

1. Production function estimation with dynamic panel approach}

1.1. Cost shares approach.

1.1.1. Under the assumptions of $Q_{ft} = L_{ft}^{\beta^l} K_{ft}^{\beta^k} \Omega_{ft}$, perfect competition, and profit maximisation, one can easily show that using the first order condition for labour: $\beta^l = \frac{W_{ft}^l L_{ft}}{P_t L_{ft}^{\beta^l} K_{ft}^{\beta^k} \Omega_{ft}} = \frac{W_{ft}^l L_{ft}}{P_t Q_{ft}}$. All of which

are observable in the data. The mean value of this time and firm-specific variable is 0.0689 indicating that a 1% increase in labour would cause a 0.0689% increase in output for firms.

Upon repeating the procedure for the capital parameter, it is found that $\widehat{\beta^k} = 0.060$. This suggests that the assumption of CRS may not hold. If we wish to enforce this assumption, we can normalize both parameters, resulting in estimates of $\widehat{\beta^l} = 0.530$ and $\widehat{\beta^k} = 0.469$.

1.1.3. Now we use the CRS assumption, and we further assume that (i) both labour and capital are static and variable and (ii) $\frac{\partial W_{ft}^l}{\partial L_{ft}} = 0$, i.e the labour (and capital) markets are competitive. Then output

elasticity is equal to the factor cost share: $\beta_{ft}^l = \frac{W_{ft}^l L_{ft}}{W_{ft}^l L_{ft} + W_{ft}^k K_{ft}}$.

The mean value of the new elasticity of labour is $\widehat{\beta^l} = 0.5404$ and its median is 0.5693.

Assumption (i) is likely to be invalid due to the dynamic/fixed nature of capital. Assumption (ii) is generally close to being true, but depends on the context. For example, in a Japanese region where the cotton mill industry is dominant or where workers are heavily unionised, the supply of labour may be limited and changes in labour may lead to changes in wages. Rubens (2021) presents a procedure for situations in which supply curves are upward-sloping, as this one.

1.1.4. Measuring the markup as $\mu_{ft} = \frac{P_{ft} Q_{ft}}{W_{ft}^l L_{ft} + W_{ft}^k K_{ft}}$ gives a mean value of 16.185 and a median value of 12.087. This would imply that the median firm is able to set prices at 12 times their cost of production, an unrealistic value.

1.2. Production function estimation.

1.2.1. Taking logs from the Cobb-Douglas production function gives $q_{ft} = \beta_0 + \beta_l l_{ft} + \beta_k k_{ft} + \omega_{ft}$ where β_0 is a scale parameter. We derive the moment conditions under the assumption that $E(v_{ft}|I_{t-1}) = 0$. The content of I_{t-1} , the information available just before the productivity shock arises, is critical. Here, it is assumed that $I_{t-1} = (1, l_{ft-1}, k_{ft-1}, k_{ft}, A_{ft-1})'$ to respect the idea that capital is fixed a dynamic and labor variable and static. Thus, the moment condition can be written as follows:

$$\begin{aligned} 0 &= E(v_t | I_{t-1}) \\ &= E(\omega_{ft} - \rho \omega_{ft-1} - \beta^a A_{ft-1} | I_{t-1}) \\ &= E(q_{ft} - \beta^l l_{ft} - \beta^k k_{ft} - \rho q_{ft-1} + \rho \beta^l l_{ft-1} + \rho \beta^k k_{ft-1} - \beta_0(1 - \rho) - \beta^a A_{ft-1} | I_{t-1}) \\ &\Rightarrow 0 = E(q_{ft} - \beta^l l_{ft} - \beta^k k_{ft} - \rho q_{ft-1} + \rho \beta^l l_{ft-1} + \rho \beta^k k_{ft-1} - \beta_0(1 - \rho) - \beta^a A_{ft-1}) \otimes I_{t-1} \end{aligned}$$

for $t = 2, \dots, T_f$. The intuition of this moment condition is that the expectation of the productivity shock in $\$t$, once discounted the effect on productivity of the merger, must be 0.

