Optimal allocation of attention in user-generated content platforms

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M2 ETE Thesis
Toulouse School of Economics

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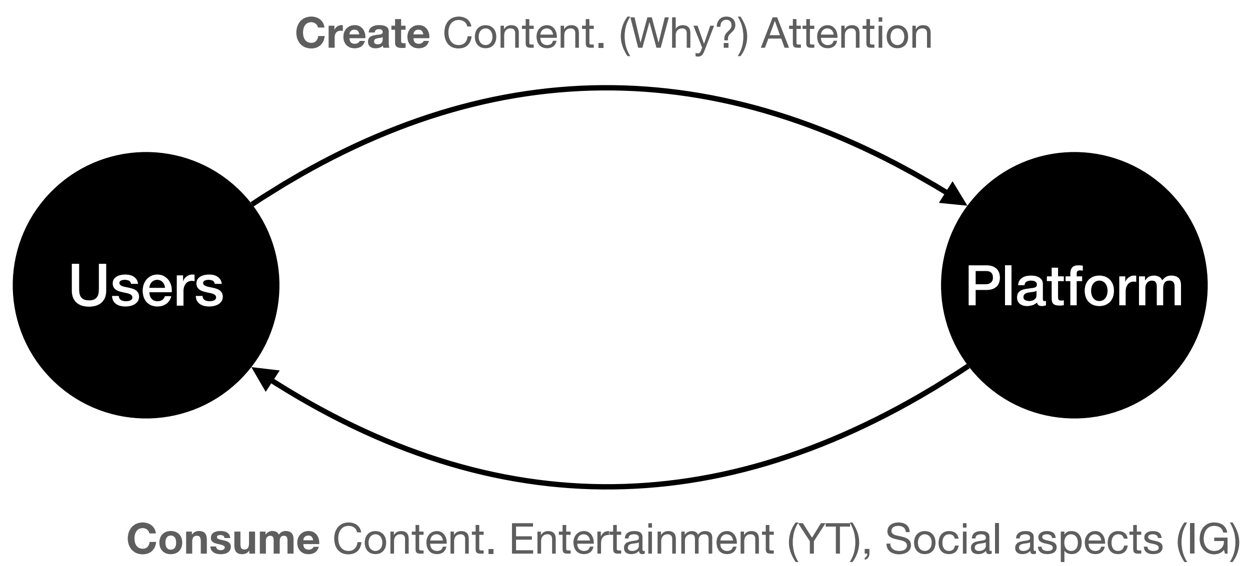

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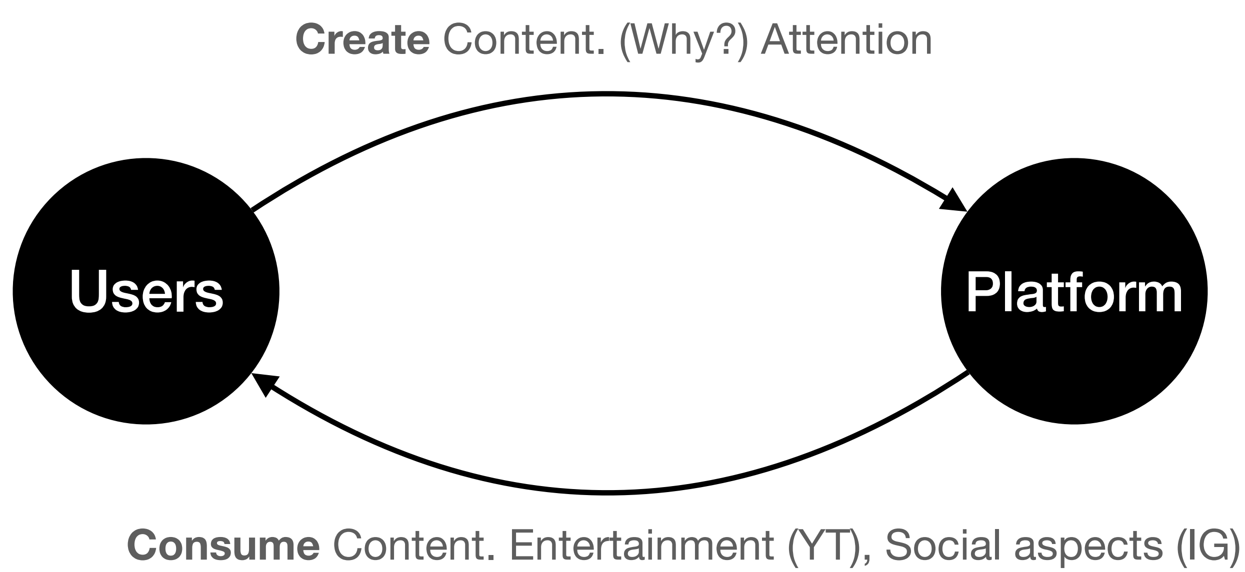

