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High-dimensional time-varying statistical factor model by analytic Bayesian filtering

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VSBFA Research (I)

- VSBFA algorithm
  - "variational sequential Bayesian factor analysis"
  - an analytic Bayesian filter
  - to estimate high-dimensional time-varying statistical factor model
- Objective
  - To make more accurate forecasts
  - than widely utilized rolling-window factor analysis (a local stationarity approximation)
- This presentation
  - is derived in part from an article yet to be published in Quantitative Finance (www.tandfonline.com/loi/rquf20)

VSBFA Research (II)

- Research paper
  - H.F. Ling and C. Franzen, 2016, “Online learning of time-varying stochastic factor structure by variational sequential Bayesian factor analysis”, *Quantitative Finance*
  - includes a long list of literature
- Researchers
  - H. “Fox” Ling and Christian Franzen
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Motivations (I)

- Common factors
  - All assets and markets are affected by common factors
  - Common factors: economical, social, technological, behavioral, political, national, global, …
  - Different assets have different sensitivities (or exposures) to various factors
- Factor models (FMs): to estimate
  - number of common factors
  - factor loadings (loadings, exposures or sensitivities)
  - factor scores (factors or factor returns)
  - covariance of factor scores
  - variance of (specific or residual) errors

Motivations (II)

- Major applications of FMs
  - Factor-based forecast of covariance matrix of large number of assets
  - Forecasting volatility of portfolio of assets
  - Portfolio selection and optimization
- Frequentist modeling of FMs
  - estimate expected values of random variables
  - estimate least-squares or optimal values of deterministic parameters
  - by methods relatively simpler (than those of Bayesian modeling)

Motivations (III)

- Model risks: errors in estimated
  - number of factors
  - expected values of random variables
  - least-squares or optimal values of parameters
  - *Q*: how to jointly model some model risks in FMs?
- Bayesian modeling of FMs
  - estimate conditional joint distribution of random variables and random parameters given the observed data
  - by joint distribution = likelihood $\times$ prior(s)
  - with the exact posterior (the solution) most likely intractable

Motivations (IV)

- Sequential Bayes for time-varying FMs
    - sensitivities of assets to factors change all the time
    - last (or current) factor model estimates to form priors for the current (or next) estimates (a Bayesian filter)
- Approximation of Bayesian posterior
    - posterior: intractable, even the simplest realistic Bayesian
    - stochastic approximation: by random sampling, MCMC, asymptotically accurate, not for large datasets
    - analytic approximation: with tractable expressions, e.g. variational Bayes (iteration), can handle large datasets
- High-dimensional FM with limited length of data history
    - high-frequency data factor modeling

VSBFA

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Factor Model Overview (I)

- Stationary factor model of vector time-series y_t

  $y_t = X\,f_t + e_t$ (with constant X)

- Time-varying factor model of data vector y_t

  $y_t = X_t\,f_t + e_t$ (X_t as function of time)

- Symbols and dimensions
  - t = time index, $t = 1, 2, \cdots$
  - y_t = n-vector for n observed time-series, $n \gg 1$
  - f_t = m-vector for m factor scores, $m \ll n$
  - X_t = $n \times m$ matrix of factor loadings (sensitivities of n assets to m factors)
  - e_t = n-vector for n error time-series

Factor Model Overview (II)

- Fundamental factor models

  $$y_t = X_t \, f_t + e_t$$

  - known factor exposures X_t, unknown factor returns f_t
  - estimation: cross-sectional regression
- Models with known factors

  $$y_t = X_t \, f_t + e_t$$

  - unknown loadings X_t, known factors f_t
  - estimation: time-series regression over a trailing period
- Statistical factor models (only data y_t is known)

  $$y_t = X_t \, f_t + e_t$$

  - unknown loadings X_t, unknown factors f_t
  - estimation: numerous approaches

Factor Model Overview (III)

- A hybrid factor model
  - fundamental: $y_t = X_t^{(F)} f_t^{(F)} + \varepsilon_t^{(F)}$
  - statistical: $\varepsilon_t^{(F)} = X_t^{(S)} f_t^{(S)} + e_t$
  - hybrid: $y_t = X_t^{(F)} f_t^{(F)} + X_t^{(S)} f_t^{(S)} + e_t$
- Explaining a statistical factor model
  - statistical FM: $y_t = X_t\, f_t + e_t$
  - explanatory indicators r_t: $r_t = X_t^{(r)} f_t + \varepsilon_t$, $n_r > m$
  - pseudoinverse $(X_t^{(r)})^+$ of $X_t^{(r)}$: $f_t = (X_t^{(r)})^+ \, (r_t - \varepsilon_t)$
  - explanation by the explanatory indicators

$$X_t f_t = X_t (X_t^{(r)})^+ \, (r_t - \varepsilon_t) = X_t (X_t^{(r)})^+ \, r_t - X_t (X_t^{(r)})^+ \, \varepsilon_t$$

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Estimations of FMs (I)

- Statistical factor model of data vector y_t

$$y_t = X_t f_t + e_t$$

- Frequentist modeling
  - random variables: f_t, e_t
  - deterministic parameters: X_t, $\mathrm{cov}(f_t)$, $\mathrm{cov}(e_t)$
  - estimating expected values and least-squares / optimal values given data y_t
- Bayesian modeling
  - random variables: X_t, f_t, e_t
  - random variables: $\mathrm{cov}(X_t)$, $\mathrm{cov}(f_t)$, $\mathrm{cov}(e_t)$
  - estimating conditional joint distribution of the random variables given data y_t

Estimations of FMs (II)