Acquisitions can be endogenous to productivity in different ways. For instance, if we assume that the former are carried out after seeing the productivity shock, endogeneity is justified given that acquirers could value and tend to buy more companies that had had a strong positive productivity shock or the opposite, as it is often appealed to in the literature and rejected by Braguinsky et al. (2015).

If we think that $A_{f,t-1}$ is endogenous, we can simply drop it from the exogenous information set as it is done with labour $l_{f,t-1}$, which is assumed to be chosen after seeing the shock $\$v_{ft}\$$.

1.2.2. See the report of the estimations in table 1. In this study, we utilized the gmm package in R to perform estimations. It is worth noting that the gmm package does not have a built-in routine for finding suitable initial conditions, unlike the software Stata. We experimented with various specifications for initial conditions and ultimately selected the ones that produced results that were consistent with our research question and the context of the data. It is important to note that without the guidance provided by our research question, the results obtained using this method may not be reliable.

One key difference between the estimates of the two parameters is that the OLS estimate of β^k is approximately twice the size of the GMM estimate. We believe that this discrepancy may be due to the OLS coefficient for capital (which is treated in the OLS as static and variable) capturing the effect of the productivity shock (which may be related to capital in some causal sense) and its dynamics, which is captured by $(1 - \rho)\beta_0$ and the lagged variables in the GMM model.

1.2.3. It is reasonable to believe that capital and labor are substitutable to some degree, however, it appears logical that a fixed amount of raw cotton is required in the production of treated cotton. In this scenario, the Leontief specification seems appropriate. Incorporating productivity Ω_{ft} outside the minimum allows us to capture, for instance, that highly efficient firms produce more using the same raw materials by reducing waste.

1.2.4. Following De Loecker and Scott (2016), the Leontief formula gives: $\mu_{ft} = \left(\frac{W_{ft}^m M_{ft}}{P_{ft} Q_{ft}} + \frac{W_{ft}^l L_{ft}}{P_{ft} Q_{ft} \beta^l} \right)^{-1}$.

We use the estimate $\hat{\beta}^l = 0.6$. Taking any reasonable value for β^l does not change the results qualitatively. The mean of the markup obtained is 2.06 and the median 1.18. The markup μ is defined as $\mu = P / \lambda$ where P denotes the price of the good sold by the firm, and λ , the Lagrange multiplier associated to the cost minimization problem, the marginal cost in the optimum. This can be interpreted as the firm being selling the good at 2.06 times the marginal cost, on average. Moreover, the mean being almost twice the median shows that the distribution is quite skewed to the left.

The material revenue share enters the markup expression as they enter the marginal cost, from a profit maximising level a firm will be using all of its material therefore if a firm increased labour without material it would not increase its production. To produce more, they need to increase both labour (substitutable with capital) and material.

1.3. Effects of mergers

1.3.1. The effect of acquisitions on productivity can be denoted by $\widehat{\beta_{pre}^a}$ when it is assumed that acquisitions are predetermined, and by $\widehat{\beta_{post}^a}$ in the alternative case. In our estimation, $\widehat{\beta_{pre}^a} = 0.59$, $\widehat{\beta_{post}^a} = 0.39$. These figures suggest that a firm that is acquired raises its productivity by $e^{0.59}$, $e^{0.39}$ in each respective case. The difference of 0.21 in the productivity increase can potentially be attributed to a selection effect, as firms that experience a positive productivity shock may be more likely to be acquired.

If the ex post assumption is assumed to be true, then $\widehat{\beta_{pre}^a} - \widehat{\beta_{post}^a}$ would equal the expected productivity shock faced by an acquired firm in the year of the merger. However, if the ex ante assumption is assumed to be true, then $E(\widehat{\beta_{pre}^a}) = E(\widehat{\beta_{post}^a})$, as the process is exogenous to the realization of the shock and the observed difference between estimators would be considered noise.

Overall, the difference between the two estimators, when considering the chaotic nature of the estimation process, is not significant enough to draw a definite conclusion.

