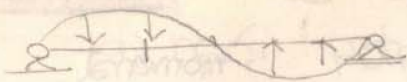


СТАТИКА КОНСТРУКЦИЈА 1

Модул: Конструкције

– материјал за вежбе –

2024.

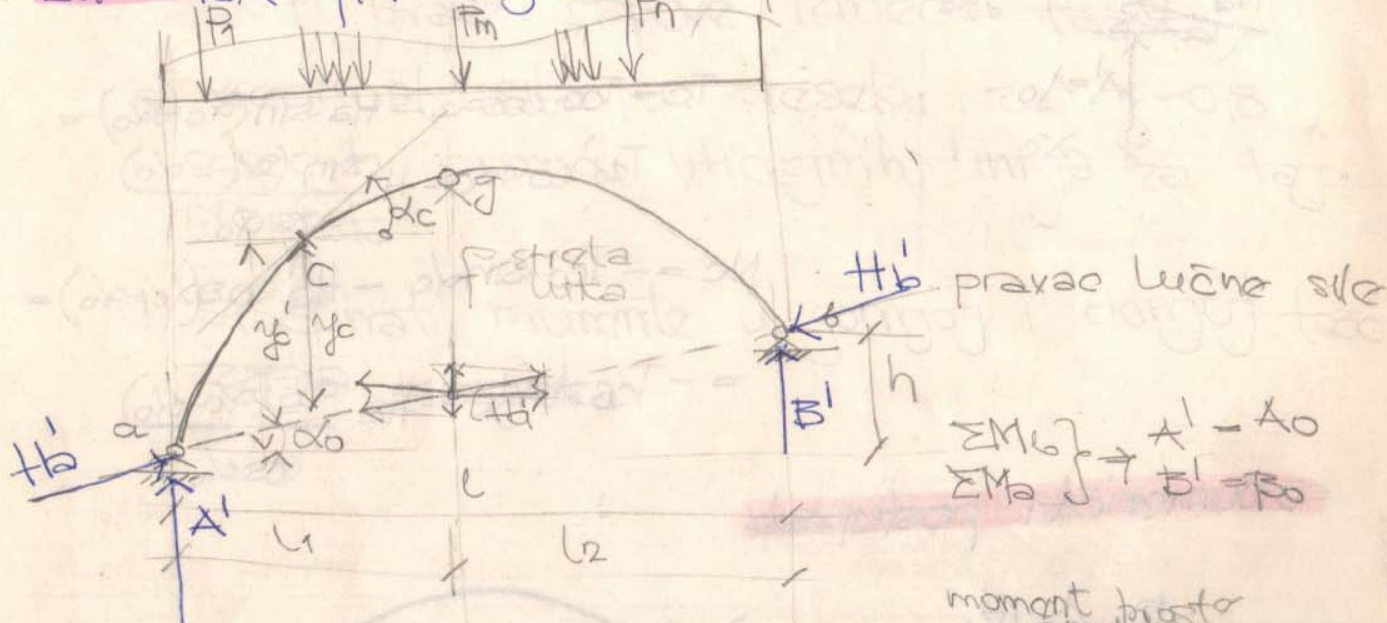


27.03.1991

nosaci koji se sastoje od dvije kinematički
krute ploče

Luk sa tri zgloba

Лук са три зглоба



$$\left. \begin{array}{l} \sum M_B = 0 \\ \sum M_A = 0 \end{array} \right\} \Rightarrow \begin{array}{l} A' = A_0 \\ B' = B_0 \end{array}$$

moment prista
gredice

$$\left. \begin{array}{l} \sum M_g = 0 \\ \sum M_g^d = 0 \end{array} \right\} \Rightarrow \begin{array}{l} M_{g0} - H'a \cos \alpha_0' \\ f = 0 \end{array}$$

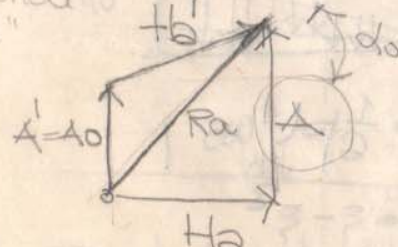
$$M_{g0} - H'b \cos \alpha_0' \cdot f = 0$$

$$H_a = \frac{M_{g0}}{f}$$

$$H_b = \frac{M_{g0}}{f}$$

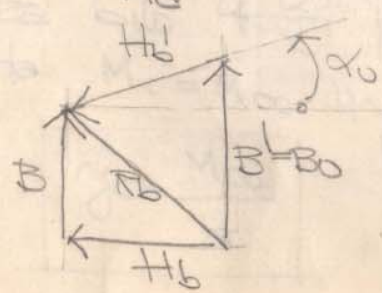
$H_a = H_b = H$ - ako imamo vertikalno opterećenje

u odnosu
"a"



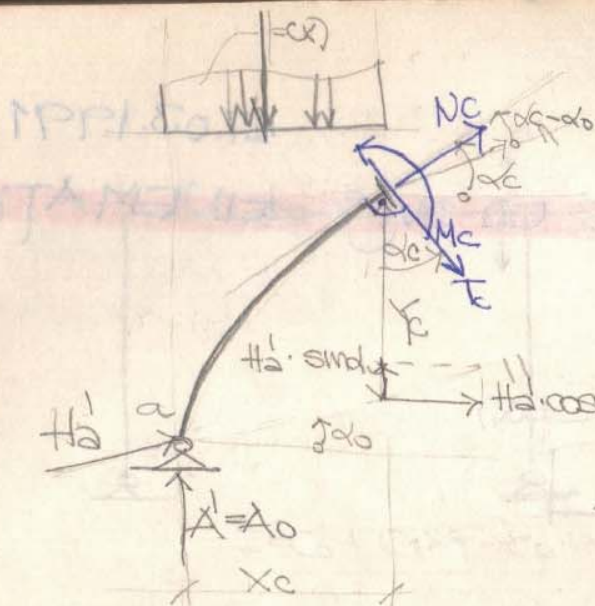
$$H_a = H'a \cos \alpha_0$$

$$A = A_0 + H'a \sin \alpha_0 = A_0 + H_a \cdot \tan \alpha_0$$



$$H_b = H'b \cos \alpha_0$$

$$B = B_0 + H'b \sin \alpha_0 = B_0 + H_b \cdot \tan \alpha_0$$



mom.
proste
grede

$$M_c = M_{c0} - H_a \cdot y_c = M_{c0} - H y_c$$

moment

$H_a = H$ po vrata je

$$T_c = T_{c0} \cos \alpha_c - H_a' \sin(\alpha_c - \alpha_0) = T_{c0} \cos \alpha_c - H \frac{\sin(\alpha_c - \alpha_0)}{\cos \alpha_0}$$

$$N_c = -T_{c0} \sin \alpha_c - H_a' \cos(\alpha_c - \alpha_0) = -T_{c0} \sin \alpha_c - H \frac{\cos(\alpha_c - \alpha_0)}{\cos \alpha_0}$$

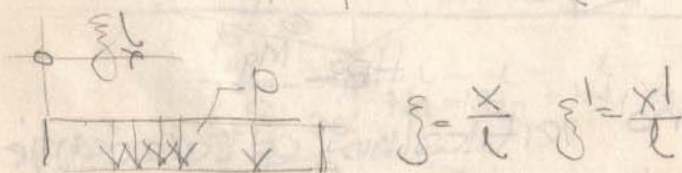
a) numerički postupak

$$P_0 = \frac{\lambda}{24} (7p_0 + 6p_1 - p_2)$$

$$P_m = \frac{\lambda}{12} (p_{m-1} + 10p_m + p_{m+1})$$

$$P_n = \frac{\lambda}{24} (7p_n + 6p_{n-1} - p_{n-2})$$

b) analitički postupak (bezdimenziionalne funkcije)



$$\xi = \frac{x}{l} \quad \xi' = \frac{x'}{l}$$

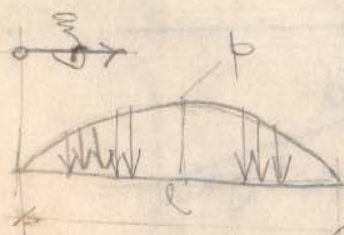
$$M_c = \frac{1}{2} p l^2 \omega_R$$

$$\omega_R = \xi - \xi^2$$



$$M_c = \frac{1}{6} p l^2 \omega_D$$

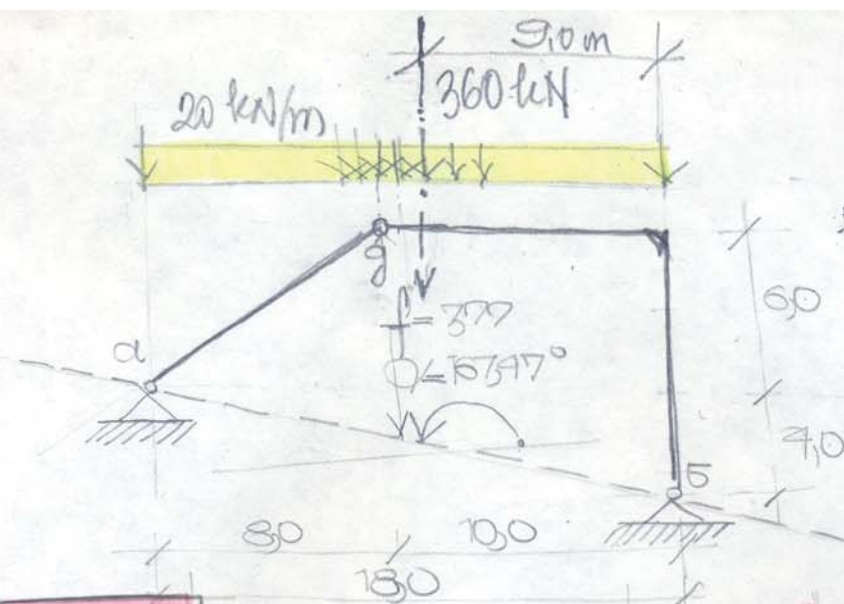
$$\omega_D = \xi - \xi^3$$



$$M_c = \frac{1}{3} p l^2 \omega_P$$

$$\omega_P = \xi - 2\xi^3 + \xi^4$$

$$\xi = 0, 0.1, 0.2, \dots, 1.0$$



* ЗА ПОСАМ СЪ ОНТЕРЕСЕНЕМ
 НАУПТАН М-АНАЛІЗАН.

