

СТАТИКА КОНСТРУКЦИЈА 1

Модул: Конструкције

- материјал за вежбе -

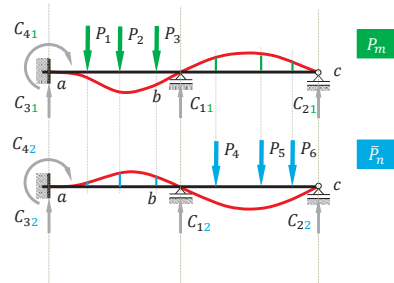
2024.

Теореме о узајамности

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Betti – јева теорема

$$\sum P_m \delta_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$



Рад спољашњих сила P_m и C_i **првог** система при померањима која изазива **други** систем једнак је раду спољашњих сила \bar{P}_n и \bar{C}_i **другог** система при померањима који изазива **први** систем утицаја.

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Maxwell – ова теорема

$$\sum P_m \delta_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

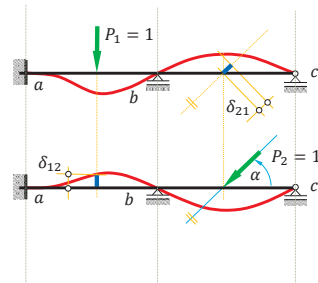
$$P_m \Rightarrow P_1 = 1 \Rightarrow \delta_n = \delta_{21}$$

$$\bar{P}_n \Rightarrow P_2 = 1 \Rightarrow \bar{\delta}_m = \delta_{12}$$

$$\sum 1 \cdot \delta_{12} + \sum C_i \bar{c}_i = \sum 1 \cdot \delta_{21} + \sum \bar{C}_i c_i$$

$$1 \cdot \delta_{12} = 1 \cdot \delta_{21}$$

$$\delta_{12} = \delta_{21}$$



Померање нападне тачке силе P_1 у правцу те силе услед силе P_2 једнако је померању нападне тачке силе P_2 у правцу силе P_2 услед силе P_1 .

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прва Rayleigh – јева теорема

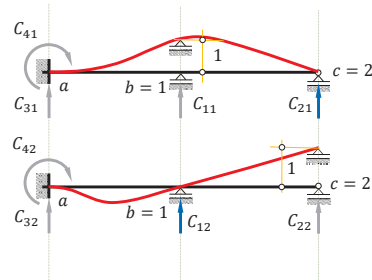
$$\sum P_m \delta_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

$$P_m \Rightarrow P_1 = 0 \quad c_i = c_1 = 1 \Rightarrow C_i = C_{12}$$

$$\bar{P}_n \Rightarrow P_2 = 0 \quad \bar{c}_i = c_2 = 1 \Rightarrow \bar{C}_i = C_{21}$$

$$\sum 0 \cdot \delta_m + \sum C_i \bar{c}_i = \sum 0 \cdot \delta_n + \sum \bar{C}_i c_i$$

$$C_{12} \cdot 1 = C_{21} \cdot 1$$



Реакција ослонца 1 услед јединичног померања ослонца 2 једнака је реакцији ослонца 2 услед јединичног померања ослонца 1.

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друга Rayleigh – јева теорема

$$\sum P_m \delta_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

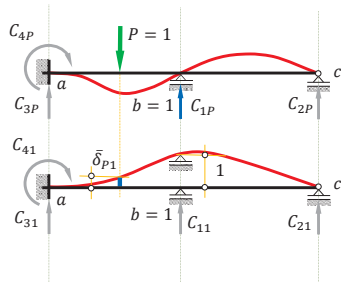
$$P_m \Rightarrow P = 1 \quad c_i = 0 \Rightarrow C_{1P}$$

$$\bar{P}_n \Rightarrow P = 0 \quad \bar{c}_i = c_1 = 1 \Rightarrow \bar{\delta}_{P1}$$

$$\sum 1 \cdot \bar{\delta}_{P1} + \sum C_{1P} \cdot 1 = \sum 0 \cdot \delta_n + \sum \bar{C}_{1P} \cdot 0$$

$$C_{1P} \cdot 1 = -\bar{\delta}_{P1} \cdot 1$$

$$C_{1P} = -\bar{\delta}_{P1}$$



Реакција ослонца 1 услед дејства јединичне силе P једнака је негативној вредности померања нападне тачке силе P у правцу те силе услед јединичног померања ослонца 1.

DEFORMACIJA STATIČKI OPREDENIH NOSAČA

$$\left. \begin{aligned} \alpha &= \frac{M}{EJ} + \alpha t \frac{\Delta t}{h} \\ \varepsilon &= \frac{N}{EF} + \alpha t \\ \varphi_T &= k \frac{T}{GF} \end{aligned} \right\} \rightarrow \Delta u_i, \Delta t_i, \Delta t_i \rightarrow C_{oi}, C_{ui} \Rightarrow u_i, v_i$$

princip virtualni radova

Princip: Algebarski zbir radova unutrašnjih i spoljasnih jednog mogućeg stanja jeste jednak nuli

$$\sum \bar{P} \delta + \sum \bar{Q} \alpha = \int_s (\bar{M} \alpha + \bar{N} \varepsilon + \bar{T} \varphi_T) ds$$

$$\bar{P}=1$$

$$\bar{M}=1$$

$$\delta = \int_s (\bar{M} \alpha + \bar{N} \varepsilon + \bar{T} \varphi_T) ds - \sum \bar{Q} \alpha$$

$$\delta = \int_s \left(\frac{M \bar{M}}{EJ} + \frac{N \bar{N}}{EF} + k \frac{T \bar{T}}{GF} + \bar{M} \alpha t \frac{\Delta t}{h} + \bar{N} \alpha t \cdot t \right) ds - \sum \bar{Q} \alpha$$

$$\delta_0 = \int_s \frac{M \bar{M}}{EJ} ds + \int_s \frac{N \bar{N}}{EF} ds + k \int_s \frac{T \bar{T}}{GF} ds$$

$$\delta_t = \int_s \bar{M} \alpha t \frac{\Delta t}{h} ds + \int_s \bar{N} \alpha t \cdot t ds$$

$$\delta_c = - \sum \bar{Q} \alpha \quad \text{pov. oslona}$$

$$EJ_c \cdot \delta_0 = \int_s M \bar{M} \frac{J_c}{J} ds + \frac{J_c}{F_c} \int_s N \bar{N} \frac{F_c}{F} ds + (2 + \nu) \frac{J_c}{F_c} \int_s T \bar{T} \frac{F_c}{F} ds$$

$$ds = \frac{J_c}{J} ds$$

$$ds = \frac{F_c}{F} ds$$

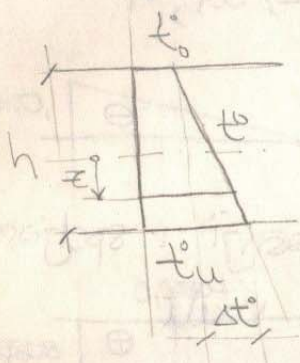
$$ds =$$

$$EJ_c \delta_0 = \int_s M \bar{M} \frac{J_c}{J} ds + \frac{J_c}{F_c} \int_s N \bar{N} \frac{F_c}{F} ds$$

usled spoljasnijeg opterećenja

$$EJ_0 \delta t = EJ_0 \int_s \bar{N} \delta t \frac{dt}{h} ds + EJ_0 \int_s \bar{N} \delta t^c ds$$

$$EJ_0 \delta \phi = - \sum Q_i \delta_i$$



$$t(x) = t_u + \Delta t \frac{x}{h}$$

$$\Delta t = t_u - t_0$$

$$t_0 = \frac{t_u + t_0}{2}$$

Mod. risultato

$$\delta = \int_s \bar{N} \epsilon ds - \sum \bar{Q}_i \delta_i$$

$$\delta = \sum_i \int_s \bar{N} \epsilon ds - \sum \bar{Q}_i \delta_i = \sum_i \bar{S}_i \Delta l - \sum \bar{Q}_i \delta_i = \sum_i \frac{\bar{S}_i}{EF} l + \sum_i \bar{S}_i \Delta t \frac{l}{h} - \sum \bar{Q}_i \delta_i$$

