
    - **Micro average:** computes a global average F1 score by counting the sums of the True Positives (TP), False Negatives (computes the proportion of correctly classified observations FN), and False Positives (FP) for all classes collectively. In other words, it out of all observations which is the **same as measuring the accuracy**.

```
from sklearn.metrics import f1_score  
f1_score(y_true, y_pred, average='macro') ← or weighted or micro
```



# F-beta score

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- The F-beta score calculation follows the same form as the F-1 score, however it also allows you to decide how to **weight the balance** between precision and recall using the beta parameter

$$\bullet \text{Fbeta score} = \frac{(1+beta^2) * Precision * Recall}{beta^2 * Precision + Recall}$$

$$Precision = \frac{TP}{TP + FP}$$

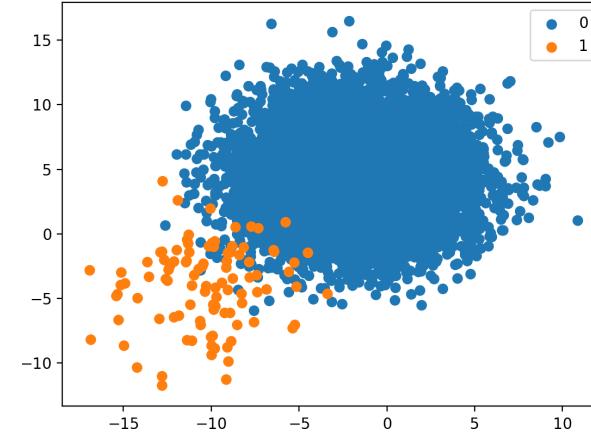
$$Recall = \frac{TP}{TP + FN}$$

- When  $\text{beta}=1$ , the F-beta score is equivalent to the F-1 Score
- When  $\text{beta}<1$ , this score is the F-0.5 score which **raises the importance of precision** and lowers the importance of recall (goal: minimize False Positives)
- When  $\text{beta}>1$ , this score is the F-2 score which **raises the importance of recall** and lowers the importance of precision (goal: minimize False Negatives)



# Balanced vs unbalanced target variable

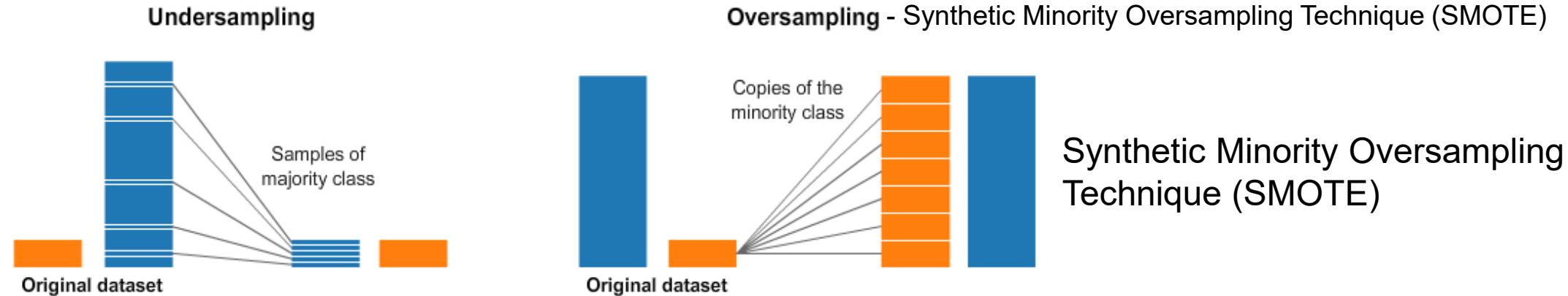
- Data imbalance: typical problem for real-world datasets:
  - number of samples between classes is uneven
    - When size of the majority class gets more than twice the size of the minority class, dataset begins to be considered unbalanced
- Machine learning model tends to be better at predicting the class with more samples (majority class) than the other with fewer samples (minority classes)
  - The greater this imbalance, the higher the bias of the model towards the majority class
- Class imbalance is affected by both the **absolute size of the minority class** and the **imbalance ratio**. Even with enough data, extreme class imbalance can still cause the model to perform poorly on the minority class, necessitating additional strategies to address it.





# Balanced vs unbalanced target variable

- Methods for balancing data are available – see [here](#)



- However, the problem of (artificially) balanced data can be worse than the unbalanced case – see [Appendix](#) (model affected by rare points)
- Effective metrics for unbalanced datasets:
  - Precision, Recall, F-1 score (with weighted average)



# When to use balancing techniques?

- Rule of thumb: you can always track performance of unbalanced classification with Precision/Recall/F-1 score metrics first and then decide whether you need to proceed towards balancing or not
  - If the F1 score is acceptably high thus both precision and recall are reasonable, you may decide that balancing isn't necessary potentially saving time and computational resources
    - **High Scores:** Typically **0.8 or above** (80%+) → strong model performance
    - **Moderate Scores:** Between **0.6 and 0.8** (60%-80%) → model is doing reasonably well but may still miss a portion of minority class instances (try applying balancing techniques)
  - if the metrics are low, they indicate that the model struggles with minority class predictions, signaling a need to consider techniques such as undersampling or oversampling (e.g., SMOTE) to help balance performance
    - **Low Scores:** Below **0.6** (less than 60%) → model struggles to recognize the minority class or is heavily biased toward the majority class



