11

СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

– материјал за вежбе –

2024.

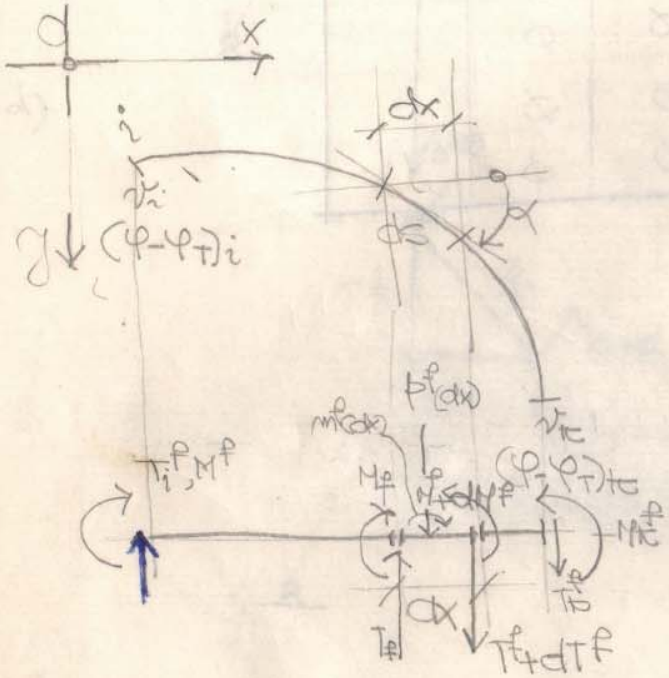
24.04.1991

DIJAGRAM POMERANJA PUNI RAVNI NOSAČA

$$du = \varepsilon dx - \varphi dy$$

$$d(\varphi - \varphi_T) = -\gamma ds \;/: \cos\alpha \Rightarrow \frac{d(\varphi-\varphi_T)}{dx} = -\frac{\gamma}{\cos\alpha}$$

$$dv = \varepsilon dy + \varphi dx \;/: dx \Rightarrow \frac{dv}{dx} = \varepsilon \cdot tg\alpha + \varphi \Rightarrow \frac{dv}{dx} = (\varphi-\varphi_T) + \varepsilon tg\alpha + \varphi_T \quad \ldots (1)$$



$$\Sigma V = 0 \;//$$

$$\frac{dT^f}{dx} = -p^f$$

$$\Sigma M_d = 0$$

$$\frac{dM^f}{dx} = T^f + m^f \quad \ldots (2)$$

$$p^f \leftrightarrow \frac{\gamma}{\cos\alpha}$$

$$m^f \leftrightarrow \varepsilon tg\alpha + \varphi_T$$

$$M^f \leftrightarrow v \;;\; T^f \leftrightarrow (\varphi-\varphi_T)$$

$$p^f = \frac{\gamma}{\cos\alpha} = \left(\frac{M}{EJ} + \alpha_t \frac{\Delta t}{h}\right) \cdot \frac{1}{\cos\alpha}$$

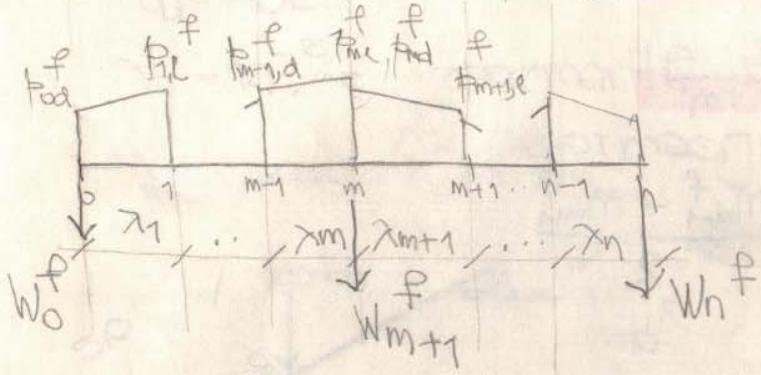
$$m^f = \varepsilon \cdot tg\alpha + \varphi_T = \left(\frac{N}{EF} + \alpha_t t^0\right) tg\alpha + t \cdot \frac{I}{GF}$$

Dijagram vertikalni pomeranja (V) grednog je dijagramu momenata m^f, a dijagram obtanja grednog je dijag. transverzalni sila T^f, fiktivnog štapa opterećenog fiktivnim raspored. silama p^f, i fiktivnim raspod. momentima m^f čiji granični uslovi po silama odgovaraju

graničnim uslovima po pomeranju na stranam nosača.

W - ELASTIČNE TEČIVE

W- a) **statička promjena opterećenja p^f**



$$W_0^f = \frac{\lambda_1}{6}\left(2p_0^f + p_{1,l}^f\right)$$

$$W_{m+1}^f = \frac{\lambda_m}{6}\left(2p_{m,l}^f + p_{m-1,d}^f\right) + \frac{\lambda_{m+1}}{6}\left(2p_{md}^f + p_{m+1,l}^f\right) \quad m = 1,2,\dots,n-1$$

$$W_n^f = \frac{\lambda_n}{6}\left(2p_n^f + p_{n-1,d}^f\right)$$

b) LINEARNA TROMJENA p^f

$\lambda = \text{const.}$

$$W_0^f = \frac{\lambda}{6}\left(2p_0^f + p_1^f\right)$$

$$W_m^f = \frac{\lambda}{6}\left(p_{m-1}^f + 4p_m^f + p_{m+1}^f\right) \quad m = 1,2,\dots,n-1$$

$$W_n^f = \frac{\lambda}{6}\left(2p_n^f + p_{n-1}^f\right)$$

c) PARABOLIČNA TROMJENA p^f

$$W_0^f = \frac{\lambda}{24}\left(7p_0^f + 6p_1^f - p_2^f\right)$$

$$W_m^f = \frac{\lambda}{12}\left(p_{m-1}^f + 10p_m^f + p_{m+1}^f\right) \quad (m = 1,2,\dots,n-1)$$

$$W_n^f = \frac{\lambda}{24}\left(7p_n^f + 6p_{n-1}^f - p_{n-2}^f\right)$$

- SKOKOVITA TRANSFORMACIJA m^f

$$W_0^{mf} = -\frac{m_0^f + m_{1,l}^f}{2} \quad ; \quad W_m^{mf} = \frac{m_{m-1,d}^f + m_{m,l}^f}{2} - \frac{m_{m,d}^f + m_{m+1,l}^f}{2}$$

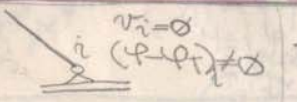
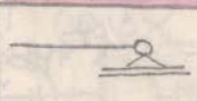
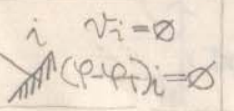
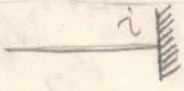
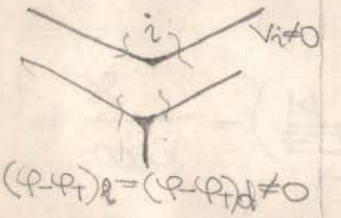
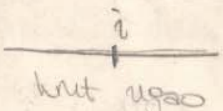
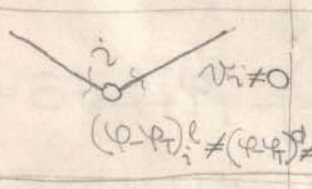
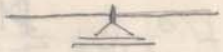
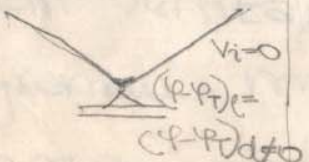
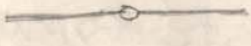
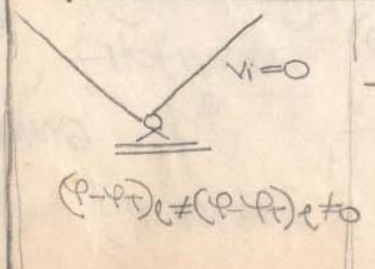
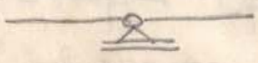
$$W_n^{mf} = \frac{m_{n-1,d}^f + m_n^f}{2}$$

- LINEARNA TRANSFORMACIJA m^f

$$W_0^{mf} = -\frac{m_0^f + m_1^f}{2} \qquad W_m^{mf} = \frac{m_{m-1}^f - m_{m+1}^f}{2}$$

$$W_n^{mf} = \frac{m_{n-1}^f + m_n^f}{2}$$

GRANIČNI USLOVI:

R. BROJ	STVARNI NOSAČ	FIKTIVNI NOSAČ
1.		 $M_i^f=0$ $T_i^f \neq 0$
2.		 $M_i^f=0$ $T_i^f=0$
3.		 $M_i^f \neq 0$ $T_i^f \neq 0$
4.		 $M_i^f \neq 0$ $T_l^f = T_d^f \neq 0$
5		 $M_i^f \neq 0$ $T_l^f \neq T_d^f \neq 0$
6		 $M_i=0$ $T_l^f = T_d^f \neq 0$
7.		 $M_f=0$ $T_l \neq T_d \neq 0$

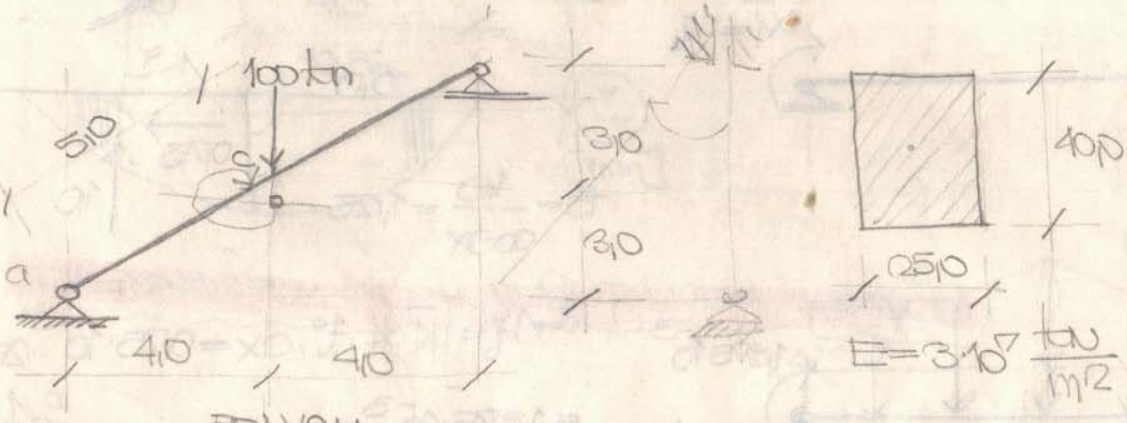
kruta veza treba da postoji

~~ZADATAK~~ Odrediti dijagrame pom. za istovremeno djelovanje opterećenja i temperature

