

8

СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

- материјал за вежбе -

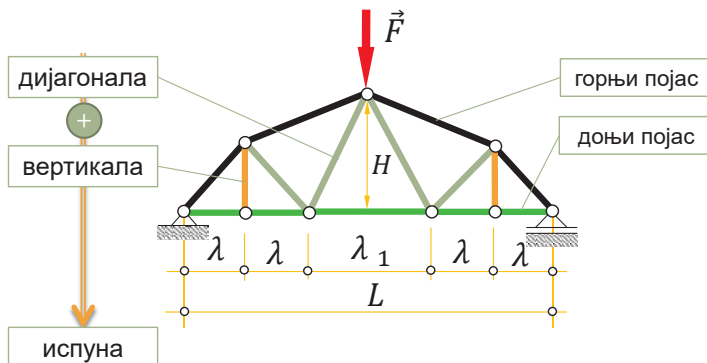
2024.

Решеткасти носачи у равни

Основне карактеристике су:

- код решетке имамо само праве зглавкасто везане штапове,
- у чвору осе свих штапова морају да се секу у једној тачки,
- активне силе делују само у чворовима решетке,
- ослонци су само у чворовима.

Елементи решеткастог носача



Обележавање:

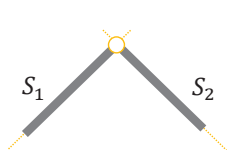
- k - број чворова решетке,

Прорачун непознатих сила у штаповима:

- Метода чворова,
- Метода пресека,
- Аналитички изрази за силе у штаповима (и њихова примена за конструисање утицајних линија),

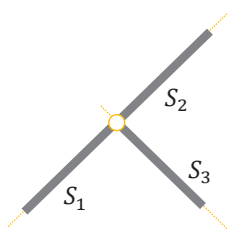
Метода чворова – „шест случајева“

не оптерећен чвор



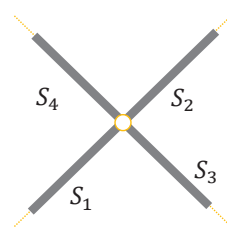
$$S_1 = 0$$

$$S_2 = 0$$



$$S_1 = S_2$$

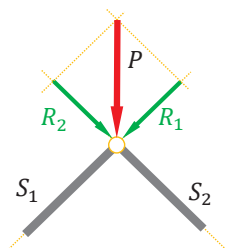
$$S_3 = 0$$



$$S_1 = S_2$$

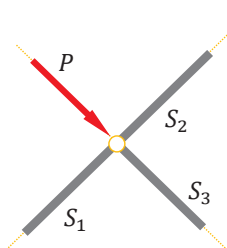
$$S_3 = S_4$$

оптерећен чвор



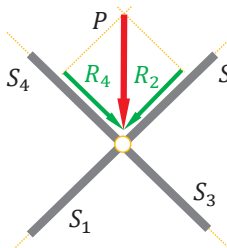
$$S_1 = R_1$$

$$S_2 = R_2$$



$$S_1 = S_2$$

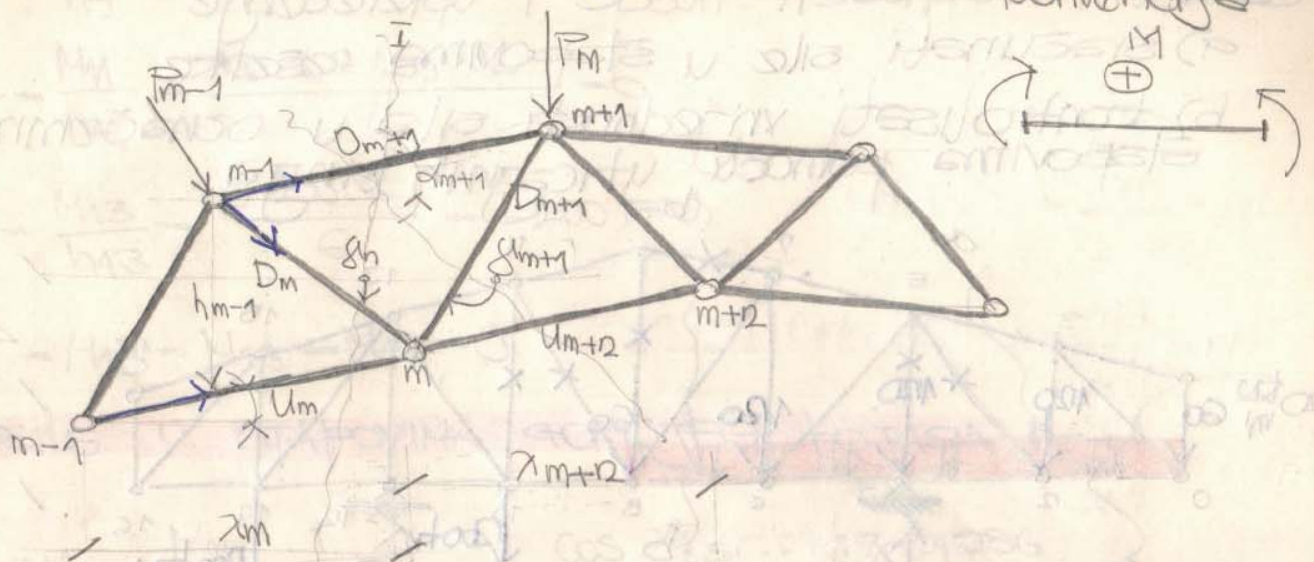
$$S_3 = P$$



$$S_1 + S_2 = R_2$$

$$S_3 + S_4 = R_4$$

- RAVNI REŠETKASTI NOSAČI -



Analitički izrazi za sile u štapovima sa trouganom ispunom

$$\sum M_m^{I-I} = 0 \Rightarrow M_m + D_{m+1} \cdot \cos \alpha_{m+1} \cdot h_m = 0$$

