Online Appendix
Frictions Lead to Sorting: a Partnership Model with On-the-Match Search

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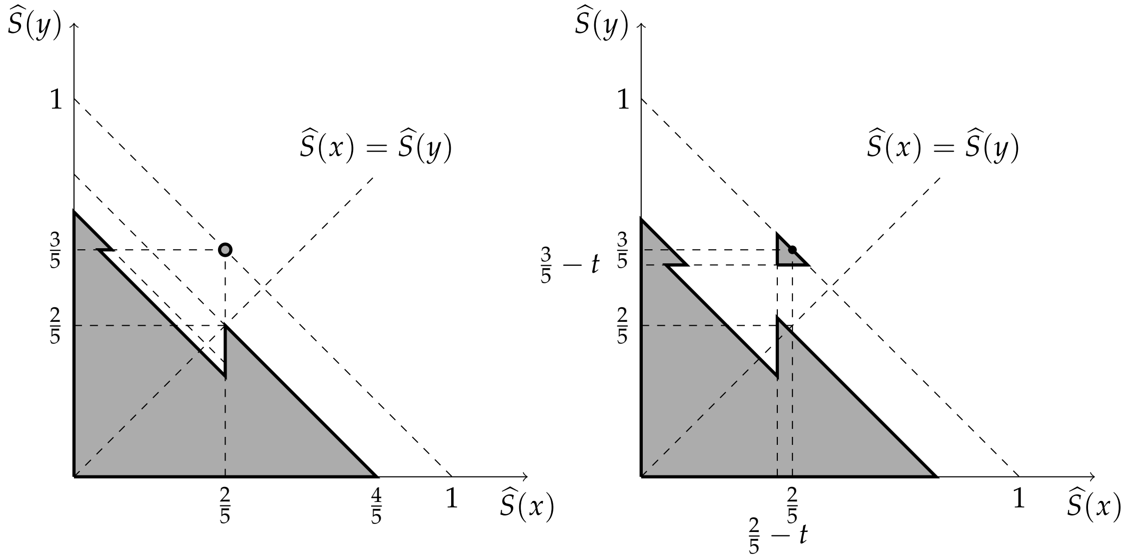
1. Multiplicity of Equilibria and Small Cost of Match-to-Match Transition: A Simple Example

As pointed out by Shimer [2006], on-the-match search leads to some uninteresting multiplicity of equilibria. To see this, consider a simple example. There are firms and workers. If a worker and a firm form a match, one unit is produced. That is, firms and workers are homogeneous. In such a symmetric setup, one would expect a symmetric outcome: in equilibrium both firms and workers get half of what is produced (and they do not leave each other). However, because of on-the-match search, there can be several other equilibria. Take the case where firms get $\frac{2}{5}$, and workers get $\frac{3}{5}$. Under this situation, an agreement leading to a (even slightly) higher payoff to the firm would mean that the worker would leave the firm whenever he gets the chance. Panel *a* in Figure 1 shows the bargaining set generated by this equilibrium (and Section 1.1 shows the details).

Figure 1: Asymmetric Equilibrium in a Simple Example

(a) No Cost of Transition (b) With Transition Cost



We are not interested in such equilibria. To see why, consider what would happen if agents had to pay an arbitrarily small cost $t > 0$ for each transition. Panel *b* in Figure 1 shows that $\left(\frac{2}{5}, \frac{3}{5}\right)$ is not an equilibrium, since it does not maximize the product of surpluses. In fact, with any $t > 0$ the only equilibrium is $\left(\frac{1}{2}, \frac{1}{2}\right)$.

1.1 Multiplicity of Equilibria. Details

Let x and y denote a firm and a worker respectively. Let $(\pi^*(x,y), \pi^*(y,x)) = \left(\frac{2}{5}, \frac{3}{5}\right)$. Nobody leaves in equilibrium: $d^* = 0$. Then $q^*(x,y) = q^*(y,x) = q^*$. In order to make the example simple, assume $q^* = r + \delta = \frac{1}{2}$, so $(r + \delta + q^*)^{-1} = 1$. Under (d^*, π^*) surplus are $S^*(x) = (r+\delta)^{-1}\left[\frac{2}{5} - q^* S^*(x)\right] = \frac{2}{5}$ and $S^*(y) = \frac{3}{5}$. This leads to a product of surpluses: $S^*(x)S^*(y) = \frac{6}{25}$.