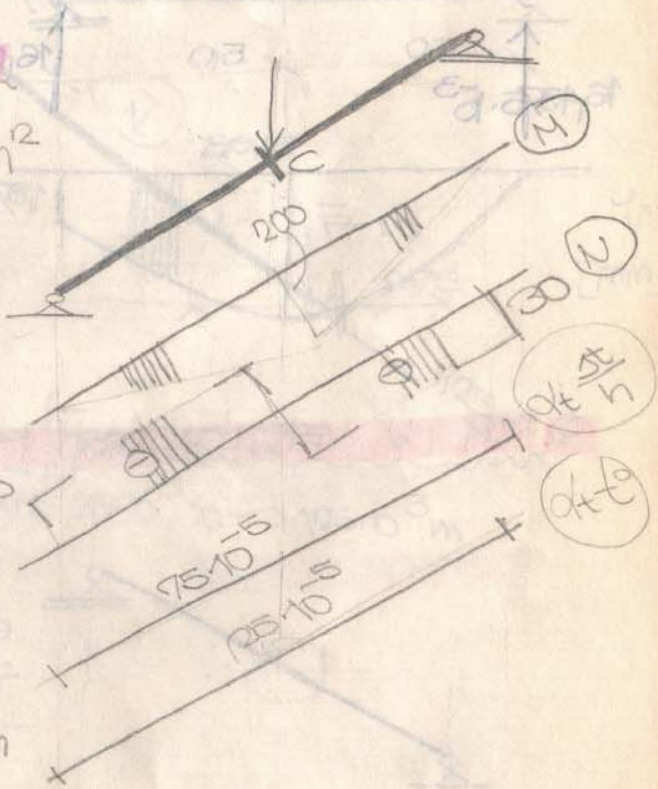
$t_0^\circ = 10^\circ C$

$t_u^\circ = 40^\circ C$

$\alpha_t = 10^{-5}/^\circ C$, zanemariti utjecaj transferzalnih sila na deformaciju.



a) POM. U TREĆEM OSE ŠTAPA



$EJ = 3 \cdot 10^7 \frac{0{,}25 \cdot 0{,}40^3}{12} = 4 \cdot 10^4 \ tm^2$

$EF = 3 \cdot 10^7 \cdot 0{,}25 \cdot 0{,}40 = 3 \cdot 10^6 \ kn$

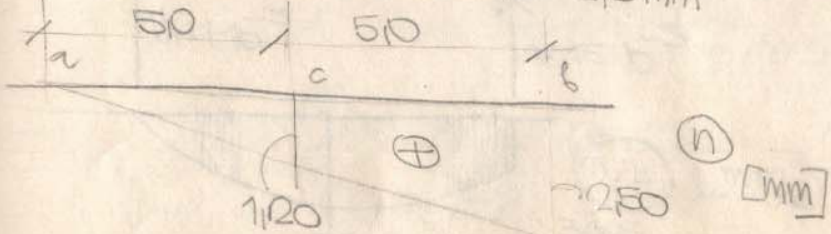
$\Delta l = \frac{S \cdot l}{EF} + \alpha_t t^\circ \cdot l$ — ИЗАЖЕЊЕ У ОСИ ШТАПА

$\Delta t^\circ = t_u - t_0 = 30^\circ C$

$t^\circ = \frac{t_u + t_0}{2} = 25^\circ C$ temp. u os štapa

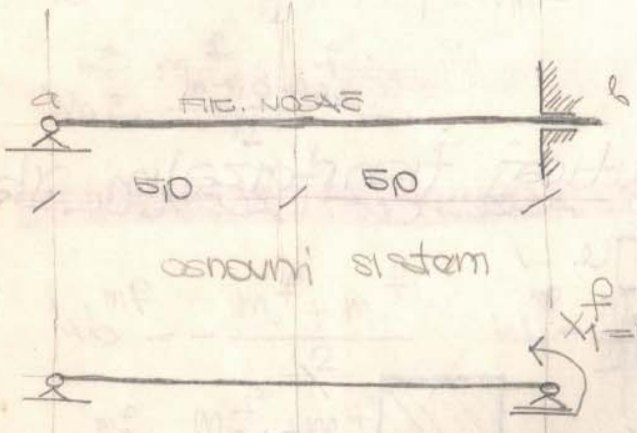
$\Delta l_{ac} = \frac{-30 \cdot 5}{3 \cdot 10^6} + 25 \cdot 10^{-5} \cdot 5 = 1{,}20 \cdot 10^{-3} m = 1{,}2 mm$

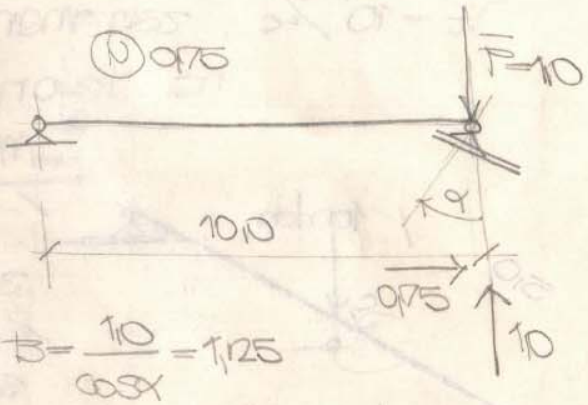
$\Delta l_{a,b} = 25 \cdot 10^{-5} \cdot 10 = 2{,}5 \cdot 10^{-3} m = 2{,}5 mm$



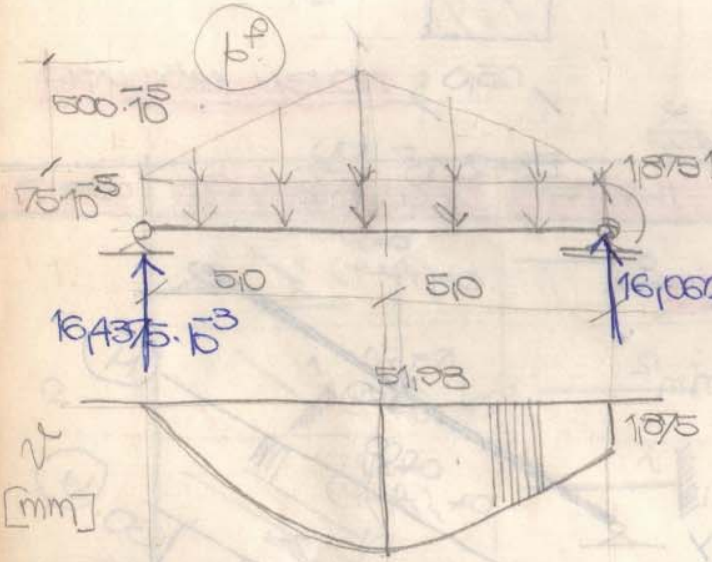
b) POM. UTJECAJ NA OSU STAPA

$\alpha = 0$





$$\beta = \frac{1,0}{\cos\alpha} = 1,125$$



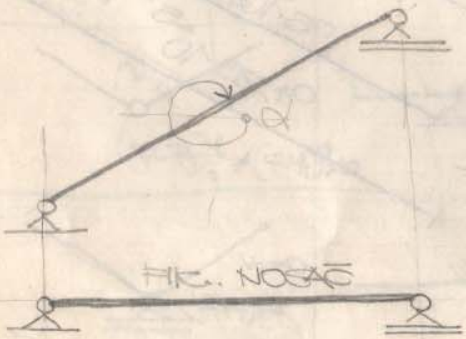
$$X_1^f = v_b = \int \bar{N} \alpha_t \cdot t^\circ \cdot dx = 0,75 \cdot 10^{-5} \cdot 25 \cdot 10,0 =$$

$$= 1,875 \cdot 10^{-3} m$$

$$b^f = \left(\frac{M}{EJ} + \alpha_t \frac{\Delta t}{h} \right) \frac{1}{\cos\alpha}$$

