

# 5

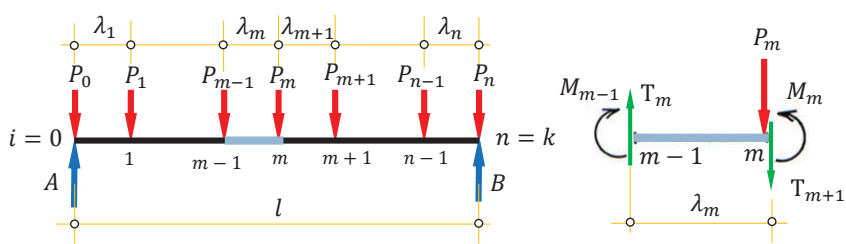
## СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

- материјал за вежбе -

2024.

## Нумерички поступак



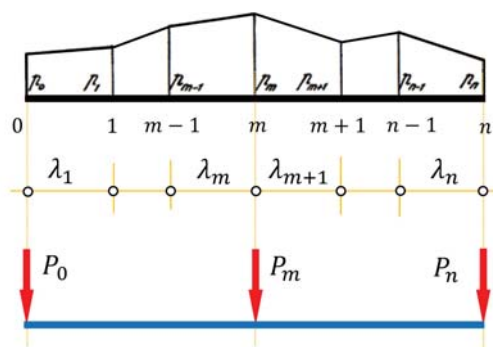
$$T_{m+1} = T_m - P_m$$

$$M_m = M_{m-1} + T_m \lambda_m$$

$$A = \frac{1}{l} \sum_{m=0}^n P_m x'_m$$

$$B = \sum_{m=0}^n P_m - A$$

## Линеарна расподела оптерећења

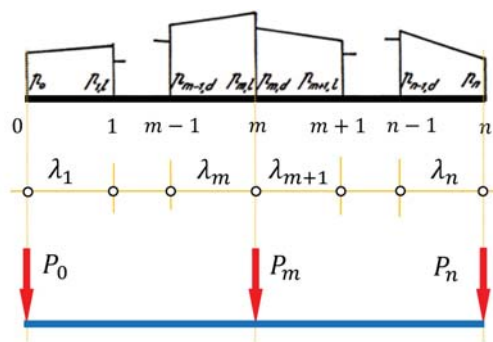


$$P_0 = \frac{\lambda}{6} (2p_0 + p_1)$$

$$P_m = \frac{\lambda}{6} (p_{m-1} + 4p_m + p_{m+1}), m = 1, 2, \dots, n-1 \quad (1)$$

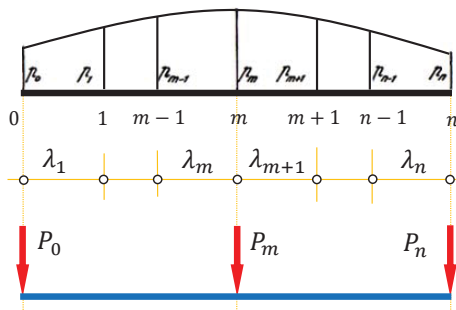
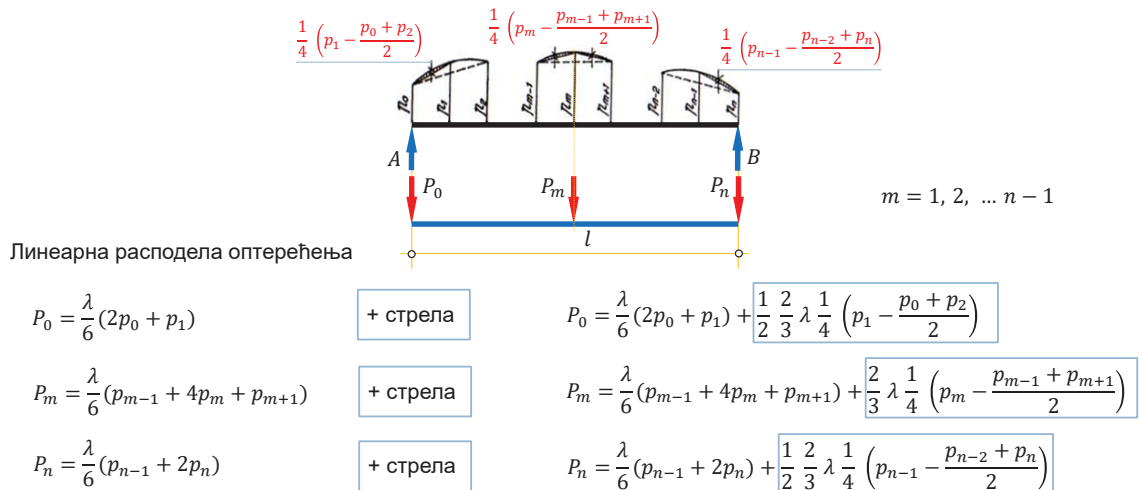
$$P_n = \frac{\lambda}{6} (p_{n-1} + 2p_n)$$

