GOVERNMENT POLYTECHNIC, DHULE

MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION, MUMBAI

A report of micro project on

"APPLICATION OF DIFFERENTIAL EQUATION IN POPULATION GROWTH"

Prepared by

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For partial fulfillment of the course 'Applied Mathematics' of Second Semester Diploma program in ...computer......Engineering during the academic year 2020 2021

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MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION, MUMBAI.

Evaluation Sheet for the Micro Project

Academic Year: 2020- 2021 **Name of Faculty:** ....
Course- Applied Mathematics **Course Code:** 22224 **Semester:** Second

Title of the Project: Application of DIFFERENTIAL EQUATION IN POPULATION GROWTH.

COs addressed by the Micro Project:

a) To learn application of differential equation in Population Growth.

Major Learning Outcomes achieved by students by doing the Project:

a) Differential equation and its application in population growth.

b) Exponential growth and Logistic growth.

c) Solving real life problem based on it.

Comments / Suggestions about team work / leadership / inter-personal communication (if any) ....

Roll No	Name of Student	Marks out of 6 for performance in group activity (D5 Col. 8)	Marks out of 4 for performance in oral/ presentation (D5 Col. 9)	Total out of 10
153	Salunke Yash Vijay			
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"APPLICATION OF DIFFERENTIAL EQUATION IN POPULATION GROWTH"

A **differential equation** is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology. In pure mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions—the set of functions that satisfy the equation. Only the simplest differential equations are solvable by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form.

Differential equations can be divided into several types. Apart from describing the properties of the equation itself, these classes of differential equations can help inform the choice of approach to a solution. Commonly used distinctions include whether the equation is: Ordinary/Partial, Linear/Non-linear, and Homogeneous/Inhomogeneous. This list is far from exhaustive; there are many other properties and subclasses of differential equations which can be very useful in specific contexts.

The study of differential equations is a wide field in pure and applied mathematics, physics, and engineering. All of these disciplines are concerned with the properties of differential equations of various types. Pure mathematics focuses on the existence and uniqueness of solutions, while applied mathematics emphasizes the rigorous justification of the methods for approximating solutions. Differential equations play an important role in modelling virtually every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods. Many fundamental laws of physics and chemistry can be formulated as differential equations. In biology and economics, differential equations are used to model the behaviour of complex systems.

Differential equations are essential in the study of pure and Applied Mathematics. We come across various situation of daily life in which one variable is depend upon one or more independent variables.

Let the functional relationship be $y = f(x)$

If the relation has one dependent and one independent variable, we obtain ordinary **First order and first degree differential equations.** In above relation x is called independent

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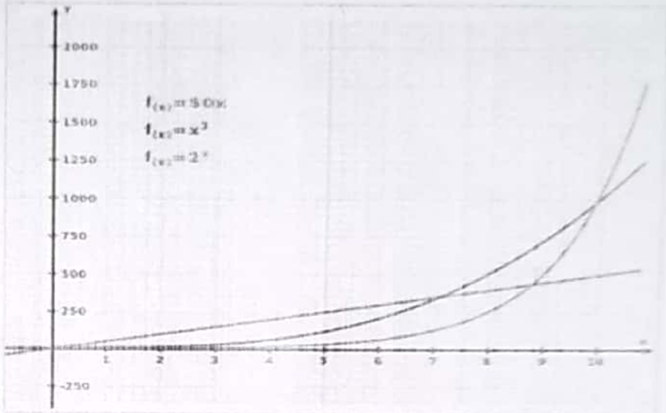
variable and y is called dependent variable. The derivative of y w.r.t. x is $\frac{dy}{dx}$. Here $\frac{dy}{dx}$ is called first order and first degree differential coefficient or derivative.

Exponential Growth:

Exponential growth is exhibited when the rate of change—the change per instant or unit of time—of the value of a mathematical function is proportional to the function's current value, resulting in its value at any time being an exponential function of time, i.e., a function in which the time value is the exponent. Exponential decay occurs in the same way when the growth rate is negative. In the case of a discrete domain of definition with equal intervals, it is also called **geometric growth** or **geometric decay,** the function values forming a geometric progression. In either exponential growth or exponential decay, the ratio of the rate of change of the quantity to its current size remains constant over time.



The graph illustrates how exponential growth (green) surpasses both linear (red) and cubic (blue) growth.

Exponential growth

Cubic growth

Linear growth

The formula for exponential growth of a variable x at the growth rate r, as time t goes on in discrete intervals (that is, at integer times 0, 1, 2, 3, ...), is

$$x_t = x_0(1+r)^t$$

where x_0 is the value of x at time 0. This formula is transparent when the exponents are converted to multiplication.

Single Species Population Models:

We need only one population variable in this case there are mainly two type of single species population model.

1) Exponential Growth: A differential equation of the separable class.

$$\frac{dP}{dt} = kP \quad \text{with} \quad P(0) = P_0$$

We can integrate this one to obtain

$$\int \frac{dP}{kP} = \int dt \implies P(t) = Ae^{kt}$$

Where A is value of P at t=0

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2) **Logistic model:** Exponential growth is not quite accurate since the environmental support system for a given species is likely not infinite.

$\frac{dP}{dt} = kP(1 - \frac{P}{M})$ here M (constant) is carrying capacity.

