# Frictions Lead to Sorting: a Partnership Model with On-the-Match Search*

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**Abstract**

We present a partnership model where heterogeneous agents bargain over the gains from trade and search on the match. Frictions allow agents to extract higher rents from more productive partners, generating an endogenous preference for high types. More productive agents upgrade their partners faster, therefore the equilibrium match distribution features positive assortative matching. Frictions are commonly understood to hamper sorting. Instead, we show how frictions generate positive sorting even with a submodular production function. Our results challenge the interpretation of positive assortative matching as evidence of complementarity.

JEL Classification: C78; D83; J63; J64

Keywords: Assortative matching; Search frictions; On-the-match search; Bargaining

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# 1. Introduction

Markets with two-sided heterogeneity are prevalent. In labor markets, firms and workers typically differ in their characteristics, quality and ability. The same is true in other markets, such as the marriage market and the market for CEOs. The evidence suggests that better CEOs sort into better corporations (Parrino [1997]), that there is positive assortative mating in the marriage market (Mare [1991]), and that more productive employees work for better firms (Bartolucci, Devicienti, and Monzón [2018]). Traditionally, positive assortative matching has been interpreted as evidence of complementarity in the production function. In this paper we argue that frictions are a natural reason for positive assortative matching to arise, even in the absence of complementarity in production.

In the presence of frictions higher types become more appealing. While in frictionless markets payoffs reflect individual contributions, the division of output becomes more even when it takes time to find a partner. To see this, assume that agents are infinitely impatient (or frictions infinitely strong), and therefore outside options are zero. In this simple case, the gains from trade are equal to the production of the match. Under standard bargaining, both agents receive an equal share of the gains from trade, so agents receive a constant fraction of the match's output. When frictions are strong enough, it is the total production of the match, rather than individual contributions, that shapes payoffs and preferences over partners. Production is increasing in the partner's type, so an endogenous preference for better types arises. Complementarity in production only plays a secondary role.

This endogenous preference for higher types shapes the sorting pattern. If the value of the match were exogenously increasing in partner type, perfect positive sorting would arise in a frictionless market (see Becker [1973] and Legros and Newman [2007]). However, this is not necessarily true in a market with frictions. Whenever agents are not allowed to search while matched, a preference for higher types can lead to different sorting patterns. If instead agents are allowed to replace their partners, more desirable agents upgrade partners faster. In this way, a preference for high type partners makes high type agents climb the ladder of partners faster. Therefore, the distribution of matches can feature positive sorting (as in Lentz [2010]).

We present a partnership model with transferable utility where agents search on the match.[1] There is a finite number of types, who must be matched to produce. A matched agent who finds a new partner can dissolve the current match and form a new one. After dissolving a match, the agent bargains with the new partner without the possibility of returning to the previous one. Our bargaining protocol prevents agents from exploiting the presence of multiple suitors to raise their payoffs. This timing makes preferences over partners simple: the value of the match to an agent depends only on her *current* partner's type.[2]

We study how frictions shape both the preferences over partners and the sorting pattern. We associate high frictions to low meeting rates, high exogenous destruction rates, and impatient agents. Our first message is simple:

**Proposition 1**. When frictions are high enough, an endogenous preference for higher types emerges.

This result holds for any number of types, any distribution over these types and any production function. Moreover, the meeting rate while matched may differ from that when unmatched. This endogenous preference for higher types affects the sorting pattern. However, it is not the only determinant of it. The underlying distribution of types and the relative meeting rates while matched and unmatched also shape the sorting pattern. We present two examples to illustrate this.

In our first example, only two types of agents are present in the population, in equal proportion. In this simple environment we solve explicitly for the primitives (in terms of production function and frictions) that lead to an endogenous prefer-

[1] In our framework match payoffs are not exogenously given, but rather endogenously determined through bargaining. Hence, in our environment utility is transferable (Smith [2006]).

[2] In some markets (like the one for academic economists) counteroffers are common practice. However, this is not the norm in most markets (see Mortensen [2005]). We also have preliminary results where we modify the bargaining protocol to allow for renegotiation (à la Kiyotaki and Lagos [2007]) and show how frictions can lead to positive sorting in this case, also without productive complementarity.

ence for higher types. We show that when the meeting rates while matched and unmatched are equal, an endogenous preference for higher types leads to positive sorting. In our second example, there are many types and production is linear. An endogenous preference for higher types emerges. In spite of this, when agents are not allowed to search while matched, no clear sorting pattern emerges. If instead the meeting rates while matched and unmatched are equal, there is positive sorting. So in general, the sorting pattern depends on the meeting rates and on the underlying distribution of types, even with an endogenous preference for higher types. Our second result provides sufficient conditions for frictions to lead to sorting.

**Proposition 2**. Assume that types are equally distributed in the population, that intensities of search while matched and unmatched are the same, and that destruction rates are high enough. Then, an endogenous preference for higher types leads to positive sorting.

The literature on assortative matching mostly focuses on how complementarity in production affects the allocation of workers to firms. In Becker's seminal partnership model [1973], a supermodular production function is necessary and sufficient for positive assortative matching. This is not true in markets with frictions. When it takes time to find a partner, agents are selective only if complementarity in production compensates the cost of waiting. In this way, frictions hamper sorting: the matching pattern becomes less ordered. In fact, Shimer and Smith [2000], Atakan [2006] and Eeckhout and Kircher [2010] present sufficient conditions on the production function so that positive (negative) sorting obtains for all primitives when frictions are present.[3]

Our results challenge the interpretation of sorting as evidence of complemen-

[3]There is no positive assortative matching in Shimer and Smith [2000] for modular and slightly supermodular production functions. In Atakan [2006], whenever the explicit cost of search is high and complementarity weak, random sorting arises in equilibrium. In Eeckhout and Kircher [2010] root-supermodularity is necessary and sufficient for positive assortative matching. Our approach differs from these papers in that we do not focus on finding sufficient conditions on the production function for sorting to obtain under all possible other primitives. Instead, we focus on how frictions shape preferences, and that translates into the sorting pattern.

tarity in production. The conventional wisdom is that stronger frictions require stronger complementarity in production for positive assortative matching to arise. Our paper highlights a different role for frictions: they modify preferences over partners. Thus, frictions can generate positive assortative matching even with a submodular production function.

Match-to-match transitions are pervasive in most developed economies. Fallick and Fleischman [2004] estimate that at least half of all new employment relationships result from job-to-job transitions. On the firm side, Albak and Sørensen [1998] and Burgess, Lane, and Stevens [2000] present empirical evidence of replacement hiring (see Kiyotaki and Lagos [2007] for a discussion). In the market for CEOs, Parrino [1997] finds that the availability of a strong outside candidate is an important consideration in the decision to replace a poor CEO. Murphy and Zabojnik [2006] report that a large proportion of managers were hired from another firm. Stevenson and Wolfers [2007] find that remarriage is one of the main determinants of divorce.

Allowing agents to search on the match adds an extra layer of difficulty to the bargaining problem: the surplus from the match depends on the bargaining outcome. Patient agents face a trade-off between per-period payoff and expected duration of the match: higher wages paid to a worker reduce the firm's per-period profits, but they decrease the likelihood that the worker quits. In fact, a higher wage may increase the value of the match both to the worker and the firm. As highlighted by Shimer [2006], bargaining sets are not necessarily convex and therefore the standard axiomatic Nash Bargaining is not applicable to this setup.

We present a solution for axiomatic bargaining when both sides can leave the match if they find a preferred option. Bargaining sets do not satisfy Nash's axioms [1950]. However, we show that bargaining sets are compact. We follow a modified version of Nash's axioms proposed by Kaneko [1980]. Kaneko shows that for compact bargaining sets the solution is exactly as in Nash [1950]: it selects the outcome which maximizes the product of agents' individual surpluses.

The rest of the paper is organized as follows. In the next section we present a partnership model with on-the-match search and present our notion of equilibrium. In Section 3 we present two motivating examples. Section 4 presents our main results. Section 5 concludes.

# 2. The Model

Consider a continuous time, infinite horizon stationary economy, populated by infinitely lived, risk neutral agents. There is a unit mass population of heterogeneous agents denoted by their fixed type $x \in X = \{1, 2, \ldots, N\}$. Types are present in the population in fractions $l(x) : X \to [0, 1]$ with $\sum_{x \in X} l(x) = 1$.

Agents can be either matched or unmatched. A match produces a flow of output $f(x, y) : X^2 \to \mathbb{R}_+$. The production function $f(x, y)$ is strictly increasing in both arguments and symmetric: $f(x, y) = f(y, x)$. Unmatched agents produce zero. Agents discount the future at rate $r > 0$.

Transitions between states occur due to exogenous destruction and match-to-match transitions. The steady state distribution $e(x, y) : X \times X \cup \{\varnothing\} \to [0, l(x)]$ specifies the number $e(x, \varnothing)$ of $x$-type agents who are unmatched, and the number $e(x, y)$ of $x$-type agents who are matched to agents of type $y \in X$. Then, $\sum_{y \in X \cup \{\varnothing\}} e(x, y) = l(x)$ for $x \in X$. Matches are exogenously destroyed at rate $\delta$.

Both matched and unmatched agents may meet potential partners (who also *themselves* may be matched or unmatched). We allow for different search efficiencies while unmatched or matched. Let $\rho_0$ denote the search efficiency of an unmatched agent and $\rho_1$ denote that of a matched one. The meeting rate is simply the product of the search efficiencies of those who meet.[4]

The set of partners an agent is willing to accept depends on her current match.

[4] For example, there are $\rho_0 \rho_1 e(x, \varnothing) e(y, y)$ unmatched $x$-type agents who meet $y$-type agents matched to other $y$-type agents. Similarly, there are $(\rho_0)^2 e(x, \varnothing) e(y, \varnothing)$ unmatched $x$-type agents who meet unmatched $y$-type agents. Our approach here is similar to Bobbio [2009].

