2nd module: Filtering in the frequency domain

Requirements:
Before starting the implementation of the tasks outlined in this file, please make sure to read Chapter 4 of *Digital Image Processing* (4th Edition) by Gonzalez.
Task 1 and Task 2 must be implemented without using any external image processing libraries, except for reading, loading, or displaying images. Task 3 must be implemented using the OpenCV library.

Taks:

1. Compute the Fourier Transform of an Image

- **Load the Image**
  - use grayscale images of 64x64 or 128x128 pixels
- **Compute the Fourier Transform:**
  - Implement the Discrete Fourier Transform (DFT) formula directly:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

  - Represent complex numbers using basic arithmetic or define a `Complex` class.
- **Visualize the Spectrum:**
  - **Shift the Origin of the Spectrum:** shift the zero-frequency component (DC component) of the Fourier Transform to the center of the spectrum. The Fourier Transform places the DC component (low frequencies) at the top-left corner by default. Centering the zero-frequency component improves visual interpretation by positioning high frequencies symmetrically around the origin.
  - **Compute the magnitude spectrum from the real and imaginary components of the Fourier transform:** the Fourier coefficients are complex numbers, containing both magnitude and phase. The magnitude spectrum shows the intensity of various frequency components, providing insight into the structure of the image.
  - **Apply a logarithmic transformation to the magnitude spectrum:** use the logarithmic scale to enhance visibility of smaller frequencies. Fourier spectra typically have a wide dynamic range, with low-frequency components dominating the values. The logarithmic transformation compresses this range, making subtle details in high frequencies more visible.
  - **Normalize and display the resulting spectrum for visual interpretation:** the transformed spectrum reveals the frequency content of the image, aiding in tasks like filtering and pattern recognition.

2. Apply a Filter in the Frequency Domain

- **Design the Filter:**
  - Create a 2D filter matrix of the same size as the image.
  - Low-pass filter
  - High-pass filter
- **Apply the Filter:**
  - Multiply the filter matrix with the Fourier-transformed image.
  - Ensure the result is computed for both real and imaginary components.
- **Inverse Transform to Spatial Domain:**
  - Implement the Inverse Discrete Fourier Transform (IDFT):

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M + vy/N)}$$

- **Visualize the Result**

3. Implement Task 1 and Task 2 using the OpenCV library.

You can use either C++ or Python for the lab tasks. If you choose C++ and need assistance configuring your workstation, feel free to contact me. For those using Python, the easiest option is to work on Google Colaboratory. Once you complete the assignment, please send me a compressed archive containing both the source code and a report. The report should include your results, along with any observations or comparisons you've made.