

# Dynamic Inefficiencies of Vertical Separation

## Evidence from the Soft Drinks Industry

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March 13, 2025

## Motivation

- Increased attention to Vertical Mergers due to
  - Can solve key inefficiencies
  - Can also lead to increased market power
  - Concerning trade-off due to rise of market power and emergence of disruptive innovations
  - Concerns that industry dynamics can exacerbate effects of lenient policy
- Inefficiencies of Vertical Separation
  - Static: Double Marginalization
  - In the presence of dynamics (**brand loyalty**), intertemporal linkages
    - **Magnify consequences** of a given above-cost input price by distorting incentives to invest
    - **Discipline** firms: adjust prices to preserve future demand and their ability to compete

## Research Question and Contribution

**Research question:** How do industry dynamics affect the magnitude of coordination problems and pass-through after integration?

### This paper:

- Provides motivating evidence that habit formation in demand affects pricing and merger outcomes
- Quantifies the
  - Magnitude of vertical separation inefficiencies in dynamic context
  - Welfare consequences of mergers
  - For different market structures and level of intertemporal linkages

### In this presentation:

- Overview of the industry and data
- Estimation of state dependence in demand at the regional level
- Structural model of vertical relations with state dependence in demand
- Estimation of structural model

# Empirical Application: The Soft Drinks Industry

- **Firms**

- Upstream: concentrate manufacturers (e.g., Coca-Cola, Pepsi)
- Downstream: Bottlers

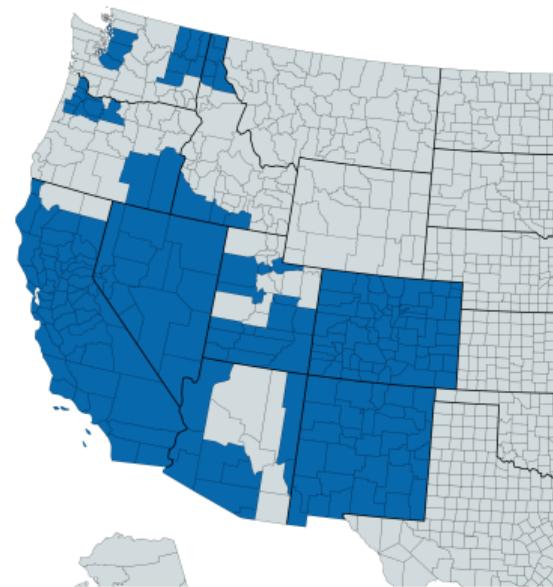
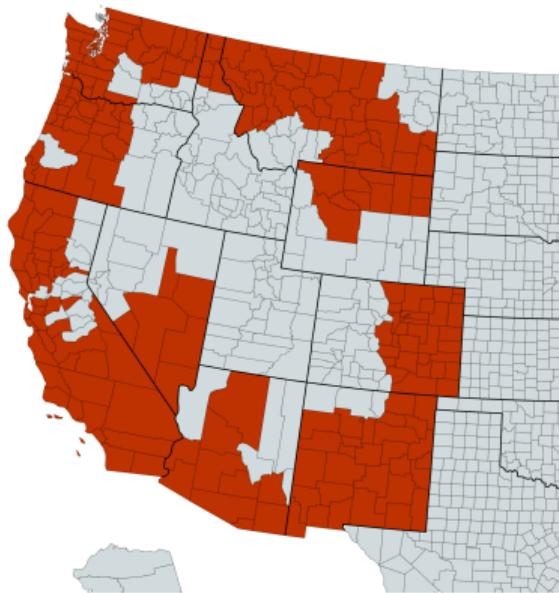
- **Industry**

- Contracts: stationary long-term linear prices with exclusive territories
- Source of dynamics: habit formation in demand (e.g., brand loyalty)
- Vertical Mergers during 2009-2010

- **Data**

- NielsenIQ - Kilts Datasets
  - Retail data: sales, characteristics, store advertisement
  - Scanner data: history of purchases, individual characteristics
- Territory maps of the U.S. bottling system (Luco and Marshall, 2020)
- Documents FTC's investigation of the mergers

## Territory Maps for Brands that Merged



**Color areas:** bottlers integrated.

**Gray areas:** bottlers remained independent.

# Overview Dynamic Model of Vertical Relations

- Setting
  - Sets  $\mathcal{N}^U$  and  $\mathcal{N}^D$  of upstream and downstream firms
  - Market structure and vertical relations are exogenously given
  - Mutually exclusive relations and perpetual
  - Intertemporal linkages: habit formation in demand
- Upstream prices
  - Before industry starts, firms set margins simultaneously
  - Firms maximize (induced) discounted flow of profits
  - Vector of marginal costs upstream  $c^U$
- Downstream competition
  - Firms compete setting prices each period
  - Vector of marginal costs upstream  $c^D$

## Model of downstream competition

- **Setting**

- Time is discrete and infinite horizon:  $t \in \{0, 1, \dots\}$
- Firm  $i \in \{1, \dots, N^D\}$  with constant component of marginal cost  $\tilde{c}_i$  (common knowledge) makes pricing decisions at each period
- Industry-wide state  $s_t$ : last period's vector of market shares
- **Privately observed** iid shocks to the marginal cost of production:  $\nu_{it} \sim \mathcal{N}(0, \sigma_{\nu_i}^2)$