A decision function $d(x, y, y') : X \times X \cup \{\varnothing\} \times X \to [0, 1]$ specifies the probability that an agent of type $x$ matched to an agent of type $y$ (or unmatched when $y = \varnothing$) would dissolve that match upon meeting a willing partner of type $y'$.[5] Define $q(x, y) : X \times X \to \mathbb{R}_+$ by

$$q(x,y) \equiv \rho_0 \, d(y, \varnothing, x) \, e(y, \varnothing) + \rho_1 \sum_{x' \in X} d(y, x', x) \, e(y, x') \, .$$

Then, an unmatched agent of type $x$ meets an agent of type $y$ who is willing to form a match with her at rate $\rho_0 q(x, y)$, while a matched agent of type $x$ does so at rate $\rho_1 q(x, y)$.

Agents only get utility from flow payoffs. Flow payoffs are constant for the duration of the match, and are determined through bargaining, as discussed in the next subsection. Let $\pi(x, y) : X \times X \to [0, f(x, y)]$, with $\pi(x, y) + \pi(y, x) \leq f(x, y)$, be the flow payoff agent $x$ receives when matched to agent $y$. Unmatched agents obtain a zero flow payoff.

We denote the value function of an $x$-type agent by $V(x, \varnothing)$ when she is unmatched and by $V(x, y)$ when she is matched to a $y$-type. Values are given by

$$\left[ r + \rho_0 \sum_{y' \in X} d(x, \varnothing, y') \, q(x, y') \right] V(x, \varnothing) = 0 + \rho_0 \sum_{y' \in X} d(x, \varnothing, y') \, q(x, y') \, V(x, y')$$

and by

$$\begin{aligned} \Big[ r + \overbrace{\delta + \rho_1 \sum_{y' \in X} d(x, y, y') q(x, y') + \rho_1 \sum_{x' \in X} d(y, x, x') q(y, x')}^{\text{all transitions}} \Big] V(x, y) = {} & \overbrace{\pi(x, y)}^{\text{flow payoff}} \\ + \underbrace{\Big[ \delta + \rho_1 \sum_{x' \in X} d(y, x, x') q(y, x') \Big] V(x, \varnothing)}_{\text{transitions to being unmatched}} + & \underbrace{\rho_1 \sum_{y' \in X} d(x, y, y') q(x, y') V(x, y')}_{\text{match to match transitions}} . \end{aligned}$$

It is usually more convenient to work directly with the surplus agents obtain relative to being unmatched. Surplus $S(x, y) : X \times X \to \mathbb{R}$ is simply given

[5] It is implicit in our formulation that a matched agent never chooses to become unmatched.

by $S(x,y) = V(x,y) - V(x,\varnothing)$. We distinguish individual surpluses $S(x,y)$ and $S(y,x)$ from the total surplus of the match $S(x,y) + S(y,x)$ because the total surplus is not necessarily split symmetrically (as we show in the next subsection).

## 2.1 Timing and Bargaining

We propose the following timing. When two agents meet they observe each others' type. Before bargaining, each agent decides whether to create a match together. If both agents are willing to form a match, a transition occurs, and any previous match is dissolved. Therefore, when an agent bargains with her partner, she cannot exploit the existence of an alternative partner to improve her bargaining position. As a result, the outside option is always the value of being unmatched.[6]

When agents search on the match, the bargaining set is non-standard. In what follows we describe the bargaining set in detail. Once agents $x$ and $y$ form a match, they bargain on how to split the output. This allocation of output remains in place until the match breaks (exogenously or endogenously). Whenever a matched agent meets a potential partner with whom she anticipates a higher surplus, she leaves her current partner. Agents cannot commit not to leave each other, and cannot engage in renegotiation when an offer arrives.

The state of the economy is summarized by a pair $(\mathcal{V}, q)$. The matrix $\mathcal{V} = \{V(x,y)\}_{(x,y)\in X\times X\cup\{\varnothing\}}$ represents the values that agents obtain in each possible match and when unmatched. The matrix $q = \{q(x,y)\}_{(x,y)\in X^2}$ represents the rates of finding willing partners. Agents take the state of the economy as given.

A possible agreement $c = (d,\pi)$ between $x$ and $y$ specifies 1) and allocation $\pi$ and 2) the decisions $d$ that agents $x$ and $y$ will make when meeting potential partners. Let $d = \left(\{d\left(x,y,y'\right)\}_{y'\in X}, \{d\left(y,x,x'\right)\}_{x'\in X}\right)$ and $\pi = (\pi(x,y), \pi(y,x))$, with $\pi(x,y) + \pi(y,x) \leq f(x,y)$. Taking the state of the economy $(\mathcal{V}, q)$ as given,

[6] The timing of match-to-match transitions follows Pissarides [1994] and several recent papers (Shimer [2006], Gautier, Teulings, and Van Vuuren [2010], and Bartolucci [2013]).

an agreement $c = (d, \pi)$ induces a surplus pair $S^c = (S^c(x,y), S^c(y,x))$.[7]

Since there is no renegotiation or commitment, we focus on agreements that agents are happy to follow ex-post.

**Definition. Consistent Agreements.** *Fix the state of the economy $(\mathcal{V}, q)$. An agreement $c = (d, \pi)$ is consistent if*

1. $(S^c(x,y), S^c(y,x)) \geq (0,0)$ *and*

2. *For all $y' \in X$,*

$$d\left(x,y,y'\right) \begin{cases} = 1 & \text{if } S(x,y') - S^c(x,y) > 0 \\ \in [0,1] & \text{if } S(x,y') - S^c(x,y) = 0 \\ = 0 & \text{if } S(x,y') - S^c(x,y) < 0 \end{cases}$$

*and the same holds for $d(y,x,x')$, for all $x' \in X$.*[8]

With this definition in hand, we can define our bargaining sets:

**Definition. Bargaining Sets $\mathcal{S}$ under On-the-Match Search.** *Fix the state of the economy $(\mathcal{V}, q)$. Agents $x$ and $y$ bargain over*

$$\mathcal{S}_{xy} = \left\{ (S^c(x,y), S^c(y,x)) \ \text{ for some consistent agreement } c \right\}.$$

Bargaining sets under on-the-match search have features that make the bargaining problem non-trivial. They may be non-convex, so Nash's assumptions [1950] are not satisfied.[9] Kaneko [1980] presents an extension of Nash's model that

[7]The surplus $S^c(x,y)$ induced by agreement $c$ is given by:

$$\begin{aligned} &\left[r + \delta + \rho_1 \sum_{x' \in X} d\left(y,x,x'\right) q\left(y,x'\right)\right] S^c(x,y) = \pi(x,y) \\ &\qquad + \rho_1 \sum_{y' \in X} d\left(x,y,y'\right) q\left(x,y'\right) \left[S(x,y') - S^c(x,y)\right] - \rho_0 \sum_{y' \in X} d\left(x,\varnothing,y'\right) q\left(x,y'\right) S(x,y') \end{aligned}$$

[8]Moreover, we say that agent $x$'s decisions when unmatched are consistent if $d(x,\varnothing,y) = 1$ if $S(x,y) \geq 0$ and zero otherwise.

[9]Bargaining sets may also be non-comprehensive. $\mathcal{S}$ is comprehensive if $0 \leq x \leq y$ and $y \in \mathcal{S}$

allows for non-convex sets. Kaneko's version of Nash's axioms permits set-valued decision functions. A decision correspondence $\phi$ assigns to each compact subset $S$ of $\mathbb{R}^2_+$ a non-empty subset $\phi(S) \subset S$.[10] Kaneko shows that a decision correspondence $\phi$ satisfies his axioms if and only if it maximizes the product of individual surpluses. In our model, the bargaining sets $\mathcal{S}_{xy}$ are compact (see Appendix A.1 for details). We thus follow Kaneko [1980]. An agreement that maximizes the product of individual surpluses solves the bargaining problem, as in Nash [1950]. However, the total surplus is not always split symmetrically because bargaining sets are non convex.[11]

## 2.2 Equilibrium

We can now formulate our notion of equilibrium in this economy.

**Definition. Equilibrium with On-the-Match Search.** *Take a pair of decision functions and allocations $(d^*, \pi^*)$, its induced state of the economy $(\mathcal{V}, q)$ and its resulting bargaining sets $\{\mathcal{S}_{xy}\}_{(x,y)\in X^2}$. We say that $(d^*, \pi^*)$ is an equilibrium if decisions are consistent when unmatched and if for all $(x, y) \in X^2$,*

1. *agreements are consistent,*
2. *surpluses solve the bargaining problem: $(S(x,y), S(y,x)) \in \phi\left(\mathcal{S}_{xy}\right)$, and*
3. *market outcomes are robust: $S(x,y) > S(y,x) \Rightarrow \exists\, y' \neq y : S(x,y) = S(x,y')$.*

implies $x \in \mathcal{S}$. Non-comprehensiveness makes the analysis in Zhou [1997] and others unapplicable in a setup with on-the-match search. See Figure 3 in Appendix A.5 for an example of how bargaining sets look with on-the-match search.

[10] There are three main differences between Nash's and Kaneko's axioms. First, Kaneko assumes strict Pareto Optimality, whereas Nash assumes a weak version. Second, the axiom of independence of irrelevant alternatives (IIA) is now: $T \subset S$, $\phi(S) \cap T \neq \varnothing \Rightarrow \phi(T) = \phi(S) \cap T$. This is consistent with Nash's IIA, but it is a fairly restrictive version. Third, Kaneko assumes a weak form of continuity in the choice correspondence $\phi$.

[11] To see why, consider an example with only two types of agents: $x, y \in \{\ell, h\}$, with $\ell$ slightly less than $h$. Assume agents produce $f(x,y) = x + y$ if matched and zero otherwise. If $\ell$ and $h$ split the total surplus symmetrically, the low-type agent makes marginally more than $\ell$ per period but is dismissed when the high-type agent finds a high-type partner. Therefore, it is more convenient for the low-type agent to receive a per-period payoff of $\ell$ and get a larger expected duration of the match. The high type also benefits from that. Then, for $\ell \approx h$, the outcome from even surplus splitting is dominated.