# Stratified Cross Validation

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- Let's assume you are doing a multiclass classification and have an imbalanced dataset that has 5 different classes. The randomized train-test split used in Kfold CV totally disregards the percentage of samples for each class in the resulting splits
- What happens in the scenario of ending up with train and a test sets with totally different data distributions for each class?
  - A model trained on a vastly different data distribution than the test set, will perform poorly at evaluation step
- The solution to this problem is called stratification
  - StratifiedKFold is a variation of KFold which preserves the percentage of samples for each class in train and test sets
- In GridSearchCV, StratifiedKFold is automatically used when estimator is classifier, and y is either binary or multiclass



# Example: Telco Dataset

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- A fictional telco company that provided home phone and Internet services to 7043 customers in California in Q3
- Dataset available [here](#)
  - 11 rows have missing values => removed
- Each row represents a customer with 21 features
  - Both categorical and numerical
- Target value: Churn – customers decision whether to leave (Yes/No)
- Binary classification problem
  - Predict whether a customer will leave or stay at the end of the quarter

# Explore dataset

df.describe()



- 4 Numerical Features
  - SeniorCitizen
    - customer is 65 or older: 1 (Yes), 0 (No)
  - Tenure
    - months that the customer has been with the company
  - MonthlyCharges
    - customer's current total monthly charge for all their services
  - TotalCharges
    - Tenure\*MonthlyCharges
- 16 Categorical Features (most of them are Yes/No, other are categorical)
  - Services that each customer has signed up for – phone, multiple lines, internet, online security, online backup, device protection, tech support, streaming TV and streaming movies
  - Customer account information – id, contract, payment method, paperless billing
  - Demographic info about customers – gender, and if they have partners and dependents
- Target Variable: Churn

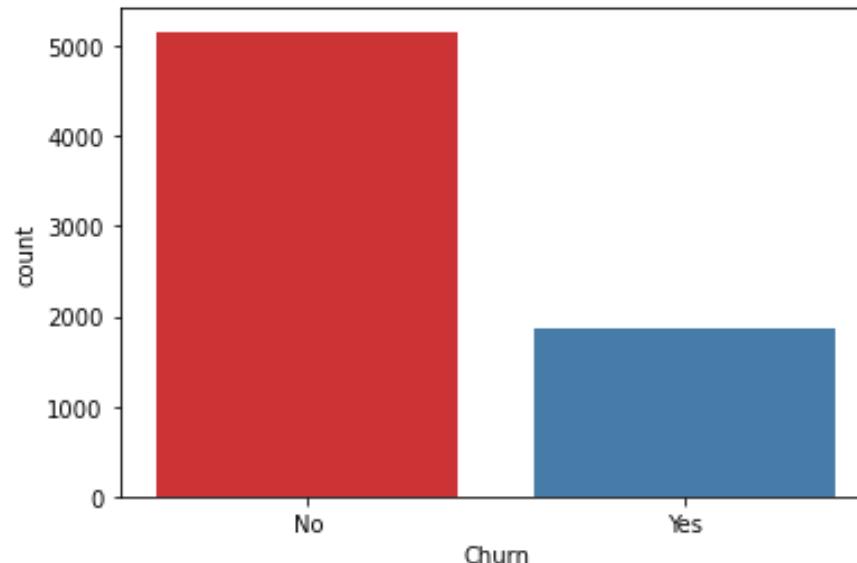
|       | SeniorCitizen | tenure      | MonthlyCharges | TotalCharges |
|-------|---------------|-------------|----------------|--------------|
| count | 7032.000000   | 7032.000000 | 7032.000000    | 7032.000000  |
| mean  | 0.162400      | 32.421786   | 64.798208      | 2283.300441  |
| std   | 0.368844      | 24.545260   | 30.085974      | 2266.771362  |
| min   | 0.000000      | 1.000000    | 18.250000      | 18.800000    |
| 25%   | 0.000000      | 9.000000    | 35.587500      | 401.450000   |
| 50%   | 0.000000      | 29.000000   | 70.350000      | 1397.475000  |
| 75%   | 0.000000      | 55.000000   | 89.862500      | 3794.737500  |
| max   | 1.000000      | 72.000000   | 118.750000     | 8684.800000  |



# Observations: Unbalanced

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- Dataset (target variable) is imbalanced
  - Churn “No” is almost 3 times as “Yes”
  - Accuracy is not the best model evaluation metric and it seems we need to consider Precision, Recall and F-1/F-beta Score

