A Scenario Approach for Parametric Markov Decision Processes

Prof. Dr. Joost-Pieter Katoen's 60th birthday

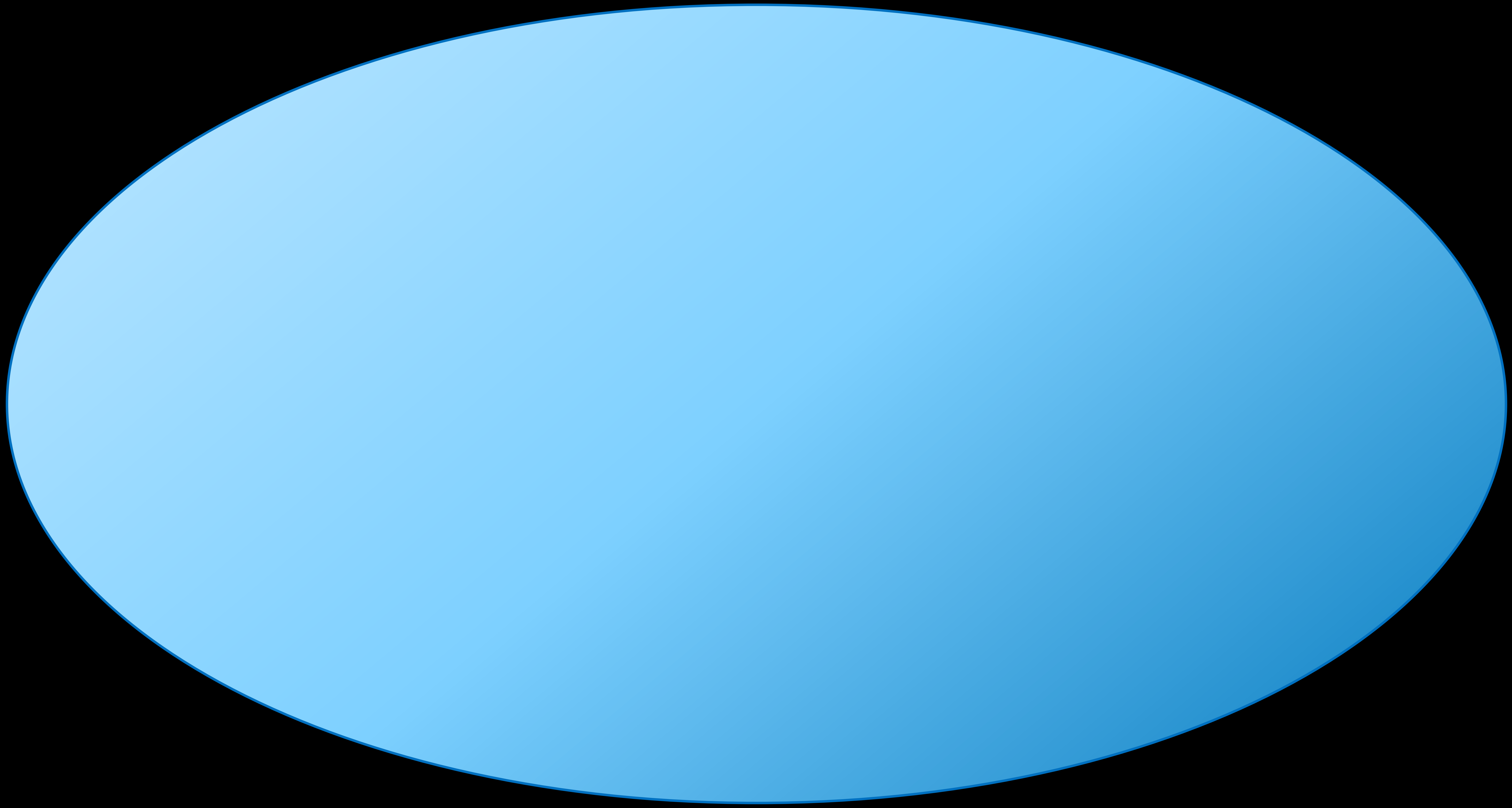
Zhiming Chi, Ying Liu, Andrea Turrini, Lijun Zhang, and David N. Jansen 🇨🇳

