'Macroeconomics with Heterogenous Agents and Input-Output Networks' Review and its Connection to 'Networks and the Macroeconomy: An Empirical Exploration'

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Abstract

The goal of this document is to connect two papers of macroeconomic theory based on networks: Daron Acemoglu, William Akcegit and William Kerr 'Networks and the Macroeconomy: An Empirical Exploration' (2015), which tried to explain empirically the geographical and economical connection between industrial sectors, and David R. Baqaee and Emmanuel Fahri's 'Macroeconomics with Heterogenous Agents and Input-Output Networks' (2019) paper, which aimed to unbundle both the representative agent and the aggregate production function. Thus, in these papers the authors caractherized the general equilibrium of the economy through a network of producers and, in the second case, also of costumers. First, I summarize the work of Baqaee and Fahri and then connect it to Acemoglu's et alter paper.

Introduction

Explicar el formato del paper etc

1 'Macroeconomics with Heterogeneous Agents and Input-Output Networks' Review

Introduction

The aim of the Baqaee and Fahri's paper is to unbundle both the representative agent and the aggregate production function simultaneously. To achieve this, they state a general model with hetereogeneus agents and consider other papers, e.g., Foerster, Sarte and Watson (2011), Di Giovanni, Levchenko and Méjean (2014) or Acemoglu, Akcigit and Kerr (2015) as special cases of this general model. They claim that these other analyses all feature symmetric propagation, due to different reasons. In Acemoglu's paper, it is because of the assumption of the Cobb-Douglas functional form and other reasons like the existence of a representative agent.

The authors also claim that the heterogenous agent analysis lead to two different approaches: one dynammical, which is preferred for macroeconomics and other statical, which is better for studies on international economics. However, both can hold a common benchmark through Arrow-Debrew view where goods can be indexed by states of nature and dates, generalizating the model.

1.1 Setup of the model

The model has a set of consumers C, producers N, factors F and supplies of factors L_f. The difference between goods and factors is that goods need from factors or other goods to be produced, whereas factors are produced *ex-nihilo*. Goods can serve as intermediate input or be destined to final consuption. Concerning agents, consumers are not owners of the factors, whereas producers are.

Households

Each consumer $c \in C$ has preferences $U_c(D_c(C_{c1}, ..., C_{cN}), L_{c1}, ..., L_{cf})$ where D_c is homothetic, which is going to be an important assumption. Thus, the budget constraint for each consumer can be specified as follows:

$$\sum_{i=1}^{N}(1+\tau_{ck})p_i c_{ci} = \sum_{f=1}^{F} w_f L_{cf} + \pi_c + \tau_c$$

where p_i and c_{ci} are respectively prices and quantities used by the consumer. w_f, L_{cf} are the price and quantity of factor f owned by c. Finally, π_c, τ_c, τ_{ck} correspond to profits, net revenues earned by taxes and subsidies and consumption taxes or subsidies on a product k for the consumer c. $L_{cf} = G_{cf}(w_f, \Upsilon_c)$, i.e., factor supplies are derived from a funcion where interact prices of factors and the consumption index.

Producers

Each good k needs to be produced with a technology constant to decreasing returns to scale. Without generallity loss, we can assume constant returns to scale. Thus, the cost fuction of each producer is written as follows:

$$\frac{y_k}{A_k} C_k((1+\tau_{k1})p_1, ..., (1+\tau_{kN})p_k, (1+\tau^f_{k1})w_1, ..., (1+\tau^f_{kF})w_F)$$

where y_k is the total output and C_k/A_k the marginal cost of producing good k. The A_k term is a classic Hicks-neutral productivity shifter.

Equilibrium

The general equilibrium is defined as usual. Given certain productivities and markups, the equilibrium is a set of prices p_i, intermediate input choices x_{ij}, factor input choices l_{if}, outputs y_i and final demands c_{ci}, for which: each producer optimizes, each household optimizes and the factor and good markets are clear.

Input-Output Definitions

Baqaee and Farhi define some input-output based measures, which allows us to connect agents each other properly. These definitions will be made both cost-based (with a tilde) or revenue-based depending on whether there is any markup/wedge acting in the economy.

