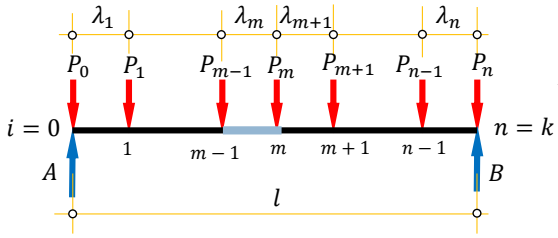
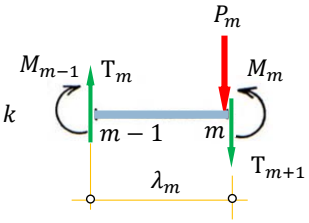
Нумерички поступак





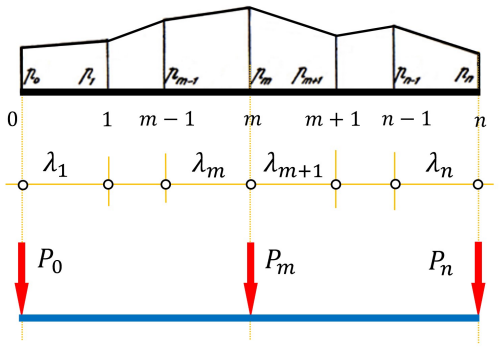
$$T_{m+1} = T_m - P_m$$

$$M_m = M_{m-1} + T_m \lambda_m$$

$$A = \frac{1}{l}\sum_{m=0}^{n} P_m x'_m$$

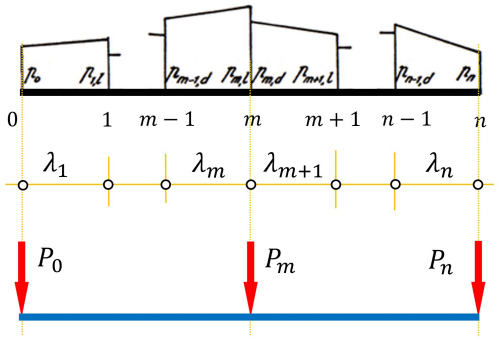
$$B = \sum_{m=0}^{n} P_m - A$$

Линеарна расподела оптерећења



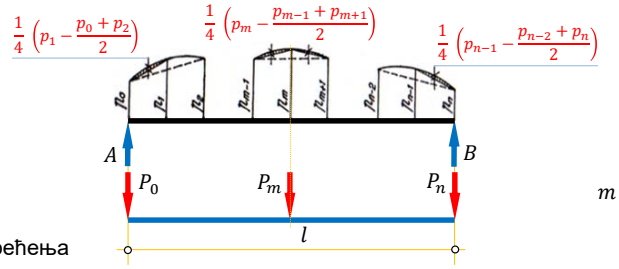
$$\begin{aligned} P_0 &= \frac{\lambda}{6}(2p_0 + p_1) \\ P_m &= \frac{\lambda}{6}(p_{m-1} + 4p_m + p_{m+1}), m = 1, 2, \ldots, n-1 \\ P_n &= \frac{\lambda}{6}(p_{n-1} + 2p_n) \end{aligned} \tag{1}$$

Линеарна скоковита расподела оптерећења



$$
\begin{aligned}
P_0 &= \frac{\lambda_1}{6}\left(2p_0 + p_1^l\right) \\
P_m &= \frac{\lambda_m}{6}\left(p_{m-1}^d + 2p_m^l\right) + \frac{\lambda_{m+1}}{6}\left(2p_m^d + p_{m+1}^l\right), m = 1, 2, ..., n-1 \\
P_n &= \frac{\lambda_n}{6}\left(p_{n-1}^d + 2p_n\right)
\end{aligned}
\tag{2}
$$

