Comparing enhancement via vacuum modes VS nanofiber modes

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One fundamental question of our nanofiber study is how strong the nanofiber can couple the scattered field into the output guided modes compared with the fraction of the fundamental modes scattered via an atom in vacuum. We can write down some important formulas for both the nanofiber case & the vacuum case, and pick out what aspects are the most important ones we care about.

Field response formula for fields scattered by only one atom located at $\vec{r}'$:

$$\vec{E}_{out}(\vec{r}) = \vec{E}_{in}(\vec{r}) + \overleftrightarrow{G}(\vec{r},\vec{r}')\cdot\overleftrightarrow{\alpha}(\vec{r}')\cdot\vec{E}_{in}(\vec{r}') \quad \leftarrow \text{Born approximation}$$

$$= \vec{E}_{in}(\vec{r}) + \alpha(\vec{r}')\overleftrightarrow{G}(\vec{r},\vec{r}')\cdot\vec{E}_{in}(\vec{r}') \quad \leftarrow \text{for simplicity, } \overleftrightarrow{\alpha} \text{ to be scalar.}$$

For the vacuum case, paraxial approximation holds, and the dyadic Green's function becomes a scalar as a function of the progating distance $(z-z')$ & $(\vec{r}_\perp - \vec{r}_\perp')$:

$\overleftrightarrow{G} \to K(\vec{r}_\perp, z; \vec{r}_\perp', z') = \frac{k_L}{i2\pi(z-z')} e^{\frac{ik_L|\vec{r}_\perp-\vec{r}_\perp'|^2}{2(z-z')}}$. Meanwhile, the output field under the Born approximation becomes

$$\vec{E}_{out}(\vec{r}) = \vec{E}_{out}(\vec{r}_\perp, z) = \vec{E}_{in}(\vec{r}_\perp, z) + i2\pi k_0 \alpha K(\vec{r}_\perp, z; \vec{r}_\perp', z')\vec{E}_{in}(\vec{r}_\perp', z').$$

(see my notes on "Green function of free-space radiation").

While for the nanofiber case, the Green's function is a complex tensor, and the amplitudes of $\overleftrightarrow{G}$ elements does not change in the propagating direction $\vec{z}$.

Mode decomposition of the two cases:

For the vacuum case:

The paraxial part of the field (expressed in K) can be expanded in a complete set of transverse and orthogonal mode set $\{\vec{u}_{mn}(\vec{r}_\perp, z)\}$ normalized via

$$\int d^2 r_\perp \, \vec{u}^*_{mn}(\vec{r}_\perp, z)\vec{u}_{m'n'}(\vec{r}_\perp, z) = A\delta_{mm'}\delta_{nn'}$$

$$A = \int d^2 r_\perp |\vec{u}_{00}(\vec{r}_\perp, z)|^2 = \frac{\pi}{2}W_0^2, \quad \vec{u}_{00}(\vec{r}_\perp, z) = \vec{\epsilon}_L u_{00}(\vec{r}_\perp, z),$$

where W_0 is the beam waist, u_{00} is the fundamental mode — TEM_{00} mode which is the only mode picked out at the detecting end.

For the nanofiber case, the propagating modes can be decomposed into scattered and guided modes, and only the guided HE_{11} modes $(m = \pm)$ can be guided to a far-field detector.

Notice that, we always assume the atom-atom interaction through scattered field is negligible.

Scattering amplitude and phase shifting:

For the vacuum case, we consider how the scattering process rotates the phase and attenuates the amplitude given a fundamental mode input. In this case, the output field can be given under the paraxial approximation as

$$\vec{E}_{out}(\vec{r}) = i2\pi k_L E_L \vec{u}_{00}(\vec{r}_\perp, z) + \alpha(\vec{r}')\, K(\vec{r}_\perp, z; \vec{r}_\perp', z')\; E_L \vec{u}_{00}(\vec{r}_\perp', z') = \vec{E}_{in}(\vec{r}) + \vec{E}_{scat}(\vec{r}).$$

where I have made the input (laser) field as

$$\vec{E}_{in}(\vec{r}) = E_L \vec{u}_{00}(\vec{r}_\perp, z).$$

Now we define the "scattering coefficient" to the mn-th mode by

$$C_{mn} = i2\pi k_L \int \frac{d^2 r_\perp}{A}\, \vec{u}^*_{mn}(\vec{r}_\perp, z)\cdot \vec{E}_{scat}(\vec{r}_\perp, z),$$

so that the output field can be written as

$$\vec{E}_{out}(\vec{r}) = E_L \vec{u}_{00}(\vec{r}_\perp, z) + \underbrace{\sum_{m,n} C_{mn} \vec{u}_{mn}(\vec{r}_\perp, z)}$$

field scattered into mn-modes.