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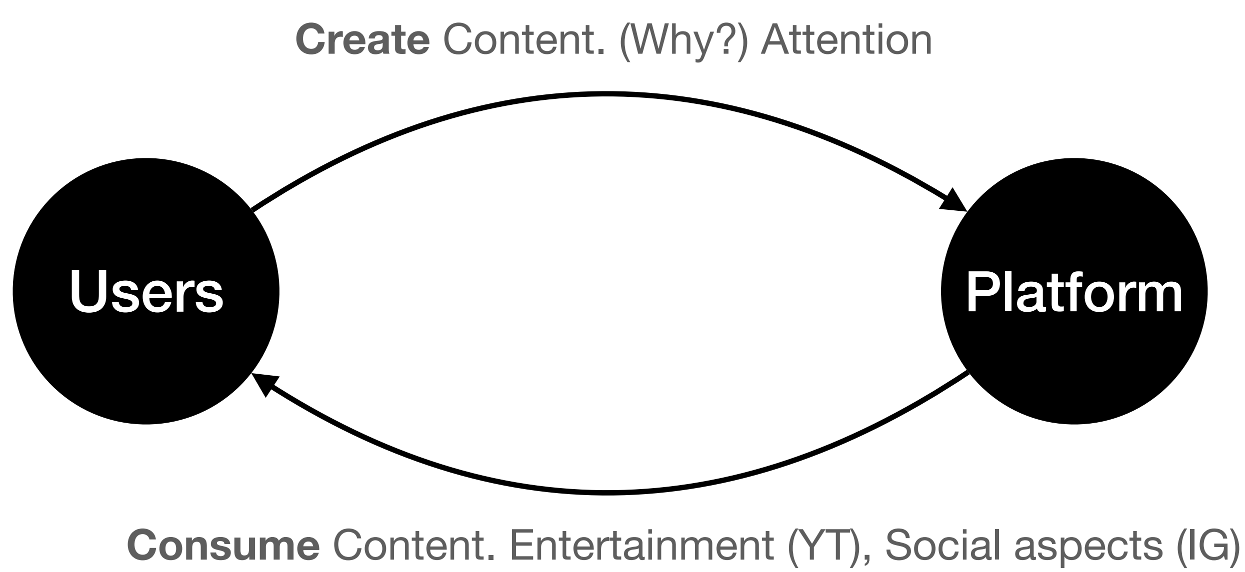