## Линеарна скоковита расподела оптерећења



$$\begin{aligned}
P_0 &= \frac{\lambda_1}{6} (2p_0 + p_1^l) \\
P_m &= \frac{\lambda_m}{6} (p_{m-1}^d + 2p_m^l) + \frac{\lambda_{m+1}}{6} (2p_m^d + p_{m+1}^l), m = 1, 2, \dots, n-1 \\
P_n &= \frac{\lambda_n}{6} (p_{n-1}^d + 2p_n)
\end{aligned} \quad (2)$$

### Параболична расподела оптерећења

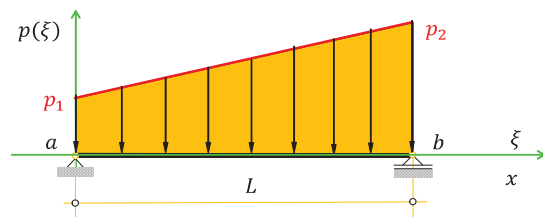


$$\begin{aligned}
P_0 &= \frac{\lambda}{24} (7p_0 + 6p_1 - p_2) \\
P_m &= \frac{\lambda}{12} (p_{m-1} + 10p_m + p_{m+1}), m = 1, 2, \dots, n-1 \\
P_n &= \frac{\lambda}{24} (-p_{n-2} + 6p_{n-1} + 7p_n)
\end{aligned} \quad (3)$$

### Примери

За све носаче са датим оптерећењима приказани на скицама потребно је срачунати реакције ослонаца и пресечне силе нумеричким поступком делећи носач у десетинама распона.

## Primer



$$p_1 := 2 \frac{\text{kN}}{\text{m}} \quad M := 0 \quad \text{kNm}$$

$$p_2 := 4 \frac{\text{kN}}{\text{m}}$$

$$L := 8 \text{ m} \quad \lambda := \frac{L}{10} \quad \lambda = 0.8$$

Funkcija opterećenja:  $p(\xi) := a \cdot \xi + b$

Given

$$1... \quad q \cdot 0 + b = 2$$

Konstante su:  $a := \frac{1}{4} \quad b := 2$

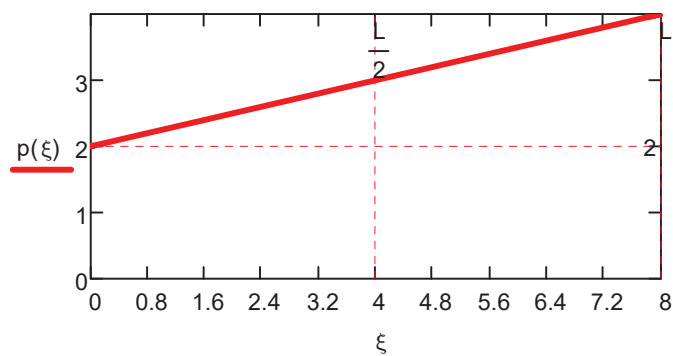
$$2... \quad a \cdot 8 + b = 4$$

$$p(\xi) := a \cdot \xi + b$$

$$\text{Find}(a, b) \rightarrow \begin{pmatrix} \frac{1}{4} \\ 2 \end{pmatrix}$$

$$p(\xi) \text{ simplify } \rightarrow \frac{\xi}{4} + 2$$

$$p(\xi) := \frac{\xi}{4} + 2$$



$$p(0) = 2$$

$$p(0.8) = 2.2$$

$$p(1.6) = 2.4$$

$$p(2.4) = 2.6$$

$$p(3.2) = 2.8$$

$$p(4) = 3$$

$$p(4.8) = 3.2$$

$$p(5.6) = 3.4$$

$$p(6.4) = 3.6$$

$$p(7.2) = 3.8$$

$$p(8) = 4$$

$$\underset{\text{w}}{p} := \begin{pmatrix} 2 \\ 2.2 \\ 2.4 \\ 2.6 \\ 2.8 \\ 3 \\ 3.2 \\ 3.4 \\ 3.6 \\ 3.8 \\ 4 \end{pmatrix} \quad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \quad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$$P_0 := \frac{\lambda}{6} \cdot (2 \cdot p_0 + p_1)$$

