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# A New Solution for the Inverse Kinematics of Concentric-Tube Robots

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***Abstract*—The continuum tubular robots (CTRs) have shown its superiority in minimally invasive medical procedures. In general, concentric tubes are consist of super-elastic nickel-titanium alloy, they can show different shapes when they extend and rotate with each other. Due to the small diameter and high degree of dexterity, CTR can perform well in complex and limited environments. However, the calculation of the inverse kinematics of concentric tube robots is a difficult problem all the time, especially when the CTRs tubes deform constantly over time. In this paper, we present a new method based on data acquisition, which is proved to be a simple and effective method through simulation and experimental verification. For any single-arm, two-tube concentric tube robots, the corresponding list of the input parameters of CTRs driving part and the tip position of CTR is first collected. And then the inverse kinematics of concentric tube robot can be solved by the algorithm in this paper. This technique can not only deal with the deformation of CTRs, but also offer a different perspective to solve the inverse kinematics problem of concentric tube robots.**

***Index Terms*—concentric-tube robot, minimally invasive surgeries, inverse kinematics.**

## I. Introduction

Comparatively speaking, concentric tube robots are a new class of miniature continuum robots[1] for minimally invasive surgery. This sort of robots is composed of concentric, telescoping, precurved superelastic nickel-titanium alloy tubes that can extend and rotate with respect to one another. CTRs derive their bending actuation from elastic interaction of curved tubes instead of tendon wires or other external function, which makes them sufficiently flexible and shapable to access confined anatomical locations. This superiority, as well as its small diameter[2],[3], makes them an ideal minimally invasive surgery robot.

There are three sorts of instruments used in minimally invasive procedures[4],[5]. Concentric tube robots hold the best properties of all three category of instruments[6]. They can control their lateral motion and force application along their arc in comparison to needles and catheters. At the same time, the hollow tube can also house some wires of tip-mounted tools. In recent years, CTR has been increasingly used into extensive clinical applications, such as otolaryngology[7], neurosurgery[8], intracardiac procedures[9],[10], thermal ablation of cancer[11], retinal vein cannulation[12], and cardiac surgery and stenting surgery[13],[14].

However, compared with traditional robots whose links are rigid, the kinematics of continuous concentric tube robot is a challenge, especially for the inverse kinematics of CTR with more than two tubes. The reason is the computation of concentric-tube robot curvature is a three dimensional beam bending problem. Normally, this problem can be regarded as a two-point boundary-value problem.

To solve this kinematic problem, researchers have proposed a constant curvature kinematic frameworks, the most well-known and widely used kinematic model for continuum robots, which means continuum robot geometry with a finite number of mutually tangent curved segments each having a constant curvature along its length. And a variety of calculating ways of the homogeneous transformation along a constant-curvature arc length have been derived in prior papers, such as Denavit-Hartenberg (D-H) parameter tables[15],[16], exponential coordinates[17], integral representation[18], Frenet-Serret frames[15].

Based on this prior assumption, as well as the application of the theory of special Cosserat rods, some modeling techniques have access to the balance between the model solving complexity, computational expense, and its accuracy. To date, the papers[19],[20] proposed a popular torsionally rigid model, which was first been introduced in [20]. In fact, torsional twist has a really significant effect on the accuracy of kinematics. Then, the model including torsional twist has also been proposed in [21]. But, all models do not take the effects of shear and axial extension of tubes for the purpose of simplifying the model formulation and convenient solution.

Up to now, no one solution to the kinematic equations of continuum robots has emerged as generally applicable method. The kinematics models of CTR consisting of two tubes can be solved using elliptic integrals[6],[22]. But, it is pretty difficult to find the analytical solution of robots with more than two tubes, or whose tubes are variable curvature.

Although there are some feasible and popular methods in terms of kinematics modeling and kinematics equation

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solving of concentric tube robot, some problems still exist in this process. These neglected problems will reduce the accuracy of motion control of concentric tube robots. The relevant questions are expressed as follows:

1) When the kinematics model is established, the concentric tube has been seen as segmented constant curvature, which is difficult to realize in the actual tube processing;
2) While the most common concentric tube material is now superelastic nickel-titanium alloy, the tube will undergo irreversible plastic deformation over time;
3) At present, all models almost do not take into account the shear and axial deformation between tubes;
4) The inverse kinematics of concentric tube robots consisting of more than two tubes is complex and there is no universally applicable solution to kinematic equation.