We provide next a short discussion of our definition of equilibrium and its properties. First, equilibrium outcomes have some straightforward properties. For all matches, allocations exhaust production: $\pi^*(x,y) + \pi^*(y,x) = f(x,y)$. Moreover, agents only perform match-to-match transitions if they are strictly better off after the transition: $d(x,y,y') = \mathbb{1}\{S(x,y') > S(x,y)\}$. These results are direct consequences of the assumption of Strict Pareto Optimality in bargaining. Second, our model is symmetric in that both sides come from the same population. Thus, by construction, a low firm matched to a high worker obtains the same surplus as a low worker matched to a high firm. Third, we focus on equilibria where behavior is a function of own type and partner's type. As a result, equilibrium outcomes with two agents of the same type are symmetric.

Condition 3 in our definition of equilibrium is desirable, although not necessary for our message. We include it for two reasons. First, when it does not hold, the equilibrium does not survive a positive cost of match-to-match transition (we elaborate on this in Appendix A.2). In that sense, it is a robustness condition which restricts the set of equilibria. Second, beyond robustness, condition 3 provides tractability to the model. It imposes symmetric surplus splitting in matches with strict preferences over partners' type.

Before focusing on the sorting pattern, we characterize the equilibrium that features an endogenous preference for higher types:

**Definition. Hyperphily.** *Take an equilibrium $(d^*, \pi^*)$ and its induced state of the economy $(\mathcal{V}, q)$. We say that the equilibrium features hyperphily whenever for any $x \in X$, $S(x,y') > S(x,y) \Leftrightarrow y' > y$.*

## 2.3 Assortative Matching

An equilibrium decision function $d^*$ induces a steady state distribution of matches $e(x,y)$. We are interested in the sorting pattern implied by this distribution.

In Becker's frictionless market [1973] there is positive assortative matching if agents only match with partners of their same type. In contrast, when it takes time to find a partner, agents may form matches with more than one type of partner; hence Shimer and Smith [2000] define sorting in terms of acceptance sets. However, if an agent can search while matched, her acceptance set depends not only on her own type, but also on her current partner's type. Since match-to-match transitions shape the steady state distribution, a characterization of the acceptance sets of unmatched agents is not enough to describe the sorting pattern. Therefore, as in Chade [2006] and Lentz [2010], we describe sorting in terms of stochastic dominance. Let $E(x, y) = \sum_{y' \leq y} e(x, y')$.

**DEFINITION. POSITIVE ASSORTATIVE MATCHING.** *Take any $x, x' \in X$ with $x > x'$. There is positive assortative matching if and only if the distribution of partners of $x$ first order stochastically dominates the distribution of partners of $x'$: $E(x, y) \leq E(x', y)$ for all $y \in X$.*

# 3. Two Motivating Examples

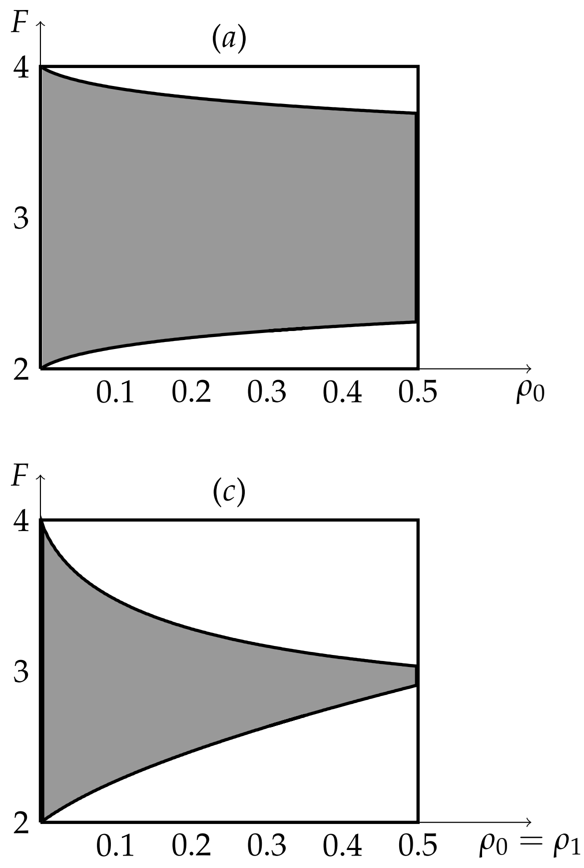
The main insight of this paper is that frictions affect the sorting pattern directly. Frictions generate rents, and rent splitting may induce hyperphily: an equilibrium preference for higher types. When agents can search while matched, hyperphily can lead to positive assortative matching, even with a submodular production function. If instead agents are not allowed to search while matched, different sorting patterns can arise. Before going over our general results, we present a simple example with only two types which captures the main intuition behind these results.

**EXAMPLE 1. TWO TYPES.** *Types from $X = \{1, 2\}$ are present in the population in equal proportion. Production is given by $f(1, 1) = 2$, $f(2, 2) = 4$, and $f(1, 2) = f(2, 1) = F$, with $2 < F < 4$.*

The parameter $F$ captures the degree of complementarity in production. A modular production function has $F = 3$, a supermodular one has $F < 3$, and $F > 3$ corresponds to the submodular cased

The simple environment of Example 1 allows us to explicitly solve the model. Figure 1 illustrates the set of primitives $(F, r, \rho_0, \rho_1, \delta)$ that lead to hyperphily.[12] We focus on two polar cases. First, we take $\rho_1 = \rho_0$, so agents search while matched with the same intensity as they do while unmatched. Panels *c* and *d* correspond to this case. Second, we take $\rho_1 = 0$, so agents cannot search while matched. Panels *a* and *b* correspond to this second case. The shaded areas in panels *a*, *b*, *c* and *d* represent the set of values of *F* consistent with an equilibrium with hyperphily.

Figure 1: Existence of Equilibrium with Hyperphily




Note: In call cases, $\delta = 0.05$. In (*a*) and (*c*), $r = 0.1$. In (*b*), $\rho_0 = 0.1$ (and $\rho_1 = 0$). In (*d*), $\rho_1 = \rho_0 = 0.1$.

Panels *c* and *d* show when hyperphily arises with on-the-match search. As we see in panel *c*, low values of $\rho_0 = \rho_1$ allow for hyperphily even when the production function is significantly submodular. As the intensity of search decreases, the

[12] We explain in Appendix A.5 how we explicitly obtain this set.

probability that a type 2 leaves the match $(1, 2)$ becomes lower, so compensating her to make her stay becomes less attractive. In the limit as $\rho_0 = \rho_1 \to 0$, hyperphily is an equilibrium for all degrees of complementarity in the production function. On the other side, as $\rho_0 = \rho_1 \to \infty$, the duration of any match with voluntary destruction approaches zero. Thus, hyperphily cannot be an equilibrium.

Panel *d* illustrates the intuition discussed in the Introduction. As agents become more impatient (higher $r$), complementarity in production becomes less important relative to rent splitting. In the limit as $r \to \infty$, hyperphily is an equilibrium for any degree of complementarity in the production function. When agents are patient, there are equilibria with hyperphily provided that the complementarity in production is not too strong.

Panels *a* and *b* show when hyperphily occurs without on-the-match search. As in the previous case, when the intensity of search decreases, or agents become more impatient, hyperphily arises even with a submodular production function. Although in both cases higher frictions lead to the same endogenous preferences over matches, its consequence over the actual sorting pattern differs. When the intensity of search while matched and unmatched is the same, hyperphily leads to positive assortative matching.

**LEMMA 1.** *Assume that there are only two types, in equal proportion in the population, as in Example 1. Moreover, let $\rho_0 = \rho_1$. Then, hyperphily leads to positive sorting.*

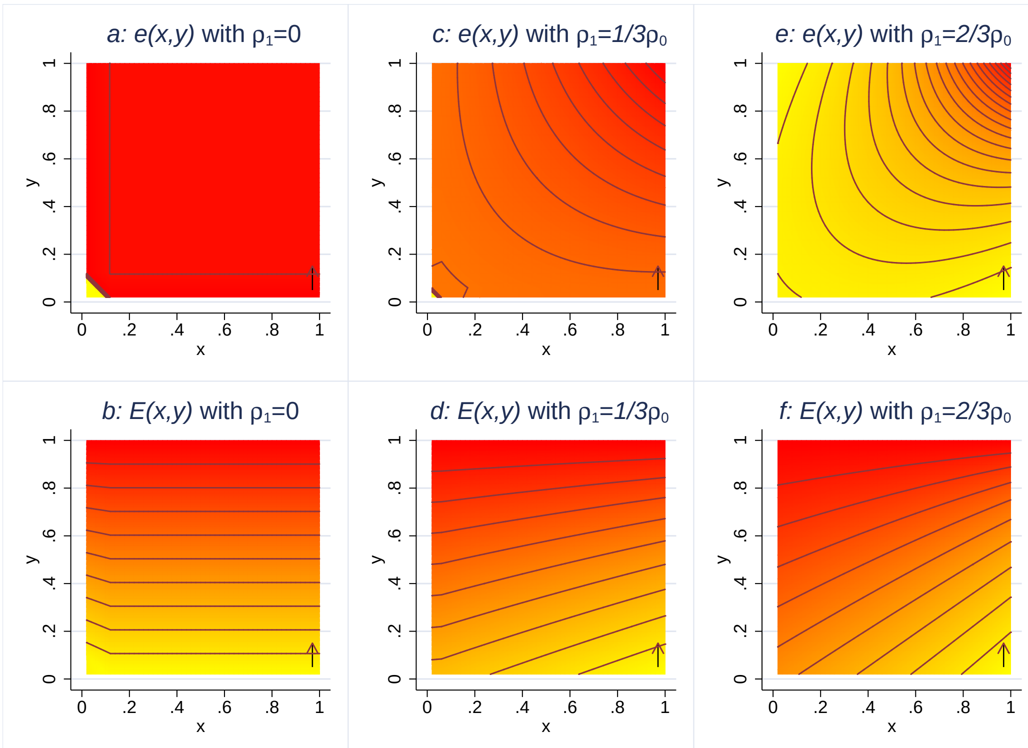
See Appendix A.4 for the proof.