Markov Decision Process

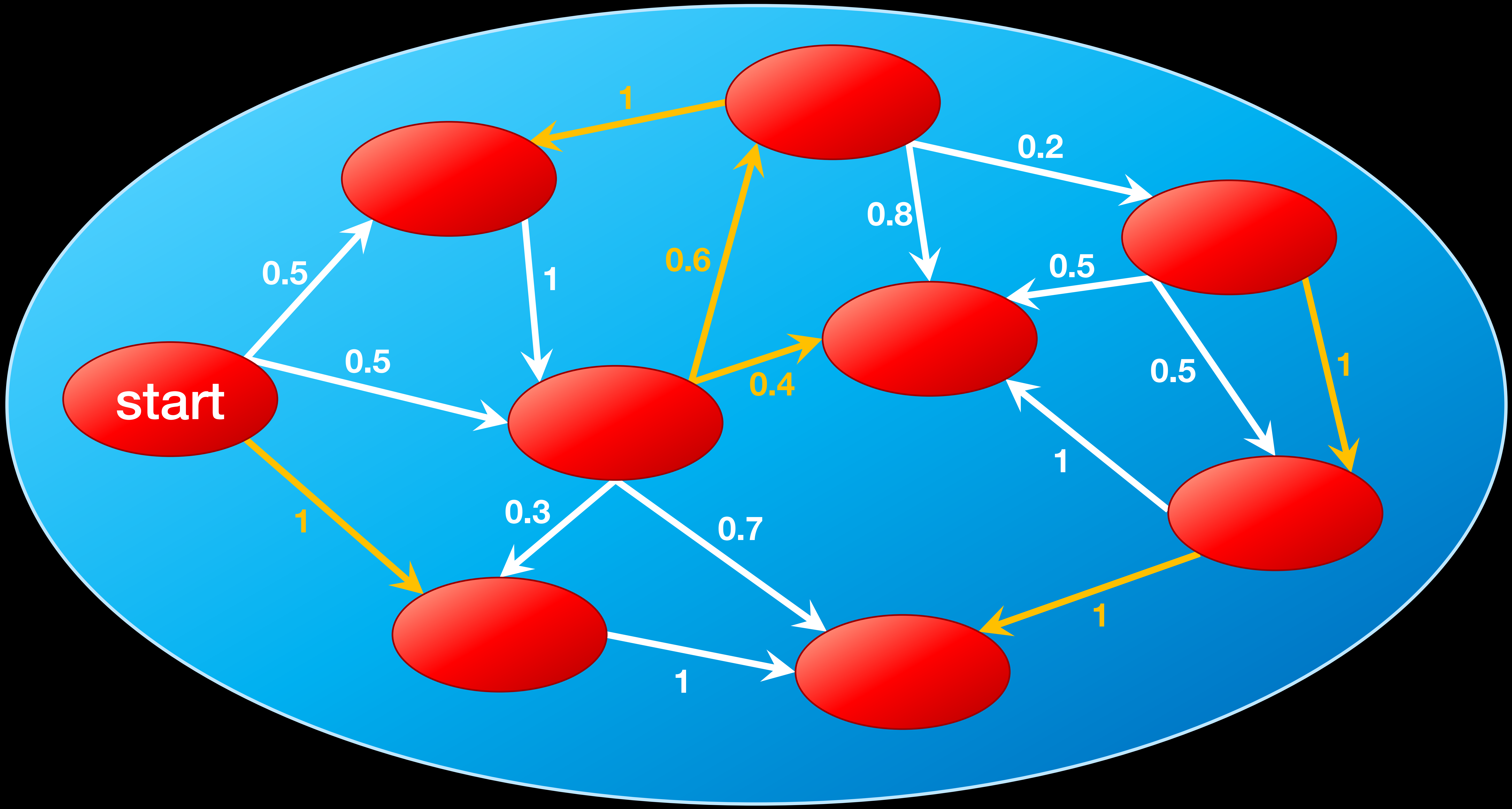
Markov Decision Process

commons.wikimedia.org/wiki/File:Amsterdam_-_Risk_players_-_1136.jpg

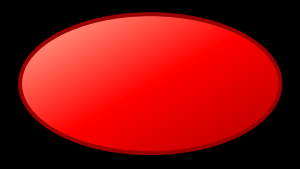
Markov Decision Process

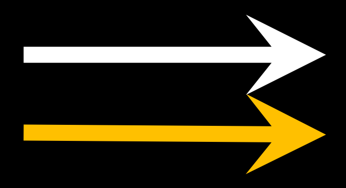


Markov Decision Process

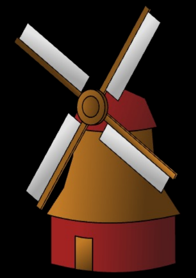


Markov Decision Process

- set of states S 
  (sometimes with initial state) 

- set of actions Σ 
- probabilistic transition relation $S \times \Sigma \rightharpoonup \mathit{Dist}(S)$ 

  can be a partial function (if not every state allows every action)
  $\mathit{Dist}(S)$ = distributions over S

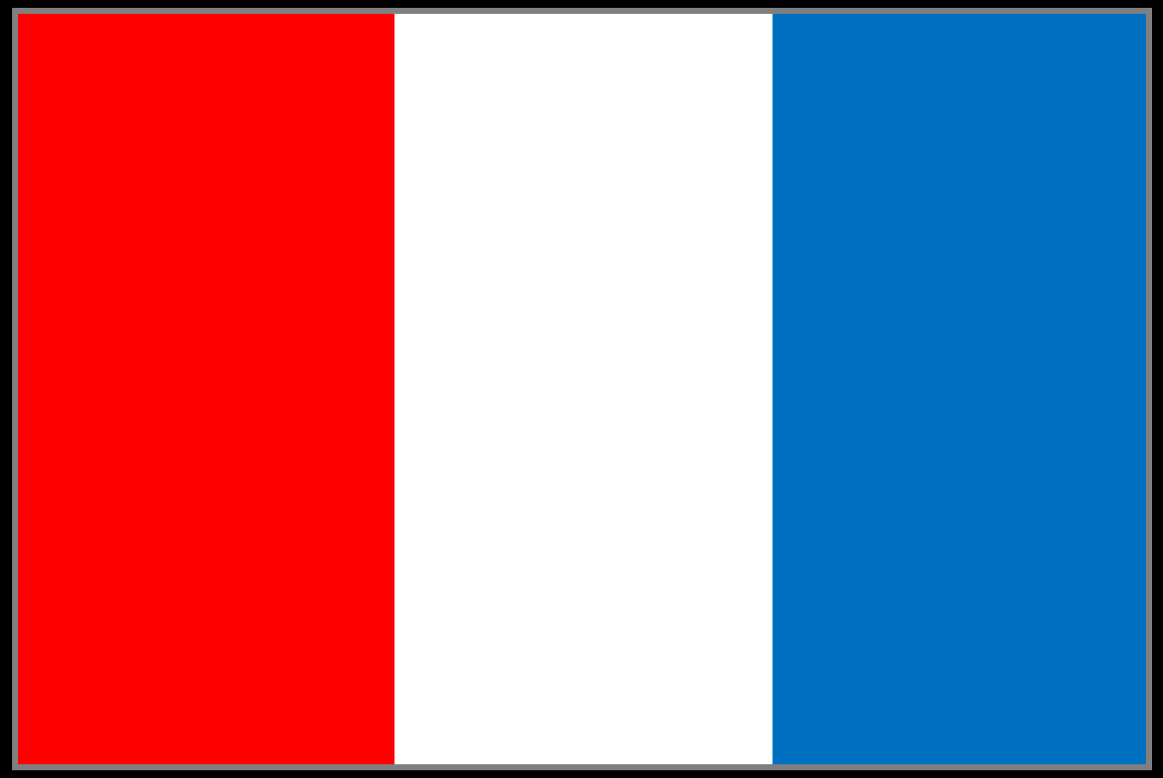
Short Break: Dutch National Flag 

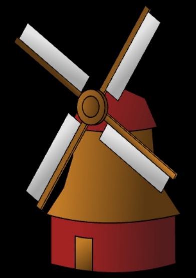
a)

b)

c)

d)