- Stationary statistical factor models

  $$y_t = X f_t + e_t$$

  - unknown constant loadings X, unknown factors f_t
- Estimation methods
  - PCA, asymptotic PCA, PCA augmented with Bayesian analysis, thresholding principal orthogonal complements
  - maximum likelihood (ML) factor analysis, expectation-maximization (EM) factor analysis, Bayesian factor analysis, ML-EM-Bayesian factor model, variational Bayesian factor analysis (VBFA), multi-step VBFA
  - variational Bayesian PCA, MCMC Bayesian PCA
  - dynamic factor model (DFM)

Estimations of FMs (III)

- Time-varying statistical factor models

  $$y_t = X_t\, f_t + e_t$$

  - unknown time-varying loadings X_t, unknown factors f_t
- Estimation methods
  - *by rolling-window, a local stationarity approximation*: all of the approaches to stationary statistical factor models
  - *by sequential Bayes, with stochastic approximation, for low-dimensional FMs*: MCMC Bayesian factor analysis, Gibbs sampling for DFM, dynamic latent factors and time-varying sparse loadings with MCMC solutions
  - *by sequential Bayes, with analytic approximation, for high-dimensional FMs*: VSBFA (this research)

VSBFA

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Random Variables (I)

- Time-varying statistical factor models

$$y_t = X_t f_t + e_t$$

- Random variables

$$f_t,\ X_t,\ e_t,\ P_t,\ \{U_{j,t}\},\ R_t$$

- Multivariate normal distributions
  - $f_t \sim N(E(f_t), P_t)$
  - $X_{j-col,t} \sim N(E(X_{j-col,t}), U_{j,t}),\ \ j = 1, 2, \cdots, m$
  - $e_t \sim N(0, R_t)$
  - P_t, $\{U_{j,t}\}$ and R_t: diagonal random matrices

Random Variables (II)

- "Diagonal inverse-gamma" distributions
  - (detailed in Appendix)
  - $P_t \sim IG_D(\alpha_t, B_{P,t})$
  - $U_{j,t} \sim IG_D(\alpha_t, B_{U,j,t}), \;\; j = 1, 2, \cdots, m$
  - $R_t \sim IG_D(\alpha_t, B_{R,t})$
  - α_t: shape parameter common to all variables
  - $B_{P,t}$, $\{B_{U,j,t}\}$ and $B_{R,t}$: diagonal matrices of scale parameters
- Denoting estimated expected values of the variables as

$$f_{t|t},\, P_{t|t},\, X_{t|t},\, \{U_{j,t|t}\},\, R_{t|t}$$

VSBFA Estimation (I)

- Time-varying statistical factor models

$$y_t = X_t f_t + e_t$$

- Variational Bayesian (VB) estimation at time t
  - optimal value of each variable is a function of optimal values of all other variables
  - iteration ($v = 1, 2, \cdots, L$): calculating the value of each variable using the latest values of all other variables
- Initial values (for the iteration at time t) by previous estimates
  - factors: $f_{t|t}^{(0)} = 0$; $P_{t|t}^{(0)} = P_{t-1|t-1}$
  - loadings: $X_{t|t}^{(0)} = X_{t-1|t-1}$; $U_{j,t|t}^{(0)} = U_{j,t-1|t-1}$, $j = 1, \cdots, m$
  - errors: $R_{t|t}^{(0)} = R_{t-1|t-1}$

VSBFA Estimation (II)

- Rescaling the diagonal $\{U_{j,t|t}^{(0)}\}$ and $R_{t|t}^{(0)}$
  - by a modeling parameter "RSVRT" h, $0 \ll h < 1$,
  - to prevent over- and under-fitting (detailed in Appendix)
- At each VB iterate $v \geq 1$ (***fully spelled out in Appendix***)
  - first, updating factors by functions $f(\cdot)$ and $P(\cdot)$

$$f_{t|t}^{(v)} = f(\, y_t,\, f_{t|t}^{(v-1)},\, P_{t|t}^{(v-1)},\, X_{t|t}^{(v-1)}, \{U_{j,t|t}^{(v-1)}\},\, R_{t|t}^{(v-1)}\,)$$

$$P_{t|t}^{(v)} = P(\, f_{t|t}^{(v)},\, P_{t|t}^{(v-1)},\, X_{t|t}^{(v-1)}, \{U_{j,t|t}^{(v-1)}\},\, R_{t|t}^{(v-1)}\,)$$

  - then, updating loadings by functions $X(\cdot)$ and $U(\cdot)$

$$X_{t|t}^{(v)} = X(\, y_t,\, f_{t|t}^{(v)},\, P_{t|t}^{(v)},\, X_{t|t}^{(v-1)}, \{U_{j,t|t}^{(v-1)}\},\, R_{t|t}^{(v-1)}\,)$$

$$U_{t|t}^{(v)} = U(\, f_{t|t}^{(v)},\, P_{t|t}^{(v)},\, X_{t|t}^{(v)}, \{U_{j,t|t}^{(v-1)}\},\, R_{t|t}^{(v-1)}\,)$$

VSBFA Estimation (III)

- At each VB iterate $v \geq 1$ (continue)
  - last, updating error variance by function $R(\cdot)$

$$R_{t|t}^{(v)} = R(\, y_t,\, f_{t|t}^{(v)},\, P_{t|t}^{(v)},\, X_{t|t}^{(v)}, \{U_{j,t|t}^{(v)}\},\, R_{t|t}^{(v-1)}\,)$$

- Making next VB iterate: $v \Leftarrow v + 1 \leq L$
- VSBFA estimates as of time t
  - factors: $f_{t|t} = f_{t|t}^{(L)}$; $P_{t|t} = P_{t|t}^{(L)}$
  - loadings: $X_{t|t} = X_{t|t}^{(L)}$; $U_{j,t|t} = U_{j,t|t}^{(L)}$, $j = 1, 2, \cdots, m$
  - errors: $R_{t|t} = R_{t|t}^{(L)}$
- VSBFA algorithm is a filter
  - $P_{t|t}$, $X_{t|t}$, $\{U_{j,t|t}\}$, $R_{t|t}$ are inputs for estimation at time t+1