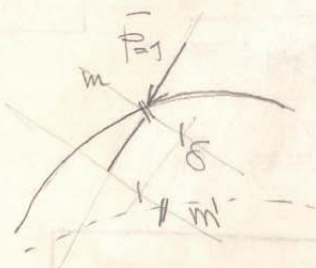
$$\int_s \bar{N} \epsilon ds = \bar{N} \epsilon \int_s ds = \bar{N} \cdot \epsilon \cdot l = \bar{S} \cdot \Delta l$$

$$\delta_0 = \sum_i \frac{\bar{S}_i}{EF} l / EF_c \rightarrow EF_c \delta_0 = \sum_i \bar{S}_i \frac{F_c}{F} \cdot l$$

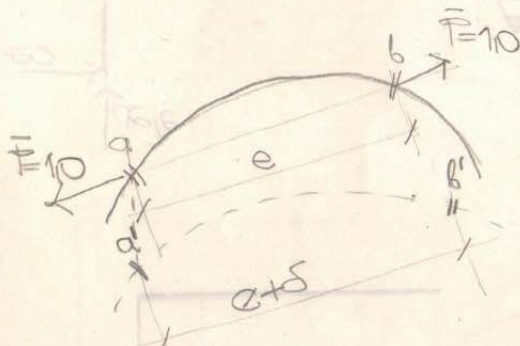
$$\delta_t = \sum_i \bar{S}_i \Delta t \frac{l}{h} / EF_c \rightarrow EF_c \delta_t = \sum_i \bar{S}_i \Delta t \frac{l}{h} EF_c$$

$$\delta_{\phi} = - \sum \bar{Q}_i \delta_i / EF_c \quad EF_c \delta_{\phi} = - \sum \bar{Q}_i \delta_i$$

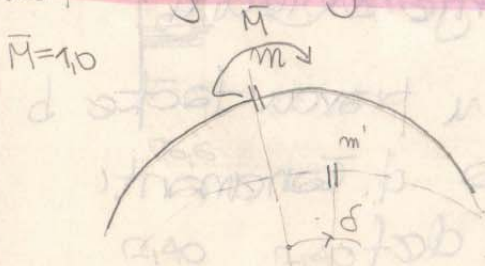
1. gen. pom. tačke u određenom pravcu



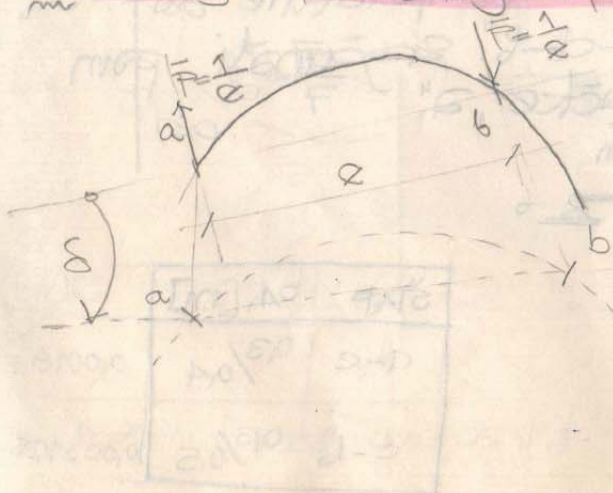
2. promjena odstojaranja između dvije tačke



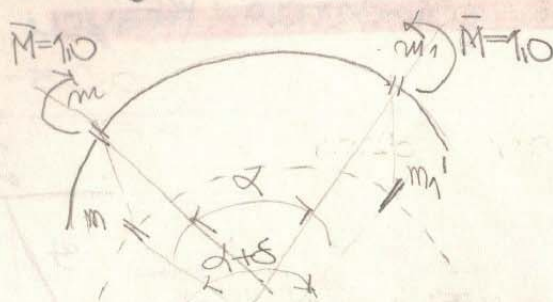
3. promjena ugla u presjeku



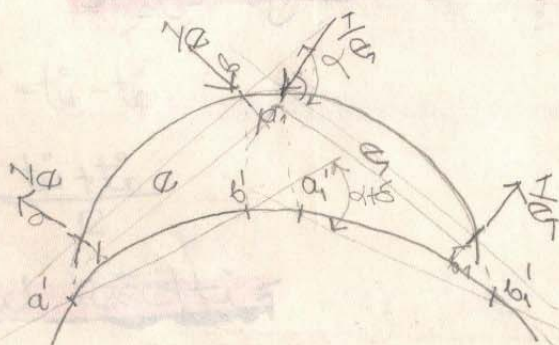
4. obrtanje prave koja prolazi kroz tačke A i B



5. promjena ugla ravnosti projekta m i m_1

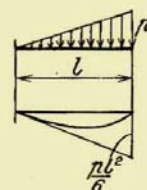
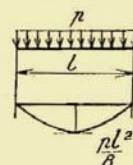
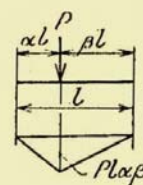
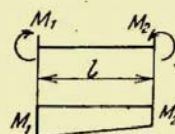