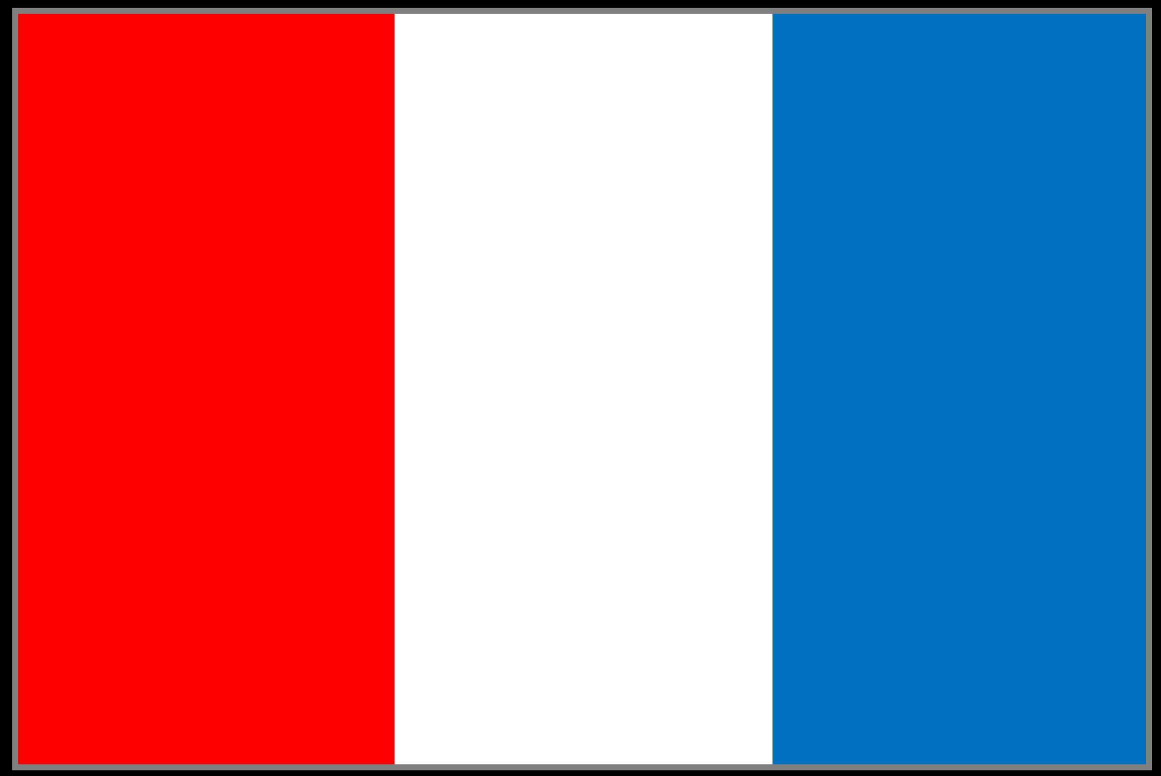


Short Break: Dutch National Flag 

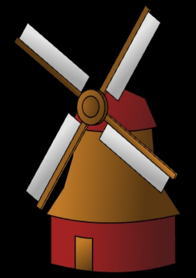
a) 

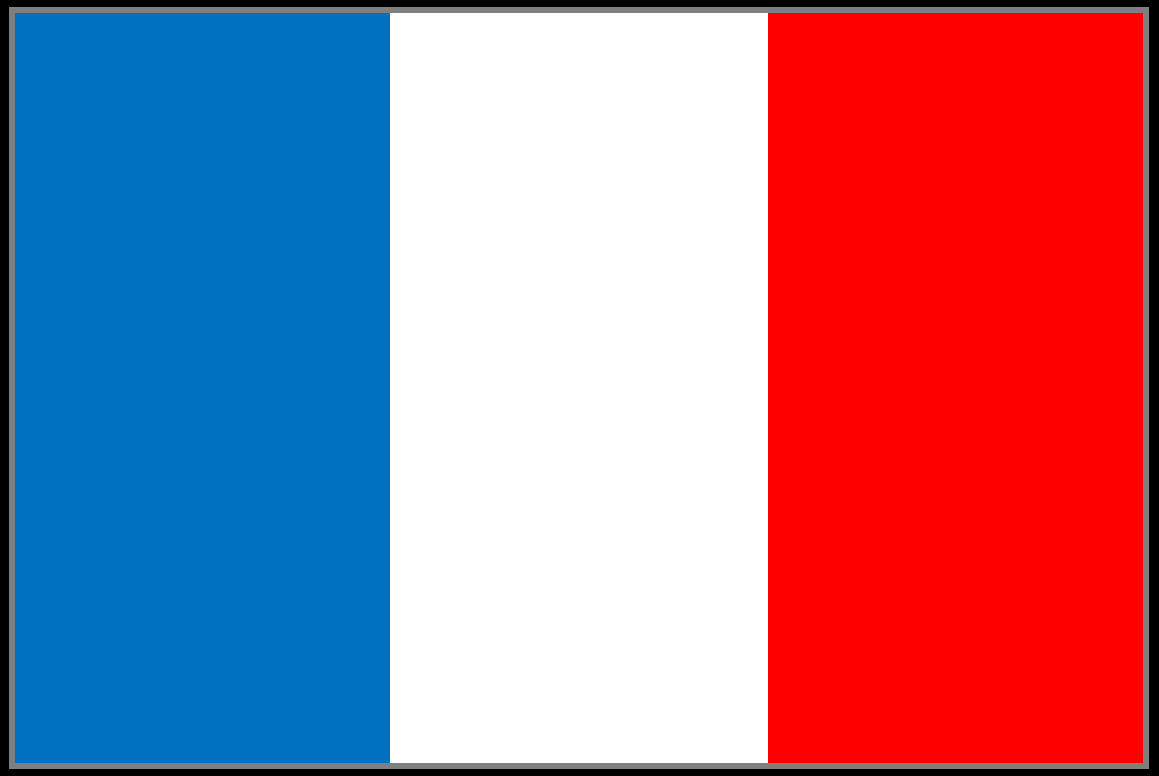
c) 