VSBFA

Cov Matrices and Portfolios (I)

- Time-varying statistical factor models

$$y_t = X_t\, f_t + e_t$$

- Expected (and least-squares / optimal) values estimated by
  - VSBFA (Bayesian): $f_{t|t}$, $P_{t|t}$, $X_{t|t}$, $\{U_{j,t|t}\}$, $R_{t|t}$
  - rolling PCA (frequentist): $f_{t|t}$, $P_{t|t}$, $X_{t|t}$, $R_{t|t}$
- Model comparison by accuracies of
  - factor-based forecasts of covariance matrices
  - covariance-based forecasts of portfolio volatilities
- Predicted covariance matrix
  - $C_{t+1|t} = E_{t+1|t}(\, y_{t+1}\, y_{t+1}^T\,)$

Cov Matrices and Portfolios (II)

- Covariance matrix forecasted by VSBFA

$$C_{t+1|t} = C_{t+1|t}^{(X)} + C_{t+1|t}^{(Spec)}$$

  - loadings-based covariance: $C_{t+1|t}^{(X)} = X_{t|t}\, P_{t|t}\, (X_{t|t})^T$
  - specific variance: $C_{t+1|t}^{(Spec)} = \sum_{j=1}^{m} P_{j,t|t}\, U_{j,t|t} + R_{t|t}$

- Portfolio of weights vector w
  - forecasted portfolio variance

$$(\sigma_{t+1|t}^{(w)})^2 = (\sigma_{t+1|t}^{(X|w)})^2 + (\sigma_{t+1|t}^{(Spec|w)})^2$$

  - loadings-based component: $(\sigma_{t+1|t}^{(X|w)})^2 = w_t^T\, C_{t+1|t}^{(X)}\, w_t$
  - specific component: $(\sigma_{t+1|t}^{(Spec|w)})^2 = w_t^T\, C_{t+1|t}^{(Spec)}\, w_t$

Difference of 2 Cov Matrices

- Difference of two distributions p and q
  - the best known measure:
    Kullback-Leibler divergence (KLD)
  - KLD is not symmetric and not a “distance”
  - symmetrized KL distance:
    KL distance = (KLD from p to q + KLD from q to p) / 2
- Difference of two covariance matrices
  - (when they represent two multivariate normal distributions)
  - can be measured by the symmetrized KL distance
  - smaller KL distance: smaller difference between the two covariance matrices

Accuracy of Portfolio Risk (I)

- Bias statistic

  = standard deviation of realized portfolio returns standardized by forecasted portfolio mean and variance

  - simple and popular
  - if less (or greater) than 1: variance is over- (or under-) predicted
  - over- and under-predictions may be cancelled out

- Log-likelihood

  = logarithm of likelihood calculated by forecasted portfolio mean and variance and realized portfolio return

  - popular and proven optimal
  - higher log-likelihood: more accurate forecasts

Accuracy of Portfolio Risk (II)

- Q-statistic
  - closely related to the log-likelihood
  - proven optimal
  - smaller Q-statistic: more accurate forecasts
- Volatility reduction

  = realized portfolio volatility reduction by variance minimization based on the forecasted covariance matrix
  - practically important and popular
  - larger volatility reduction: more accurate forecasts
- The four measures
  - largely agree with each other

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Estimated Cov Matrices (I)

- Simulated data
  - 200 simulated daily time-series of 9 years in length
  - known daily DGM (data generating model): daily factor model of 10 factors
  - the daily DGM: 10-factor rolling 65-day PCA on 9-year daily returns of 200 Russell Top Index stocks
- Tested values
  - known daily covariance matrices by the DGM
  - daily covariance estimated by 5-, 10- and 15-factor rolling PCA of various window size T_{MW}
  - daily covariance estimated by 5-, 10- and 15-factor VSBFA of various RSVRT h

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Estimated Cov Matrices (II)

- Test statistic
  - 8-Year and annual averages of symmetrized KL distances
  - between the known daily covariance matrix and those estimated by rolling PCA and VSBFA
- Results
  - under-specification (model m < "true" $m = 10$) results in more accurate forecasts
  - the best rolling PCA is of $T_{MW} = 65$
  - the best VSBFA is of $h = 0.94$
- **Conclusion**
  - **Covariance matrices estimated by VSBFA are more accurate than those by rolling PCA**

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Estimated Cov Matrices (III)

	Rolling PCA			VSBFA			
T_{MW}	$m = 5$	$m = 10$	$m = 15$	h	$m = 5$	$m = 10$	$m = 15$
25	0.737	1.034	1.568	0.86	0.634	0.639	0.647
30	0.696	0.908	1.241	0.87	0.631	0.636	0.643
35	0.673	0.837	1.081	0.88	0.628	0.633	0.640
40	0.661	0.793	0.987	0.89	0.626	0.630	0.638
45	0.654	0.765	0.927	0.90	0.624	0.628	0.635
50	0.653	0.748	0.887	0.91	0.623	0.626	0.633
55	0.654	0.738	0.859	0.92	0.621	0.625	0.632
60	0.657	0.732	0.840	0.93	0.620	0.624	0.631
65	0.662	0.730	0.827	0.94	0.620	0.625	0.631
70	0.668	0.730	0.818	0.95	0.621	0.625	0.632

Estimated Cov Matrices (IV)