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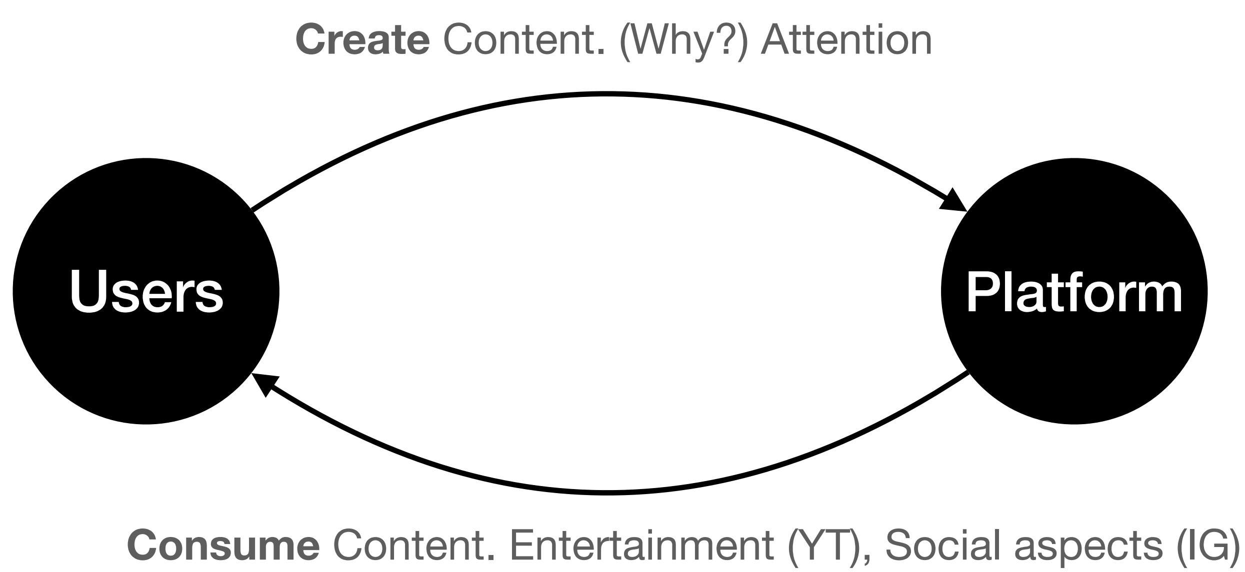

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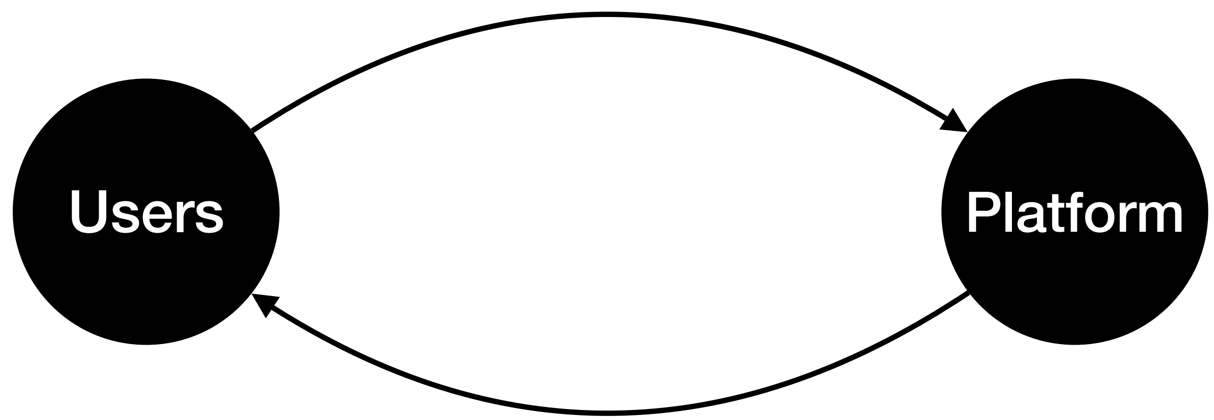
Create Content. (Why?) Attention



Consume Content. Entertainment (YT), Social aspects (IG)