Example 1 and Lemma 1 together illustrate the main message of this paper. With two types and $\rho_0 = \rho_1$, hyperphily arises when frictions are large, and leads to positive sorting. Proposition 1 in the next section extends the first part of this message to a general environment: hyperphily always emerges when frictions are large. However, the effect of hyperphily on the sorting pattern depends in general on whether agents can search while matched, and the relative intensity of search. The following example highlights this point.

**EXAMPLE 2.** *Types are uniformly distributed in a* 100*-point grid between* 0 *and* 1*. Pro-*

*duction is modular:* $f(x,y) = x + y$. ($\delta = 0.05, r = 0.1$ *and* $\rho_0 = \sqrt{0.1}$). *Consider three cases:* $(i) : \rho_1 = 0, (ii) : \rho_1 = \frac{1}{3}\rho_0$, *and* $(iii) : \rho_1 = \frac{2}{3}\rho_0$.

Figure 2: Different On-the-Match and Out-of-the-Match Search Intensity in Example 2




Note: Panels $a$, $c$ and $e$ present the density of matches $e(x,y)$ for different values of $\rho_1$. Panels $b$, $d$ and $f$ present the cumulative distribution of type x's partners $E(x,y)$ for different values of $\rho_1$. Darker points correspond to higher values.

Positive sorting does not hold for low values of $\rho_1$, but it does as $\rho_1$ increases. Upper panels in Figure 2 show densities $e(x,y)$ while lower panels show cumulative distribution of type x's partners $E(x,y) \equiv \sum_{\tilde{y} \in X \cup \{\varnothing\}, \tilde{y} < y} e(x, \tilde{y})$. Without on-the-match search ($\rho_1 = 0$) the equilibrium features hyperphily but no positive assortative matching: the contour lines of $E(x,y)$ are decreasing in $x$ in panel $b$. When we allow agents to search on the match but with low search efficiency, there is assortative matching for agents of high type, but low-type agents still prefer to wait unmatched for more profitable partners. Therefore matching is not positively assortative for low type agents. When instead $\rho_1 = 2/3\rho_0$ there is no difference in acceptance sets between unmatched agents of different types. All unmatched agents accept all partners, and since agents search on the match and prefer better partners, there is positive assortative matching in the whole support of types.[13]

Example 2 shows how hyperphily can arise with many types. It also illustrates that hyperphily does not imply positive sorting when on-the-match search is not allowed. In this example, as the intensity of search increases, positive sorting eventually arises. In the following section, we provide sufficient conditions for positive sorting to arise in general.

# 4. Main Results

We extend now the intuition described in the introduction and illustrated by Examples 1 and 2. We first show that when frictions are high, hyperphily is the unique equilibrium.

We associate high frictions to impatient agents (high values of $r$), high exogenous destruction rates (high values of $\delta$), or low meeting rates (low values of $\rho_0$ and $\rho_1$). Formally, let $\theta \equiv \max\{\rho_0, \rho_1\}^2/(r+\delta)$ denote an index for frictions. We show next that for low enough $\theta$, hyperphily is an equilibrium, and no other equilibrium exists.

The intuition behind the emergence of an endogenous preference for higher types is as follows. When $\theta$ is low, continuation values become less relevant. Therefore, individual surpluses depend mostly on current payoffs, which are close to an equal split of output. Payoffs then depend on total output, which increases in the partner's type. As a result, surplus is increasing in the partner's type for low values of $\theta$. Then, no equilibrium other than hyperphily can exist. Additionally, we show that hyperphily is an equilibrium. In that respect, note first that consistency is straightforward for small $\theta$, given our previous argument. Second, we show in

[13] With a large number of types one cannot characterize equilibrium behavior for all parameter values. We solve the model by a nested fixed point algorithm. See Appendix A.6 for a short description of this procedure.

Appendix A.3 how no alternative consistent agreement leads to a higher product of individual surpluses. Proposition 1 summarizes these findings.

**PROPOSITION 1. FRICTIONS LEAD TO HYPERPHILY.** *Consider an economy with N types, distributed by $l(x)$, and a production function $f(x, y)$. There exists $\widetilde{\theta} > 0$ such that for all $\theta \leq \widetilde{\theta}$ there is a unique equilibrium. This equilibrium features hyperphily.*

See Appendix A.3 for the proof.

We provide next sufficient conditions for positive sorting to emerge. As examples 1 and 2 highlight, even with hyperphily positive sorting may not emerge, for instance when agents are not allowed to search while matched.

**PROPOSITION 2. SUFFICIENT CONDITIONS FOR HYPERPHILY TO LEAD TO POSITIVE SORTING.** *Consider an economy with N types who are equally distributed in the population. Assume moreover that $\rho_0 = \rho_1$ and that $\rho_0^2/\delta < 2$. Then, any equilibrium with hyperphily features positive sorting.*

See Appendix A.4 for the proof.

As Example 2 hints, the sufficient conditions in Proposition 2 are far from necessary. In fact, we have computed the steady state distribution under hyperphily for $1,000$ values of $\rho_0^2/\delta \in (0,1)$ and $1,000$ values of $(\rho_0^2/\delta)^{-1} \in (0,1)$. We have done this for $N = 10, 20$ and $100$ number of types. In all cases we find positive assortative matching.

# 5. Discussion

In this paper we show how frictions can lead to positive assortative matching. While in frictionless markets payoffs reflect individual contributions, the division of output becomes more even when it takes time to find a partner. The total production of the match becomes the main determinant of preferences over partners when frictions are large. Production increases in partner's type, so an endogenous preference for better types (hyperphily) arises. When individuals search while matched, more productive agents upgrade their partners faster. The steady state distribution thus becomes positively assortative.

We show that high frictions lead to an endogenous preferences for higher types, and provide sufficient conditions for this preference to translate into positive sorting. A key element in our analysis is that agents are allowed to search while matched. Match-to-match transitions are pervasive in markets with two-sided heterogeneity. We present a partnership model that includes the key elements of Shimer and Smith [2000] and allows for bilateral on-the-match search.[14]

The conventional wisdom states that stronger frictions require stronger complementarity in production for positive assortative matching to arise. The intuition behind this view is straightforward: with frictions, agents only wait for their preferred partners if the complementarity is strong enough to compensate for the waiting cost. Our paper highlights a different role for frictions. Frictions modify the division of output and therefore shape preferences over partners. If frictions are strong, agents prefer higher types. Therefore, frictions can lead to positive sorting. Our result challenges the interpretation of sorting as evidence of complementarity in production in markets with frictions.

Future work should address the connection between on-the-match search and investment decisions. In our paper, agents' productivities (types) are predetermined. Lower types can extract part of higher types' contribution. This hold-up problem distorts investment decisions when types are endogenous (see Acemoglu and Shimer [1999] or Flinn and Mullins [2015]). However, if agents are allowed to replace their partners, high-type agents can sort to avoid being held up. Under positive sorting, high types end up matched to other high types often and hence they have higher incentives to invest. Therefore, we expect that allowing agents to replace their partners can reduce investment distortions produced by frictions. A rigorous analysis of this point is beyond the scope of this paper, but we believe it deserves further investigation.

[14] Most recent studies on assortative matching in markets with frictions and transferable utility take the canonical model of Shimer and Smith [2000] as a starting point (see Lopes de Melo [2018], Hagedorn, Law, and Manovskii [2017], and Lise, Meghir, and Robin [2016]).

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# A. Appendix

## A.1 Bargaining Sets are Compact

**LEMMA 2.** *Take any state of the economy $(\mathcal{V}, q)$. Then, bargaining sets $\mathcal{S}_{xy}$ under on-the-match search are compact.*

*Proof.* Since $r > 0$ and $f(x, y)$ is finite, $\mathcal{S}_{xy}$ is bounded. We show next that $\mathcal{S}_{xy}$ is also closed. Take a convergent sequence $\{(S^n(x,y), S^n(y,x))\}_{n=1}^{\infty} \in \mathcal{S}_{xy}$, generated by a sequence of consistent agreements $\{(d^n, \pi^n)\}_{n=1}^{\infty}$. Denote this sequence by $S^n$. Let $\left(\overline{S}(x,y), \overline{S}(y,x)\right) = \lim_{n\to\infty} \left(S^n(x,y), S^n(y,x)\right)$. We show next that a consistent agreement generates $\left(\overline{S}(x,y), \overline{S}(y,x)\right)$, so $\left(\overline{S}(x,y), \overline{S}(y,x)\right) \in \mathcal{S}_{xy}$.

We present first a simple intermediate result. Consider a subsequence of $S^n$ with a component that converges to its limit either from below or above, without reaching it. To fix ideas, consider a subsequence with elements that satisfy

$S^n(x,y) < \overline{S}(x,y)$. In this subsequence, agent $x$'s decision function must eventually be constant. Formally, let $\{(S^{n_m}(x,y), S^{n_m}(y,x))\}_{m=1}^{\infty}$ be some subsequence with $S^{n_m}(x,y) < \overline{S}(x,y)$ for all $m$. Then, there exists $M < \infty$ such that $\forall m > M$,

$$\max_{y' \in X: S(x,y') < \overline{S}(x,y)} S(x,y') < S^{n_m}(x,y) < \overline{S}(x,y) \leq \min_{y' \in X: S(x,y') \geq \overline{S}(x,y)} S(x,y')$$

Then, $d^{n_m}(x,y,y') = \overline{d}$ for all $m > M$. It is straightforward to show that the same result holds for any subsequence with $S^{n_m}(x,y) > \overline{S}(x,y)$ for all $m$.