Year	Lower Bound	Rolling PCA of $T_{MW} = 65$			VSBFA of $h = 0.94$			VSBFA vs PCA	
		m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10
2007	0.399	0.648	0.725	0.829	0.614	0.619	0.627	-13%	-32%
2008	0.412	0.680	0.768	0.882	0.650	0.660	0.669	-11%	-30%
2009	0.408	0.715	0.759	0.837	0.642	0.653	0.664	-24%	-30%
2010	0.393	0.650	0.713	0.805	0.606	0.611	0.616	-17%	-32%
2011	0.405	0.644	0.718	0.819	0.615	0.623	0.630	-12%	-30%
2012	0.401	0.660	0.725	0.816	0.616	0.618	0.622	-17%	-33%
2013	0.402	0.641	0.710	0.804	0.604	0.603	0.607	-16%	-35%
2014	0.405	0.660	0.725	0.823	0.613	0.610	0.614	-19%	-36%
’07-’14	0.403	0.662	0.730	0.827	0.620	0.625	0.631	-16%	-32%

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Global Stocks (I)

- Global stock data
  - 807 stocks chosen from (1610 stocks in) MSCI World Index on 01/31/2014
  - all stocks have complete 25-year history of monthly total returns in USD from Jan 1989 to Dec 2013

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Global Stocks (II)

Country of Domicile	GICS Sectors										Total
	CD	CS	En	Fin	HC	Ind	IT	Mat	Tel	Ut	
Australia		1	4	8		3		8		1	25
Austria			1								1
Belgium		2		3	1			1			7
Canada			1					1			2
Denmark				1	1	2					4
Finland				1		2					3
France	8	4	2	10	2	9	2	3	1		41
Germany	5	2		4	2	5		5		2	25
Hong Kong				13		3				3	19
Ireland				1				1			2
Italy	2		1	7		1			1		12
Japan	37	15	2	23	15	57	27	30	1	12	219
Luxembourg	1										1
Netherlands	2	4	1	2		2		1			12
Norway		1						1			2
Singapore	2			7		4					13
Spain				4		2			1	2	9
Sweden	2	1		3		5	1				12
Switzerland	2	2		5	1	5		3			18
U.K.	12	9	6	15	2	14		5	2		65
U.S.	41	31	27	52	29	47	32	22	4	30	315
Total	114	72	45	159	53	161	62	81	10	50	807

Portfolio Volatility by PCA (I)

- Random portfolios
  - 1000 long-only portfolios; 1000 long/short portfolios
- Tested values
  - portfolio volatilities forecasted by 10-factor rolling PCA of various window size T_{MW}
- Test statistic:
  - 240-month and 60-month values of
  - bias statistic, $BS^{(PCA)}$
  - log-likelihood, $LL^{(PCA)}$
  - Q-statistic, $QS^{(PCA)}$
  - volatility-reduction, $VM^{(PCA)}$

Portfolio Volatility by PCA (II)

- Results
  - rolling PCA of $T_{MW} = 37$ can be regarded as the best
  - for forecasting volatilities of both long-only and long/short portfolios

Portfolio Volatility by PCA (III)

T_{MW}	Long-Only Portfolios				Long/Short Portfolios			
	$BS^{(PCA)}$	$LL^{(PCA)}$	$QS^{(PCA)}$	$VM^{(PCA)}$	$BS^{(PCA)}$	$LL^{(PCA)}$	$QS^{(PCA)}$	$VM^{(PCA)}$
30	1.143	1.6436	2.5879	-4.09%	1.060	2.9000	2.4428	-0.56%
31	1.141	1.6441	2.5837	-4.11%	1.059	2.8998	2.4442	-0.56%
32	1.141	1.6423	2.5953	-4.11%	1.057	2.8995	2.4473	-0.57%
33	1.140	1.6422	2.5945	-4.14%	1.055	2.9000	2.4453	-0.57%
34	1.137	1.6430	2.6036	-4.17%	1.053	2.9004	2.4448	-0.58%
35	1.133	1.6460	2.6105	-4.17%	1.052	2.9003	2.4457	-0.59%
36	1.136	1.6409	2.6207	-4.16%	1.050	2.9005	2.4467	-0.59%
37	1.132	1.6433	2.6224	-4.18%	1.049	2.8999	2.4465	-0.60%
38	1.135	1.6380	2.6285	-4.21%	1.048	2.8992	2.4485	-0.60%
39	1.136	1.6352	2.6351	-4.20%	1.047	2.8989	2.4495	-0.60%
40	1.136	1.6335	2.6413	-4.24%	1.046	2.8981	2.4524	-0.61%
41	1.135	1.6318	2.6527	-4.24%	1.045	2.8973	2.4543	-0.61%
42	1.135	1.6299	2.6722	-4.27%	1.044	2.8966	2.4557	-0.61%

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Portfolio Volatility by PCA (IV)

Rolling PCA $T_{MW} = 37$	Long-Only Portfolios				Long/Short Portfolios			
	$BS^{(PCA)}$	$LL^{(PCA)}$	$QS^{(PCA)}$	$VM^{(PCA)}$	$BS^{(PCA)}$	$LL^{(PCA)}$	$QS^{(PCA)}$	$VM^{(PCA)}$
1994 - 1998	1.222	1.799	2.616	-2.47%	1.115	2.912	2.444	-0.59%
1999 - 2003	0.995	1.698	2.314	-3.14%	1.029	2.692	2.364	-0.72%
2004 - 2008	1.350	1.603	3.161	-3.97%	1.058	3.039	2.479	-0.41%
2009 - 2013	0.938	1.474	2.398	-6.84%	1.004	2.956	2.499	-0.61%
1994 - 2013	1.132	1.643	2.622	-4.18%	1.049	2.900	2.447	-0.60%

Portfolio Volatility by VSBFA (I)