$$N^f = \left(\frac{N}{EF} + \alpha_t t^\circ \right) \frac{1,0}{\cos\alpha}$$

c) POM. U VERTIKALNOM PRAVCU

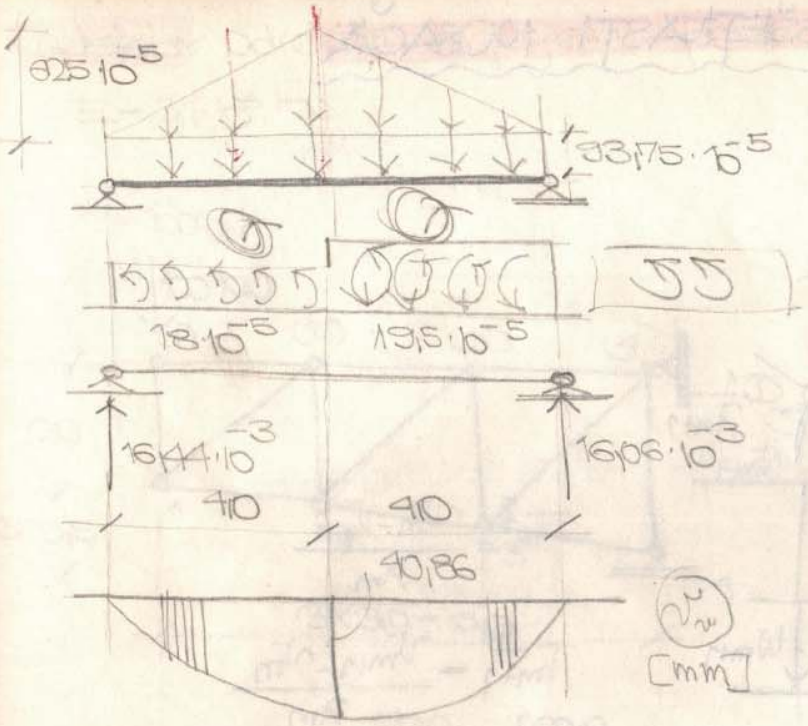


$\sin\alpha = -0,6$

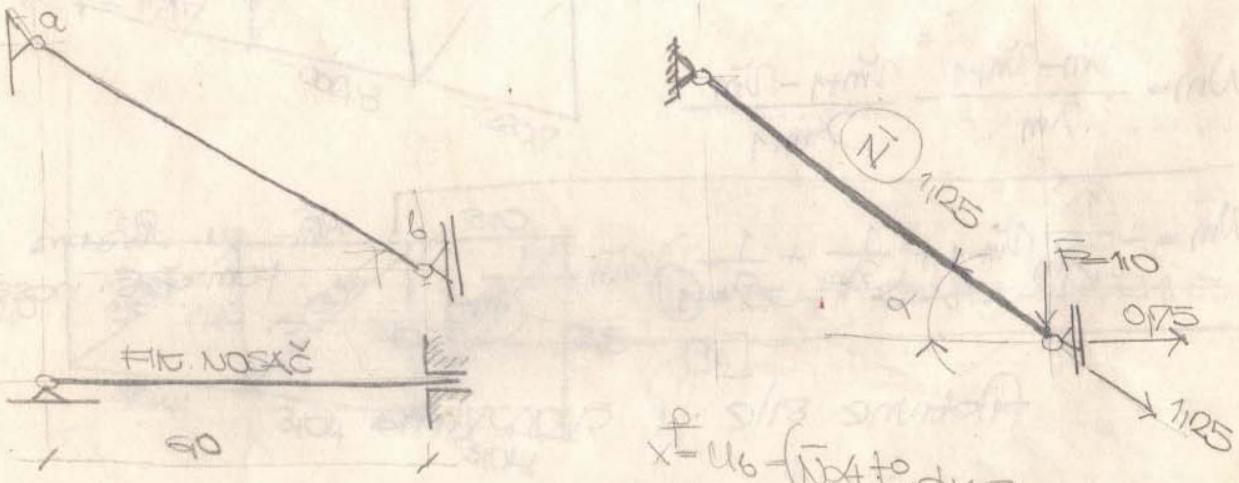
$\cos\alpha = 0,8$

$\operatorname{tg}\alpha = -0,75$

$$b^f = \left(\frac{M}{EJ} + \alpha_t \cdot \frac{\Delta t}{h} \right) \frac{1}{\cos\alpha}$$

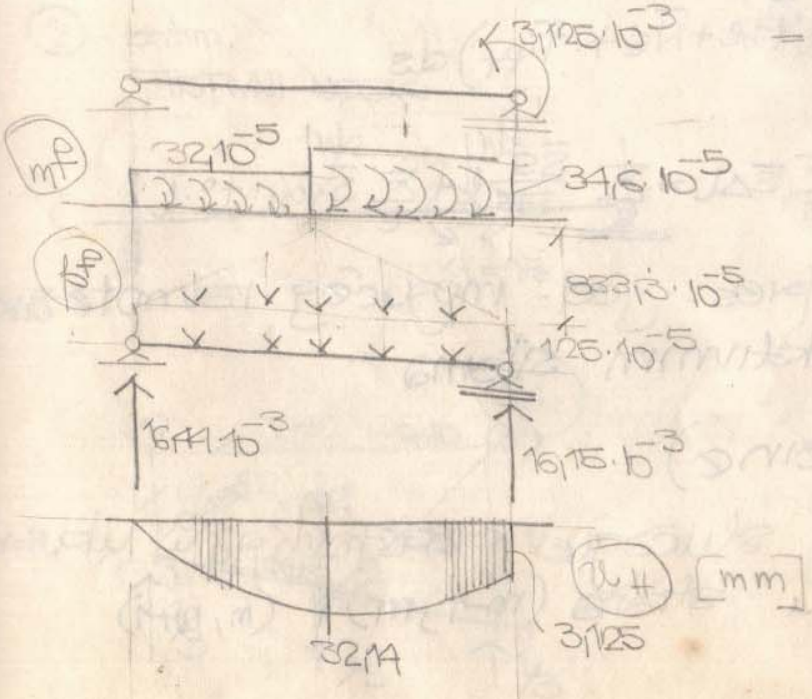


d) DIJAGRAM POM. U HORIZONTALNOM PRAVCU

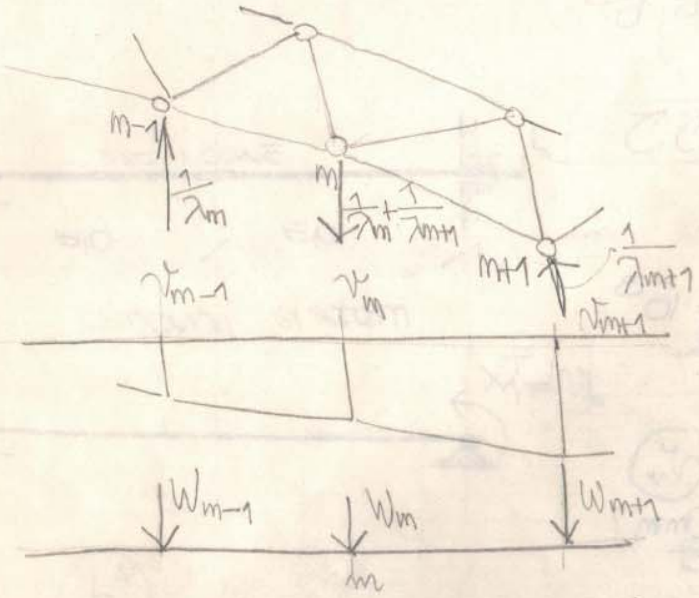


$$x = u_6 = \int \bar{N} \Delta t^{\circ} \cdot dx =$$

$$= 1{,}25 \cdot 25 \cdot 10^{-5} \cdot 10 = 3{,}125 \cdot 10^{-3}\ m$$

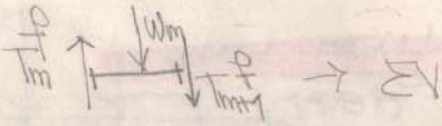


DIJAGRAM TOM. REŠETKASTI NOSAČA



$$T_m^f = \frac{v_m - v_{m-1}}{\lambda_m}$$

$$T_{m+1}^f = \frac{v_{m+1} - v_m}{\lambda_m}$$



$$\rightarrow \Sigma V = 0 \Rightarrow W_m = T_m^f - T_{m+1}^f$$

$$W_m = \frac{v_m - v_{m-1}}{\lambda_m} - \frac{v_{m+1} - v_m}{\lambda_{m+1}}$$

$$W_m = -\frac{1}{\lambda_m} v_{m-1} + \left(\frac{1}{\lambda_m} + \frac{1}{\lambda_{m+1}}\right) v_m - \frac{1}{\lambda_{m+1}}$$

v_m - su strana pom reš. nosača

fiktivne sile u čvorovima

- PUNE NOSAČE: $W_m = \int_S (\bar{M}\varkappa + \bar{N}\varepsilon + \bar{T}\varphi_T)\, ds$

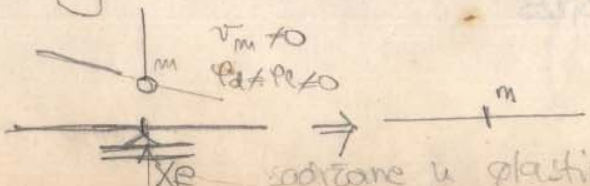
REŠETKAST NOSAČ:

$$W_m = \sum_S \bar{S}\Delta l = \sum_S \frac{\bar{S}S}{EF} l + \sum_S \bar{S} \cdot \alpha_t \cdot t^\circ \cdot l$$

- $\bar{M}; \bar{N}; \bar{T}; \bar{S}$; - sile u pres. jed. mogućeg ravnotežnog stanja opterećenja fiktivnim silama

- ($\varkappa, \varepsilon, \varphi_T$ - def. veličine)

- El. težine u ovom slučaju predstavljaju ukupne promjene ugla između strana $(m-1, m)$ i $(m, m+1)$

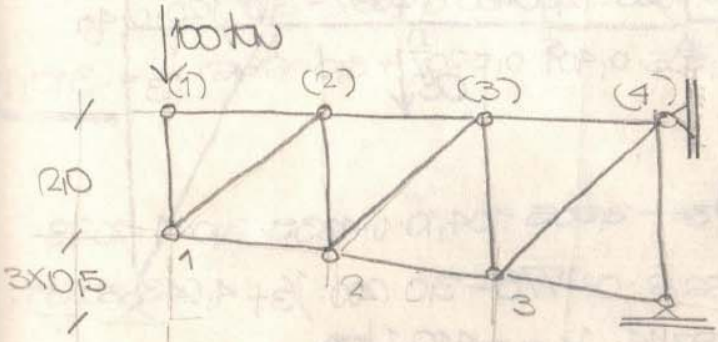


sadržane u plastičnim težinama

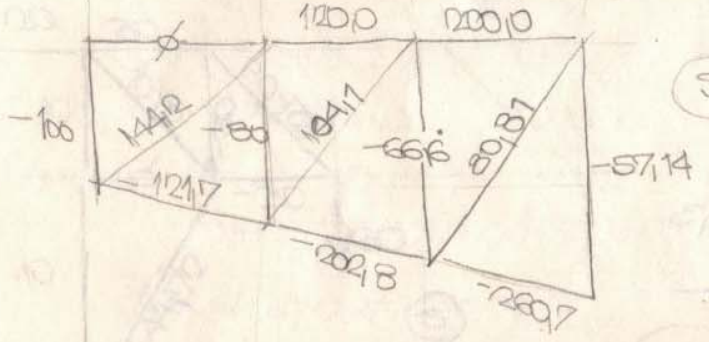
zadatak: odrediti dijagram pom. donjeg pojasa Ves. nosača