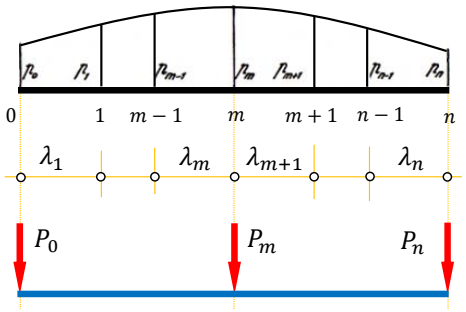
Параболична расподела оптерећења



$m = 1,\ 2,\ \ldots\ n-1$

Линеарна расподела оптерећења

$P_0 = \frac{\lambda}{6}(2p_0 + p_1)$	+ стрела	$P_0 = \frac{\lambda}{6}(2p_0 + p_1) + \frac{1}{2}\frac{2}{3}\lambda\frac{1}{4}\left(p_1 - \frac{p_0+p_2}{2}\right)$
$P_m = \frac{\lambda}{6}(p_{m-1} + 4p_m + p_{m+1})$	+ стрела	$P_m = \frac{\lambda}{6}(p_{m-1} + 4p_m + p_{m+1}) + \frac{2}{3}\lambda\frac{1}{4}\left(p_m - \frac{p_{m-1}+p_{m+1}}{2}\right)$
$P_n = \frac{\lambda}{6}(p_{n-1} + 2p_n)$	+ стрела	$P_n = \frac{\lambda}{6}(p_{n-1} + 2p_n) + \frac{1}{2}\frac{2}{3}\lambda\frac{1}{4}\left(p_{n-1} - \frac{p_{n-2}+p_n}{2}\right)$

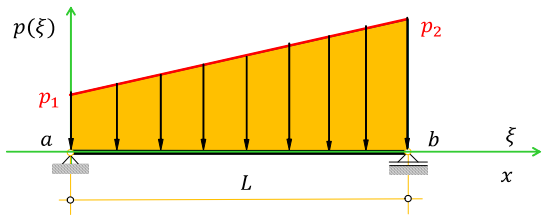


$$
\begin{aligned}
P_0 &= \frac{\lambda}{24}(7p_0 + 6p_1 - p_2) \\
P_m &= \frac{\lambda}{12}(p_{m-1} + 10p_m + p_{m+1}), m = 1, 2, ..., n-1 \\
P_n &= \frac{\lambda}{24}(-p_{n-2} + 6p_{n-1} + 7p_n)
\end{aligned}
\tag{3}
$$

Примери

За све носаче са датим оптерећењима приказани на скицама потребно је срачунати реакције ослонаца и пресечне силе нумеричким поступком делећи носач у десетинама распона.

Primer



$p_1 := 2 \ \frac{kN}{m}$ $M := 0$ kNm

$p_2 := 4 \ \frac{kN}{m}$

$L := 8$ m $\lambda := \frac{L}{10}$ $\lambda = 0.8$

Funkcija opterećenja: $p(\xi) := a \cdot \xi + b$

Given

1... $q \cdot 0 + b = 2$

2... $a \cdot 8 + b = 4$

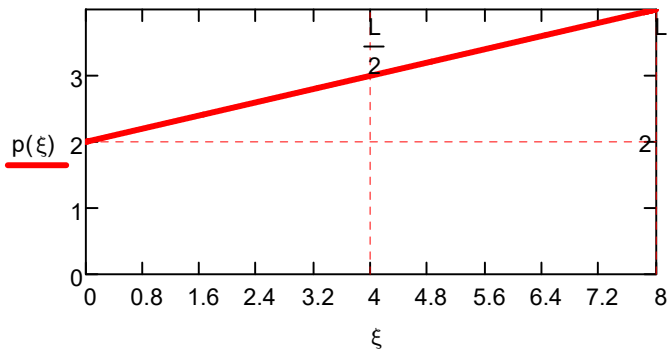
Konstante su: $a := \frac{1}{4}$ $b := 2$

$p(\xi) := a \cdot \xi + b$

$\text{Find}(a, b) \rightarrow \begin{pmatrix} \frac{1}{4} \\ 2 \end{pmatrix}$

$p(\xi) \text{ simplify } \rightarrow \frac{\xi}{4} + 2$

$p(\xi) := \frac{\xi}{4} + 2$



$p(0) = 2$

$p(0.8) = 2.2$

$p(1.6) = 2.4$

$p(2.4) = 2.6$

$p(3.2) = 2.8$

$p(4) = 3$

$p(4.8) = 3.2$ $p(5.6) = 3.4$ $p(6.4) = 3.6$ $p(7.2) = 3.8$ $p(8) = 4$

$$p := \begin{pmatrix} 2 \\ 2.2 \\ 2.4 \\ 2.6 \\ 2.8 \\ 3 \\ 3.2 \\ 3.4 \\ 3.6 \\ 3.8 \\ 4 \end{pmatrix} \qquad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \qquad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$P_0 := \frac{\lambda}{6} \cdot (2 \cdot p_0 + p_1)$ $P_0 = 0.827$ $P_8 := \frac{\lambda}{6} \cdot (p_7 + 4 p_8 + p_9)$ $P_8 = 2.88$