- **Final Expenditure Shares:** $b_{ci} = \dfrac{p_i c_{ci}}{\sum_{i \in N} p_i c_{ci}}$
- **Consumer c's share of total expenditure:** $\chi_c = \dfrac{\sum_{i \in N} p_i c_{ci}}{\sum_{j \in N} \sum_{d \in C} p_j c_{dj}}$
- **HA-IO matrix:** $\Omega : (\Omega)_{ij} = \dfrac{p_j x_{ij}}{p_i y_i}$**.** The ijth element is equal to i's expenditures on inputs from j as a share of its total revenues.
- **Factor distribution matrix:** $\Phi_{cf} = \dfrac{w_f L_{cf}}{w_f L_f}$**,** i.e., this is the share of factor f accruing to consumer c.
- **Cost-based HA-IO matrix:** $\tilde{\Omega} : (\tilde{\Omega})_{ij} = \dfrac{p_j x_{ij}}{\sum_l p_l x_{il}}$. By Shephard's Lemma, this ijth term is also the elasticity of the cost of i to the price of j.

Now we can define the Leontief Inverse Matrix, which collects direct but also indirect effects of network propagation:

- **HA-IO Leontief inverse matrix:** $\Psi \equiv (1-\Omega)^{-1} = Id + \Omega + \Omega^2 + ...$
- **HA-IO Leontief inverse matrix (cost-based):** $\tilde{\Psi} \equiv (1-\tilde{\Omega})^{-1} = Id + \tilde{\Omega} + \tilde{\Omega}^2 + ...$
- **GDP:** $GDP = \sum_{i \in N} \sum_{c \in C} p_i c_{ci}$.
- **Domar Weights:** $\lambda_i \equiv \dfrac{p_i y_i}{GDP}$, that are the sales share of a producer i as a fraction of GDP.

Leontief Inverse and Domar Weights are linked via:

$$\lambda^T = b^T \Psi = b^T Id + b^T \Omega + b^T \Omega^2 + ...$$

Then, to finish with definitions, authors state **real GDP change** as:

$$d \log \Upsilon = \sum_i b_i d \log c_i$$

and **GDP deflator** as:

$$d\log P = \sum_i b_i d\log p_i$$

N.B.: The original paper has lots of explanations about interpretation, some considerations about certain equivalencies or how to construct the model in terms of non-neutral shocks. Furthermore, it specify the funcional form of the producion of these nested-CES Economies. I will take it as known *and to carry on showing up the most important results of the paper, which enable the comparation with the paper written by Acemoglu et al.*

1.2 Symmetric Propagation

This section's aim is to explain where and why we find the most likely contrafactural symmetry results when there is a **representative agent** and **balanced growth** preferences. Note that it is not imposed a nested CES economy. This section's fundamental result is the following:

Proposition 1. (Symmetric Propagation). *Consider the efficient model without wedges or markups. For two producers i and j, symmetric propagation*

$$\frac{d\lambda_j}{d\log A_i} = \frac{d\lambda_i}{d\log A_j}$$

holds in equilibrium if ***either*** *of the following conditions is satisfied:*
(i) There is a ***representative agent*** *with* ***balanced-growth preferences:***

$$U(D(c_1, ..., c_N), L_1, ..., L_F) = U(D(c), v(L))$$

where D is ***homogeneus*** *degree-one and factor supply is* ***derived from these preferences****.*
(ii) There is a ***single primary factor****, denoted by L, and preferences are:*

$$U(D(c_1, ..., c_N), L)$$

with D ***homothetic****.*

However, we are able to state a new proposition where preferences are not neccesarily homothetic and concerns not sales-shares but total sales.

Proposition 2. *Consider a modification of the efficient model withput markups or wedges with a representative agent whose preferences are given by:*

$$U(c_1, ..., c_N, L_1, ..., L_F),$$

where preferences over consumption goods are ***not necessarily homothetic****. Then, there exists some prices index P_u such that*

$$\frac{dp_i y_i}{d\log A_j} = \frac{dp_j y_j}{d\log A_i}$$

when P_u is used as numeraire. If U is homothetic, then P_u is the ***ideal index*** *associated with U. If U is homothetic and factors are* ***inelastically supplied****, then P_u is the GPD Deflator.*

Interestingly, we can see that the logic for simmetry in sales is stronger than for sales shares. This result of simmetry derives from the simmetry of the partial derivates.

