Solve the boundary condition problem of nanofiber

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0115. Starting from Equ. (1.121–122) of my scattering note.

For simplicity, I use $r \leftarrow r_\perp$, $H_m \leftarrow H_m^{(1)}$, $E_z \leftarrow \mathcal{E}^{(0)}_{z,m\beta}(r_\perp=a)$, $B_z \leftarrow \mathcal{B}^{(0)}_{z,m\beta}$, $E_\phi \leftarrow \mathcal{E}^{(0)}_{\phi,m\beta}$, $B_\phi \leftarrow \mathcal{B}^{(0)}_{\phi,m\beta}$

To make equations symmetric, we multiply $ah^2p^2 J_m(ha)$ on both sides of Equs (1.121–122) to give

$$\begin{cases} m\beta(h^2-p^2)J_m(ha)H_m(pa)\,a_{m\beta} + ika\left[h^2 J_m(ha)\frac{\partial H_m(pa)}{\partial a} - p^2\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right]b_{m\beta} \\ = m\beta p^2 E_z J_m(ha) + ikap^2\frac{\partial J_m(ha)}{\partial a}B_z + ah^2p^2J_m(ha)E_\phi \quad (1) \\ m\beta(h^2-p^2)J_m(ha)H_m(pa)\,b_{m\beta} + ika\left[h^2 J_m(ha)\frac{\partial H_m(pa)}{\partial a} - \varepsilon p^2\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right]a_{m\beta} \\ = m\beta p^2 B_z J_m(ha) + ika\epsilon p^2\frac{\partial J_m(ha)}{\partial a}E_z - ah^2p^2J_m(ha)B_\phi. \quad (2) \end{cases}$$

Note that $h^2 = \varepsilon k^2-\beta^2$, $p^2 = k^2-\beta^2 \Rightarrow h^2-p^2 = k^2(\varepsilon-1)$. So, the first coefficient of both equations can be written as

$$\mathbb{P} = \beta m k^2 J_m(ha)H_m(pa)(\varepsilon-1)$$ — same as defined in Klimov's paper.

For the second coefficient of the two equations, we can similiarly define

$$\left.\begin{aligned} Q &= ka\left[h^2 J_m(ha)\frac{\partial H_m(pa)}{\partial a} - p^2\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right] \\ R &= ka\left[h^2 J_m(ha)\frac{\partial H_m(pa)}{\partial a} - \varepsilon p^2\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right] \end{aligned}\right\}$$ — equivalent to Klimov's definition

Symmetrically, we define

$$S = kahp\left[J_m(ha)\frac{\partial H_m(pa)}{\partial a} - \frac{\partial J_m(ha)}{\partial a}H_m(pa)\right]$$

$$R = kahp\left[J_m(ha)\frac{\partial H_m(pa)}{\partial a} - \frac{\partial J_m(ha)}{\partial a}H_m(pa)\right].$$

Now, Equs. (1) & (2) can be rewritten as

$$\begin{cases} \mathbb{P}a_{m\beta} + iQ\,b_{m\beta} = m\beta p^2 E_z J_m(ha) + ikap^2\frac{\partial J_m(ha)}{\partial a}B_z + ah^2p^2J_m(ha)E_\phi = U \quad (3) \\ \mathbb{P}b_{m\beta} + iR\,a_{m\beta} = m\beta p^2 B_z J_m(ha) + ika\epsilon p^2\frac{\partial J_m(ha)}{\partial a}E_z - ah^2p^2J_m(ha)B_\phi = V \quad (4) \end{cases}$$

(3) $\times i\mathbb{P}$ + (4) $\times Q$ gives

$$\begin{aligned} i(\mathbb{P}^2+QR)a_{m\beta} &= i\mathbb{P}U + QV \\ &= i\left[m\beta J_m(ha)\mathbb{P} + ka\epsilon\frac{\partial J_m(ha)}{\partial a}Q\right]p^2E_z + \left[m\beta J_m(ha)Q - ka\frac{\partial J_m(ha)}{\partial a}\mathbb{P}\right]p^2B_z \\ &\quad + iah^2p^2J_m(ha)\mathbb{P}E_\phi - ah^2p^2J_m(ha)Q\,B_\phi \end{aligned}$$

Using the fact that

$$\begin{aligned} &\left[m\beta J_m(ha)Q - ka\frac{\partial J_m(ha)}{\partial a}\mathbb{P}\right]p^2 \\ &= m\beta ka J_m(ha)p^2\left[h^2J_m(ha)\frac{\partial H_m(pa)}{\partial a} - p^2\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right] - \beta mkaJ_m(ha)p^2H_m(pa)\frac{\partial J_m(ha)}{\partial a}(\varepsilon-1)k^2 \\ &= m\beta kap^2J_m(ha)\left[h^2J_m(ha)\frac{\partial H_m(pa)}{\partial a} - (p^2+h^2-p^2)\frac{\partial J_m(ha)}{\partial a}H_m(pa)\right] \end{aligned}$$

$= m\beta h p J_m(ha) S$

$$\Rightarrow a_{m\beta} = \frac{na}{p^2 + QR}$$

with $na = [m\beta J_m(ha)\mathbb{P} + ka\epsilon \frac{\partial J_m(ha)}{\partial a} Q] p^2 E_z - i m\beta p h J_m(ha) S$

$+ ah^2 p^2 J_m(ha)\mathbb{P} E_\phi + i a h^2 p^2 J_m(ha) Q B_\phi.$

Similarly, $b_{m\beta} = \frac{nb}{p^2 + QR}$. with nb defined as in Klimov's paper:

$nb = h^2 p^2 a J_m(ha)\mathbb{P} B_\phi + p^2 (J_m(ha)\beta m \mathbb{P} - ak \frac{\partial J_m(ha)}{\partial a} R) B_z$

$+ i h^2 p^2 a J_m(ha) R E_\phi + i m \beta h p J_m(ha) T E_z.$