- Random portfolios
  - 1000 long-only portfolios; 1000 long/short portfolios
- Tested values
  - portfolio volatilities forecasted by 10-factor VSBFA of various RSVRT h
- Test statistic
  - 240-month and 60-month values of
  - bias statistic, $BS^{(VSB)}$
  - log-likelihood, $LL^{(VSB)}$
  - Q-statistic, $QS^{(VSB)}$
  - volatility-reduction, $VM^{(VSB)}$

Portfolio Volatility by VSBFA (II)

- Results
  - VSBFA of $h = 0.86$ can be regarded as the best
  - for forecasting volatilities of both long-only and long/short portfolios

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Portfolio Volatility by VSBFA (III)

h	Long-Only Portfolios				Long/Short Portfolios			
	$BS^{(VSB)}$	$LL^{(VSB)}$	$QS^{(VSB)}$	$VM^{(VSB)}$	$BS^{(VSB)}$	$LL^{(VSB)}$	$QS^{(VSB)}$	$VM^{(VSB)}$
0.80	0.970	1.7485	2.4121	-4.44%	1.030	2.9453	2.3559	-0.69%
0.81	0.976	1.7487	2.4116	-4.43%	1.029	2.9454	2.3556	-0.69%
0.82	0.981	1.7489	2.4112	-4.42%	1.028	2.9455	2.3553	-0.69%
0.83	0.987	1.7490	2.4110	-4.41%	1.027	2.9456	2.3551	-0.69%
0.84	0.993	1.7490	2.4110	-4.39%	1.025	2.9457	2.3549	-0.69%
0.85	1.000	1.7489	2.4113	-4.37%	1.024	2.9458	2.3547	-0.69%
0.86	1.007	1.7486	2.4118	-4.34%	1.022	2.9459	2.3546	-0.69%
0.87	1.015	1.7482	2.4126	-4.32%	1.020	2.9460	2.3545	-0.69%
0.88	1.023	1.7476	2.4138	-4.29%	1.018	2.9460	2.3544	-0.69%
0.89	1.032	1.7468	2.4154	-4.25%	1.016	2.9460	2.3544	-0.69%
0.90	1.041	1.7457	2.4176	-4.22%	1.014	2.9460	2.3544	-0.69%
0.91	1.051	1.7443	2.4203	-4.17%	1.012	2.9460	2.3545	-0.68%
0.92	1.062	1.7426	2.4239	-4.13%	1.010	2.9459	2.3546	-0.68%
0.93	1.074	1.7404	2.4283	-4.08%	1.007	2.9458	2.3548	-0.67%
0.94	1.087	1.7376	2.4339	-4.03%	1.004	2.9456	2.3551	-0.67%
0.95	1.102	1.7340	2.4409	-3.97%	1.001	2.9455	2.3555	-0.66%

Portfolio Volatility by VSBFA (IV)

VSBFA $h = 0.86$	Long-Only Portfolios				Long/Short Portfolios			
	$BS^{(VSB)}$	$LL^{(VSB)}$	$QS^{(VSB)}$	$VM^{(VSB)}$	$BS^{(VSB)}$	$LL^{(VSB)}$	$QS^{(VSB)}$	$VM^{(VSB)}$
1994 - 1998	1.076	1.903	2.408	-3.17%	1.076	2.957	2.354	-0.65%
1999 - 2003	0.961	1.641	2.427	-3.24%	0.982	2.711	2.325	-0.81%
2004 - 2008	1.110	1.911	2.545	-3.25%	1.076	3.110	2.339	-0.51%
2009 - 2013	0.890	1.539	2.267	-7.45%	0.959	3.006	2.400	-0.76%
1994 - 2013	1.007	1.749	2.412	-4.34%	1.022	2.946	2.355	-0.69%

VSBFA vs. Rolling PCA (I)

- Random portfolios
  - 1000 long-only portfolios
- Tested values
  - portfolio volatilities forecasted by 10-factor
  - VSBFA of $h = 0.86$ and rolling PCA of $T_{MW} = 37$
- Test statistic
  - rolling-60-month and 240-month values of
  - bias statistic, $BS^{(VSB)}$ and $BS^{(PCA)}$
  - log-likelihood difference, $\Delta LL = LL^{(VSB)} - LL^{(PCA)}$
  - Q-statistic difference, $\Delta QS = QS^{(VSB)} - QS^{(PCA)}$
  - volatility-reduction difference, $\Delta VM = VM^{(VSB)} - VM^{(PCA)}$

VSBFA vs. Rolling PCA (II)

- **Model comparison**
  - VSBFA can make more accurate portfolio volatility forecasts than rolling PCA in
  - 11 or 14 out of the 16 rolling 60-month periods and
  - the entire 240-month period
- Conclusion
  - variances of long-only portfolios are overwhelmingly dominated by loadings-based (or systematic) component
  - **implication: VSBFA can make more accurate forecasts in factor-based covariance than rolling PCA**

VSBFA vs. Rolling PCA (III)