$$\tan \alpha = -0.22$$

$$\sum M_b = 0$$

$$A = \frac{1}{18.0} [360 \cdot 9.0] = 180 \text{ kN}$$

$$\sum M_a = 0$$

$$B = \frac{1}{18.0} [360 \cdot 9.0] = 180.0 \text{ kN}$$

$$\sum M_g = 0$$

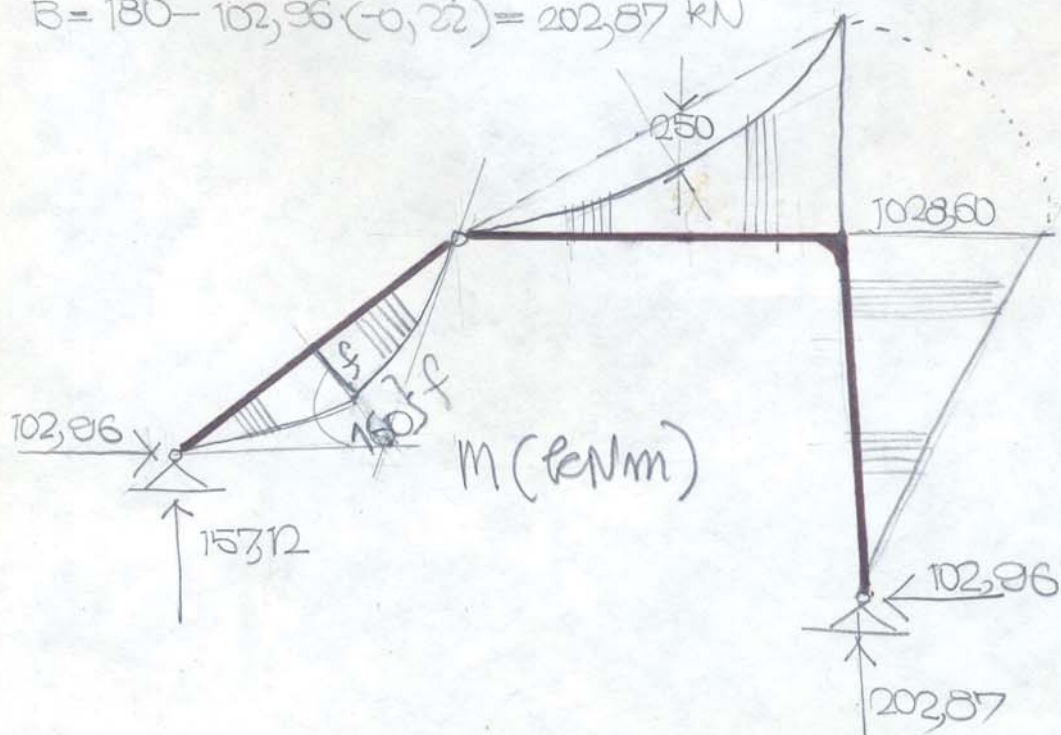
$$H_a = \frac{1}{8.77} [180 \cdot 8.0 - 20 \cdot 8.0 \cdot 4.0] = 102.96 \text{ kN}$$

$$\sum M_g = 0$$

$$H_b = \frac{1}{8.77} [180 \cdot 10.0 - 20 \cdot 10.0 \cdot 5.0] = -102.96 \text{ kN}$$

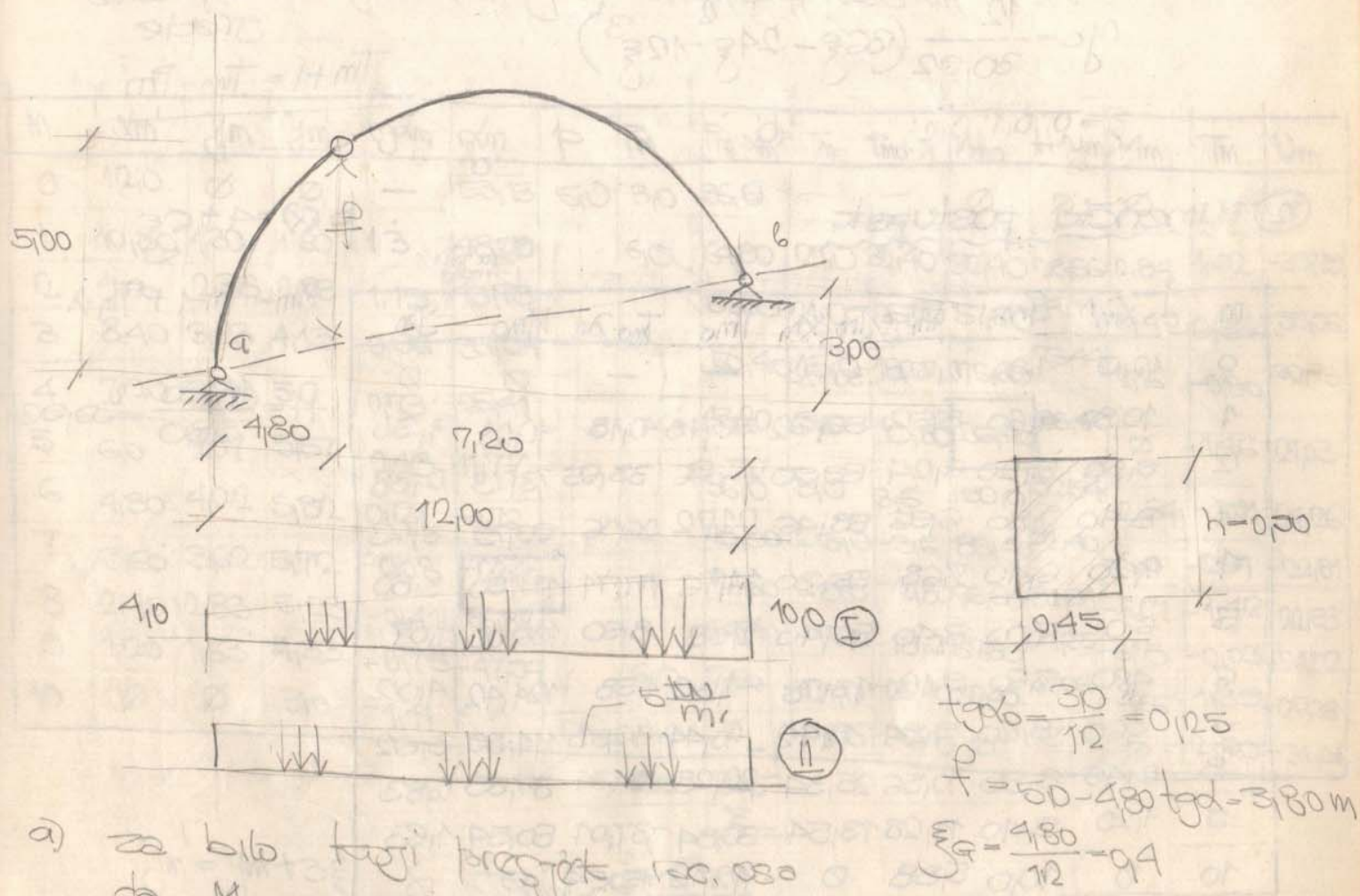
$$A = 180 + 102.96(-0.22) = 153.12 \text{ kN}$$

$$B = 180 - 102.96(-0.22) = 202.87 \text{ kN}$$



zadatak Za luku na tri zgloba:

- naći potpormu linije (racionalnu osu) sa ordinatama u desetinama raspona za opterećenje I
- za opterećenje II nacrtati dijagrame sila u presjecima u desetinama raspona (pretpostaviti prave stepove između taceta)
- vrednost sila u presjeku za $\xi = 0,8$ odrediti pomoću uticajnih linija za taj presjek
- izračunati momente u gornjoj i donjoj tački jezgra presjeka

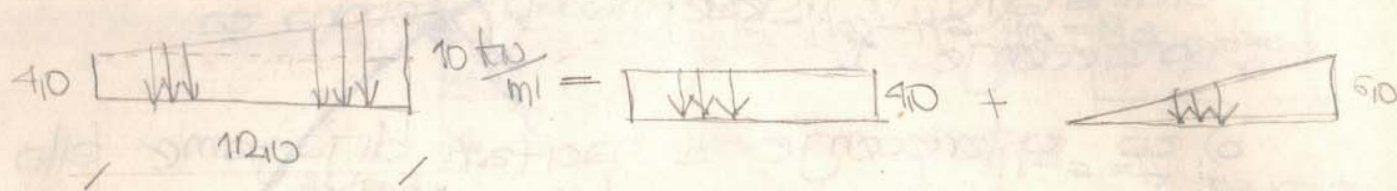


- a) za bilo koji presjek rac. osa
da $M_c = M_{co} - H y_c = 0$

$$y_c = \frac{M_{co}}{H}$$

ordinate
racionalne ose

① Pomocu beadm. funkcija



$$M_0 = \frac{1}{2} \cdot p \cdot l^2 \cdot \omega_R + \frac{1}{6} p l^2 \omega_b = \frac{1}{2} 40 \cdot 12 \cdot 0^2 \cdot (\xi - \xi^2) + \frac{1}{6} 60 \cdot 12 \cdot 0^2 (\xi - \xi^3) =$$

$$M_0 = (36\xi - 24\xi^2 - 12\xi^3) \cdot 12$$

potrebno

luka - $H = \frac{M_0}{f} = \frac{M_0(\xi=0,4)}{3,80} = \frac{(36 \cdot 0,4 - 24 \cdot 0,4^2 - 12 \cdot 0,4^3) \cdot 12}{3,80} =$

$$H = 30,92 \text{ kN}$$

T_{m0} transferzalna sila prste grade

$$\eta_c = \frac{12}{30,92} (36\xi - 24\xi^2 - 12\xi^3)$$

$$\xi = 0,1; 0,2; \dots; 1,0$$

$$T_{m+1} = T_m - P_m$$

② numerički postupak

$$p(x) = 4 + 5\xi$$

$$M_m = M_{m-1} + T_m \cdot \lambda$$

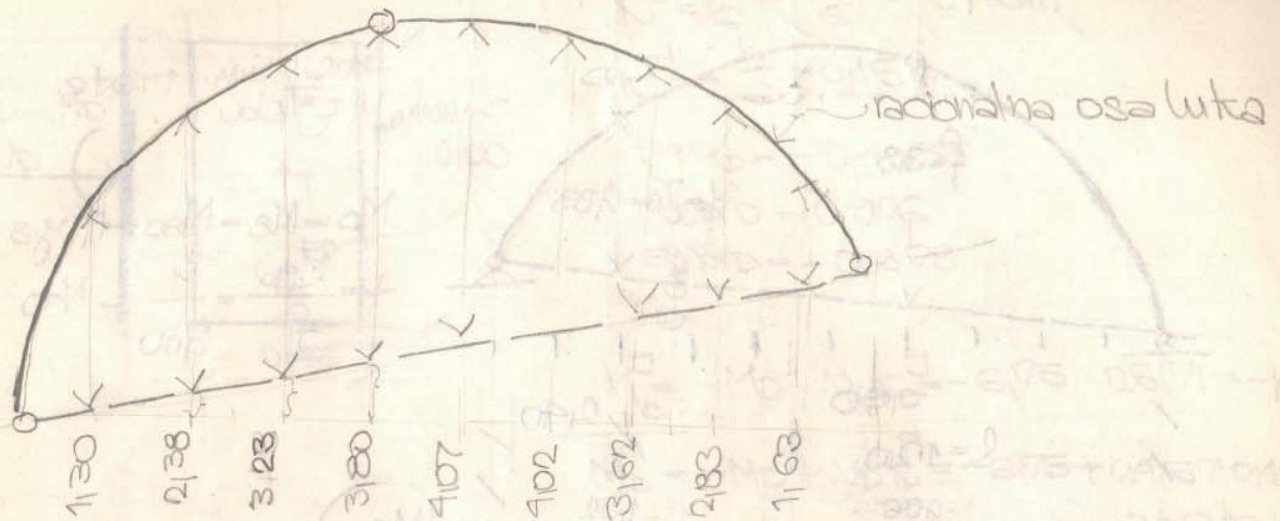
$$H = \frac{117,50}{3,80} = 30,92$$

m	x_m	b_m	P_m	$T_m \cdot x_m$	T_{m0}	$T_{m0} \cdot \lambda$	M_{m0}	y_m
0	12,0	4,0	2,52	30,24	←	—	0	0
1	10,80	4,60	5,52	59,62	33,48	40,18	40,18	1,30
2	9,60	5,20	6,24	59,50	27,96	33,55	73,73	2,38
3	8,40	5,80	6,56	58,46	21,72	26,06	39,79	3,23
4	7,20	6,40	7,68	55,30	14,76	17,71	117,50	3,80
5	6,0	7,00	8,40	50,40	7,08	8,50	126,00	4,07
6	4,80	7,60	9,12	43,78	-1,32	1,58	124,42	4,02
7	3,60	8,20	9,84	35,92	-10,44	-12,53	111,89	3,62
8	2,40	8,80	10,56	25,34	-20,28	-24,34	87,55	2,83
9	1,20	9,40	11,28	13,54	-30,84	-37,01	50,54	1,63
10	0	10,0	5,88	0	-42,12	-50,54	0	0