$P_1 := \frac{\lambda}{6} \cdot (p_0 + 4 p_1 + p_2)$ $P_1 = 1.76$ $P_9 := \frac{\lambda}{6} \cdot (p_8 + 4 p_9 + p_{10})$ $P_9 = 3.04$

$P_2 := \frac{\lambda}{6} \cdot (p_1 + 4 p_2 + p_3)$ $P_2 = 1.92$ $P_{10} := \frac{\lambda}{6} \cdot (2 \cdot p_{10} + p_9)$ $P_{10} = 1.573$

$P_3 := \frac{\lambda}{6} \cdot (p_2 + 4 p_3 + p_4)$ $P_3 = 2.08$

$P_4 := \frac{\lambda}{6} \cdot (p_3 + 4 p_4 + p_5)$ $P_4 = 2.24$

$P_5 := \frac{\lambda}{6} \cdot (p_4 + 4 p_5 + p_6)$ $P_5 = 2.4$

$P_6 := \frac{\lambda}{6} \cdot (p_5 + 4 p_6 + p_7)$ $P_6 = 2.56$

$P_7 := \frac{\lambda}{6} \cdot (p_6 + 4 p_7 + p_8)$ $P_7 = 2.72$

Reakcije oslonaca:

1. od opterećenja: $A_P := \sum_{m=0}^{10} (P_m \xi d_m)$ $A_P = 10.667$ kN

$B_P := \sum_{m=0}^{10} (P_m \xi_m)$ $B_P = 13.333$ kN

2. od koncentrisanog momenta:

$A_M := \frac{-M}{L}$ $A_M = 0$ kN

$B_M := \frac{M}{L}$ $B_M = 0$ kN

Reakcije su: $A := A_P + A_M$ $A = 10.667$ kN

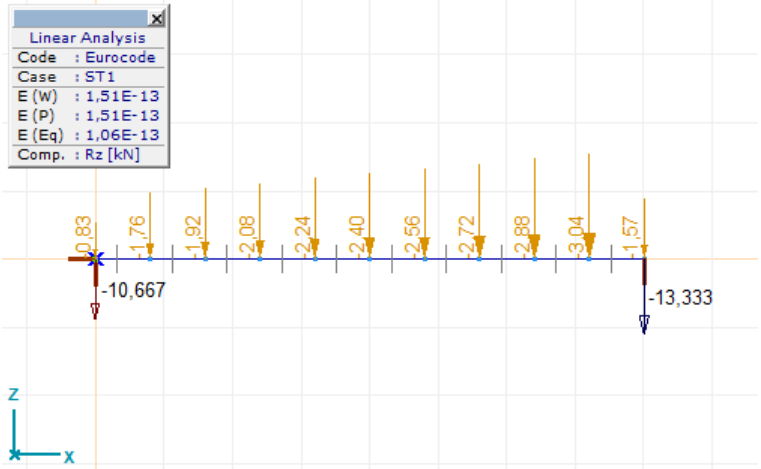
$B := B_P + B_M$ $B = 13.333$ kN

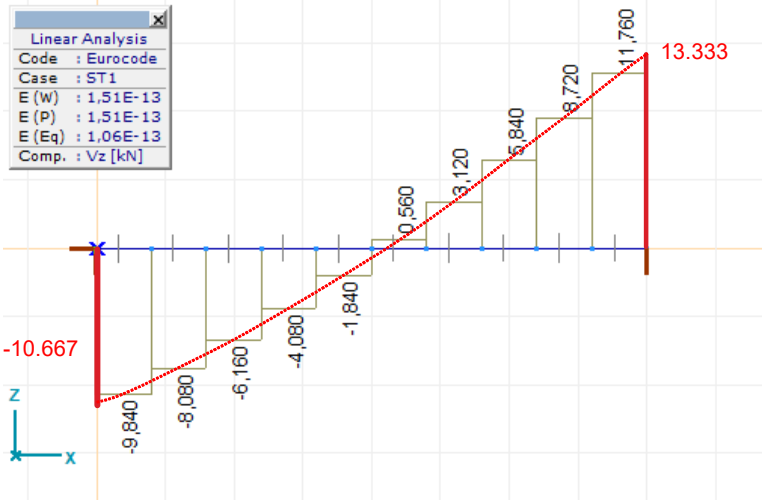
Presečne sile: $T_m := T_{m-1} - P_{m-1}$ $T_0 := A$