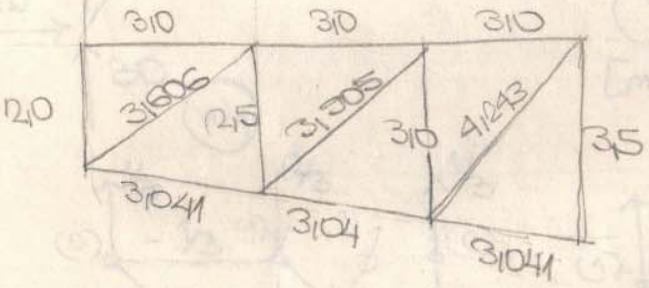
$E = 21 \cdot 10^3 \frac{kN}{mm^2}$

$F = 100\,cm^2$

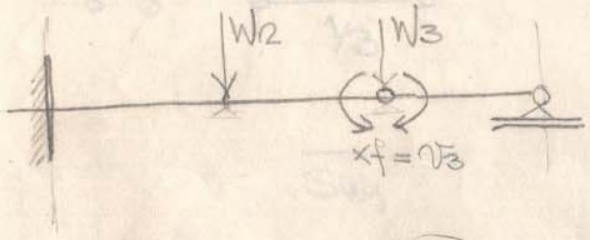


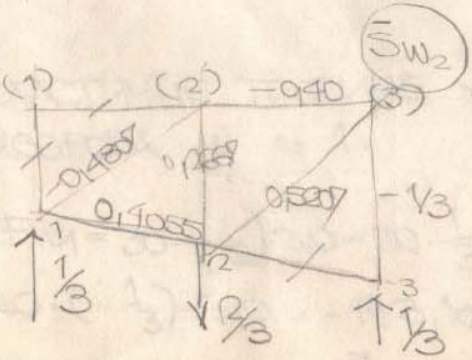
① – sile u štap od opterećenja

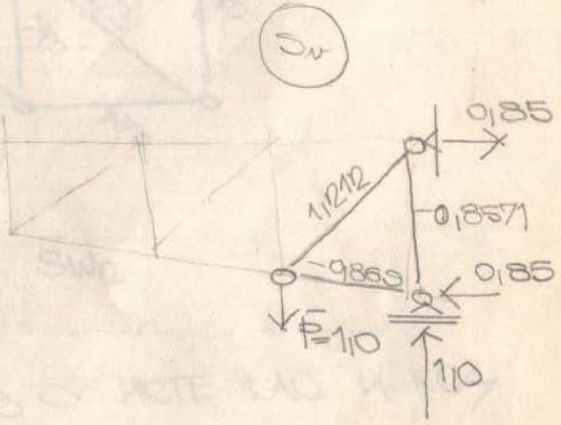




② – zadtm.
FIKTIVNI NOSAČ



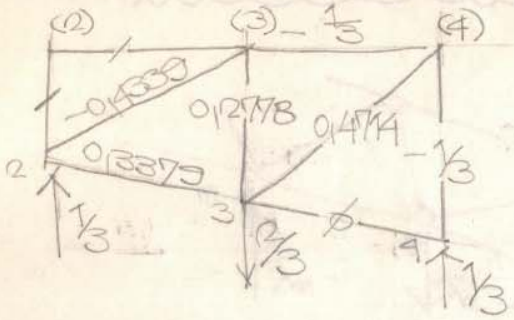




$EF V_3 = \sum_s S \bar{S}_v \cdot l$

$EF V_3 = 3,041 \cdot 2,80,7 \cdot 0,8580 + 3,50 \cdot 57,14 \cdot 0,8571 = 1276\ tm$

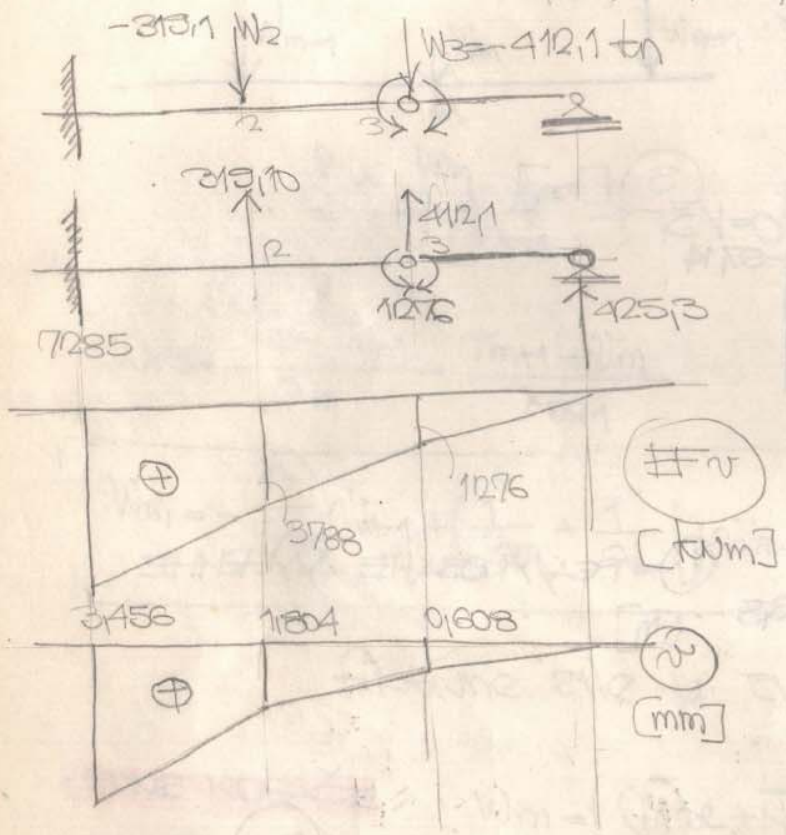
$V_3 = 0,000608\,m = 0,608\,mm$

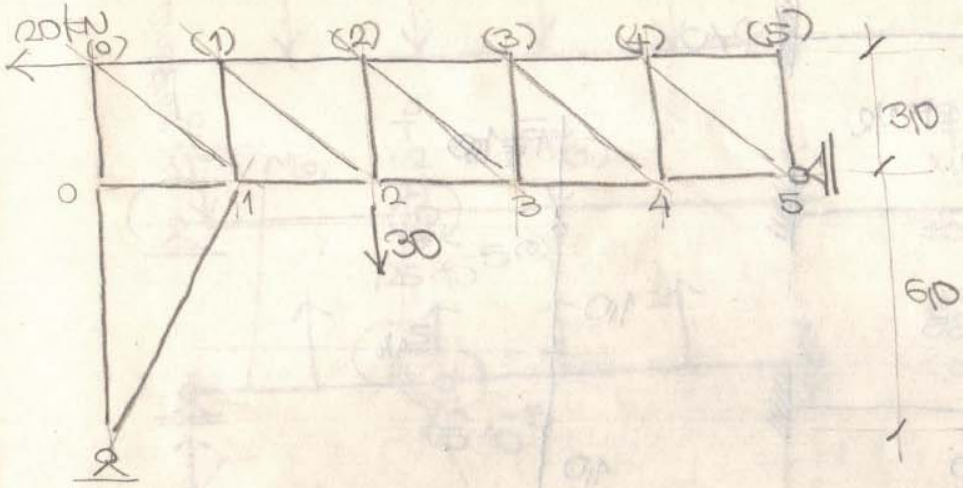


$\overline{5}_{W3}$

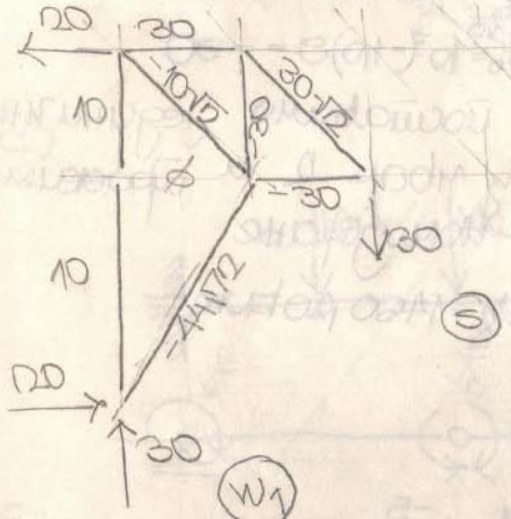
$\Sigma F W_2 = -3,606 \cdot 144,12 \cdot 0,4807 - 3,041 \cdot 121,7 \cdot$
$0,4055 - 2,50 \cdot 80 \cdot 0,2667 - 3,0 \cdot 120 \cdot 0,44$
$+ 3,205 \cdot 0,401 \cdot 0,5207 + 3,0 \cdot 665 \cdot \frac{1}{3} = -313,1$ tm

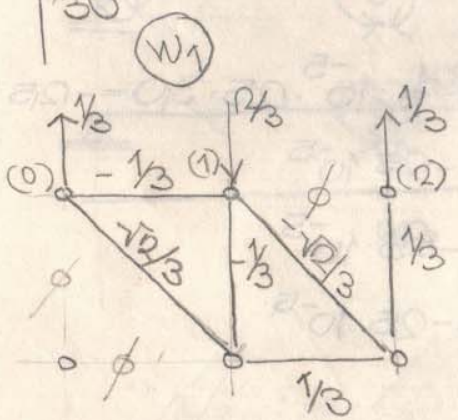
$\Sigma F W_3 = -3,205 \cdot 104,10 \cdot 0,4833 \cdot 3,041 \cdot 202,8 \cdot$
$\cdot 0,3379 - 3,0 \cdot 66,6 \cdot 0,12778 - 3,0 \cdot 200 \cdot \frac{1}{3} + 4,243 \cdot 80,81 \cdot$
$\cdot 0,4774 + 3,5 \cdot 87,14 \cdot \frac{1}{3} = -412,1$ tm