$$84,00 \quad 43,20$$

$$A_0 = \frac{\sum P_m \cdot x_m}{l} = \frac{432,0}{12} = 36,0 \text{ kN}$$

b)



$$M_m = M_{m0} - H \cdot y_m$$

$$T_m = (T_{m0} + H \cdot \operatorname{tg} \alpha_m) \cdot \cos \alpha_m - H \cdot \sin \alpha_m$$

$$N_m = - (T_{m0} + H \cdot \operatorname{tg} \alpha_m) \cdot \sin \alpha_m - H \cdot \cos \alpha_m$$

M	x_m'	y_m	y_m'	$\operatorname{tg} \alpha_m$	α_m	p	P_m	$P_m \cdot x_m'$	T_{m0}	$T_{m0} \cdot \lambda$	M_{m0}	$H \cdot y_m$	M_m	T_m	N_m
0	12.0	0	0	—	53.13	5.0	3.0	36.0	—	—	0	0	0	—	—
1	10.80	1.20	1.60	1.3	48.69		6.0	64.80	27.0	32.40	32.40	2.86	2.89	1.42	-33.75
2	9.60	2.38	2.98	1.15	43.78			57.60	21.0	25.20	57.60	5.11	3.40	0.35	-35.06
3	8.40	3.23	4.13	0.96	35.34			50.40	15.0	18.0	17.60	7.34	2.16	-0.80	-30.78
4	7.20	3.80	5.0	0.73	25.41			43.20	9.0	10.80	86.40	8.40	0	-1.46	-21.03
5	6.0	4.07	5.57	0.43	11.77			36.0	3.0	3.6	50.0	9.25	-2.54	-1.91	-21.26
6	4.80	4.02	5.82	0.21	-4.76			28.80	-3.0	-3.6	86.40	9.40	-5.0	-2.01	-22.81
7	3.60	3.62	5.72	-0.08	-22.10			21.60	-3.0	-10.80	75.60	8.23	-6.71	-1.42	-22.33
8	2.40	2.83	5.123	-0.41	-36.87			14.40	-15.0	-18.0	57.60	6.435	-6.75	-0.03	-24.57
9	1.20	1.63	4.33	-0.75	-47.34			7.20	-21.0	-25.20	32.40	3.706	-4.66	1.39	-27.38
10	0	0	3.0	-1.11			5.0	3.0	-27	-32.40	0	0	0	2.60	-31.06
								60							
								360							
								Σ							

$$y_m' = y_m + 3\xi$$

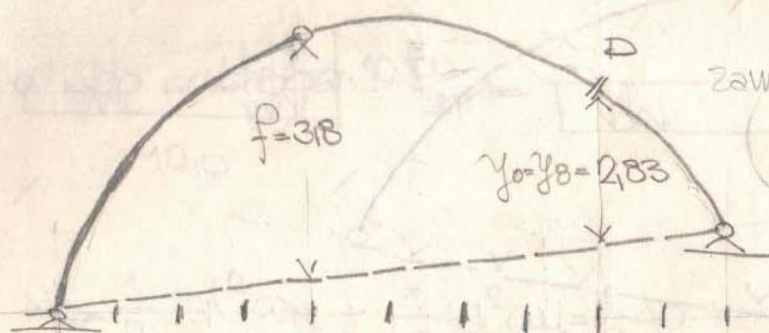
$$\operatorname{tg} \alpha_m = \frac{y_m' - y_{m-1}'}{\lambda}$$

$$H_0 = \frac{\Sigma P_m \cdot x_m'}{l} = 30 \text{ t}$$

$$H = \frac{M_{90}}{f} = \frac{86.40}{3.80} = 22.74$$

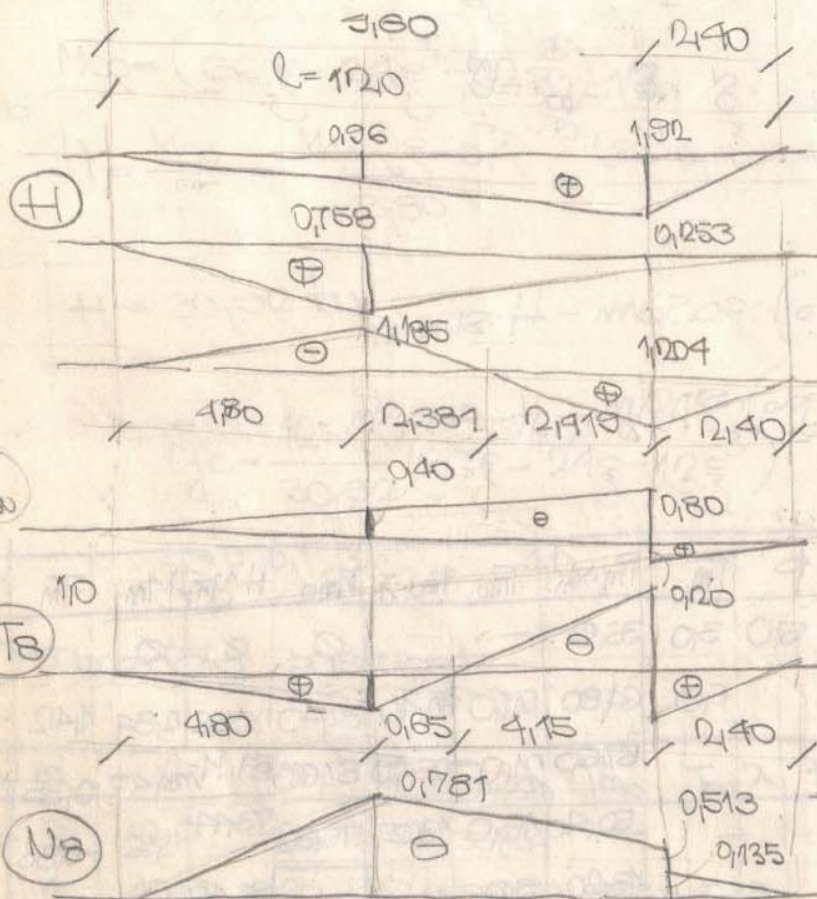
c)

atc. lim. proste grede
za moment u globu



$$M_b = M_s = M_{s0} - H \cdot y_s =$$

$$H = \frac{M_{s0}}{f} = \frac{1}{3.80} = 0.263$$



M_{s0}

$$= M_{s0} - 2.83H$$

M_b

$$M_b = \frac{5.0}{2} (-1.185 \cdot 7.181 + 1.204 \cdot 4.813) = -6.77 \text{ kNm}$$

$$T_b = T_{s0} \cdot \cos \alpha + H (\cos \alpha \cdot \tan \phi_0 - \sin \alpha)$$

$$\tan \phi_0 = 0.25 \quad \text{iz tabele } \alpha$$

$$\sin \alpha = 0.37803$$

$$\cos \alpha = 0.92579$$

$$T_b = 0.92579 \cdot T_{s0} + 0.60348H$$

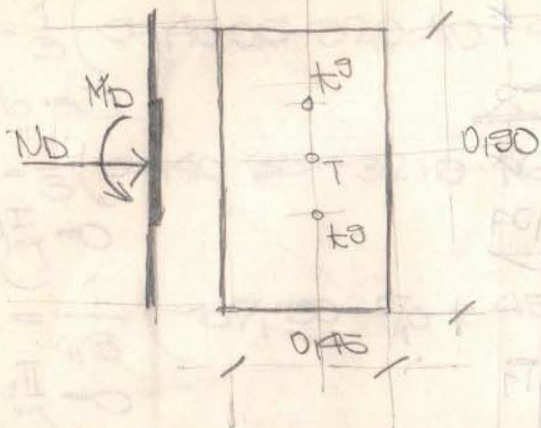
$$T_b = \frac{5.0}{2} (0.92579 \cdot 5.45 + 0.339 \cdot 2.40 - 0.586 \cdot 4.15) = -2.70 \text{ kN}$$

$$N_b = -T_{s0} \cdot \cos \alpha - H (\cos \alpha + \sin \alpha \cdot \tan \phi_0)$$

$$N_b = 0.37803 \cdot T_{s0} - (0.83128 \cdot H)$$

$$N_b = \frac{5.0}{2} (0.3781 \cdot 4.80 + (0.3781 + 0.513) \cdot 4.80 + 0.135 \cdot 2.40) = -25.71 \text{ kN}$$

d)