Consider an alternative agreement $\left(\widehat{d}, \widehat{\pi}\right)$, leading to surplus $\left(\widehat{S}(x), \widehat{S}(y)\right)$. Since in (d^*, π^*) agents do not leave each other, there is no alternative agreement with $\widehat{S}(x) \geq S^*(x)$ and $\widehat{S}(y) \geq S^*(y)$. Let us study possible alternative agreements case by case. First, assume $\widehat{S}(x) \geq S^*(x)$ and $\widehat{S}(y) < S^*(y)$. Then, $\widehat{S}(y) = \frac{\widehat{\pi}(y,x) - q^* \widehat{S}(y)}{r+\delta} = \widehat{\pi}(y,x)$, and $\widehat{S}(x) = \left(\widehat{\pi}(x,y) - \frac{1}{5}\right)$. For this alternative agreement to be consistent, it must be the case that $\widehat{\pi}(x,y) > \frac{3}{5}$. The first area is given by:

$$\mathcal{S}_1 = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(x,y) \geq \frac{2}{5}, \widehat{S}(y,x) \leq \frac{4}{5} - \widehat{S}(x,y) \right\}.$$

Second, assume $\widehat{S}(x) < S^*(x)$ and $\widehat{S}(y) \geq S^*(y)$. Then $\widehat{S}(x) = \frac{\widehat{\pi}(x,y) - q^* \widehat{S}(x)}{r+\delta} = \widehat{\pi}(x,y)$ and $\widehat{S}(y) = (r + \delta + q^*)^{-1}\left[\widehat{\pi}(y,x) - q^* S^*(y)\right] = \widehat{\pi}(y,x) - \frac{3}{10}$. For this alternative agreement to be consistent, it must be the case that $\widehat{\pi}(y,x) > \frac{9}{10}$. The second area is given by:

$$\mathcal{S}_2 = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(y,x) \geq \frac{3}{5}, \widehat{S}(x,y) \leq \frac{7}{10} - \widehat{S}(y,x) \right\}.$$

Finally, assume $\widehat{S}(x) < S^*(x)$ and $\widehat{S}(y) < S^*(y)$. Then $\widehat{S}(y) = \frac{\widehat{\pi}(y,x) - q^* \widehat{S}(y)}{r+\delta} = \frac{2}{3}\widehat{\pi}(y,x)$ and $\widehat{S}(x) = \frac{2}{3}\widehat{\pi}(x,y)$. This can only hold if $\widehat{\pi}(x,y) < \frac{3}{5}$ and $\widehat{\pi}(y,x) < \frac{9}{10}$. The third area is given by:

$$\mathcal{S}_3 = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(x,y) \leq \frac{2}{5}, \widehat{S}(y,x) \leq \frac{3}{5}, \widehat{S}(x,y) + \widehat{S}(y,x) \leq \frac{2}{3} \right\}.$$

Then, $\mathcal{S}_{xy} = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3$.

Now, consider the case when firms have to pay a cost t if they switch costs. The new sets become:

$$\mathcal{S}_1^t = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(x,y) \geq \frac{2}{5} - t, \widehat{S}(y,x) \leq \frac{4}{5} - \frac{t}{2} - \widehat{S}(x,y) \right\},$$

$$\mathcal{S}_2^t = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(y,x) \geq \frac{3}{5} - t, \widehat{S}(x,y) \leq \frac{7}{10} - \frac{t}{2} - \widehat{S}(y,x) \right\},$$
$$\mathcal{S}_3^t = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(x,y) \leq \frac{2}{5} - t, \widehat{S}(y,x) \leq \frac{3}{5} - t, \widehat{S}(x,y) + \widehat{S}(y,x) \leq \frac{2}{3} - t \right\},$$
$$\mathcal{S}_4^t = \left\{ \left(\widehat{S}(x,y), \widehat{S}(y,x)\right) : \widehat{S}(x,y) \geq \frac{2}{5} - t, \widehat{S}(y,x) \geq \frac{3}{5} - t, \widehat{S}(x,y) + \widehat{S}(y,x) \leq 1 \right\}.$$

2. Equilibrium with Hyperphily when $\rho_0 = \rho_1$. Details

In what follows we explicitly calculate the parameters that make an equilibrium with hyperphily feasible. Let the intensity of search be the same both while matched and unmatched: $\rho_0 = \rho_1 = \rho$. We first compute the steady state conditions under hyperphily.

2.1 Steady State Conditions

Under hyperphily $d(\ell,\ell,h) = d(h,\ell,h) = 1$ and $d(\ell,h,\ell) = d(h,h,\ell) = 0$. Then, the steady state conditions become:

$$\begin{aligned}
e(\ell,\ell)\left[\delta + q(\ell,h)\right] + e(\ell,h)\left[\delta + q(h,h)\right] &= e(\ell,\varnothing)\left[q(\ell,\ell) + q(\ell,h)\right] \\
e(\ell,\varnothing)q(\ell,\ell) &= e(\ell,\ell)\left[\delta + 2q(\ell,h)\right] \\
\left[e(\ell,\varnothing) + e(\ell,\ell)\right]q(\ell,h) &= e(\ell,h)\left[\delta + q(h,h)\right] \\
\delta\left[e(h,\ell) + e(h,h)\right] &= e(h,\varnothing)\left[q(h,\ell) + q(h,h)\right] \\
\left[e(h,\varnothing) + e(h,\ell)\right]q(h,h) &= \delta e(h,h)
\end{aligned}$$