In consideration of the current research status and existing problems of continuous robot kinematics, a new method of solving inverse kinematics of robots is proposed in this paper. This is an approach based on data table, so it is called data-driven approach (DDA) in this article. The basic idea of this method is the information table including the end point coordinates information of CTR and its corresponding tube parameters is obtained by data acquisition. Then, based on this table and solution algorithm presented in this paper the inverse kinematics of CTR will be solved easily. This method effectively deal with these problems mentioned above, so as to accurately and reliably control the motion of robot.

This paper is organized as follows. In Section II, the problem statement of data-driven approach including the generation and usage of DDA table will be introduced and the proposed DDA algorithm is described in detail in Section III. Then, simulation and experimental results based on this method will be presented in Section IV. Final,this paper is concluded in Section V.

## II. The Problem Statement of Date-Driven Approach

Theoretically, inverse kinematics makes use of the kinematics equations to determine the joint parameters that provide a desired position for the robot's end-effectors. In other words, the kinematics is a bridge between the input parameters and the end position of robots, as visualized in Fig.1. In this figure, the letter A represents forward kinematics, which is a mapping from actuator space to task space. On the contrary, the mapping from task space to actuator space is inverse kinematics, indicated by the latter B. The kinematics of CTR can also be expressed as,

$$X_i = F(P_i) \tag{1}$$

where $F$ is the mapping between task space $X_i$ and the actuator space $P_i$.

Based on the understanding of the essence of forward and inverse kinematics of concentric tube robots, an alternative and effective inverse kinematics solution method is proposed, which is called DDA in this paper. Through the introduction of this method, the spatial mapping relationship of kinematics showed in Fig.1 can be represented by Fig.2 The design process of this method consists of two parts. The first part is how to generate the DDA table, and the second is how to use the DDA table, indicated by letter C and letter D respectively in Fig.2. This method will be illustrated by an example of concentric tube robot consisting of two tubes, one of which is straight, and the other is part straight and part curved. The details will be described below.

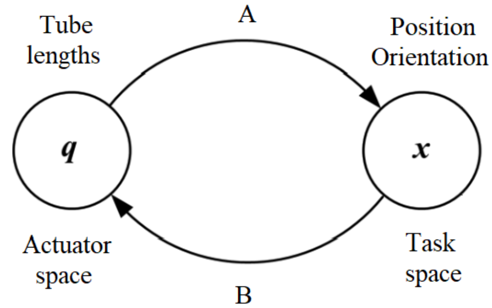


Fig. 1. The relationship between robot motion input parameters and robot end position. The letter A represents forward kinematics, which is a mapping from actuator space to task space. On the contrary, the mapping from task space to actuator space is inverse kinematics, indicated by the latter B.

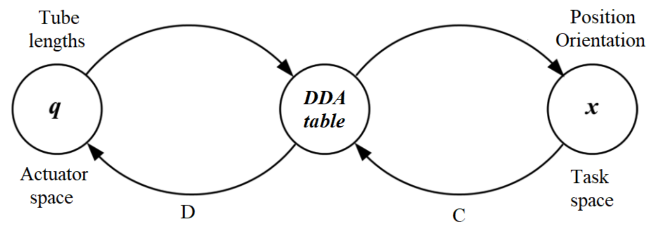


Fig. 2. The relationship between robot motion input parameters, robot end position and the DDA table. While the letter C represents the generation of DDA table as a result of forward kinematics of CTR, the letter D indicates the solving process from this table to input parameters.

### A. The Generation of DDA Table

The first part is the generation of DDA table. The simplest and more direct way of thinking is to traverse all points in the feasible space of concentric tube robots, generated a huge table made up of a lot of data. Then, the value of input parameters can be obtained directly through data matching. Obviously, this mean is totally impracticable and meaningless. Therefore, this paper adopts such data acquisition method. While the length of outer tube of CTR keeps $l_1 = l_{1max}$, which is maximum elongation of outer tube $l_1$, the length $l_2$ of inner tube moves from $l_{1max}$ to $l_{2max}$ and the rotation angle between the coordinate frame $F_2(0)$ of inner tube and the world coordinate frame $W(0)$ of CTR keeps 0(The method of coordinate system establishment is shown in literature [19]). Meanwhile, the coordinates of end points of CTR and the corresponding input parameters are collected during the movement process. Finally, DDA table is obtained.