1.3 Basic Model

The basic model that the authors use as benchmark has a representative consumer, inelastic factors and no distorsions, as well as nested-CES economies. All I will write down is the comparative statics of this model. In the original paper, there are some other sections like examples or, in this case, an explanation about how to use the results in a regression. However, I will atend uniquely to the most important results which are theoretical and they all lie on the **comparative statics** section.

Aggregate Output and Shares

Proposition 3. *(Aggregate Output and Shares) The elasticities of aggregate output to the different productivities are given by*

$$\frac{d\log\Upsilon}{d\log A_k}=\lambda_k$$

The elasticities of the sales shares or Domar weights of i is given by

$$\frac{d\log\lambda_i}{d\log A_k}=\sum_j(\theta_j-1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\Psi_{(k)},\Psi_{(i)})-\sum_g\sum_j(\theta_j-1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\Psi_{(g)},\Psi_{(i)})\frac{d\log\Lambda_g}{d\log A_k}$$

where $\Lambda_g=\lambda_g$ when g is a factor, $\theta_j>1$ a parameter which allows that, in response to a positive shock in A_k with fixed prices, a producer j substitutes (in shares) towards *those inputs i that are more reliant on producer k, captured by Ψ_{ik}.*

We need to specify the covariance operator. It is defined as follows by the authors in Baqaee and Farhi (2017a):

$$Cov_{\Omega^{(j)}}(\Psi_{(k)},\Psi_{(l)})=\sum_i\Omega_{ji}\Psi_{ik}\Psi_{il}-(\sum_i\Omega_{ji}\Psi_{ik})(\sum_i\Omega_{ji}\Psi_{il})$$

We can rewrite the linear system for factor share as follows:

$$\frac{d\log\Lambda}{d\log A_k}=\Gamma\frac{d\log\Lambda}{d\log A_k}+\delta_{(k)}$$

The authors call the δ term the *factor share impulse matrix*, its kth column encodes the direct and first-round effects of a shock to the productivity of producer k in income shares, taking relative prices as given. They also call Γ the *factor share propagation matrix.* It encondes the efects of changes in relative factor prices on factor income shares, and it is independent of the source k.

Prices

Proposition 4. *(Prices) The elasticities of the prices of the different producers to the different productivities are given by:*

$$\frac{d\log w_f}{d\log A_k}=\frac{d\log\Lambda_f}{d\log A_k}+\frac{d\log\Upsilon}{d\log A_k}$$

$$\frac{d\log p_i}{d\log A_k}=-\Psi_{ik}+\sum_g\Psi_{ig}\frac{d\log w_g}{d\log A_k}$$

Note that the first equation is neccesary to the second one and it is needed to recover the general equilibrium.

Sales and Quantities

This corollary is direct from the past equations.

Corollary. *(Sales and Quantities) THe elasticities of the sales and output quantities of the different producers to the different productivities are given by:*

$$\frac{d\log p_iy_i}{d\log A_k}=\frac{d\log\lambda_i}{d\log A_k}+\frac{d\log\Upsilon}{d\log A_k}$$

$$\frac{d\log y_i}{d\log A_k}=\frac{d\log p_iy_i}{A_k}-\frac{d\log p_i}{d\log A_k}$$

Symmetric Propagation

Since the economy satisfies the conditions of the **proposition 1**, it features symmetric propagation, i.e.:

$$\frac{d\lambda_i}{d\log A_j}=\frac{d\lambda_j}{d\log A_i}=\frac{d^2\log\Upsilon}{d\log A_id\log A_j}$$

1.4 Heterogeneous Agents

In this section, authors focus on the case where there are heterogeneous consumers. However, they mantain inelastically supplied factors and no distorsions.