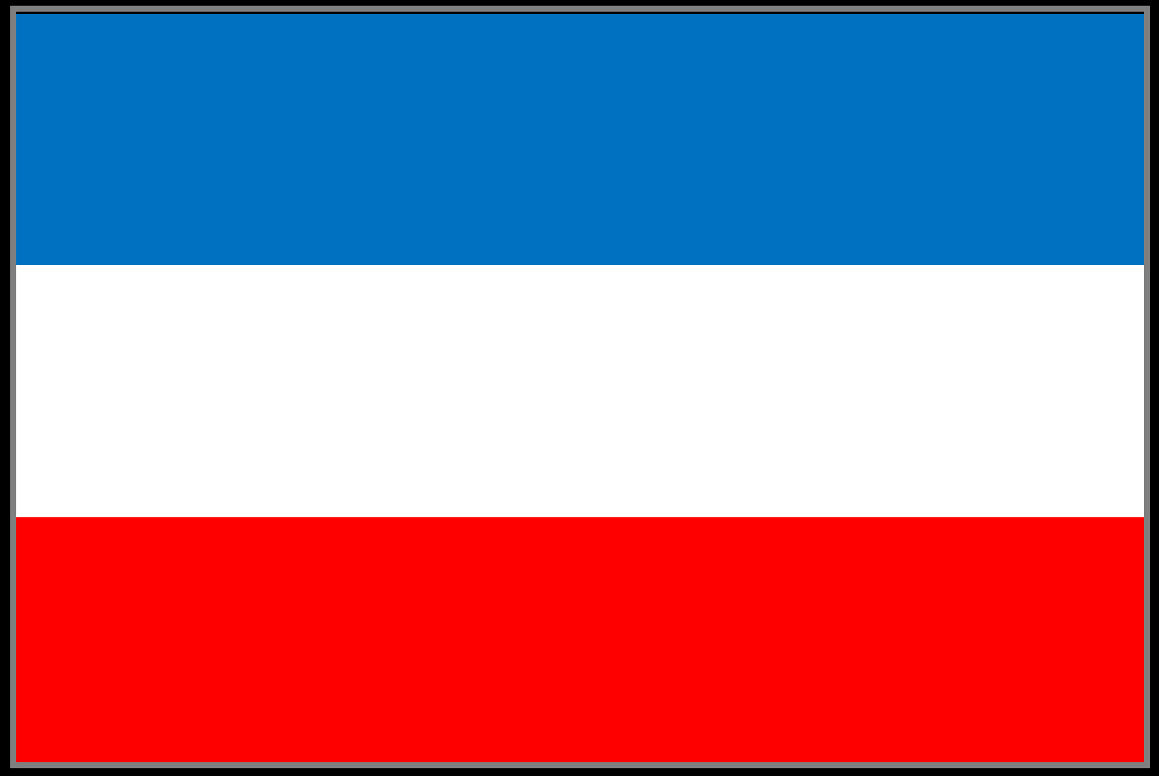
b) 

d) 

https://openclipart.org/detail/270085/windmill-by-egore911

Short Break: Dutch National Flag 

a) 

b) 

c) 

d) 