- **Timing**

- 1 Given  $s_t$ , each firm  $i$  receives a shock to the marginal cost of production  $\nu_{it}$
- 2 Firms make pricing decisions simultaneously:  $p_t = (p_{1t}, \dots, p_{Nt}) \in \mathbb{R}^N$
- 3 Demands for all firms are realized. Firms receive period payoffs:  $\pi_i(p_t, s_t, \nu_{it})$
- 4 Industry state evolves according the the law of motion:  $s_{t+1} = D(p_t, s_t; \theta^D)$

## Firms' Dynamic Problem

- Restrict attention to Markov Perfect Equilibria
- Let the profile of Markov strategies be denoted by:  $\sigma = (\sigma_1, \dots, \sigma_N)$
- The value function of a firm, given strategy profile of other firms, can be written recursively as

$$V_i(s, \nu_i; \sigma) = \max_p \left[ \pi_i(p, \sigma_{-i}(s, \nu), s, \nu_i) + \beta \mathbb{E} [V_i(s', \nu'_i; \sigma)] \right],$$

where

- Law of motion of state variable:  $s_{t+1} = D(p_t, s_t; \theta^D)$
- Period profit:  $\pi_i(p_t, s_t, \nu_{it}) = [p_{it} - (\tilde{c}_i + \nu_{it})] D(p_t, s_t; \theta^D)$

## Upstream prices

- Upstream firms  $i \in \{1, \dots, N^U\}$  make simultaneous TIOPI offers before downstream competition starts
- Firm  $i$  sets (stationary) margin  $m$  to maximize

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t m D(\sigma(s_t, \nu_t), s_t; \theta^D) \mid s_0; \theta^S \right]$$

- For each downstream firm  $i \in \mathcal{N}^D$  supplied by  $j \in \mathcal{N}^U$ , the constant component of  $i$ 's marginal cost is

$$\tilde{c}_i = c_j^U + c_i^D + m_j,$$

- where  $c_j^U + c_i^D = c_{ji}^{U+D}$  represents the fixed production cost of that vertical structure.
- Let  $\mu(c^{U+D}, \sigma_\nu)$  be the equilibrium upstream margins.
- Model parameters:  $\theta = (\theta^D, \theta^S)$ , where  $\theta^S = (c^{U+D}, \sigma_\nu)$

- Need for a suboptimal solution
  - Exact Solution Method infeasible due to continuous state and action spaces
  - Solution: Approximate Dynamic Programming with Neural Networks
- Rewrite the problem with Action-value function

$$Q(\mathbf{s}, \boldsymbol{\nu}, p; \boldsymbol{\sigma}) = \pi_i(p, \boldsymbol{\sigma}_{-i}(\mathbf{s}, \boldsymbol{\nu}), \mathbf{s}, \boldsymbol{\nu}) + \beta \mathbb{E} [V_i(\mathbf{s}', \boldsymbol{\nu}'; \boldsymbol{\sigma})]$$

$$V_i(\mathbf{s}, \boldsymbol{\nu}; \boldsymbol{\sigma}) = \max_p Q(\mathbf{s}, \boldsymbol{\nu}, p; \boldsymbol{\sigma})$$

- For each firm
  - **Value Function Approximation:** minimize loss  $J(\rho) = \mathbb{E}[(Q(\mathbf{s}, \boldsymbol{\nu}, p) - Q_\rho(\mathbf{s}, \boldsymbol{\nu}, p))^2]$
  - $Q_\rho(\mathbf{s}, \boldsymbol{\nu}, p)$ : Neural Network
  - True Value Function  $Q(\mathbf{s}, \boldsymbol{\nu}, p)$  replaced by target:  $y = \pi_i(p, \boldsymbol{\sigma}, \mathbf{s}, \boldsymbol{\nu}) + \beta \max_{p'} Q_\rho(\mathbf{s}', \boldsymbol{\nu}', p')$
  - Problematic with continuous control → Solution: learn a maximizer (**Policy Function Approximation**)

- From downstream solver we have:  $\sigma_i^{\phi_i}(s, \nu_i; \tilde{c}, \sigma_\nu)$
- From upstream solver we have:  $\mu_i^{\psi_i}(c^{U+D}, \sigma_\nu)$
- Estimation by MSM with prices implied by the model given by

$$\sigma_i^{\phi_i}(s, \nu_i; c^{U+D} + \mu_i^{\psi_i}(c^{U+D}, \sigma_\nu), \sigma_\nu)$$

- Moments used so far: average price and standard deviation.

## Estimation of demand-side parameters

- State dependence in demand:
  - Exogenous: heterogeneity in demographics, historical product availability... Purchase patterns
  - Endogenous: advertisement
- Utility of individual  $i$  from product  $j$  in period  $t$

$$U_{ijt} = \phi_t - \alpha_i p_{jt} + \xi_{ij} + \gamma_i \mathbb{1}\{j \in s_{it}\} + \epsilon_{ijt}$$

- $s_{it}$ : set of products purchased last period (observed in the data)
- $\gamma$ : state dependence coefficient
- Random coefficients:  $\alpha_i, \xi_{ij}, \gamma_i$
- $\epsilon_{ijt}$ : standard iid EV Type 1 error term
- Let  $\theta^D$  denote the vector of demand parameters to be estimated

# Conclusion

**Research question:** How do industry dynamics affect the magnitude of coordination problems and pass-through after integration?