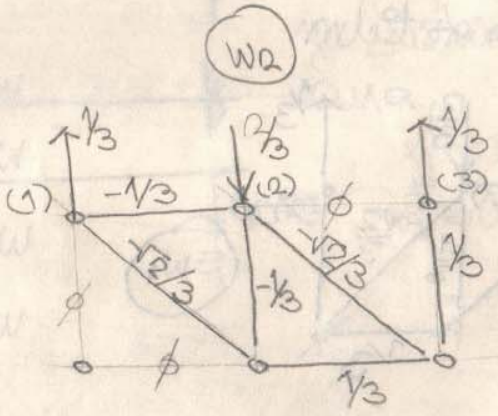




a) Силе у штаповима од спољњег оптерећења







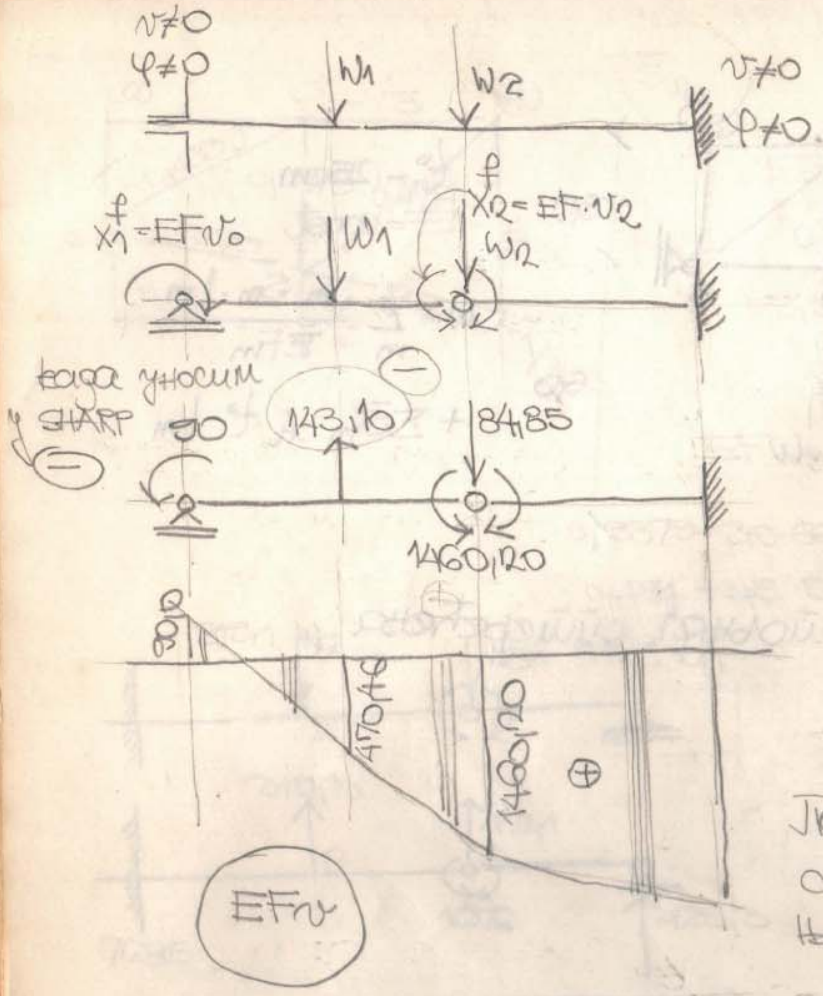
$\bar{S}_{W1}$ $\bar{S}_{W2}$

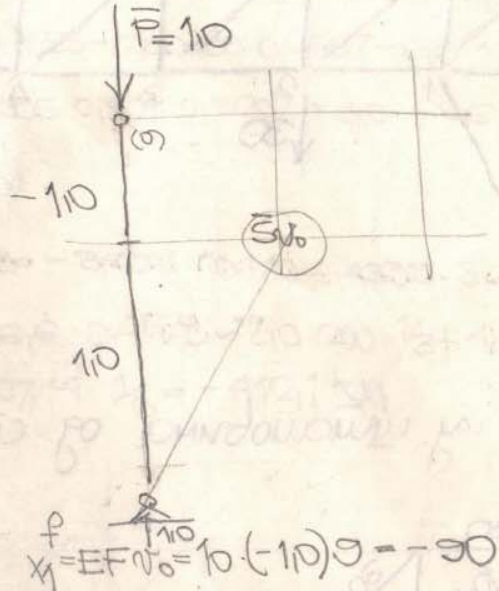
- ЕКСТЕРНЕ ТЕГЛЕЊЕ У ЧВОРУ 4 И 3 СУ ИСТЕ ЛЕВО И ДОЛЕ У ЧВОРОВА 2 И 1.

$EF_{W1} = 30 \cdot \left(-\frac{1}{3}\right) 310 - 30 \cdot \frac{1}{3} \cdot 310 - 10 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{3} \cdot 3\sqrt{2} + 30 \cdot \sqrt{2} \left(-\frac{\sqrt{2}}{3}\right) 310 -$

$- 30 \cdot \left(-\frac{1}{3}\right) \cdot 310 = -143,10.$

$EF_{W2} = 30\sqrt{2} \cdot \frac{\sqrt{2}}{3} \cdot 3\sqrt{2} = 84,85$





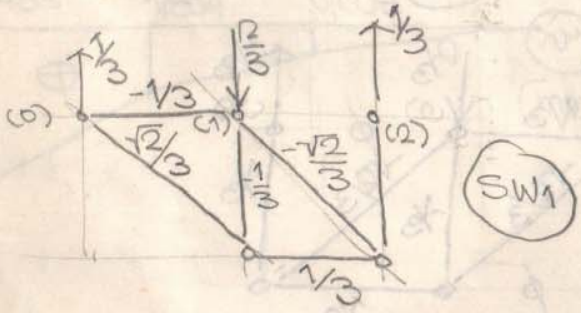
$X_1^f = EFv_0 = 10 \cdot (-1,0) \cdot 9 = -90$

Тамље постављамо јединичну силу у чвор 2 и трашимо њихово померање

$X_2^f = EFv_2 = 1460$ потум

B) $t = 25^\circ C$

$W_m = \sum \bar{S}_m \cdot \alpha t \cdot t \cdot l_m$

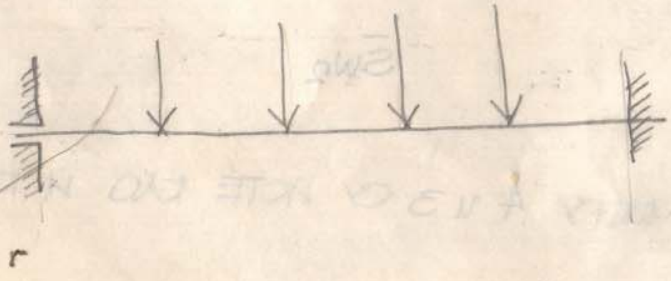


$W_1 = -\frac{1}{3} \cdot 10^{-5} \cdot 25 \cdot 30 = -25 \cdot 10^{-5}$

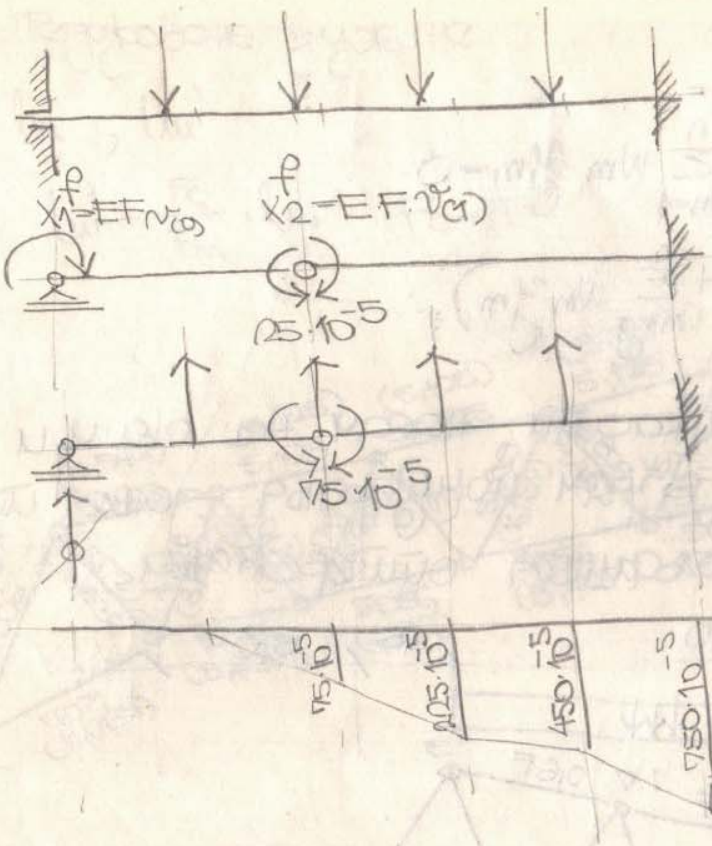
$W_2 = -25 \cdot 10^{-5}$

$W_3 = -25 \cdot 10^{-5}$

$W_4 = -25 \cdot 10^{-5}$



r

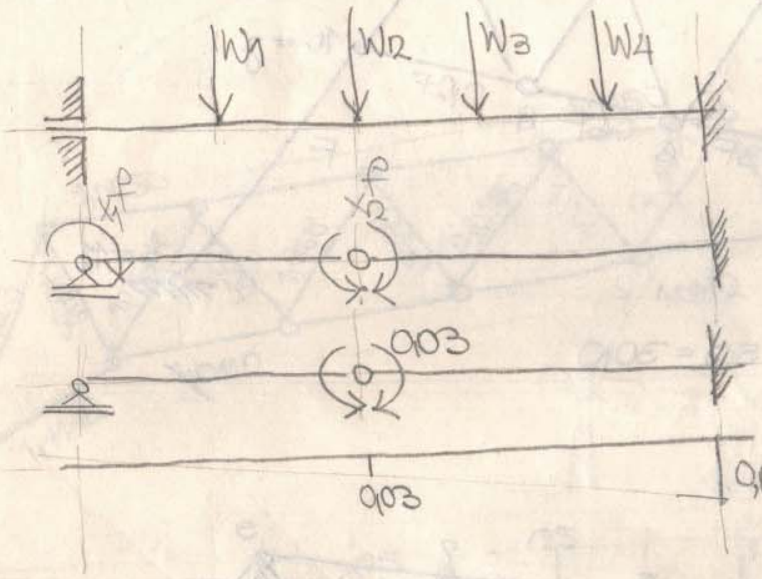


$X_1^f = EF\nu(0) = 0$

zbog toga što usled jedinične sile u čvoru (c) nema sila u štapu gornjeg pojasa

$X_2^f = EF\nu(1) - 75\cdot10^{-5}$

c)

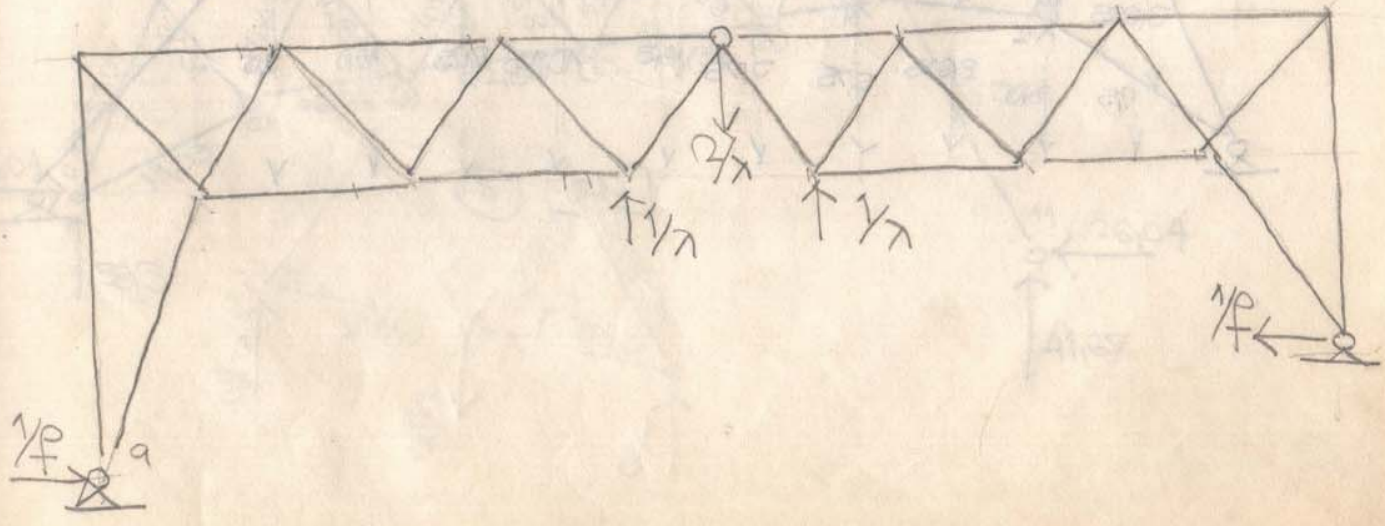


sve su elastične veze jednake su nuli zbog toga što usled jedinične raspodeljena opterećenja u štapovima nema sila