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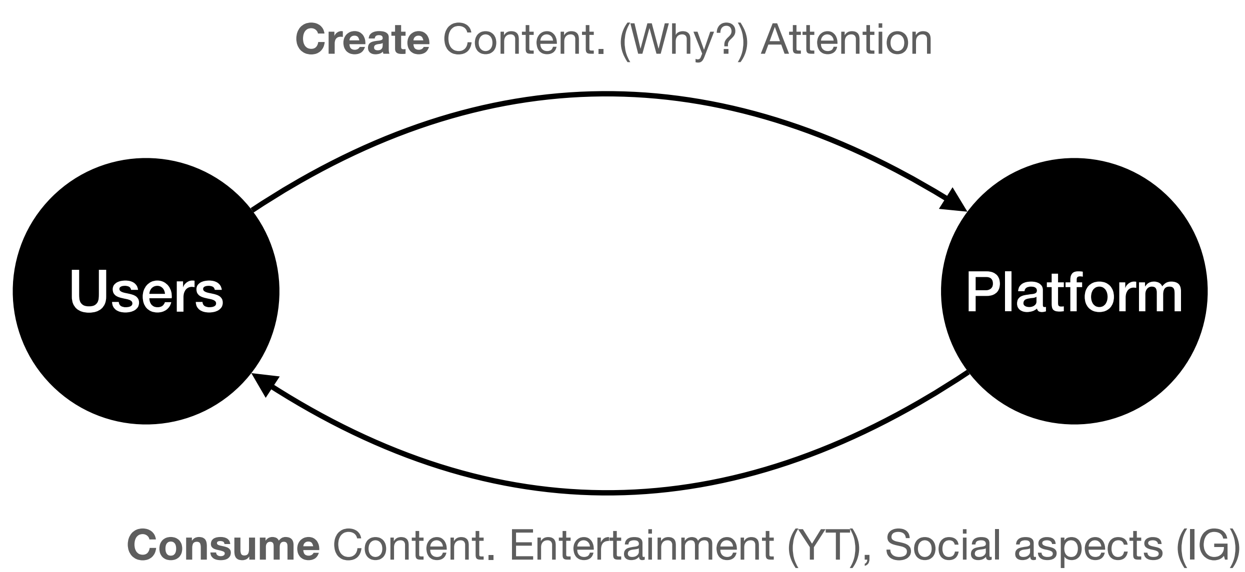

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    - Others tried to incentivise the creation of content from all users and social interaction
        - E.g. Instagram (stories introduction), BeReal… <— “Low quality” content, attention more distributed

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Question to answer in this thesis: For a monopolist platform, which is the **best way to distribute attention** across users such that they are incentivised to create content in a way that maximises its utility (profit/sum of users utilities)?

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Social platforms (IG, Snapchat): users prefer to consume content from a lot of creators
VS Entertainment platforms (Youtube, Twitch)… where quality comes first

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1. **No monetary transfers available**: payment of attention (which is **bounded**)
2. **Utility** of any user in the platform **depends on the transfers** (attention) **paid to the rest** of users.

Outline

1. General Model (theoretical framework)

2. Binary Model (more results)

3. Ad-funded Platforms (application)

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  - Total attention is bounded: $\Sigma_{i=1}^{N} A_i \leq 1$.

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- In the consumption utility, I use $A_j q_j$ and not just q_j because the relevant variable is the **perceived quality**. Otherwise, users derive utility from quality they are not paying attention to.

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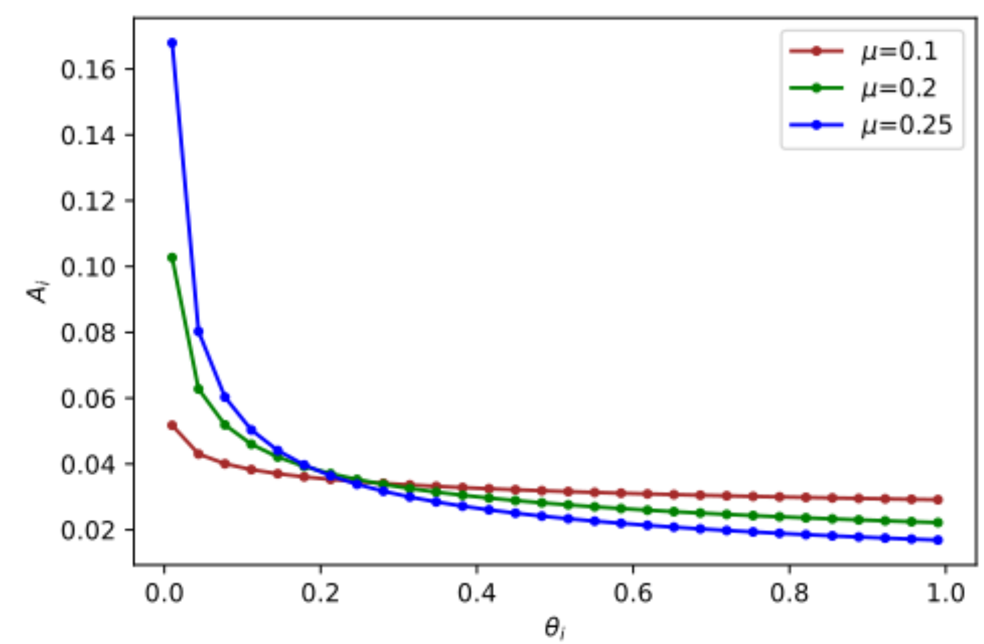
- **Optimal attention shares and qualities:**