## In this presentation:

- Overview of the industry and data
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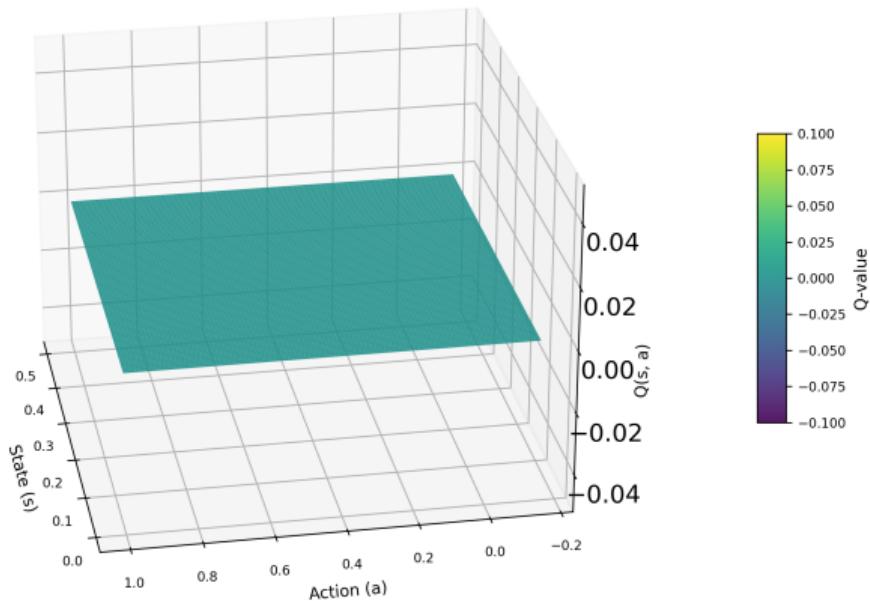
## Next:

- Provide theoretical model to illustrate effect of dynamics on coordination problem and pass-through
- Counterfactuals will
  - Quantify magnitude of coordination problem in dynamic context
  - Show dynamic pass-through incentives
  - Welfare consequences of mergers
  - For different market structures and level of intertemporal linkages

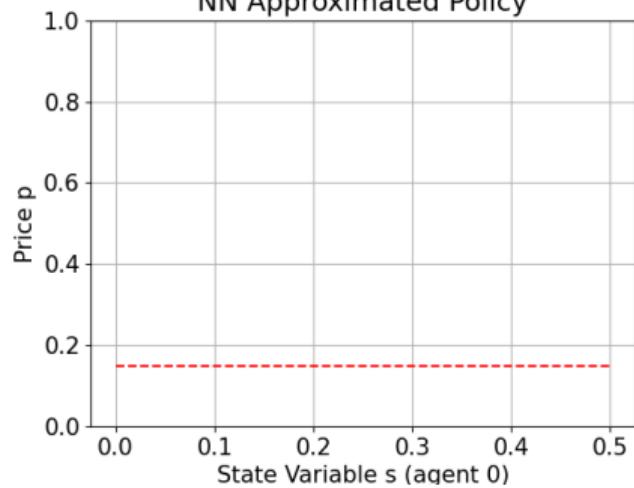
# Training: initialize neural networks

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Action-Value Function of Agent 0's