VSBFA vs. PCA	Random Long-Only Portfolios							
	Bias Statistic		Log-Likelihood		Q-Statistic		Volatility Minim	
	$BS^{(VSB)}$	$BS^{(PCA)}$	ΔLL	p-Val	ΔQS	p-Val	ΔVM	p-Val
1994 - 1998	1.076	1.222	0.104	0.0000	-0.208	0.0000	-0.71%	0.0000
1995 - 1999	1.035	1.211	0.090	0.0000	-0.179	0.0000	-0.66%	0.0000
1996 - 2000	1.025	1.223	0.079	0.0000	-0.158	0.0000	-0.44%	0.0000
1997 - 2001	1.061	1.263	0.054	0.0000	-0.108	0.0000	-0.53%	0.0000
1998 - 2002	1.046	1.218	0.034	0.0000	-0.068	0.0000	-0.52%	0.0000
1999 - 2003	0.961	0.995	*-0.057*	0.0000	*0.113*	0.0000	-0.10%	0.0000
2000 - 2004	0.989	0.974	*-0.023*	0.0000	*0.046*	0.0000	-0.17%	0.0000
2001 - 2005	0.996	0.939	0.003	0.0000	-0.007	0.0000	-0.34%	0.0000
2002 - 2006	0.952	0.898	0.017	0.0000	-0.034	0.0000	-0.10%	0.0000
2003 - 2007	0.954	0.867	0.026	0.0000	-0.053	0.0000	-0.04%	0.0022
2004 - 2008	1.110	1.350	0.308	0.0000	-0.616	0.0000	*0.72%*	0.0000
2005 - 2009	1.112	1.476	0.339	0.0000	-0.677	0.0000	*0.32%*	0.0000
2006 - 2010	1.115	1.494	0.312	0.0000	-0.624	0.0000	*0.41%*	0.0000
2007 - 2011	1.111	1.473	0.330	0.0000	-0.660	0.0000	*0.34%*	0.0000
2008 - 2012	1.088	1.438	0.319	0.0000	-0.638	0.0000	*0.33%*	0.0000
2009 - 2013	0.890	0.938	0.066	0.0000	-0.131	0.0000	-0.60%	0.0000
1994 - 2013	1.007	1.132	0.105	0.0000	-0.211	0.0000	-0.16%	0.0000

VSBFA vs. Rolling PCA (IV)

- Random portfolios
  - 1000 long/short portfolios
- Tested values
  - portfolio volatilities forecasted by 10-factor
  - VSBFA of $h = 0.86$ and rolling PCA of $T_{MW} = 37$
- Test statistic
  - rolling-60-month and 240-month values of
  - bias statistic, $BS^{(VSB)}$ and $BS^{(PCA)}$
  - log-likelihood difference, $\Delta LL = LL^{(VSB)} - LL^{(PCA)}$
  - Q-statistic difference, $\Delta QS = QS^{(VSB)} - QS^{(PCA)}$
  - volatility-reduction difference, $\Delta VM = VM^{(VSB)} - VM^{(PCA)}$

VSBFA vs. Rolling PCA (V)

- **Model comparison**
  - VSBFA can make more accurate portfolio volatility forecasts than rolling PCA in
  - all of the 16 rolling 60-month periods and
  - the entire 240-month period
- Conclusion
  - variances of long/short portfolios are dominated by specific component
  - **implication: VSBFA can make more accurate forecasts in specific variance as well than rolling PCA**

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VSBFA vs. Rolling PCA (VI)

VSBFA vs. PCA	Random Long/Short Portfolios							
	Bias Statistic		Log-Likelihood		Q-Statistic		Volatility Minim	
	$BS^{(VSB)}$	$BS^{(PCA)}$	ΔLL	p-Val	ΔQS	p-Val	ΔVM	p-Val
1994 - 1998	1.076	1.115	0.045	0.0000	-0.090	0.0000	-0.05%	0.0000
1995 - 1999	1.087	1.175	0.047	0.0000	-0.093	0.0000	-0.07%	0.0000
1996 - 2000	1.076	1.215	0.051	0.0000	-0.103	0.0000	-0.07%	0.0000
1997 - 2001	1.054	1.210	0.052	0.0000	-0.104	0.0000	-0.07%	0.0000
1998 - 2002	1.025	1.152	0.047	0.0000	-0.094	0.0000	-0.09%	0.0000
1999 - 2003	0.982	1.029	0.019	0.0000	-0.038	0.0000	-0.09%	0.0000
2000 - 2004	0.970	0.944	0.027	0.0000	-0.055	0.0000	-0.10%	0.0000
2001 - 2005	0.979	0.865	0.033	0.0000	-0.065	0.0000	-0.11%	0.0000
2002 - 2006	1.003	0.865	0.033	0.0000	-0.066	0.0000	-0.11%	0.0000
2003 - 2007	1.019	0.900	0.033	0.0000	-0.066	0.0000	-0.06%	0.0000
2004 - 2008	1.076	1.058	0.070	0.0000	-0.140	0.0000	-0.09%	0.0000
2005 - 2009	1.080	1.187	0.091	0.0000	-0.181	0.0000	-0.13%	0.0000
2006 - 2010	1.029	1.179	0.078	0.0000	-0.157	0.0000	-0.14%	0.0000
2007 - 2011	1.016	1.159	0.087	0.0000	-0.174	0.0000	-0.15%	0.0000
2008 - 2012	1.001	1.128	0.089	0.0000	-0.179	0.0000	-0.16%	0.0000
2009 - 2013	0.959	1.004	0.050	0.0000	-0.100	0.0000	-0.15%	0.0000
1994 - 2013	1.022	1.049	0.046	0.0000	-0.092	0.0000	-0.10%	0.0000

VSBFA

- Introduction
- Factor models
- VSBFA method
- Model comparison
- Case 1: simulations
- Case 2: global stocks
- **Q&A**
- Appendix
- Disclaimer

VSBFA Method

- Questions ?
- Discussions

- **Thank You !**

- (VSBFA methodology is detailed in the Appendix next)

VSBFA

Bayesian Modeling (I)

- Time-varying statistical factor models

$$y_t = X_t f_t + e_t$$

- The simplest distributions to be assumed for random variables

$$f_t,\ X_t,\ e_t,\ P_t,\ \{U_{j,t}\},\ R_t$$

- Multivariate normal distributions
  - maximum entropy (i.e. the most appropriate assumption) given only mean and covariance
  - $f_t \sim N(E(f_t), P_t)$
  - $X_{j-col,t} \sim N(E(X_{j-col,t}), U_{j,t}),\ \ j = 1, 2, \cdots, m$
  - $e_t \sim N(0, R_t)$
  - elements of f_t, X_t and e_t independent of each other
  - P_t, $\{U_{j,t}\}$ and R_t: diagonal random matrices