$X_1^f(c=0) = 0$

$X_2^f = \nu(2) = -\sum \bar{C}_i c_i =$

$= -(-1)\cdot 0{,}03 = 0{,}03$

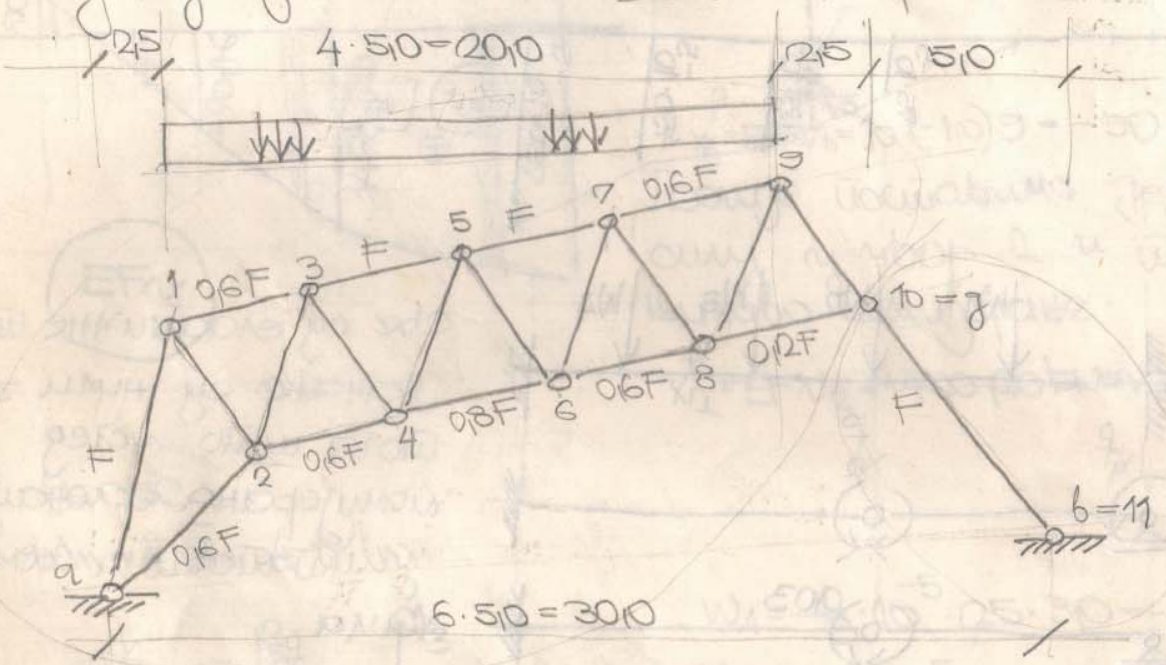


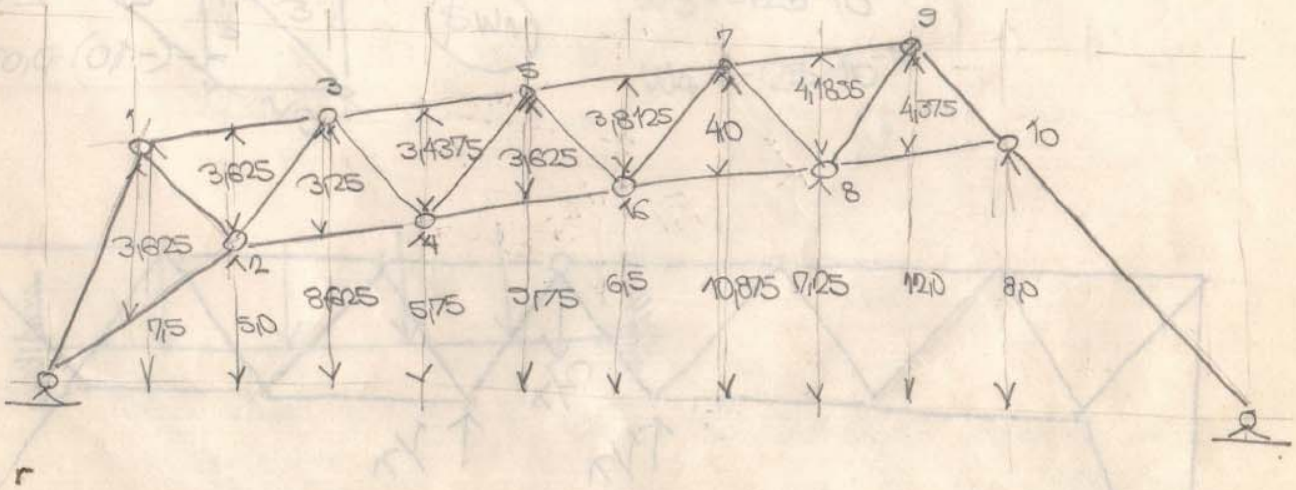
$$\Delta l_{ab} = 0$$

$$\Delta l_{ab} = \sum_{m=1}^{n} \Delta l_m \cdot \sec\alpha_m + \sum_{m=1}^{n} w_m \cdot y_m = 0$$

$$w_g = -\frac{1}{y_g}\left(\sum_{m=1}^{n} \Delta l_m \cdot \sec\alpha_m + \sum_{m=j}^{n} w_m y_m\right)$$

ЗАДАТАК: За решеткасти носач на слици нацртати дијаграм помјерања доњег појаса услед дејства сопственог оптерећења.