$$t^g = \frac{h}{6} = \frac{0.130}{6} = 0.0217m$$

$$t^d = \frac{b}{2} = 0.075m$$

$$\tan \alpha_D = -0.4083$$

$$\cos \alpha_D = 0.926$$

$$\sin \alpha_D = -0.378$$

M_D

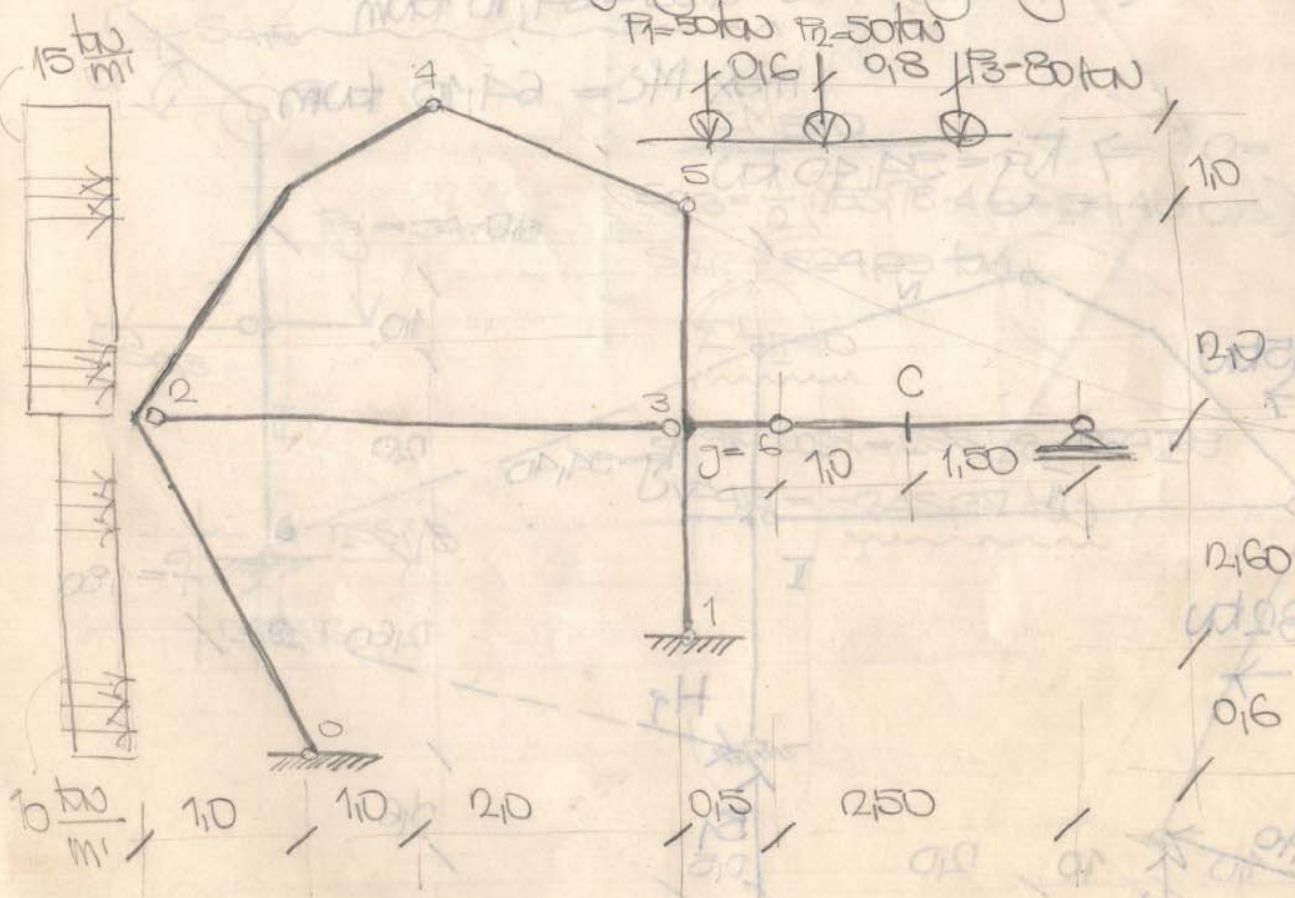
$$M_D^g = -M_D - N_D \cdot t^g = -6.75 - 25.71 = -32.46$$

$$M_D^d = -M_D + N_D \cdot t^d = -6.75 + 25.71 \cdot 0.075 = -3.07 \text{ tnm}$$

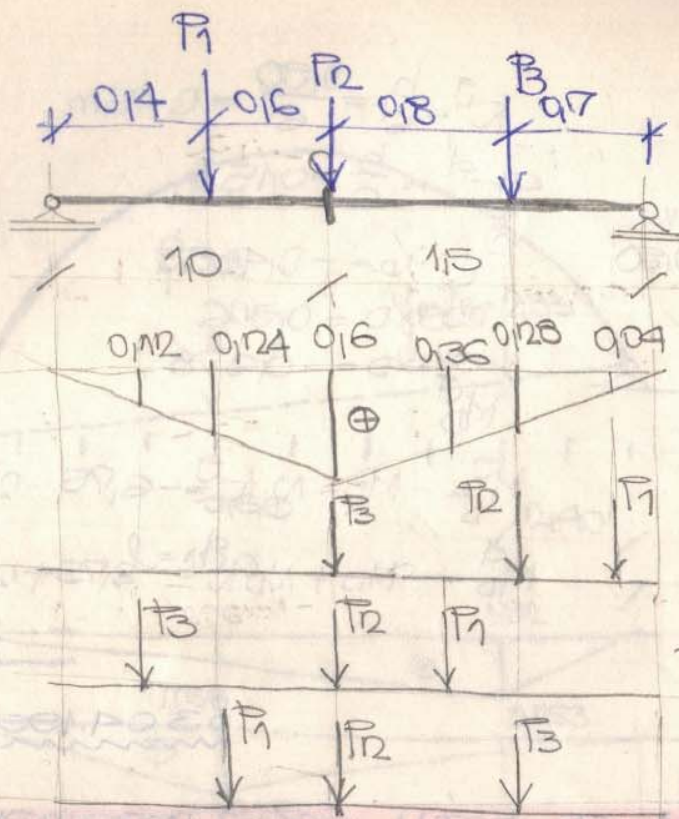
03.04.1991

NOSAČI SA 3 ZGLOBA OD KOJIH JE 1 IMAGINARAN

1) Za položaj sistema koncentrisanih sila koji na postojećoj gredi razvija max. moment u preseku C i pod opterećenjem sa lijeve strane nacrtati dijagram savijanja nosača.



a)



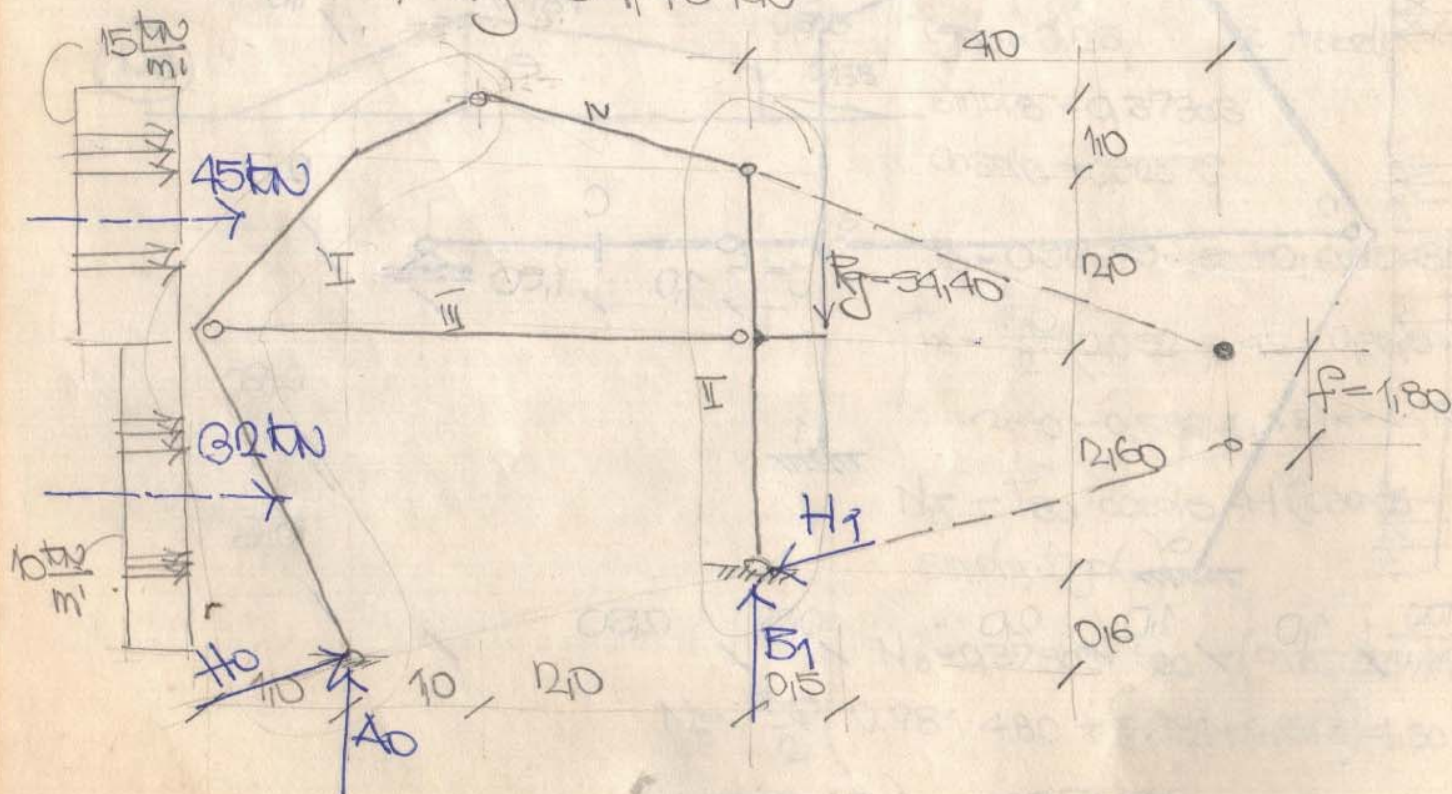
$$I: M_C = 80 \cdot 0.16 + 50(0.128 + 0.04) = 64 \text{ kNm}$$

$$II: M_C = 80 \cdot 0.12 + 50(0.16 + 0.36) = 57.60 \text{ kNm}$$

$$III: M_C = 50(0.124 + 0.16) + 80 \cdot 0.128 = 64.10 \text{ kNm}$$

$$\max M_C = 64.10 \text{ kNm}$$

$$\sum M_7 = 0 \Rightarrow R_g = 34.40 \text{ kN}$$



$$\Sigma M_1 = 0$$

$$A_0 = \frac{1}{3}(-3440 \cdot 0.5 - 320 \cdot 1.0 - 450 \cdot 4.1) = -87.30 \text{ kN}$$

$$\Sigma M_0 = 0$$

$$B_1 = \frac{1}{3}(3440 \cdot 3.5 + 320 \cdot 1.6 + 450 \cdot 4.7) = 157.70 \text{ kN}$$

$$\Sigma M_I = 0$$

$$H_0 = \frac{1}{1.8}(-87.30 \cdot 7.0 + 45 \cdot 1.5 - 32 \cdot 1.60) = -332.78 \text{ kN}$$