The successful meeting rates are $q(\ell,\ell) = \rho e(\ell,\varnothing)$, $q(\ell,h) = \rho e(h,\varnothing)$, $q(h,\ell) = \rho[e(\ell,\varnothing) + e(\ell,\ell)]$ and $q(h,h) = \rho[e(h,\varnothing) + e(h,\ell)]$. Substituting these into the steady state conditions, dividing by ρ, and setting $\kappa = \frac{\rho}{\delta}$, we get

$$e(\ell,\ell)\left[\frac{1}{\kappa} + e(h,\varnothing)\right] + e(\ell,h)\left[\frac{1}{\kappa} + \frac{1}{2} - e(h,h)\right] = e(\ell,\varnothing)\left[e(\ell,\varnothing) + e(h,\varnothing)\right] \tag{1}$$

$$e(\ell,\varnothing)^2 = e(\ell,\ell)\left[\kappa^{-1} + 2e(h,\varnothing)\right] \tag{2}$$

$$\left[e(\ell,\varnothing) + e(\ell,\ell)\right]e(h,\varnothing) = e(\ell,h)\left[\kappa^{-1} + e(h,\varnothing) + e(h,\ell)\right] \tag{3}$$

$$\kappa^{-1}\left[e(h,\ell) + e(h,h)\right] = e(h,\varnothing)\left[e(\ell,\varnothing) + e(\ell,\ell) + e(h,\varnothing) + e(h,\ell)\right] \tag{4}$$

$$\left[e(h,\varnothing) + e(h,\ell)\right]^2 = \kappa^{-1}e(h,h) \tag{5}$$

Solving equation (4) for $e(h,\varnothing)$, gives a quadratic equation in the unknown $e(h,\varnothing)$, $\kappa^{-1}\left[\frac{1}{2}-e(h,\varnothing)\right]=e(h,\varnothing)\left[\frac{1}{2}+e(h,\varnothing)\right]$, the positive solution of which is

$$e(h,\varnothing)=\frac{1}{2}\left(\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}-\kappa^{-1}-\frac{1}{2}\right)$$

A similar procedure on equation (5), gives

$$e(h,h)=\frac{1}{2}\left(1+\kappa^{-1}-\sqrt{\kappa^{-2}+2\kappa^{-1}}\right)$$

Using these two results together with the normalization condition gives

$$e(h,\ell)=e(\ell,h)=\frac{1}{2}\left(\frac{1}{2}+\sqrt{\kappa^{-2}+2\kappa^{-1}}-\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\right)$$

Solving equation (1) for $e(\ell,\varnothing)$ yields the following quadratic equation (the $e(h,\cdot)$ are all known by now):

$$e(\ell,\varnothing)^2+\left[\kappa^{-1}+2e(h,\varnothing)\right]e(\ell,\varnothing)-\frac{1}{2}\kappa^{-1}-\frac{1}{2}e(h,\varnothing)-e(h,\ell)^2=0$$

Its positive solution, after substituting in the values of $e(h,\varnothing)$ and $e(h,\ell)$, is

$$\begin{aligned}e(\ell,\varnothing)=&\frac{1}{4}-\frac{1}{2}\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\\&+\frac{1}{2}\sqrt{\frac{3}{\kappa^2}+\frac{9}{\kappa}+\frac{1}{2}-\sqrt{\frac{1}{\kappa^2}+\frac{3}{\kappa}+\frac{1}{4}}\left(2\sqrt{\frac{1}{\kappa^2}+\frac{2}{\kappa}}+1\right)+\sqrt{\frac{1}{\kappa^2}+\frac{2}{\kappa}}}\end{aligned}$$

Finally, since $e(\ell,\ell)=\frac{1}{2}-e(\ell,\varnothing)-e(h,\ell)$ we get

$$\begin{aligned}e(\ell,\ell)=&\sqrt{\frac{1}{\kappa^2}+\frac{3}{\kappa}+\frac{1}{4}}-\frac{1}{2}\sqrt{\frac{1}{\kappa^2}+\frac{2}{\kappa}}\\&-\frac{1}{2}\sqrt{\frac{3}{\kappa^2}+\frac{9}{\kappa}+\frac{1}{2}-\sqrt{\frac{1}{\kappa^2}+\frac{3}{\kappa}+\frac{1}{4}}\left(\sqrt{\frac{2}{\kappa^2}+2\frac{1}{\kappa}}+1\right)+\sqrt{\frac{1}{\kappa^2}+\frac{2}{\kappa}}}\end{aligned}$$

2.2 Individual Surpluses

Under hyperphily, surpluses are:

$$S^*(h,\ell)=\left[r+\delta+q^*(h,\ell)+q^*(h,h)\right]^{-1}\left[F-\pi^*(\ell,h)\right]$$
$$S^*(h,h)=\left[r+\delta+q^*(h,h)\right]^{-1}\left[h-q^*(h,\ell)S^*(h,\ell)\right]$$

$$S^*(\ell,h) = [r+\delta+q^*(\ell,h)+q^*(h,h)]^{-1}[\pi^*(\ell,h)-q^*(\ell,\ell)S^*(\ell,\ell)]$$
$$S^*(\ell,\ell) = [r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\,\ell$$

Surplus equalization $S^*(h,\ell) = S^*(\ell,h)$ requires:

$$\pi^*(\ell,h) = \frac{[r+\delta+q^*(\ell,h)+q^*(h,h)]\,F + \frac{r+\delta+q^*(h,\ell)+q^*(h,h)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}q^*(\ell,\ell)\ell}{2\,(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)}$$

A pair (d^*,π^*) is consistent in an equilibrium with hyperphily if $G^1_{HYP} \equiv S^*(h,h)-S^*(h,\ell) > 0$ and $G^2_{HYP} \equiv S^*(\ell,h)-S^*(\ell,\ell) > 0$.