$$P_0 = 0.827$$

$$P_8 := \frac{\lambda}{6} \cdot (p_7 + 4 p_8 + p_9)$$

$$P_8 = 2.88$$

$$P_1 := \frac{\lambda}{6} \cdot (p_0 + 4 p_1 + p_2)$$

$$P_1 = 1.76$$

$$P_9 := \frac{\lambda}{6} \cdot (p_8 + 4 p_9 + p_{10})$$

$$P_9 = 3.04$$

$$P_2 := \frac{\lambda}{6} \cdot (p_1 + 4 p_2 + p_3)$$

$$P_2 = 1.92$$

$$P_{10} := \frac{\lambda}{6} \cdot (2 \cdot p_{10} + p_9)$$

$$P_{10} = 1.573$$

$$P_3 := \frac{\lambda}{6} \cdot (p_2 + 4 p_3 + p_4)$$

$$P_3 = 2.08$$

$$P_4 := \frac{\lambda}{6} \cdot (p_3 + 4 p_4 + p_5)$$

$$P_4 = 2.24$$

$$P_5 := \frac{\lambda}{6} \cdot (p_4 + 4 p_5 + p_6)$$

$$P_5 = 2.4$$

$$P_6 := \frac{\lambda}{6} \cdot (p_5 + 4 p_6 + p_7)$$

$$P_6 = 2.56$$

$$P_7 := \frac{\lambda}{6} \cdot (p_6 + 4 p_7 + p_8)$$

$$P_7 = 2.72$$

Reakcije oslonaca:

1. od opterećenja:

$$A_P := \sum_{m=0}^{10} (P_m \xi_{d_m}) \quad A_P = 10.667 \quad \text{kN}$$

$$B_P := \sum_{m=0}^{10} (P_m \xi_m) \quad B_P = 13.333 \quad \text{kN}$$

2. od koncentrisanog momenta:

$$A_M := \frac{-M}{L} \quad A_M = 0 \quad \text{kN}$$

$$B_M := \frac{M}{L} \quad B_M = 0 \quad \text{kN}$$

Reakcije su:

$$A := A_P + A_M \quad A = 10.667 \quad \text{kN}$$

$$B := B_P + B_M \quad B = 13.333 \quad \text{kN}$$

Presečne sile:

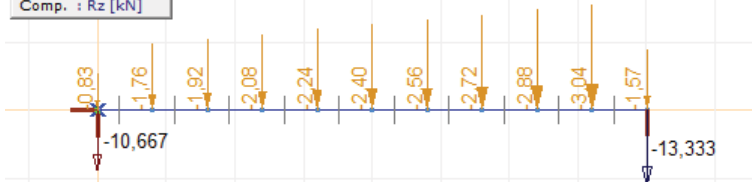
$$T_m := T_{m-1} - P_{m-1} \quad T_0 := A$$

