

The impact of interparticle correlations on the behavior of Bose-Einstein Condensates (BECs) is discussed using two approaches. In the first approach, the wavefunction of a BEC is encoded in the N -particle sector of an extended “catalytic state”. Going to a time-dependent interaction picture, we can organize the effective Hamiltonian by powers of $N^{-\frac{1}{2}}$. Requiring the terms of order $N^{\frac{1}{2}}$ to vanish, we get the Gross-Pitaevskii Equation (GPE). Going to the next order, N^0 , we obtain the number-conserving Bogoliubov approximation. Our approach allows one to stay in the Schrödinger picture and to apply many techniques from quantum optics. Moreover, it is much easier to track different orders in the Hamiltonian and to generalize to the multi-component case. In the second approach, I consider a state of $N = l \times n$ bosons that is derived by symmetrizing the n -fold tensor product of an arbitrary l -boson state. The rationale behind this approach is the BBGKY hierarchy: errors to the many-particle Reduced Density Matrices (RDMs) only weakly affect the few-particle RDMs. Particularly, we are interested in the pure state case for $l = 2$, which we call the Pair-Correlated State (PCS). I show that PCS reproduces the number-conserving Bogoliubov approximation; moreover, it also works in the strong interaction regime where the Bogoliubov approximation fails. For the two-site Bose-Hubbard model, I find numerically that the error (measured by trace distance of the two-particle RDMs) of PCS is less than two percent over the entire parameter space, thus making PCS a bridge between the superfluid and Mott insulating phases. Amazingly, the error of PCS does not increase, in the time-dependent case, as the system evolves for longer times. I derive both time-dependent and -independent equations for the ground state and the time evolution of the PCS ansatz, along with a condition for fragmentation of BECs. The time complexity of simulating PCS does not depend on N and is linear in the number of orbitals in use. Compared to other methods, e.g., the Jastrow wavefunction, the Gutzwiller wavefunction, and the multi-configurational time-dependent Hartree method, our approach does not require quantum Monte Carlo nor very demanding computational power.