$$D_{m+1} = -\frac{M_m}{P_m} \cdot \frac{1}{\cos \alpha_{m+1}}$$

$$\sum M_{m-1}^{I-I} = 0 \Rightarrow M_{m-1} - U_m \cdot \cos \beta_m \cdot h_{m-1} = 0$$

$$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \frac{1}{\cos \beta_m}$$

$$\sum H^{I-I} = 0 \Rightarrow D_{m+1} \cos \alpha_{m+1} + D_m \cos \gamma_m \cdot \cos \gamma_m + U_m \cos \beta_m + H_m = 0$$

$$D_m = \left(\frac{M_m}{h_m} - \frac{M_{m-1}}{h_{m-1}} - H_m \right) \cdot \frac{1}{\cos \gamma_m}$$

dijagonala pada s lijeva na desno

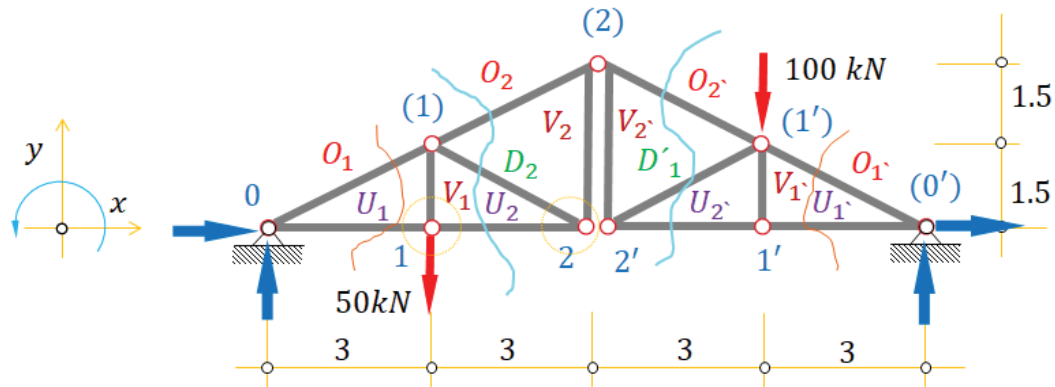
$$\sum H^{II-II} = 0 \Rightarrow D_{m+1} \cos \alpha_{m+1} + D_{m+1} \cos \gamma_{m+1} + U_{m+2} \cos \beta_{m+2} + H_{m+2} = 0$$

$$D_{m+1} = \left(\frac{M_m}{h_m} - \frac{M_{m+1}}{h_{m+1}} - H_{m+2} \right) \cdot \frac{1}{\cos \gamma_{m+1}}$$

dijagonala pada s desna na lijevo

Пример

За носач са датим оптерећењем приказ на скици срачунати силе у штаповима на основу аналитичких израза.



Реакције веза:

$$\Sigma H = 0$$

Given

$$P_1 := 50 \text{ kN}$$

$$P_2 := 100 \text{ kN}$$

$$H_a + H_b = 0$$

$$\Sigma V = 0$$

$$V_a + V_b - P_1 - P_2 = 0$$

$$\Sigma M_a = 0$$

$$V_b \cdot 12 - P_1 \cdot 3 - P_2 \cdot 9 = 0$$

$$\Sigma M_{2D} = 0$$

$$V_b \cdot 6 - H_b \cdot 3 - P_2 \cdot 3 = 0$$

$$\text{Find}(H_a, H_b, V_a, V_b) \rightarrow \begin{pmatrix} -75 \\ 75 \\ \frac{125}{2} \\ \frac{175}{2} \end{pmatrix}$$

$$H_a := -75 \text{ kN}$$

$$H_b := 75 \text{ kN}$$

$$V_a := \frac{125}{2} = 62.5 \text{ kN}$$

$$V_b := \frac{175}{2} = 87.5 \text{ kN}$$

Моменти савијања у чворовима решеткастог носача:

$$M_1 := V_a \cdot 3 \quad M_1 = 187.5$$

$$M_{1G} := V_a \cdot 3 + H_a \cdot 1.5 \quad M_{1G} = 75$$

$$M_2 := V_a \cdot 6 - P_1 \cdot 3 \quad M_2 = 225$$

$$M_{2G} := V_a \cdot 6 - P_1 \cdot 3 + H_a \cdot 3 \quad M_{2G} = 0$$

$$M_{2'} := V_b \cdot 6 - P_2 \cdot 3 \quad M_{2'} = 225$$

$$M_{1'} := V_b \cdot 3 \quad M_{1'} = 262.5$$

$$M_{1'G} := V_b \cdot 3 - H_b \cdot 1.5 \quad M_{1'G} = 150$$

Силе у штаповима на основу аналитичких израза

$$\alpha := 26.57 \text{deg} \quad \beta := 0 \text{deg} \quad h_1 := 1.5 \quad h_2 := 3$$

$$U_2 := \frac{M_{1G}}{h_1} \cdot \frac{1}{\cos(\beta)} \quad U_2 = 50 \text{ kN} \quad U_1 = U_2$$

$$U_{1'} := \frac{M_{1'G}}{h_1} \cdot \frac{1}{\cos(\beta)} \quad U_{1'} = 100 \text{ kN} \quad U_{1'} = U_{2'}$$

$$O_2 := \frac{-M_2}{h_2} \cdot \frac{1}{\cos(\alpha)} \quad O_2 = -83.86 \text{ kN}$$

$$O_1 := \frac{-M_1}{h_1} \cdot \frac{1}{\cos(\alpha)} \quad O_1 = -139.76 \text{ kN}$$