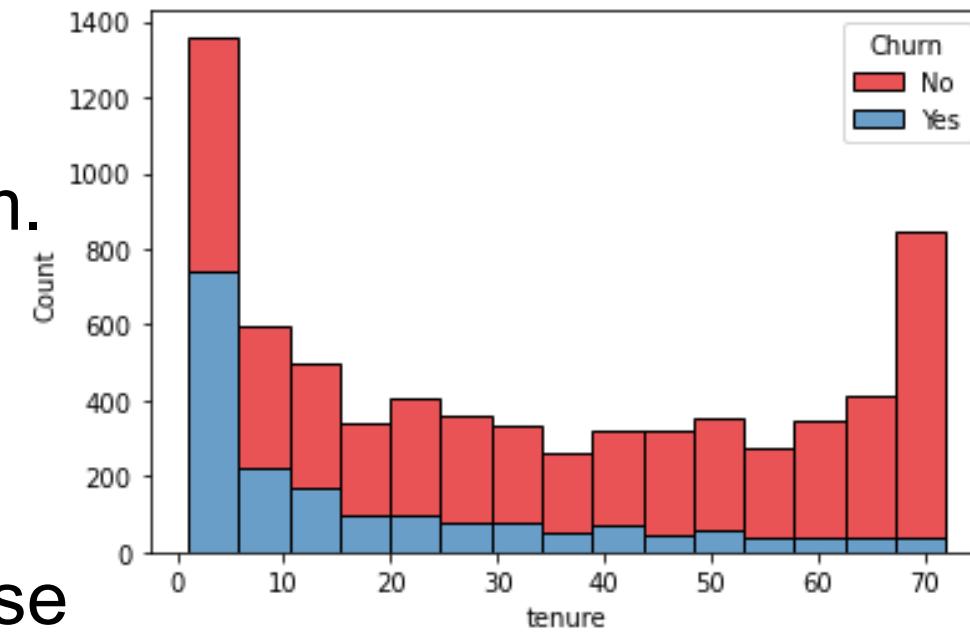




# Observations: Tenure vs Churn

- Most customers who churned, left within the first 5 months (~600+), and churn rates decline steadily over time.
- Customers retained for 10-20 months are much more likely to remain long-term.
- Nearly all customers with 72 months of tenure have not churned.
- Tenure is a strong predictor of churn (considered as important feature) because it shows clearly different patterns between customers who stayed and those who left.

Histogram of tenure for different the values of the target variable



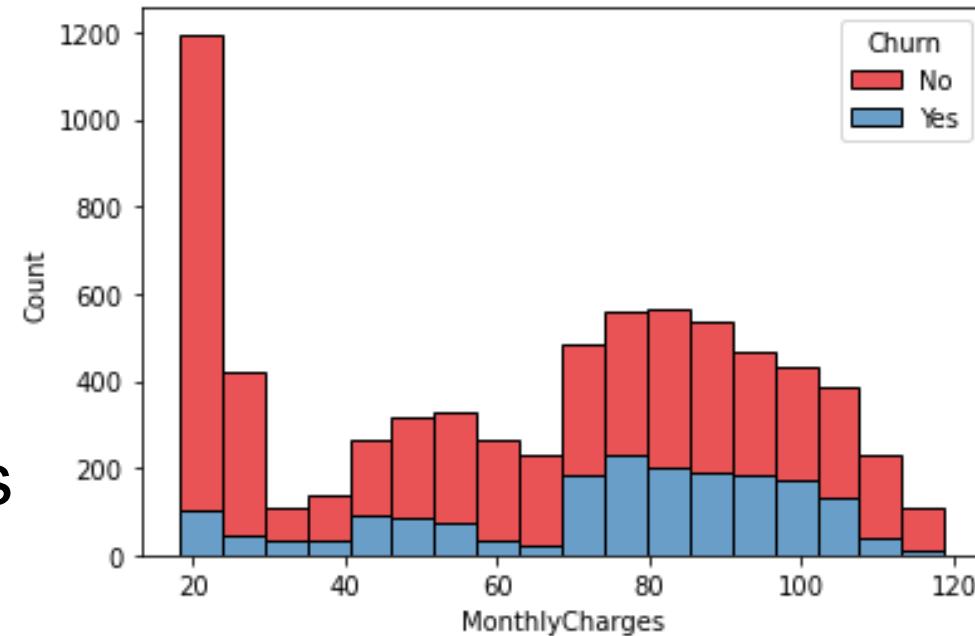
(number of months with the company)



# Observations: MonthlyCharges vs Churn

- Majority of customers pay low small monthly charges (\$18 – 20) and tend to be loyal
- Customer leaving are mostly in the band of \$75-100 who have opted for multiple services
- MonthlyCharges seems to be also a significant predictor of churn – it exhibits different patterns between customers who stayed and those who left