### B. The Usage of DDA Table

As mentioned above, given the target position of the robot, the input data required to reach this point is obtained by some ways, which is the process of inverse kinematics solution. Thus, this process can be expressed in equation (2), which is the embodiment of equation (1).

$$((l_{1i},\theta_{1i}),(l_{2i},\theta_{2i})) = F^{-1}(x_i,y_i,z_i) \tag{2}$$

where $(x_i,y_i,z_i)$ represents any point in the task space of CTR, $(l_{1i},\theta_{1i})$ and $(l_{2i},\theta_{2i})$ mean the axial length and rotation angle of the outer and inner tube relative to the world coordinate system respectively, when the robot is in this configuration. What we then should figure out from the DDA table are $l_{1i}$, $l_{2i}$ and $\theta_{2i}$. Because the outer tube is straight, the rotation angle $\theta_{1i}$ is not to be determined. The usage of DDA table will be discussed in three aspects below.

*1) Determining the right column from DDA table:* When the inner tube of the CTR rotates one revolution around its axis relative to the world coordinate system, the points at any arc length $s$ on the inner tube draw a circle in space, as shown in Fig.3. It is not difficult to find in this figure that the arc length of tube 2 determines the size of this circle and thus determines the radius of it. According to Fig.4, the following formulas can be given,

$$\begin{aligned} \theta &= \frac{180^o \cdot s \cdot k}{\pi} \\ \psi &= \frac{\theta}{2} \\ b &= \frac{2}{k} \cdot sin(\frac{\theta}{2}) \end{aligned} \tag{3}$$

where, $s$ is the arc length of inner tube, $k$ is the curvature of its curved part, and $b$ is the chord length corresponding to the arc length of inner tube. Combined with the above formulas, the relationship between arc length $s$ and radius $r$ of the circle can be derived,

$$r = \frac{2}{k} \cdot sin^2(\frac{90^o \cdot s \cdot k}{\pi}) \tag{4}$$

where $r$ represents the radius of this circle drawn by the end point of inner tube around its axis. It can be seen from equation (4) that the value of radius $r$ is only determined by the arc length $s$ of inner tube and its curvature $k$. For the processed concentric tube, the curvature $k$ is a fixed value. That is to say, the value of $r$ only depends on the arc length $s$. Therefore, when the robot is in the target configuration, we can calculate the only rotation radius $r_i$ of end point. Compared with the counterpart of every column of DDA table, the column number of this table closest to $r_i$ can be determined. Final, the robot input parameters $(l_{1i},l_{2i})$ in this column are obtained to prepare for subsequent data processing.

*2) Determining the length of outer and inner tube $l_1$ and $l_2$:* In the previous step, one column data from DDA table has been selected in accordance with the end point coordinate. This column include the specific input value ($l_{1i}$, $l_{2i}$) of CTR when it is in the target configuration. However, based on the knowledge of the feasible space of CTR with two tubes,

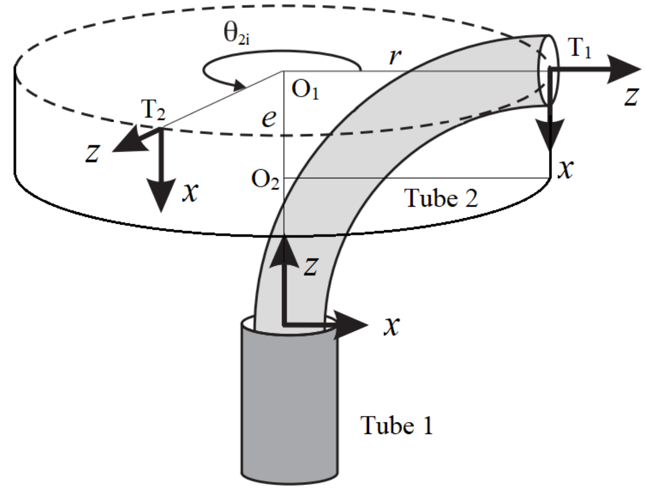


Fig. 3. The dotted line in the figure represents the circle drawn by end point in inner tube rotating around its axis.