Consider next a subsequence with $S^n(x,y) \neq \overline{S}(x,y)$ and $S^n(y,x) \neq \overline{S}(y,x)$ for all $n$. In this subsequence itself, (at least) one of the following four cases occurs infinitely often: 1) $S^{n_m}(x,y) > \overline{S}(x,y)$ and $S^{n_m}(y,x) > \overline{S}(y,x)$, or 2) $S^{n_m}(x,y) > \overline{S}(x,y)$ and $S^{n_m}(y,x) < \overline{S}(y,x)$, or 3) $S^{n_m}(x,y) < \overline{S}(x,y)$ and $S^{n_m}(y,x) > \overline{S}(y,x)$, or finally 4) $S^{n_m}(x,y) < \overline{S}(x,y)$ and $S^{n_m}(y,x) < \overline{S}(y,x)$. In any such subsequence, for $m$ big enough $d^{n_m} = \overline{d}$ is constant. So $S^{n_m}(x,y)$ and $S^{n_m}(y,x)$ are simply linear functions of $\pi^{n_m}(x,y)$ and $\pi^{n_m}(y,x)$. Since $S^{n_m}$ converges, so does $\pi^{n_m} \to \overline{\pi}$. Moreover, since $\pi^{n_m}(x,y) + \pi^{n_m}(y,x) \leq f(x,y) \ \forall \ m$, then also $\overline{\pi}(x,y) + \overline{\pi}(y,x) \leq f(x,y)$. Thus, the agreement $(\overline{d}, \overline{\pi})$ generates $\left(\overline{S}(x,y), \overline{S}(y,x)\right)$ and is consistent.

The only case left is one with either $S^n(x,y) = \overline{S}(x,y)$ or $S^n(y,x) = \overline{S}(y,x)$ i.o. If $(S^n(x,y), S^n(y,x)) = \left(\overline{S}(x,y), \overline{S}(y,x)\right)$ for some $n$, then $\left(\overline{S}(x,y), \overline{S}(y,x)\right) \in \mathcal{S}_{xy}$. Without loss of generality, assume that $S^n(x,y) = \overline{S}(x,y)$ i.o. Then there is a subsequence $\{(S^{n_m}(x,y), S^{n_m}(y,x))\}_{m=1}^{\infty}$ with $S^{n_m}(x,y) = \overline{S}(x,y)$ and either always $S^{n_m}(y,x) > \overline{S}(y,x)$, or always $S^{n_m}(y,x) < \overline{S}(y,x)$. In any such subsequence for $m$ big enough $d^{n_m}(y,x) = \overline{d}_y$. Since $S^{n_m}(x,y) = \overline{S}(x,y)$, $\pi^n(x,y) = \overline{\pi}(x,y)$ is also constant. Define $\overline{\pi}(y,x) = f(x,y) - \overline{\pi}(x,y) \geq \pi^n(y,x)$. Let $\overline{d}_x$ be the most beneficial to agent $y$ (so $x$ does not leave if indifferent). Let $\widetilde{S} = (\overline{S}(x,y), \widetilde{S}(y,x))$ be induced by $(\overline{d}_x, \overline{d}_y)$ and $(\overline{\pi}(x,y), \overline{\pi}(y,x))$. $\overline{d}_x$ is no worse than what $y$ gets in the subsequence, and $\overline{\pi}(y,x) \geq \pi^n(y,x)$. Then $S^{n_m}(y,x) \leq \widetilde{S}(y,x)$. Thus, $\overline{S}(y,x) = \lim_{n\to\infty} S^n(y,x) \leq \widetilde{S}(y,x)$. If $\overline{S}(y,x) = \widetilde{S}(y,x)$ we are done. Otherwise, decrease $\overline{\pi}(y,x)$ to make it so. ■

## A.2 Details on Multiplicity of Equilibria

Conditions 1 and 2 in our definition of equilibrium are not enough to weed out some fragile outcomes under on-the-match search.[15] Several divisions of output can satisfy these two conditions for a given decision function $d^*$, but not all of them are robust. Consider $(d^*, \pi^*)$ satisfying conditions 1 and 2 and leading to equal surplus splitting in match $(x,y)$. Take an alternative $(d^*, \pi^{**})$ with the *same* decision function and a small perturbation *only* in match $(x,y)$'s payoffs. Individual

[15] In the Online Appendix we provide a simple example of how on-the-match search can lead to some uninteresting multiplicity of equilibrium.

surpluses change only marginally, so agreements can still be consistent. Regarding condition 2, note that under the alternative $(d^*, \pi^{**})$ agent $x$ expects $\pi^{**}(x,y)$ when matched to *any* type-$y$ agent. If $y$ offers $x$ less than that, $x$ breaks the match whenever she finds *another* type-$y$ agent. Such an offer increases $y$'s flow payoff marginally while the probability that $x$ leaves increases discretely, making both partners worse off. To sum up, once $(\pi^{**}(x,y), \pi^{**}(y,x))$ is expected, any small deviation from it leads to a lower surplus for both partners. This example highlights that even keeping $d^*$ and payoffs in all other matches fixed, several divisions of production in match $(x,y)$ can satisfy conditions 1 and 2.

We include condition 3 to rule out fragile cases like $(d^*, \pi^{**})$, which would not survive a positive cost of transition. If breaking a match were costly, agent $x$ would not leave for *another* type-$y$ agent when receiving slightly less than the expected $\pi^{**}(x,y)$. So slight deviations from $(\pi^{**}(x,y), \pi^{**}(y,x))$ would increase the surplus of one agent while reducing the surplus of the other one *in the same amount* (as long as these slight deviations do not make agents leave for *other different types*). Thus, only symmetric surplus splitting would maximize the product of individual surpluses. Our third condition states that an agent can get a higher surplus than her partner only if she is indifferent between her current partner and a partner of a *different* type. This condition guarantees that equilibria are robust in the following sense.

Assume that agents have to pay a small cost $t > 0$ each time they quit their current partner to form a new match. Surplus from matches are then given by the following slightly modified version of (1):

$$S(x,y) = \Big[r + \delta + \rho_1 \sum_{x' \in X} d(y,x,x')q(y,x')\Big]^{-1} \Bigg[\pi(x,y) \\ + \rho_1 \sum_{y' \in X} d(x,y,y')q(x,y') \left[S(x,y') - S(x,y) - t\right] - \rho_0 \sum_{y' \in X} d(x,\varnothing,y')q(x,y')S(x,y')\Bigg]$$

Take a pair $(d^*, \pi^*)$ satisfying the first two conditions in our equilibrium definition. We show next that $S(x,y) > S(y,x) \Rightarrow \exists\ y' : S(x,y) = S(x,y') - t$ must be satisfied. Assume it is not. Then, there exists an alternative consistent agreement between $x$ and $y$ which leads to a higher product of individual surpluses. To build it, keep the decision function unchanged but pick $\tilde{\pi}(x,y) = \pi^*(x,y) - \varepsilon$ and $\tilde{\pi}(y,x) = \pi^*(y,x) + \varepsilon$. For small $\varepsilon > 0$, agent $x$ does not change his behavior. Thus, the new pair $(d^*, \tilde{\pi})$ is consistent. Moreover, again for small $\varepsilon > 0$, the product of individual surpluses is larger. Then, the original pair $(d^*, \pi^*)$ does not solve the bargaining problem.

## A.3 Proof of Proposition 1

Individual surpluses are given by

$$
\begin{aligned}
rS(x,y) = {} & \pi(x,y) - \Big[\delta + \rho_1 \sum_{x' \in X} d(y,x,x')q(y,x')\Big] S(x,y) \\
& + \rho_1 \sum_{y' \in X} d(x,y,y')q(x,y') \left[S(x,y') - S(x,y)\right] - \rho_0 \sum_{y' \in X} d(x,\varnothing,y')q(x,y')S(x,y')
\end{aligned} \tag{1}
$$

The maximum possible production of a match is $\overline{F} = \max_{(x,y)\in X^2} f(x,y)$. The following intermediate lemma highlights several bounds for the individual surplus:

**LEMMA 3. BOUNDS ON SURPLUS.** *Fix the state of the economy $(\mathcal{V}, q)$.*

1. *Under any consistent agreement $c$, individual surpluses are bounded above by the present value of the flow of the maximum possible product: $S^c(x,y) \leq \overline{F}/(r+\delta)$.*
2. *This results in the following bounds for any consistent agreement $c = (d,\pi)$:*

$$
\pi(x,y) - \theta\overline{F} \leq (r+\delta+\rho)S^c(x,y) \leq \pi(x,y) + \theta\overline{F}. \tag{2}
$$

3. *For any match with positive surplus in equilibrium, there is a consistent agreement $c = (d,\pi)$ where production is split evenly: $\pi(x,y) = \pi(y,x) = f(x,y)/2$. Then,*

$$
f(x,y)/2 - \theta\overline{F} \leq (r+\delta+\rho)S^c(x,y). \tag{3}
$$

*Proof.* Let $S(x,\overline{y}) = \max_y S(x,y)$. Take a consistent agreement $c = (d,\pi)$ in pair $(x,\overline{y})$. Then,

$$
\begin{aligned}
rS(x,\overline{y}) &= \pi(x,y) - \Big[\delta + \rho_1 \sum_{x' \in X} d(\overline{y},x,x')q(\overline{y},x')\Big] S(x,\overline{y}) \\
&\quad - \rho_0 \sum_{y' \in X} d(x,\varnothing,y')q(x,y')S(x,y') \\
&\leq \pi(x,y) - \delta S(x,\overline{y}) \leq f(x,\overline{y}) - \delta S(x,\overline{y}) \leq \overline{F} - \delta S(x,\overline{y})
\end{aligned}
$$

This shows part 1. Consider again equation (1) and a consistent agreement $c = (d,\pi)$ for pair $(x,y)$.