6. promjena ugla ravnosti duge krive



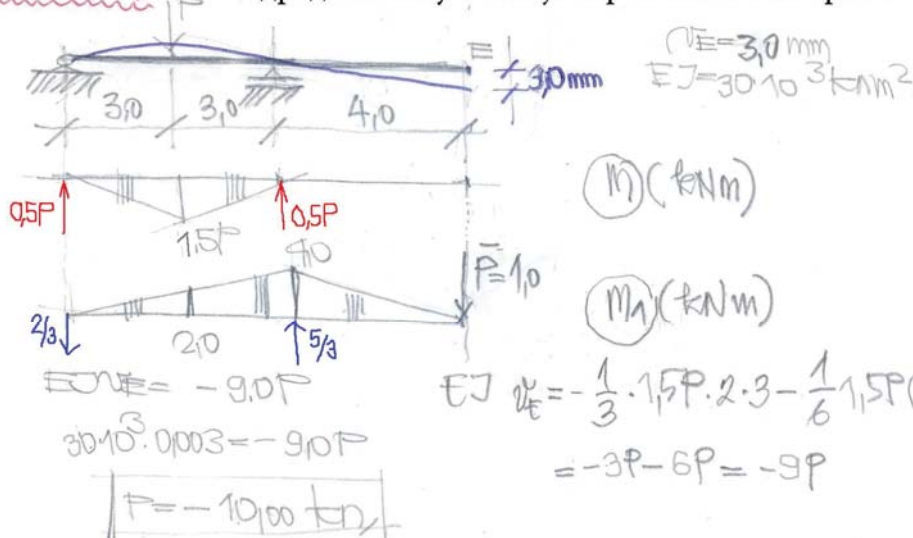
TABLICA 1

$\frac{1}{l} \int_0^l M \bar{M} ds$	\bar{i}	\bar{k}	\bar{l} \bar{k}	$\alpha \bar{l}$ $\beta \bar{l}$		\bar{k}
i	$i \bar{i}$	$\frac{1}{2} i \bar{k}$	$\frac{1}{2} i (\bar{l} + \bar{k})$	$\frac{1}{2} i \bar{m}$	$\frac{2}{3} i \bar{m}$	$\frac{1}{4} i \bar{k}$
k	$\frac{1}{2} k \bar{i}$	$\frac{1}{3} k \bar{k}$	$\frac{1}{6} k (\bar{l} + 2\bar{k})$	$\frac{1}{6} k \bar{m} (1 + \alpha)$	$\frac{1}{3} k \bar{m}$	$\frac{2}{15} k \bar{k}$
i	$\frac{1}{2} i \bar{i}$	$\frac{1}{6} i \bar{k}$	$\frac{1}{6} i (2\bar{l} + \bar{k})$	$\frac{1}{6} i \bar{m} (1 + \beta)$	$\frac{1}{3} i \bar{m}$	$\frac{2}{60} i \bar{k}$
i k	$\frac{1}{2} (i + k) \bar{i}$	$\frac{1}{6} (i + 2k) \bar{k}$	$\frac{1}{6} [i(2\bar{l} + \bar{k}) + k(\bar{l} + 2\bar{k})]$	$\frac{1}{6} [i(1 + \beta) + k(1 + \alpha)] \bar{m}$	$\frac{1}{3} (i + k) \bar{m}$	$\frac{1}{60} (7i + 8k) \bar{k}$
$\alpha \bar{l}$ $\beta \bar{l}$	$\frac{1}{2} m \bar{i}$	$\frac{1}{6} m \bar{k} (1 + \alpha)$	$\frac{1}{6} m [\bar{l}(1 + \beta) + \bar{k}(1 + \alpha)]$	$\frac{1}{3} m \bar{m}$	$\frac{1}{3} m \bar{m} (1 + \alpha \beta)$	$\frac{1}{20} m \bar{k} (1 + \alpha) \cdot (\frac{7}{3} - \alpha^2)$
	$\frac{2}{3} m \bar{i}$	$\frac{1}{3} m \bar{k}$	$\frac{1}{3} m (\bar{l} + \bar{k})$	$\frac{1}{3} m \bar{m} (1 + \alpha \beta)$	$\frac{8}{15} m \bar{m}$	$\frac{1}{5} m \bar{k}$
k	$\frac{1}{4} k \bar{i}$	$\frac{2}{15} k \bar{k}$	$\frac{1}{60} k (7\bar{l} + 8\bar{k})$	$\frac{1}{20} k \bar{m} (1 + \alpha) \cdot (\frac{7}{3} - \alpha^2)$	$\frac{1}{5} k \bar{m}$	$\frac{8}{105} k \bar{k}$
i	$\frac{1}{4} i \bar{i}$	$\frac{2}{60} i \bar{k}$	$\frac{1}{60} i (8\bar{l} + 7\bar{k})$	$\frac{1}{20} i \bar{m} (1 + \beta) \cdot (\frac{7}{3} - \beta^2)$	$\frac{1}{5} i \bar{m}$	$\frac{31}{420} i \bar{k}$



ЗАДАЧА

Одредити силу P ако је вертикално померање тачке "E" 3mm.

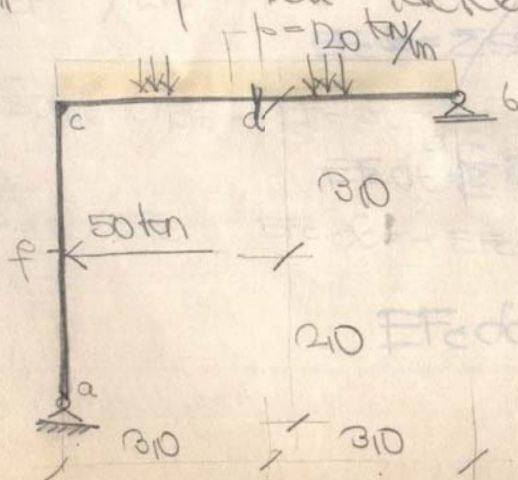


задатак

За дати носач и оптерећење одредити:

1.
 - a) померање тачке f у правцу тачке b
 - b) обртање пресека d. тангенти
- утицај транс. сила на деф.

2. При задатој промени температуре $t_0 = -5^\circ\text{C}$, $t_1 = 25^\circ\text{C}$ дуж стуба "c-d-b" израчунати пом. тачке "d" у правцу тачке "a"



СТАП	b/h [m]	J (m ⁴)
a-c	0.3/0.4	0.0016
c-b	0.3/0.5	0.003125

$E = 2 \cdot 10^7 \text{ t/m}^2$

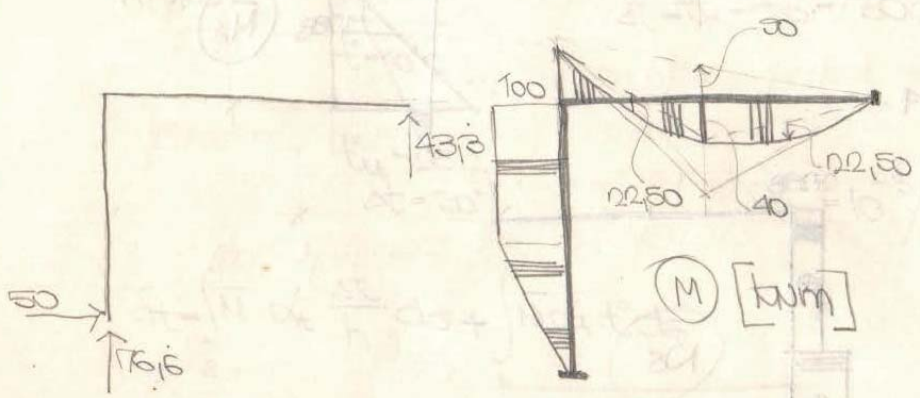
$F(m^2)$
0,12
0,15

$$F_c = 0,12 m^2$$

$$J_c = 0,0016 m^4$$

$$\frac{J_c}{F_c} = 0,0133 m^2$$

$$EJ_c = 2 \cdot 10^7 \cdot 0,0016 = 32000 \text{ kNm}^2$$



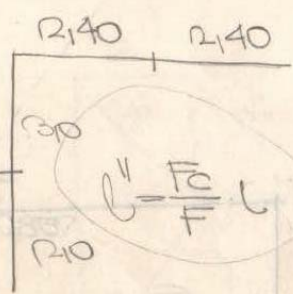
$$(M) [kNm]$$



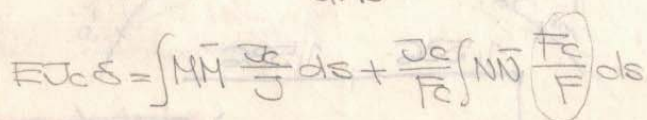
$$(N) [kN]$$

$$1,536 \quad 1,536$$

$$l' = \frac{J_c}{J} l$$

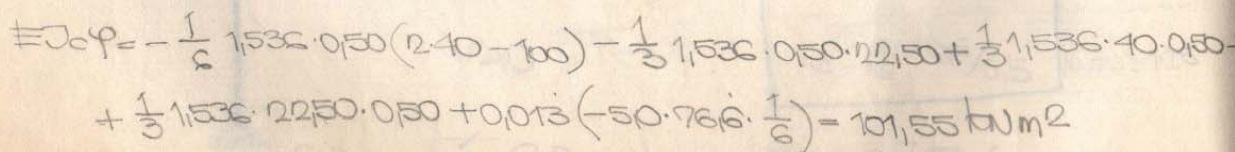


$$l'' = \frac{F_c}{F} l$$



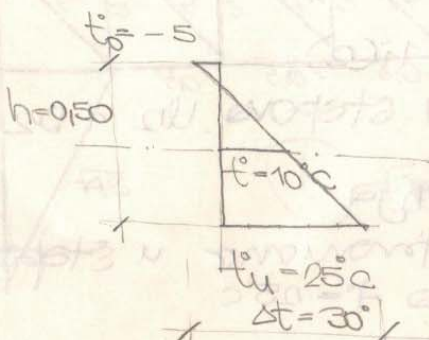
$$EJ \cdot \delta = - 676,35 \frac{\text{kg}}{\text{m}^3}$$