1.3.2. The model was estimated in first differences, both with and without year fixed effects. The estimation results show that the logarithm of the markup would increase by 0.99 and 0.87, respectively. Generally, it is expected that competition will decrease when a firm is acquired, especially for the acquirer - whose market power increases - when a direct competitor is acquired. However, this increase in the logarithm of the markup could also be explained by the productivity gains discussed in the previous section or simply by economies of scale.

1.3.3. The model was estimated using first differences. When we include year fixed effects, our estimation for the coefficient associated with the M/Q ratio is -0.3608 and -0.3334 when we do not. This indicates that for each unit produced, there is a 33-36% reduction in the use of intermediate inputs once a firm has been acquired. This suggests the existence of a significant productivity gain and supports the ideas presented in sections 3(a) and 3(b).

1.3.4. The model was estimated using first differences. When we include year fixed effects, our estimation for the coefficient associated with the M/Q ratio is approximately 0. When we do not include year fixed effects, the estimation for this coefficient is -0.0479, with a standard deviation estimate of 0.064936. This suggests that prices do not tend to significantly increase once a firm is acquired. This supports the idea that the acquisition is driven by productivity gains rather than anti-competitive motives. The exogenous input price assumption therefore appears to be reasonable, as bigger firms, presumably with more bargaining power, do not obtain *any* benefit in this sense.

1.4. Reallocation, productivity and markup growth.

1.4.1. See Figure 1. It shows that aggregate and mean productivity increases until the beginning of WW1. However, this increase is not evenly distributed among firms, as the median indicates that most firms do not experience any growth in productivity. It appears that some highly efficient firms are driving the productivity growth in the industry. Additionally, it is possible that resource reallocation through mergers and acquisitions contributes to this growth, as previously discussed. However, this information cannot be definitively concluded based on this graph alone.

1.4.2. We can see in Figure 2 that aggregate and mean markups increase until the beginning of WW1. However, the median markup remains stable. This is the analogous to what we observed in section 4.a, where some firms were increasing their productivity while the majority were maintaining a constant one. Our analysis in the whole problem set suggests that these firms are able to raise their markups due to their greater efficiency (lowering their costs) rather than a decrease in competition or any sort of price increase.

The absence of the competitive issue is also supported by the fact that the least efficient firms do not raise the markup, as they would do if the whole industry had become less competitive. To sum up, our work here supports the claims of the original paper (Braguinsky et al, 2015).

2. Production Function Estimation OP/ACF

2.1. Estimating the production function using OLS on the unbalanced panel gives an elasticity of labour of 0.5433 and capital of 0.4272, while the balanced panel gives elasticities of 0.429 and 0.533. These results are shown in Table 1. The fact that the elasticity of capital is higher in the balanced sample is

surprising as the probability of exit is generally decreasing in productivity and capital, therefore low-capital firms that survive tend to be highly productive, whilst low-productivity firms only tend to survive if they have lots of capital. This creates a downward bias on the returns to capital that using a balanced panel should amplify.

The coefficients for the year and industry dummies indicate how efficient firms in a particular year or industry have been, relative to some base year or industry, holding capital and labour constant. This could therefore capture higher returns to labour or capital for these values or increased productivity from other sources.

2.2. Compared to the OLS results we see the returns to labour rise, and capital fall, as can be seen in Table 2. This is suggestive of a known bias from OLS, that firms that are consistently more productive than the average, will be able to invest more and accumulate more capital. Therefore overestimating the returns to capital. It is however surprising the large difference between the first difference and the fixed effects results, returns to capital are much smaller and statistically insignificant from 0 in the first difference approach. Part of this difference could come from the different samples as taking first differences reduces the sample by half. Another explanation is that by only using the change in capital over one period does not allow for enough variation in the variable to estimate the returns, however as one time period is 5 years in the data this explanation seems inadequate.

2.3. The results can be seen in Table 3. As expected we find a significant positive coefficient for log of deflated investment, firms that expect to survive the next five years are more likely to invest and doing so may make them more likely to survive, as they will have more capital in order to be able to survive negative shocks. We find a small and statistically insignificant parameter for capital, whereas theory expects this to also be important in firm survival.

