Date.....

Expt. No..... Page No.....

Karan Agarwal Tutorial-4 CS-D-(12)

1- $T(n) = 3T(n/2) + n^2$

Sol^n $T(n) = aT(n/b) + f(n)$

$a \geq b$, $b > 1$

On comparing

$a=3, b=2, f(n) = n^2$

Now,

$c = \log_b a$, $\log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

2- $T(n) = 4T(n/2) + n^2$

Sol^n $a \geq 1$, $b > 1$

$a=4, b=2, f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

3- $T(n) = T(n/2) + 2^n$

Sol^n $a=1, b=2$ $f(n) = 2^n$

$c = \log_b a = \log_2 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$.

4- $T(n) = 2^n T(n/2) + n^n$

Sol^n: $a = 2^n$, $b=2$, $f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c = n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \theta(n^2 \log_2 n)$

5- $T(n) = 16T(n/4) + n$

Sol^n: $a=16$, $b=4$ $f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$ $f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

6- $T(n) = 2T(n/2) + n \log n$

Sol^n: $a=2$, $b=2$ $f(n) = n \log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n \log n > n$

$\therefore f(n) > n^c$

$\therefore f(n) = \theta(n \log n)$

Teacher's Signature.....

Date.....

Expt. No.....

Page No.....

7- $T(n) = 2T(n/2) + n/\log n$

Soln:- $a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore f(n) < \Theta(n)$

8- $T(n) = 2T(n/4) + n^{0.51}$

Soln:- $a = 2,\ b = 4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

$\therefore n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9- $T(n) = 0.5T(n/2) + 1/n$

Soln:- $a = 0.5,\ b = 2$

Since acc. to Master theorem $a \geq 1$, but here a is 0.5 so we cannot apply master theorem.

10- $T(n) = 16T(n/4) + n!$

Soln:- $a = 16,\ b = 4,\ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

11- $4T(n/2) + \log n$

Soln:- $a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$ $f(n) = \log n$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

12- $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

Soln:- $a = \sqrt{n},\ b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log n < \log n$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log(n))$.

Teacher's Signature.....

Expt. No.....

Page No.....

13- $T(n) = 3T(n/2) + n$

Sol^n:- $a=3, b=2, f(n)=n$

$c = \log_b a = \log_2 3 = 1.584$

$\therefore n^c = n^{1.58}$

$\therefore n < n^{1.584}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.584})$

14- $T(n) = 3T(n/3) + \text{sqrt}(n)$

Sol^n:- $a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

15- $T(n) = 4T(n/2) + cn$

Sol^n:- $a=4, b=2$ $f(n)=cn$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn < n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

16- $T(n) = 3T(n/4) + n\log n$

Sol^n:- $a=3, b=4, f(n) = n\log n$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n \log n)$

17- $T(n) = 3T(n/3) + n/2$

Sol^n:- $a=3, b=3$

$c = \log_3 3 = 1$ $f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$\therefore f(n) < an^c$

$\therefore T(n) = \Theta(n)$.

18- $T(n) = 6T(n/3) + n^2\log n$

Sol^n:- $a=6, b=3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

19- $T(n) = 4T(n/2) + n/\log n$

Sol^n: $a=4, b=2, f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2$ $\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

Teacher's Signature.....

Date.....

Expt. No..... Page No.....

20- $T(n) = 64\,T(n/8) - n^2 \log n$

Soln:- $a = 64,\ b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \theta(n^2 \log n)$

21- $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$

22- $T(n) = T(n/2) + n(2 - \cos n)$

Soln:- $a = 1,\ b = 2$

$c = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \theta(n(2 - \cos n))$.

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