5



$\varphi = 0,0032 \text{ rad.}$

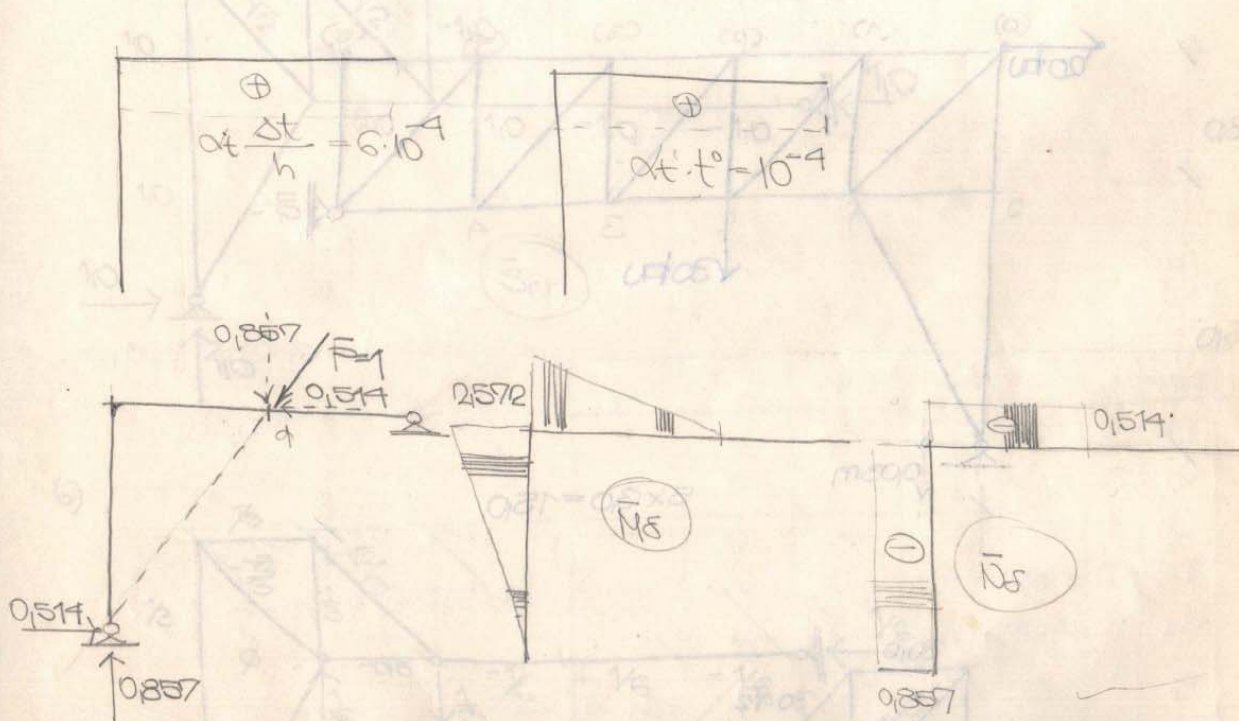
(2.)



$t^\circ = \frac{t_u + t_p}{2} = 10^\circ\text{C}$
 $\Delta t = t_u - t_p = 30^\circ\text{C}$

$\alpha_t = 10^{-5}/^\circ\text{C} - \text{za beton}$

$$\delta_t = \int_s \bar{N} \cdot \alpha_t \frac{\Delta t}{h} ds + \int_s \bar{N} \alpha_t t^\circ ds$$

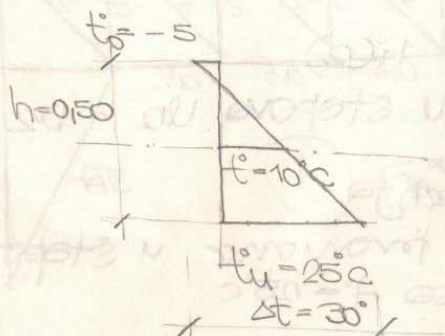


$$\int_s \bar{N} \alpha_t \frac{\Delta t}{h} ds = \left(-\frac{1}{2} \cdot 3 \cdot 0,2572 \cdot 6 \cdot 10^{-4} - 3 \cdot 0,1514 \cdot 10^{-4} \right) \cdot E \cdot \delta_t$$

$$\delta_t = -2,161 \cdot 10^{-3} \text{ m} \Rightarrow \delta = -2,16 \text{ mm}$$

$$\varphi = 0,0032 \text{ rad.}$$

(2.)

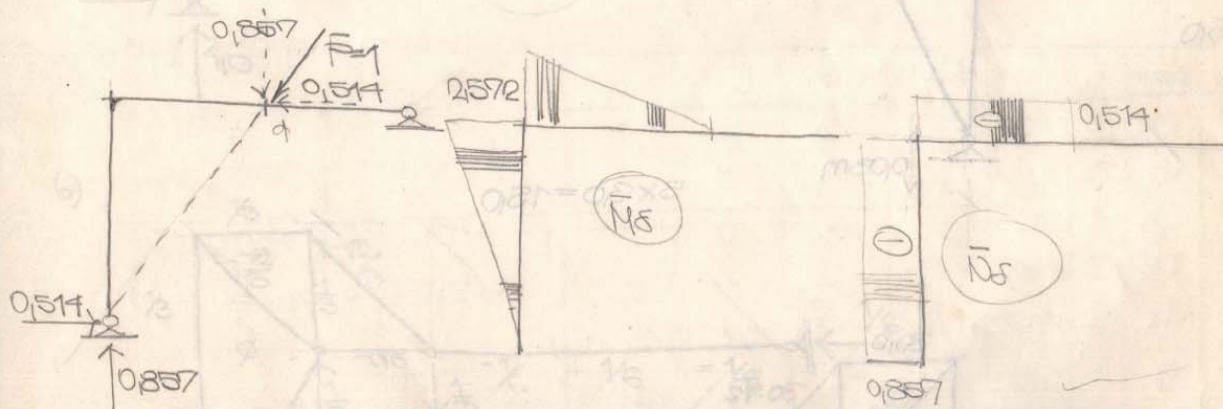
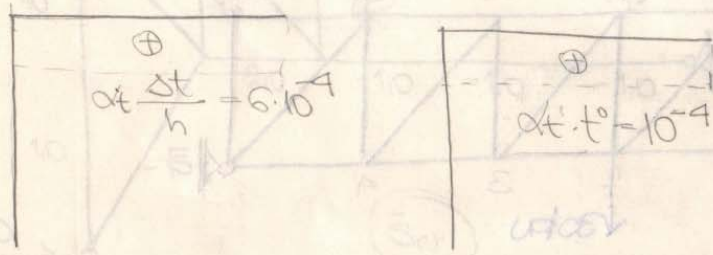


$$t^\circ = \frac{t_1 + t_0}{2} = 10^\circ$$

$$\Delta t = t_1 - t_0 = 30^\circ$$

$$\alpha_t = 10^{-5}/^\circ\text{C} - \text{za beton}$$

$$\delta_t = \int_s \bar{N} \cdot \alpha_t \frac{\Delta t}{h} ds + \int_s \bar{N} \alpha_t t^\circ ds$$



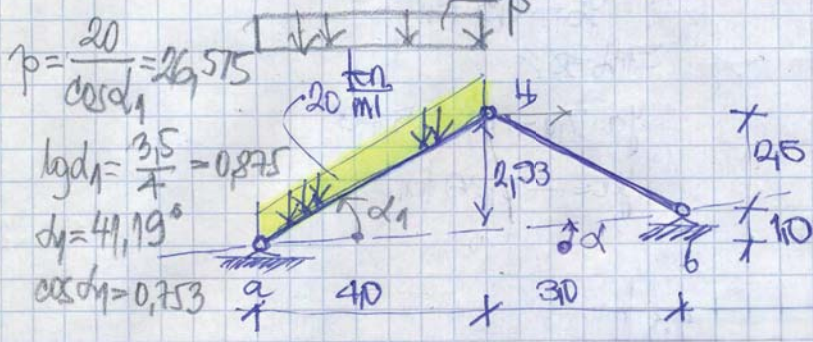
$$EJ_c \delta_t = \left(-\frac{1}{2} \cdot 3.0 \cdot 2572 \cdot 6 \cdot 10^{-4} - 3 \cdot 0.514 \cdot 10^{-4} \right) \cdot EJ_c$$

$$\delta_t = -2,181 \cdot 10^{-3} \text{ m} \rightarrow \delta = -2,16 \text{ mm}$$