$$
\begin{aligned}
rS^c(x,y) &\leq \pi(x,y) - \delta S^c(x,y) + \rho_1 \max\{\rho_0,\rho_1\} \left[\overline{F}/(r+\delta) - S^c(x,y)\right] \\
S^c(x,y) &\leq (r+\delta+\rho)^{-1}\left[\pi(x,y) + \rho\overline{F}/(r+\delta)\right]
\end{aligned}
$$

And,

$$
\begin{aligned}
rS^c(x,y) &\geq \pi^c(x,y) - (\delta+\rho)S^c(x,y) - \rho\overline{F}/(r+\delta) \\
S^c(x,y) &\geq (r+\delta+\rho)^{-1}\left[\pi^c(x,y) - \rho\overline{F}/(r+\delta)\right]
\end{aligned}
$$

This shows part 2. For the last part, consider an equal split of production. Assume that agent $y$ *always* leaves agent $x$. Compute $x$'s surplus and his resulting decision, that is, who would $x$ actually leave $y$ for. Considering this decision from $x$, compute $y$'s surplus and $y$'s resulting decision. Keep doing this until decisions are consistent. Note that at each step the computed surplus is weakly larger. As a result, the set of alternative agents who are preferred must always weakly shrink. Since there is a finite number of agents, the process eventually stops. By then, there is a consistent agreement $c$ with surplus $S^c(x,y)$ bounded below as follows:

$$rS^c(x,y) \geq f(x,y)/2 - (\delta+\rho)S^c(x,y) - \rho\overline{F}/(r+\delta)$$

This completes the proof. ■

We show next that individual payoffs must approach an even split of production as $\theta$ approaches zero. Take equations (2) and (3) from Lemma 3. Since $S(x,y)S(y,x) \geq S^c(x,y)S^c(y,x)$ for all consistent agreements $c=(d,\pi)$, then:

$$\left[f(x,y)/2 - \theta\overline{F}\right]^2 \leq \left(\pi^*(x,y)+\theta\overline{F}\right)\left(\pi^*(y,x)+\theta\overline{F}\right)$$
$$\left[f(x,y)/2 - \theta\overline{F}\right]^2 \leq \pi^*(x,y)\pi^*(y,x)+\theta\overline{F}\left(\theta\overline{F}+f(x,y)\right)$$

Then, since $\pi^*(x,y)\pi^*(y,x) \leq [f(x,y)/2]^2$,

$$\underbrace{\left[\frac{f(x,y)}{2}-\theta\overline{F}\right]^2}_{\to\frac{f(x,y)}{2}} - \underbrace{\left[f(x,y)+\theta\overline{F}\right]}_{\to\frac{f(x,y)}{2}}\underbrace{\theta\overline{F}}_{\to 0} \leq \pi^*(x,y)\pi^*(y,x) \leq \left[\frac{f(x,y)}{2}\right]^2.$$

This directly implies that $\lim_{\theta\to 0}\pi^*(x,y)\pi^*(y,x) - [f(x,y)/2]^2 = 0$, so

$$\lim_{\theta\to 0}\pi^*(x,y) = f(x,y)/2 \quad \forall (x,y)\in X^2. \tag{4}$$

Payoffs approach an even split of production for *all* sequences of equilibria. Consider then two possible matches $(x,y)$ and $(x,y+1)$. Equation (2) implies that

$$[S(x,y+1)-S(x,y)] \geq \pi^*(x,y+1)-\theta\overline{F}-\left(\pi^*(x,y)+\theta\overline{F}\right)$$
$$[S(x,y+1)-S(x,y)] \geq \underbrace{\pi^*(x,y+1)-\pi^*(x,y)}_{\to[f(x,y+1)-f(x,y)]/2} - \underbrace{2\theta\overline{F}}_{\to 0}$$

Thus, for $\theta$ low enough, $S(x,y+1)-S(x,y)>0$, so only hyperphily can be an equilibrium.

We show next that for small $\theta$, hyperphily is indeed an equilibrium. Consider $(d^*,\pi^*)$ under hyperphily. Condition 3 in our equilibrium definition is always satisfied for hyperphily since agents split surplus evenly. Condition 1 (consistency) is guaranteed for $\theta$ small, as shown above. Then, we only need to verify that no consistent agreement $c = (d, \pi)$ leads to a higher product of individual surpluses (condition 2). We do this next.

The bounds in equation (2) from Lemma 3 hold for any consistent agreement $c = (d, \pi)$:

$$\left|(r+\delta+\rho)\left[S^c(x,y)+S^c(y,x)\right]-\pi(x,y)+\pi(y,x)\right| \leq 2\theta\overline{F}.$$

Thus $\lim_{\theta\to 0}(r+\delta+\rho)\left[S^c(x,y)+S^c(y,x)\right] = f(x,y)$. Hyperphily is consistent, and $S(x,y) = S(y,x)$. Then, $\lim_{\theta\to 0}(r+\delta+\rho)S(x,y) = f(x,y)/2$.

Assume there is an alternative consistent agreement $c = (d, \pi)$ that leads to a higher product of individual surpluses:

$$\begin{aligned} S^c(x,y)S^c(y,x) &> S(x,y)S(y,x) \\ (r+\delta+\rho)^2 S^c(x,y)S^c(y,x) &> (r+\delta+\rho)^2 S(x,y)S(y,x) \\ (r+\delta+\rho)^2\left[S^c(x,y)+S^c(y,x)-S^c(y,x)\right]S^c(y,x) &> (r+\delta+\rho)^2 S(x,y)S(y,x) \end{aligned} \tag{5}$$

Any agreement leading to a higher product of individual surpluses must have $S^c(x,y) \geq S(x,y+1)$ or $S^c(y,x) \geq S(y,x+1)$ or both.[16] Assume without loss of generality that $S^c(x,y) \geq S(x,y+1)$ and recall that $\lim_{\theta\to 0}(r+\delta+\rho)S(x,y+1) = f(x,y+1)/2$. Then,

$$\lim_{\theta\to 0}(r+\delta+\rho)\left[S^c(x,y)+S^c(y,x)-S^c(y,x)\right] = f(x,y) - f(x,y+1)/2 < f(x,y)/2$$

Thus,

$$\lim_{\theta\to 0}(r+\delta+\rho)^2\left[S^c(x,y)+S^c(y,x)-S^c(y,x)\right]S^c(y,x) < \left[f(x,y)/2\right]^2$$

But $\lim_{\theta\to 0}(r+\delta+\rho)^2 S(x,y)S(y,x) = \left[f(x,y)/2\right]^2$. This contradicts equation (5). So for $\theta$ small enough there cannot be a consistent agreement which leads to a higher product of individual surpluses. Condition (2) thus holds. ■

[16] To see this, note that if $S^c(x,y) < S(x,y+1)$ and $S^c(y,x) < S(y,x+1)$ then neither agent leaves the other less often. Then $S^c(x,y)+S^c(y,x) \leq 2S(x,y)$.

## A.4 Proofs of Lemma 1 and Proposition 2

We study next the steady state conditions for each match. Let $\rho = \rho_0^2 = \rho_1^2$. Take match $(x, y) \in X \times X$. Inflow $I(x, y)$ and outflow $O(x, y)$ are given by:[17]

$$I(x,y) = \rho \left( \sum_{\tilde{y}<y} e(x,\tilde{y}) \right) \left( \sum_{\tilde{x}<x} e(\tilde{x},y) \right)$$

$$O(x,y) = \delta e(x,y) + \rho e(x,y) \left[ \sum_{\tilde{y}>y} \sum_{\tilde{x}<x} e(\tilde{x},\tilde{y}) + \sum_{\tilde{x}>x} \sum_{\tilde{y}<y} e(\tilde{x},\tilde{y}) \right]$$

Let $\kappa = \frac{\rho}{\delta}$. In steady state, $I(x, y) = O(x, y)$, so

$$\left( \sum_{\tilde{y}<y} e(x,\tilde{y}) \right) \left( \sum_{\tilde{x}<x} e(\tilde{x},y) \right) = e(x,y) \left[ \kappa + \sum_{\tilde{y}>y} \sum_{\tilde{x}<x} e(\tilde{x},\tilde{y}) + \sum_{\tilde{x}>x} \sum_{\tilde{y}<y} e(\tilde{x},\tilde{y}) \right] \quad (6)$$

We express sorting in terms of stochastic dominance. We then focus on the term $\sum_{\tilde{y}<y} e(x+1,\tilde{y}) - e(x,\tilde{y})$. Equation (6) leads to

$$\left( \sum_{\tilde{y}<y} e(x+1,\tilde{y}) - \sum_{\tilde{y}<y} e(x,\tilde{y}) \right) \sum_{\tilde{x}<x} e(\tilde{x},y) = e(x,y) \left[ \sum_{\tilde{y}>y} e(x,\tilde{y}) - 2 \sum_{\tilde{y}<y} e(x+1,\tilde{y}) \right]$$
$$+ [e(x+1,y) - e(x,y)] \left[ \kappa + \sum_{\tilde{y}>y} \sum_{\tilde{x}<x+1} e(\tilde{x},\tilde{y}) + \sum_{\tilde{x}>x+1} \sum_{\tilde{y}<y} e(\tilde{x},\tilde{y}) \right]. \quad (7)$$

Similarly, for unmatched agents $I(x, \varnothing) = O(x, \varnothing)$. Then,

$$\sum_{y\in X} e(x,y) \sum_{\tilde{x}>x} \sum_{\tilde{y}<y} e(\tilde{x},\tilde{y}) = -\frac{\kappa}{N} + e(x,\varnothing) \left[ \kappa + \sum_{\tilde{x}\in X} \sum_{y<x} e(\tilde{x},y) \right] \quad (8)$$

I present next the proof of Lemma 1.