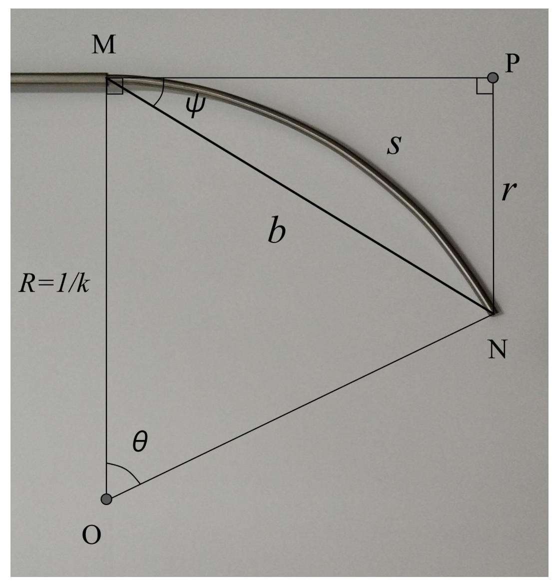


Fig. 4. Geometrical relationship diagram of rotation radius $r$ and arc length $s$.

it can be known that this set of values is not necessarily the correct answer. As mentioned earlier, the prerequisite for DDA table generation is that the length of inner tube reaches its maximum, that is, $l_1 = l_{max}$. Therefore, if the target point does not satisfy this condition, the values of $l_{1i}$ and $l_{2i}$ need to be corrected. As shown in Fig.3, it is assumed that the dotted line circle is the circle obtained by the rotation of inner tube when its length is maximum, and the solid line circle is the circle obtained by the rotation of target point. So, the correct value should be,

$$\begin{aligned} l_1 &= l_{1i} + e \\ l_2 &= l_{2i} + e \end{aligned} \tag{5}$$

where $e$ is the modified parameter, represented in Fig.3 the distance between two centers $O_1$ and $O_2$.

*3) Determining the rotation angle $\theta_2$ :* The inverse kinematics derivation process of concentric tube robot composed of two tubes is to solve the corresponding input ($l_1$, $l_2$, $\theta_2$) on the basis of the target position of the robot. The first two parts have given the value of $l_1$ and $l_2$, and now we need to figure out the rotation angle $\theta_2$ of tube 2 relative to the world coordinate system. When determining the column number in the DDA table, we use the circle information drawn by the inner tube rotating around its axis, which can also be used to determine the rotation angle. As Fig.3 shows, if $T_2$ is the target point, the angle between the vector $\vec{O_1 T_1}$ and the vector $\vec{O_1 T_2}$ needs to be determined.

$$\theta_{2i} = \arccos\left(\frac{\vec{O_1 T_1} \cdot \vec{O_1 T_2}}{\left\|\vec{O_1 T_1}\right\| \cdot \left\|\vec{O_1 T_2}\right\|}\right) \tag{6}$$

The data range of this angle right here is 0 to $\pi$. In order to directly determine the rotation direction of inner tube during experiment, the inner tube can reach the specified target point with shortest path, the positive and negative of this angle need to be determined too. If the target point is within the range of counterclockwise rotation, this angle $\theta_{2i}$ regards as positive, and vice versa. Then,

$$\theta_2 = \begin{cases} \theta_{2i}, & \alpha \geq 0 \\ -\theta_{2i}, & \alpha < 0 \end{cases} \tag{7}$$

where, $\alpha$ here is a coordinate value, which can distinguish the both side spaces. If the coordinate system is established as shown in Fig.3, it is just the $y$ value, otherwise, it is the projected value on this axis.

## III. The Algorithm of Date-Driven Approach

In section II, the main problems and solutions of DDA method have been described. The specific algorithm is now described, which is called DDA Algorithm, in Algorithm 1. The function of this algorithm is to solve the input parameters $P(l_1, l_2, \theta_2)$ of concentric tube robot with known DDA table and target position.

In the algorithm, $\mathbf{T}_{dda}$ indicates the DDA table with $n$ columns and $m$ rows, where $n$ is determined by the length increment of inner tube extended out of the outer tube in the process of data collection each time for DDA table. In theory, the higher the value of $n$, the higher the precision of inverse kinematics. And, the number of rows $m$ is 6. One row records the number of columns, two rows record the lengths of two tubes, and another three rows are used to store the end position coordinates of robot. Given the goal position $x_{goal}$, $Radius(x_{goal})$ can be used to calculate the radius $r_{goal}$ of circle corresponding to this point. The radius $r_i$ of each coordinate information in DDA table is calculated by the same way. Combined with these radius information, the right column number $N$ from DDA table is calculated using $Coulmn(r_{goal}, r_i)$. Then, the correction coefficient $e$ is determined by comparing the components on the $Z$-axis of centers $O_N$ and $O_{goal}$ corresponding to this two positions $x_N$ and $x_{goal}$. Final, the values of $l_1$ and $l_2$ are determined. The rotation angle $\theta_{2i}$ is then calculated by $Angle(x_{goal})$ with the known center $O_{goal}$. The right angle $\theta_2$ is given on the basis of judging the sign of $\alpha$, which is the projection value of goal point $x_{goal}$ on the $Y$-axis.

