Fiber modes and scattering formalism

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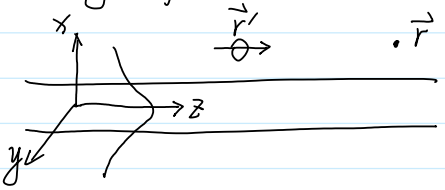
20140106-1 This part of work is to find how the Klimov's work fit into the scattering formalism.



One atom condition is assumed. Using a scattering theory with Born approximation, the electrical field at an arbitrary position $\vec{r}$ can be given by

$$\vec{E}(\vec{r},\omega) = \vec{E}_0(\vec{r},\omega) + \overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot\vec{E}_0(\vec{r}',\omega)\cdot\frac{4\pi\omega^2}{c^2}\alpha \quad \leftarrow \text{polarizability, treat as scalar first.}$$

$$= E_0\,\vec{u}(\vec{r},\omega) + \overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot E_0\,\vec{u}(\vec{r}',\omega)\cdot C$$

where $\vec{E}_0$ is the field strength without any atom/excitation or bare field vector; $\vec{u}$ is the normalized bare field vector; E_0 is the field amplitude of the bare field; $\overleftrightarrow{G}(\vec{r},\vec{r}',\omega)$ is the Green's function dyadic.

Let us only pick up the z-component of the E-field from Klimov's result for a z-dipole case.

$$E_z^{(R)}(\rho>a,\omega)$$

$$= \sum_{m=-\infty}^{\infty}\int d\beta\cdot e^{im\varphi+i\beta z}H_m^{(1)}(q\beta)\,A_{m\beta} \qquad (1)$$

where β is the wave number in z direction, m is the mode index, $q=\sqrt{k^2-\beta^2}$ is the radial wave number outside fiber. $A_{m\beta}$ is a constant determined by the boundary condition, which is given as Equ. (47) of Klimov's paper.

We treat the nanofiber as a lossless medium, and hence the field can be decomposed into guided and radiation contributions by

$$\vec{E}_z^{(R)}(\vec{r},\omega) = \sum_{m=\pm1}\oint_{\beta_0} d\beta\cdot e^{im\varphi+i\beta z}H_m^{(1)}(q\beta)\,A_{m\beta}$$

$$+\sum_{m=-\infty}^{\infty}\int_{-k}^{k} d\beta\cdot e^{im\varphi+i\beta z}H_m^{(1)}(q\beta)\,A_{m\beta}$$

$$= E^{(R)}_{z,\text{guided}}(\vec{r},\omega) + E^{(R)}_{z,\text{radiation}}(\vec{r},\omega). \qquad (2)$$

where $\oint_{\beta_0}$ is the contour integral around the pole for $m=\pm1$.

$k=\frac{\omega}{c}$ is the wave number @ ω.

In contrast, the guided and radiation bare field components have been given in Le Kien's paper (PRA, 72, 032509 (2005)). The z-components of the bare fields can be given by

$$E^{0}_{z,\text{guided}}(\rho>a,\omega) = i\sum_{f,p}\sqrt{\beta'}\, e_z^{(u)} e^{i(f\beta z+p\varphi)}$$

$$= i\sum_{f,p}\sqrt{\beta'}\cdot Af\frac{2q}{\beta}K_1(qr)\,e^{i(f\beta z+p\varphi)} \quad (3)$$

$$E^{0}_{z,\text{radiation}}(\rho>a,\omega) = i\sum_{m,p}\int_{-k}^{k} d\beta\cdot e_z^{(v)} e^{+i(\beta z+m\varphi)}$$

$$= i\sum_{m,p}\int_{-k}^{k} d\beta\cdot\sum_{j=1,2} C_j H_m^{(j)}(qr)\,e^{+i(\beta z+m\varphi)} \quad (4)$$

with
- constants A & C_j given is Le Kien's paper.
- β' is the derivative of β with respect to β,
- $f=\pm1$ denotes forward or backword propagation, → Is "f" necessary?
- $p=\pm1$ denotes the counter-clockwise or clockwise rotation of polarization.