3.3.3.3.3.3

ЗА НОСАМ НА СВИЦИ УОБЕЗ ЗАДАТОГ ОПТЕРЕЋЕЊА ОДРЕДИТИ:

- 1° - ПРОВОДНИ УГЛА χ "II"
- 2° - ХОРИЗОНТАЛНО ПОМИЉЕЊЕ ЧЕОРА "II" УОБЕЗ ВЕРТИКАЛНОГ ПОМИЉЕЊА ОСИОНИЦА "a" ЗА ОПГП

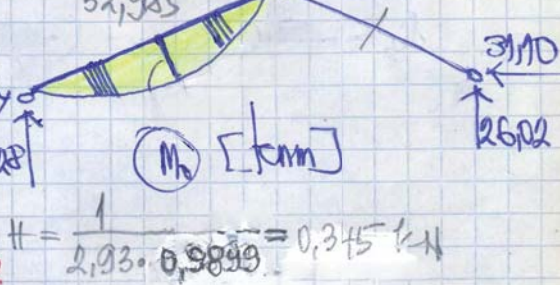
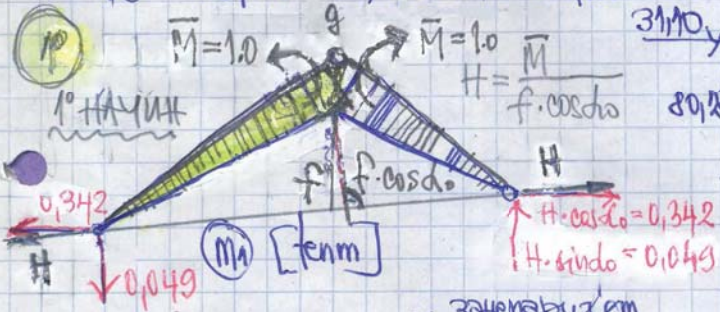


$E = 2 \cdot 10^7 \text{ kNm}^2$
 $J = 0.0072 \text{ m}^4$
 $\tan \alpha_1 = \frac{3.5}{4} = 0.875$
 $\alpha_1 = 41.19^\circ$
 $\cos \alpha_1 = 0.753$
 $\tan \alpha = \frac{1}{7} = 0.14285 = 0.14$
 $\alpha = 8.13^\circ$

$\sum M_a = 0 \Rightarrow y_b \cdot 70 - 106.30 \cdot 20 = 0 \Rightarrow y_b = 30.37 \text{ tcn}$
 $\sum M_b = 0 \Rightarrow -y_a \cdot 70 + 106.30 \cdot 50 = 0 \Rightarrow y_a = 75.93 \text{ tcn}$
 $H_a = \frac{1}{2.33} (75.93 \cdot 40 - 106.30 \cdot 20) \Rightarrow H_a = 31.10 \text{ tcn}$
 $H_b = \frac{1}{2.33} (30.37 \cdot 30) \Rightarrow H_b = 31.10 \text{ tcn}$

$V_a = 75.93 + 31.10 \cdot 0.14 = 80.28 \text{ tcn}$
 $V_b = 30.37 - 31.10 \cdot 0.14 = 26.92 \text{ tcn}$

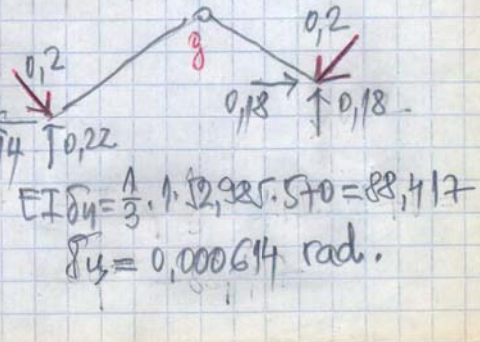
$M = 80.28 \cdot 2 - 31.10 \cdot 5.35 - 26.575 \cdot 2 \cdot 1 = 52.985$



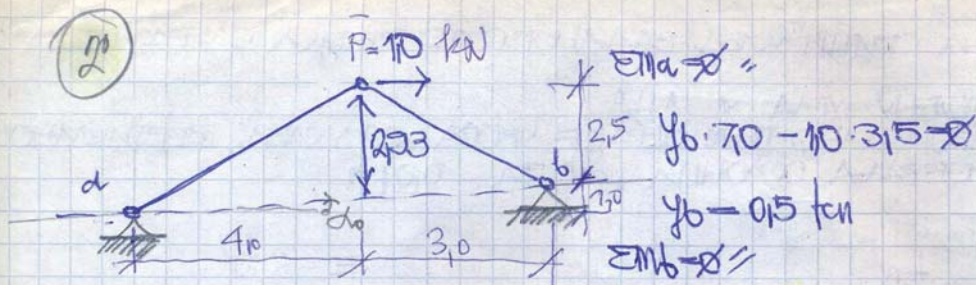
$EI \delta_y = \int \frac{M M_0}{EI} ds + \int \frac{N N_0}{EI} ds + \int \frac{T T_0}{EI} ds$
 $EI \delta_y = -\frac{1}{3} \cdot 1 \cdot 52.985 \cdot 5 + 0 = 88.417$

$\delta_y = \frac{88.417}{2 \cdot 10^7 \cdot 0.0072}$
 $\delta_y = 0.000614 \text{ rad.}$

→ ПРОВЕРА СА 2° НАМНОМ.