**Algorithm 1** DDA Algorithm

**Input:**

$\mathbf{T}_{dda}$ : DDA table, a $m*n$ matrix
$x_{goal}$ : goal position

**Output:**

$P(l_1, l_2, \theta_2)$ : the input parameters of CTR
$l_1$ : the length of outer tube
$l_2$ : the length of inner tube
$\theta_2$ : the rotation angle of inner tube

$r_{goal} \leftarrow Radius(x_{goal})$
$x_i = (\mathbf{T}_{dda}[m-3][i-1], \mathbf{T}_{dda}[m-2][i-1], \mathbf{T}_{dda}[m-1][i-1])$, where $m$=6 and $1 \leq i \leq n$
$r_i \leftarrow Radius(x_i)$
$N \leftarrow Column(r_i, r_{goal})$
$x_N = (\mathbf{T}_{dda}[m-3][N-1], \mathbf{T}_{dda}[m-2][N-1], \mathbf{T}_{dda}[m-1][N-1])$
$l_{1i} = \mathbf{T}_{dda}[m-4][N-1]$
$l_{2i} = \mathbf{T}_{dda}[m-5][N-1]$
$O_N \leftarrow Center(x_N)$
$O_{goal} \leftarrow Center(x_{goal})$
$e \leftarrow Distance(O_N, O_{goal})$
**if** $x_{goal}(3) < x_N(3)$ **then**
    $l_1 = l_{1i} + e$
    $l_2 = l_{2i} + e$
**else**
    $l_1 = l_{1i}$
    $l_2 = l_{2i}$
**end if**
$\theta_{2i} \leftarrow Angle(x_{goal})$
$\alpha \leftarrow$ the projected component on $Y-$axis of $x_{goal}$
**if** $\alpha > 0$ **then**
    $\theta_2 = \theta_{2i}$
**else**
    $\theta_2 = -\theta_{2i}$
**end if**
**return** $P$

## IV. Experimental Results and Discussion

The results of simulation and experiment will be shown in part A and part B respectively. At last, the reflection and discussion on experiment will be given in part C.

### *A. Simulation Results*

The simulation is carried out on MATLAB, and the graph generated by simulation are shown in the Fig.5. The blue line in this figure are drawn by the end point of inner tube of CTR when the DDA table is obtained. As discussed earlier, if the rotation axis of inner tube is known, one blue line can meet the calculation requirement. In this case, every time the inner tube rotates $120^o$, the coordinate data of its terminal point is collected once. Therefore, three points are obtained by

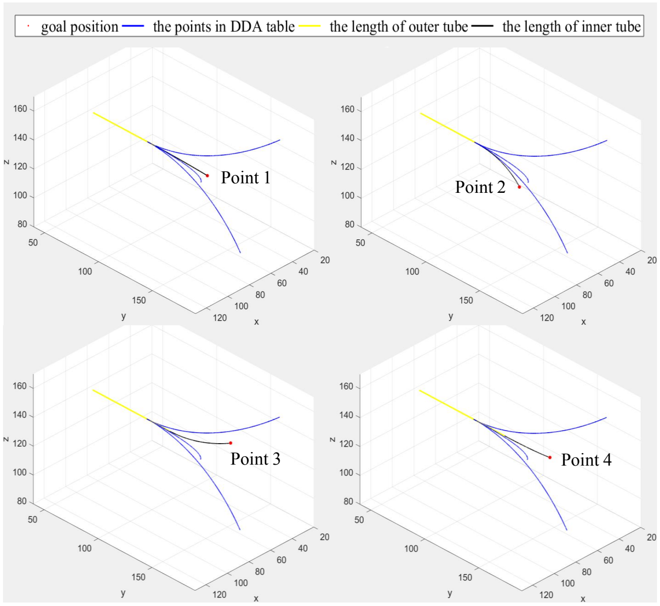


Fig. 5. The simulation results of four goal positions

rotating one revolution, thus three blue lines appear in the figure. In this way, the center and radius of the corresponding circle can be determined directly from these points. In addition, the red dot represents the target position, and the black and yellow lines represent the inner and outer tubes respectively when robot reaches the target position.

