Exercise sheet 6: Unification (Solutions)

Exercise 1

(a) 1.
$$(X \equiv 2) \land (X \equiv 2)$$
 (by decompose)
2. $(2 \equiv 2) \land (X \equiv 2)$ (by eliminate)
3. $X \equiv 2$ (by delete)

Succeeds with X=2

(b) 1.
$$(X \equiv 2+2) \land (X \equiv 4)$$
 (by decompose)
2. $(4 \equiv 2+2) \land (X \equiv 4)$ (by eliminate)

Fails (by conflict)

(c) 1.
$$(X \equiv a) \land (Y \equiv g(b)) \land (Y \equiv g(b))$$
 (by decompose)
2. $(X \equiv a) \land (g(b) \equiv g(b)) \land (Y \equiv g(b))$ (by eliminate)
3. $(X \equiv a) \land (Y \equiv g(b))$ (by delete)

Succeeds with X=a and Y=g(b)

(d) 1.
$$(X \equiv a) \land (b \equiv Y)$$
 (by decompose)

Fail as target contains a variable.

Exercise 2

(a) 1.
$$(X \equiv a) \land (b \equiv Y)$$
 (by decompose)
2. $(X \equiv a) \land (Y \equiv b)$ (by switch)

Succeeds with X=a and Y=b

(b) 1.
$$(X \equiv Y) \land (b \equiv a)$$
 (by decompose)

Fails

(c) 1.
$$(X \equiv f(Y)) \land (a \equiv Y)$$
 (by decompose)
2. $(X \equiv f(Y)) \land (Y \equiv a)$ (by switch)

3.
$$(X \equiv f(a)) \land (Y \equiv a)$$
 (by eliminate)

Succeeds with $X \equiv f(a)$ and $Y \equiv a$.

$$\begin{array}{ll} \text{(d) 1. } (X \equiv f(Y)) \wedge (g(X) \equiv Y) & \text{(by $decompose$)} \\ 2. \ (X \equiv f(Y)) \wedge (g(f(Y)) \equiv Y) & \text{(by $coalesce$)} \\ 3. \ (X \equiv f(Y)) \wedge (Y \equiv g(f(Y))) & \text{(by $switch$)} \end{array}$$

Fails due to occurs check.

(e) 1.
$$(a + X \equiv a) \land (b \equiv Y)$$
 (by decompose)
2. $(a \equiv a + X) \land (b \equiv Y)$ (by switch)

Fails due to occurs check.