Bargaining in match (ℓ,h)

Condition 1 in the main text lists the three kinds of agreement which may prevent (d^*,π^*) from solving the bargaining problem in the match (ℓ,h). We check next when these agreements are not feasible. First, in agreement c_1, neither ℓ nor h leave each other, and h is made indifferent. ℓ obtains $S^{c_1}(\ell,h)$. We need then $G^3_{HYP} \equiv S^*(\ell,h)-S^{c_1}(\ell,h) > 0$. In agreement c_2, $S^{c_2}(h,\ell) = S^*(h,h)$ (thus h never leaves) and ℓ only leaves when she finds an h, leading to surplus $S^{c_2}(\ell,h)$. We need then $G^4_{HYP} \equiv S^*(\ell,h)S^*(h,\ell)-S^{c_2}(\ell,h)S^*(h,h) \geq 0$. Finally, agreement c_3 also has $S^{c_3}(h,\ell) = S^*(h,h)$, but now ℓ always leaves. We need $G^5_{HYP} \equiv S^*(\ell,h)S^*(h,\ell)-S^{c_3}(\ell,h)S^*(h,h) \geq 0$.

Bargaining in match (ℓ,ℓ)

Condition 2 lists the three kinds of agreements which may prevent (d^*,π^*) from solving the bargaining problem in match (ℓ,ℓ). We check next when these agreements are not feasible. First, let $S^{c_4}(\ell,\ell)$ be the surplus obtained by either agent in match (ℓ,ℓ) when they do not leave each other. If c_4 were consistent, it would lead to a higher product of surpluses, as both agents would receive a higher surplus. Therefore $G^6_{HYP} \equiv S^*(\ell,h)-S^{c_4}(\ell,\ell) > 0$ must hold for hyperphily to be an equilibrium. Consider next agreement c_5. One agent (let us call him ℓ_1) obtains $S^{c_5}(\ell_1,\ell_2) = S^*(\ell,h)$ and does not leave. The second one (ℓ_2) leaves only when meeting agent h, so $S^{c_5}(\ell_2,\ell_1) \geq S^*(\ell,\ell)$. We need then $G^7_{HYP} \equiv S^*(\ell,\ell)-S^{c_5}(\ell_2,\ell_1) > 0$. Finally, consider agreement c_6. One agent (let us call him ℓ_1)

obtains $S^{c_6}(\ell_1, \ell_2)$ and *always* leaves. The second one (ℓ_2) never leaves, since he is indifferent between this match and one with h: $S^{c_6}(\ell_2, \ell_1) \geq S^*(\ell, h)$. For hyperphily to solve the bargaining problem, it must be the case that $G^8_{HYP} \equiv S^*(\ell, \ell)^2 - S^{c_6}(\ell_1, \ell_2) S^{c_6}(\ell_2, \ell_1) \geq 0$.

Bargaining in match (h, h)

In match (h, h) there is no endogenous destruction and agents equalize surplus, therefore they are maximizing the product of surpluses.

Details on equilibrium conditions

We characterize each condition as a function of primitives. Let us start with match (ℓ, h). In alternative agreement c_1 for match (ℓ, h), we have $d^{c_1}(x, y, y') = 0$ for $x \in \{\ell, h\}, y \in \{\ell, h\}, y' \in \{\ell, h\}$. Moreover, $\pi^{c_1}(h, \ell) = h$. For any lower $\pi^{c_1}(h, \ell)$, h would leave. Any higher $\pi^{c_1}(h, \ell)$ would lead to a lower product of surpluses. It suffices then to focus on this agreement. We need to show that $G^3_{HYP} \equiv S^*(\ell, h) - S^{c_1}(\ell, h) > 0$.

$$S^{c_1}(\ell, h) = (r+\delta)^{-1} \left[F - h - q^*(\ell, \ell) S^*(\ell, \ell) - q^*(\ell, h) S^*(\ell, h)\right] \quad \text{and}$$
$$S^*(\ell, h) = \frac{F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}}{2\left(r+\delta+q^*(h,h)\right) + q^*(\ell,h) + q^*(h,\ell)}$$

Therefore, we need

$$F - h - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)} < \frac{\left[r+\delta+q^*(\ell,h)\right]\left[F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right]}{2[r+\delta+q^*(h,h)] + q^*(\ell,h) + q^*(h,\ell)}$$

Thus

$$F < h\left(1 + \frac{r+\delta+q^*(\ell,h)}{r+\delta+2q^*(h,h)+q^*(h,\ell)}\right) + \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)} \tag{HYP 1}$$

