DSC 534–Lab 2

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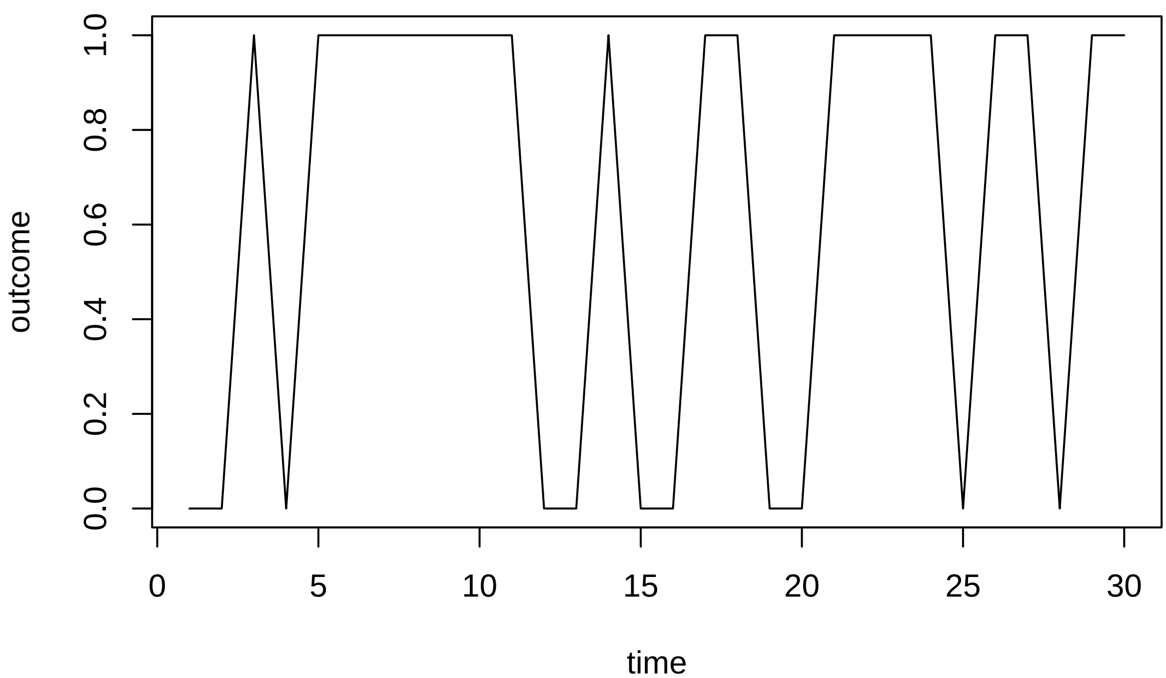
Bernoulli sequence (iid example)

A sequence of independent Bernoulli random variables, say X_t, with probability of success p is a trivial example of a time series. In this $X_t = 0$ or 1 and $F_t = Bernoulli(p)$.

```
library(astsa)
```

```
## Warning: package 'astsa' was built under R version 4.3.3
```