$$A_i^* = \frac{\theta_i^{\frac{\mu}{2\mu-1}}}{\sum_{j=1}^{N} \theta_j^{\frac{\mu}{2\mu-1}}} \qquad q_i^* = \frac{A_i^*}{\theta_i} = \frac{\theta_i^{\frac{1-\mu}{2\mu-1}}}{\sum_{j=1}^{N} \theta_j^{\frac{\mu}{2\mu-1}}}$$

Example. $N = 30$ and equidistant costs.



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- **Proposition 2**:
- In the First Best setting, the optimal attention allocation with respect to qualities follows a **Generalized Tullock Contest:**

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(more elegant and tractable seems to be the case of the continuum of agents explored minimally in Appendix B, but its study is left out of this thesis)

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$$\max_{A_L, A_H, q_L, q_H} N_L \left(A_L - q_L\theta_L + (N-1)\left(A_L q_L\right)^{\mu} \right) + N_H \left(A_H - q_H\theta_H + (N-1)(A_H q_H)^{\mu} \right) \qquad \equiv \max_{\mathbf{A},\mathbf{q}} \Sigma_{i=1}^{N} U_i$$

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Proposition 3:

Under standard assumptions, quality of the low-cost type is distorted

$$\exists\, \mu \in \left(0, \frac{1}{2}\right) \;:\; q_L(\mu)^{FB} \neq q_L(\mu)^{SB}$$

Binary Model

(The first best is a special case of the general case)

- Platform P knows $\theta_L < \theta_H$ and proportions f_L, f_H of users of each type, but **not which type each user is.**
- Assume N is large enough such that it is admissible consider that the actual proportions are the theoretical ones and $N_k = f_k N$ for $k \in \{L, H\}$.
- The platform offers a menu of contracts $\{A_i, q_i\}_{i=L,H}$ where allocates A_i in exchange for production of q_i and each user chooses its preferred one. They are designed incentivising self-revelation through IC constraint(s)

$$\max_{A_L, A_H, q_L, q_H} \; N_L\left(A_L - q_L\theta_L + (N-1)\left(A_L q_L\right)^{\mu}\right) + N_H\left(A_H - q_H\theta_H + (N-1)(A_H q_H)^{\mu}\right) \qquad \equiv \max_{\mathbf{A},\mathbf{q}} \Sigma_{i=1}^{N} U_i$$

$$\text{s.t.} \begin{cases} A_L - q_L\theta_L \geq A_H - q_H\theta_L & (\mathrm{IC_L}) \\ A_H - q_H\theta_H \geq 0 & (\mathrm{IR_H}) \\ N_L A_L + N_H A_H \leq 1 & (\text{Feasibility } 1) \\ 0 \leq A_i \leq 1 & (\text{Feasibility } 2),\;\; \mathrm{i} \in \{\mathrm{L}, \mathrm{H}\} \end{cases}$$