*Proof.* Let there be two types, $\ell$ and $h$. In equation (7), let $x = \ell, x+1 = h$ and $y = h$. Then,

$$\left( \sum_{y\in\{\varnothing,\ell\}} e(h,y) - e(\ell,y) \right) [e(h,\varnothing) + \kappa] = -2e(\ell,h) \sum_{y\in\{\varnothing,\ell\}} e(h,y)$$

Then, $\sum_{y\in\{\varnothing,\ell\}} e(h,y) - e(\ell,y) < 0$. Next, consider the steady state conditions for

[17] $X$ does not include $\varnothing$. But $\sum_{\tilde{y}<y}$ does include it.

$e(h,\varnothing)$ and $e(\ell,\varnothing)$. Given (8), they are respectively given by:

$$0=-\frac{\kappa}{N}+e\left(h,\varnothing\right)\left[\kappa+\sum_{\tilde{x}\in\{\ell,h\}}\sum_{y\in\{\varnothing,\ell\}}e\left(\tilde{x},y\right)\right]\quad\text{and}$$

$$\sum_{y\in\{\varnothing,\ell\}}e\left(\ell,y\right)e\left(h,\varnothing\right)=-\frac{\kappa}{N}+e\left(\ell,\varnothing\right)\left[\kappa+\sum_{\tilde{x}\in\{\ell,h\}}e\left(\tilde{x},\varnothing\right)\right].$$

Thus, $e\left(\ell,\varnothing\right)>e\left(h,\varnothing\right)$. ■

In what follows, I present the proof of Proposition 2. We need to show that for any two types $x, x+1\in X$, $x+1$'s distribution of partners first order stochastically dominates $x$'s:

$$\sum_{\tilde{y}<y}e\left(x+1,\tilde{y}\right)-\sum_{\tilde{y}<y}e\left(x,\tilde{y}\right)<0\qquad\text{for all } x\in X\setminus\{N\}.$$

In order to express equation (7) in simpler terms, we first define:

$$\begin{aligned}
A_x(y)&\equiv\sum_{\tilde{y}<y}e\left(x+1,\tilde{y}\right)-\sum_{\tilde{y}<y}e\left(x,\tilde{y}\right)\\
a_x(y)&\equiv e\left(x+1,y\right)-e\left(x,y\right),\\
b_x(y)&\equiv\sum_{\tilde{x}<x}e\left(\tilde{x},y\right)>0\\
c_x(y)&\equiv\kappa+\sum_{\tilde{y}>y}\sum_{\tilde{x}<x+1}e\left(\tilde{x},\tilde{y}\right)+\sum_{\tilde{x}>x+1}\sum_{\tilde{y}<y}e\left(\tilde{x},\tilde{y}\right)>0\\
D_x(y)&\equiv e(x,y)\left[\sum_{\tilde{y}>y}e\left(x,\tilde{y}\right)-2\sum_{\tilde{y}<y}e\left(x+1,\tilde{y}\right)\right]
\end{aligned}$$

So equation (7) becomes:

$$A_x(y)b_x(y)=a_x(y)c_x(y)+D_x(y)\tag{9}$$

After these preliminaries, we turn now to the main content of the proof. This proof consists of six steps:

1. $\kappa>\frac{1}{2}\Rightarrow e\left(N,\varnothing\right)>\frac{1}{3N}$

   To see this, write (8) for the highest type $N$:

$$0=-\frac{\kappa}{N}+e\left(N,\varnothing\right)\left[\kappa+\sum_{\tilde{x}\in X}\sum_{y<N}e\left(\tilde{x},y\right)\right]$$

Then,

$$e(N, \varnothing) = \frac{\kappa}{N} \frac{1}{\kappa + \sum_{\tilde{x} \in X} \sum_{y < N} e(\tilde{x}, y)} > \frac{\kappa}{N} \frac{1}{\kappa + 1} = \frac{1}{N} \frac{1}{1 + \frac{1}{\kappa}} > \frac{1}{3N}$$

2. Next, we show that $A_x(N) < 0$ for all $x \in X \setminus \{N\}$.

   To see this, note $D_x(N) < 0$. Moreover, $A_x(N) = -a_x(N)$. Given (9), then, $A_x(N) < 0$.

3. If $A_x(y+1) < 0$ and $e(x+1, \varnothing) > \frac{1}{3N}$, then $D_x(y) < 0$. Note,

$$\begin{aligned} A_x(y+1) < 0 &\Leftrightarrow \sum_{\tilde{y} < y+1} e(x+1, \tilde{y}) - e(x, \tilde{y}) < 0 \\ &\Leftrightarrow \sum_{\tilde{y} > y} e(x+1, \tilde{y}) - e(x, \tilde{y}) > 0 \end{aligned}$$

   Then

$$\begin{aligned} \sum_{\tilde{y} > y} e(x, \tilde{y}) - 2\sum_{\tilde{y} < y} e(x+1, \tilde{y}) &< \sum_{\tilde{y} > y} e(x+1, \tilde{y}) - 2\sum_{\tilde{y} < y} e(x+1, \tilde{y}) \\ &< \frac{1}{N} - 3\sum_{\tilde{y} < y} e(x+1, \tilde{y}) \leq \frac{1}{N} - 3e(x+1, \varnothing) < 0 \end{aligned}$$

   which proves $D_x(y) < 0$.

4. If $A_x(y+1) < 0$ and $D_x(y) < 0$, then $A_x(y) < 0$.

   To see this, note that if $A_x(y+1) = A_x(y) + a_x(y)$. Assume $A_x(y) \geq 0$. Then, $a_x(y) < 0$. But if $a_x(y) < 0$ and $D_x(y) < 0$, then (9) guarantees $A_x(y) < 0$.

5. Whenever $A_x(N) < 0$ and $e(x+1, \varnothing) > \frac{1}{3N}$, then $A_x(y) < 0$ for all $y \in X$ and $e(x, \varnothing) > \frac{1}{3}$. Given the previous two points, this step is straightforward. Note that $A_x(1) < 0$ implies $A_x(1) = e(x+1, \varnothing) - e(x, \varnothing) < 0$. This guarantees also $e(x, \varnothing) > \frac{1}{3}$.

6. Steps 1 and 2 guarantee that the assumptions of step 5 hold for $x = N - 1$. Then, we can apply step 5 repeatedly to get our result. ■

## A.5 Hyperphily. The Two Type Case

Under hyperphily, since no agent is indifferent between partners of different types, the total surplus of the match is split evenly in all matches (see equilibrium definition). Therefore, the equilibrium allocations are given by $\pi^*(\ell, \ell) = \ell$, $\pi^*(h, h) = h$, and $\pi^*(\ell, h)$ is set so that $S(\ell, h) = S(h, \ell)$.

Agents' transitions must be consistent with the surplus they obtain in each match. Moreover, we require that, for each match, no consistent agreement leads to a higher product of individual surpluses. Thus, the agreement between agents must be a global maximum in the bargaining set. This is a restrictive condition, which is not easy to check in general. We check each match step by step.

Pair $(d^*, \pi^*)$ is consistent in an equilibrium with hyperphily if the resulting surpluses satisfy

$$S(h,h) > S(h,\ell) \quad \text{and} \quad S(\ell,h) > S(\ell,\ell). \tag{10}$$

We discuss next when $(d^*, \pi^*)$ solves the bargaining problem for each possible match.

### Bargaining Solution in Match $(\ell, h)$

Total surplus is split evenly between $\ell$ and $h$. An agreement leading to a higher product of individual surpluses can only exist if it also induces a larger *total* surplus. Since $\ell$ does not leave the match $(\ell, h)$ under hyperphily, a larger total surplus can only be reached in the match $(\ell, h)$ if $h$ chooses not to leave. Thus, we study consistent agreements between $\ell$ and $h$ where $h$ does not leave. Let $(S^c(\ell,h), S^c(h,\ell))$ denote the surplus in some alternative agreement $c$. $h$ does not leave for a high-type agent only if $S^c(h,\ell) \geq S(h,h)$.

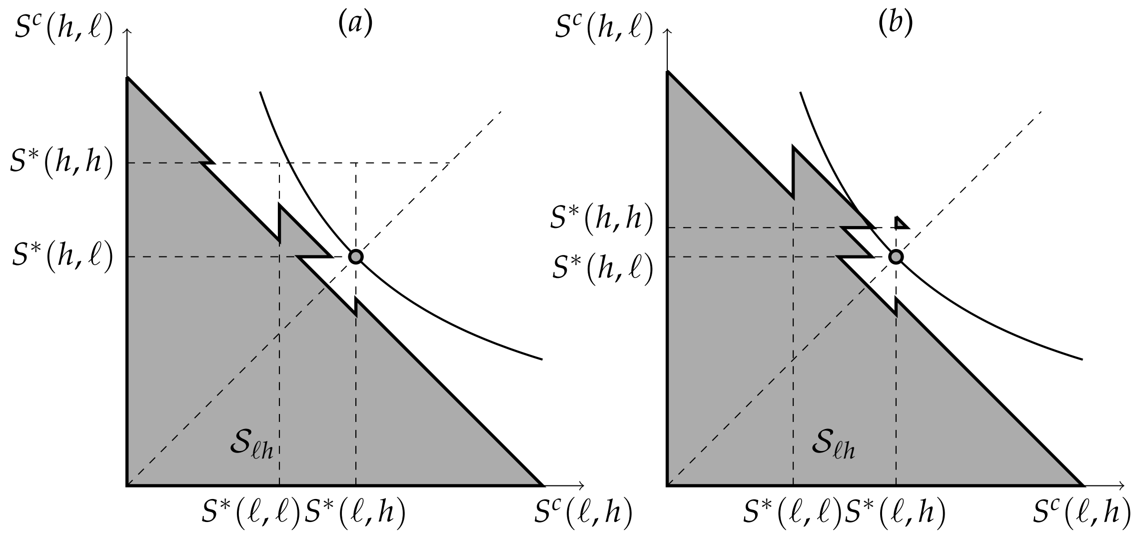
There are three possible kinds of agreements with $h$ staying. In the first kind of agreement ($c_1$), both $\ell$ and $h$ choose not to leave each other. In the second one ($c_2$), $h$ always stays, but $\ell$ leaves when she finds a new $h$. In the third one ($c_3$), $h$ always stays, but $\ell$ leaves when she finds *any* new partner. If the first kind of agreement exists, it makes both agents better off, so our original candidate is not an equilibrium. The second and third cases involve $\ell$ obtaining a lower surplus. We need to check whether a higher product of individual surpluses is attained in these cases. Pair $(d^*, \pi^*)$ solves the bargaining problem in match $(\ell, h)$ if and only if Condition 1 holds.