2



$$H_b = \frac{1}{2.53} (0.15 \cdot 3.0) = 0.151 \text{ kN}$$

$$H_a = \frac{1}{2.53} (-0.36 \cdot 4.0) = -0.49 \text{ kN}$$

$$V_a = -0.36 + (-0.49) \cdot 0.14 = -0.43 \text{ kN}$$

$$V_b = 0.15 - 0.151 \cdot 0.14 = 0.143$$

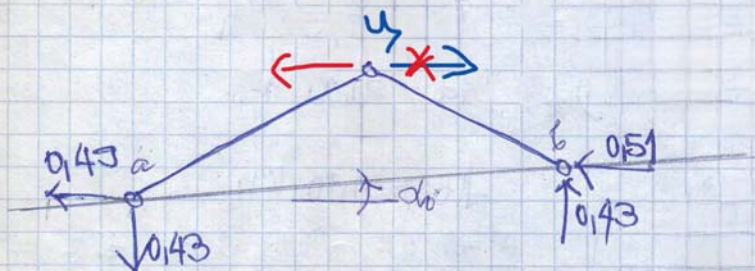
$$\sum M_a = 0$$

$$y_b \cdot 7.0 - 10 \cdot 3.5 = 0$$

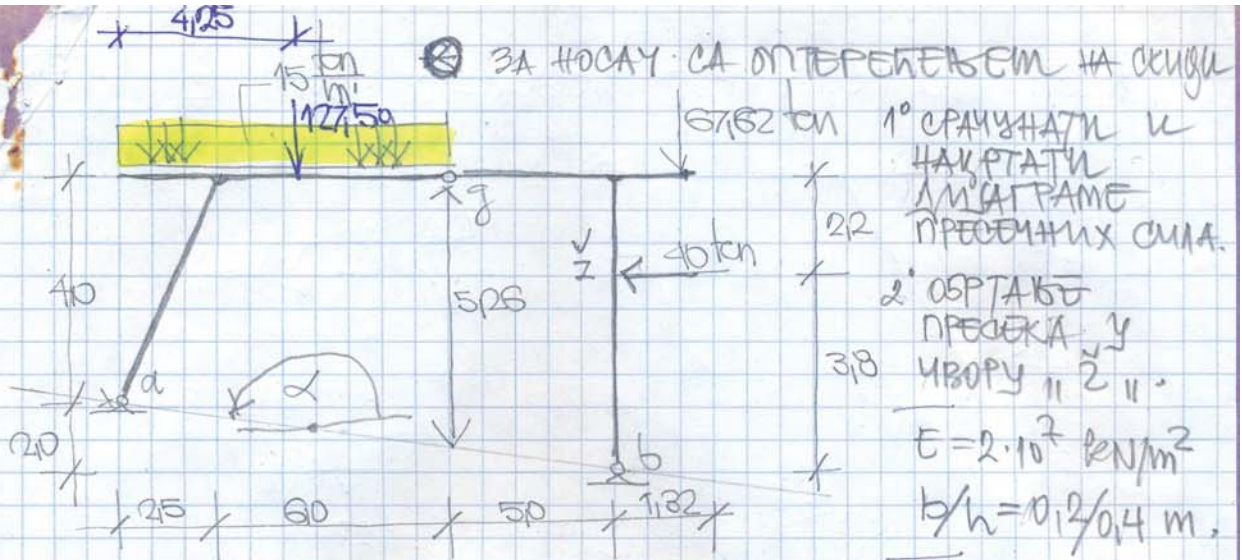
$$\sum M_b = 0$$

$$y_a \cdot 7.0 + 10 \cdot 2.5 = 0$$

$$y_a = -0.36 \text{ kN}$$



$$\delta_y = -\sum \bar{C} \cdot c_i = -(0.43) \cdot 0.01 = -0.0043 \text{ m} = -4.3 \text{ mm}$$



$$\sum M_a = 0: y_b \cdot 13.5 + 40 \cdot 1.8 - 67.62 \cdot 14.82 - 127.5 \cdot 4.25 = 0$$

$$y_b = \frac{-40 \cdot 1.8 + 67.62 \cdot 14.82 + 127.5 \cdot 4.25}{13.5} \quad y_b = 108.71 \text{ kN}$$

$$\sum M_b = 0:$$

$$-y_a \cdot 13.5 + 127.5 \cdot 5.25 + 40 \cdot 3.8 - 67.62 \cdot 1.32$$

$$y_a = \frac{127.5 \cdot 5.25 + 40 \cdot 3.8 - 67.62 \cdot 1.32}{13.5}$$

$$y_a = 32.00 \text{ kN.}$$

$$\sum M_g = 0:$$

$$-y_a \cdot 3.5 + 127.5 \cdot 4.25 + H_a \cdot 5.25 = 0$$

$$H_a = \frac{320.85 - 127.5 \cdot 4.25}{5.25} \quad H_a = 45.65 \text{ kN}$$

$$\sum M_z = 0:$$

$$H_b \cdot 5.25 - y_b \cdot 5.0 + 40 \cdot 2.2 + 67.62 \cdot 6.32 = 0$$

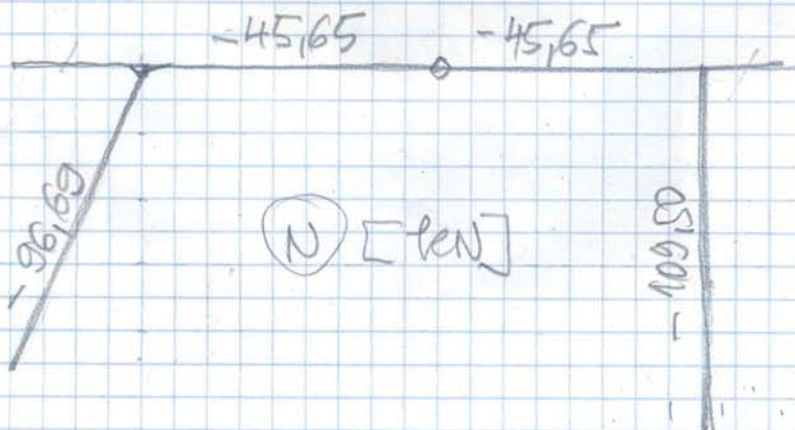
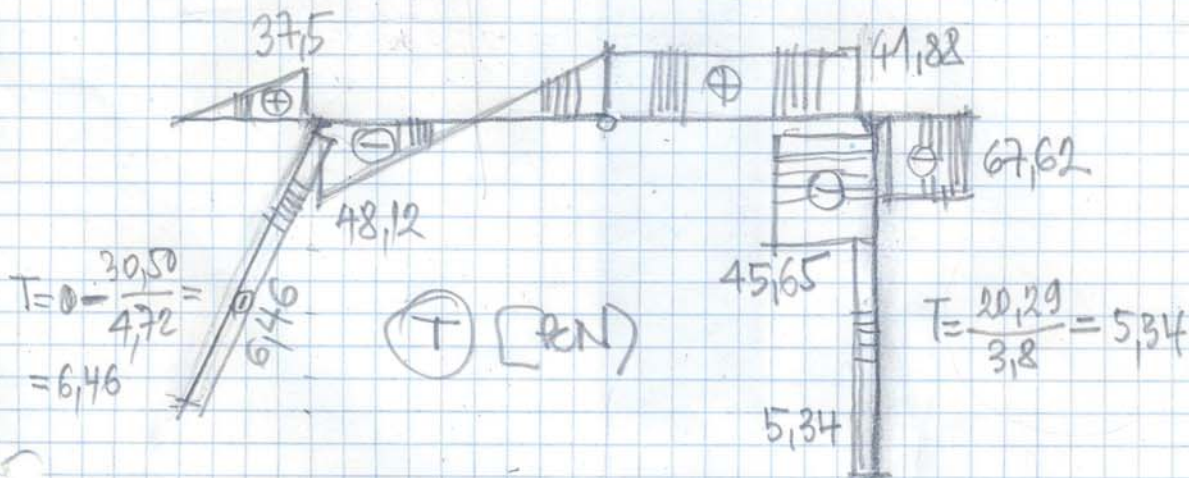
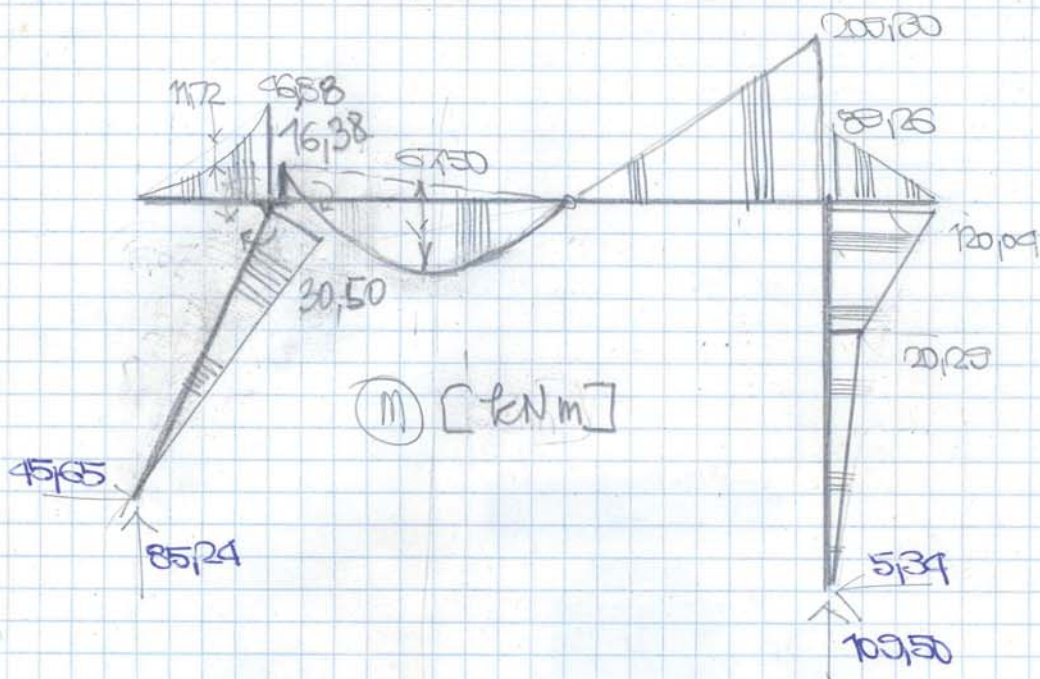
$$H_b = \frac{108.71 \cdot 5.0 - 40 \cdot 2.2 - 67.62 \cdot 6.32}{5.25} \quad H_b = 5.34 \text{ kN}$$