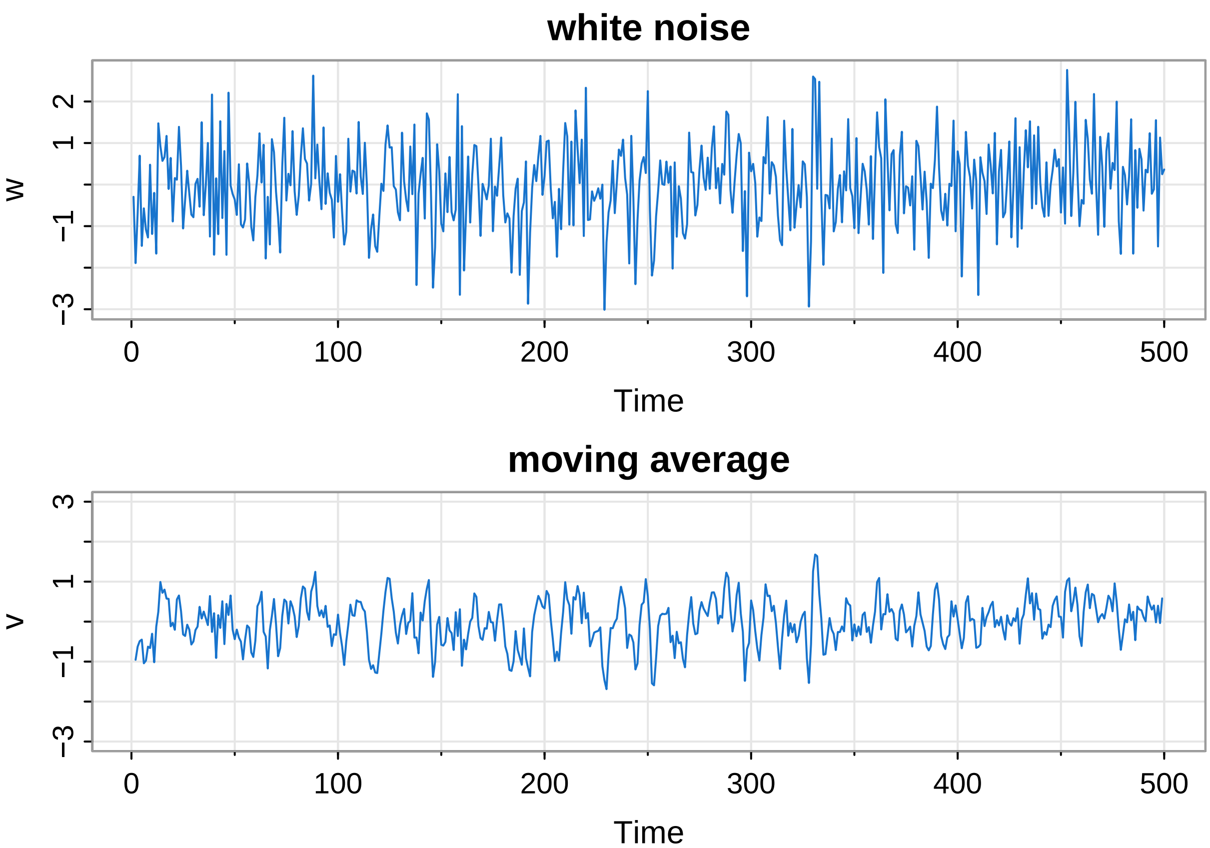
```
set.seed(1245)
p=6/10
n=30
x=rbinom(n, 1, p)
ts.plot(x, xlab="time", ylab="outcome")  ##not that informative
```



Normal sequence (iid example)

A sequence of independent normal random variables, say X_t, with mean μ and variance σ^2 is a trivial example of a white noise series. In this $X_t \in \mathbb{R}$ and $F_t = N(\mu, \sigma^2)$. See the effect of filtering on the observed series.

```
par(mfrow=2:1)
w = rnorm(500) # 500 N(0,1) variates
v = filter(w, sides=2, filter=rep(1/3,3)) # moving average
tsplot(w, col=4, main="white noise")
tsplot(v, ylim=c(-3,3), col=4, main="moving average")
```



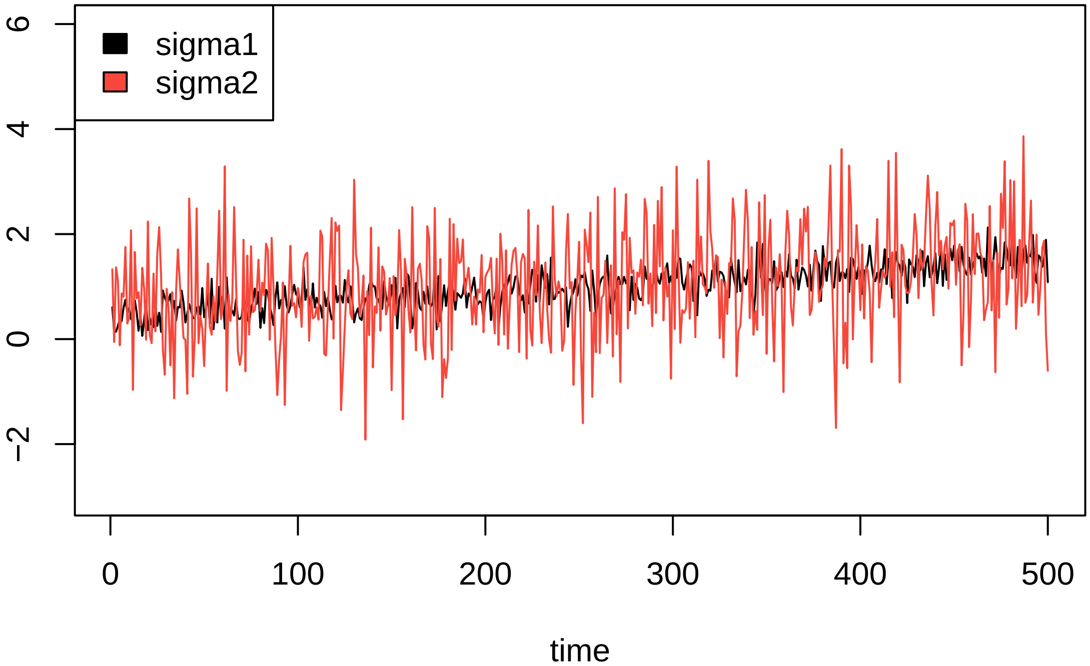
Normal sequence (slowly varying trend)

A sequence of independent normal random variables, say X_t, with mean

$$\mu_t = \alpha + \beta \frac{t}{n}$$

and variance σ^2 is another example of a time series. In this X_t is real valued and $F_t = Normal(\mu_t, \sigma^2)$. Below is an example of two time series with different variances so that you can examine how the variance affects the observed data.

