Institute for Applied and Numerical Mathematics
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Geometric Numerical Integration

Summer Semester 2016

Assignment 3

The Outer Solar system

Simulate the motion of the outer planets (Jupiter, Saturn, Uranus, Neptune and Pluto) with the explicit Euler, the symplectic Euler and the Störmer-Verlet method. This N-body problem can be formulated as a Hamiltonian system with $N = 5$ and

$$H(p,q) = \frac{1}{2}\sum_{i=1}^{N}\frac{1}{m_i}{p^{(i)}}^T p^{(i)} - G\sum_{i=1}^{N}\sum_{j=0}^{i-1}\frac{m_i m_j}{||q^{(i)} - q^{(j)}||}.$$

Above, $q = (q^{(1)}, \dots, q^{(N)})^T \in \mathbb{R}^{3N}$ and $p = (p^{(1)}, \dots, p^{(N)})^T \in \mathbb{R}^{3N}$ are composed of the vectors $q^{(i)}, p^{(i)} = m_i v^{(i)} \in \mathbb{R}^3$, which describe the positions and momenta of the i-th planet, with $m_i > 0$ denoting its mass and velocity $v^{(i)}$.

The initial data[1] is given in Table 1 and corresponds to September 5th, 1994 at 00:00h[2]. The masses are taken relative to the Sun, distances are in A.U. (astronomical units), time in Earth days and the gravitational constant is

$$G = 2.95912208286e - 4.$$

Note that the Sun mass m_0 takes into account also the mass of the inner planets.

(a) Apply the explicit Euler method to the outer solar system with $h = 10$ over a time period of 200.000 Earth days.

(b) Apply the symplectic Euler method to the outer solar system with $h = 100$ over a time period of 200.000 Earth days.

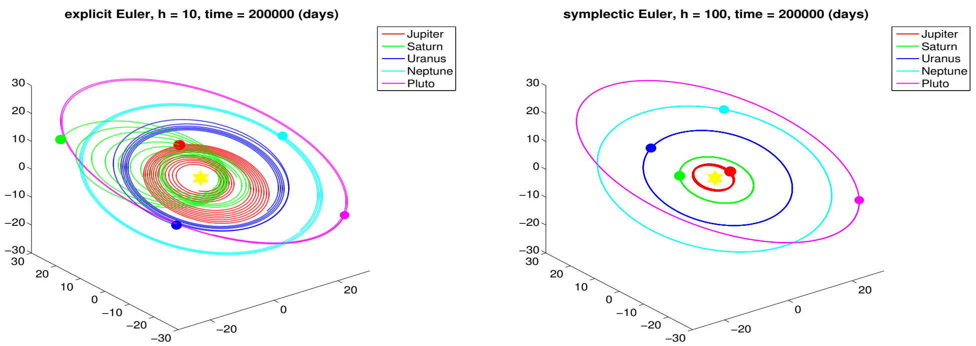
(c) Apply the Störmer-Verlet method to the outer solar system with $h = 100$ over a time period of 200.000 Earth days.

(d) Compare the results by plotting the orbits of all planets. Observe the energy drift by plotting the relativ energy error over time. Examine which methods conserve the total angular momentum by plotting the relativ angular momentum error over time.

[1] Taken from *Ernst Hairer, Christian Lubich, and Gerhard Wanner*, Geometric numerical integration. Structure-preserving algorithms for ordinary differential equations. Second edition, Springer, 2006.
[2] A .txt file with these values can be downloaded from the course website.

planet	mass	initial position	initial velocity
Sun	$m_0 = 1.00000597682$	0 0 0	0 0 0
Jupiter	$m_1 = 9.54786104043e - 4$	-3.5023653 -3.8169847 -1.5507963	0.00565429 -0.00412490 -0.00190589
Saturn	$m_2 = 2.85583733151e - 4$	9.0755314 -3.0458353 -1.6483708	0.00168318 0.00483525 0.00192462
Uranus	$m_3 = 4.37273164546e - 5$	8.3101420 -16.2901086 -7.2521278	0.00354178 0.00137102 0.00055029
Neptune	$m_4 = 5.17759138449e - 5$	11.4707666 -25.7294829 -10.8169456	0.00288930 0.00114527 0.00039677
Pluto	$m_5 = 1/(1.3 \cdot 10^8)$	-15.5387357 -25.2225594 -3.1902382	0.00276725 -0.00170702 -0.00136504

Table 1: Data for the outer solar system



Assistance with this programming exercise will be provided in the problem sessions on **8th June, 2016** and **22nd June, 2016**.

Homepage:

The link http://www.math.kit.edu/ianm3/lehre/geomnumint2016s/en leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the programming exercise.