Consider next agreement c_2: $d^{c_2}(h, \ell, \ell) = d^{c_2}(h, \ell, h) = d^{c_2}(\ell, h, \ell) = 0$ and $d^{c_2}(\ell, h, h) = 1$. Payoffs $\pi^{c_2}(h, \ell)$ are such that $S^{c_2}(h, \ell) = S^*(h, h)$. Any lower $\pi^{c_2}(h, \ell)$ would make h leave instead. It suffices then to focus on this agreement. We need to show that $G^4_{HYP} \equiv S^*(\ell, h) S^*(h, \ell) - S^{c_2}(\ell, h) S^{c_2}(h, \ell) \geq 0$. Surpluses are:

$$S^{c_2}(h, \ell) = \frac{F - \pi^{c_2}(\ell, h) - q^*(h,h) S^*(h,h) - q^*(h,\ell) S^*(h,\ell)}{r+\delta+q^*(\ell,h)} = S^*(h,h)$$

$$S^{c_2}(\ell,h) = (r+\delta+q^*(\ell,h))^{-1}\left(\pi^{c_2}(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)\right)$$

We can recover $\pi^{c_2}(\ell,h)$ from the previous expressions:

$$\pi^{c_2}(\ell,h) = F - [r+\delta+q^*(h,h)]^{-1} \times \\ \left[(r+\delta+q^*(\ell,h)+q^*(h,h))\,h - \frac{q^*(h,\ell)q^*(\ell,h)(F-\pi^*(\ell,h))}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]$$

From now on, we work with $\pi^*(\ell,h) = A_1 + B_1F$ and $\pi^{c_2}(\ell,h) = A_2 + B_2F$, with:

$$\begin{aligned}
A_1 &= \frac{\left[r+\delta+q^*(h,\ell)+q^*(h,h)\right]q^*(\ell,\ell)\ell}{[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]\,[2\,(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)]} \\
B_1 &= \frac{r+\delta+q^*(\ell,h)+q^*(h,h)}{2\,(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)} \\
A_2 &= -\frac{r+\delta+q^*(\ell,h)+q^*(h,h)}{r+\delta+q^*(h,h)}h \\
&\quad - \frac{q^*(h,\ell)q^*(\ell,h)A_1}{(r+\delta+q^*(h,\ell)+q^*(h,h))\,(r+\delta+q^*(h,h))} \\
B_2 &= 1 + \frac{q^*(h,\ell)q^*(\ell,h)(1-B_1)}{(r+\delta+q^*(h,h))\,(r+\delta+q^*(h,\ell)+q^*(h,h))}
\end{aligned}$$

We first check that $S^{c_2}(\ell,h) \geq S^*(\ell,\ell)$. This occurs whenever

$$F \geq B_2^{-1}\left(\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right)$$

If agreement c_2 is consistent we still have to check that F is large enough to make the product of surpluses larger, that is, $S^*(h,\ell)S^*(\ell,h) \geq S^{c_2}(h,\ell)S^{c_2}(\ell,h)$:

$$\begin{aligned}
[(1-B_1)F - A_1]^2 \geq C_1 &\left(B_2F + A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right) \\
&\times \left(h - q^*(h,\ell)\frac{(1-B_1)F - A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) \quad \text{with} \\
C_1 &= \frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h))(r+\delta+q^*(h,h))}.
\end{aligned}$$

The previous expression holds with equality for F given by:

$$\begin{aligned}
&\left[(1-B_1)^2 + C_1B_2\frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]F^2 \\
&+ \left[-2(1-B_1)A_1 - C_1B_2\left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\right]F \\
&+ \frac{C_1q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\left(A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right)F
\end{aligned}$$

$$+A_1^2 - C_1\left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\left(A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right) = 0$$

Since $(1-B_1)$, C_1 and B_2 are positive, G^4_{HYP} is a convex function of F. In order to have an equilibrium with hyperphily, F has to be smaller than the lower root or larger than the higher one. Only the first of these two conditions is relevant. To see this, note that there exists an $\widehat{F}$ such that $S^*(\ell,h) = S^*(h,h)$. For $F=\widehat{F}$, $S^{c_2}(\ell,h)S^{c_2}(h,\ell) > S^*(\ell,h)S^*(h,\ell)$ holds.[1] Therefore, F larger than the large root of $G^4_{HYP}=0$ requires that $F>\widehat{F}$. However, consistency condition G^1_{HYP} states than an equilibrium with hyperphily requires $F<\widehat{F}$. Therefore if F^4_{HYP} is the smaller root of $G^4_{HYP}=0$, an equilibrium with hyperphily requires:

$$F \leq \max\left(F^4_{HYP}, B_2^{-1}\left[\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right]\right) \tag{HYP 2}$$

We move next to alternative agreement c_3. Surpluses are:

$$S^{c_3}(h,\ell) = \frac{F-\pi^{c_3}(\ell,h) - q^*(h,h)S^*(h,h) - q^*(h,\ell)S^*(h,\ell)}{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)} = S^*(h,h)$$
$$S^{c_3}(\ell,h) = [r+\delta+q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\pi^{c_3}(\ell,h)$$