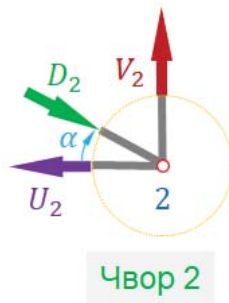
$$O_{2'G} := \frac{-M_{2'}}{h_2} \cdot \frac{1}{\cos(\alpha)} \quad O_{2'G} = -83.86 \text{ kN}$$

$$O_{1'G} := \frac{-M_{1'}}{h_1} \cdot \frac{1}{\cos(\alpha)} \quad O_{1'G} = -195.66 \text{ kN}$$

$$D_2 := \left(\frac{M_2}{h_2} - \frac{M_1 G}{h_1} + H_a \right) \cdot \frac{1}{\cos(\alpha)} \quad D_2 = -55.9 \text{ kN}$$

$$D_{1'} := \left(\frac{M_{2'}}{h_2} - \frac{M_{1'} G}{h_1} - H_b \right) \cdot \frac{1}{\cos(\alpha)} \quad D_{1'} = -111.81 \text{ kN}$$

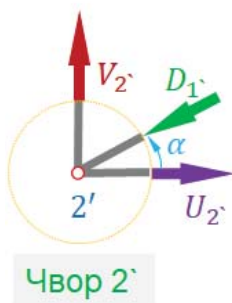
Чвор 1 $\Sigma V = 0$ given $V_1 - 50 = 0$ find(V_1) $\rightarrow 50 \text{ kN}$



Чвор 2 $\Sigma V = 0$ given $V_2 - D_2 \cdot \sin(\alpha) = 0$

$$\text{find}(V_2) \text{ simplify } \rightarrow -\frac{50.0 \cdot \sin(26.57 \cdot \text{deg})}{\cos(26.57 \cdot \text{deg})^{1.0}}$$

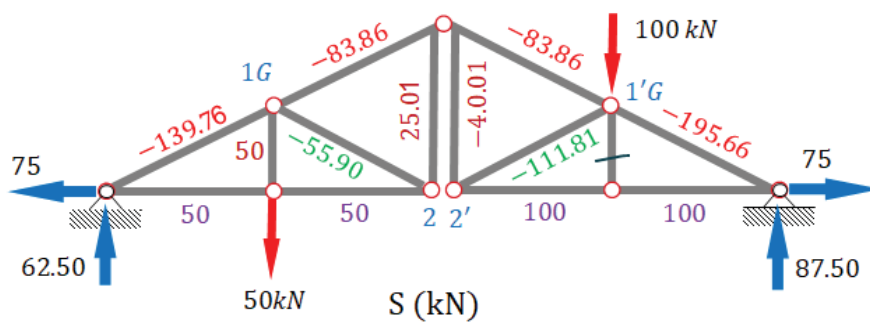
$$\frac{50.0 \cdot \sin(26.57 \cdot \text{deg})}{\cos(26.57 \cdot \text{deg})^{1.0}} = 25.01 \text{ kN}$$



Чвор 2' $\Sigma V = 0$ Given $V_{2'} - D_{1'} \cdot \sin(\alpha) = 0$

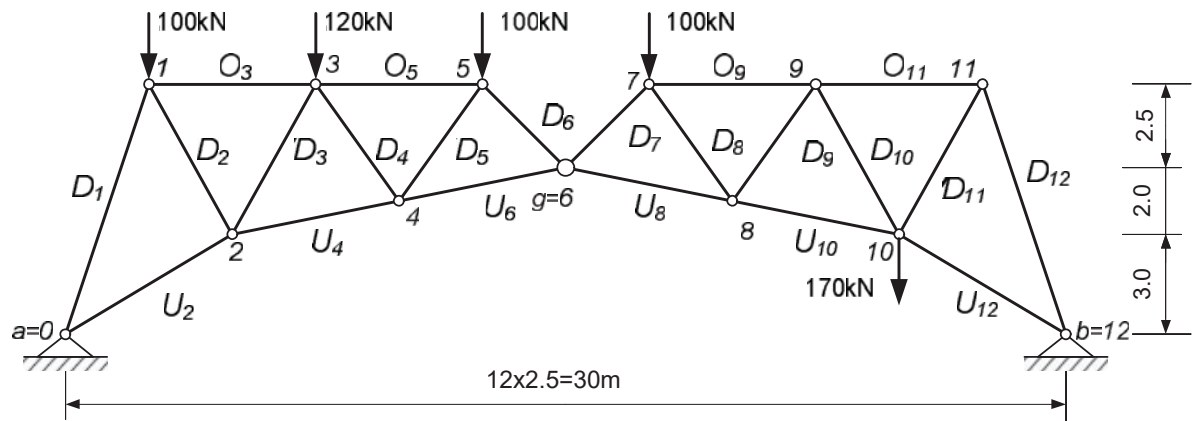
$$\text{find}(V_{2'}) \text{ simplify } \rightarrow -\frac{100.0 \cdot \sin(26.57 \cdot \text{deg})}{\cos(26.57 \cdot \text{deg})^{1.0}}$$

$$-\frac{80.0 \cdot \sin(26.57 \cdot \text{deg})}{\cos(26.57 \cdot \text{deg})^{1.0}} = -40.01 \text{ kN}$$



Пример

За носач са датим оптерећењем приказ на скици срачунати силе у штаповима на основу аналитичких израза.