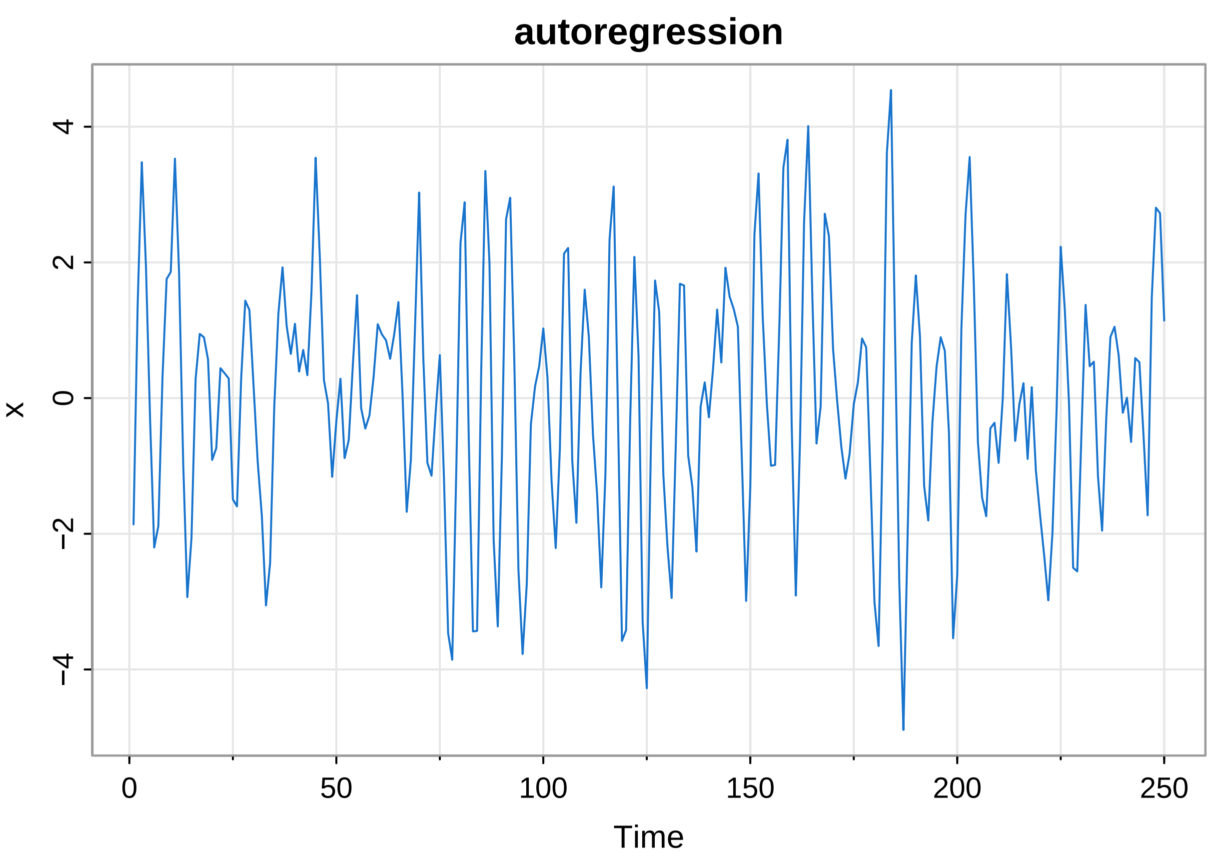
```
set.seed(1245)
alpha=0.5
beta=1
n=500
mu.t= alpha+beta*(1:n)/n
sigma1=0.25
sigma2=1
x1=rnorm(n, mu.t, sigma1)
x2=rnorm(n, mu.t, sigma2)
ts.plot(cbind(x1,x2), col=1:2, xlab="time", ylim=c(-3,6))
leg.txt <- c("sigma1", "sigma2")
legend("topleft", leg.txt, fill=1:2)
```



Autoregressions

This is an example of an autoregressive process of order 2.