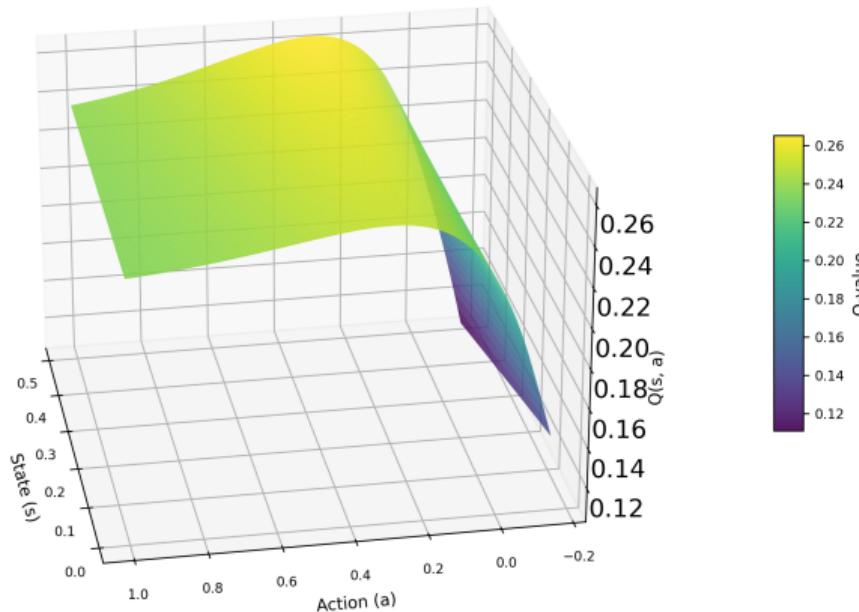


NN Approximated Policy

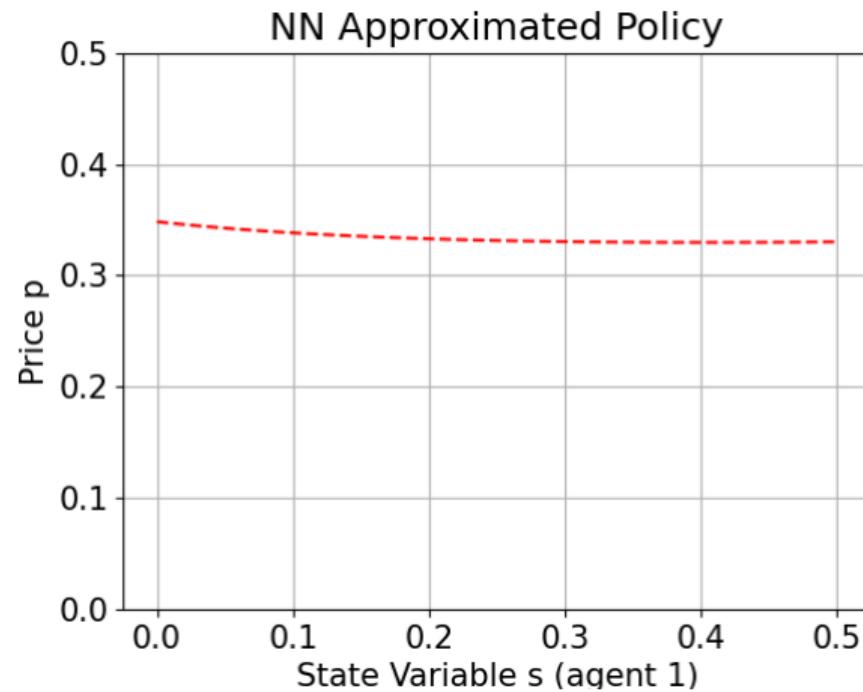


## Training: fix policies and train VF until convergence

Action-Value Function of Agent 0's



Training: fix VF and train policy until convergence





## Appendix





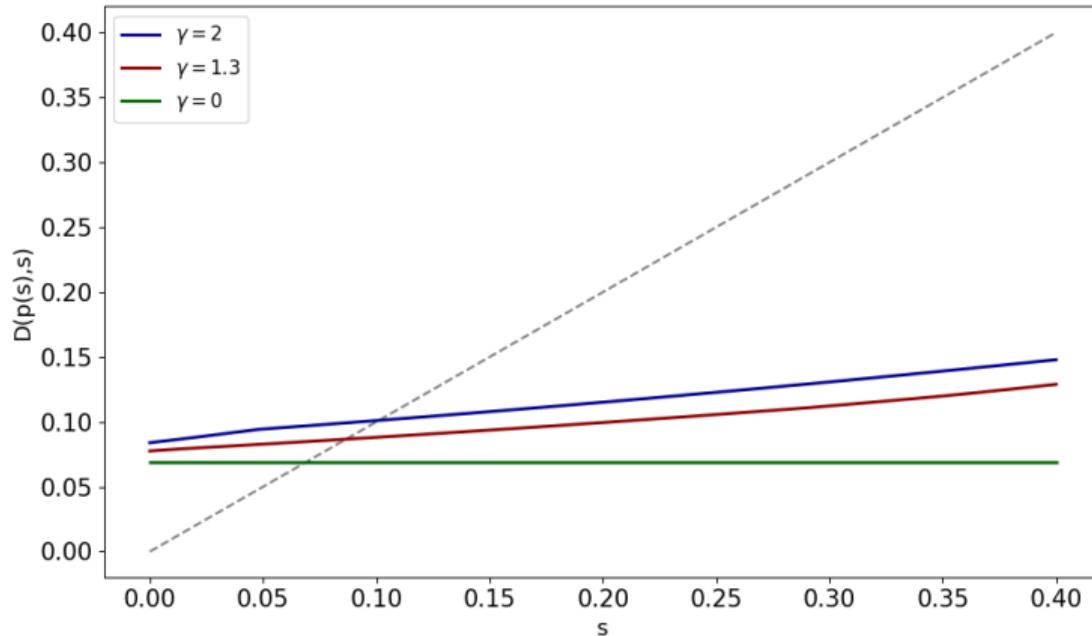








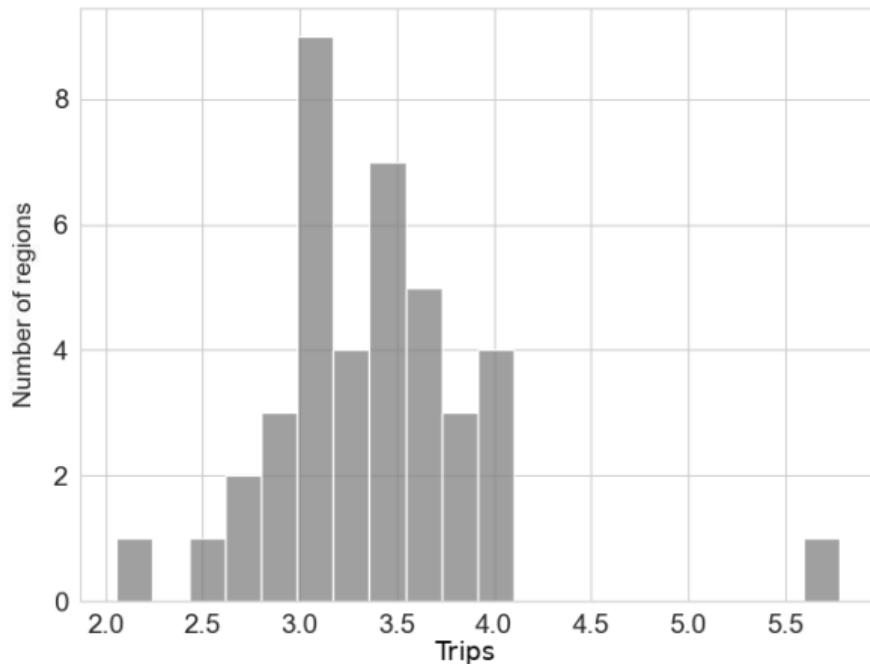
## Solution: demand dynamics and stationary point