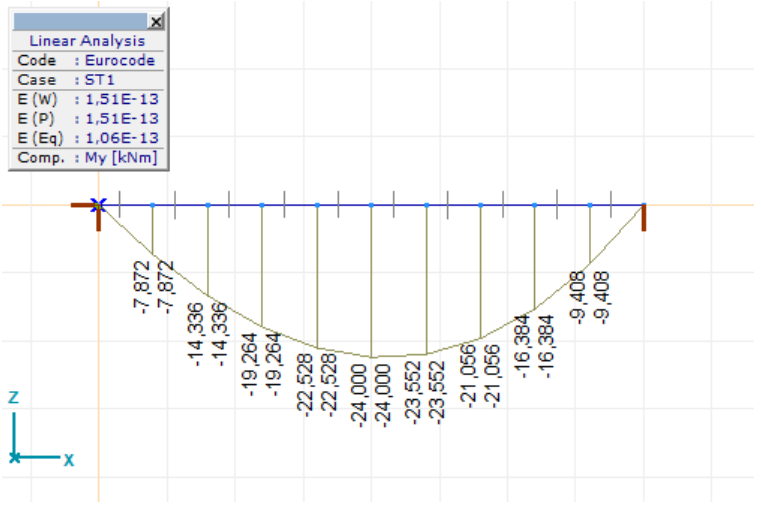
$M_m := M_{m-1} + T_m \cdot \lambda_m$

m	λ_m	ξ_m	p_m	P_m	ξ'_m	$P_m\xi'_m$	T_m	$T_m\lambda_m$	$M_m^{(M)}$	M_m
0	0,8	0	2	0,827	1	0,827	10,667	0		0
1	0,8	0,1	2,2	1,76	0,9	1,584	9,840	7,872		7,872
2	0,8	0,2	2,4	1,92	0,8	1,536	8,080	6,464		14,336
3	0,8	0,3	2,6	2,08	0,7	1,456	6,160	4,928		19,264
4	0,8	0,4	2,8	2,24	0,6	1,344	4,080	3,264		22,528
5	0,8	0,5	3	2,4	0,5	1,200	1,840	1,472		24,000
6	0,8	0,6	3,2	2,56	0,4	1,024	-0,560	-0,448		23,552
7	0,8	0,7	3,4	2,72	0,3	0,816	-3,120	-2,496		21,056
8	0,8	0,8	3,6	2,88	0,2	0,576	-5,840	-4,672		16,384
9	0,8	0,9	3,8	3,04	0,1	0,304	-8,720	-6,976		9,408
10	0,8	1	4	1,573	0	0	-11,760	-9,408		0,000
			$\sum P_m =$	24	$A_P =$	10,667	-13,333	$=B$		

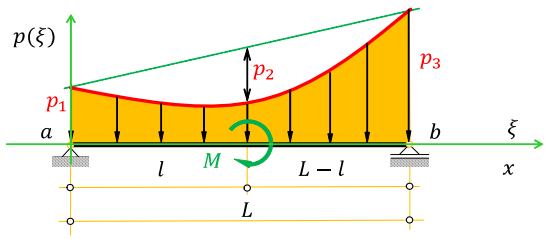
Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila







Primer



$p_1 := 6 \ \frac{kN}{m}$ $\quad M := 50 \ kNm$

$p_2 := 7 \ \frac{kN}{m}$

$p_3 := 12 \ \frac{kN}{m}$

$L := 8 \ m$ $\quad l := 4 \ m$ $\quad \lambda := \frac{L}{10}$ $\quad \lambda = 0.8$