https://openclipart.org/detail/27085/windmill-by-egore911

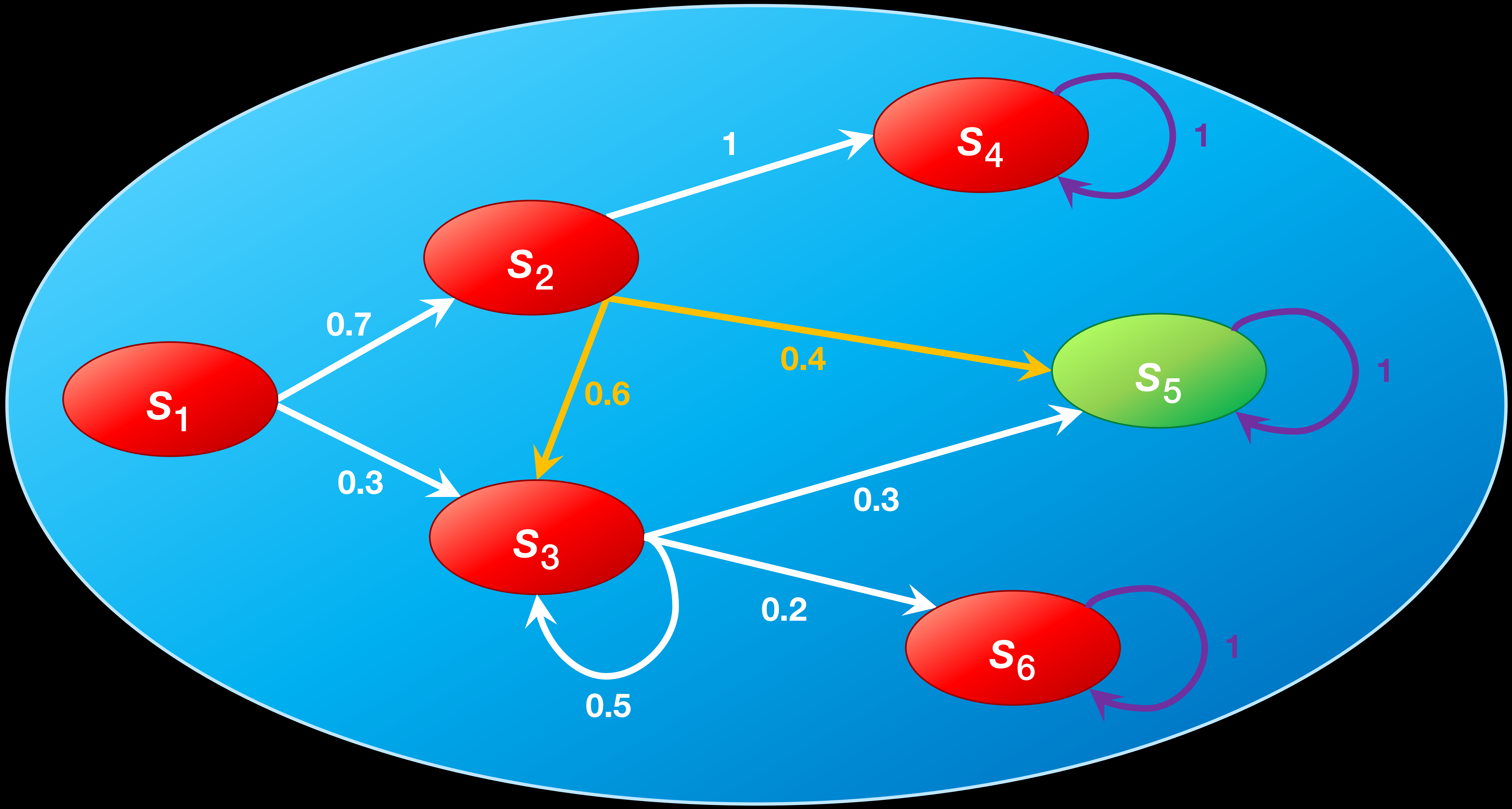
PRCTL – logic for MDPs

- PRCTL = probabilistic and reward CTL
- Example properties:
  - “What is the highest probability to win the game?”
  - “How much will the taxi ride from the station to the university cost on average?”

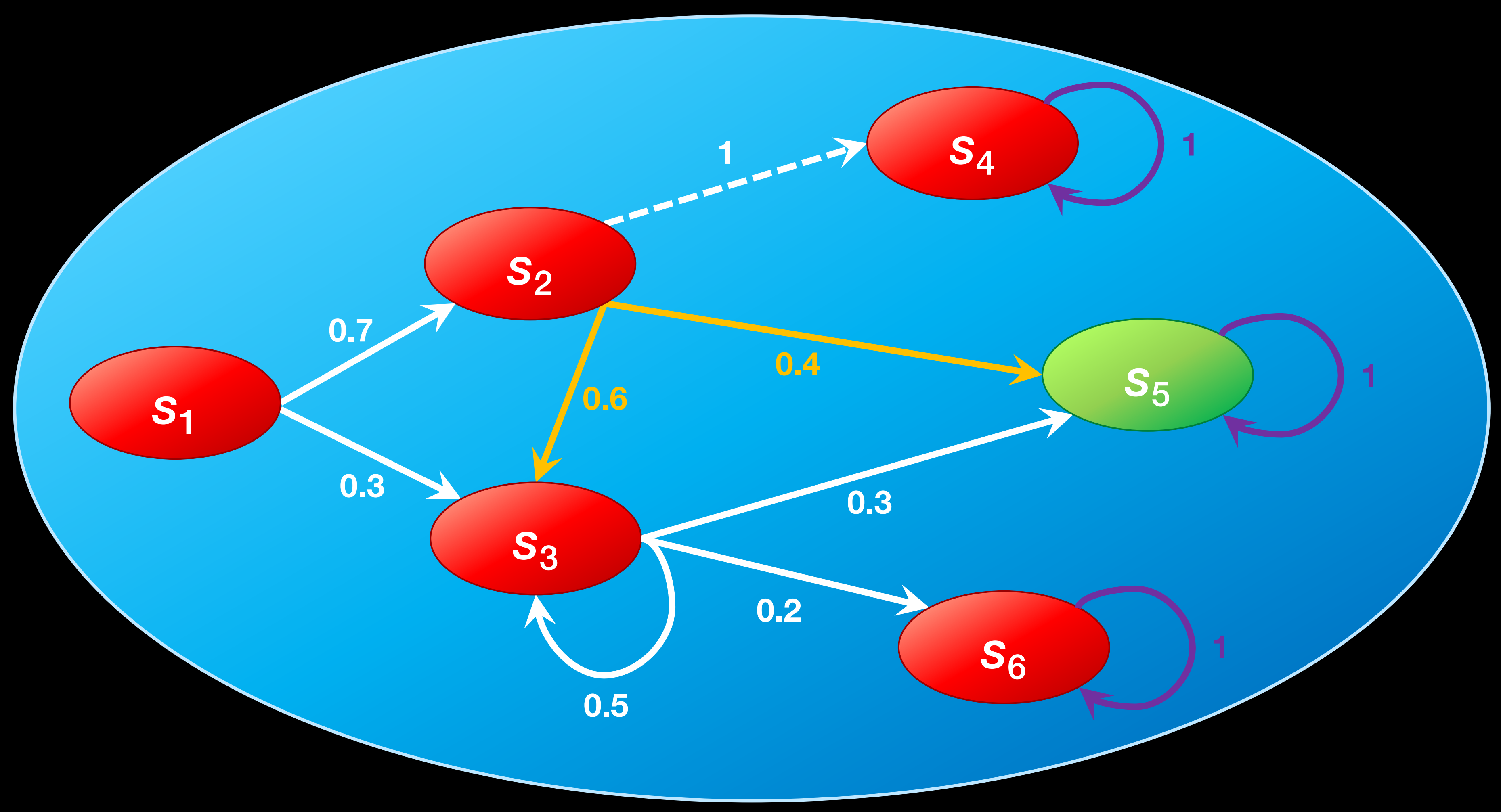
MDP Model Checking

- Given a MDP and a PRCTL property
- Idea of model checking:
  1. choose a policy (that decides which action to take in which state)
  2. calculate the probability (by solving a linear equation system)
  3. adapt the policy if it’s not yet optimal and go back to step 2.

MDP Model Checking



MDP Model Checking



- What is the probability to reach the green state s_5?
- Two strategies: in s_2, either choose $\dashrightarrow$ or $\rightarrow$.