2.4. The Olley-Pakes approach is a method attempting to remove the omitted variable bias coming from unobserved productivity. The approach is to assume that what a firm knows of its productivity can be summarised into one variable and that investment (or some other observed variable) is a strictly monotone function of this productivity. This function can therefore be inverted to control for unobserved productivity, and this new function can be substituted into the production function.

More formally:

$$i_t = i_t(k_t, \omega_t) \text{ under some assumptions } \Rightarrow \omega_t = h_t(k_t, i_t)y_{jt} = \beta_l l_{jt} + \beta_0 + \beta_k k_{jt} + h_t(k_{jt}, i_{jt})$$

The returns to labour can be estimated in this first stage, whilst $h_t(k_t, i_t)$ can be approximated using fourth-order polynomials. Note that $\widehat{\Phi}_t(k_t, i_t) = \beta_0 + \beta_k + h_t(k_t, i_t)$.

A second stage is then performed to estimate probabilities that firms exit the market, similar to the previous question but again using fourth-order polynomials of investment and capital, with different coefficients for every time period, and industry dummies. $P_{jt} = p_t(k_{jt}, i_{jt}, \text{industry}_j)$.

$$\begin{aligned} \text{We can therefore write: } y_{jt+1} - \beta_l l_{jt+1} &= \beta_0 + \beta_k k_{jt+1} + g(P_{jt}, \omega_{jt}) + \epsilon_{jt+1} + v_{jt+1} \\ y_{jt+1} - \widehat{\beta}_l l_{jt+1} &= \beta_0 + \beta_k k_{jt+1} + g(\widehat{P}_{jt}, \widehat{\Phi}_t(k_t, i_t) - \beta_0 - \beta_k k_{jt}) + \epsilon_{jt+1} + v_{jt+1} \end{aligned}$$

With $g(\cdot, \cdot)$ representing the Markov process for productivity, we approximate using fourth-order polynomials. This second equation can then be estimated using OLS, the results are in Table 4.

Ironically our new estimated returns to capital and labour are very close to the returns found in the unbalanced panel OLS estimation. This is surprising as the Olley Pakes method should account for two

forms of bias in the OLS estimation, simultaneity bias, with firms choosing labour after observing productivity, and the previously discussed survival bias. That our estimates don't change much could imply that these biases were not present in our data, or that the OP approach is not successful in correcting for them. We have not bootstrapped our second-stage estimation therefore the second-stage standard errors are not correct.

2.5. When including both returns to capital and returns to RnD capital as state variables in the OP estimation it is not clear that using both forms of investment to control for productivity makes sense. The argument for the approach comes from being able to invert a strictly increasing function of investment. If we have two functions for different types of investment it is unclear to us that a similar approach would make mathematical sense. We have therefore opted to use only one of the investments at a time and have presented results for both methods in Table 5.

Compared to our previous results we now find much lower returns to capital, with these returns seemingly being captured by the returns to RnD capital. It may well have been that firms that tended to have high capital also tended to have high RnD capital biasing our estimates. The estimated returns to a form of capital are higher when productivity is controlled for using the investment for the same type of capital, i.e. returns to RnD capital are higher when we use RnD investment to control for productivity. It is not immediately clear to us why this is the case.

2.6. We show the OP productivity decomposition for the whole economy in Table 6, by year. The first row shows the economy-wide mean productivity, weighted by the log of deflated sales, whilst the second shows the unweighted mean productivity. In both the weighted and unweighted means we see productivity rise in general over the 20-year period, but with a big drop in 1983. The third row gives the covariance between output and productivity and shows if more productive firms tend to produce more. We find a positive number in the first two periods indicating that the more productive firms tend to produce more. This number increases over the five-year period, indicating that output was moving to more productive firms. However, this number goes negative in 1983, before returning to a positive but lower number in 1988. The final row shows the correlation between capital and productivity. Here we see a negative number that gets more extreme over time, indicating that the more productive firms actually have less capital with this trend increasing over time. This could be due to high-capital firms being more able to survive low productivity as discussed earlier.