Demand dynamics for different values of state dependence in demand

## Purchase Behavior Across Regions 1/4: re-purchase path duration

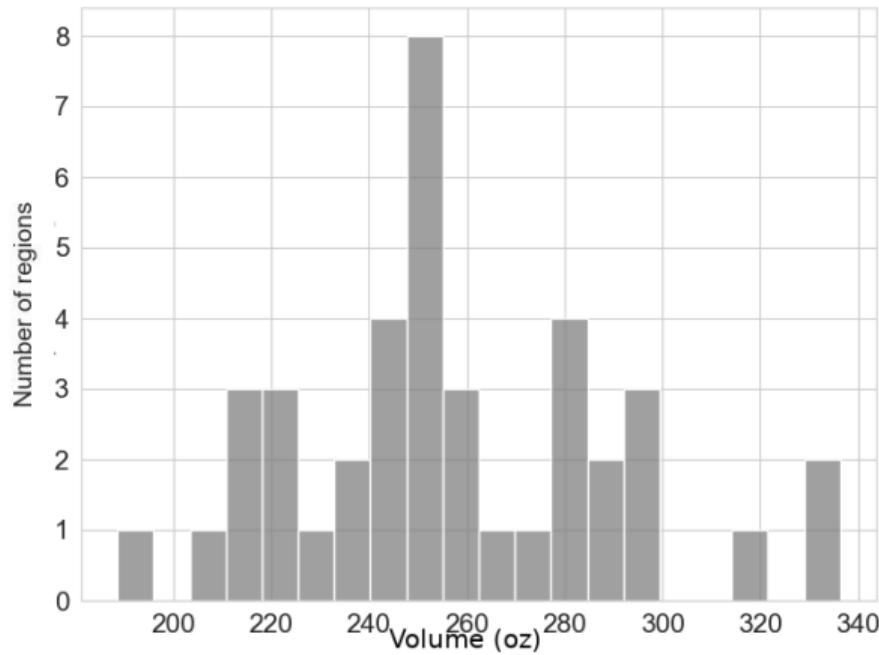
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Average Consecutive Repurchase Path Duration Across Regions

## Purchase Behavior Across Regions 2/4: purchase volume per trip

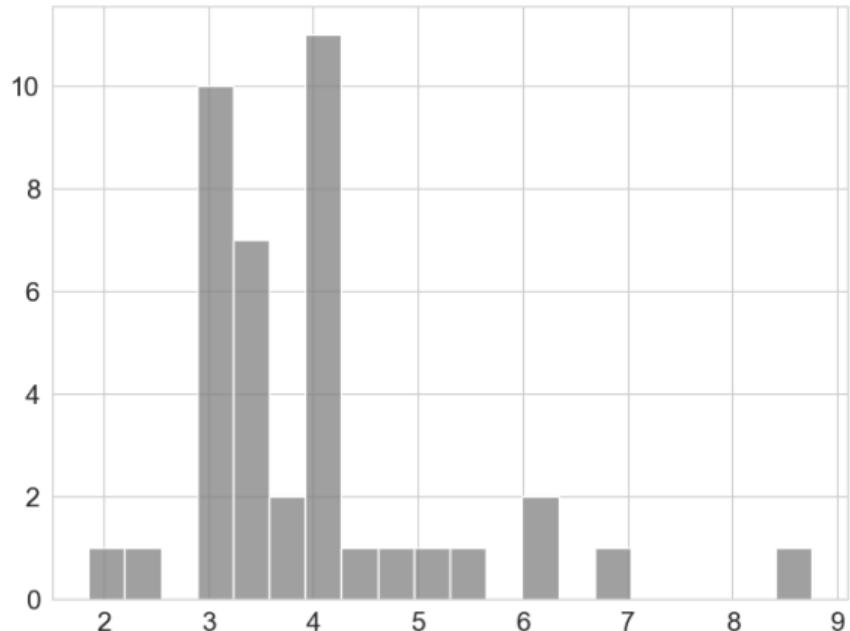
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Average Purchase Volume Across Regions (oz)