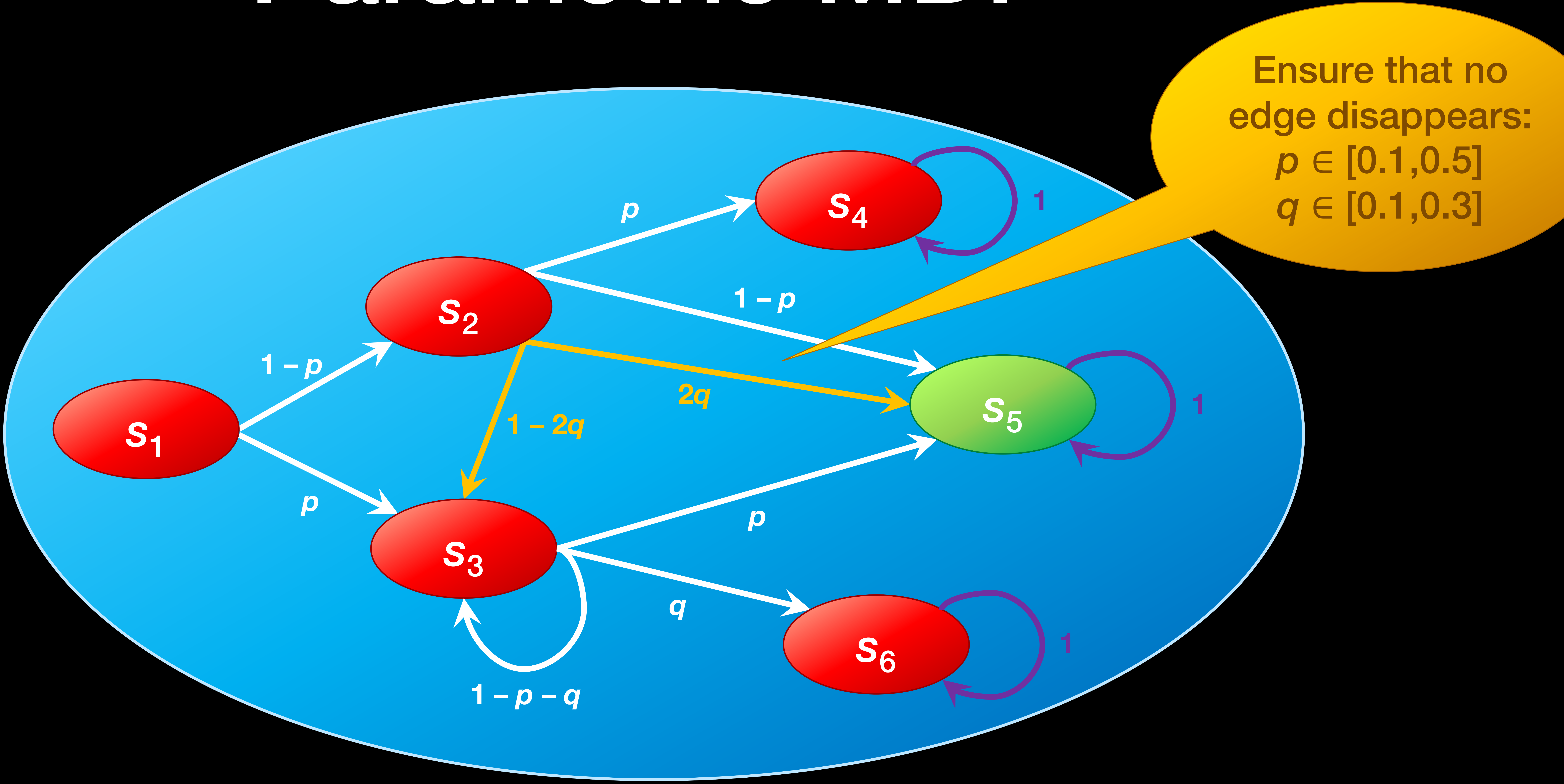
Probability if $\dashrightarrow$ is chosen: $\Pr(s_1 \rightarrow s_3\ [\circlearrowright] \rightarrow s_5) = 0.3 \times \frac{0.3}{0.3 + 0.2} = 0.18$

Probability if $\rightarrow$ is chosen: $\Pr(s_1 \rightarrow s_3\ [\circlearrowright] \rightarrow s_5)\ +$

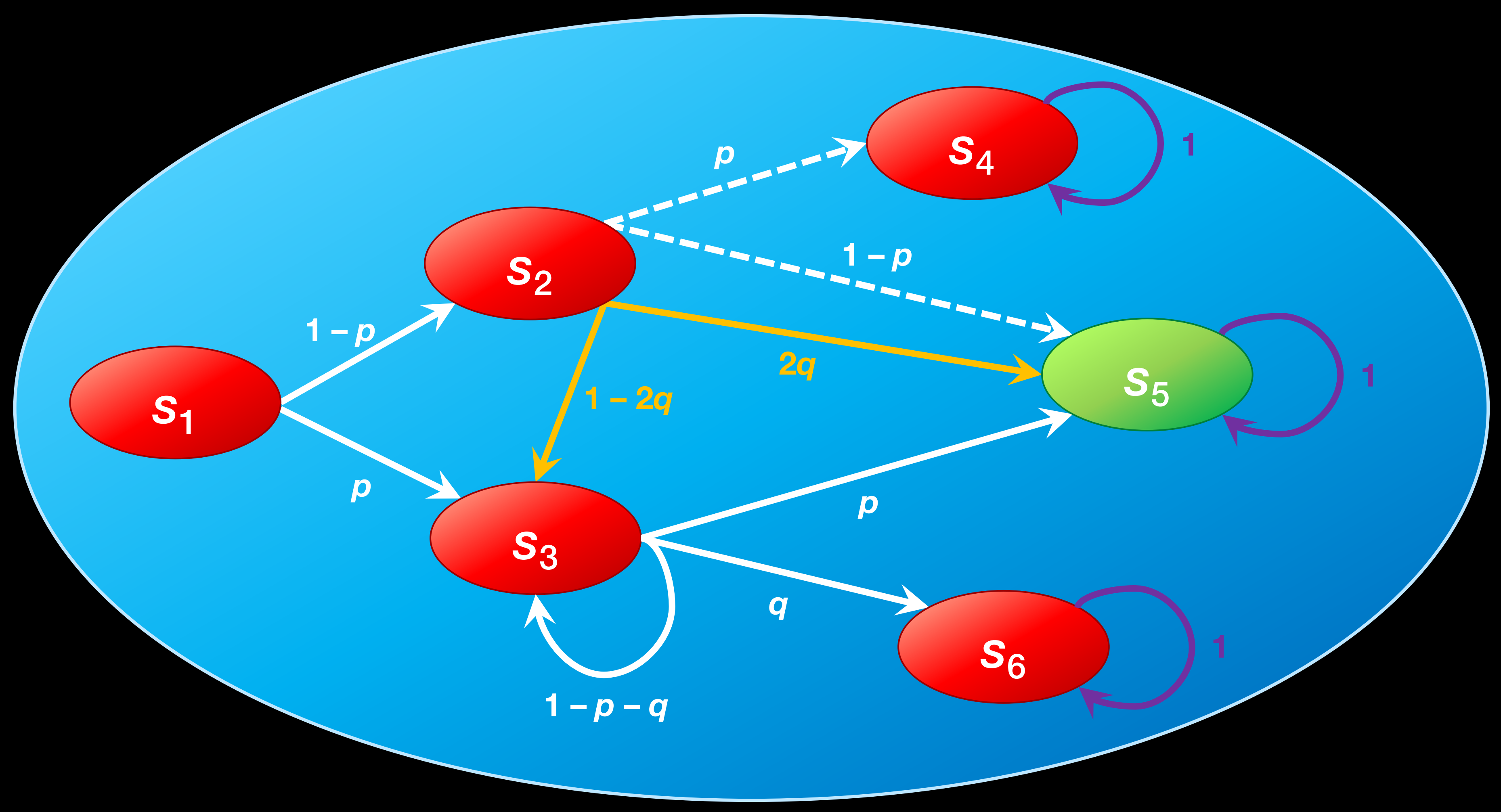
$\Pr(s_1 \rightarrow s_2 \rightarrow s_5)\ +$