Focussing on sector 357 we find different results, productivity still increases but we no longer see the dip. The covariance between productivity and sales is now negative and decreasing over time, a surprising result and the correlation between productivity and capital is negative but moving towards zero.

2.7. Looking at Table 7 the first thing to note is that we see the same overall trends across the different methods, productivity was generally increasing over time, with a big decrease in 1983.

2.8. An issue that could come here is that as price is included in sales, we probably have markups entering into sales in a way that it should not enter quantity. For example, if a firm has a high markup it will raise its sales variable, relative to the quantity produced, creating bias in our proxy. Even worse, this bias will probably be correlated with productivity. If a firm is consistently productive over time it may drive out competitors, increasing its market power and markup, therefore, increasing the bias in our proxy variable. This could lead us to overestimate the impact of productivity on quantity produced. It may be possible to control for this using the market power data that we have available, through controlling something like the number of competitors or market share, although more thought would have to be given to the best way to do this.

2.9. Performing the ACF method, to get returns to labour, capital, and RnD capital involves estimating known productivity as a function of labour, capital, investment and RnD capital in the first stage. This time we use a third-level polynomial. Again, calling this function ϕ_t and assuming that productivity follows an AR1 process allows the second stage to be estimated using GMM on the equation below.

$$E[y_{jt} - \beta_0 - \beta_k k_{jt} - \beta_l l_{jt} - \beta_{rd} r_{djt} - \rho(\widehat{\phi_{jt-1}} - \beta_0 \beta_k k_{jt-1} - \beta_l l_{jt-1} - \beta_{rd} r_{djt-1})] \\ \otimes \begin{bmatrix} 1 \\ k_{jt} \\ l_{jt-1} \\ r_{djt} \widehat{\phi_{jt-1}} \end{bmatrix}$$

The results are shown in Table 9. The coefficient for labour is now smaller than in the OP approach, whilst RnD has stayed roughly the same, and returns to capital have risen.

2.10. We then compute the production function using the Woolridge one-step method, rather than in two steps as in the normal OP approach. We find results comparably similar to using the OP method, but with larger standard errors. This should not normally be the case, however, our OP second-stage standard errors have not been corrected in that they are estimated in a second stage but treated as if it is the first stage. Also, our OP estimation uses non-linear least squares and not GMM.

References

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De Loecker, Jan and Paul T Scott (2016) “Estimating market power Evidence from the US Brewing Industry,” Technical report, National Bureau of Economic Research.

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Appendix

	β^L	β^K	ρ	A_{ft-1}	intercept (β_0)
OLS	0.39853	0.81169	-	0.06843	-2.39605
GMM (Exogenous)	0.59596	0.44622	0.50671	0.59599	-1.05961
GMM (Endogenous)	0.62945	0.44489	0.73359	0.39196	-1.75928

Table 1: Estimation Report.