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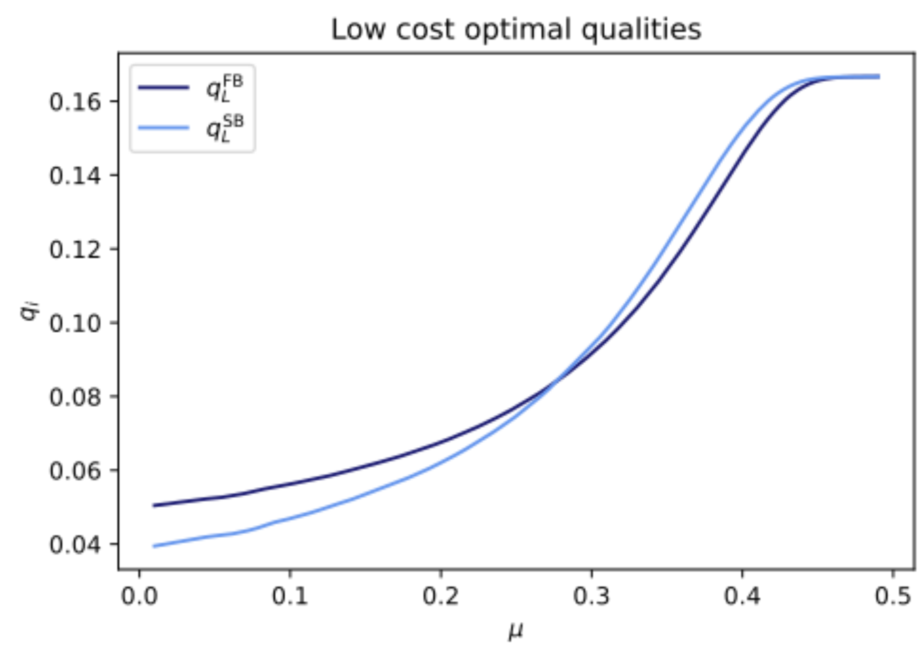
Conjecture 1:

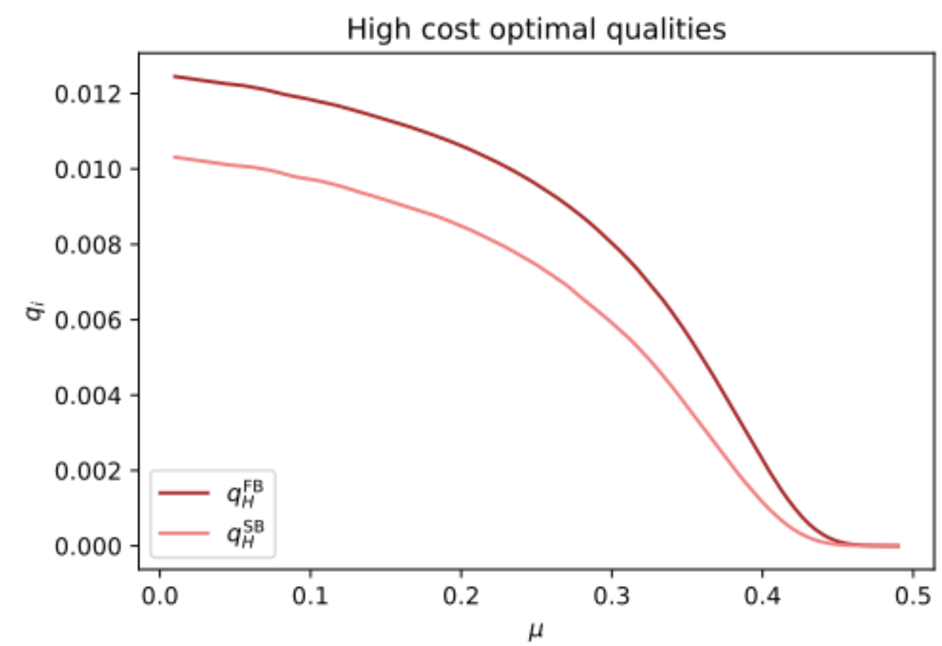
Moreover, the direction of the distortion depends on the preference for variety μ