$$M_m := M_{m-1} + T_m \cdot \lambda_m$$

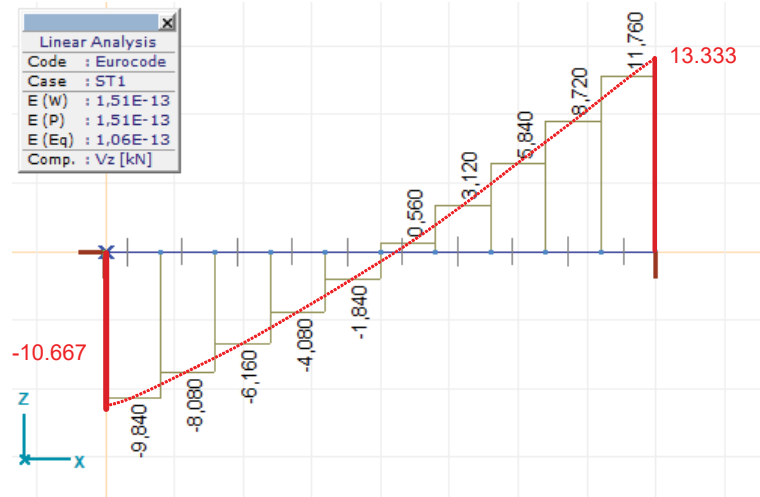
$m$	$\lambda_m$	$\xi_m$	$p_m$	$P_m$	$\xi'_m$	$P_m \xi'_m$	$T_m$	$T_m \lambda_m$	$M_m^{(M)}$	$M_m$
0	0,8	0	2	0,827	1	0,827	10,667	0		0
1	0,8	0,1	2,2	1,76	0,9	1,584	9,840	7,872		7,872
2	0,8	0,2	2,4	1,92	0,8	1,536	8,080	6,464		14,336
3	0,8	0,3	2,6	2,08	0,7	1,456	6,160	4,928		19,264
4	0,8	0,4	2,8	2,24	0,6	1,344	4,080	3,264		22,528
5	0,8	0,5	3	2,4	0,5	1,200	1,840	1,472		24,000
6	0,8	0,6	3,2	2,56	0,4	1,024	-0,560	-0,448		23,552
7	0,8	0,7	3,4	2,72	0,3	0,816	-3,120	-2,496		21,056
8	0,8	0,8	3,6	2,88	0,2	0,576	-5,840	-4,672		16,384
9	0,8	0,9	3,8	3,04	0,1	0,304	-8,720	-6,976		9,408
10	0,8	1	4	1,573	0	0	-11,760	-9,408		0,000
$\sum P_m =$				24	$A_P =$	10,667	-13,333	$=B$		

## Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila

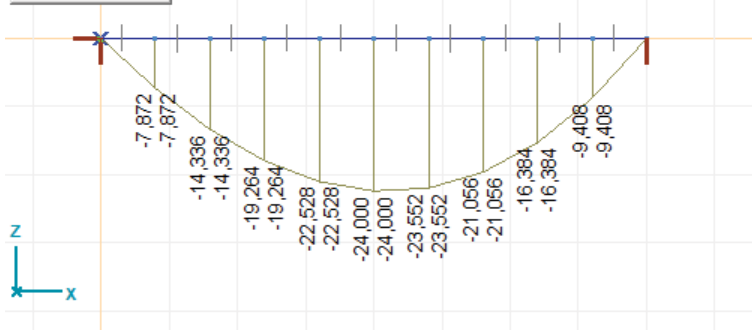
Linear Analysis
Code : Eurocode
Case : ST1
E (w) : 1,51E-13
E (p) : 1,51E-13
E (Eq) : 1,06E-13
Comp. : Rz [kN]



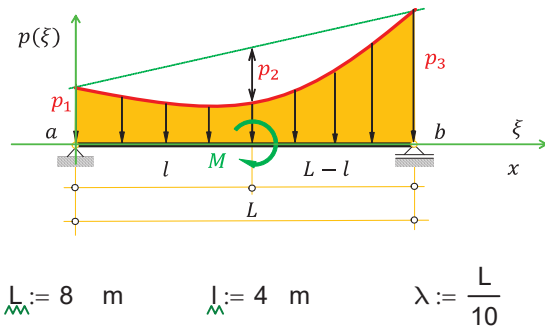
Linear Analysis
Code : Eurocode
Case : ST1
E (w) : 1,51E-13
E (p) : 1,51E-13
E (Eq) : 1,06E-13
Comp. : Vz [kN]



Linear Analysis
Code : Eurocode
Case : ST1
E (w) : 1,51E-13
E (p) : 1,51E-13
E (Eq) : 1,06E-13
Comp. : My [kNm]