Аналитички изрази за силе у штаповима решеткастог носача.

за..... $m=3,5,9,11$

$$O_m = -\frac{M_{m-1}}{h_{m-1}} \cdot \sec \alpha_m$$

за..... $m=2,4,6,8,10,12$

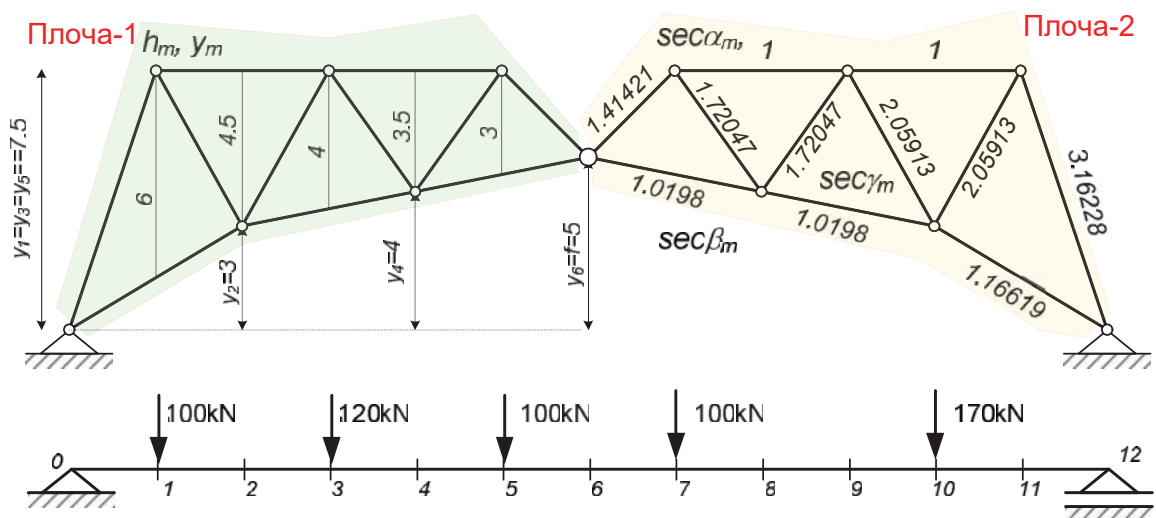
$$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \sec \beta_m$$

за..... $m=1,3,5,7,9,11$

$$D_m = \left(\frac{M_{m-1}}{h_{m-1}} - \frac{M_m}{h_m} - H_m \right) \cdot \sec \gamma_m$$

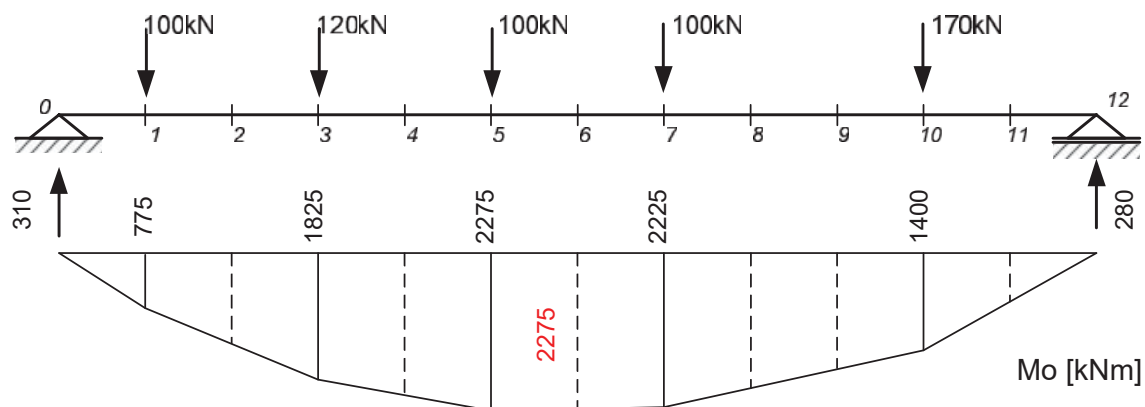
за..... $m=2,4,6,8,10,12$

$$D_m = \left(\frac{M_m}{h_m} - \frac{M_{m-1}}{h_{m-1}} - H_m \right) \cdot \sec \gamma_m$$



