Q1) a) $w_1 = w \quad z_1 = z$

$w_2 = 1 \quad z_2 = i$

$w_3 = +i \quad z_3 = 0$

$w_4 = -1 \quad z_4 = -i$

$$\frac{(w-i)(1+1)}{(w+1)(1-i)} = \frac{(z)(i+i)}{(z+i)(i)}$$

$$\frac{2(w-i)}{(w+1)(1-i)} = \frac{2zi}{(zi+1)}$$

$$(2w-2i)(zi-1) = 2zi(w+1)(1-i)$$

$$2wzi - 2w + 2z + 2i = 2zi(w - wi + 1 - i)$$

$$\cancel{2wzi} - 2w + \cancel{2z} + 2i = \cancel{2zwi} + 2wz + 2zi + \cancel{2z}$$

$$\cancel{2}i - \cancel{2}w = \cancel{2}wz + \cancel{2}zi$$

$$i - zi = wz + w$$

$$i(1-z) = w(z+1)$$

$$\boxed{w = \frac{i(1-z)}{(z+1)}}$$

Q2) a) $f(z) = \sqrt{|xy|}$

$\longrightarrow f(z) = \sqrt{|xy|} + 0i$

$u + iv = \sqrt{|xy|} + 0i$

$u = \sqrt{xy} \quad + v = 0$

$$\left(\frac{\partial u}{\partial x}\right)_{(0,0)} = \lim_{h \to 0} \frac{u(h,0) - u(0,0)}{h} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$$

der For some derivative is exist

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z}$$

$$= \lim_{\substack{x \to 0 \\ y \to 0}} \frac{\sqrt{|xy|} - 0}{x + iy}$$

$$= \quad y = mx \quad x \to 0$$

$$\lim_{x \to 0} \frac{\sqrt{mx^2}}{x + mxi}$$

$$f'(z) = \lim_{x \to 0} \frac{\cancel{x}\sqrt{m}}{\cancel{x}(1+mi)}$$ — depend on m der .....

Q2) b) $\cancel{f(z)}\ u = e^{-x}(x\cos y + y\sin y)$

$$\frac{\partial u}{\partial x} = e^{-x}[\cos y] + [x\cos y + y\sin y]e^{-x}(-1)$$

$$\frac{\partial u}{\partial y} = e^{-x}[-x\sin y + y\cos y + \sin y] + [0]$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

derivative

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$= e^{-x}[\cos y - x\cos y - y\sin y]$$

$$e^{-x}[-x\sin y + y\cos y + \sin y]$$

$$= e^{-x}[\cos y - x\cos y - y\sin y - x\sin y + y\cos y + \sin y)$$

By milne's method

$x = z \quad y = 0$

$$\int f'(z) e^{-z}[1 - z]$$

$$f(z) = \left[[1-z]\frac{e^{-z}}{-1}\right] - \left[[-1]\frac{e^{-z}}{(-1)(-1)}\right]$$

$$= -[1-z]e^{-z} + e^{-z} \qquad = e^{-z}[-\cancel{1} + z + \cancel{1}] \qquad = +ze^{-z} + c$$

put $f(z) = u+iv$ $z = x+iy$ $c = A+iB$

$$u+iv = (x+iy)e^{-(x+iy)} + A+iB$$

$$= (x+iy)e^{-x}, e^{-iy} + A+iB$$

$$= (x+iy)e^{-x}[\cos y - i\sin y] + A+iB$$

$$= e^{-x}[x\cos y - xi\sin y + iy\cos y + y\sin y] + A+iB$$

$$= e^{-x}\underbrace{[x\cos y + y\sin y] + A}_{u} + ie^{-x}\underbrace{[y\cos y - x\sin y] + B]}_{v}$$