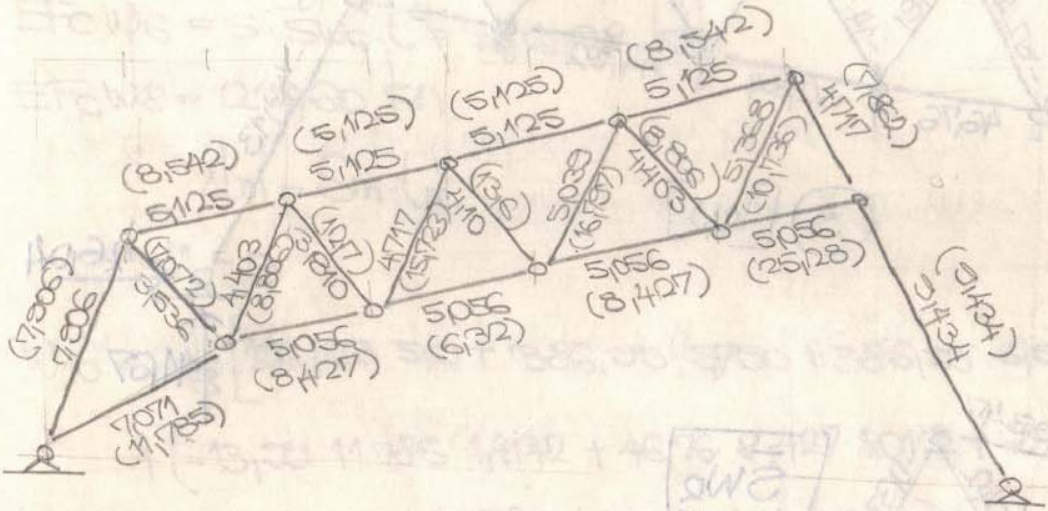
г

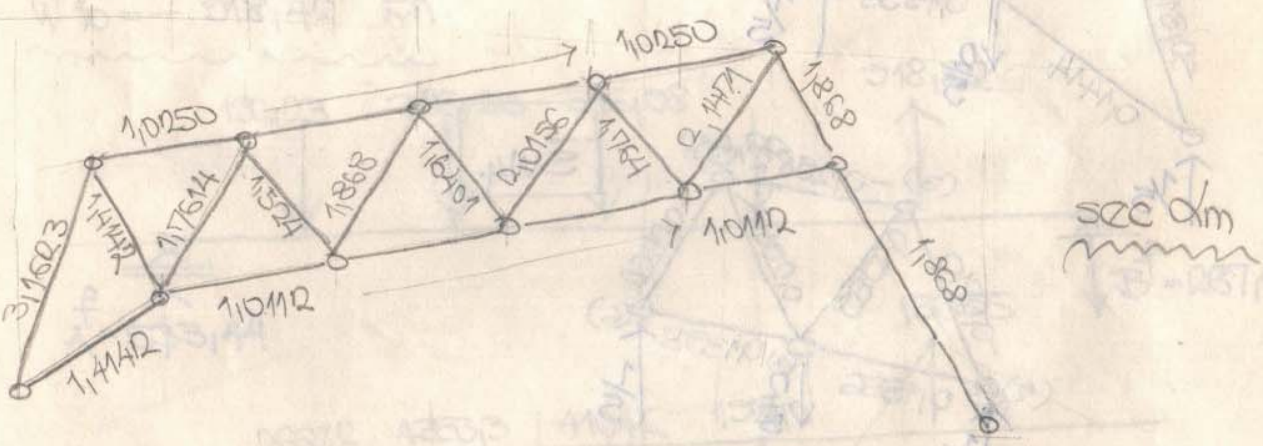
Регуляторные усилия

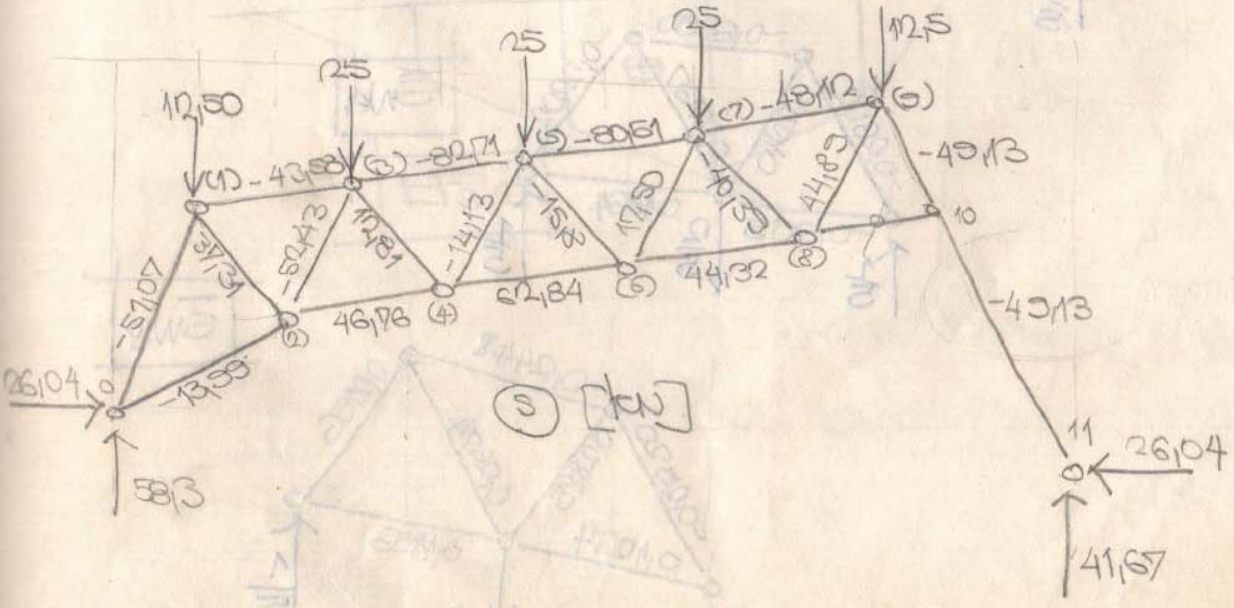
l_w , l_w'

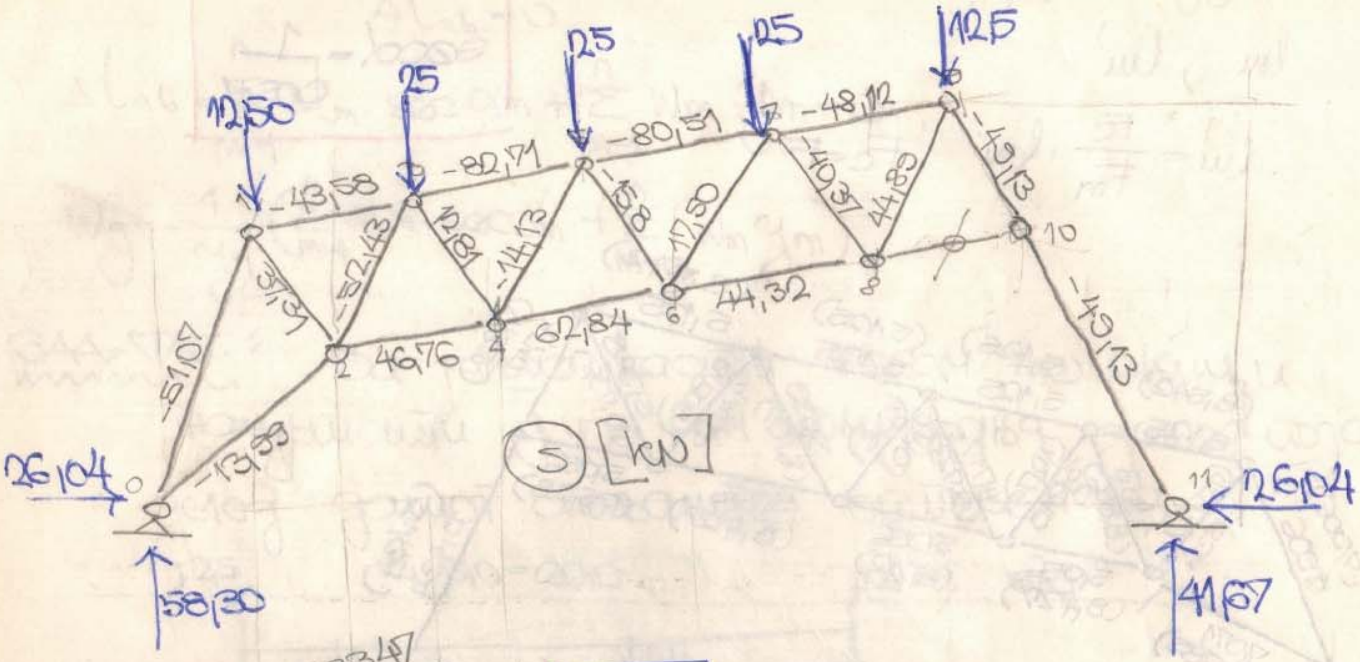
$l_w' = \frac{F_c}{F_m} \cdot l_w$ $F_c = F$

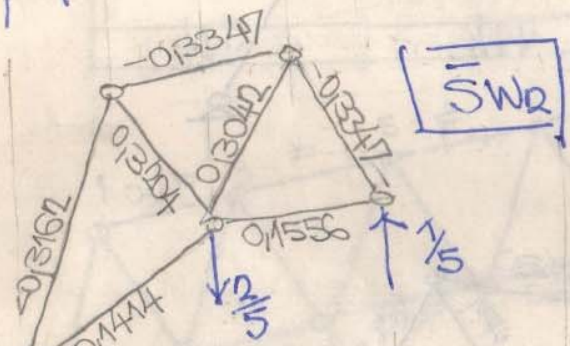
$\sec\alpha = \frac{1}{\cos\alpha}$

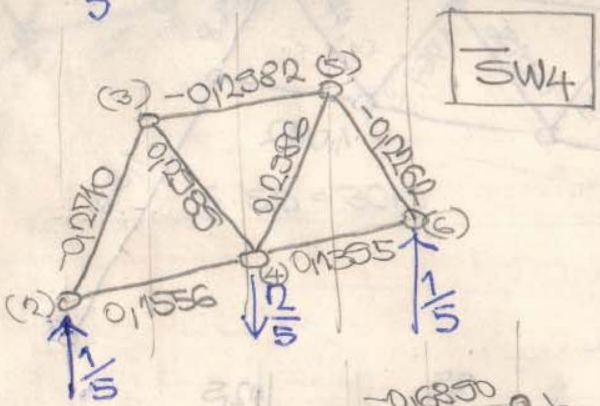


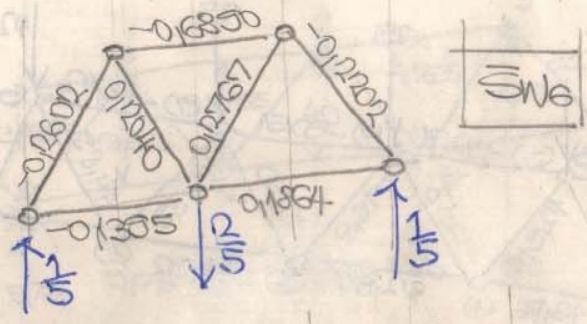


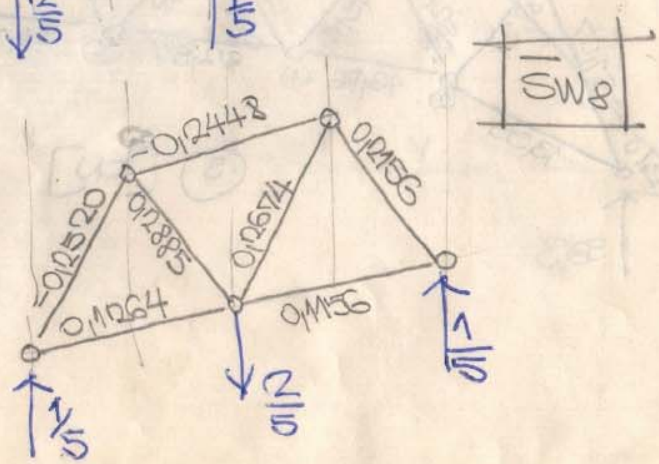



12,50
25
25
25
12,5
-43,58
-82,71
-80,57
-48,12
-43,13
-52,43
12,84
-14,13
-15,8
17,50
-40,37
44,83
44,32
-43,13
-51,07
-13,99
46,76
62,84
26,04
26,04
58,30
41,67
S [kN]