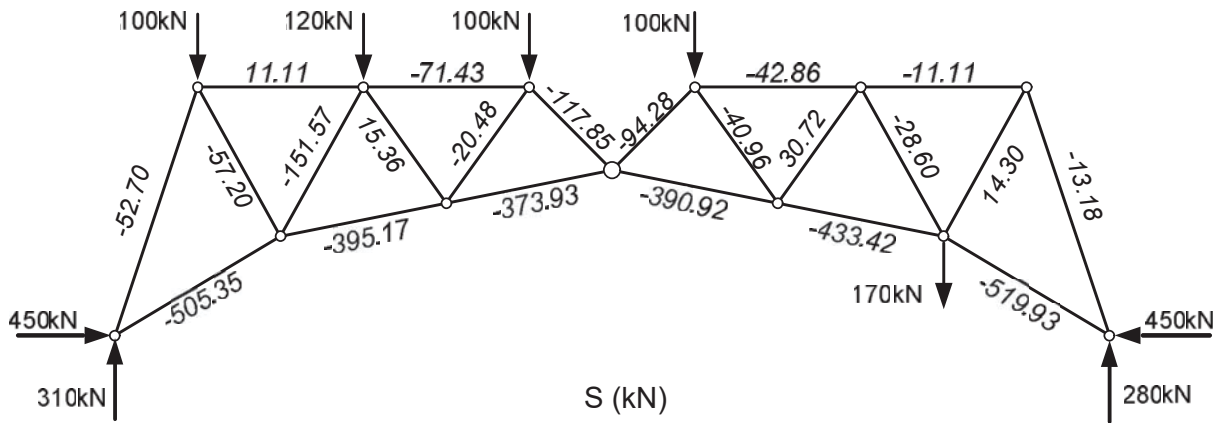
$$M_m = M_{m0} - H \cdot y_m$$

M_{m0} - моменти савијања одговарајуће просте греде

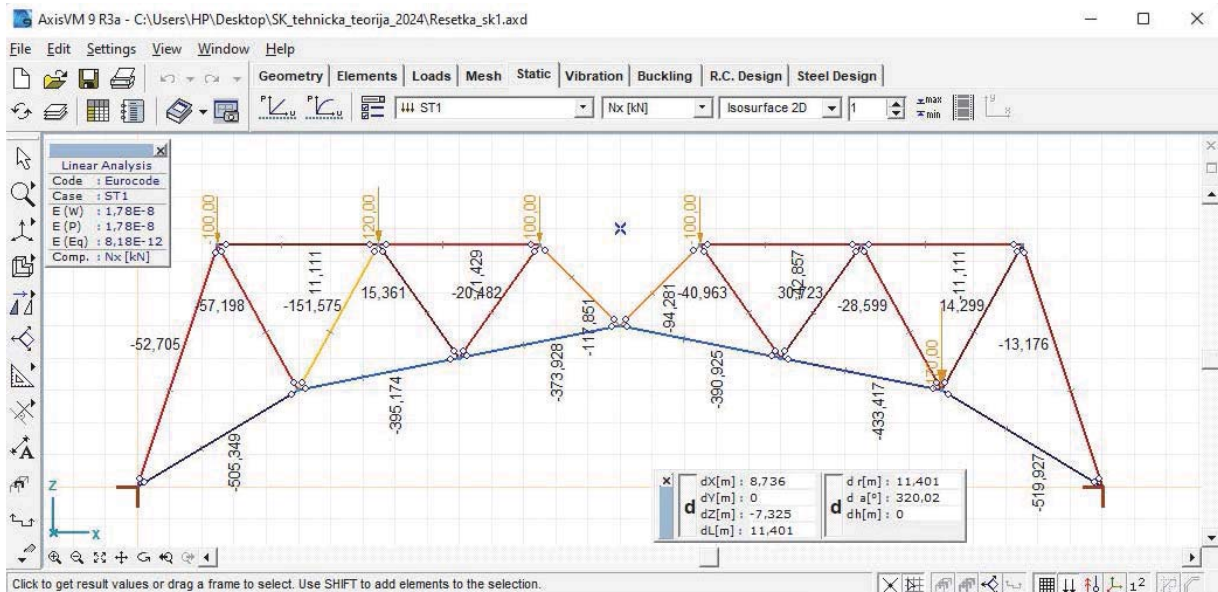
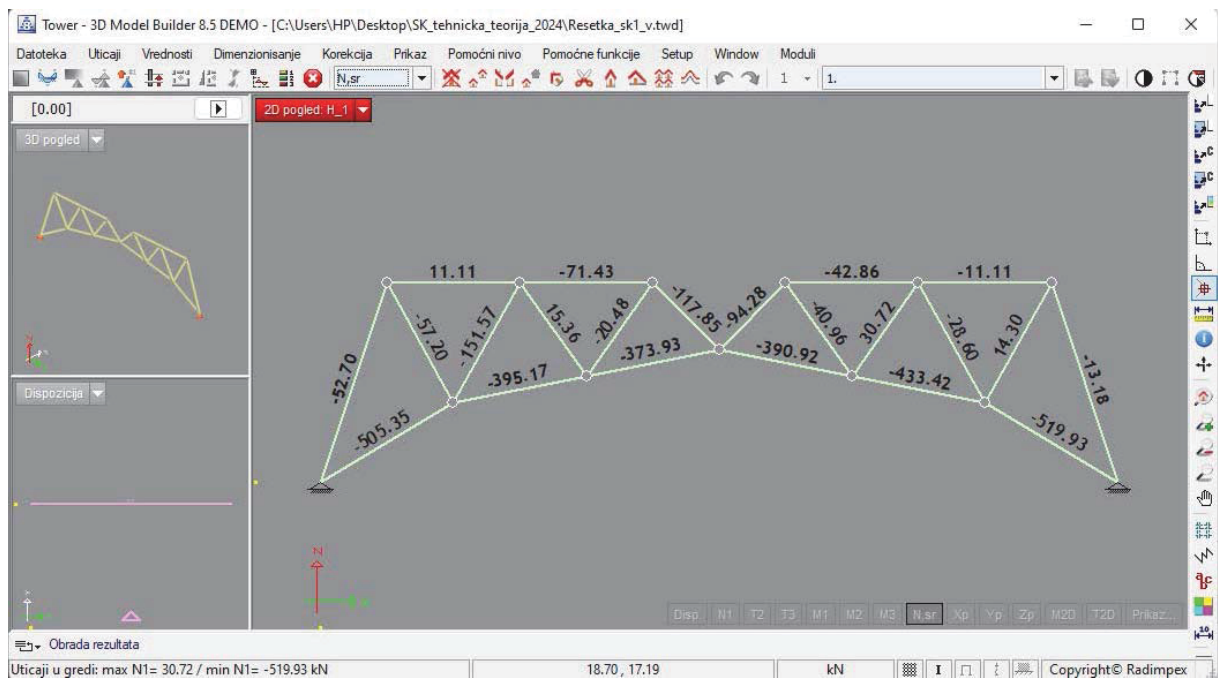


