

① Find Composition table, of $\langle \mathbb{Z}_7, * \rangle$ and find left coset of $[0]_7$ in \mathbb{Z}_7 .

→

$* \backslash \mathbb{Z}_7$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

For

$* \backslash \mathbb{Z}_7$	0	4
0	0	0
4	0	2

multiplication

$$a * H = \{ a * h \mid a \in G, h \in H \}$$

$$a [0]_7 = \{ 0 \times 0, 0 \times 4 \}$$

$$[0]_7 = \{ 0, 0 \}$$

$$[1]_7 = \{ 0, 4 \}$$

$$[2]_7 = \{ 0, 2 \}$$

$$[3]_7 = \{ 0, 5 \}$$

$$[4]_7 = \{ 0, 2 \}$$

$$[5]_7 = \{ 0, 6 \}$$

$$\textcircled{1} [6]_7 = \{ 0, 3 \}$$

Q2) ^{even} B code parity of 1011001

→ Given message = 1011001

$$\text{No of bits} = 7 = m$$

$$\text{No of parity bits} \therefore 2^p \geq m+p+1$$

$$2^p \geq 7+p+1$$

① $p=3$

$$2^3 \geq 7+3+1$$

$$8 \geq 11$$

② $p=4$

$$2^4 \geq 7+4+1$$

$$16 \geq 12$$

$p=4$

$$1 - 0001$$

$$2 - 0010$$

$$3 - 0011$$

$$4 - 0100$$

$$5 - 0101$$

$$6 - 0110$$

$$7 - 0111$$

$$8 - 0000$$

$$9 - 1001$$

$$10 - 1010$$

$$11 - 1011$$

Total No of bits = $m+p$
 $= 7+4$
 $= 11$

1011001

11	10	9	8	7	6	5	4	3	2	1
m_7	m_6	m_5	p_4	m_4	m_3	m_2	p_3	m_1	p_2	p_1
1	0	1		1	0	0		1		

$$p_1 = 1, 3, 5, 7, 9, 11$$

$$= p_1 10111$$

$p_1 = 0$

$$p_2 = 2, 3, 6, 7, 10, 11$$

$$= p_2 10101$$

$p_2 = 1$

$$p_3 = 4, 5, 6, 7$$

$$= p_3 001$$

$p_3 = 1$

$$p_4 = 8, 9, 10, 11$$

$$= p_4 001$$

$p_4 = 0$

message 10101001110

$$\begin{aligned}\text{error in message} &= P_4 P_3 P_2 \\ &= (0110)_{10} \\ &= (6)_{10}\end{aligned}$$

10101001110
↑

Replace 1 by 0 or 0 by 1

10101101110

Hamming distance (H/d)

mismatching

① $C = \{(0000), (0101), (1011), (0111)\}$

→ $x = 0000$

$y = 0101$

$z = 0011$

$w = 0111$

$(x, y) = 2$

$(x, z) = 3$

$(x, w) = 3$

$(y, z) = 3$

$(y, w) = 1$

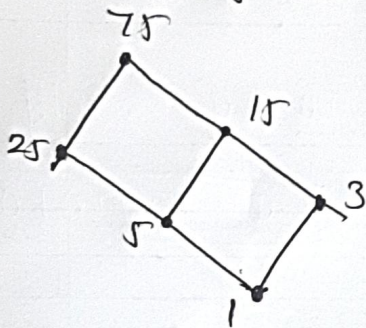
$(z, w) = 2$

minimum Hamming distance →

$H(y, w) = 1$

Q7 Draw Lattices $\langle S, \cap \rangle$ $n = \underline{75}$ also find GLB, LUB, complement of every element?

→ Lattice diagram



GLB

$\#/\wedge$	1	3	5	15	25	75	$\#/\vee$	LUB	1	3	5	15	25	75
1	1	1	1	1	1	1	1	1	1	3	5	15	25	75
3	1	3	3	3	3	3	3	3	3	3	5	15	25	75
5	1	3	5	5	5	5	5	5	5	5	5	15	25	75
15	1	3	5	15	15	15	15	15	15	15	15	15	25	75
25	1	3	5	15	25	25	25	25	25	25	25	25	25	75
75	1	3	5	15	25	75	75	75	75	75	75	75	75	75

$$LUB = 75 \quad GLB = 1$$

$$a \oplus b = 75 \quad a * b = 1$$

$$\{1, 75\}$$

$$1 \oplus 3 = 3 \quad 1 * 3 = 1$$

Canonical form

① $f(x, y, z) = xy' + y'z$

$$\rightarrow xy'(z+z') + y'z'(x+x')$$

$$xy'z + xy'z' + x'y'z' + x'y'z$$

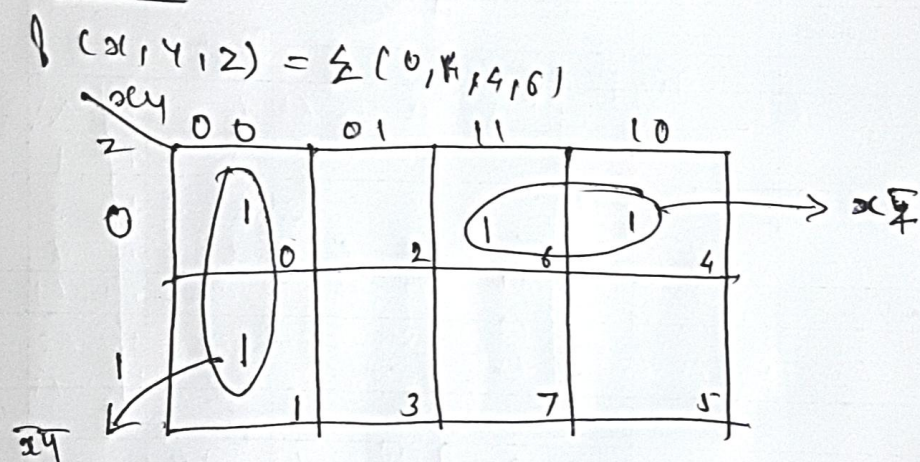
$$xy'z + xy'z' + x'y'z'$$

1	0	1		
	1	0	0	
		0	0	0

⊕ 5, 4, 0

⊕ 0, 4, 5

K-map



⊕ $y = x\bar{x} + \bar{x}\bar{y}$

circuit diagram

$f = x + y + z$