To calculate the Green's functions, we need to calculate

$$\begin{cases} C\cdot\overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot\vec{E}^{0}_{\text{guided}} = \vec{E}_{\text{guided}}(\vec{r},\omega) - \vec{E}^{0}_{\text{guided}}(\vec{r},\omega) \\ C\cdot\overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot\vec{E}^{0}_{\text{radiation}} = \vec{E}_{\text{radiation}}(\vec{r},\omega) - \vec{E}^{0}_{\text{radiation}}(\vec{r},\omega) \end{cases} \quad (5)$$

Notice that

$$\overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot\vec{E} = \begin{pmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{pmatrix}\cdot\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} G_{xx}E_x + G_{xy}E_y + G_{xz}E_z \\ G_{yx}E_x + G_{yy}E_y + G_{yz}E_z \\ G_{zx}E_x + G_{zy}E_y + G_{zz}E_z \end{pmatrix}$$

$$\Rightarrow G_{ix}(\vec{r},\vec{r}',\omega)E^{0}_x(\vec{r}',\omega) + G_{iy}(\vec{r},\vec{r}',\omega)E^{0}_y(\vec{r}',\omega) + G_{iz}(\vec{r},\vec{r}',\omega)E^{0}_z(\vec{r}',\omega) = \left[E_i(\vec{r},\omega) - E^{0}_i(\vec{r},\omega)\right]/C \quad (6)$$

The elements of the Green's function dyadic (tensor) can be calculated analytically by matching up some patterns. But this is not easy or even unique.

However, we can numerically calculate the Green's functions for the nanofiber system with one dipole, and check the equivalence of Klimov's formalism and Le Kien & Hakuta's formulism backward.

0106-2. Below, I will give the computation framework for solving the Green's function tensor including one dipole starting from Klimov's conclusion or a numerical solver.

With a dipole source at $\vec{r}'$, the field at an arbitrary position $\vec{r}$ can be given by

$$\vec{E}(\vec{r},\omega) = \overleftrightarrow{G}(\vec{r},\vec{r}',\omega)\cdot\vec{d}(\vec{r}',\omega) \quad (7)$$

→ dipole momentum.

where we have assumed there is no initial field while the dipole is excited.

$$\Rightarrow E_i(\vec{r},\omega) = G_{ix}(\vec{r},\vec{r}',\omega)\,d_x(\vec{r}',\omega) + G_{iy}(\vec{r},\vec{r}',\omega)\,d_y(\vec{r}',\omega) + G_{iz}(\vec{r},\vec{r}',\omega)\,d_z(\vec{r}',\omega). \quad (8)$$

Since the dipole momentum $\vec{d}(\vec{r},\omega)$ is artificially given, we can use a dipole merely orientated in j direction and measure the field components at position $\vec{r}$, which is exactly Klimov's paper has solved or can be solved numerically using some software.

The elements of the Green's function can then be given by

$$G_{ij}(\vec{r},\vec{r}',\omega) = \frac{E_i(\vec{r},\omega)}{d_j(\vec{r}',\omega)}. \quad (9)$$

By orientating the dipole in $j = x, y, z$ directions, all components of the Green function tensor $\vec{\vec{G}}(\vec{r},\vec{r}',\omega)$ can be calculated.
Actually, using the results of Klimov's paper, the Green's function tensor in the cylindrical coordinate can be written down immediately, which can be used to standardize any numerical calculation method through some software.

0106-3. As long as we used a proper dipole momentum function in the last part, we should be approximately able to replace the Green's function tensor in Equ. (6) with the one obtained in Equ. (9), and verify the validity of Equ. (6) backward.

More rigorously, the Green's function obtained in Equ. (9) should be a bare-dipole Green's function. That is to say, the Green's function tensor in Equ (9) does not consider the interaction between the dipole and the waveguide field. while we expect the Green's function tensor in the formalism of Equation (6) should be the one including the scattering effect from the atom (actually, we did not include this effect when we write Equ (6) down, and hence Equ. (6) does not really associate Klimov's formalism and Le Kien & Hakuta's formalism toughtly).

The correct way of using Equ. (6) is to use the one-dipole Green's function tensor which can be obtained from the bare-dipole Green's function tensor through Dyson Equation or other methods, and the total field should be the field considering both the fiber modes & the presence of the atom.