Automated Reasoning
The University of Edinburgh

Solutions to Exercise sheet 1: Propositional Logic

Exercise 1

1. Cats chase mice or birds, but not at the same time.
   This can be represented as: $(M \vee B) \wedge \neg(M \wedge B)$
   where M: Cats chase mice B: Cats chase birds

M	B	$M \vee B$	$\neg(M \wedge B)$	$(M \vee B) \wedge \neg(M \wedge B)$
t	t	t	f	f
t	f	t	t	t
f	t	t	t	t
f	f	f	t	f

2. If it rains the beach will be empty.
   This can be represented as: $R \longrightarrow E$
   where R: It rains E: Beach is empty

R	E	$R \longrightarrow E$
t	t	t
t	f	f
f	t	t
f	f	t

3. If Jane bought a piano today, she either sold her old one or took out a bank loan.
   This can be represented as: $P \longrightarrow S \vee B$
   where P: Jane bought a piano today S: Jane sold her old piano
   B: Jane took out a bank loan

P	S	L	$P \longrightarrow S \vee B$
t	t	t	t
t	t	f	t
t	f	t	t
t	f	f	f
f	t	t	t
f	t	f	t
f	f	t	t
f	f	f	t

Exercise 2

The proposition $P \wedge (P \longrightarrow Q)$ is satisfiable if there is some interpretation which evaluates to *true*. It is valid if all interpretations evaluate to *true*

$$
\begin{aligned}
& P \wedge (P \longrightarrow Q) \\
= & P \wedge (\neg P \vee Q) \\
= & (P \wedge \neg P) \vee (P \wedge Q) \\
= & \bot \vee (P \wedge Q) \\
= & P \wedge Q
\end{aligned}
$$

$P \wedge Q$ is satisfiable, since it evaluates to *true* when P is *true* and Q is *true*. Thus $P \wedge (P \longrightarrow Q)$ is satisfiable.

$P \wedge Q$ is not valid, since it evaluates to *false* when P is *false* and Q is *false*. Thus $P \wedge (P \longrightarrow Q)$ is not valid.

Exercise 3

CONNECTIVE	EXPRESSION WITH | ALONE
$\neg$	$p \mid$ p
$\wedge$	$(\ p \mid q\ ) \mid (p \mid q)$
$\vee$	$(p \mid p\ ) \mid (\ q \mid q\ )$
$\longrightarrow$	$p \mid (q \mid q)$

Some notes:

- $p \wedge q$ is the same as $\neg\ (p \mid q)$
- $p \vee q$ is the same as $\neg(\neg\ p \wedge \neg\ q)$
- $p \longrightarrow q$ is the same as $\neg\ p \vee q$