

The goal of this note is to derive the polarizability formula for trapped atoms.

First, we start with a two-level atom interacting with a far-off resonance light field.

$$\begin{array}{c} \xrightarrow{\quad} |e\rangle \\ \uparrow \quad \downarrow \Omega \quad \leftarrow \vec{E} = \frac{1}{2} \vec{E}_0 \vec{u} e^{-i\omega t} + \frac{1}{2} \vec{E}_0 \vec{u}^* e^{i\omega t} \\ \xrightarrow{\quad} |g\rangle \end{array} \quad \int d^3r \cdot \vec{u}_m^*(\vec{r}) \cdot \vec{u}_n(\vec{r}) = \delta_{mn}, \quad \vec{u}_m(\vec{r}) = \vec{u}_m(\vec{r}_\perp) e^{i\beta_0 z - i\omega t}$$

$$\hat{H} = \hat{H}_F + \hat{H}_A + \hat{H}_{int},$$

with  $\left\{ \begin{array}{l} \hat{H}_F = \sum_{\vec{k}} \hbar \omega_{\vec{k}} (\hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}} + \frac{1}{2}) \\ \hat{H}_A = \hbar \omega_0 |ex\rangle \langle ex| \rightarrow \text{chose the ground state as the zero energy state.} \\ \hat{H}_{int} = \hbar \sum_{\vec{k}} (g_{\vec{k}} \hat{a}_{\vec{k}} \hat{\sigma}_+ + g_{\vec{k}}^* \hat{a}_{\vec{k}}^\dagger \hat{\sigma}_-) \end{array} \right.$