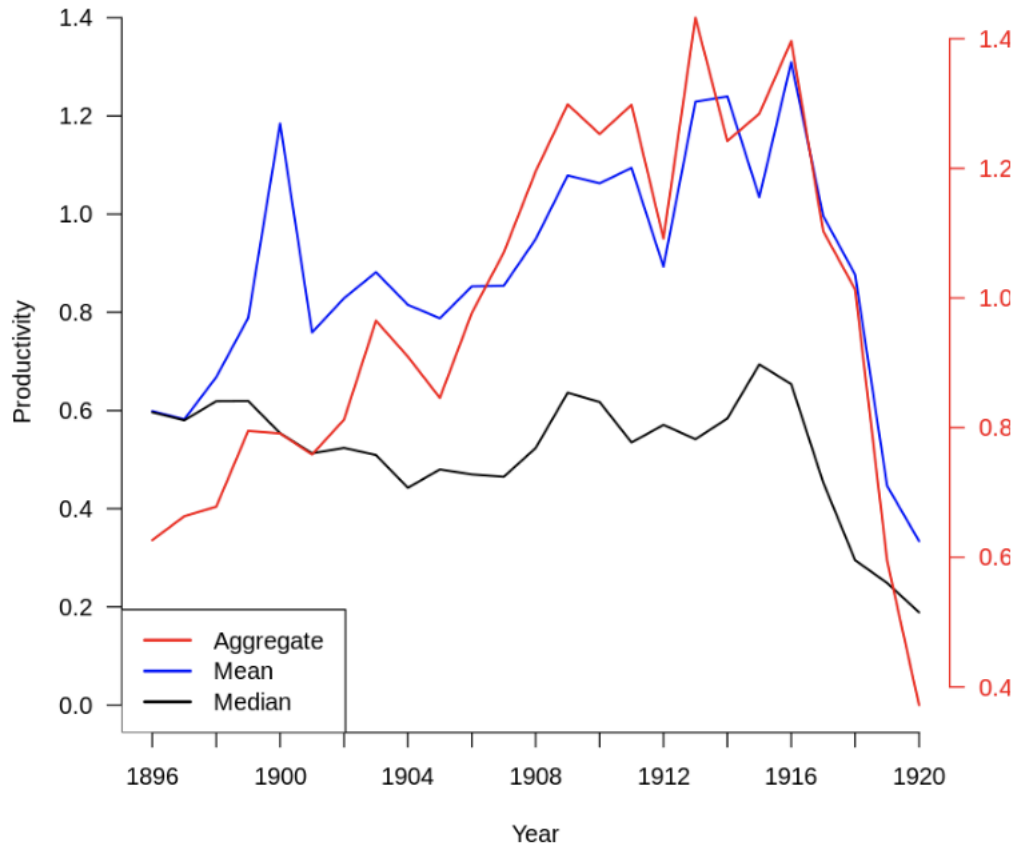


Figure 1: Productivity year by year.

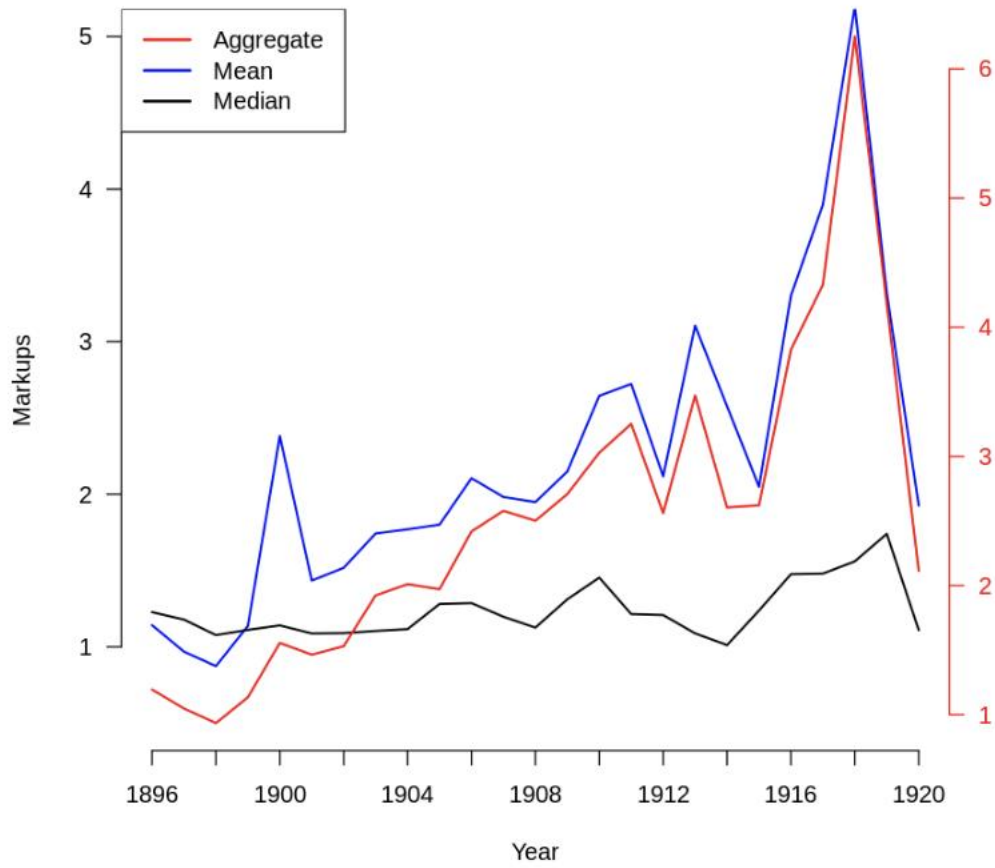


Figure 2: Markups year by year.