Bayesian Modeling (II)

- Inverse-gamma distribution
  - conjugate prior of variance of normal distribution
  - diagonal elements of P_t, $\{U_{j,t}\}$ and R_t: independent inverse-gamma variables
- A "diagonal inverse-gamma" distribution $IG_D(\cdot)$
  - = product of independent inverse-gamma distributions $IG(\cdot)$
  - example: $IG_D(P_t; \alpha_t, B_{P,t}) = \prod_{j=1}^{m} IG((P_t)_{jj}; \alpha_t, (B_{P,t})_{jj})$
- "Diagonal inverse-gamma" distributions
  - $P_t \sim IG_D(\alpha_t, B_{P,t})$
  - $U_{j,t} \sim IG_D(\alpha_t, B_{U,j,t}),\ \ j = 1, 2, \cdots, m$
  - $R_t \sim IG_D(\alpha_t, B_{R,t})$

Bayesian Modeling (III)

- “Diagonal inverse-gamma” (continue)
  - α_t: shape parameter common to all variables
  - $B_{P,t}$, $\{B_{U,j,t}\}$ and $B_{R,t}$: diagonal matrices of scale parameters
- Bayesian modeling
  - joint distribution = likelihood × (sequential) priors

$$p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t) =$$

$$= p(y_t|X_t, f_t, R_t)p(f_t|P_t)p(P_t)p(X_t|\{U_{j,t}\})p(\{U_{j,t}\})p(R_t)$$

- Bayesian solution
  - posterior

$$p(f_t, P_t, X_t, \{U_{j,t}\}, R_t|y_t)$$

Sequential Bayes (I)

- Time-varying statistical factor models

$$y_t = X_t f_t + e_t$$

- Random variables: f_t, P_t, X_t, $\{U_{j,t}\}$, R_t
- Estimated means: $f_{t|t}$, $P_{t|t}$, $X_{t|t}$, $\{U_{j,t|t}\}$, $R_{t|t}$
- Predicted means: $f_{t+1|t}$, $P_{t+1|t}$, $X_{t+1|t}$, $\{U_{j,t+1|t}\}$, $R_{t+1|t}$
- Predicting "slow-changing" variables by random walks
  - $P_{t+1|t} = P_{t|t}$
  - $X_{t+1|t} = X_{t|t}$
  - $U_{j,t+1|t} = U_{j,t|t}$
  - $R_{t+1|t} = R_{t|t}$

Sequential Bayes (II)

- Predicting of “fast-changing” variable by long-term mean
  - $f_{t+1|t} = 0$
- Likelihood at time t
  - $p(y_t | X_t, f_t, R_t) = N(y_t; X_t f_t, R_t)$
- “Sequential priors” at time t
  - expected value of a prior = predicted mean of the variable
  - $p(f_t | P_t) = N(f_t; f_{t|t-1}, P_t)$
  - $p(P_t) = IG_D(P_t; \alpha_{t|t-1}, B_{P,t|t-1})$
  - $p(X_t | \{U_{j,t}\}) = \prod_{j=1}^{m} N(X_{j-col,t}; X_{j-col,t|t-1}, U_{j,t})$
  - $p(\{U_{j,t}\}) = \prod_{j=1}^{m} IG_D(U_{j,t}; \alpha_{t|t-1}, B_{U,j,t|t-1})$
  - $p(R_t) = IG_D(R_t; \alpha_{t|t-1}, B_{R,t|t-1})$

Sequential Bayes (III)

- Sequential Bayesian posterior at time t

  $p(f_t, P_t, X_t, \{U_{j,t}\}, R_t | y_t)$

  - estimated by the joint distribution = likelihood × sequential priors
- The posterior
  - analytically intractable
  - even though only the simplest likelihood and priors are assumed
  - to be approximated

Approximated Posterior

- Stochastic approximation of intractable posterior
  - numeric distributions by stochastic sampling
  - approximated posterior → exact posterior
    when sample size → ∞
  - example: Markov Chain Monte Carlo (MCMC) method
  - algorithm: particle filtering
  - model dimension, n, can not be large
- Analytic approximation of intractable posterior
  - approximated by simpler tractable distributions
  - example: variational Bayes (VB)
  - algorithm: analytic filtering
  - model dimension, n, can be very large

Variational Bayes (I)

- Variational Bayesian approximation (VBA)
  - developed long ago in quantum mechanics
  - now widely applied in science and engineering
  - on its way to economics and finance
- To approximate posterior $p(\theta|y)$ by factorized distribution $q(\theta)$

$$p(\theta|y) = p(\theta_1, \theta_2, \cdots, \theta_k|y) \approx$$

$$\approx q(\theta) = q_1(\theta_1)\, q_2(\theta_2) \cdots q_k(\theta_k)$$

  - factorization, or separable distribution, is the only simplification assumption
  - no need to pre-identify the function forms of $q_j(\theta_j)$
  - iterative solutions always converge to local optima (with reasonable initial distributions)

Variational Bayes (II)