Then $\pi^{c_3}(\ell,h) = A_3 + B_3F$, with

$$\begin{aligned} A_3 = & -\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)+q^*(h,h)}{r+\delta+q^*(h,h)}h \\ & - \frac{q^*(h,\ell)(q^*(\ell,h)+q^*(\ell,\ell))A_1}{[r+\delta+q^*(h,h)][r+\delta+q^*(h,h)+q^*(h,\ell)]} \\ B_3 = & \ 1 + \frac{q^*(h,\ell)(q^*(\ell,h)+q^*(\ell,\ell))B_1}{[r+\delta+q^*(h,h)]\,[r+\delta+q^*(h,h)+q^*(h,\ell)]} \end{aligned}$$

Condition $G^5_{HYP} \equiv S^*(h,\ell)S^*(\ell,h) - S^{c_3}(h,\ell)S^{c_3}(\ell,h) \geq 0$ holds if:

$$[(1-B_1)F - A_1]^2 \geq C_2\,(B_3F + A_3)\left(h - q^*(h,\ell)\frac{(1-B_1)F-A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)$$
$$\text{with } C_2 = \frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h)+q^*(\ell,\ell))(r+\delta+q^*(h,h))}$$

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[1] If $F=\widehat{F}$, this is equivalent to $S^{c_2}(\ell,h) > S^*(\ell,h)$ because $S^{c_2}(h,\ell) = S^*(h,h) = S^*(h,\ell)$. Add $S^{c_2}(h,\ell) = S^*(h,\ell)$ on both sides of the inequality and rearrange terms to get:

$$\begin{aligned} & \frac{\widehat{F} - q^*(h,h)S^*(h,h) - q^*(h,\ell)S^*(h,\ell) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,h)} \\ > & \frac{\widehat{F} + [q^*(\ell,h) - q^*(h,\ell) - q^*(h,h)]S^*(h,\ell) - q^*(\ell,\ell)S^*(\ell,\ell) - q^*(h,h)S^*(\ell,h)}{r+\delta+q^*(\ell,h)} \end{aligned}$$

Comparing numerators gives $q^*(h,h) > q^*(\ell,h)$, which indeed holds.

Therefore:

$$\begin{aligned}
G^5_{HYP} = &\left[(1-B_1)^2 + C_2 B_3 \frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right] F^2 \\
&+ \left[-2(1-B_1)A_1 - C_2 B_3 \left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\right. \\
&\left. + \frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)} C_2 A_3\right] F \\
&+ A_1^2 - \left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) C_2 A_3 \geq 0
\end{aligned}$$

Since $(1-B_1)$, C_2, and B_3 are positive, G^6_{HYP} is a convex function of F. There are two values $F^5_{HYP} < F^{5'}_{HYP}$ of F that equalize the product of the surplus. In order to have an equilibrium with hyperphily F has to be smaller than F^5_{HYP} or larger then $F^{5'}_{HYP}$:

$$F \notin \left(F^5_{HYP}, F^{5'}_{HYP}\right) \tag{HYP 3}$$

We move next to match (ℓ,ℓ). Consider alternative agreement c_4. We need to show that $G^6_{HYP} \equiv S^*(\ell,h) - S^{c_4}(\ell,\ell) > 0$, with

$$S^{c_4}(\ell,\ell) = (r+\delta)^{-1}\left[\ell - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)\right]$$

This occurs whenever

$$F > \frac{[2(r+\delta+q^*(h,h)) + q^*(\ell,h) + q^*(h,\ell)][r+\delta+2q^*(\ell,h)] + [r+\delta+q^*(\ell,h)]q^*(\ell,\ell)}{r+\delta+q^*(\ell,h)][r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)]}\ell \tag{HYP 4}$$

Next, consider agreement c_5. Agent ℓ_2 does not leave so $S^{c_5}(\ell_2,\ell_1) = S^*(\ell,h)$, with

$$S^{c_5}(\ell_2,\ell_1) = (r+\delta+q^*(\ell,h)^{-1}[\pi^{c_5}(\ell_2,\ell_1) - q^*(\ell,\ell)S^*(\ell,\ell) - q^*(\ell,h)S^*(\ell,h)]$$

This requires

$$\begin{aligned}
\pi^{c_5}(\ell_2,\ell_1) = &\frac{r+\delta+2q^*(\ell,h)}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)} F \\
&+ \frac{\left(r+\delta+2q^*(h,h)-q^*(\ell,h)+q^*(h,\ell)\right)q^*(\ell,\ell)\ell}{[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)][2(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)]}
\end{aligned}$$

We need to verify now that $G^7_{HYP} \equiv S^*(\ell,\ell) - S^{c_5}(\ell_1,\ell_2) > 0$ with

$$S^{c_5}(\ell_1,\ell_2) = \frac{2\ell - \pi^{c_5}(\ell_2,\ell_1) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,h)} < S^*(\ell,\ell)$$

If condition G^6_{HYP} holds, it must be the case that $S^{c_5}(\ell_2, \ell_1) < S^*(\ell, h)$. We look for the maximum F that makes the agreement $(S^{c_5}(\ell_1, \ell_2), S^{c_5}(\ell_2, \ell_1))$ consistent:

$$\begin{aligned} F > & \frac{[r+\delta+3q^*(\ell,h)+q^*(\ell,\ell)][2(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)]}{[r+\delta+2q^*(\ell,h)][r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]} \\ & - \frac{[r+\delta+2q^*(h,h)-q^*(\ell,h)+q^*(h,\ell)]}{[r+\delta+2q^*(\ell,h)][r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]}\ell \end{aligned} \tag{HYP 5}$$