Through repeated experiments, it is found that the main factor that affects the accuracy of this method is the length increment of inner tube when data is collected. The precision is expressed by the linear distance $d$ between the target point and the end position of robot when it reaches the target configuration according to the DDA method. Table I shows the accuracy of four target points in two different increment situations. The simulation results show that the smaller the increment, the higher the precision of this algorithm.

TABLE I
THE SIMULATION PRECISION OF TWO DIFFERENT INCREMENTS

| Configuration | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Coordinate | (6,6,130) | (8,-8,130) | (-8,8,150) | (-9,-9,160) |
| Increment 1.0$mm$ | 0.1872 | 0.1444 | 0.1444 | 0.2048 |
| Increment 0.5$mm$ | 0.0043 | 0.0762 | 0.0762 | 0.0279 |

## B. Experimental Results

In addition to the simulation experiments, the actual experiments based on the concentric tube robot platform are also carried out. The experimental platform is shown in the Fig.6 . In the experiment, NDI Electromagnetic Tracking System generate DDA table by collecting location information of its senor which is mounted at the end of concentric tube robot. Through calculation, the motion parameters of robot are obtained. When the robot reaches goal position, the coordinates of end point are recorded by NDI, so as to calculate the precision.

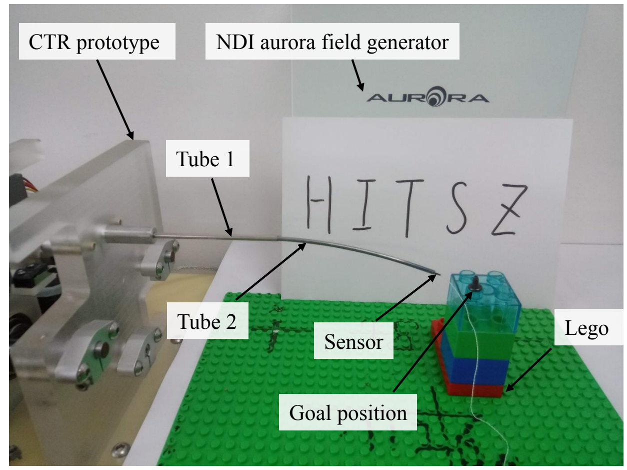


Fig. 6. The experimental platform

In this experiment, five end points in different CTR spatial configuration were selected for experimental verification. The corresponding parameters data in these configurations are shown in Table II. Each point was repeated 10 times, and the analysis of experimental errors are shown in the Table III. It can be seen from this results that although the accuracy of the experiment is not as high as that of simulation, it is enough to prove the feasibility and effectiveness of the DDA method.

TABLE II
THE PARAMETERS OF FIVE CTR CONFIGURATION

| Configuration | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $l_1(mm)$ | 40 | 40 | 70 | 70 | 70 |
| $l_2(mm)$ | 60 | 90 | 30 | 60 | 90 |
| $\theta_2$ | $-45^o$ | $90^o$ | $180^o$ | $0^o$ | $45^o$ |

TABLE III
THE DATA ANALYSIS OF EXPERIMENTAL RESULTS

| Configuration | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Mean(mm) | 2.1662 | 2.0085 | 0.4920 | 0.2010 | 0.1408 |
| Std(mm) | 0.1108 | 0.0656 | 0.0972 | 0.0489 | 0.0292 |
| Max(mm) | 2.4975 | 2.0795 | 0.6818 | 0.3015 | 0.1871 |
| Min(mm) | 2.1150 | 1.8209 | 0.4210 | 0.1338 | 0.1082 |

## C. Discussion

Simulation results prove that DDA algorithm theory is correct and reliable, and the feasibility and effectiveness of the proposed method are certified by the experimental. It is worth mentioning that the following points should be noted when using this method.

1) The circle obtained by rotating the inner tube around its axis plays a vital role in this algorithm;
2) The judgment of the this circle center directly affects the calculation accuracy of inner tube rotation angle;

3) The radius of this circle affects the column number determined from the DDA table, thus affecting the accuracy of inner and outer tube lengths.

## V. Conclusion

In this paper, we propose a new calculation method of inverse kinematics of concentric tube robots based on data-driven theory, which it is called DDA. And the feasibility of this method is proved by several simulation and experiment results. The DDA method can effectively avoid the problems existing in the popular inverse kinematics solutions and solve the plastic deformation problem of concentric tubes over time. At present, this method is only applicable to those concentric tube robots consisting of two tubes. Therefore, in order to expand the application scope of this method, we will continue to study and design a simple and effective DDA algorithm for CTRs with more than two tubes.

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