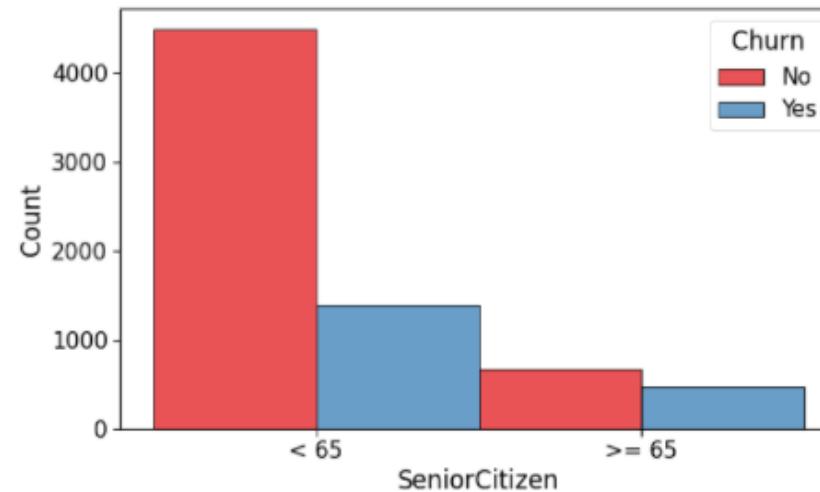
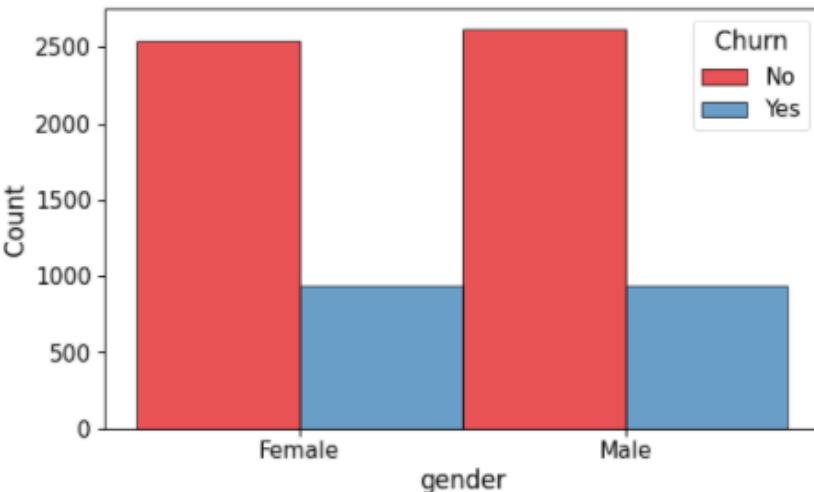
Histogram of monthly charges for different the values of the target variable



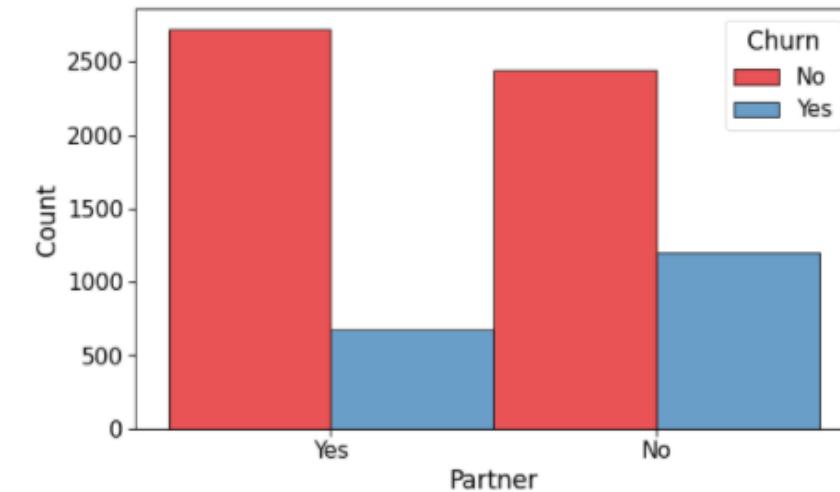


# Observations

- gender: Difficult to determine churn using this field. Counts are almost same in either category – not significant feature
- SeniorCitizen: Almost 50% of senior customers tend to leave
  - Since the share of senior customers is 16% of the total amount of customers, this indicator requires further research with additional data
- Partner: Customers with partner have lower chance of leaving