$$\Sigma M_{II} = 0$$

$$H_1 = \frac{1}{1.8}(-157.70 \cdot 4.0 + 3440 \cdot 3.5) = -225.78 \text{ kN}$$

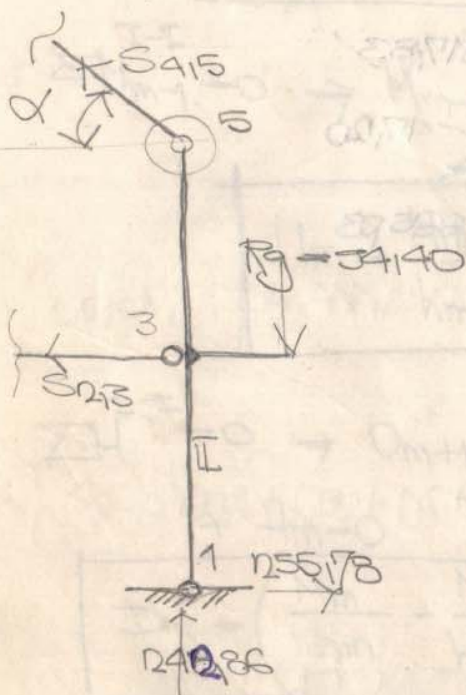
$$A = A_0 + H_0 \cdot \tan \alpha$$

$$\tan \alpha = 0.120$$

$$A = -87.30 - 332.78 \cdot 0.120 = -154.46 \text{ kN}$$

$$B = B_0 - H_1 \cdot \tan \alpha$$

$$B = 157.70 + 225.78 \cdot 0.120 = 242.86 \text{ kN}$$



$$\Sigma M_5 = 0$$

$$S_{23} = \frac{1}{2}(255.78 \cdot 4.6 - 3440 \cdot 0.15) = -564.69 \text{ kN}$$

$$\Sigma H_{II} = 0$$

$$S_{45} \cos \alpha = 255.78 - 564.69$$

$$S_{45} = -345.37 \text{ kN}$$

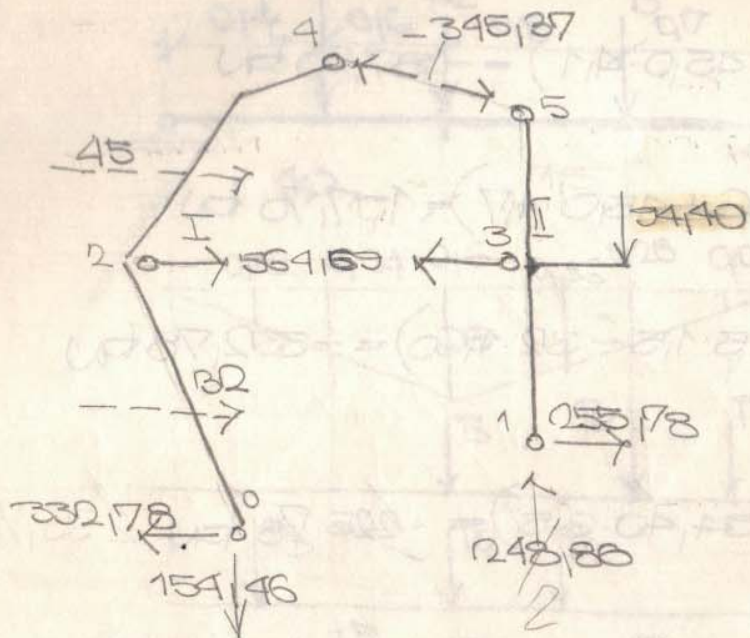
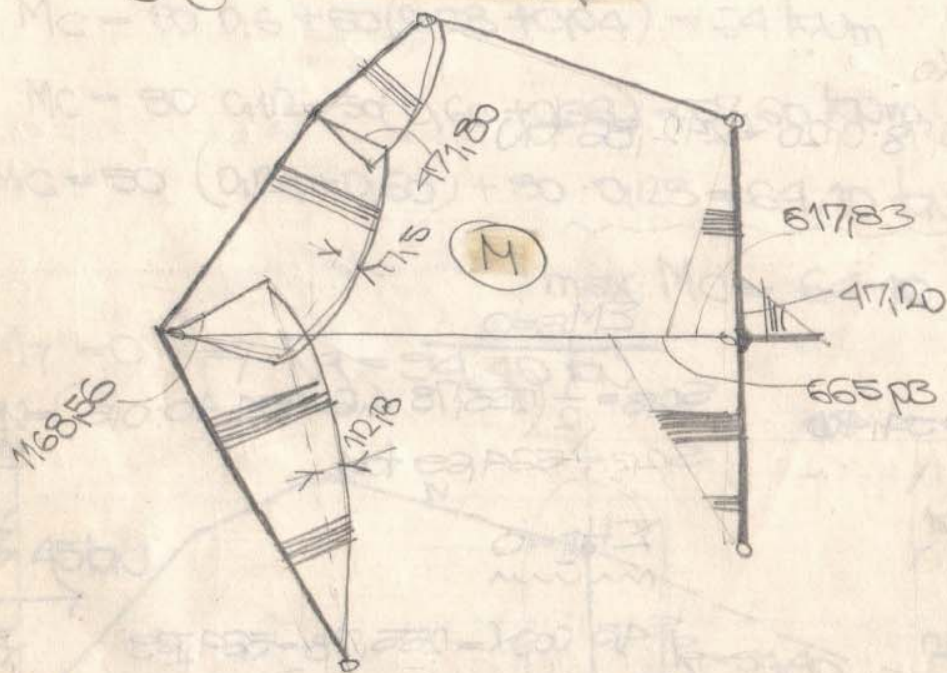
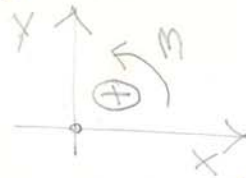


diagrama momente



ЗАДАЧА (Звук и Губы, стр. 122)



Нацртати (м) зграда за
задатим носач;

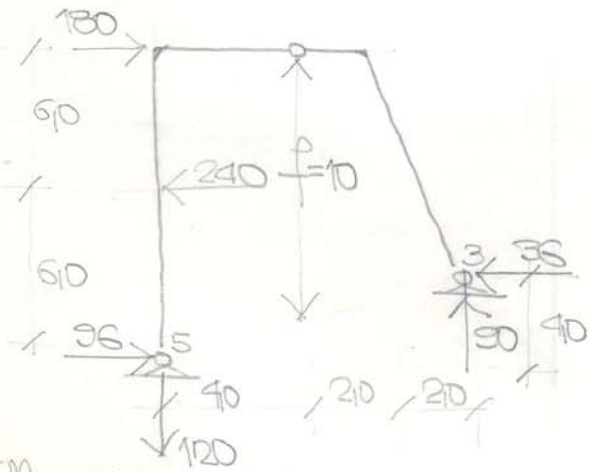
$$\underline{IMa = 0}$$

$$Y_b \cdot 240 - 240 \cdot 60 = 0 \quad Y_b = \frac{240 \cdot 60}{240} = 60$$

$$Y_6 = 60 \text{ kn}$$

$$\underline{\Sigma M_b = 0} //$$

$$x_2 = 180 \text{ t/n}$$



$$\Sigma M_3 = 0 //$$

$$-15 \cdot B_p + 240 \cdot 2_p - 180 \cdot 8_p = 0$$

$$-Y_5 = \frac{180 \cdot 80 - 240 \cdot 20}{80}$$

$$-y_5 = 120 \quad y_5 = -120 \text{ cm}$$

$$\frac{EM_g = \phi''}{\quad}$$

$$120 \cdot 40 - 240 \cdot 60 + 115 \cdot 10 = 0$$

$$H_5 = \frac{240 \cdot 610 - 120 \cdot 410}{10}$$

$$H_0 = 96 \text{ kN}$$

$$\underline{\Sigma M_5 = 0}$$

$$13 \cdot 8,0 + 20,0 \cdot 6,0 - 180 \cdot 12,0 = 0$$

$$Y_3 = \frac{180 \cdot 120 - 240 \cdot 60}{80} = 90$$

$$Y_3 = 30,0 \text{ ton}$$

$$\sum m_i \vec{g} = 0 //$$

$$50.910 - 113.10 = 0$$

$$H_3 = 38 \text{ km,}$$

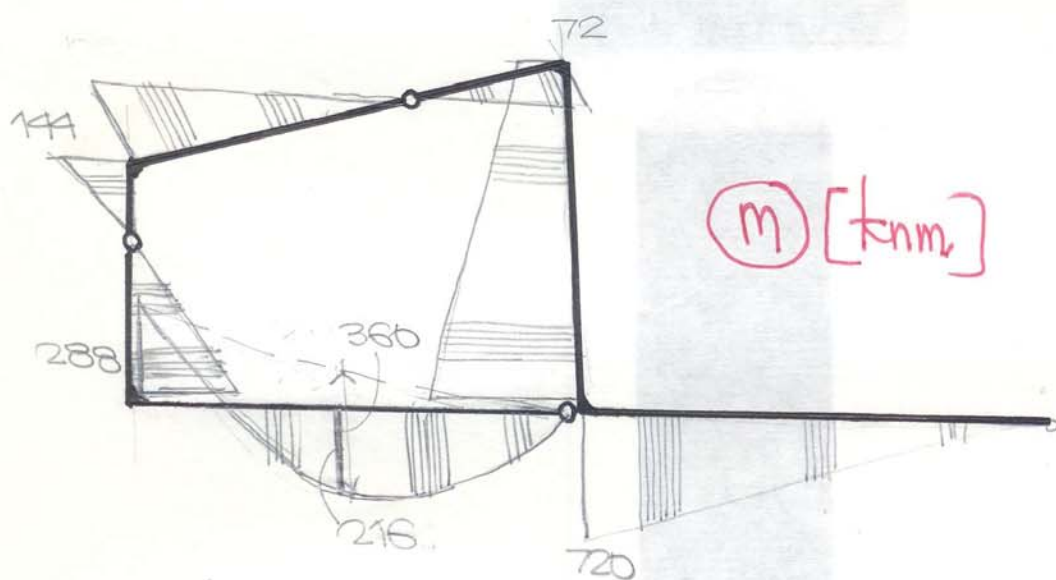
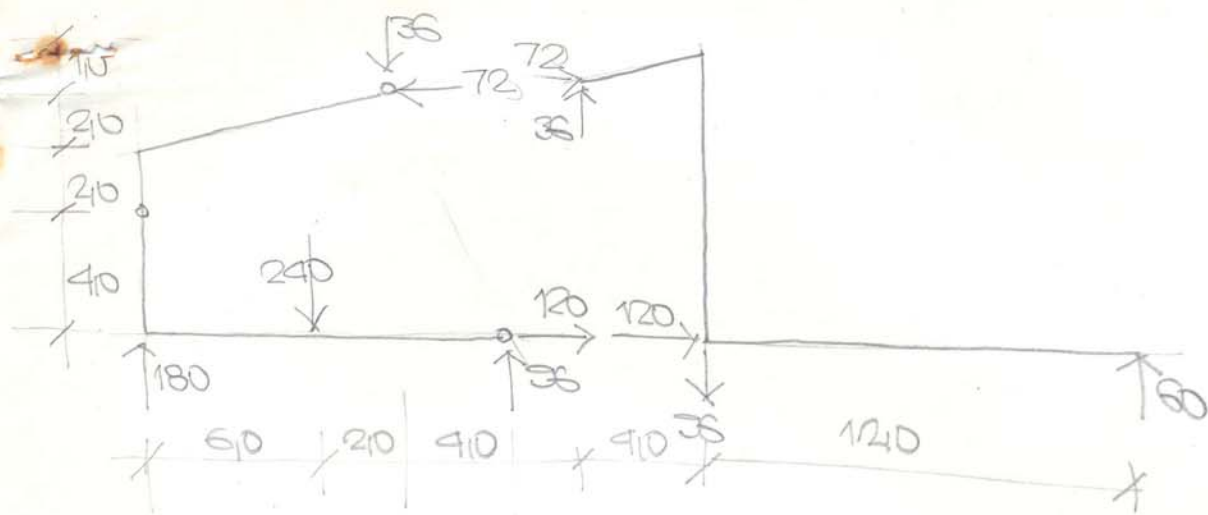
$$t_{5\alpha} = 0,5$$