**CONDITION 1.** *Let $c_1, c_2$ and $c_3$ be defined as stated. No allocation generates*

1. $S^{c_1}(h,\ell) \geq S(h,h)$ *and* $S^{c_1}(\ell,h) \geq S(\ell,h)$, *or*
2. $S^{c_2}(h,\ell) \geq S(h,h)$, $S(\ell,\ell) \leq S^{c_2}(\ell,h) < S(\ell,h)$ *and* $S^{c_2}(\ell,h)S^{c_2}(h,\ell) > S(\ell,h)S(h,\ell)$, *or*
3. $S^{c_3}(h,\ell) \geq S(h,h)$, $S^{c_3}(\ell,h) < S(\ell,\ell)$ *and* $S^{c_3}(\ell,h)S^{c_3}(h,\ell) > S(\ell,h)S(h,\ell)$.

Figure 3 presents two examples to illustrate how bargaining sets are built and how to verify Condition 1. The trade-off between expected duration and flow payoff makes the bargaining sets non-convex. To see why, take the boundary of bargaining set $\mathcal{S}_{\ell h}$ in panel *a* in Figure 3. Consider first the point that gives $\ell$ zero surplus and $h$ his maximum possible surplus on $\mathcal{S}_{\ell h}$. At this point, $h$ never leaves the match, while $\ell$ gets $\pi(\ell, h) = 0$ so she leaves for any alternative partner (of either type). An increase in $\pi(\ell, h)$, together with its corresponding decrease in $\pi(h, \ell)$, increases $S^c(\ell, h)$ in the same amount as $S^c(h, \ell)$ decreases. Thus, for small changes in flow payoffs, the boundary of the bargaining set is linear. However, consider now the point where $S^c(h, \ell) = S(h, h)$. A further increase in $\pi(\ell, h)$ makes $S^c(h, \ell) < S(h, h)$, so $h$ starts leaving whenever she finds another $h$. The expected duration of the match decreases, and although $\pi(\ell, h)$ is higher, $S^c(\ell, h)$ decreases discretely. It is this jump that generates a non-convexity in the bargaining set. In general, bargaining sets are non-convex in the neighborhood of agreements leading to indifference.

Figure 3: Bargaining Sets $\mathcal{S}_{\ell h}$




Note: $\rho = 0.1$, $r = 0.1$ and $\delta = 0.05$. In (a), $F = 3$. In (b), $F = 3.6$.

As Figure 3 illustrates, bargaining sets are built from potentially disjoint compact sets. In fact, agreement $(d^*, \pi^*)$ maps to an isolated point in the bargaining set. Any marginal deviation from $\pi^*$ decreases the expected duration of the match discretely. This occurs because the partner whose flow payoff has been reduced now leaves when she finds a new partner of the *same* type as her current one.

Panel *a* shows a case where condition 1 holds: hyperphily solves the bargaining problem in the match $(\ell, h)$. The shaded area in panel *a* represents the bargaining set $\mathcal{S}_{\ell h}$ under hyperphily and a modular production function. The curve through $(S(\ell, h), S(h, \ell))$ indicates all points attaining product $S(\ell, h) \times S(h, \ell)$. No element in the bargaining set attains a higher product of individual surpluses. Note this occurs without complementarity in production and with patient agents.

Panel *b* shows a case where condition 1 does not hold. When the production function is sufficiently submodular hyperphily is no longer an equilibrium. An alternative consistent agreement leads to a higher product of individual surpluses *and* to a higher individual surplus for both agents. Agent $\ell$ receives *less* than half of a *larger* surplus in order to make her partner indifferent. Still, agent $\ell$ is better off. This violates the first line of Condition 1.

In the example presented in panel $b$ the second line of Condition 1 is also violated. An agreement that makes 1) $h$ indifferent to a match with another $h$ and 2) $\ell$ worse off than in a match to a different $h$ is also consistent and leads to a larger product of individual surpluses.

### Bargaining Solution in Match $(\ell, \ell)$

As in match $(\ell, h)$, there are three cases to consider. In the first ($c_4$), both agents choose not to leave each other. In the second ($c_5$), one agent ($\ell_1$) never leaves while the second one ($\ell_2$) leaves only when finding a willing $h$. In the third ($c_6$), one agent ($\ell_1$) never leaves while the other one ($\ell_2$) leaves when finding *any* willing partner. Pair $(d^*, \pi^*)$ solves the bargaining problem in match $(\ell, \ell)$ if and only if Condition 2 holds.

**CONDITION 2.** *Let $c_4, c_5$ and $c_6$ be defined as stated. No allocation generates*

1. $S^{c_4}(\ell, \ell) \geq S(\ell, h)$, *or*
2. $S^{c_5}(\ell_1, \ell_2) \geq S(\ell, h)$ *and* $S(\ell, \ell) \leq S^{c_1}(\ell_2, \ell_1) < S(\ell, h)$, *or*
3. $S^{c_6}(\ell_1, \ell_2) \geq S(\ell, h)$, $S^{c_6}(\ell_2, \ell_1) < S(\ell, \ell)$ *and* $S^{c_6}(\ell_1, \ell_2) S^{c_6}(\ell_2, \ell_1) > [S(\ell, \ell)]^2$.

We present again two examples to illustrate bargaining, this time on match $(\ell, \ell)$. Panels $a$ and $b$ in Figure 4 present bargaining set $\mathcal{S}_{\ell\ell}$ with hyperphily and a modular production function. In panel $b$, types are closer: $\ell = 1.66$ and $h = 2$, whereas in panel $a$, $\ell = 1$ and $h = 2$. It is easy to see that hyperphily solves the bargaining problem in panel $a$. In panel $b$, however, an alternative agreement with both $\ell$ agents choosing not to leave each other makes them better off, so hyperphily does not solve the bargaining problem.

In the example presented in panel $b$, the second line in Condition 2 is also violated. An agreement that makes 1) one $\ell$ indifferent to a match with $h$ and 2) the second $\ell$ at least as well off as before is also feasible.

### Bargaining Solution in Match $(h, h)$

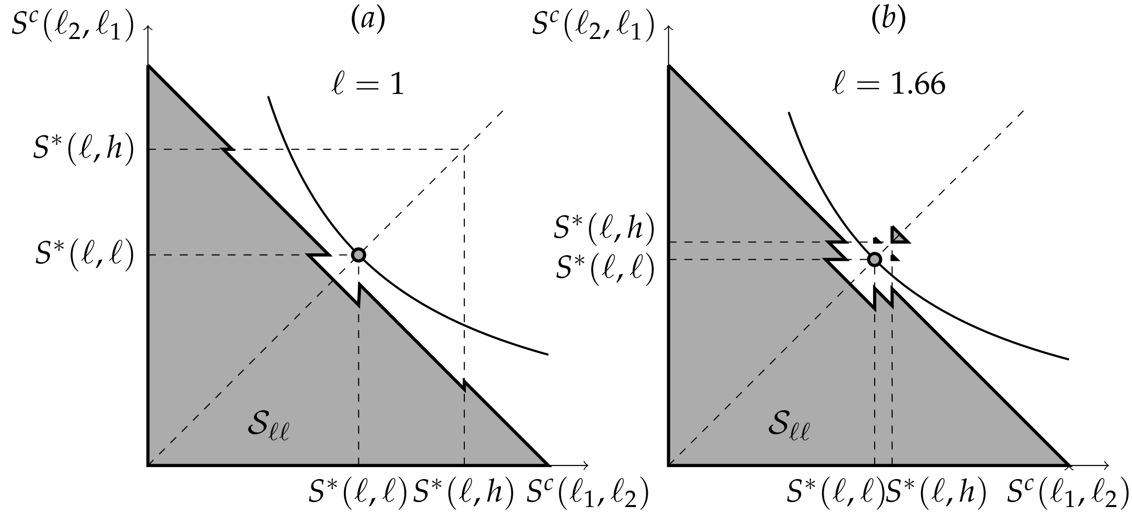
There is no endogenous destruction in match $(h, h)$ and agents split the surplus evenly. Therefore, no consistent alternative agreement leads to a higher product of individual surpluses.

In the Online Appendix we solve explicitly for the set $(\ell, h, F, r, \rho, \delta)$ of primitives such that an equilibrium with hyperphily exists.

## A.6 Numerical Simulations

When the number of types is only two, we can solve explicitly for equilibria. However, with a larger number of types, that becomes impossible. We solve the model

Figure 4: Bargaining Sets $\mathcal{S}_{\ell\ell}$




Note: $\rho = 0.1$, $r = 0.1, \delta = 0.05, h = 2$ and $F = \ell + h$.

by a nested fixed point algorithm. We start from a flat distribution of matches and calculate value functions for all possible matches. These first value functions induce preferences over partner types which we use to update the steady state distribution of matches. With the updated steady state distribution, we update the value functions. We iterate this process until we find a fixed point for both the steady state distribution of matches and the value functions.

We search specifically for equilibria without indifference over partners. When no agent is indifferent, symmetric surplus splitting solves the bargaining problem in all matches. Once we find a candidate set of value functions and distribution of matches that solves the model, we check that the solution maximizes the product of surpluses in all matches. To do this, for each match we evaluate all possible consistent agreements $c = (d, \pi)$, given our candidate. Our candidate solves the model if it maximizes the product of surpluses in every match.