## Primer



$$p_1 := 6 \frac{\text{kN}}{\text{m}} \quad M := 50 \text{ kNm}$$

$$p_2 := 7 \frac{\text{kN}}{\text{m}}$$

$$p_3 := 12 \frac{\text{kN}}{\text{m}}$$

$$L := 8 \text{ m}$$

$$l := 4 \text{ m}$$

$$\lambda := \frac{L}{10}$$

$$\lambda = 0.8$$

Funkcija opterećenja:  $p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

Given

$$1... \quad a \cdot 0^2 + b \cdot 0 + c = 6$$

Konstante su:  $a := \frac{7}{16} \quad b := -\frac{11}{4} \quad c := 6$

$$2... \quad a \cdot 4^2 + b \cdot 4 + c = 2$$

$$p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$$

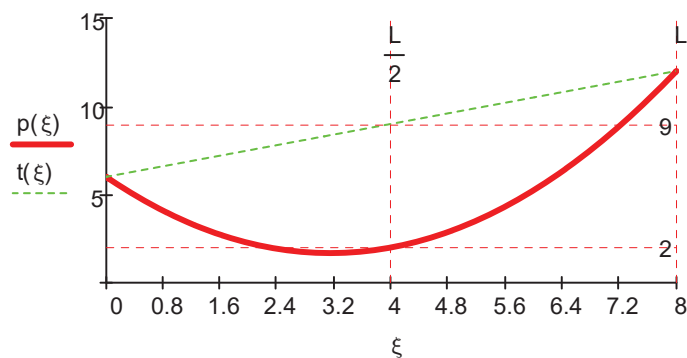
$$3... \quad a \cdot 8^2 + b \cdot 8 + c = 12$$

$$p(\xi) \text{ simplify } \rightarrow \frac{7 \cdot \xi^2}{16} - \frac{11 \cdot \xi}{4} + 6$$

$$\text{Find}(a, b, c) \rightarrow \begin{pmatrix} \frac{7}{16} \\ -\frac{11}{4} \\ 6 \end{pmatrix}$$

$$p(\xi) := \frac{7}{16} \cdot \xi^2 - \frac{11}{4} \cdot \xi + 6$$

$$t(\xi) := 6 + \frac{6}{8} \cdot \xi$$



$$p(0) = 6$$

$$p(0.8) = 4.08$$

$$p(1.6) = 2.72$$

$$p(2.4) = 1.92$$

$$p(3.2) = 1.68$$

$$p(4) = 2$$



$$p(4.8) = 2.88$$

$$p(5.6) = 4.32$$

$$p(6.4) = 6.32$$

$$p(7.2) = 8.88$$

$$p(8) = 12$$

$$\underset{\text{w}}{p} := \begin{pmatrix} 6 \\ 4.08 \\ 2.72 \\ 1.92 \\ 1.68 \\ 2 \\ 2.88 \\ 4.32 \\ 6.32 \\ 8.88 \\ 12 \end{pmatrix} \quad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \quad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$$P_0 := \frac{\lambda}{24} \cdot (7 \cdot p_0 + 6 p_1 - p_2) \quad P_0 = 2.125$$

$$P_8 := \frac{\lambda}{12} \cdot (p_7 + 10 p_8 + p_9) \quad P_8 = 5.093$$

$$P_1 := \frac{\lambda}{12} \cdot (p_0 + 10 p_1 + p_2) \quad P_1 = 3.301$$

$$P_9 := \frac{\lambda}{12} \cdot (p_8 + 10 p_9 + p_{10}) \quad P_9 = 7.141$$

$$P_2 := \frac{\lambda}{12} \cdot (p_1 + 10 p_2 + p_3) \quad P_2 = 2.213$$

$$P_{10} := \frac{\lambda}{24} \cdot (7 \cdot p_{10} + 6 p_9 - p_8) \quad P_{10} = 4.365$$

$$P_3 := \frac{\lambda}{12} \cdot (p_2 + 10 p_3 + p_4) \quad P_3 = 1.573$$

$$P_4 := \frac{\lambda}{12} \cdot (p_3 + 10 p_4 + p_5) \quad P_4 = 1.381$$

$$P_5 := \frac{\lambda}{12} \cdot (p_4 + 10 p_5 + p_6) \quad P_5 = 1.637$$

$$P_6 := \frac{\lambda}{12} \cdot (p_5 + 10 p_6 + p_7) \quad P_6 = 2.341$$

$$P_7 := \frac{\lambda}{12} \cdot (p_6 + 10 p_7 + p_8) \quad P_7 = 3.493$$

Reakcije oslonaca:

1. od opterećenja:

$$A_P := \sum_{m=0}^{10} (P_m \xi_m) \quad A_P = 13.333 \quad \text{kN}$$

$$B_P := \sum_{m=0}^{10} (P_m \xi_m) \quad B_P = 21.333 \quad \text{kN}$$

2. od koncentrisanog momenta:

$$A_M := \frac{-M}{L} \quad A_M = -6.25 \quad \text{kN}$$

$$B_M := \frac{M}{L} \quad B_M = 6.25 \quad \text{kN}$$

Reakcije su:

$$A := A_P + A_M \quad A = 7.083 \quad \text{kN}$$

$$B := B_P + B_M \quad B = 27.58 \quad \text{kN}$$

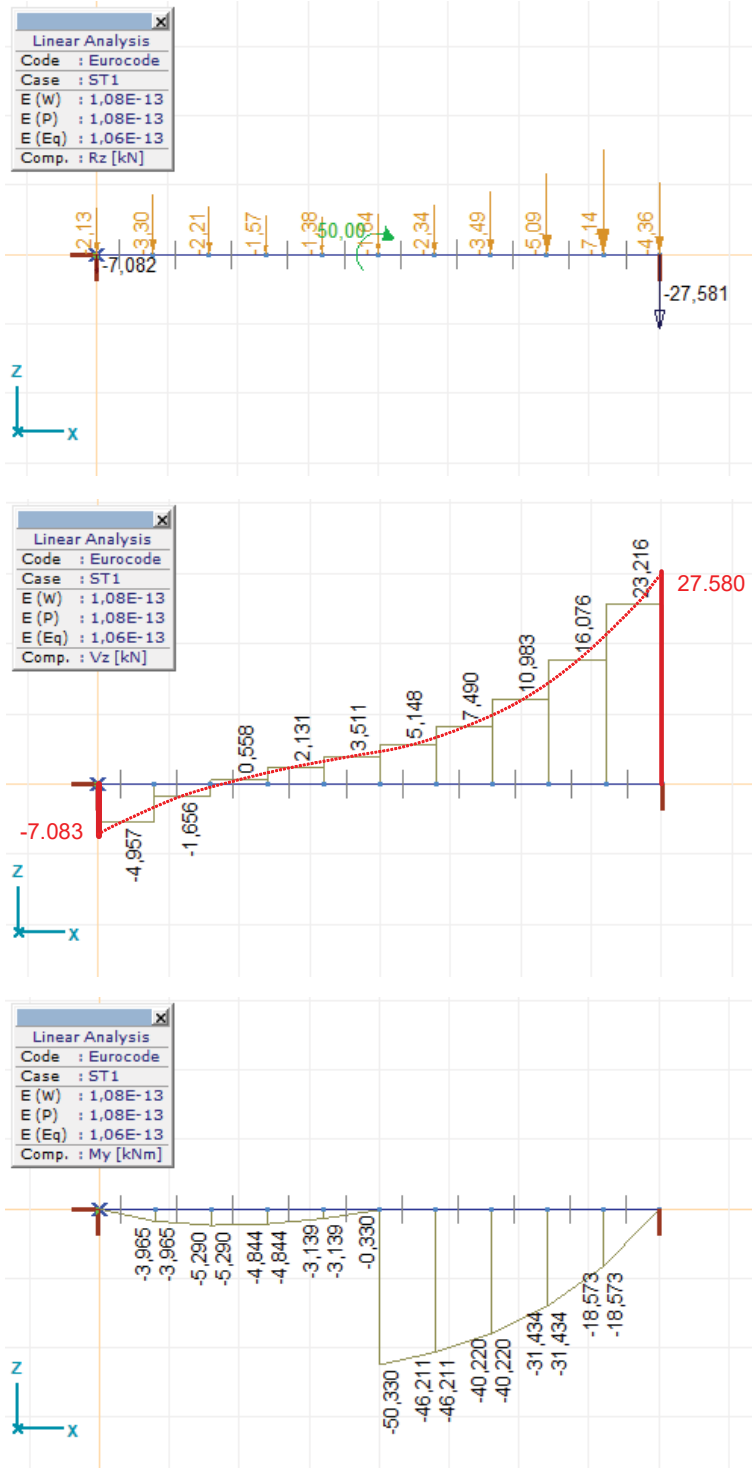
Presečne sile:

$$T_m := T_{m-1} - P_{m-1} \quad T_0 := A$$

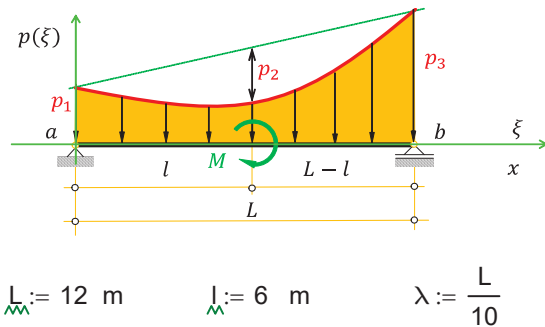
$$M_m := M_{m-1} + T_m \cdot \lambda_m$$