$$H = M_{g0}/f = 2250/5 = 450 \text{ kN}$$

m	M_{m0}	$H \cdot y_m$	M_m	M_m/h_m	U_m	O_m	D_m
0	0	0	0	0	-	-	-
1	775	3375	-2600	-433.33	-	-	-52.70
2	1300	1350	-50	-11.11	-505.35	-	-57.20
3	1825	3375	-1550	-387.5	-	11.11	-151.57
4	2050	1800	250	71.43	-395.17	-	15.36
5	2275	3375	-1100	-366.67	-	-71.43	-20.48
6	2250	2250	0	0	-373.93	-	-117.85
7	2225	3375	-1150	-383.33	-	-	-94.28
8	1950	1800	150	42.86	-390.92	-	-40.96
9	1675	3375	-1700	-425.00	-	-42.86	30.72
10	1400	1350	50	11.11	-433.42	-	-28.60
11	700	3375	-2675	-445.83	-	-11.11	14.30
12	0	0	0	0	-519.93	-	-13.18

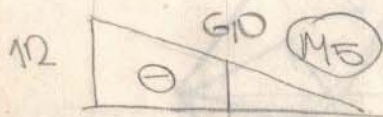
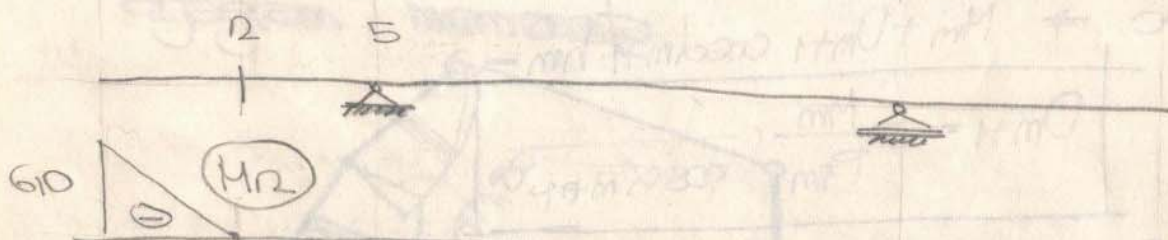
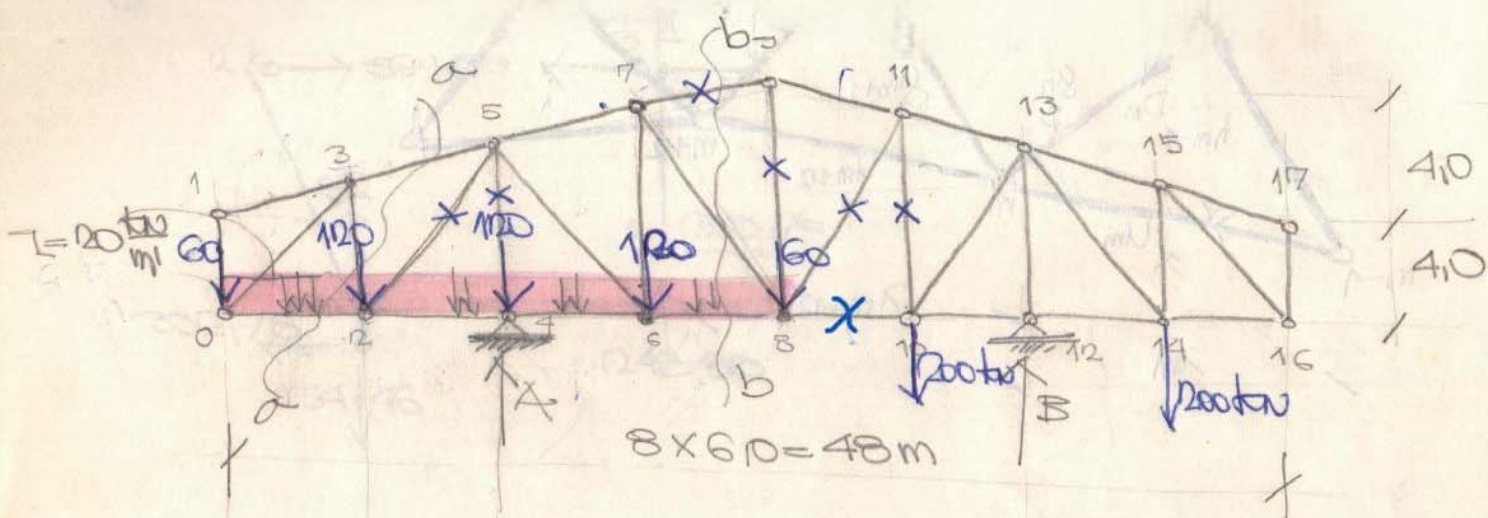