## Purchase Behavior Across Regions 3/4: purchase frequency

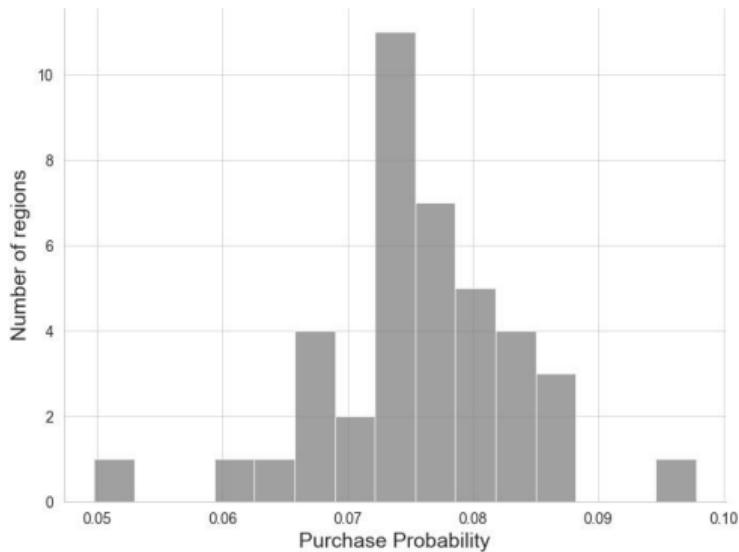
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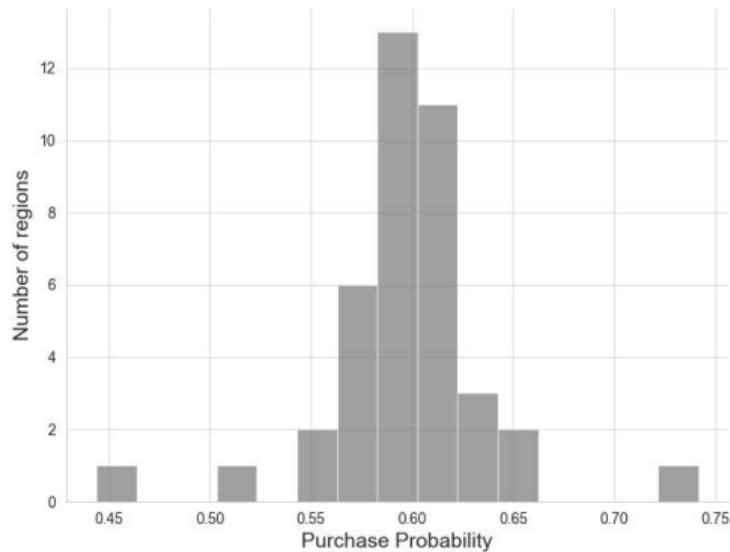
Average Week Difference Between Trips Across Regions

## Purchase Behavior Across Regions 4/4: purchase probability

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Conditional on no re-purchase



Conditional on re-purchase

- 1: **Input:** initial policy parameters  $\phi$ , Q-function parameters  $\rho$ , empty replay buffer  $\mathcal{D}$
- 2: Set target parameters equal to main parameters  $\rho_{\text{targ}} \leftarrow \rho$ ,  $\phi_{\text{targ}} \leftarrow \phi$
- 3: **repeat**
- 4:     Observe state  $s$  and select action  $a = \text{clip}(\mu_\phi(s) + \epsilon, a_{\text{low}}, a_{\text{high}})$ , where  $\epsilon \sim \mathcal{N}$
- 5:     Execute  $a$  in the environment
- 6:     Observe next state  $s'$  and reward  $r$
- 7:     Store  $(s, a, r, s')$  in replay buffer  $\mathcal{D}$
- 8:     **if** it's time to update **then**
- 9:         **for** however many updates **do**
- 10:             Randomly sample a batch of transitions,  $B = \{(s, a, r, s')\}$  from  $\mathcal{D}$
- 11:             Compute targets

$$y(r, s', d) = r + \gamma(1 - d)Q_{\rho_{\text{targ}}}(s', \mu_{\phi_{\text{targ}}}(s'))$$

12:

Update Q-function by one step of gradient descent using

$$\nabla_{\rho} \frac{1}{|B|} \sum_{(s,a,r,s') \in B} (Q_{\rho}(s, a) - y(r, s'))^2$$

13:

Update policy by one step of gradient ascent using

$$\nabla_{\rho} \frac{1}{|B|} \sum_{s \in B} Q_{\rho}(s, \mu_{\phi}(s))$$

14:

Update target networks by Polyak averaging

$$\phi_{\text{targ}} \leftarrow \eta \phi_{\text{targ}} + (1 - \eta) \phi$$

$$\rho_{\text{targ}} \leftarrow \eta \rho_{\text{targ}} + (1 - \eta) \rho$$

15:

**end for**

16:

**end if**

17: **until** convergence

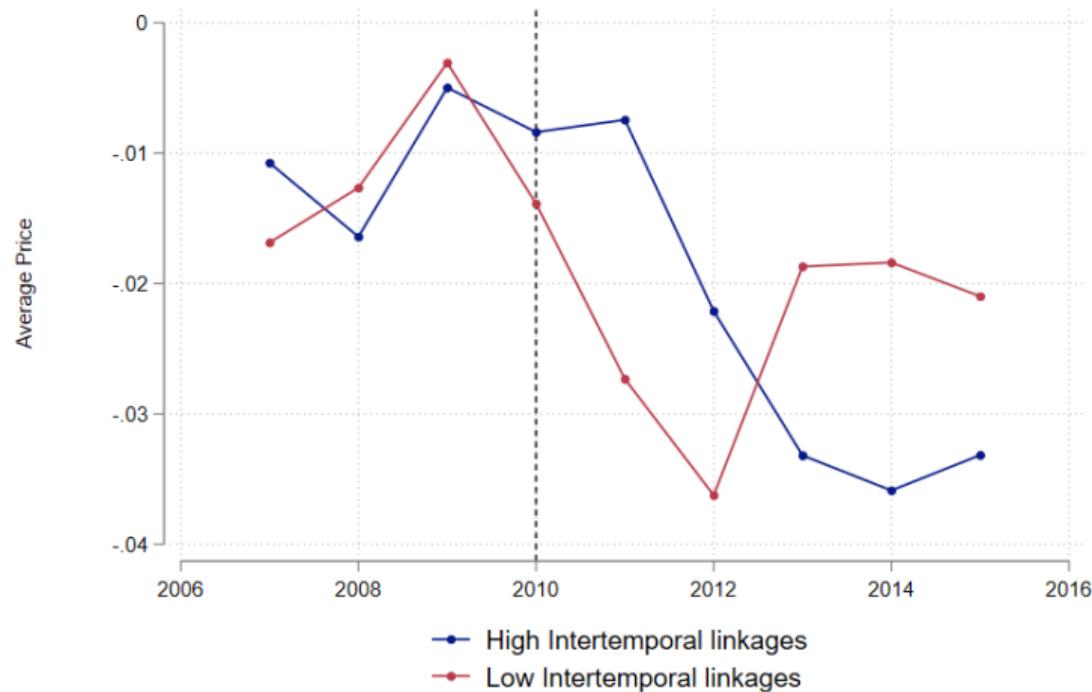
- Let  $\kappa = \{c, \sigma\}$  be the vector of parameters to be estimated
- Draw a set of marginal cost shocks  $\tilde{\nu} = \{\tilde{\nu}_1, \dots, \tilde{\nu}_J\}$  from the distribution  $\mathcal{N}(0, \sigma^2)$ .
- Simulated moments:  $\hat{m}(s, \kappa) = \{\hat{m}_1(s, \kappa), \dots, \hat{m}_R(s, \kappa)\}$  with  $r^{th}$  element:  $\hat{m}_r(\mathbf{s}, \kappa) = \frac{1}{J} \sum_{j=1}^J m_r(s, \tilde{\nu}_j, \kappa)$
- $e(\mathbf{s}, \kappa) = \{e_1(\mathbf{s}, \kappa), \dots, e_R(\mathbf{s}, \kappa)\}$
- $e_r(\mathbf{s}, \kappa) = \hat{m}_r(\mathbf{s}, \kappa) - m_r(\mathbf{s})$
- $\hat{\kappa}_{MSM} = \arg \min_{\kappa} e(\mathbf{s}, \kappa)' \mathbf{W} e(\mathbf{s}, \kappa)$

- Let  $\kappa_0 = \{c_0, \sigma_0\}$  be the initial guess of the vector of parameters
- Draw a set of marginal cost shocks  $\tilde{\nu} = \{\tilde{\nu}_1, \dots, \tilde{\nu}_S\}$  from the distribution  $\mathcal{N}(0, \sigma^2)$ .
- For each state and shock, compute prices implied by the model:  $p^{NN}(s, \tilde{\nu}_i, c, \sigma)$
- Compute moments observed in the data and predicted by the moment for each value of the state  $s$ 
  - $\bar{p}^{NN}(s, \kappa) = \frac{1}{S} \sum_{i=1}^S p^{NN}(s, \tilde{\nu}_i, \kappa)$
  - $p_{SD}^{NN}(s, \kappa) = \sqrt{\frac{1}{S} \sum_{i=1}^S (p^{NN}(s, \tilde{\nu}_i, \kappa) - \bar{p}^{NN}(s, \kappa))^2}$
- Define the loss function

$$L(\kappa) = \sum_s \left[ (\bar{p}^{NN}(s, \kappa) - \bar{p}^{obs}(s))^2 + (p_{SD}^{NN}(s, \kappa) - p_{SD}^{obs}(s))^2 \right]$$

- Update parameters, go back to first step, until convergence.

## Differential trends TD



## Diff trends Event Study TD

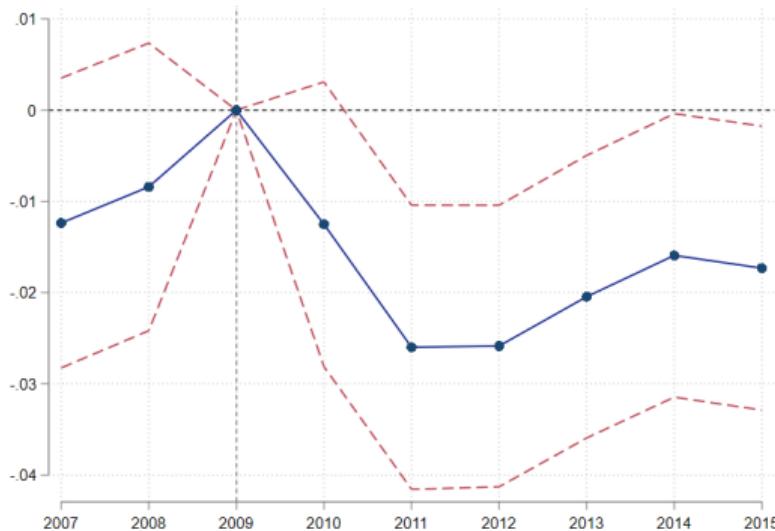


## Event Study TD, by level of intertemporal linkages

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- High intertemporal linkages