SW2
-0,13347
-0,13347
0,13042
0,13042
-0,13162
0,14414
0,11556
1/5
2/5
1/5
1/5


SW4
(3)
(5)
-0,12382
-0,12740
0,12386
0,12386
-0,12160
(2)
(4)
(6)
0,11556
0,11505
1/5
2/5
1/5


SW6
-0,16830
-0,12202
-0,12602
0,12040
0,12767
-0,11305
0,11864
1/5
2/5
1/5


SW8
-0,12448
0,12156
-0,12520
0,12885
0,12674
0,11264
0,11156
1/5
2/5
1/5

$$W_m = \sum_{m=1}^{n} \frac{S_m \cdot \bar{S}_m}{EF} \cdot l_m$$

$EF_c \cdot W_2 = 136{,}23$ tm

$EF_c W_4 = 386{,}35$ tm

$EF_c W_6 = S \cdot \bar{S}_{W_6} \cdot l = 386{,}38$ tm

$EF_c W_8 = 214{,}60$ tm

$$\Delta l_m = S_m \cdot l_m'$$

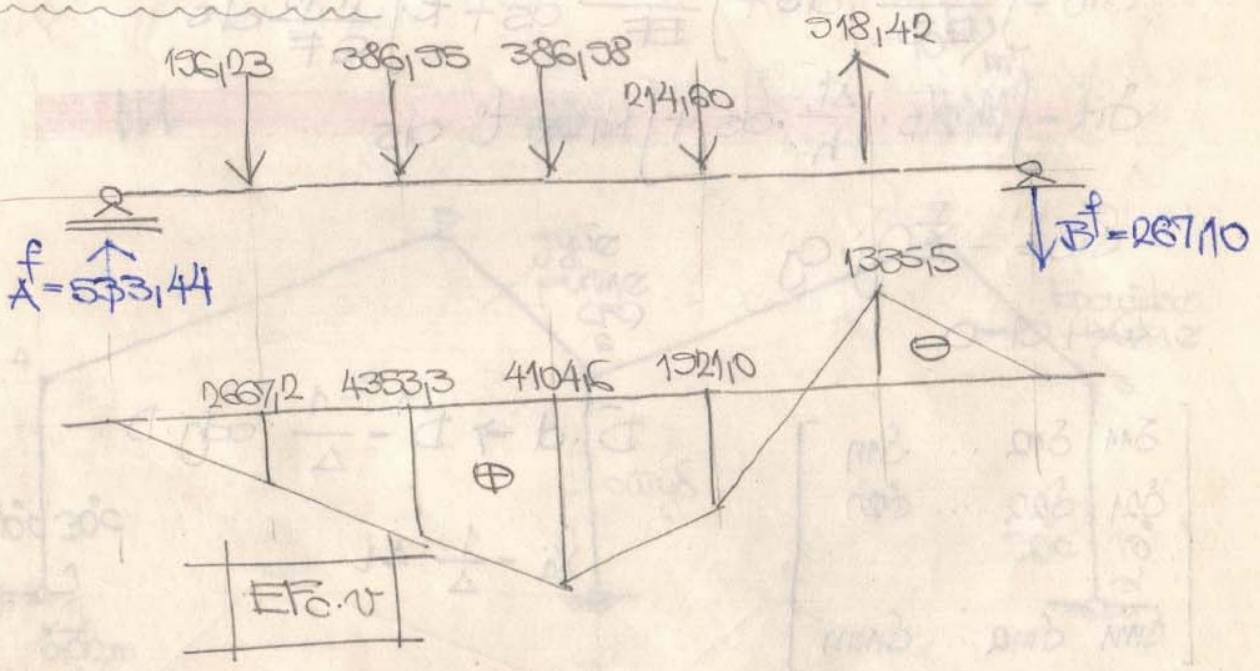
$\Delta l_{ab} = 0$

$$W_{10} = -\frac{1}{8}\Big[136{,}23 \cdot 5{,}10 + 386{,}35 \cdot 9{,}750 + 386{,}38 \cdot 6{,}50 + 214{,}6 \cdot 7{,}25 +$$

$$+ \big(-13{,}30 \cdot 11{,}785 \cdot 1{,}4142 + 46{,}76 \cdot 8{,}427 \cdot 1{,}0112 + 62{,}84 \cdot 6{,}32 \cdot 1{,}012 +$$

$$+ 44{,}32 \cdot 8{,}427 \cdot 1{,}0112 - 49{,}13 \cdot 9{,}434 \cdot 1{,}8868\big)\Big] =$$

$W_{10} = -\ 918{,}42$ tm



ЗАДАТАК:

ЗА НОСАЧ ПРЕМА СКИЦИ УСЛЕД ЗАГРИЈАВАЊА ГОРЊЕГ ВЛАКНА ЗА $t_0 = 20°C$

1, НАЦРТАТИ ДИЈАГРАМ ВЕРТИКАЛНИХ ПОМЕРАЊА СА ОРДИНАТАМА У ЧВОРОВИМА РАСПОНА И ОДРЕДИТИ ХОРИЗОНТАЛНО ПОМЕРАЊЕ ОСЛОНЦА Б. НОСАЧ ЈЕ ПРАВОУГАОНОГ ПОПРЕЧНОГ ПРЕСЕКА И ТО: ОД 0–2 $b/h = 0{,}4/1{,}0$ m

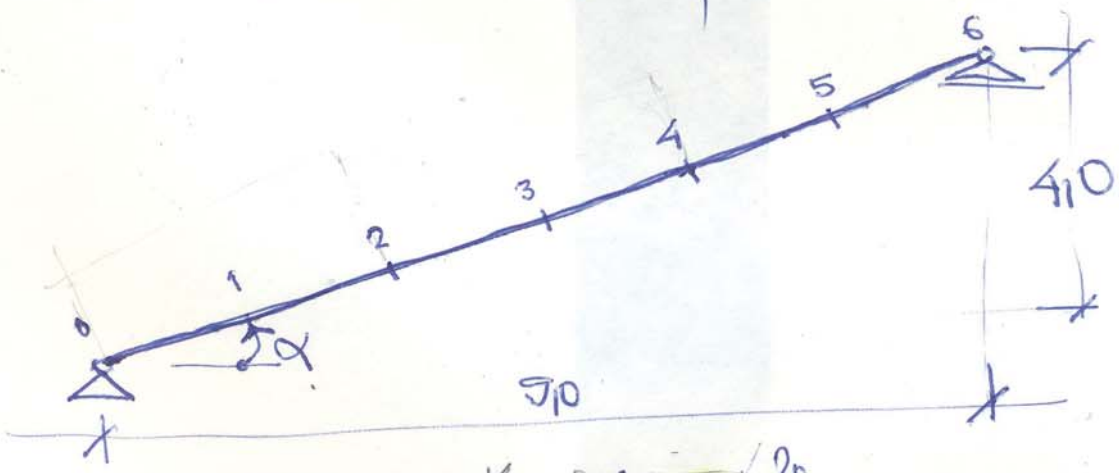
2–4 $b/h = 0{,}4/0{,}8$ m

4–6 $b/h = 0{,}4/0{,}6$ m

НАПОМЕНА:

ТЕМПЕРАТУРА СЕ ПО ВИСИНИ ПОПРЕЧНОГ ПРЕСЕКА МИЊА ЛИНЕАРНО ОД $t_0 = 20°C$ НА ГОРЊЕМ ВЛАКНУ ДО $t_u = 0°C$ НА ДОЊЕМ ВЛАКНУ

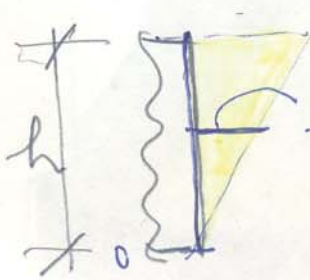
$\alpha_t = 10^{-5} \frac{1}{°C}$ $E = 3 \cdot 10^7 kN/m^2$ · $t_0 = 20°C$



$\cos\alpha = 0{,}9138$

$\sin\alpha = 0{,}4061$

$tg\alpha = -0{,}444$



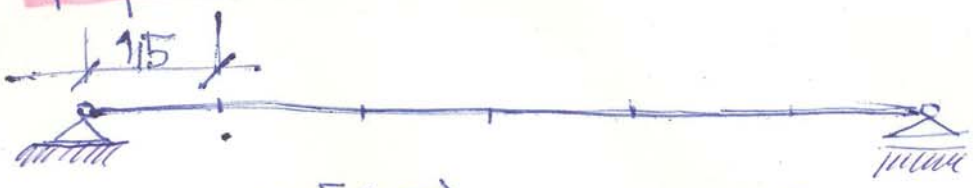
$t^\circ = \frac{t_0 + t_u}{2} = 10°C$

$\Delta t = 0 - 20 = -20°C$

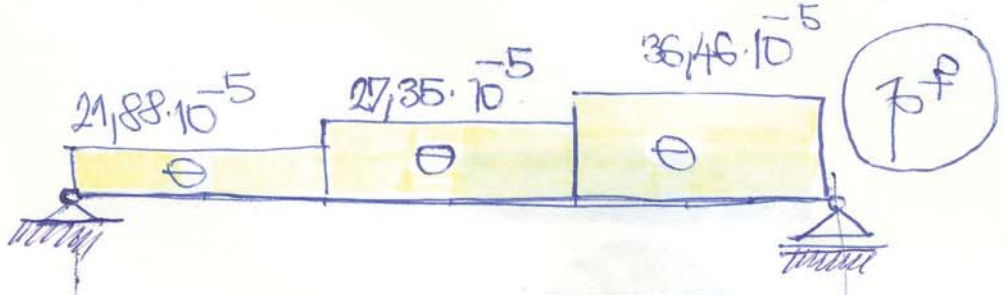
$f = \frac{\alpha_t \cdot \Delta t}{h \cdot \cos\alpha}$

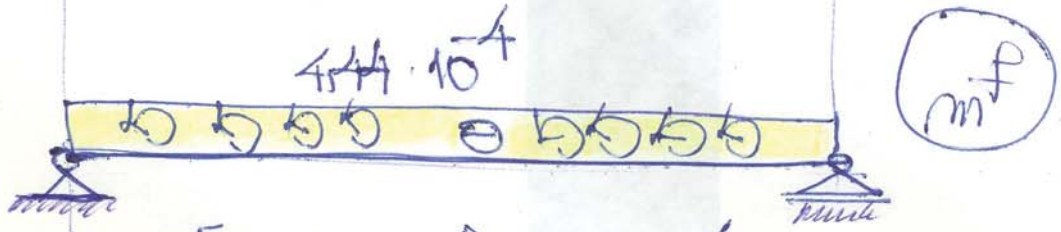
$m_f = \alpha_t \cdot t^\circ \cdot tg\alpha$

ФИКТИВНИ НОСАЧ

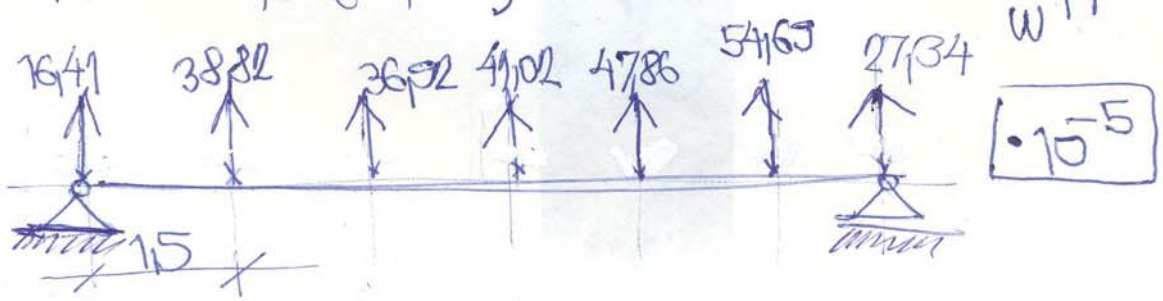


$$\frac{\alpha_t \cdot \Delta t}{h \cdot \cos\alpha} = \frac{10^{-5}(-20)}{1{,}0 \cdot 0{,}9138} = -21{,}88 \cdot 10^{-5}$$





$$m^f = 10^{-5} \cdot 10 \,(-0{,}4442) = -44 \cdot 10^{-4}$$



$$w_1^{mf} = \frac{4{,}44 + 4{,}44}{2} - \frac{4{,}44 + 4{,}44}{2} = 0$$