Seems significant feature

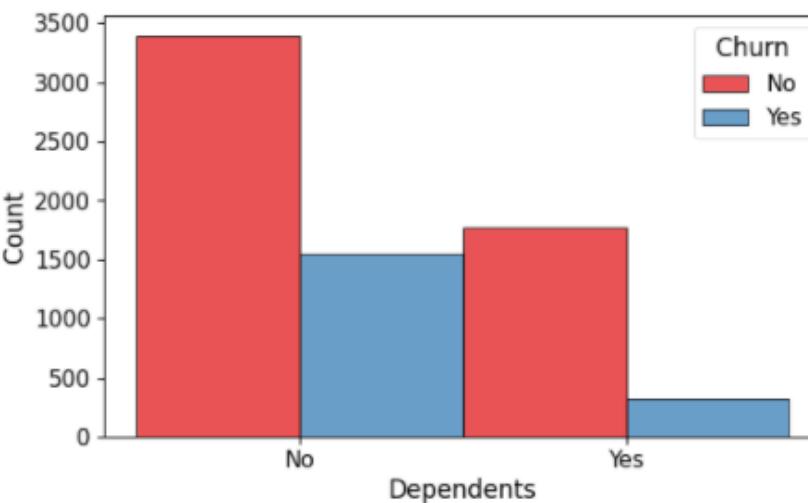


Significant feature

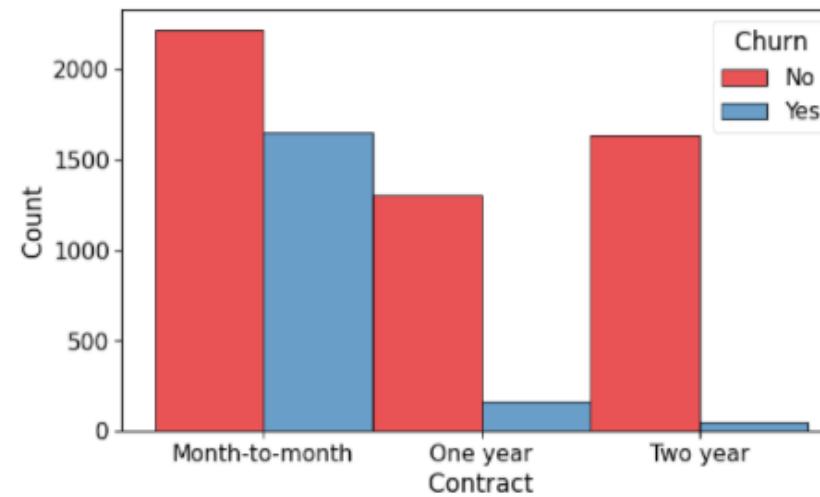


# Observations

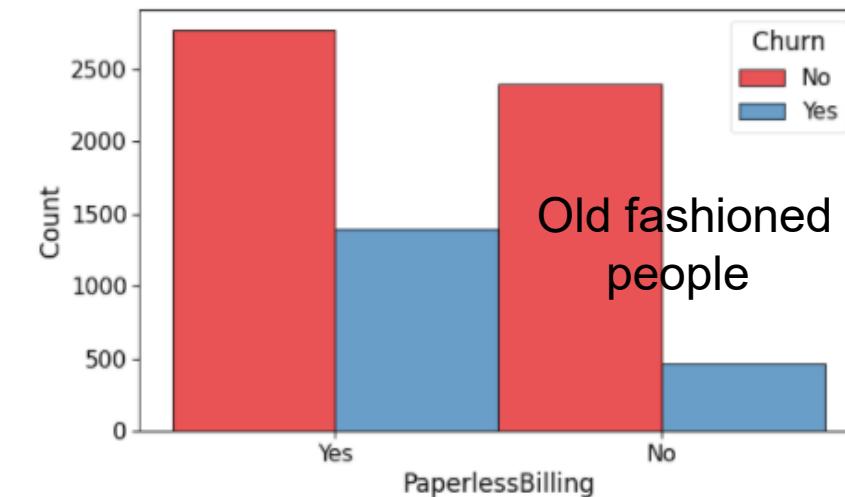
- Dependents: Customer with dependents have lower chance of leaving
- Contract: Month to Month customers have likely higher chances to leave; Old customers are more likely to stay
- PaperlessBilling: Customers with paperless billing have higher chances of leaving compared to more customers preferring traditional paper billing