Next, consider agreement c_6. We need to verify that

$$G^8_{HYP} \equiv S^*(\ell,\ell)^2 - S^{c_6}(\ell_1,\ell_2)S^{c_6}(\ell_2,\ell_1) \geq 0 \text{ with}$$

$$\begin{aligned} S^{c_6}(\ell_1,\ell_2) &= \frac{2\ell - \pi^{c_6}(\ell_2,\ell_1)}{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)} \quad \text{ and} \\ S^{c_6}(\ell_2,\ell_1) &= \frac{\pi^{c_6}(\ell_2,\ell_1) - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)} \end{aligned}$$

Note that

$$\begin{aligned} S^{c_6}(\ell_1,\ell_2) + S^{c_6}(\ell_2,\ell_1) &= \frac{2\ell - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)} \quad \text{ and} \\ 2S^*(\ell,\ell) &= \frac{2\ell}{r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)} = \frac{2\ell - 2q^*(\ell,h)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)} \end{aligned}$$

Since $S^*(\ell,\ell) < S^*(\ell,h)$ and $q^*(\ell,\ell) = \rho e(\ell,\varnothing) > \rho e(h,\varnothing) = q^*(\ell,h)$, then $2S^*(\ell,\ell) > S^{c_6}(\ell_1,\ell_2) + S^{c_6}(\ell_2,\ell_1)$. Both ℓ agents equalize surplus in $S^*(\ell,\ell)$, and no agreement in the same segment of the frontier or in an interior segment of the frontier can generate a larger product of surpluses. Therefore condition G^8_{HYP} always holds.

Finally, we check consistency of the equilibrium with hyperphily. For condition G^1_{HYP}, note that $S^*(h,h) > S^*(h,\ell)$ if and only if $h > \pi^*(h,\ell)$. The following expression always holds:

$$S^*(\ell,h) - S^{c_1}(\ell,h) = (r+\delta)^{-1}\left[\pi^*(\ell,h) - (F-h) - q^*(h,h)S^*(\ell,h)\right].$$

Condition G^3_{HYP} implies the previous expression is positive. Thus $\pi^*(\ell,h) - F + h > 0 \Rightarrow F - \pi^*(h,\ell) - F + h > 0 \Rightarrow h > \pi^*(h,\ell)$, so G^1_{HYP} holds.

G^2_{HYP} holds whenever G^6_{HYP} holds. To see this, note that

$$S^{c_4}(\ell,\ell) = S^*(\ell,\ell) + \frac{q^*(\ell,h)\left[2S^*(\ell,\ell) - S^*(\ell,h)\right]}{r+\delta}.$$

Then, whenever $S^*(\ell,\ell) - S^*(\ell,h) \geq 0$, also $S^{c_4}(\ell,\ell) - S^*(\ell,h) \geq 0$.

3. Equilibrium with Hyperphily when $\rho_0 = 0$. Details

In what follows we explicitly calculate the parameters such that an equilibrium with hyperphily can arise when there is no on-the-match-search. In such a case, surplus splitting occurs. If a match gets formed, then $S(\ell, h) = S(h, \ell)$. As a result, there are two kinds of equilibria that feature hyperphily:

1. All possible matches get formed: $S(h,h) > S(h,\ell) = S(\ell,h) > S(\ell,\ell) > 0$.
2. Two ℓ-type agents do not form a match: $S(h,h) > S(h,\ell) = S(\ell,h) > 0 > S(\ell,\ell)$.

The surplus obtained by the partners in a match (x,y) are given by:

$$(r+\delta)S(x,y) = \pi(x,y) - rV(x)$$
$$(r+\delta)S(y,x) = F(x,y) - \pi(x,y) - rV(y)$$

Therefore, due to surplus equalization: $S(x,y) = \frac{1}{r+\delta}\frac{F(x,y)-rV(x)-rV(y)}{2}$.

3.1 All Matches Get Formed

3.1.1 Steady State Conditions

Note that $e(\ell,\varnothing) = e(h,\varnothing) \equiv u$ and let $\kappa = \rho/\delta$. The steady state condition is:

$$\delta\left[e\left(h,\ell\right) + e\left(h,h\right)\right] = \rho e\left(h,\varnothing\right)\left[e\left(\ell,\varnothing\right) + e\left(h,\varnothing\right)\right]$$

which implies that $u = (\sqrt{1+4\kappa} - 1)/4\kappa$.