Funkcija opterećenja: $p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

Given

1... $a \cdot 0^2 + b \cdot 0 + c = 6$

2... $a \cdot 4^2 + b \cdot 4 + c = 2$

3... $a \cdot 8^2 + b \cdot 8 + c = 12$

Konstante su: $a := \frac{7}{16}$ $\quad b := -\frac{11}{4}$ $\quad c := 6$

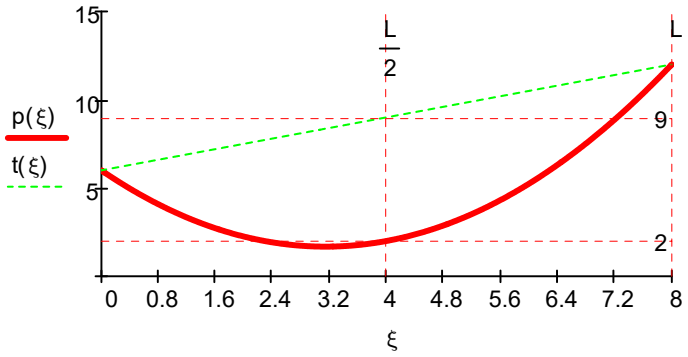
$p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

$p(\xi) \text{ simplify } \rightarrow \frac{7 \cdot \xi^2}{16} - \frac{11 \cdot \xi}{4} + 6$

$\text{Find}(a, b, c) \rightarrow \begin{pmatrix} \frac{7}{16} \\ -\frac{11}{4} \\ 6 \end{pmatrix}$

$p(\xi) := \frac{7}{16} \cdot \xi^2 - \frac{11}{4} \cdot \xi + 6$

$t(\xi) := 6 + \frac{6}{8} \cdot \xi$



$p(0) = 6$

$p(0.8) = 4.08$

$p(1.6) = 2.72$

$p(2.4) = 1.92$

$p(3.2) = 1.68$

$p(4) = 2$

$p(4.8) = 2.88 \qquad p(5.6) = 4.32 \qquad p(6.4) = 6.32 \qquad p(7.2) = 8.88 \qquad p(8) = 12$

$$p := \begin{pmatrix} 6 \\ 4.08 \\ 2.72 \\ 1.92 \\ 1.68 \\ 2 \\ 2.88 \\ 4.32 \\ 6.32 \\ 8.88 \\ 12 \end{pmatrix} \qquad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \qquad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$P_0 := \frac{\lambda}{24} \cdot (7 \cdot p_0 + 6\,p_1 - p_2) \qquad P_0 = 2.125$

$P_1 := \frac{\lambda}{12} \cdot (p_0 + 10\,p_1 + p_2) \qquad P_1 = 3.301$

$P_2 := \frac{\lambda}{12} \cdot (p_1 + 10\,p_2 + p_3) \qquad P_2 = 2.213$

$P_3 := \frac{\lambda}{12} \cdot (p_2 + 10\,p_3 + p_4) \qquad P_3 = 1.573$

$P_4 := \frac{\lambda}{12} \cdot (p_3 + 10\,p_4 + p_5) \qquad P_4 = 1.381$

$P_5 := \frac{\lambda}{12} \cdot (p_4 + 10\,p_5 + p_6) \qquad P_5 = 1.637$

$P_6 := \frac{\lambda}{12} \cdot (p_5 + 10\,p_6 + p_7) \qquad P_6 = 2.341$

$P_7 := \frac{\lambda}{12} \cdot (p_6 + 10\,p_7 + p_8) \qquad P_7 = 3.493$

$P_8 := \frac{\lambda}{12} \cdot (p_7 + 10\,p_8 + p_9) \qquad P_8 = 5.093$

$P_9 := \frac{\lambda}{12} \cdot (p_8 + 10\,p_9 + p_{10}) \qquad P_9 = 7.141$

$P_{10} := \frac{\lambda}{24} \cdot (7 \cdot p_{10} + 6\,p_9 - p_8) \qquad P_{10} = 4.365$

Reakcije oslonaca:

1. od opterećenja:

$$A_P := \sum_{m=0}^{10} \left(P_m \xi d_m\right) \qquad A_P = 13.333 \quad kN$$

$$B_P := \sum_{m=0}^{10} \left(P_m \xi_m\right) \qquad B_P = 21.333 \quad kN$$

2. od koncentrisanog momenta:

$$A_M := \frac{-M}{L} \qquad A_M = -6.25 \quad kN$$

$$B_M := \frac{M}{L} \qquad B_M = 6.25 \quad kN$$

Reakcije su:

$$A := A_P + A_M \qquad A = 7.083 \quad kN$$

$$B := B_P + B_M \qquad B = 27.58 \quad kN$$

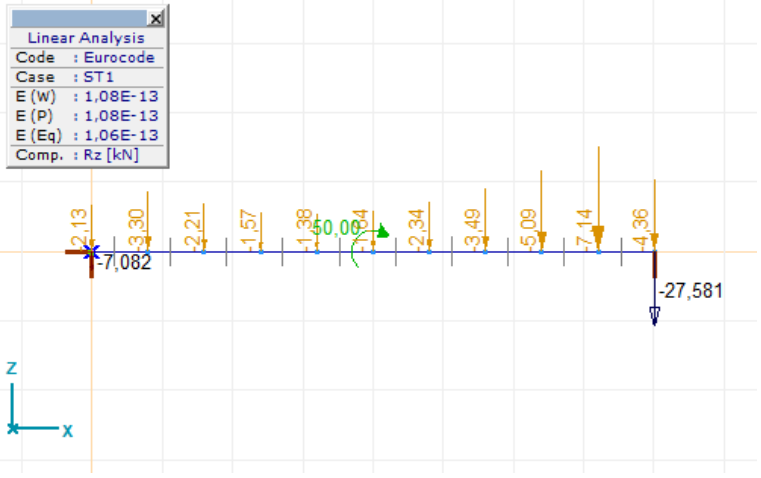
Presečne sile:

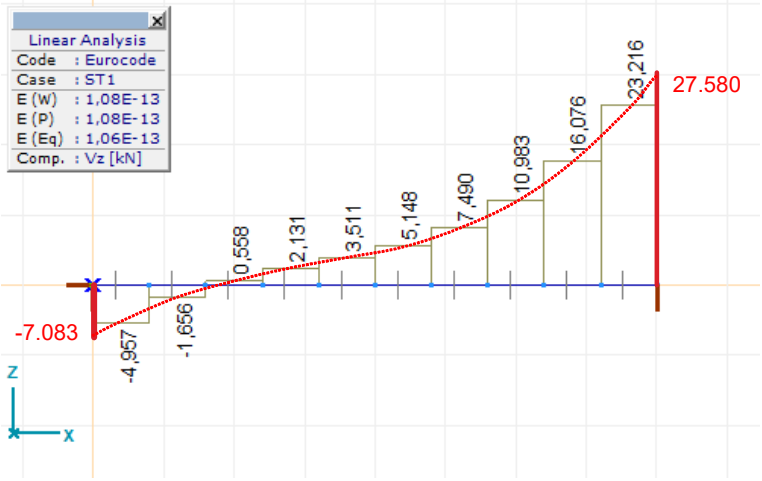
$$T_m := T_{m-1} - P_{m-1} \qquad T_0 := A$$

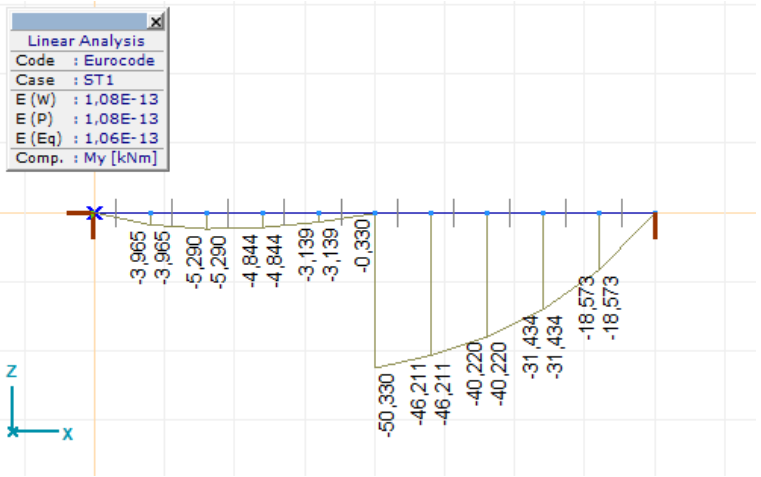
$$M_m := M_{m-1} + T_m \cdot \lambda_m$$