For the foundamental mode input case, the scattering coefficient for the u_{00} mode is

$$\vec{C}_{00} = i2\pi k_L \underbrace{\int \frac{d^2 r_\perp}{A}\, \vec{u}^*_{00}(\vec{r}_\perp, z)\cdot K(\vec{r}_\perp, z; \vec{r}_\perp', z')}\, \alpha(\vec{r}') \cdot E_L \vec{u}_{00}(\vec{r}_\perp', z)$$

$$= \left(\int \frac{d^2 r_\perp}{A}\, \vec{u}_{00}(\vec{r}_\perp, z) K^*(\vec{r}_\perp, z; \vec{r}_\perp', z')\right)^* = \vec{u}^*_{00}(\vec{r}_\perp', z')/A$$

$$= i2\pi k_L \frac{\alpha(\vec{r}')}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 E_L$$

$\Rightarrow$ The output field with the foundamental mode input, $\vec{E}_{in}(\vec{r}_\perp, z) = \vec{\epsilon}_L E_L u_{00}(\vec{r}_\perp, z)$, is

$$\vec{E}_{out}(\vec{r}_\perp, z) \doteq E_L \vec{u}_{00}(\vec{r}_\perp, z) + \frac{C_{00}}{E_L} E_L \vec{u}_{00}(\vec{r}_\perp, z) + \text{other terms}^{\text{ignored.}}$$

$$= \left(1 + i2\pi k_L \frac{\alpha(\vec{r}')}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2\right) \cdot E_L \vec{u}_{00}(\vec{r}_\perp, z)$$

$$\approx e^{i2\pi \frac{\alpha(\vec{r}')}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 k_L} \vec{E}_{in}(\vec{r}_\perp, z)$$

$$= e^{i2\pi \frac{Re(\alpha)}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 k_L}\, e^{-2\pi \frac{Im(\alpha)}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 k_L}\, \vec{E}_{in}(\vec{r}_\perp, z).$$

Therefore, the atom, as a scatter, gives the initial input field a phase shift

$$\delta\phi = \frac{2\pi Re(\alpha(\vec{r}'))}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 k_L \leftarrow 2\pi \cdot Re(\alpha(\vec{r}')) k_L \int \frac{d^2 r_\perp}{A}\, \vec{u}^*_{00}(\vec{r}_\perp, z)\cdot K(\vec{r}_\perp - \vec{r}_\perp', z - z')\cdot \vec{u}_{00}(\vec{r}_\perp', z').$$

and an attenuation to the amplitude

$$a = \frac{2\pi \cdot Im(\alpha(\vec{r}'))}{A} |\vec{u}_{00}(\vec{r}_\perp', z')|^2 k_L. \leftarrow 2\pi \cdot Im(\alpha(\vec{r}')) k_L \int \frac{d^2 r_\perp}{A}\, \vec{u}^*_{00}(\vec{r}_\perp, z)\cdot K(\vec{r}_\perp - \vec{r}_\perp', z - z')\, \vec{u}_{00}(\vec{r}_\perp', z').$$

To make the paraxial approximation work, we choose

$$z \Rightarrow z_R = \frac{\pi W_0^2}{\lambda}.$$

For the nanofiber case, we can go through a similar deriving process to conclude that:

① Given a right-circular-polarized forward-propagating HE_{11} $(m=1)$ mode input field $E_+\vec{U}_+(\vec{r}_\perp, z)$, the output field can be expressed as

$$\vec{E}_{out}(\vec{r}_\perp, z) = E_+\vec{U}_+(\vec{r}_\perp, z) + C_{++}\cdot\vec{U}_+(\vec{r}_\perp, z) + C_{-+}\vec{U}_-(\vec{r}_\perp, z) + \text{other-mode terms.}$$