Q3 a) $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$

$\bar{F} = \Delta \times \bar{F} \rightarrow$ Irrotational

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - yz & y^2 - zx & z^2 - xy \end{vmatrix}$$

$$i(-x+x) + j(-y+y) + k(-z+z)$$

$$= 0 + 0 + 0$$

$$= 0$$

$\phi(x,y,z)$ is a scalar potential function

$$\bar{F} = \Delta \phi$$

$$(x^2 - yz)i + (y^2 zx)j + (z^2 - xy) = \left(\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k\right)$$

$$\int \frac{\partial \phi}{\partial x} = \int x^2 dx - \int yz\, dx$$

$$= \frac{x^3}{3} - xyz + c_1$$

$$= \frac{y^3}{3} - xyz + c_2$$

$$= \frac{z^3}{3} - xyz + c_3$$

$$\phi = \phi_1 \cup \phi_2 \cup \phi_3$$

$$= \frac{x^3}{3} + \frac{y^3}{3} + \frac{z^3}{3} - 3xyz + c$$

Q3 (5) $\because \bar{F} = 2zi - xj + 4k$ (Volume)

$$\int_0^2 \int_0^4 \left[\int_{x^2}^{2} 2zi - xj + 4k \, dz\right] dy\, dx$$

$$\int_0^2 \int_0^4 \left[\frac{2z^2}{2} i - xzj + 4zk\right]_{x^2}^{2} dy\, dx$$

$$\int_0^2 \int_0^4 \left[(4 - x^4) i + (-2x + x^3) j + (24 - x^2 4) k\right] dy\, dx$$

$$\int_0^2 \left[(4y - x^4 y) i + (-2xy + x^3 y) j + \left(\frac{2y^2}{2} - \frac{x^2 y^2}{2}\right) k\right]_0^4 dx$$

$$\int_0^2 \left[(16 - 4x^4) i + (-8x + 4x^3) j + (16 - 8x^2) k\right] dx$$

$$\left[\left(16x - \frac{4x^5}{5}\right) i + \left(-\frac{8x^2}{2} + \frac{4x^4}{4}\right) j + \left(16x - \frac{8x^3}{3}\right) k\right]_0^2$$

$$\left(32 - \frac{4 \times 32}{5}\right) i + \left(\cancel{-16} + \cancel{16}\right) j + \left(32 - \frac{8 \times 8}{3}\right) k$$

$32 - \frac{64}{3}$

$32(1 - \frac{2}{3}$

$$\frac{32}{5} i + \frac{32}{3} j$$

$$\underline{\underline{32\left(\frac{i}{5} + \frac{j}{3}\right)}}$$

Q4 b) $\Delta \cdot (r^n \bar{r})$

$= (\Delta r^n) \cdot \bar{r} + (r^n (\Delta \cdot \bar{r})$

$= n r^{n-2} \bar{r} \cdot \bar{r} + r^n \cdot 3$

$= n r^{n-2} r^2 + r^n \cdot 3$

$= n r^n + 3 r^n$

$= (n+3) r^n$

(2) $\Delta \times (r^n \bar{r}) = 0$

$(\Delta r^n) \times \bar{r} + r^n \cdot (\Delta \times \bar{r})$

$n r^{n-2} \bar{r} \times \bar{r} + r^n \cdot (\Delta \times \bar{r})$

$0 + 0$

$= \underline{\underline{0}}$

(3) Δr^n

$f(r) = r^n$

$f'(r) = n r^{n-1}$

$\Delta r = \frac{\bar{r}}{r}$

$\Delta f(r) = f'(r) \Delta r = f'(r) \frac{\bar{r}}{r}$

$= n r^{n-1} \frac{\bar{r}}{r} = n r^{n-1} r^{-1} \bar{r} = \underline{\underline{n r^{n-2} \bar{r}}}$

4. $\Delta^2 r^n$.

$\Delta(\Delta r^n)$

$\Delta(n r^{n-2} \bar{r})$

$(\Delta n r^{n-2} \cdot \bar{r}) + n r^{n-2}(\Delta \cdot \bar{r})$

$n(n-2) r^{n-2-2} \bar{r} \cdot \bar{r} + 3 n r^{n-2}$

$n(n-2) r^{n-4} r^2 + 3n r^{n-2}$

$n(n-2) r^{n-2} + 3n r^{n-2}$

$(n^2 - 2n + 3n) r^{n-2}$

$(n^2 + n) r^{n-2}$

$\underline{\underline{n(n+1) r^{n-2}}}$

Q4 a) $\phi = x^2 y^2 z^2$ at $(1, 1, -2)$

$x = e^{-t}$ $y = 2\sin t$ $z = t - \cos t$

$\longrightarrow$

$\Delta \phi = x^2 y^2 z^2$

$\Delta \phi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial x} j + \frac{\partial}{\partial z} k\right) x^2 y^2 z^2$

$\Delta \phi_{(1,1,-2)} = (2xy^2z^2) i + (2x^2yz^2) j + (2x^2y^2z) k$

$\Delta \phi = (\underline{8i + 8j - 4k})$

tangent: $\bar{r}$

$\bar{r} = xi + yj + zk$

$\bar{r} = e^{-t} i + 2\sin t\, j + (t - \cos t) k$

$\left(\frac{d\bar{r}}{dt}\right)_{t=0} = (-e^{-t}) i + (2\cos t) j + (1 + \sin t) k$

$= \underline{(-1) i + 2j + k}$

directional derivative $= (8i + 8j - 4k) \frac{(-i + 2j + k)}{\sqrt{1 + 4 + 1}} = \frac{-8 + 16 - 4}{\sqrt{5}} = \boxed{\frac{4}{\sqrt{5}}}$

Q 1) b) $\frac{1}{z^2-3z+2}$ ① $|z|<1$ ② $1<|z|<2$

$\longrightarrow \frac{1}{z^2-3z+2} = \frac{1}{z-1} + \frac{1}{z-2}$

$z^2-z-2z+2$
$z(z-1)-2(z-1)$
$(z-1)(z-2)$

$= \frac{A}{z-1} + \frac{B}{z-2}$

multiply by A by $(z-2)$ & B by $(z-1)$

$1 = A(z-2) + B(z-1)$

$A = \left(\frac{1}{z-2}\right)_{z=1} = \frac{1}{1-2} = -1$

$B = \left(\frac{1}{z-1}\right)_{z=2} = \frac{1}{2-1} = 1$

$= \frac{-1}{z-1} + \frac{1}{z-2}$

① $|z|<1$

$= \frac{-1}{-1(1-z)} + \frac{1}{-2(1-\frac{z}{2})}$

$= \frac{+1}{+1}(1-z)^{-1} + \frac{1}{-2}\left(1-\frac{z}{2}\right)^{-1}$

$= \left[1+z^2+\frac{z^2}{2}+\frac{z^3}{3}\cdots\right] \frac{-1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3\cdots\right]$

$= \left[1-\frac{1}{2}\right] + \left[1-\frac{1}{4}\right]z + \left[\frac{1}{2}-\frac{1}{8}\right]z^2 \cdots$

$= \frac{1}{2} + \frac{3}{4}z + \frac{7}{8}z^2 \cdots \frac{z^n}{n}$

$= \cancel{\frac{-1}{z\left[1-\frac{1}{z}\right]}} +$

(2) $1 < |z| < 2$

$$= \frac{-1}{z-1} + \frac{1}{z-2}$$

$$= \frac{-1}{z\left(1-\frac{1}{z}\right)} + \frac{1}{-2\left(1-\frac{z}{2}\right)}$$

$$= \frac{-1}{z}\left(1-\frac{1}{z}\right)^{-1} + \frac{1}{-2}\left(1-\frac{z}{2}\right)^{-1}$$

$$= \frac{-1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots\right] + \frac{1}{-2}\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}\cdots\right]$$

$$= -\left[\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}\cdots\right] + \frac{1}{-2}\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}\cdots\right]$$

$$= \left[-\frac{1}{2}-\frac{z}{4}-\frac{z^2}{8}-\frac{z^3}{16}\cdots-\frac{1}{z}-\frac{1}{z^2}-\frac{1}{z^3}-\frac{1}{z^4}\right].$$