3.1.2 Individual Surpluses

The value of unemployment is given by:

$$rV(\ell) = \rho e(\ell,\varnothing)S(\ell,\ell) + \rho e(h,\varnothing)S(\ell,h) = \rho u\left[S(\ell,\ell) + S(\ell,h)\right]$$
$$rV(h) = \rho e(\ell,\varnothing)S(h,\ell) + \rho e(h,\varnothing)S(h,h) = \rho u\left[S(h,\ell) + S(h,h)\right]$$

which imply:

$$rV(h) = \frac{1}{2}\frac{1}{2+\frac{1}{\theta u}}\left[F + \frac{3+\frac{2}{\theta u}}{1+\frac{1}{\theta u}}h - \frac{1}{1+\frac{1}{\theta u}}\ell\right]$$
$$rV(\ell) = \frac{1}{2}\frac{1}{2+\frac{1}{\theta u}}\left[F + \frac{3+\frac{2}{\theta u}}{1+\frac{1}{\theta u}}\ell - \frac{1}{1+\frac{1}{\theta u}}h\right]$$

Let us go up to individual surpluses:

$$\rho S(\ell,\ell) = \theta\left[2\ell - 2rV(\ell)\right]/2 = \theta[\ell - rV(\ell)]$$
$$\rho S(h,h) = \theta\left[2h - 2rV(h)\right]/2 = \theta[h - rV(h)]$$
$$\rho S(\ell,h) = \theta\left[F - rV(h) - rV(\ell)\right]/2$$

3.1.3 Conditions for Consistency

We check first that $S(h,h) \geq S(\ell,h)$:

$$S(h,h) \geq S(\ell,h) \Leftrightarrow \rho S(h,h) \geq \rho S(\ell,h) \Leftrightarrow \theta[h - rV(h)] \geq \theta\left[F - rV(h) - rV(\ell)\right]/2$$

When $\ell = 1$ and $h = 2$, this requires $F \leq 4 - \left(1 + \frac{1}{\theta u}\right)^{-1}$.

We check next that $S(\ell,h) \geq S(\ell,\ell)$:

$$S(\ell,h) \geq S(\ell,\ell) \Leftrightarrow \rho S(\ell,h) \geq S\rho(\ell,\ell) \Leftrightarrow \theta\left[F - rV(h) - rV(\ell)\right]/2 \geq \theta[\ell - rV(\ell)]$$

The last condition ($S(\ell,\ell) \geq 0$) holds every time that $S(h,h) \geq S(\ell,h)$ holds.

To sum up,

$$F \in \left[2 + \frac{1}{1+\frac{1}{\theta u}}, 4 - \frac{1}{1+\frac{1}{\theta u}}\right] \quad \text{with } u = \frac{\sqrt{1+4\kappa}-1}{4\kappa}$$

3.2 Low Types (ℓ) Only Accept High Types (h)

3.2.1 Steady State Conditions

The steady state conditions are given by:

$$\delta[e(\ell,h) + e(h,h)] = \rho e(h,\varnothing)[e(\ell,\varnothing) + e(h,\varnothing)]$$
$$\delta e(\ell,h) = \rho e(l,\varnothing)e(h,\varnothing)$$

These conditions require:

$$0 = \kappa^2 u_h^3 + 2\kappa u_h^2 + u_h - \tfrac{1}{2} \quad \text{and}$$
$$u_l = [2(1+\kappa u_h)]^{-1}$$

3.2.2 Individual Surpluses

In this case:

$$\begin{aligned}
rV(\ell) &= \rho u_h S(\ell,h) \\
rV(h) &= \rho u_l S(\ell,h) + \rho u_h S(h,h) \\
\rho S(\ell,h) &= \theta[F - rV(\ell) - rV(h)]/2 \\
\rho S(\ell,\ell) &= \theta[\ell - rV(\ell)] \\
\rho S(h,h) &= \theta[h - rV(h)]
\end{aligned}$$

The value of unemployment is given by:

$$\begin{aligned}
rV(h) &= \frac{(2+\theta u_h)u_h h + u_l F}{(2+\theta u_h)(1+\theta u_h)/\theta + u_l} \\
rV(\ell) &= \frac{(1+\theta u_h)u_h F - \theta u_h^2 h}{(2+\theta u_h)(1+\theta u_h)/\theta + u_l}
\end{aligned}$$

3.2.3 Conditions for Consistency

We check first that $S(h,h) \geq S(\ell,h)$. This requires that:

$$F < \left(1 - \frac{u_l/2}{(1+\theta u_h)/\theta + u_l}\right) 2h$$

The condition $S(\ell,h) > 0$ always holds.

We finally check that $S(\ell,\ell) < 0$. With $\ell = 1$ and $h = 2$, this requires that

$$F > \frac{(2+\theta u_h)(1+\theta u_h)/\theta + u_l + 2\theta u_h^2}{(1+\theta u_h)u_h}$$

To sum up, (with $\ell = 1$ and $h = 2$),

$$F \in \left[\frac{(2+\theta u_h)(1+\theta u_h)/\theta + u_l + 2\theta u_h^2}{(1+\theta u_h)u_h}, 4 - \frac{2u_l}{(1+\theta u_h)/\theta + u_l}\right]$$

With

$$
\begin{aligned}
0 &= \kappa^2 u_h^3 + 2\kappa u_h^2 + u_h - \tfrac{1}{2} \quad \text{and} \\
u_l &= [2(1 + \kappa u_h)]^{-1}
\end{aligned}
$$