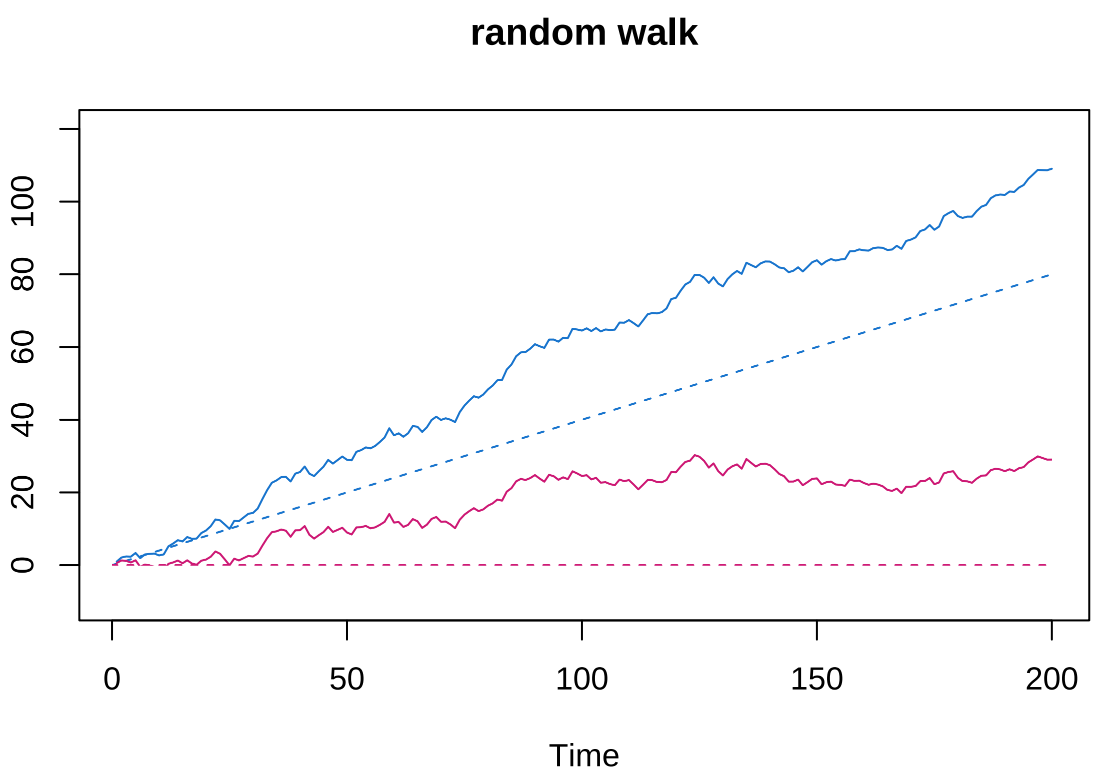
```
w = rnorm(250 + 50) # 50 extra to avoid startup problems
x = filter(w, filter=c(1,-.80), method="recursive")[-(1:50)] ##remove the first 50 observations
tsplot(x, main="autoregression", col=4)
```



Random walk with drift

Two examples of a random walk with drift. The first example (variable X) is an example of model with $\delta = 0$. The other example (variable X_d) is an example of radom walk model with $\delta = 4/10$.
We impose the lines $y = (4/10) * x$ and $y = 0$.

```
set.seed(12345)
w  = rnorm(200)
x  = cumsum(w)
wd = w + 4/10
xd = cumsum(wd)
ts.plot(xd, ylim=c(-10,120), main="random walk", ylab="", col=4)
clip(0, 200, 0, 120)
abline(a=0, b=.4, lty=2, col=4) # drift
lines(x, col=6)
clip(0, 200, 0, 120)
abline(h=0, col=6, lty=2)
```



Signal in noise model

Here we have a signal plus noise model of the form

$$X_t = 2 * \cos\Big(2\pi \frac{(t+15)}{50}\Big), \quad t = 1, 2, 3, \ldots, 500.$$

An additive noise term was taken to be white noise with $\sigma_w^2 = 1$ (middle panel) and $\sigma_w^2 = 5$ (bottom panel), drawn from a normal distribution. Adding the two together contaminates the signal, as shown in the lower panels. Of course, the degree to which the signal is contaminated depends on the amplitude of the signal and the size of σ_w^2. The ratio of the amplitude of the signal to σ_w^2 (or some function of the ratio) is sometimes called the signal-to-noise ratio (SNR); the larger the SNR, the easier it is to detect the signal. Note that the signal is easily discoverable in the middle panel whereas the signal is obscured in the bottom panel. Typically, we will not observe the signal but the signal obscured by noise.

```r
library(astsa)
cs = 2*cos(2*pi*(1:500)/50 + .6*pi)
cs = 2*cos(2*pi*(1:500+15)/50)          # same thing
w  = rnorm(500,0,1)
par(mfrow=c(3,1))
tsplot(cs, ylab="", main = expression(x[t]==2*cos(2*pi*t/50+.6*pi)))
tsplot(cs + w, ylab="", main = expression(x[t]==2*cos(2*pi*t/50+.6*pi)+N(0,1)))
tsplot(cs + 5*w, ylab="", main = expression(x[t]==2*cos(2*pi*t/50+.6*pi)+N(0,25)))
```