$$y_5 = -120 + 0,5 \cdot 96 = -72$$

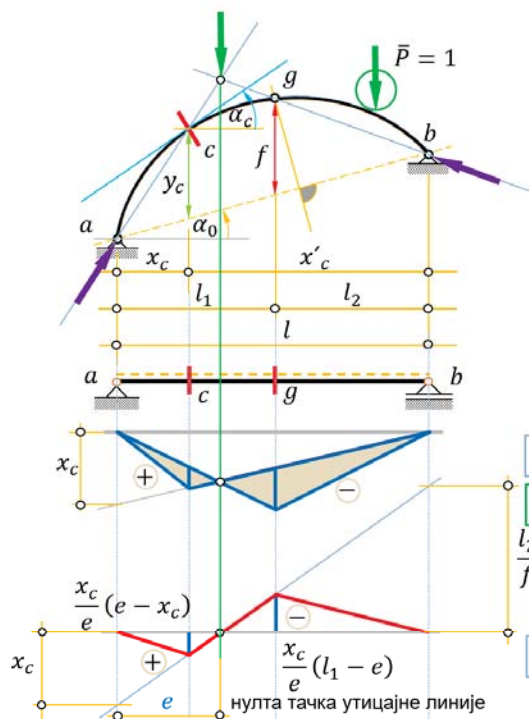
$$Y_5 = -72.10 \text{ t/m}$$

$$\sqrt[3]{3} = 90 - 0,5 \cdot 36 = 72$$

$$Y_3 = 72,0 \text{ ton}$$



Утицајна линија за момент савијања у пресеку "с"



$$M_c = M_{c0} - H \cdot y_c$$

$$M_{g0} = B_0 \cdot l_2 = \frac{e \cdot l_2}{l} \quad H = \frac{M_{g0}}{f} = \frac{e \cdot l_2}{f \cdot l}$$

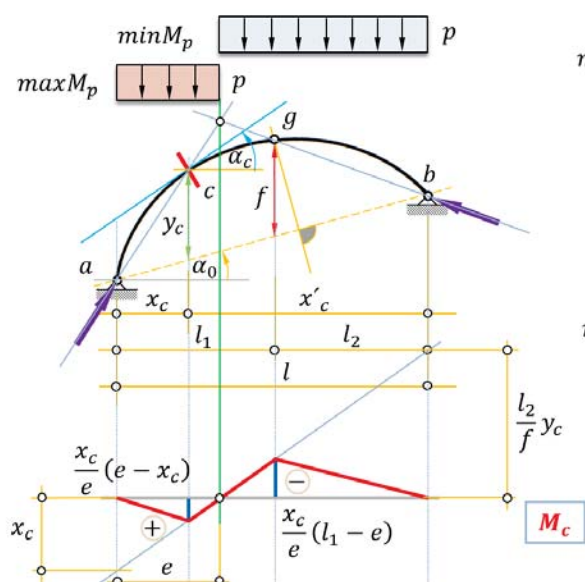
$$V'_a = A_0 = \frac{l - e}{l} \quad V'_b = B_0 = \frac{e}{l}$$

$$M_c = A_0 x_c - H y_c = 0 \quad \text{УСЛОВ}$$

$$= \frac{l - e}{l} \cdot x_c - \frac{e \cdot l_2}{f \cdot l} \cdot y_c = 0$$

$$e = \frac{l}{1 + \frac{l_2 y_c}{f x_c}}$$

Екстремне вредности момента савијања



$$\max M_p^c = \frac{p \cdot x_c (e - x_c)}{2}$$

$$T_{c0} = \frac{pl}{2} \left[2 \left(\frac{e}{l} - \frac{x_c}{l} \right) - \left(\frac{e}{l} \right)^2 \right]$$

$$H = \frac{pl}{2} \cdot \frac{l_2}{f} \left(\frac{e}{l} \right)^2$$

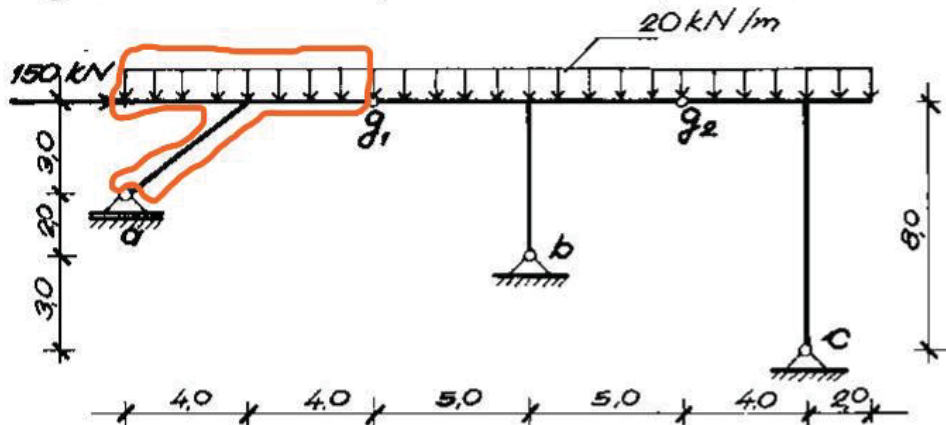
$$\min M_p^c = \frac{p \cdot x_c (l_1 - e)(l - e)}{2 \cdot e}$$

$$T_{c0} = \frac{pl}{2} \left(1 - \frac{e}{l} \right)^2$$

$$H = \frac{pl}{2} \left[\left(1 - \frac{e}{l} \right)^2 \frac{l_1}{f} - \frac{(l_1 - e)^2}{l \cdot f} \right]$$

Пример

Za dati nosač i opterećenje sračunati i nacrtati dijagrame sila u presecima: M , T i N .



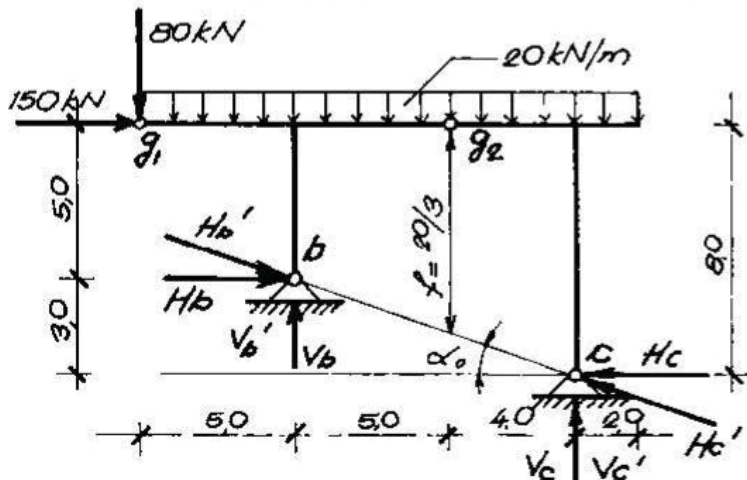
Rešenje :

Odrediti vanje reakcija ostataka

$$\sum M_{g1} = 0 \Rightarrow V_a \cdot 80 = 20 \cdot 80 \cdot 40 \Rightarrow V_a = 80 \text{ kN}$$

$$H_{g1} = 150 \text{ kN} \quad ; \quad V_{g1} = 208,0 - 80 = 80 \text{ kN}$$

Deo $g_1 - g_2, a, b$ predstavlja luk na tri zgloba.



$$\frac{1}{9} \alpha_0 = \frac{3}{9} = \frac{1}{3}$$

$$f = 80 - 40 \cdot \frac{1}{3}$$

$$f = \frac{20}{3}$$

$$\sum M_c = 0 \Rightarrow V_b' = \frac{1}{9} (80 \cdot 14 - 150 \cdot 8 + 16 \cdot 20 \cdot 6) = \frac{1340}{9}$$

$$\Sigma M_b = 0 \Rightarrow V_c' = \frac{1}{9} (20 \cdot 16 \cdot 3 + 150 \cdot 5 - 80 \cdot 5) = \frac{1310}{9}$$

$$\sum M_2 = 0 \Rightarrow H_b = \frac{1}{f} \left(\frac{1840}{9} \cdot 5 - 80 \cdot 10 - 20 \cdot 10 \cdot 5 \right)$$

$$H_b = \frac{3}{20} \cdot \left(-\frac{7000}{9} \right) = -\frac{700}{6} \text{ kN}$$

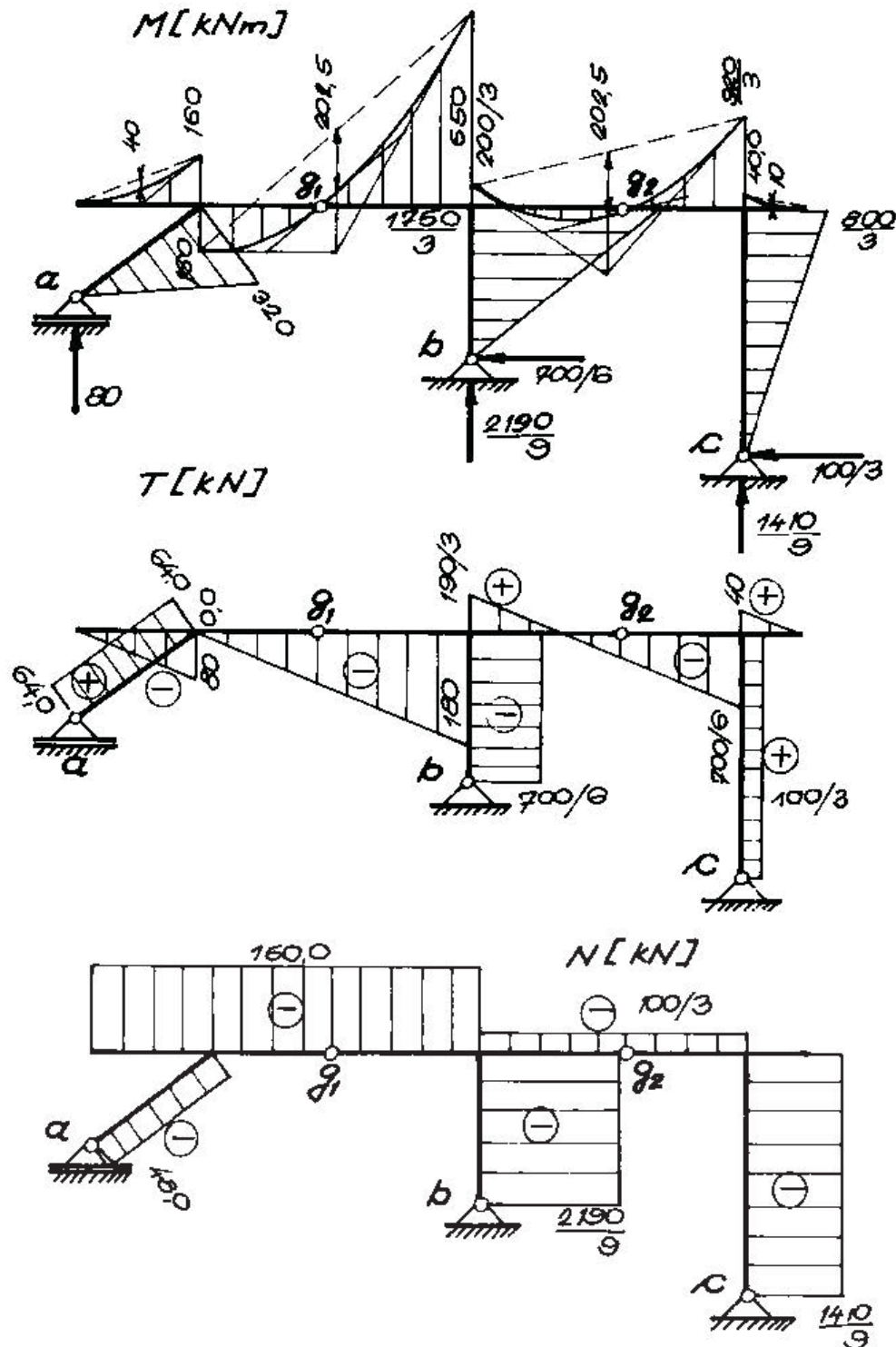
$$\Sigma M_{g_2}^d = 0 \Rightarrow H_c = \frac{1}{f} \left(\frac{1310}{9} \cdot 4 - 6 \cdot 20 \cdot 3 \right)$$