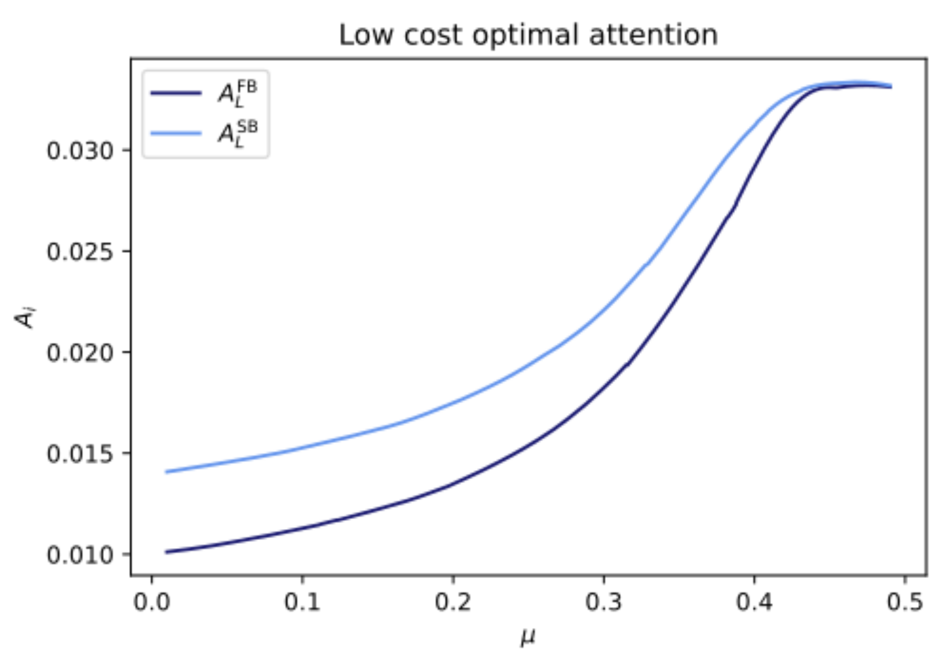
$$\begin{cases} q_L(\mu)^{SB} < q_L(\mu)^{FB} & \text{iff} \quad \mu < \mu^* \\ q_L(\mu)^{SB} = q_L(\mu)^{FB} & \text{iff} \quad \mu = \mu^* \\ q_L(\mu)^{SB} > q_L(\mu)^{FB} & \text{iff} \quad \mu > \mu^* \end{cases}$$

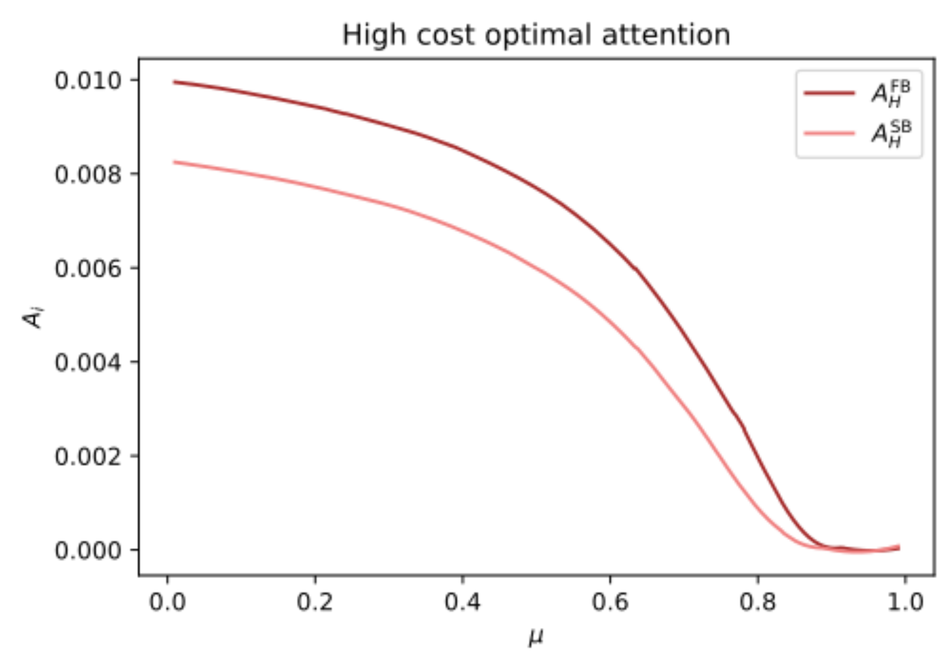
Example: Numerical Solutions

2 options to meet IC: $\begin{cases} \downarrow q_L \\ \uparrow A_L \downarrow A_H \end{cases}$ Optimal one depends on preference for variety μ !









$N_L = 30,\ N_H = 70,\ \theta_L = 0.2,\ \theta_H = 0.8$

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Does not depend on the info context !

Does not depend in any parameter but μ !

Conclusion

Takeaways:

- In complete information, distribution of attention shaped by preferences on diversity
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Different directions in **future research**:

- Make the model continuous
- Heterogeneous μ_i across agents and platforms
- Behavioural aspects (e.g. addiction)