P(t) is size of population at time t.

k(constant) : constant proportionality or growth parameter, k large population grow quickly and for small k population grow slowly(k>0). M

M is size of population that environment can sustain.

After solving differential equation

$P(t) = \frac{M}{1+Ae^{-kt}}$ where is $A = \frac{M-P(0)}{P(0)}$, P(t) at t = 0

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Ex. 1. A population of bacteria grows logistically. Suppose the initial population is 3 mg of bacteria, the carrying capacity is 100 mg, and the growth parameter is 0.2 per hour. Then find

i) Logistic equation for population growth
ii) When will population will reach 20 mg.
iii) When will population will reach 200 mg.
iv) Find the value for t = 5, 10,15,20,30,40,50,60 hour and plot a graph (t and x axis and population of bacteria at y axis).

Solution:-

Initial Population (P_0) = 3mg
Carrying Capacity (m) = 100mg
Growth Parameter (k) = 0.2/hr.

$\therefore A = \frac{M - P_0}{P_0} \quad \therefore \frac{100-3}{3}$

$\boxed{\therefore A = 32.33}$

i] Logistic equation for population growth

$\therefore P(t) = \frac{M}{1+Ae^{-kt}}$

$\boxed{\therefore P(t) = \frac{100}{1+32.33e^{-0.2t}}}$

ii] Find t when P(t) = 20mg

$\therefore 20 = \frac{100}{1+32.33e^{-0.2t}}$

$\therefore 20 \times (1+32.33e^{-0.2t}) = 100$

$\therefore 20 + 646.6e^{-0.2t} = 100$

$\therefore 646.6e^{-0.2t} = 100 - 20 = 80$

$\therefore e^{-0.2t} = \frac{80}{646.6}$

$\boxed{\therefore t = 10}$

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iii] Find t when $P(t) = 200mg$

$$200 = \frac{100}{1 + 32.33e^{-0.2t}}$$

$$= 200 \times (1 + 32.33e^{-0.2t}) = 100$$

$$1 + 32.33e^{-0.2t} = 0.5$$

$$32.33e^{-0.2t} = 0.5^{-1}$$

$$e^{-0.2t} = 0.5$$

$$\boxed{e^{-0.2t} = -0.0154}$$

iv] For $t = 5$

$$P[t] = \frac{100}{1 + 32.33e^{-0.2t \times 5}}$$

$\therefore P(5) = 7.15mg$

For $t = 10$

$P(10) = 18.6mg$ $\boxed{P(10) = 18.6mg}$

For $t = 15$

$P(15) = 38.91mg$ $\boxed{P(15) = 38.91mg}$

For $t = 20$

$P(20) = 62.80mg$ $\boxed{P(20) = 62.80mg}$

For $t = 30$

$P(30) = 92.58mg$ $\boxed{P(30) = 92.58mg}$

For $t = 40$

$P(40) = 98.92mg$ $\boxed{P(40) = 98.92mg}$

For $t = 50$

$P(50) = 99.85mg$ $\boxed{P(50) = 99.85mg}$

For $t = 60$

$P(60) = 99.898mg$ $\boxed{P(60) = 99.898mg}$

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Ex. 2. A herd of llamas has 1000 llamas in it initially and the population is growing exponentially. At time t=4 it has 2000 llamas. Write a formula for the number of llamas at *arbitrary* time t.

→ Solution :-

$t = 0 = 1000$

$t = 4 = 2000$

C = value of f at $t = 0 = 1000$

$$k = \frac{\ln F(t_1) - \ln F(t_2)}{t_1 - t_2}$$

$$= \frac{\ln 1000 - \ln 2000}{0 - 4}$$

$$= \ln \frac{1000}{2000} - 4 = \frac{\ln \frac{1}{2}}{-4} = \frac{(\ln 2)}{4}$$

Therefore

$$\boxed{\begin{aligned} F(t) &= 1000\, e^{\frac{\ln 2}{4} t} \\ &= 1000 \cdot 2^{t/4} \end{aligned}}$$

This is desired formula of number of llamas at orbitary time t

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Ex. 3. A herd of elephants is growing exponentially. At time t=2 it has 1000 elephants in it, and at time t=4 it has 2000 elephants. Write a formula for the number of elephants at *arbitrary* time t and find population of elephants at time t = 5.

→ Solution

$P(2) = 1000$

$P(4) = 2000$

$P(2) = A \cdot e^{2k}$

$\therefore 1000 = A \cdot e^{2k}$

$\therefore 2000 = A \cdot e^{4k}$

$$\frac{P(2)}{P(4)} = \frac{1000}{2000} = \frac{A \cdot e^{2k}}{A \cdot e^{4k}} = e^{-2k}$$

$\therefore 0.5 = e^{-2k}$

$\boxed{\therefore k = 0.34465}$

Now, Find A

$P(2) = A \cdot e^{kt}$

$\therefore 1000 = A \cdot e^{0.9465 \times 2}$

$\boxed{\therefore A = 500}$

Now Find P(5)

$P(5) = A \cdot e^{5k}$

$= 500 \times e^{5 \times 0.3465}$

$\therefore P(5) = 2827$

$\boxed{\therefore t = 5 = 2827}$

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