$$= \frac{3}{20} \cdot \frac{2000}{9} = \frac{100}{3} \text{ kN}$$

$$V_b = V_b' - H_b \tan \alpha = \frac{1840}{9} + \frac{700}{6} \cdot \frac{1}{3} = \frac{2190}{9} = 243,333 \text{ kN}$$

$$V_c = V_c' + H_c \tan \alpha = \frac{1310}{9} + \frac{700}{3} \cdot \frac{1}{3} = \frac{1410}{9} = 156,666 \text{ kN}$$

Sa nađenim reakcijama osbnaca lako se dobijaju traženi dijagrami M , T i N .



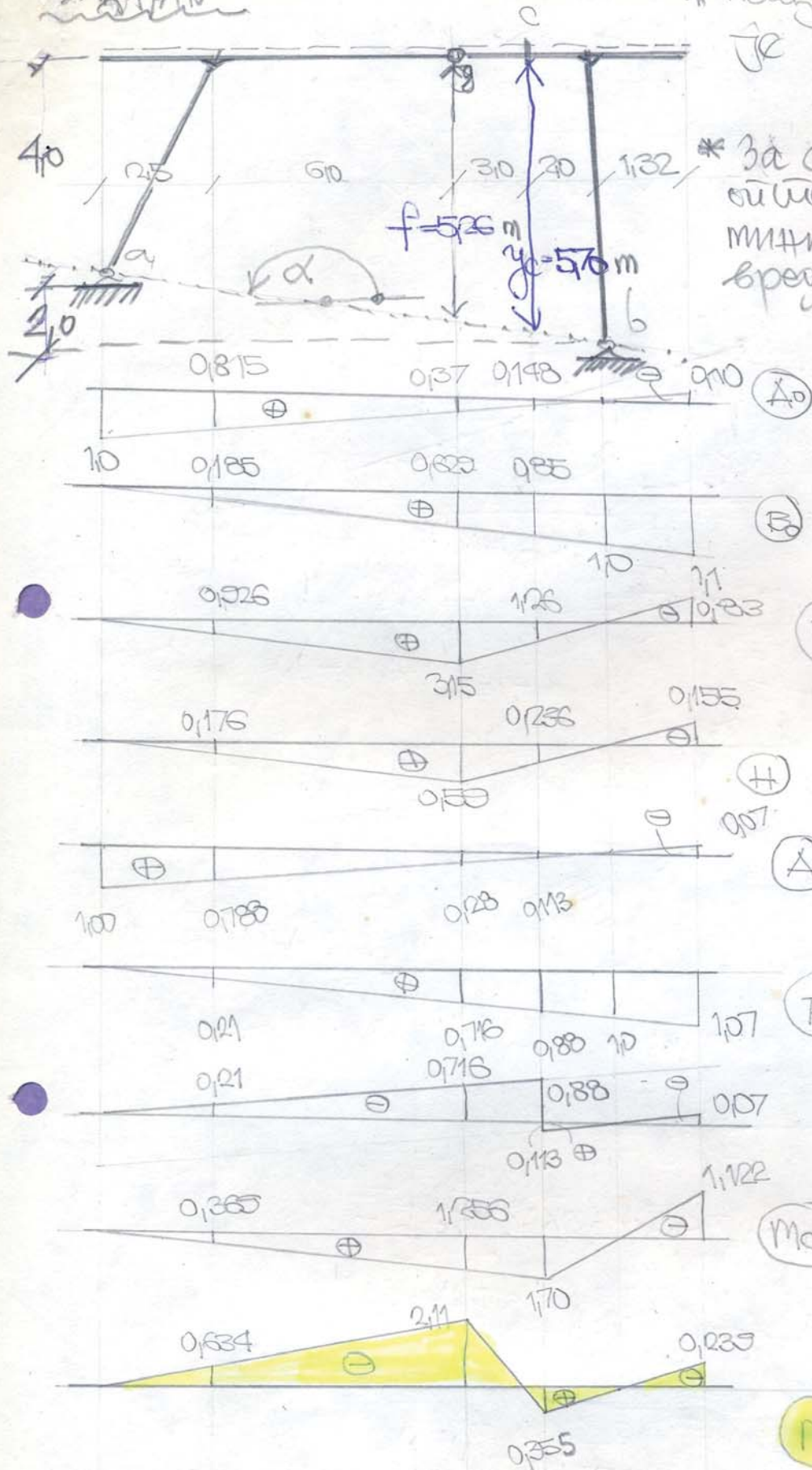
ЗАДАЧА

* Напряжения упираются

je 22

A, B, C, Mc

* За систему укрепитель
определяем сбалансирован
минимумы абсолютности
вращения Mc.



$$\tan \alpha = \frac{2}{13.5} = 0.148148 =$$

$$\alpha = 8.42696^\circ$$

$$\alpha = 180 - 8.42696 = 171.57^\circ$$

$$\tan \alpha = -0.148148$$

$$H = \frac{M_{B0}}{F} = \frac{3.15}{5.26} = 0.5988$$

$$d_1 = \alpha$$

$$\sin d_1 = 0.146601 = 0.147$$

$$T_c = T_{c0} \cos d_1 - H \cdot \frac{\sin(d_1 - d_0)}{\cos d_0}$$

$$T_c = T_{c0} - H \cdot \sin(-d_1)$$

или по пересечению
лево $\rightarrow -B, 09$
пересечение право $\rightarrow A$

$$T_c = -0.85 + 0.236 \cdot (-0.147)$$

$$T_c = -0.88$$

$$M_c = M_{c0} - H \cdot y_c$$

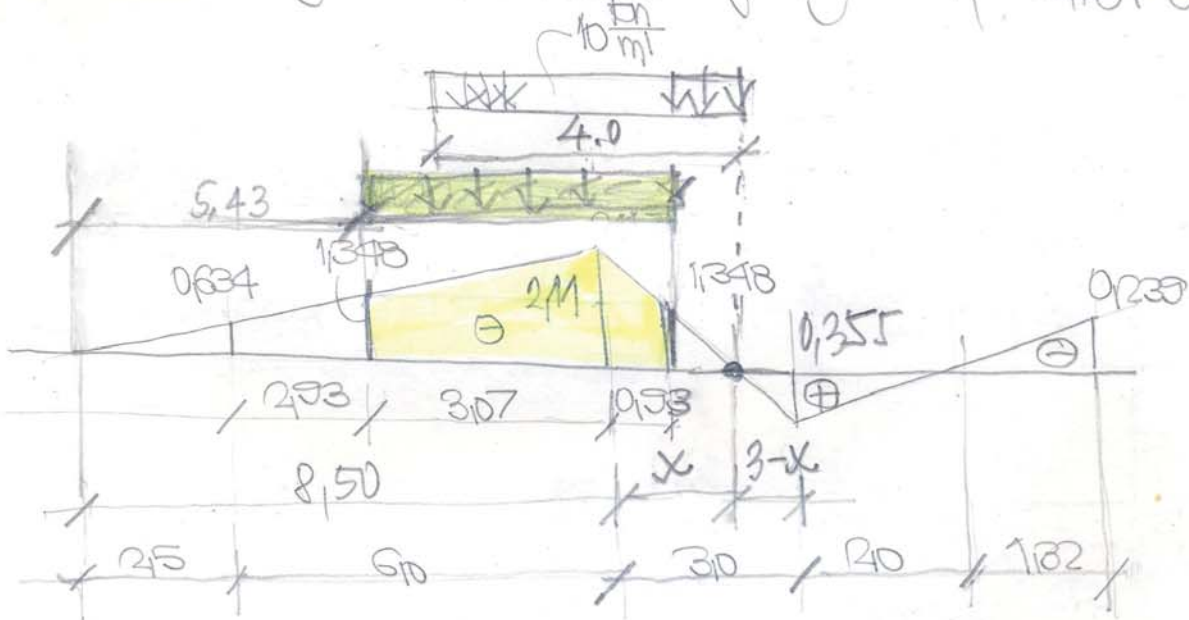
$$M_c = 1.70 - 0.236 \cdot 5.70 = 0.355$$

$$M_c = 1.256 - 0.59 \cdot 5.70 = 2.107 = 2.11$$

$$M_c = 0.369 - 0.176 \cdot 5.70 = 0.634$$

$$M_c = 1.122 - 0.155 \cdot 5.70 = 0.239$$

Minimumi aksijalni momentu koji se pojavljuje na osovini.