Контрола резултата методом коначних елемената



ZADATAK

Za ravni rešetkasti most i opterećenje
sračunati sile u štapovima rešetke



$$D_5 = 60 \cdot 1,131 + 120 \cdot 1,416 = 237,70 \text{ kN}$$

$$\sum M_{112} = 0 \Rightarrow A = \frac{1}{24} [60(36+12) + 120(30+24+18) + 200(6-6)] =$$

$$A = 480 \text{ kN}$$

$$\sum V = 0 \quad B = 2 \cdot 60 + 3 \cdot 120 + 2 \cdot 200 - 480 = 400 \text{ kN}$$

$$\frac{M_2}{h_2} = \frac{M_3}{h_3} = \frac{-60 \cdot 6}{5} = -72 \text{ kN}$$

$$\frac{M_4}{h_4} = \frac{M_5}{h_5} = \frac{-60 \cdot 12 - 120 \cdot 6}{6} = -240 \text{ kN}$$

$$\frac{M_6}{h_6} = \frac{M_7}{h_7} = \frac{-60 \cdot 18 - 120(12+6) + 480 \cdot 6}{7} = 51,43 \text{ kN}$$

$$\frac{M_8}{h_8} = \frac{M_9}{h_9} = \frac{400 \cdot 12 - 200(6+18)}{8} = 0$$

$$\frac{M_{10}}{h_{10}} = \frac{M_{11}}{h_{11}} = \frac{400 \cdot 6 - 200 \cdot 12}{7} = 0$$

$$\frac{M_{12}}{h_{12}} = \frac{M_{13}}{h_{13}} = \frac{-200 \cdot 6}{6} = -200 \text{ kN}$$

$$M_{14} = M_{15} = M_{16} = M_{17} = 0$$

→ SILE U ŠTAPOVIMA GORNJEG POJASA →

$$O_{m+1} = -\frac{M_m}{h_m} \cdot \frac{1}{\cos \alpha_{m+1}}$$

$$\cos \alpha_{3,5} \dots 17 = 0,586$$

$$O_3 = -\frac{M_0}{h_0} \cdot \frac{1}{\cos \alpha_3} = 0$$

$$O_5 = -\frac{M_2}{h_2} \cdot \frac{1}{\cos \alpha_5} = -(-720) \cdot \frac{1}{0,586} = 730,2 \text{ kN}$$

$$O_7 = -\frac{M_6}{h_6} \cdot \frac{1}{\cos \alpha_7} = -(51,43) \cdot \frac{1}{0,586} = 52,16 \text{ kN}$$

$$O_9 = -\frac{M_8}{h_8} \cdot \frac{1}{\cos \alpha_9} = 0$$

$$O_{11} = -\frac{M_{10}}{h_{10}} \cdot \frac{1}{\cos \alpha_{11}} = 0$$

↳ Riterova tačka za step 11

$$O_{13} = -\frac{M_{12}}{h_{12}} \cdot \frac{1}{\cos \alpha_{13}} = 0$$

$$O_{15} = O_{17} = 0$$

→ SILE U ŠTAPOVIMA DONJEG POJASA →

$$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \frac{1}{\cos \beta_m}$$

$$\cos \beta_{2,4} \dots 16 = 1,0$$

$$U_2 = \frac{M_3}{h_3} = -72 \text{ kN}$$

↳ Riterova tačka za step 2

$$U_4 = \frac{M_5}{h_5} = -240 \text{ kN}$$

$$U_6 = \frac{M_7}{h_7} = -240 \text{ kN}$$

$$U_8 = \frac{M_7}{h_7} = -51,43 \text{ kN}$$

$$U_{10} = \frac{M_{11}}{h_{11}} = 0$$

$$U_{12} = \frac{M_{13}}{h_{13}} = -200 \text{ kN}$$

$$U_{14} = \frac{M_{13}}{h_{13}} = -200 \text{ kN}$$

$$U_{16} = \frac{M_{15}}{h_{15}} = 0$$

$$Z_3 + Z_0 = 2t_0$$

t - broj čvorova

Z_3 = sila u štapu

Z_0 = reakcija oslonaca

+ sile u dijagonalama +

$$\cos \varphi_3 = \cos \varphi_{16} = 0,768$$

$$\cos \varphi_5 = \cos \varphi_6 = \cos \varphi_{13} = \cos \varphi_{14} = \frac{\sqrt{2}}{2}$$

$$\cos \varphi_8 = \cos \varphi_{11} = 0,651$$

$$\checkmark D_3 = \left(\frac{M_0}{h_0} - \frac{M_3}{h_3} - \frac{M_2}{h_2} \right) \cdot \frac{1}{\cos \varphi_3} = (-720) \cdot \frac{1}{0,768} = 93,75 \text{ kN}$$