$$\frac{H_b}{y_b} = -0.148$$

$$A = 32.00 + 45.65(-0.148) = 25.24 \text{ kN}$$

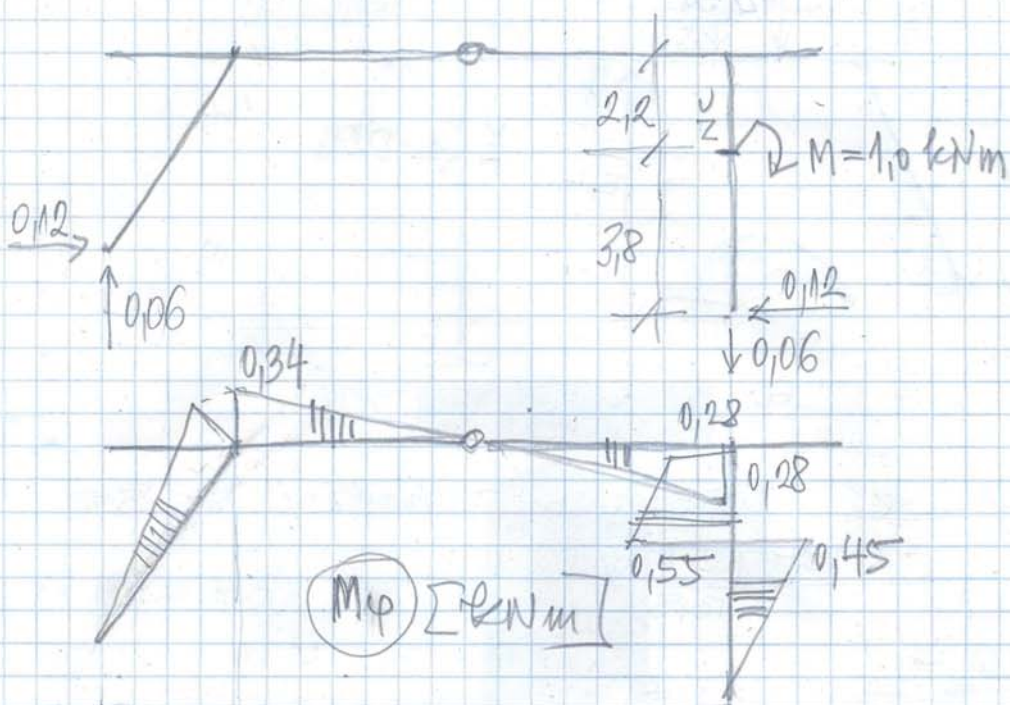
$$B = 108.71 + 5.34 \cdot 0.148 = 109.50 \text{ kN.}$$

А.А.А.



Simbol

* ОБРАТНО ПОПРЕЧНОТ ПРЕСЯКА У ЧВОРУ "Z"



$$EI\delta\varphi = \int_s M \cdot M_\varphi ds =$$

$$\begin{aligned}
 &= -\frac{1}{3} 30.51 \cdot 0.34 \cdot 4.72 + \\
 &+ \frac{1}{3} 0.34 \cdot 16.38 \cdot 6 - \frac{1}{3} 0.34 \cdot 67.5 \cdot 6 - \\
 &- \frac{1}{3} 209.30 \cdot 0.28 \cdot 5 - \\
 &- \frac{1}{6} [0.55 \cdot (2 \cdot 20.29 + 120.04) + 0.28 (20.29 + 2 \cdot 120.04)] \cdot 2.2 + \\
 &+ \frac{1}{3} 0.45 \cdot 20.29 \cdot 3.8 = -196.308
 \end{aligned}$$

$$EI\delta\varphi = -196.308$$

$$\delta\varphi = -\frac{196.308}{3 \cdot 10^7 \cdot \frac{0.2 \cdot 0.14^3}{12}} = -\frac{196.308}{3.2 \cdot 10^4} =$$

$$\delta\varphi = -0.006135 \text{ rad.}$$

ZADATAK: Za rešetku na slici izračunati

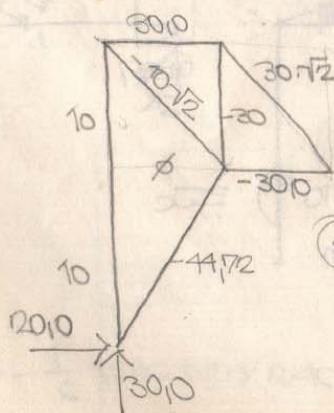
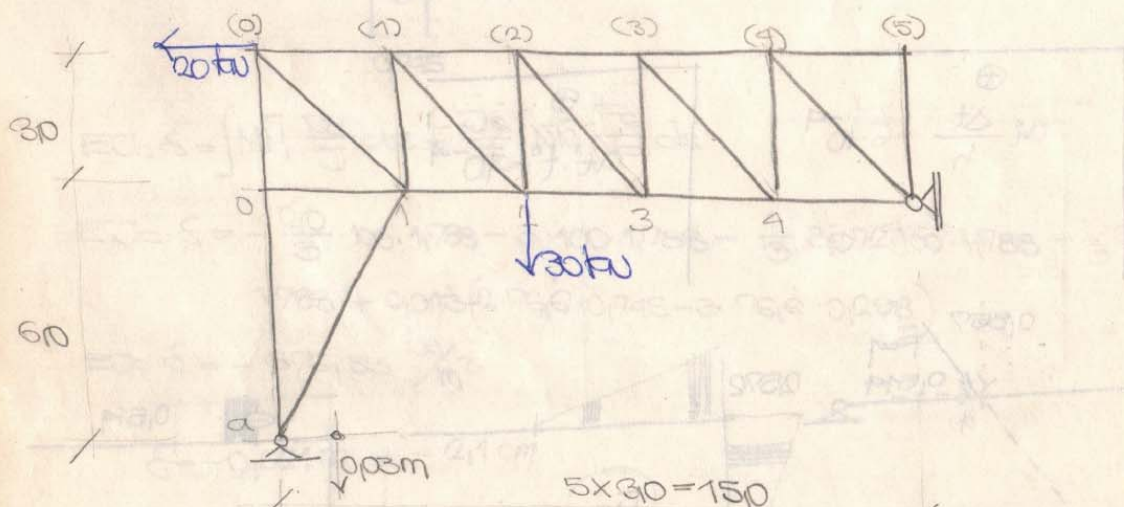
- vertikalno pom. čvora (2)
- obrtanje stepa U_2
- razmicanje čvorova 1 i (2)
- promena ugla između stepova U_2 i D_2