- Low intertemporal linkages

## Triple Difference Estimator

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Average Price	(1)	(2)	(3)	(4)	(5)	(6)
D	-0.0150 (0.0194)	-0.0290*** (0.0103)	-0.0332*** (0.0092)	-0.01202 (0.0078)	-0.0127*** (0.0041)	-0.0107*** (0.0039)
$D^*\gamma$	0.0016 (0.0165)	0.0160* (0.0088)	0.0240*** (0.0075)			
$D^*\gamma^H$				-0.0012 (0.0109)	0.0035 (0.0058)	0.0098* (0.0051)
Product, county, period FE	✓	✓	✓	✓	✓	✓
Product-by-county FE		✓				✓
Product-by-period FE		✓				✓
County-by-period FE		✓				✓
$R^2 adj$	0.0859	0.7437	0.9173	0.0849	0.7433	0.9171
N	5,730	2,910	2,820	5,730	5,730	5,730

## Non-linearities

Average Price	(1)	(2)	(3)	(4)	(5)	(6)
D	0.0144 (0.1079)	0.0631 (0.0576)	0.1774*** (0.053)	-0.0064 (0.0131)	-0.0033 (0.0070)	-0.0008 (0.0061)
D* $\gamma$		-0.0562 (0.1949)	-0.1558 (0.1042)	-0.3636*** (0.0967)		
D* $\gamma^2$		0.0256 (0.0817)	0.0735* (0.0437)	0.1638*** (0.0407)		
D* $\gamma^{L.2}$				-0.0132 (0.0153)	-0.0192* (0.0082)	-0.0169** (0.0073)
D* $\gamma^{L.3}$				-0.0033 (0.0162)	0.0030 (0.0086)	0.0073 (0.0073)
Product, county, period FE	✓	✓		✓	✓	
Other FE		✓			✓	
R <sup>2</sup> adj	0.0868	0.7438	0.9178	0.0894	0.7437	0.9175
N	5,730	5,730	5,628	5,730	5,730	5,628

- Bellman equation for firm  $i$ , given strategy profile of other firms:

$$V_i(\mathbf{s}; \boldsymbol{\sigma}) = \max_{(p_t)} \left[ \pi_i(p_i, \boldsymbol{\sigma}_{-i}(\mathbf{s}), \mathbf{s}) + \beta V_i(\mathbf{s}'; \boldsymbol{\sigma}) \right]. \quad (1)$$

- The first order condition for the optimization problem of firm  $i$  is

$$\frac{\partial \pi_i}{\partial p_i}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) + \beta \frac{dV_i}{d\mathbf{s}'}(\mathbf{s}') \frac{\partial \mathbf{s}'}{\partial p_i}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) = 0 \quad (2)$$

- Differentiate equation 1 in order to get an equation for the partial derivatives of the value function.

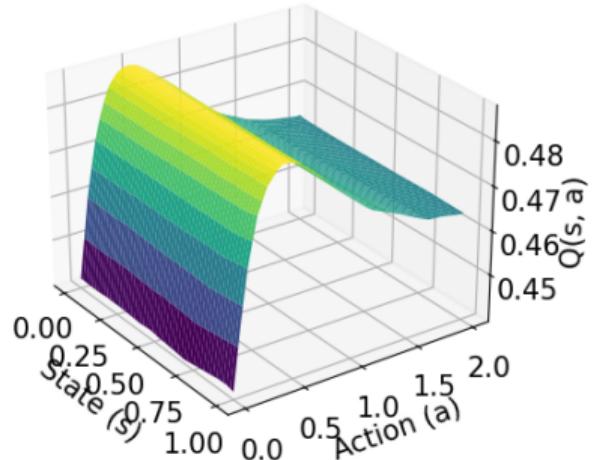
$$\frac{dV_i}{d\mathbf{s}}(\mathbf{s}) = \frac{\partial \pi_i}{\partial \mathbf{s}}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) + \beta \frac{dV_i}{d\mathbf{s}'}(\mathbf{s}') \frac{\partial \mathbf{s}'}{\partial \mathbf{s}}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) \quad (3)$$

- One can use to substitute into , in order to substitute back into and get:

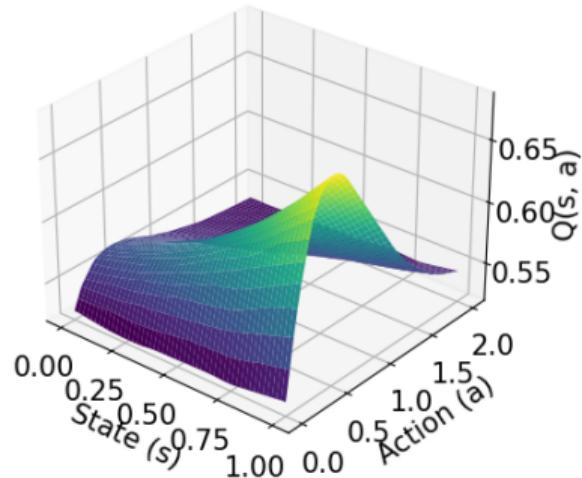
$$\frac{\partial \pi_i}{\partial p_i}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) + \beta \left[ \frac{\partial \pi_i}{\partial \mathbf{s}} - \frac{\partial \pi_i}{\partial p_i} \left( \frac{\partial \mathbf{s}'}{\partial \mathbf{p}} \right)^{-1} \frac{\partial \mathbf{s}'}{\partial \mathbf{s}} \right] (p_i', \boldsymbol{\sigma}_{-i}', \mathbf{s}') \frac{\partial \mathbf{s}'}{\partial p_i}(p_i, \boldsymbol{\sigma}_{-i}, \mathbf{s}) = 0 \quad (4)$$

## Solution: action-value function

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- No state dependence in demand



- Average state dependence in demand