$m$	$\lambda_m$	$\xi_m$	$p_m$	$P_m$	$\xi_m$	$P_m \xi_m$	$T_m$	$T_m \lambda_m$	$M_m^{(M)}$	$M_m$
0	0,8	0	6	2,125	1	2,125	7,083	0		0
1	0,8	0,1	4,08	3,301	0,9	2,971	4,958	3,966		3,966
2	0,8	0,2	2,72	2,213	0,8	1,770	1,657	1,326		5,292
3	0,8	0,3	1,92	1,573	0,7	1,101	-0,556	-0,445		4,847
4	0,8	0,4	1,68	1,381	0,6	0,829	-2,129	-1,703		3,144
5	0,8	0,5	2	1,637	0,5	0,819	-3,510	-2,808		0,336
									50	50,336
6	0,8	0,6	2,88	2,341	0,4	0,936	-5,147	-4,118		46,218
7	0,8	0,7	4,32	3,493	0,3	1,048	-7,488	-5,990		40,228
8	0,8	0,8	6,32	5,093	0,2	1,019	-10,981	-8,785		31,443
9	0,8	0,9	8,88	7,141	0,1	0,714	-16,074	-12,859		18,584
10	0,8	1	12	4,365	0	0	-23,215	-18,572		0
				$\sum P_m = 34,663$	$A_P =$	13,332	-27,580	$=B$		

## Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila



## Primer



$$p_1 := 6 \frac{\text{kN}}{\text{m}} \quad M := 60 \text{ kNm}$$

$$p_2 := 7 \frac{\text{kN}}{\text{m}}$$

$$p_3 := 12 \frac{\text{kN}}{\text{m}}$$

$$L := 12 \text{ m} \quad l := 6 \text{ m} \quad \lambda := \frac{L}{10}$$

$$\lambda = 1.2$$

Funkcija opterećenja:  $p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

Given

$$1... \quad a \cdot 0^2 + b \cdot 0 + c = 6$$

Konstante su:  $a := \frac{7}{36} \quad b := -\frac{11}{6} \quad c := 6$

$$2... \quad a \cdot 6^2 + b \cdot 6 + c = 2$$

$$p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$$

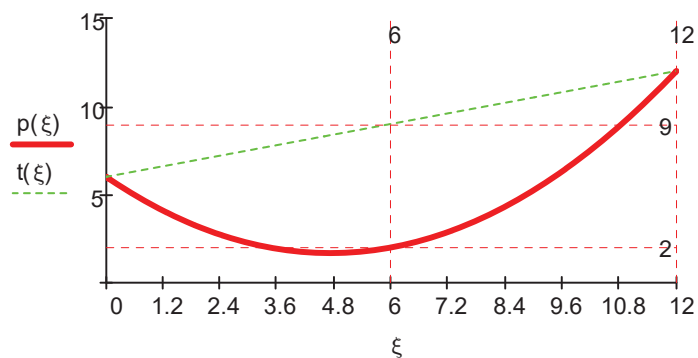
$$3... \quad a \cdot 12^2 + b \cdot 12 + c = 12$$

$$p(\xi) \text{ simplify } \rightarrow \frac{7 \cdot \xi^2}{36} - \frac{11 \cdot \xi}{6} + 6$$

$$\text{Find}(a, b, c) \rightarrow \begin{pmatrix} \frac{7}{36} \\ -\frac{11}{6} \\ 6 \end{pmatrix}$$

$$p(\xi) := \frac{7}{36} \cdot \xi^2 - \frac{11}{6} \cdot \xi + 6$$

$$t(\xi) := 6 + 0.5 \cdot \xi$$



$$p(0) = 6$$

$$p(1.2) = 4.08$$

$$p(2.4) = 2.72$$

$$p(3.6) = 1.92$$

$$p(4.8) = 1.68$$

$$p(6) = 2$$

$$p(7.2) = 2.88 \quad p(8.4) = 4.32 \quad p(9.6) = 6.32 \quad p(10.8) = 8.88 \quad p(12) = 12$$