$\Pr(s_1 \rightarrow s_2 \rightarrow s_3\ [\circlearrowright] \rightarrow s_5) = 0.712$

Parametric MDP



Parametric MDP

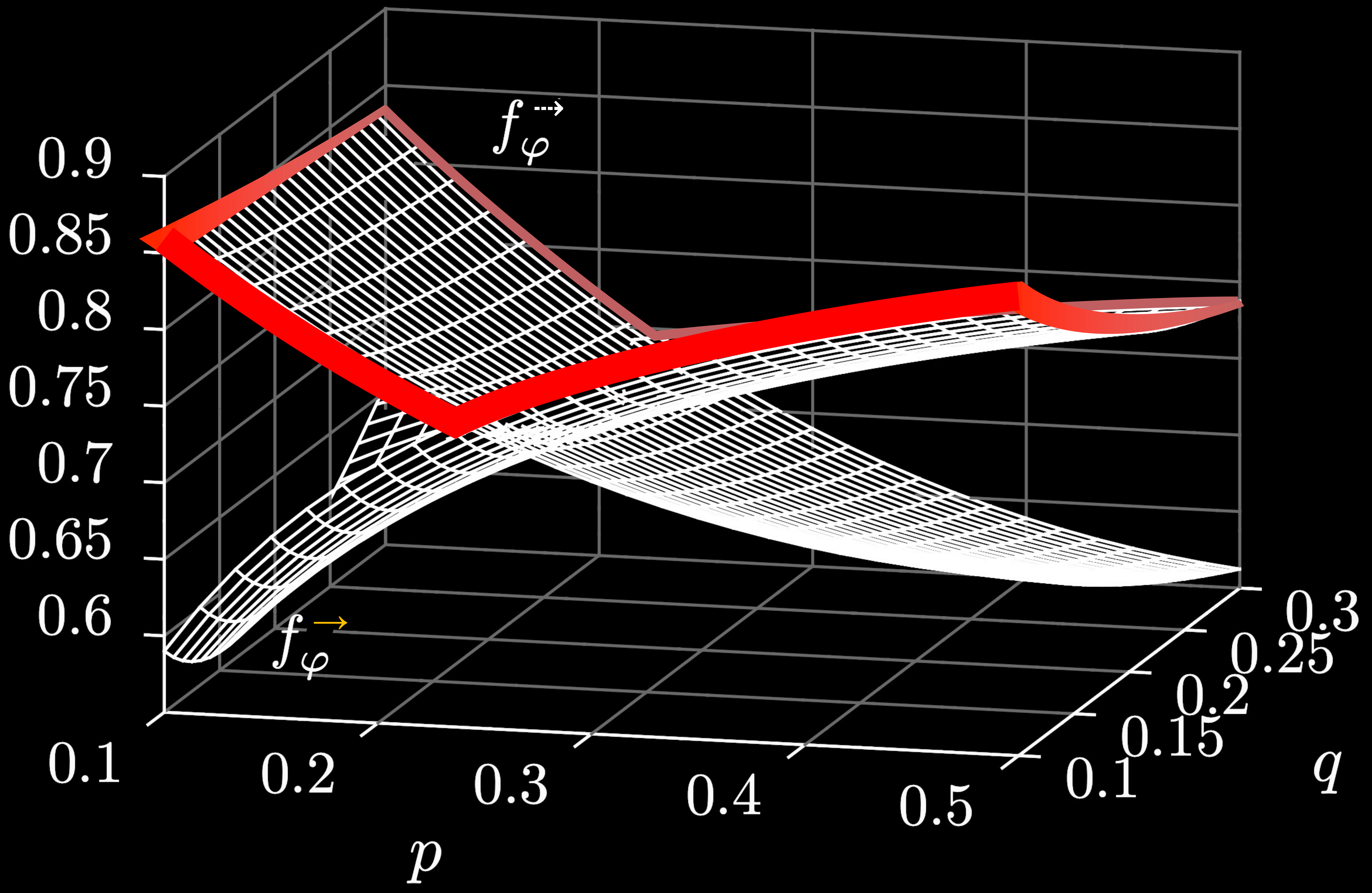


- What is the probability to reach the green state s_5?
- Two strategies: in s_2, either choose $\dashrightarrow$ or $\longrightarrow$.