Aggregate Output and Shares

Proposition 5. *(Aggregate Output and Shares) The elasticities of aggregate output to the different productivities are given by*

$$\frac{d\log\Upsilon}{d\log A_k} = \lambda_k$$

The elasticities of the sales shares or Domar weights of i is given by

$$\frac{d\log\lambda_i}{d\log A_k} = \sum_j(\theta_j-1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\Psi_{(k)},\Psi_{(i)}) - \sum_g\sum_j(\theta_j-1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\Psi_{(g)},\Psi_{(i)})\frac{d\log\Lambda_g}{d\log A_k}+$$

$$+\frac{1}{\lambda_i}\sum_g\sum_c(\lambda_i^c-\lambda_i)\Phi_{cg}\Lambda_g\frac{d\log\Lambda_g}{d\log A_k}$$

where $\Lambda_g=\lambda_g$ when g is a factor, $\theta_j>1$ a parameter which allows that, in response to a positive shock in A_k with fixed prices, a producer j substitutes (in shares) towards *those inputs i that are more reliant on producer k, captured by Ψ_{ik}.*

We can rewrite the linear system for factor share as follows:

$$\frac{d\log\Lambda}{d\log A_k} = \Gamma\frac{d\log\Lambda}{d\log A_k}+\delta_{(k)}+\Theta\frac{d\log\Lambda}{d\log A_k} \quad (12)$$

The new term Θ captures how changes in the price of the factor g change the distribution of income across consumers, and how this change in this distribution of income, in turn, affects demand for the factor f (since different hpuseholds are differently exposed, directly and indirectly, to the different factors). So that, $\exists l : \lambda_l^c=\lambda_l$, i.e., all consumers are symmetrically exposed to certain good, then the new term will disappear as the changes in the distribution of income will have no effect. **Here is where asymmetric propagation appears**. The section of prices is exactly the same than in the previous section so it is not specified here.

Asymmetric Propagation

Models with Heterogeneous Agents present asymmetric propagations because of non-homotheticies in final demand generated by the income distribution channel.

E.g.: There are two goods i and j such that $\lambda_i^c=\lambda_i \ \forall c\in C$ and $\lambda_j^c\neq\lambda_j$ for certain j. In (12), the income distribution term (which starts with Θ) disappears if the shock goes from i to j, but it does not if it goes j to i.

Id est: for symmetry it is needed that each costumer's share λ is fixed. With this hypothesis, it is possible to recover one representative agent.

1.5 Elastic Factor Supplies

In this section, authors explain results concerning the case with elastic factor supplies. They mantein, for simplicity, a representative agent and no distorsions. They assume the supply of factor f as $L_f=G_f(w_f,\Upsilon)$, where the price of the factor and the aggregate output take part.

Authors denote some special elasticities: $\zeta_f=\dfrac{\partial\log G_f}{\partial\log w_f}$ and $\gamma_f=-\dfrac{\partial\log G_f}{\partial\log\Upsilon}$. Note that, we are in the case of the basic model wether $\zeta_f=\gamma_f=0$.

Aggregate Output and Shares

Proposition 6. *(Aggregate Output and Shares) The elasticities of aggregate output to the different productivities are given by*

$$\frac{d\log\Upsilon}{d\log A_k} = \rho[\lambda_k-\sum_g\frac{1}{1+\zeta_g}\Lambda_g\frac{d\log\Lambda_g}{d\log A_k}]$$

with

$$\frac{d\log\lambda_i}{d\log A_k} = \sum_j(\theta_j - 1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\sum_k \Psi_{(k)}, \Psi_{(i)}) -$$
$$-\sum_j(\theta_j - 1)\frac{\lambda_j}{\lambda_i}Cov_{\Omega^{(j)}}(\sum_g \Psi_{(g)}\frac{1}{1+\zeta_g}\frac{d\log\Lambda_g}{d\log A_k} + \sum_g \Psi_{(g)}\frac{\gamma_g - \zeta_g}{1+\zeta_g}\frac{d\log\Upsilon}{d\log A_k}, \Psi_{(i)})$$

where $\Lambda_g = \lambda_g$ when g is a factor, $\theta_j > 1$ a parameter which allows that, in response to a positive shock in A_k with fixed prices, a producer j substitutes (in shares) towards *those inputs i that are more reliant on producer k, captured by Ψ_{ik}.*

The term ρ indicates the relative strength of the income and subtitution effects on factor supply. If we think about this model with only one factor it is easier to see as $\rho = \frac{1+\zeta_f}{1+\gamma_f}$. We also would get $\frac{d\log\Upsilon}{d\log A_k} = \rho\lambda_k$ and $d\log L = \zeta_L d\log w - \gamma_L d\log\Upsilon$. If we assume a productivity shock $d\log A_k > 0$: the shock increases output and so reduces factor supply via income effect, the strength of which dependes on γ_L. Furthermore, it increases real wage of this factor and so increases factor supply via a subtitution effect, which dependes on γ_L. So that, we only recover the Hulten's theorem (i.e., $\frac{d\log\Upsilon}{d\log A_k} = \lambda_k$) if $\zeta_L = \gamma_L$ and, thus, $\rho = 1$.

Prices

Proposition 7. *(Prices) The elasticities of the prices of the different producers to the different productivities are given by:*

$$\frac{d\log w_f}{d\log A_k} = \frac{1}{1+\zeta_f}\frac{d\log\Lambda_f}{d\log A_k} + \frac{1+\gamma_f}{1+\zeta_f}\frac{d\log\Upsilon}{d\log A_k}$$
$$\frac{d\log p_i}{d\log A_k} = -\Psi_{ik} + \sum_g \Psi_{ig}\frac{d\log w_g}{d\log A_k}$$

Note that this is exaclty the same result as Proposition 4, excepts for the subtitution and income elasticities that now are present, which are necessary to descpmpose the elasticity of factor share into elasticities both of the factor price and the factor quantity to productivities.

Asymmetric Propagation

With elastic factors, only in the case where $\gamma_f = \zeta_f\ \forall f$ the propagation is symmetric.

1.6 Distorsions

In this last sections of modifications of the basic model, we mantein the representative agent and inelastically supplied factors, however, we add a set of wedges and markups (exogen). As we now treat with markups we need to use the definitions based on cost, not in revenues.

Aggregate Output and Shares

Proposition 8. *(Aggregate Output and Shares) The elasticities of aggregate output to the different productivities are given by*

$$\frac{d\log\Upsilon}{d\log A_k} = \tilde{\lambda}_k - \sum\tilde{\Lambda}_g\frac{d\log\Lambda_g}{d\log A_k}$$

The elasticities of the sales shares or Domar weights of i is given by

$$\frac{d\log\lambda_i}{d\log A_k} = \sum_j(\theta_j - 1)\frac{\mu_j^{-1}\lambda_j}{\lambda_i}Cov_{\tilde{\Omega}^{(j)}}(\tilde{\Psi}_{(k)} - \sum_g\tilde{\Psi}_{(g)}\frac{d\log\Lambda_g}{d\log A_k}, \Psi_{(i)})$$

where $\Lambda_g = \lambda_g$ when g is a factor, $\theta_j > 1$ a parameter which allows that, in response to a positive shock in A_k with fixed prices, a producer j substitutes (in shares) towards *those inputs i that are more reliant on producer k, captured by Ψ_{ik}.*

In efficient economies, cost and revenue are the same and therefore: (i), $\lambda_k = \tilde{\lambda}_k$, (ii) $\sum_g \tilde{\Lambda}_g \frac{d \log \Lambda_g}{d \log A_k} = 0$, i.e., changes in technology are captured by revenue-based Domar weights and changes in allocative efficiency are zero so that we recover Hulten's theorem.

Prices

Proposition 9. *(Prices) The elasticities of the prices of the different producers to the different productivities are given by:*

$$\frac{d \log w_f}{d \log A_k} = \frac{d \log \Lambda_f}{d \log A_k} + \frac{d \log \Upsilon}{d \log A_k}$$

$$\frac{d \log p_i}{d \log A_k} = -\tilde{\Psi}_{ik} + \sum_g \tilde{\Psi}_{ig} \frac{d \log w_g}{d \log A_k}$$

The only difference, as we can see, is the cost-based instead of revenue-based HA-IO matrices in the second equation.

Asymmetric Propagation

As authors stated in the subsection 1.2, in inefficient economies there are no symmetry. The reason is that wedges sever connection between the elasticities of the output function and the sales shares. One sympton can be seen directly in the Proposition 8 where Γ involves not only Ψ but also $\tilde{\Psi}$.

1.7 Industry Level Aggregation

blah, blah, blah...