Here, we ignore the unguided modes, and only focus on the $m=1$ HE_{11} mode terms, in order to determine phase shift & amplitude modulation. We define the scattering coefficients

$$C_{++} = \int d^2r_\perp \vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')E_+\alpha(\vec{r}')$$

$$C_{-+} = \int d^2r_\perp \vec{U}_-^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')E_+\alpha(\vec{r}')$$

or, in general,

$$C_{m+} = E_+\int d^2r_\perp \vec{U}_m^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')\alpha(\vec{r}'),$$

where $\overleftrightarrow{G}^g$ is the dyadic Green's function only with the guided mode contribution; $\vec{U}_+$ & $\vec{U}_-$ are the guided HE_{11} modes with mode indices $m=+1$ & -1, respectively.

② If we only consider the $m=1$ HE_{11} mode output, the output field can be written as

$$\vec{E}_{out}(\vec{r}_\perp, z) \doteq E_+\vec{U}_+(\vec{r}_\perp, z) + \frac{C_{++}}{E_+}E_+\vec{U}_+(\vec{r}_\perp, z)$$

$$= \left(1 + \alpha\int d^2r_\perp \vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')\right)E_+\vec{U}_+(\vec{r}_\perp, z)$$

$$\approx e^{i\delta\phi}e^{a}\vec{E}_{in}(\vec{r}_\perp, z)$$

with phase shift

$$\delta\phi = \mathrm{Im}\left[\alpha\cdot\int d^2r_\perp\cdot\vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')\right]$$

$$\cancel{\simeq \mathrm{Im}(\alpha)\cdot\int d^2r_\perp \vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')}$$

$$a = \mathrm{Re}\left[\alpha\cdot\int d^2r_\perp \vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')\right]$$

$$\cancel{\simeq \mathrm{Re}(\alpha)\cdot\int d^2r_\perp \vec{U}_+^*(\vec{r}_\perp, z)\cdot\overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp', z')\cdot\vec{U}_+(\vec{r}_\perp', z')}$$

To make the approximation that $\overleftrightarrow{G} \simeq \overleftrightarrow{G}^g$, as we used above, it requires $z \gg \lambda$.

If we simply make the polarizability α the same one for both vacuum & nanofiber cases, the relative strength of phase shifts between the two cases is approximately determined via nanofiber cases, the relative strength of phase shifts between the two cases is approximately determined via

$$C_{n/v} = A \frac{\mathrm{Im}\left[\alpha \int d^2 r_\perp \, \vec{u}_+^*(\vec{r}_\perp, z) \cdot \overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp{}', z') \cdot \vec{u}_+(\vec{r}_\perp{}', z')\right]}{2\pi k_L \, \mathrm{Re}(\alpha) \, |u_{00}(\vec{r}_\perp{}', z')|^2}$$

$$= \frac{W_0^2 \cdot \mathrm{Im}\left[\alpha \cdot \int d^2 r_\perp \cdot \vec{u}_+^*(\vec{r}_\perp, z) \cdot \overleftrightarrow{G}^g(\vec{r}_\perp, z; \vec{r}_\perp{}', z') \cdot \vec{u}_+(\vec{r}_\perp{}', z')\right]}{4 k_L \, \mathrm{Re}(\alpha) \cdot |u_{00}(\vec{r}_\perp{}', z')|^2} \quad \leftarrow \alpha \text{ cannot cancel!}$$

We can use

$$u_{00}(\vec{r}_\perp, z) = \frac{W_0}{W(z)} e^{-\frac{|\vec{r}_\perp|^2}{|W(z)|^2}} e^{\frac{i k_L |\vec{r}_\perp|^2}{2R(z)}} e^{-i\phi(z)},$$

with

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

$$R(z) = z\left(1 + \left(\frac{z_R}{z}\right)^2\right)$$

$$\phi(z) = \tan^{-1}(z/z_R)$$

$$z_R = \frac{k_L W_0^2}{2}.$$

To be effective, the positions of the atom in the vacuum and nanofiber cases can be different, and should be chosen to be the respective typical values.