$$x = \frac{2.57 \cdot 4.0}{8.5 + 2.57} = 0.93$$

$$2.11 : x = 0.355 \cdot (3 - x)$$

$$\min M_c = p \cdot F^- =$$

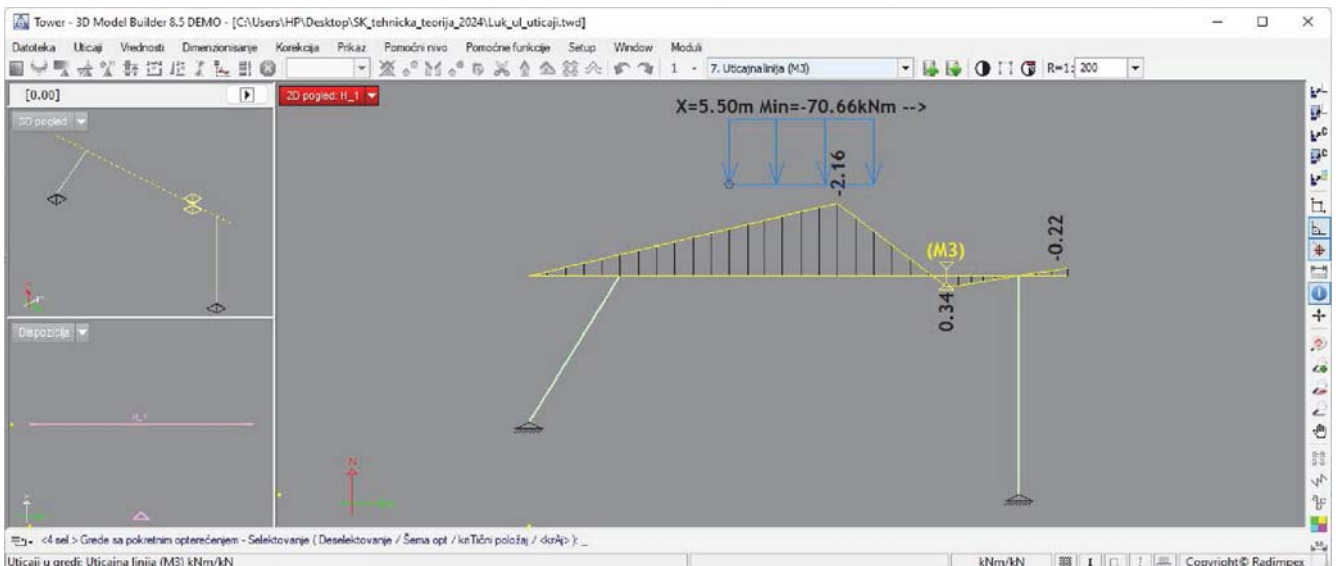
$$x = 2.57$$

$$F^- = \frac{1}{2} (1.348 + 2.11) \cdot 3.07 +$$

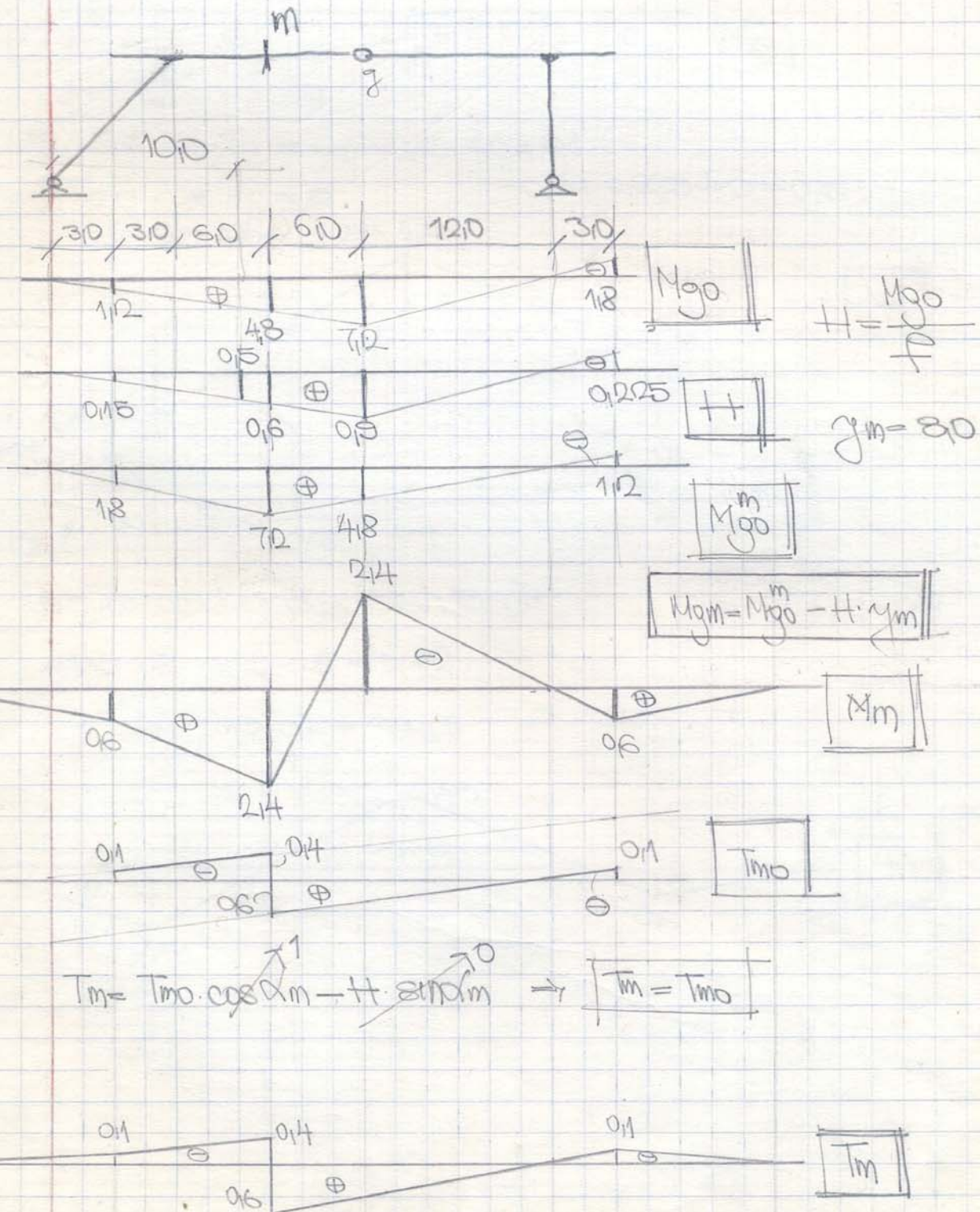
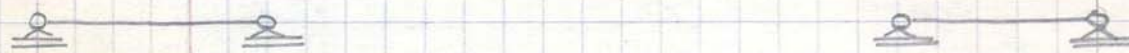
$$+ \frac{1}{2} (2.11 + 1.348) \cdot 0.93 = 6.916$$

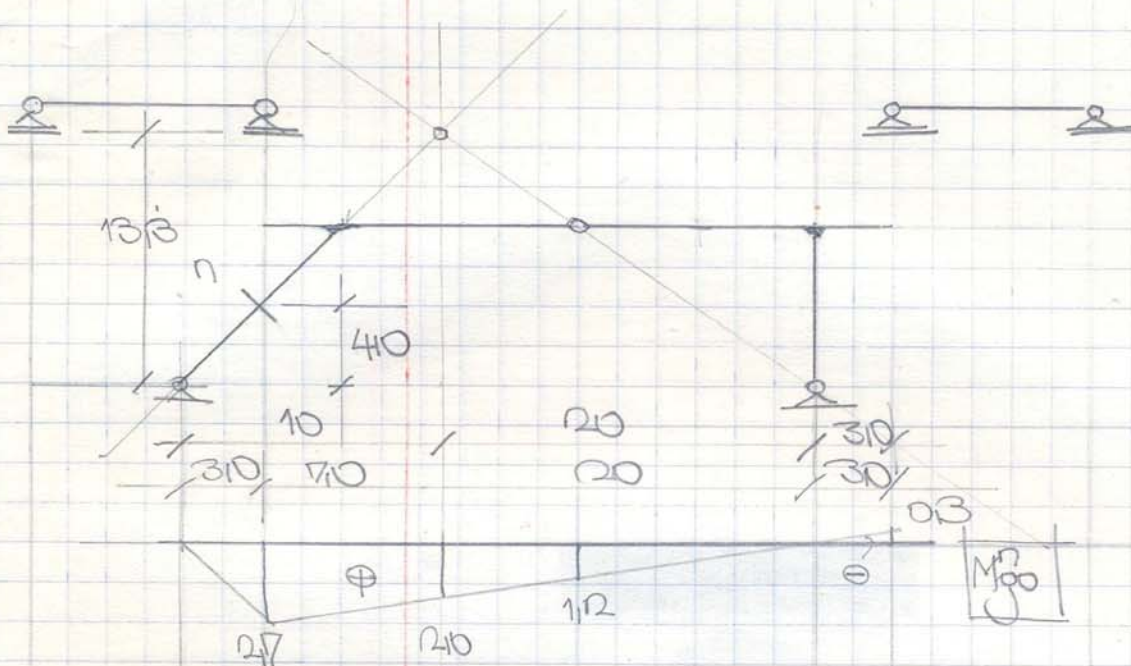
$$\min M_c = 10 \cdot 6.916 = 69.16 \text{ kNm}$$

Контрола резултата - методом коначних елемената



За носач приказан на скици конструисати утицајне линије за M_m , T_m , M_n , T_n .



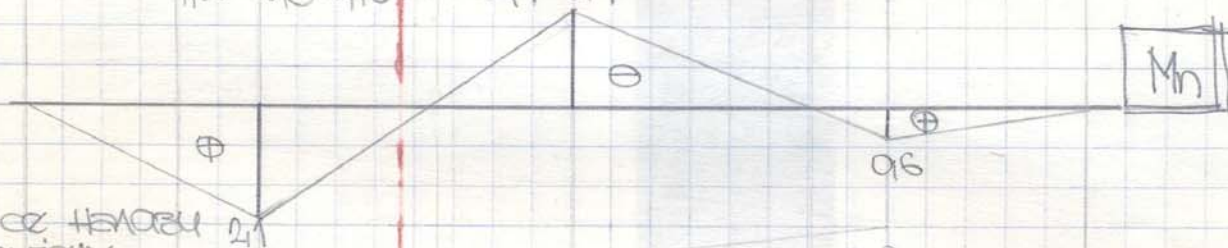


$$M_n = M_{00} - H \cdot y_n = 2.7 - 0.15 \cdot 40 = 2.1$$

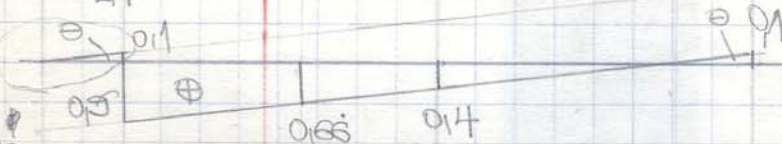
$$2.0 - 0.15 \cdot 40 = 0$$

$$-0.3 + 0.225 \cdot 40 = 0.6$$

$$1.2 - 0.15 \cdot 40 = -2.4 \quad 2.4$$



обоим направлениям
и горизонтально
вертикально
и горизонтально
и вертикально



$$T_n = T_{n0} \cdot \cos 53.13 - H \cdot \sin 53.13$$

$$\cos 53.13 = 0.6$$

$$\sin 53.13 = 0.8$$

$$= 0.3 \cdot 0.6 - 0.15 \cdot 0.8 = 0.42$$

$$0.4 \cdot 0.6 - 0.3 \cdot 0.8 = -0.48$$