Significant feature



Significant feature

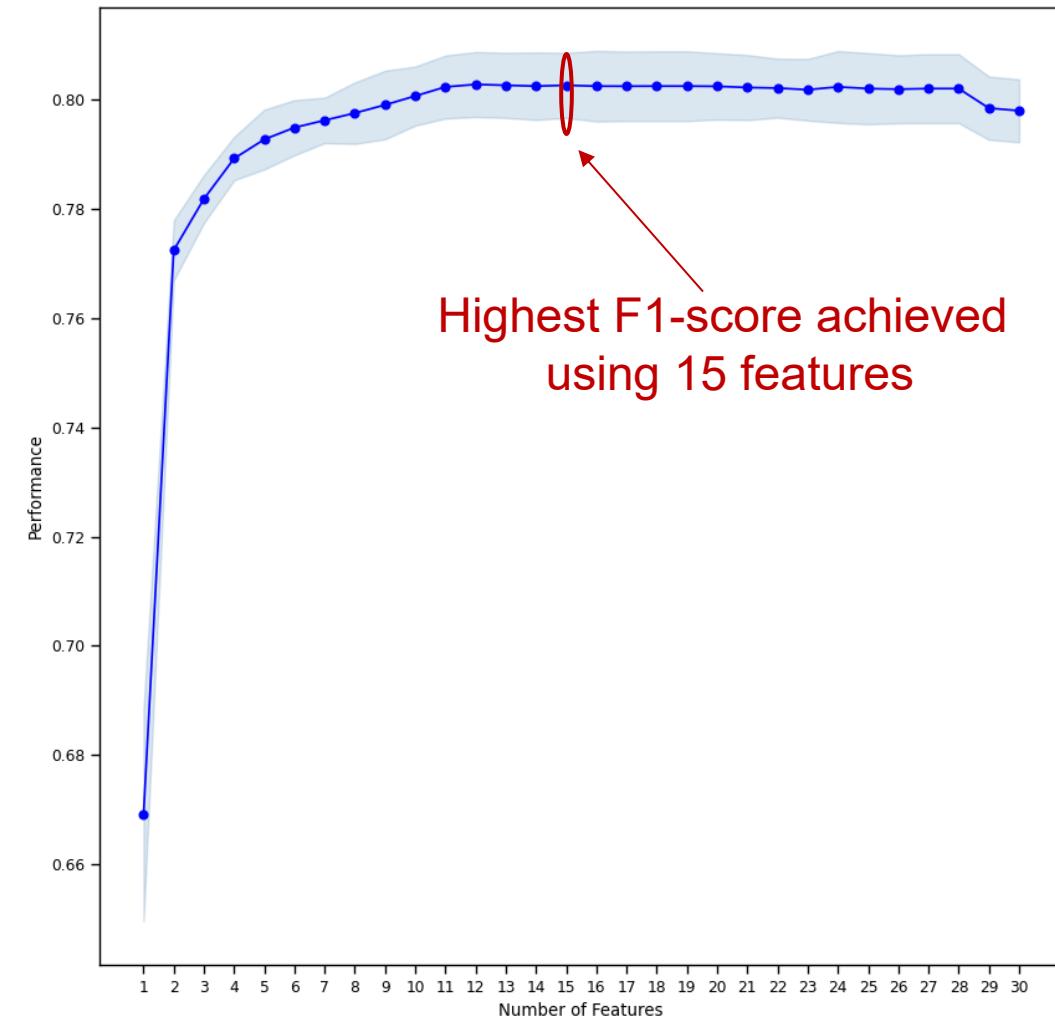


Significant feature



# Feature Selection (SFS technique)

- Categorical data are converted to numerical using one hot encoding
  - End up with 30 numerical features
- Sequential Forward Selection (SFS) technique used for feature selection (elimination)
  - keep 15 most important features that maximize the weighted F-1 score (weighted average)