Table 2: Basic Production Function Results

	(1)	(2)	(3)	(4)
log of deflated capital	0.427*** (0.014)	0.533*** (0.024)	0.189*** (0.020)	0.271*** (0.037)
log of employment	0.543*** (0.018)	0.429*** (0.031)	0.771*** (0.022)	0.707*** (0.036)
Observations	2971	856	2971	856
Dummies	No	No	Yes	Yes
Balanced	No	Yes	No	Yes

Standard errors in parentheses

Robust Standard Errors. Year and Industry Dummies.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Production Function Results

	(First Difference)	(Fixed Effects)
Returns to Capital	0.043 (0.028)	0.311*** (0.037)
Returns to Labour	0.910*** (0.048)	0.753*** (0.061)
Constant	0.162*** (0.010)	3.335*** (0.145)
Observations	1502	2971

Robust Standard Errors. Firm level fixed effects. Unbalanced Panel

Table 4: Survival Function

	(1)
survive	
log of deflated investment	0.152*** (0.047)
log of deflated capital	0.036 (0.049)
Observations	2256

Robust Standard Errors. Dummies for year and industries. Unbalanced Panel

Table 5: OP Estimation

	(First Stage)	(Third Stage)
Returns to Labour	0.575*** (0.017)	
Returns to Capital		0.437*** (0.010)
Observations	2971	1486

Second stage standard errors not corrected. Unbalanced Panel

Table 6: OP Estimation with RnD

	(First Stage)	(Third Stage)	(First Stage)	(Third Stage)
Returns to Labour	0.593*** (0.019)		0.609*** (0.019)	
Returns to Capital		0.274*** (0.021)		0.225*** (0.023)
Returns to RnD Capital		0.189*** (0.025)		0.261*** (0.030)
Observations	2971	1486	2971	1486
Using Capital Investment	X	X		
Using RnD Investment			X	X

Second stage standard errors not corrected. Unbalanced Panel

Table 7: Productivity Decomposition

	1973	1978	1983	1988
Economy Wide				
Weighted Prod	27.88	28.46	21.00	31.08
Unweighted Prod	24.04	24.18	21.65	29.25
$\Sigma_i \Delta s_{it} \Delta p_{it}$	3.84	4.28	-0.64	1.83
$\rho(p_t, k_t)$	-0.08	-0.05	-0.18	-0.20
Sector 357				
Weighted Prod	0.61	2.03	6.81	26.04
Unweighted Prod	0.84	3.03	9.71	41.13
$\Sigma_i \Delta s_{it} \Delta p_{it}$	-0.23	-1.00	-2.90	-15.09
$\rho(p_t, k_t)$	-0.74	-0.55	-0.46	-0.32

Table 8: Productivity by Estimation Method

	1973	1978	1983	1988
Productivity OLS	92.37	97.50	76.06	127.33
Productivity FE	112.69	119.57	93.57	158.12
Productivity OP	25.38	25.96	19.96	31.75
Productivity OP with RnD	27.88	28.46	21.00	31.08

Table 9: ACF Estimation with RnD

	(1)
Constant	2.725*** (0.176)
Returns to Capital	0.403*** (0.044)
Returns to Labour	0.315*** (0.080)
Returns to RnD Capital	0.244*** (0.045)
ρ	0.610*** (0.068)
Observations	1502
Unbalanced Panel	

Table 10: Wooldridge Method

	(1)
log of employment	0.522*** (0.021)
log of deflated capital	0.215*** (0.030)
log of deflated R&D capital	0.210*** (0.031)
Observations	1502
Unbalanced Panel	