Probability if $\dashrightarrow$ is chosen: $\Pr(s_1 \longrightarrow s_2 \dashrightarrow s_5) + \Pr(s_1 \longrightarrow s_3\ [\circlearrowright] \longrightarrow s_5) = \dfrac{p + q - p^2 - 2pq + p^3 + p^2q}{p + q}$

Probability if $\longrightarrow$ is chosen: $\Pr(s_1 \longrightarrow s_3\ [\circlearrowright] \longrightarrow s_5) + \Pr(s_1 \longrightarrow s_2 \longrightarrow s_5) + \Pr(s_1 \longrightarrow s_2 \longrightarrow s_3\ [\circlearrowright] \longrightarrow s_5) = \dfrac{p + 2q^2 - 2pq^2}{p + q}$

Parametric MDP



Exact solution of pMDP?

- a piecewise (for every policy a piece)
  rational (cycles require to divide polynomials)
  function

- easily becomes too complex...

Short Break: South California 🌅

How warm is the Pacific Ocean near South California?
(no phones please 📵)

a) 15 °C

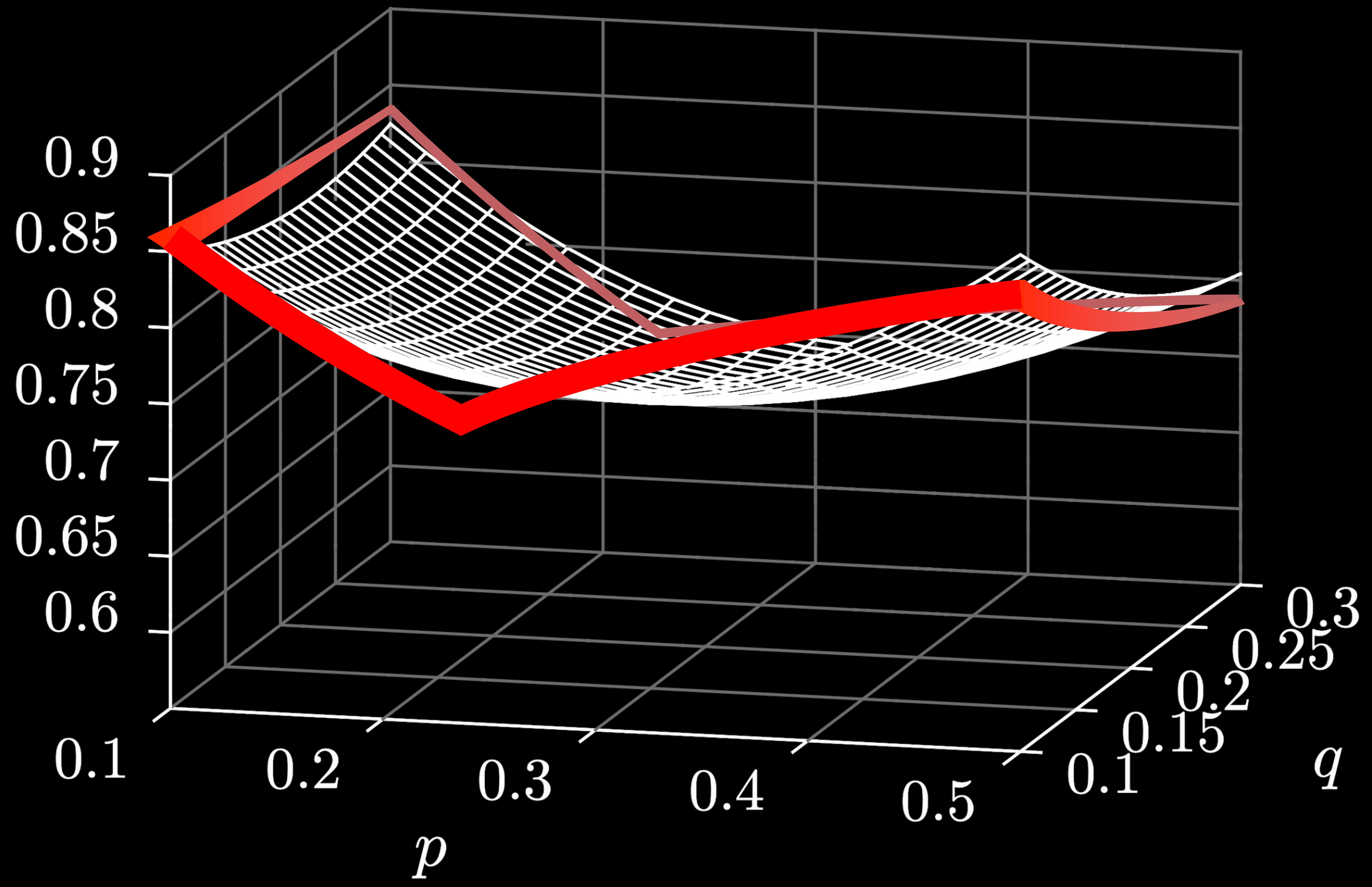
b) 18 °C

c) 21 °C

Scenario Approach

- The parameter space is continuous, makes an exact solution difficult
- Choose a few scenarios: sample from the parameter space, to reach confidence $1–\eta$ and error probability ε.
- Solve the (non-parametric) MDP model checking problem for all scenarios
- Select a shape for an approximation function (e.g. polynomial of degree $\leq d$), find the best coefficients and the error margin λ

Scenario Approach



Probably Approximately Correct

- What is the quality of the approximation function?
- We can guarantee that it's PAC:
  If the number of iid samples is $\ell \geq 2(m - \ln \eta) / \varepsilon$,
  then with confidence $1 - \eta$,
  the approximation error is at most λ with probability at least $1 - \varepsilon$.

m = number of parameters of the approximation problem = $\binom{n+d}{n}$

Probably Approximately Correct

- What is the quality of the approximation function?

- We can guarantee that it's PAC:
  If the number of iid samples is $\ell \geq 2(m - \ln \eta) / \varepsilon$,
  then with confidence $1 - \eta$,
  the probability to hit a parameter value with a large approximation error is at most ε.

m = number of parameters of the approximation problem = $\binom{n+d}{n}$

约瑟彼得，万岁！