m	λ_m	ξ_m	p_m	P_m	ξ'_m	$P_m\xi'_m$	T_m	$T_m\lambda_m$	$M_m^{(M)}$	M_m
0	0,8	0	6	2,125	1	2,125	7,083	0		0
1	0,8	0,1	4,08	3,301	0,9	2,971	4,958	3,966		3,966
2	0,8	0,2	2,72	2,213	0,8	1,770	1,657	1,326		5,292
3	0,8	0,3	1,92	1,573	0,7	1,101	-0,556	-0,445		4,847
4	0,8	0,4	1,68	1,381	0,6	0,829	-2,129	-1,703		3,144
5	0,8	0,5	2	1,637	0,5	0,819	-3,510	-2,808		0,336
									50	50,336
6	0,8	0,6	2,88	2,341	0,4	0,936	-5,147	-4,118		46,218
7	0,8	0,7	4,32	3,493	0,3	1,048	-7,488	-5,990		40,228
8	0,8	0,8	6,32	5,093	0,2	1,019	-10,981	-8,785		31,443
9	0,8	0,9	8,88	7,141	0,1	0,714	-16,074	-12,859		18,584
10	0,8	1	12	4,365	0	0	-23,215	-18,572		0
			$\sum P_m =$	34,663	$A_P =$	13,332	-27,580	$=B$		

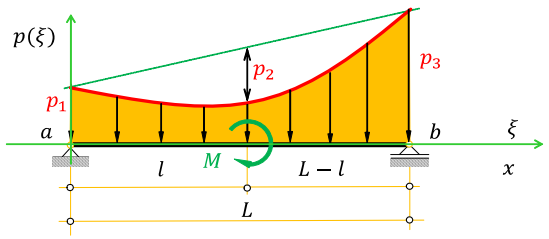
Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila







Primer



$p_1 := 6 \ \frac{kN}{m}$ $\quad M := 60 \ kNm$

$p_2 := 7 \ \frac{kN}{m}$

$p_3 := 12 \ \frac{kN}{m}$

$L := 12 \ m$ $\quad l := 6 \ m$ $\quad \lambda := \frac{L}{10}$ $\quad \lambda = 1.2$

Funkcija opterećenja: $p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

Given

1... $a \cdot 0^2 + b \cdot 0 + c = 6$

2... $a \cdot 6^2 + b \cdot 6 + c = 2$

3... $a \cdot 12^2 + b \cdot 12 + c = 12$

Konstante su: $a := \frac{7}{36}$ $\quad b := -\frac{11}{6}$ $\quad c := 6$

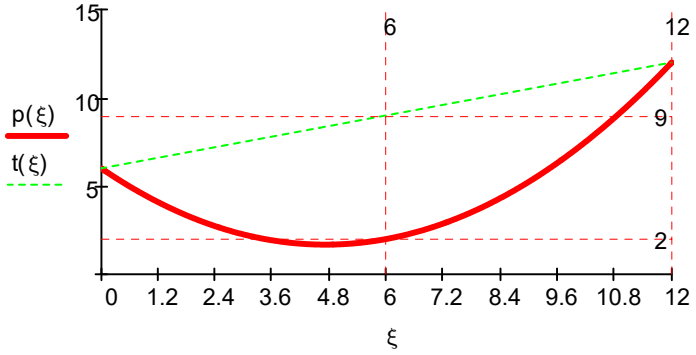
$p(\xi) := a \cdot \xi^2 + b \cdot \xi + c$

$p(\xi) \text{ simplify } \rightarrow \frac{7 \cdot \xi^2}{36} - \frac{11 \cdot \xi}{6} + 6$

$\text{Find}(a, b, c) \rightarrow \begin{pmatrix} \frac{7}{36} \\ -\frac{11}{6} \\ 6 \end{pmatrix}$