$$\underset{\text{w}}{p} := \begin{pmatrix} 6 \\ 4.08 \\ 2.72 \\ 1.92 \\ 1.68 \\ 2 \\ 2.88 \\ 4.32 \\ 6.32 \\ 8.88 \\ 12 \end{pmatrix} \quad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \quad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$$P_0 := \frac{\lambda}{24} \cdot (7 \cdot p_0 + 6 p_1 - p_2) \quad P_0 = 3.188 \quad P_8 := \frac{\lambda}{12} \cdot (p_7 + 10 p_8 + p_9) \quad P_8 = 7.64$$

$$P_1 := \frac{\lambda}{12} \cdot (p_0 + 10 p_1 + p_2) \quad P_1 = 4.952 \quad P_9 := \frac{\lambda}{12} \cdot (p_8 + 10 p_9 + p_{10}) \quad P_9 = 10.712$$

$$P_2 := \frac{\lambda}{12} \cdot (p_1 + 10 p_2 + p_3) \quad P_2 = 3.32 \quad P_{10} := \frac{\lambda}{24} \cdot (7 \cdot p_{10} + 6 p_9 - p_8) \quad P_{10} = 6.548$$

$$P_3 := \frac{\lambda}{12} \cdot (p_2 + 10 p_3 + p_4) \quad P_3 = 2.36$$

$$P_4 := \frac{\lambda}{12} \cdot (p_3 + 10 p_4 + p_5) \quad P_4 = 2.072$$

$$P_5 := \frac{\lambda}{12} \cdot (p_4 + 10 p_5 + p_6) \quad P_5 = 2.456$$

$$P_6 := \frac{\lambda}{12} \cdot (p_5 + 10 p_6 + p_7) \quad P_6 = 3.512$$

$$P_7 := \frac{\lambda}{12} \cdot (p_6 + 10 p_7 + p_8) \quad P_7 = 5.24$$

Reakcije oslonaca:

1. od opterećenja:

$$A_P := \sum_{m=0}^{10} (P_m \xi_m) \quad A_P = 20 \quad \text{kN}$$

$$B_P := \sum_{m=0}^{10} (P_m \xi_m) \quad B_P = 32 \quad \text{kN}$$

2. od koncentrisanog momenta:

$$A_M := \frac{-M}{L} \quad A_M = -5 \quad \text{kN}$$

$$B_M := \frac{M}{L} \quad B_M = 5 \quad \text{kN}$$

Reakcije su:

$$A := A_P + A_M \quad A = 15 \quad \text{kN}$$

$$B := B_P + B_M \quad B = 37 \quad \text{kN}$$

Presečne sile:

$$T_m := T_{m-1} - P_{m-1} \quad T_0 := A$$

$$M_m := M_{m-1} + T_m \cdot \lambda_m$$

$m$	$\lambda_m$	$\xi_m$	$p_m$	$P_m$	$\xi'_m$	$P_m \xi'_m$	$T_m$	$T_m \lambda_m$	$M_m^m$	$M_m$
0	1,2	0	6	3,188	1	3,188	15,000	0		0
1	1,2	0,1	4,08	4,952	0,9	4,457	11,812	14,174		14,174
2	1,2	0,2	2,72	3,32	0,8	2,656	6,860	8,232		22,406
3	1,2	0,3	1,92	2,36	0,7	1,652	3,540	4,248		26,654
4	1,2	0,4	1,68	2,072	0,6	1,243	1,180	1,416		28,070
5	1,2	0,5	2	2,456	0,5	1,228	-0,892	-1,070		27,000
6	1,2	0,6	2,88	3,512	0,4	1,405	-3,348	-4,018	60	87,000
7	1,2	0,7	4,32	5,24	0,3	1,572	-6,860	-8,232		82,982
8	1,2	0,8	6,32	7,64	0,2	1,528	-12,100	-14,520		74,750
9	1,2	0,9	8,88	10,712	0,1	1,071	-19,740	-23,688		60,230
10	1,2	1	12	6,548	0	0	-30,452	-36,542		36,542
				$\sum P_m = 52$	$A_P = 20$		$-37 = B$			

## Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila

