Solutions to Artin's Algebra Second Ed

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Contents

Chapter 2

Groups

2.1 Laws of Composition

Exercise 1.1

Proof. For any $a, b, c \in S$, we have

$$(ab)c = ac = a = ab = a(bc),$$

which implies that the law of composition is associative. $\square$

Let a be an arbitrary element in the set for which the law has an identity. Then, we have

$$a = a1 = 1a = 1,$$

which implies that the set must be $\{1\}$.

Exercise 1.2

(1) *Proof.* $la = 1$ and $ar = 1$ imply $l = r = a^{-1}$. $\square$

(2) *Proof.* Suppose that both a' and a'' are the inverses of a. Then $a' = a''$ by part (1) and so the inverse is unique. $\square$

(3) *Proof.* $(ab)(b^{-1}a^{-1}) = a(bb^{-1})a^{-1} = aa^{-1} = 1$ implies that $(ab)^{-1} = b^{-1}a^{-1}$. $\square$

(4) See Exercise 1.3.

Exercise 1.3

Proof. s has no right inverse because there is no inverse when $n = 1$. However, s has infinitely many left inverses because there are infinitely many mappings sending $n + 1$ to n for $n \in \mathbb{N}$. $\square$