$$D_5 = \left(\frac{M_2}{h_2} - \frac{M_5}{h_5} \right) \cdot \frac{1}{\cos \varphi_5} = (-720 - (-240)) \cdot \frac{1}{\frac{\sqrt{2}}{2}} = -237,70 \text{ kN}$$

$$\checkmark D_6 = \left(\frac{M_5}{h_5} - \frac{M_6}{h_6} \right) \cdot \frac{1}{\cos \varphi_6} = (-51,43 - (-240)) \cdot \frac{1}{\frac{\sqrt{2}}{2}} = 266,72 \text{ kN}$$

$$D_8 = \left(\frac{M_8}{h_8} - \frac{M_7}{h_7} \right) \cdot \frac{1}{\cos \varphi_8} = (0 - (-51,43)) \cdot \frac{1}{0,651} = 79,0 \text{ kN}$$

$$\checkmark D_{11} = \left(\frac{M_8}{h_8} - \frac{M_{11}}{h_{11}} \right) \cdot \frac{1}{\cos \varphi_{11}} = (0 - 0) \cdot \frac{1}{0,651} = 0$$

$$D_{13} = \left(\frac{M_0}{h_0} - \frac{M_{13}}{h_{13}} \right) \cdot \frac{1}{\cos \varphi_{13}} = (0 - (-200)) \cdot \frac{1}{\frac{\sqrt{2}}{2}} = 282,84 \text{ kN}$$

$$\checkmark D_{14} = \left(\frac{M_{14}}{h_{14}} - \frac{M_{13}}{h_{13}} \right) \cdot \frac{1}{\cos \varphi_{14}} = (0 - (-200)) \cdot \frac{1}{\frac{\sqrt{2}}{2}} = 282,84 \text{ kN}$$

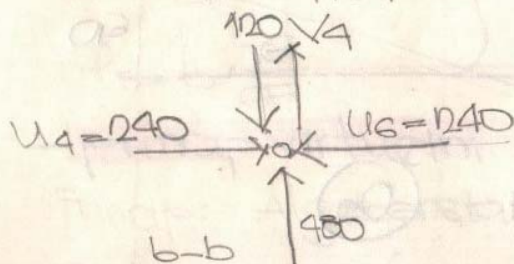
$$D_{16} = 0$$

SILE U VERTIKALAMA

$$V_0 = V_{16} = V_{10} = V_8 = V_{14} = 0$$

$$V_2 + 60 - 0.5 \cdot \sin 65 = 0$$

$$V_2 = 73.02 \cdot 0.164 - 60 = -48.02 \text{ kN}$$



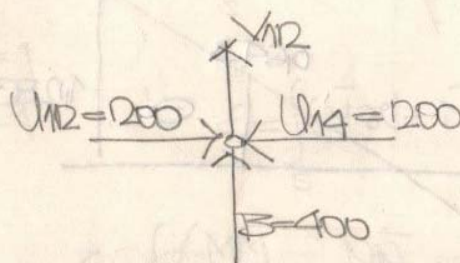
$$\sum V_A = 0$$

$$V_4 = -360 \text{ kN}$$

$$\sum V_{\text{levo}} = 0$$

$$V_6 + 0.7 \cdot \sin 67 + 60 = 0$$

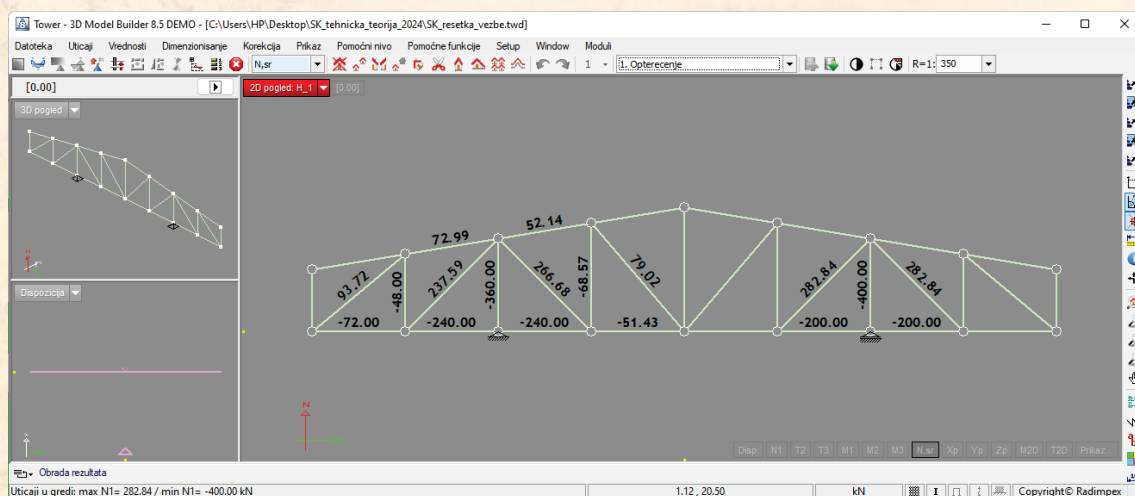
$$V_6 = -52.16 \cdot 0.164 - 60 = -68.55 \text{ kN}$$



$$\sum V = 0$$

$$V_{12} = -400 \text{ kN}$$

Преглед резултата - метода коначних елемената



Слика 1: $S(kN)$