usled

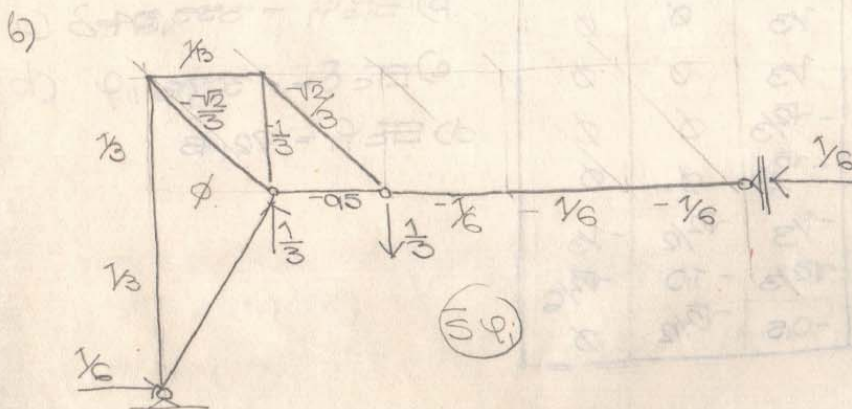
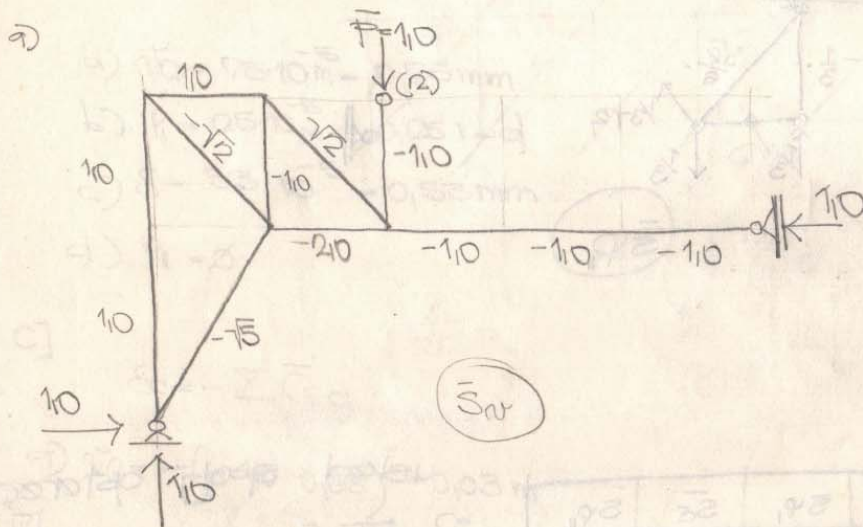
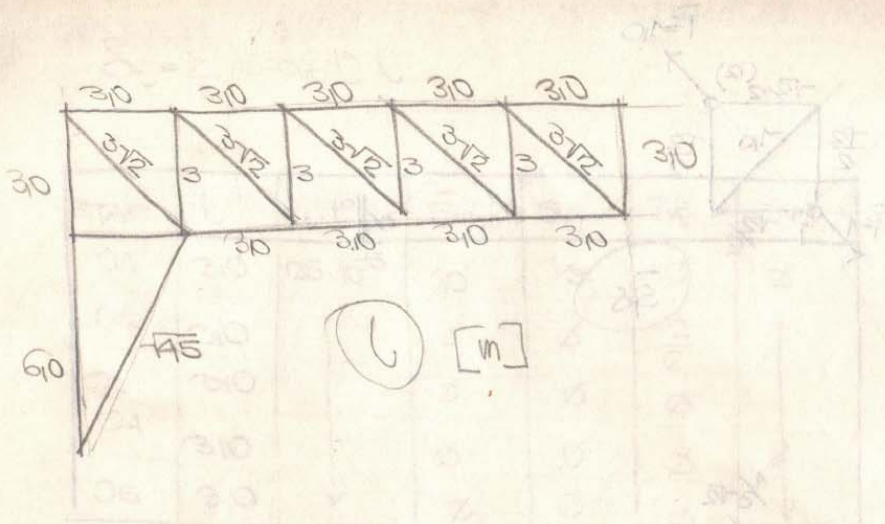
A - datog opterećenja

B - temperaturne promene u stepovima
gornjeg pojasa $t = 25^\circ\text{C}$

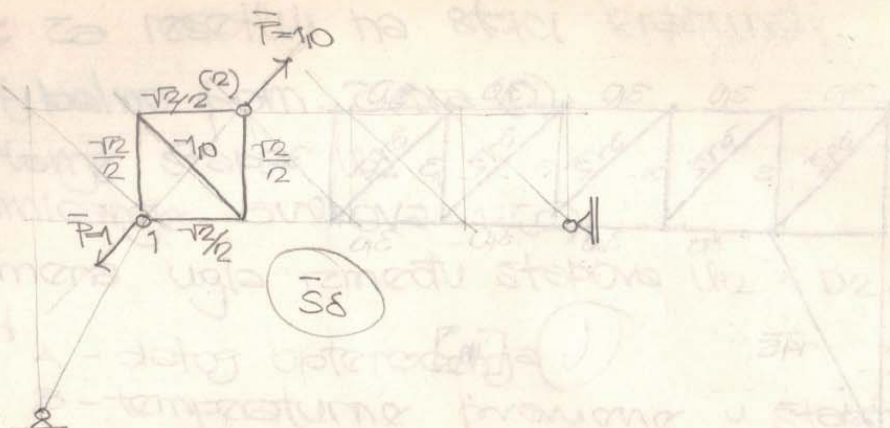
C - pom. oslonca A za $0,03\text{m}$



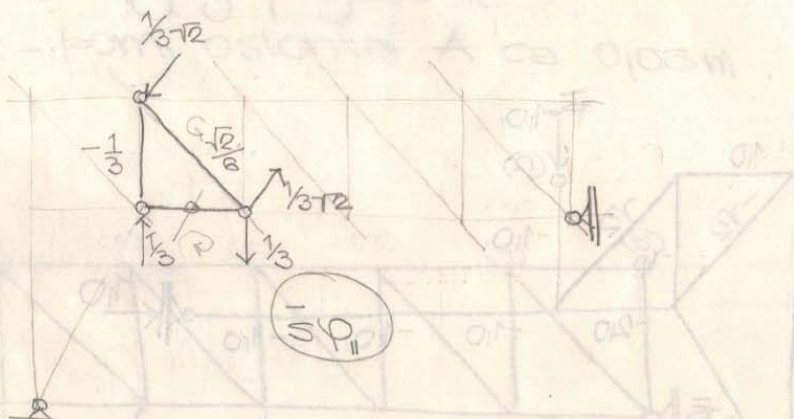
(5) [kN]



c)



d)



A]

STAP	L	S	\bar{S}_v	\bar{S}_{φ_1}	\bar{S}_δ	S_{φ_1}
V2	6	10	1	$1/3$	\emptyset	\emptyset
V0	3	10	1	$1/3$	\emptyset	\emptyset
O1	3	30	1	$1/3$	\emptyset	\emptyset
D1	$3\sqrt{2}$	$-10\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}/3$	\emptyset	\emptyset
U1	3	$-41\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}/6$	\emptyset	\emptyset
V1	3	-30	-1	$-1/3$	$\sqrt{2}/2$	$-1/3$
D2	$3\sqrt{2}$	$-30\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}/3$	-1.0	$-\sqrt{2}/6$
U2	3	-30	-2	-0.5	$-\sqrt{2}/2$	\emptyset

used složi: opterećenje

a) $EF\bar{S}_\delta = 1430,20$

b) $EF\bar{S}_{\varphi_1} = 259,04$

c) $EF\bar{S}_\delta = -307,3$

d) $EF\bar{S}_{\varphi_1} = 72,48$

B]

$$\bar{S}_c = \sum \bar{S}_i \cdot \Delta t_i \cdot l_i$$

STAP	l	Δt_i	\bar{S}_i	S_{φ_i}	\bar{S}_S	$S_{\varphi_{II}}$
01	310	$25 \cdot 10^{-5}$	10	$1/3$	\emptyset	\emptyset
02	310		\emptyset	\emptyset	$\frac{\sqrt{2}}{2}$	
03	310		\emptyset	\emptyset	\emptyset	
04	310		\emptyset	\emptyset	\emptyset	
05	310	\checkmark	\emptyset	\emptyset	\emptyset	\checkmark

$$a) \bar{v}_2 = 75 \cdot 10^{-5} = 0,75 \text{ mm}$$

$$b) \varphi_1 = 25 \cdot 10^{-5} = 0,25 \text{ rad}$$

$$c) \delta = 53 \cdot 10^{-5} = 0,53 \text{ mm}$$

$$d) \varphi_{II} = \emptyset$$

C]

$$\bar{S}_c = - \sum \bar{Q}_i a_i$$

$$a) \bar{v}_2 = -(-10 \cdot 0,03) = 0,03 \text{ m}$$

$$b) \varphi_1 = \emptyset$$

$$c) \delta = \emptyset$$

$$d) \varphi_{II} = \emptyset$$