Phase shift from Heisenberg-Langevin equation

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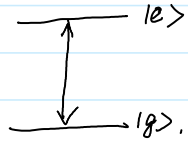
0702-1. We start our discussion from F. Le Kien. etc. PRA. 72, 032509. (2005).
Let's focus on a two level system at this moment.
The Heisenberg-Langevin equations read



$$\begin{cases} \dot{\sigma}_{ge} = -\frac{1}{2}\Gamma_{ee}\sigma_{ge} + \xi_{ge} & ① \\ \dot{\sigma}_{ee} = -\Gamma_{ee}\sigma_{ee} + \xi_{ee} & ② \\ \dot{\sigma}_{gg} = \Gamma_{eegg}\sigma_{ee} + \xi_{gg} \rightleftharpoons \Gamma_{ee}\sigma_{ee} + \xi_{gg}. & ③ \end{cases}$$

where $\begin{cases} \Gamma_{ee} = \gamma_{ee}^{(g)} + \gamma_{ee}^{(r)} \\ \Gamma_{eegg} = \gamma_{eegg}^{(g)} + \gamma_{eegg}^{(r)} = \gamma_{ee}^{(g)} + \gamma_{ee}^{(r)} = \Gamma_{ee} \end{cases}$

$\gamma_{ee}^{(r)}$: ↳ spontaneous emission into radiation modes.

$\gamma_{ee}^{(g)}$: ↳ spontaneous emission into guide modes.

$\sigma_{ge} = \hat{\sigma}_{ge} = |g\rangle\langle e|$.

In our case, our "atom" – the 2-level system — is in a far-detuning zone, and hence

$$\begin{cases} \langle\sigma_{ee}\rangle(t) \simeq 0 \\ \langle\sigma_{gg}\rangle(t) \simeq 1. \end{cases}$$

Here, we only interested in the evolution of σ_{ge} operator, in order to retrieve

$$\begin{cases} a_\mu(t) = a_\mu(t_0) + \sum_{e,g} G_{\mu eg}^{*}\int_{t_0}^{t} dt'\, \sigma_{ge}(t')\, e^{i(\omega-\omega_0)t'} & ④ \\ a_\nu(t) = a_\nu(t_0) + \sum_{eg} G_{\nu eg}^{*}\int_{t_0}^{t} dt'\, \sigma_{ge}(t')\, e^{i(\omega-\omega_0)t'} & ⑤ \end{cases}$$

and hence

$$\begin{cases} \hat{E}_{guided}^{(+)} = i\sum_{fp}\int_0^\infty d\omega \sqrt{\frac{\hbar\omega\beta'}{4\pi\epsilon_0}}\, a_\mu \vec{e}^{(\mu)} e^{-i(\omega t - f\beta z - p\varphi)} & ⑥ \\ \hat{E}_{rad}^{(+)} = i\sum_{mp}\int_0^\infty d\omega \int_{-kn_2}^{kn_2} d\beta \sqrt{\frac{\hbar\omega}{4\pi\epsilon_0}}\, a_\nu \vec{e}^{(\nu)} e^{-i(\omega t - \beta z - m\varphi)}. & ⑦. \end{cases}$$

← We are more interested in this one.

The formal solution for equation ① is

$$\sigma_{ge}(t) = \frac{\int_0^t e^{\frac{1}{2}\int_0^{t'}\Gamma_{ee}dt''}\cdot \xi_{ge}\,dt'}{e^{\int_0^t \frac{1}{2}\Gamma_{ee}dt'}} + C$$

$$\doteq \frac{\int_0^t e^{\frac{1}{2}\Gamma_{ee}t'}\xi_{ge}\cdot dt'}{e^{\frac{1}{2}\Gamma_{ee}t}} + C$$

From my derivation to the Heisenberg-Langevin equations, we have

$$\xi_{ge} = -\sum_{fp}\int_0^\infty d\omega\, G_{\mu eg}\,(\sigma_{gg}-\sigma_{ee})\, a_\mu(t_0)\, e^{-i(\omega-\omega_0)t}$$

$$-\sum \int_0^\infty d\omega \int^{kn_2} d\beta\, G_{\nu eg}\,(\sigma_{gg}-\sigma_{ee})\, a_\nu(t_0)\, e^{-i(\omega-\omega_0)t}$$

$$
\begin{aligned}
\sigma_{ge} = & -\sum_{fp}\int_0 d\omega\, G_{ueg}\,(\sigma_{gg}-\sigma_{ee})\, a_u(t_0)\, e \\
& -\sum_{mp}\int_0^\infty d\omega \int_{-kn_2}^{kn_2} d\beta\; G_{\nu eg}\,(\sigma_{gg}-\sigma_{ee})\, a_\nu(t_0)\, e^{-i(\omega-\omega_0)t} \\
& + i\sum_{fp}\int_0^\infty d\omega\; \mathbb{P}\left(\frac{1}{\omega-\omega_0}\right)\cdot G_{ueg}\, G_{ueg}^{*}\, \sigma_{ge} \\
& + i\sum_{mp}\int_0^\infty d\omega \int_{-kn_2}^{kn_2} d\beta\; \mathbb{P}\left(\frac{1}{\omega-\omega_0}\right)\cdot G_{\nu eg}\, G_{\nu eg}^{*}\, \sigma_{ge}
\end{aligned}
$$