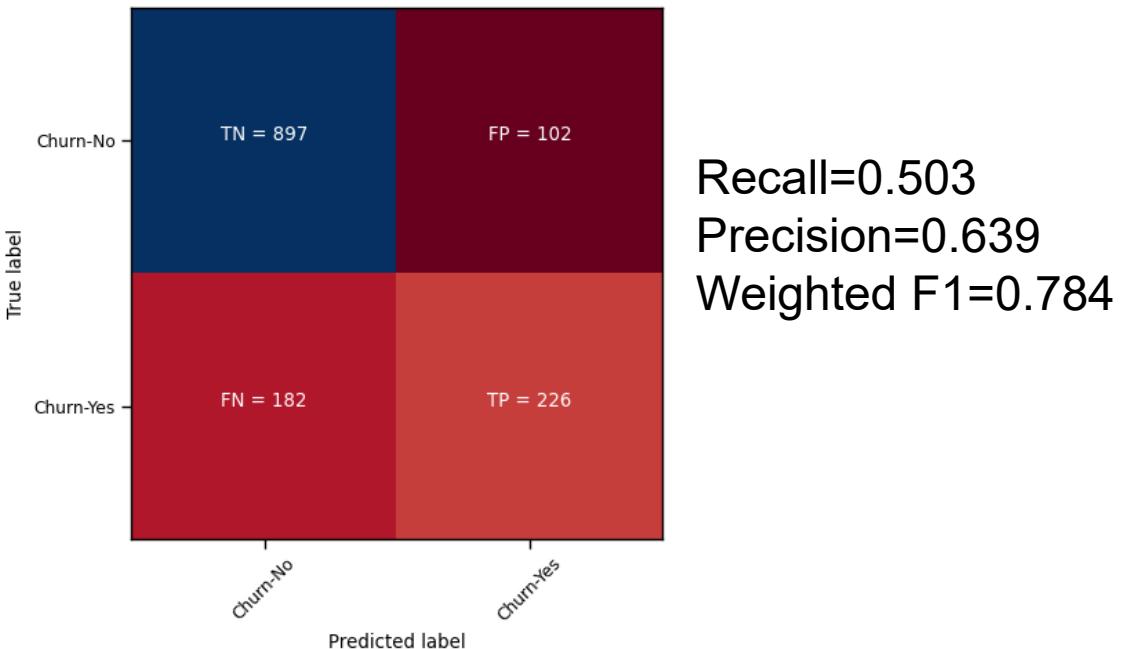




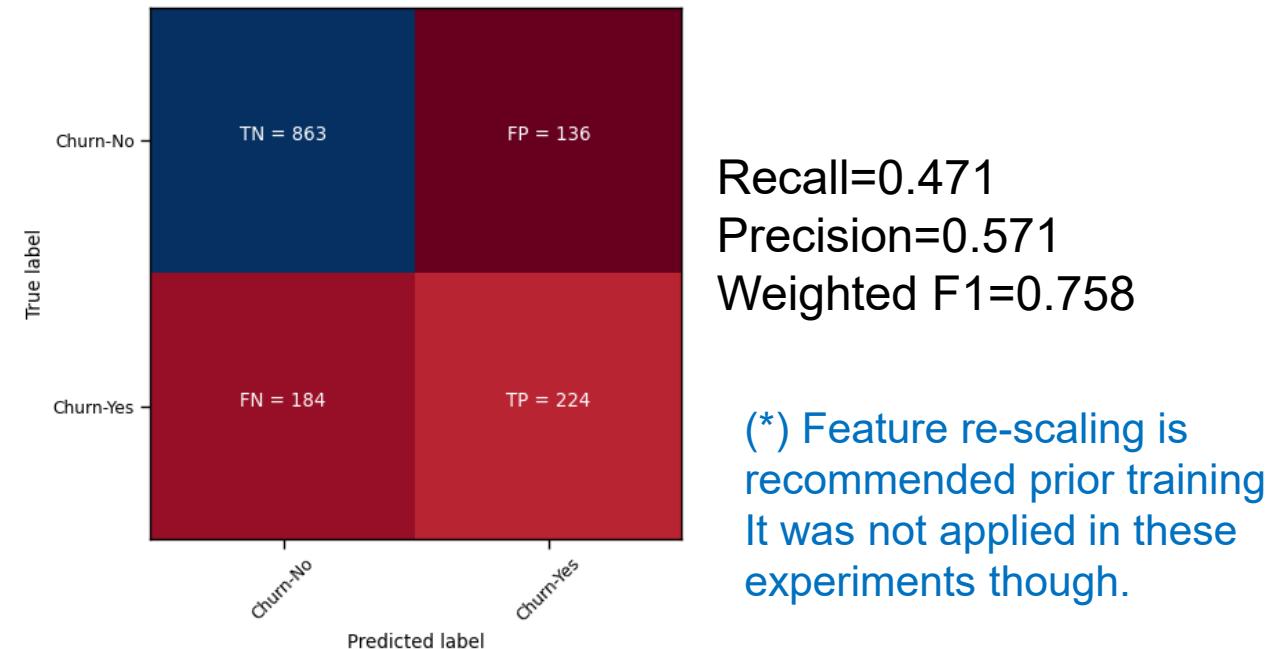
# Initial Evaluation

- Run a large set of classifiers using default hyper-parameters values (no GridSearchCV) with 10-fold CV **to have an initial feeling of the performance**
  - LogisticRegression, KNeighborsClassifier\*, SVC\*, DecisionTreeClassifier, RandomForestClassifier, AdaBoostClassifier
  - Consider confusion matrix, Precision, Recall, F-1 score (weighted average)

Logistic Regression Results



K Nearest Neighbors Results



(\*) Feature re-scaling is recommended prior training. It was not applied in these experiments though.



# Initial Evaluation: results

|   | Model                  | Cross Val Score | Test Accuracy | Average_Accuracy | Precision | Recall   | Avg Precision Recall | F1 Score |
|---|------------------------|-----------------|---------------|------------------|-----------|----------|----------------------|----------|
| 0 | LogisticRegression     | 0.808539        | 0.7925        | 0.800519         | 0.639456  | 0.502674 | 0.621877             | 0.783907 |
| 7 | AdaBoostClassifier     | 0.802848        | 0.7846        | 0.793724         | 0.620339  | 0.489305 | 0.636841             | 0.775899 |
| 3 | SVC                    | 0.776897        | 0.7754        | 0.776148         | 0.619835  | 0.401070 | 0.615200             | 0.758089 |
| 1 | KNeighborsClassifier   | 0.776892        | 0.7655        | 0.771196         | 0.571429  | 0.470588 | 0.519334             | 0.757740 |
| 5 | RandomForestClassifier | 0.766932        | 0.7548        | 0.760866         | 0.545455  | 0.465241 | 0.540537             | 0.748246 |
| 2 | GaussianNB             | 0.769602        | 0.7491        | 0.759351         | 0.521561  | 0.679144 | 0.566332             | 0.758317 |
| 6 | SGDClassifier          | 0.747868        | 0.7420        | 0.744934         | 0.511066  | 0.679144 | 0.541105             | 0.752055 |
| 4 | DecisionTreeClassifier | 0.737422        | 0.7299        | 0.733661         | 0.491329  | 0.454545 | 0.371390             | 0.726476 |

- Logistic Regression and Adaboost classifier model look promising achieving highest Average Accuracy and Weighted F1 score
- Let's try to improve both models by selecting the best combination of hyper-parameter values (use of GridSearchCV)



# Best model selection

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- GridSearchCV on a grid hyper parameters
    - Logistic Regression and Ada Boost Classifier do not require feature scaling
  - Logistic Regression Classifier

Final accuracy score on the testing data: 0.7925  
Final F-score on the testing data: 0.7839  
`LogisticRegression(C=10, max_iter=10000, solver='newton-cg')`
  - AdaBoost Classifier

Final accuracy score on the testing data: 0.7846  
Final F1-score on the testing data: 0.7748  
`AdaBoostClassifier(algorithm='SAMME', n_estimators=500)`
  - Source code for all classification experimentations can be found [here](#)
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# Appendix – Problem with artificial balancing

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- Let's say you're recognizing hand-written letters from English alphabet (26 letters). Overbalancing every letter appearance will give every letter a probability of being classified (correctly or not) roughly  $1/26$ , so classifier will forget about actual distribution of letters in the original sample. And it's ok when classifier is able to generalize and recognize every letter with high accuracy.
- But if accuracy and most importantly generalization isn't "so high" (I can't give you a definition - you can think of it just as a "worst case") - the misclassified points will most-likely equally distribute among all letters, something like:
  - "A" was misclassified 10 times
  - "B" was misclassified 10 times
  - "C" was misclassified 11 times
  - "D" was misclassified 10 times
  - ...and so on



# Appendix – Problem with artificial balancing

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- As opposed to without balancing (assuming that "A" and "C" have much higher probabilities of appearance in text)
  - "A" was misclassified 3 times
  - "B" was misclassified 14 times
  - "C" was misclassified 3 times
  - "D" was misclassified 14 times
  - ...and so on
- So frequent cases will get fewer misclassifications. Whether it's good or not depends on your task. For natural text recognition, one could argue that letters with higher frequencies are more viable, as they would preserve semantics of the original text, bringing the recognition task closer to prediction (where semantics represent tendencies). But if you're trying to recognize something like screenshot of [ECDSA-key](#) (more entropy -> less prediction) - keeping data unbalanced wouldn't help. So, again, it depends.