$p(\xi) := \frac{7}{36} \cdot \xi^2 - \frac{11}{6} \cdot \xi + 6$

$t(\xi) := 6 + 0.5 \cdot \xi$



$p(0) = 6$

$p(1.2) = 4.08$

$p(2.4) = 2.72$

$p(3.6) = 1.92$

$p(4.8) = 1.68$

$p(6) = 2$

$p(7.2) = 2.88 \qquad p(8.4) = 4.32 \qquad p(9.6) = 6.32 \qquad p(10.8) = 8.88 \qquad p(12) = 12$

$$p := \begin{pmatrix} 6 \\ 4.08 \\ 2.72 \\ 1.92 \\ 1.68 \\ 2 \\ 2.88 \\ 4.32 \\ 6.32 \\ 8.88 \\ 12 \end{pmatrix} \qquad \xi := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \qquad \xi d := \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0 \end{pmatrix}$$

$P_0 := \frac{\lambda}{24} \cdot (7 \cdot p_0 + 6\,p_1 - p_2) \qquad P_0 = 3.188$

$P_1 := \frac{\lambda}{12} \cdot (p_0 + 10\,p_1 + p_2) \qquad P_1 = 4.952$

$P_2 := \frac{\lambda}{12} \cdot (p_1 + 10\,p_2 + p_3) \qquad P_2 = 3.32$

$P_3 := \frac{\lambda}{12} \cdot (p_2 + 10\,p_3 + p_4) \qquad P_3 = 2.36$

$P_4 := \frac{\lambda}{12} \cdot (p_3 + 10\,p_4 + p_5) \qquad P_4 = 2.072$

$P_5 := \frac{\lambda}{12} \cdot (p_4 + 10\,p_5 + p_6) \qquad P_5 = 2.456$

$P_6 := \frac{\lambda}{12} \cdot (p_5 + 10\,p_6 + p_7) \qquad P_6 = 3.512$

$P_7 := \frac{\lambda}{12} \cdot (p_6 + 10\,p_7 + p_8) \qquad P_7 = 5.24$

$P_8 := \frac{\lambda}{12} \cdot (p_7 + 10\,p_8 + p_9) \qquad P_8 = 7.64$

$P_9 := \frac{\lambda}{12} \cdot (p_8 + 10\,p_9 + p_{10}) \qquad P_9 = 10.712$

$P_{10} := \frac{\lambda}{24} \cdot (7 \cdot p_{10} + 6\,p_9 - p_8) \qquad P_{10} = 6.548$

Reakcije oslonaca:

1. od opterećenja: $A_P := \sum_{m=0}^{10} (P_m \xi d_m)$ $A_P = 20$ kN

$B_P := \sum_{m=0}^{10} (P_m \xi_m)$ $B_P = 32$ kN

2. od koncentrisanog momenta:

$A_M := \frac{-M}{L}$ $A_M = -5$ kN

$B_M := \frac{M}{L}$ $B_M = 5$ kN

Reakcije su: $A := A_P + A_M$ $A = 15$ kN

$B := B_P + B_M$ $B = 37$ kN

Presečne sile: $T_m := T_{m-1} - P_{m-1}$ $T_0 := A$

$M_m := M_{m-1} + T_m \cdot \lambda_m$

m	λ_m	ξ_m	p_m	P_m	ξ'_m	$P_m\xi'_m$	T_m	$T_m\lambda_m$	$M_m^{\mathfrak{M}}$	M_m
0	1,2	0	6	3,188	1	3,188	15,000	0		0
1	1,2	0,1	4,08	4,952	0,9	4,457	11,812	14,174		14,174
2	1,2	0,2	2,72	3,32	0,8	2,656	6,860	8,232		22,406
3	1,2	0,3	1,92	2,36	0,7	1,652	3,540	4,248		26,654
4	1,2	0,4	1,68	2,072	0,6	1,243	1,180	1,416		28,070
5	1,2	0,5	2	2,456	0,5	1,228	-0,892	-1,070		27,000
									60	87,000
6	1,2	0,6	2,88	3,512	0,4	1,405	-3,348	-4,018		82,982
7	1,2	0,7	4,32	5,24	0,3	1,572	-6,860	-8,232		74,750
8	1,2	0,8	6,32	7,64	0,2	1,528	-12,100	-14,520		60,230
9	1,2	0,9	8,88	10,712	0,1	1,071	-19,740	-23,688		36,542
10	1,2	1	12	6,548	0	0	-30,452	-36,542		0
			$\sum P_m =$	52	$A_P =$	20	-37	$=B$		

Ekvivalentno statičko opterećenje sa reakcijama i dijagramima presečnih sila