# Appendix – Problem with artificial balancing

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- **The most important distinction is that the accuracy estimate is, itself, getting biased** (as you can see in the balanced alphabet example), so you don't know how the model's behavior is getting affected by most rare or most frequent points.
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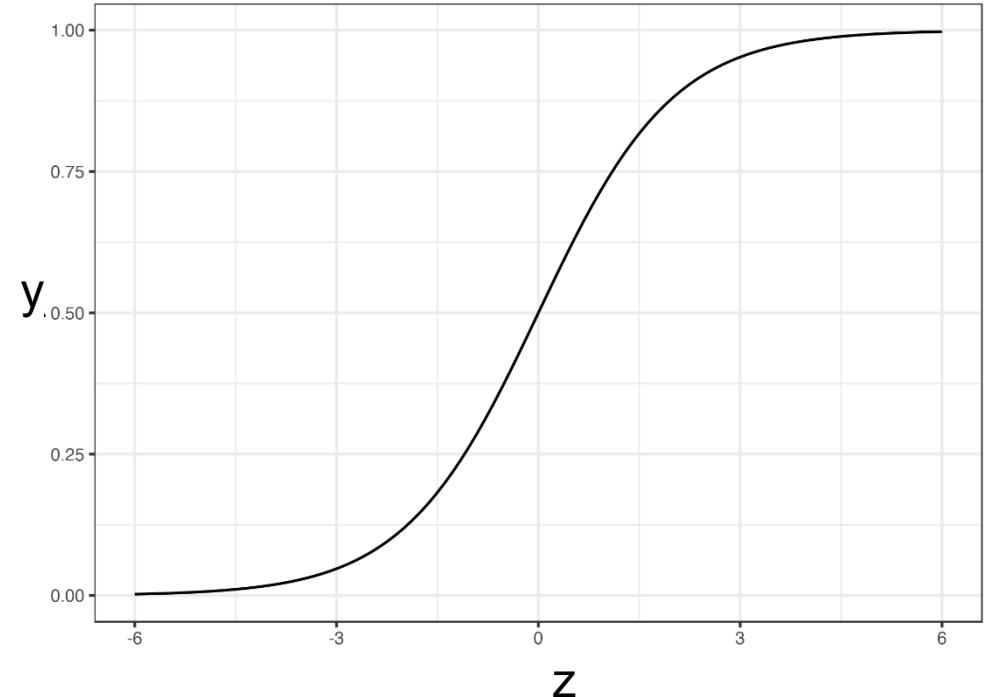


# Appendix – Logistic Regression

- Logistic regression name comes from the logistic sigmoid function

$$y = \text{logistic}(z) = \frac{1}{1 + e^{-z}}$$

- Logistic function outputs a value (a probability) between 0 and 1
  - Output  $y$  can be seen as the probability of belonging to the positive class,  $P_{(y=1)}$
  - The returned probability can be converted to a binary category
    - $z \geq 0 \Rightarrow P_{(y=1)} \geq 0.5 \Rightarrow \text{Sample belongs to positive class (e.g. spam)}$
    - $z < 0 \Rightarrow P_{(y=1)} < 0.5 \Rightarrow \text{Sample belongs to negative class (e.g. not spam)}$



- Input  $z$  can be expressed as  $z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n$ , where  $X_i$  are independent variables (features) of the classification problem

# Appendix – Logistic Regression Interpretation

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- The interpretation of the coefficients ( $\beta_0$ ,  $\beta_1$ , etc.) in logistic regression differs from the interpretation of the coefficients in linear regression
- Coefficients do not influence the probability linearly any longer
- Reformulate the equation so that only the linear term is on the right side of the formula

$$y = P_{(y=1)} = \frac{1}{1 + e^{-z}} \Rightarrow \ln\left(\frac{P_{(y=1)}}{1 - P_{(y=1)}}\right) = \ln\left(\frac{P_{(y=1)}}{P_{(y=0)}}\right) = z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n$$

- We call the term in the  $\ln()$  function “odds” and wrapped in the logarithm it is called log odds
- This formula shows that the logistic regression model is a linear model for the log odds

# Appendix – Logistic Regression Assumptions

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$$y = \frac{1}{1 + e^{-z}} \Rightarrow \ln\left(\frac{y}{1 - y}\right) = z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n$$

- Linear relationships between X and  $\ln(y)$
- No or little multicollinearity
  - Multicollinearity: two or more of the independent variables are highly correlated to one another