- VBA of posterior for time-varying statistical factor models

$$p(f_t, P_t, X_t, \{U_{j,t}\}, R_t | y_t) \approx$$

$$\approx q(f_t, P_t, X_t, \{U_{j,t}\}, R_t) =$$

$$= \prod_{j=1}^{m} q_{f,j}(f_{j,t}) \times \prod_{j=1}^{m} q_{P,j}(P_{j,t}) \times \prod_{j=1}^{m} q_{X,j}(X_{j-col,t}) \times$$

$$\times \prod_{j=1}^{m} q_{U,j}(U_{j,t}) \times q_R(R_t)$$

- Denoting operators of “expectation w.r.t. $q_j(\theta_j)$”
  - $E_{f,j}\, z = E_{f,j}(z) = \int z\, q_{f,j}(f_{j,t})\, df_{j,t}$
  - $E_{P,j}\, z = E_{P,j}(z) = \int z\, q_{P,j}(P_{j,t})\, dP_{j,t}$
  - $E_{X,j}\, z = E_{X,j}(z) = \int z\, q_{X,j}(X_{j-col,t})\, dX_{j-col,t}$
  - $E_{U,j}\, z = E_{U,j}(z) = \int z\, q_{U,j}(U_{j,t})\, dU_{j,t}$
  - $E_R\, z = E_R(z) = \int z\, q_R(R_t)\, dR_t$

Variational Bayes (III)

- Denoting combined expectation operators
  - $E_f\, z = \left(\prod_{j=1}^{m} E_{f,j}\right) z = \int z \prod_{j=1}^{m} q_{f,j}(f_{j,t})\, df_{j,t}$
  - $E_P\, z = \left(\prod_{j=1}^{m} E_{P,j}\right) z = \int z \prod_{j=1}^{m} q_{P,j}(P_{j,t})\, dP_{j,t}$
  - $E_X\, z = \left(\prod_{j=1}^{m} E_{X,j}\right) z = \int z \prod_{j=1}^{m} q_{X,j}(X_{j-col,t})\, dX_{j-col,t}$
  - $E_U\, z = \left(\prod_{j=1}^{m} E_{U,j}\right) z = \int z \prod_{j=1}^{m} q_{U,j}(U_{j,t})\, dU_{j,t}$
- Having the known joint distribution (likelihood × priors)

$$p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t) =$$

$$= p(y_t|X_t, f_t, R_t)p(f_t|P_t)p(P_t)p(X_t|\{U_{j,t}\})p(\{U_{j,t}\})p(R_t)$$

optimal approximating distributions $q_j(\theta_j)$ are the solutions to a set of VBA optimization equations

Variational Bayes (IV)

- VBA optimization equations, for $j = 1, 2, \cdots, m$
  - $\ln q_{f,j}(f_{j,t}) = const +$
    $$+ \left(\prod_{k \neq j} E_{f,k}\right) E_P\, E_X\, E_U\, E_R \ln p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t)$$
  - $\ln q_{P,j}(P_{j,t}) = const +$
    $$+ E_f \left(\prod_{k \neq j} E_{P,k}\right) E_X\, E_U\, E_R \ln p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t)$$
  - $\ln q_{X,j}(X_{j-col,t}) = const +$
    $$+ E_f\, E_P \left(\prod_{k \neq j} E_{X,k}\right) E_U\, E_R \ln p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t)$$
  - $\ln q_{U,j}(U_{j,t}) = const +$
    $$+ E_f\, E_P\, E_X \left(\prod_{k \neq j} E_{U,k}\right) E_R \ln p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t)$$

Variational Bayes (V)

- VBA optimization equations (continue)
  - $\ln q_R(R_t) = const +$
    $$+ E_f\, E_P\, E_X\, E_U\, \ln p(y_t, f_t, P_t, X_t, \{U_{j,t}\}, R_t)$$
- The VBA optimization equations
  - interrelated: each optimal solution $q_j(\theta_j)$ is expressed in terms of all others
  - can identify the function forms of $q_j(\theta_j)$, $j = 1, 2, \cdots, k$
  - can be solved iteratively: estimating each $q_j(\theta_j)$ with the latest estimates of all others
  - with reasonable initial distributions, the iterative solutions always converge to local optima
  - the sequential priors offer reasonable initial distributions

VSBFA Algorithm (I)

- Time-varying statistical factor models

$$y_t = X_t f_t + e_t$$

- VBA iteration at each time t
  - an executable iteration, with iteration label $v = 1, 2, \cdots, L$, to solve the set of VBA optimization equations
- Initial values for the VBA iteration at time t
  - factors: $f_{t|t}^{(0)} = 0$; $P_{t|t}^{(0)} = P_{t-1|t-1}$
  - loadings: $X_{t|t}^{(0)} = X_{t-1|t-1}$; $U_{j,t|t}^{(0)} = U_{j,t-1|t-1}$, $j = 1, \cdots, m$
  - errors: $R_{t|t}^{(0)} = R_{t-1|t-1}$
  - additional: $Q_{t|t}^{(0)} = P_{t|t}^{(0)}$; $V_{j,t|t}^{(0)} = U_{j,t|t}^{(0)}$, $j = 1, \cdots, m$

VSBFA Algorithm (II)

- Rescaling the diagonal $\{U_{j,t|t}^{(0)}\}$ and $R_{t|t}^{(0)}$
  - to prevent over- and under-fitting
  - by keeping specific variance $\sum_{j=1}^{m} P_{j,t|t}^{(0)} U_{j,t|t}^{(0)} + R_{t|t}^{(0)}$ unchanged
  - while targeting residual-to-specific variance ratios $R_{t|t}^{(0)} (\sum_{j=1}^{m} P_{j,t|t}^{(0)} U_{j,t|t}^{(0)} + R_{t|t}^{(0)})^{-1} = h\, I_n$
  - with a “residual-to-specific variance ratio target” (or RSVRT, a modeling parameter) h: $0 \ll h < 1$
  - smaller h (smaller $R_{t|t}^{(0)}$) causing over-response to data
  - larger h (smaller $U_{j,t|t}^{(0)}$) causing under-response to data

VSBFA Algorithm (III)

- At each VBA iterate $v \geq 1$ at time t
  - *Quite easy to code (e.g. in MATLAB)*
  - The first group of statements: updating factors one by one, for $j = 1, 2, \cdots, m$,
  - $Q_{j,t|t}^{(v)} = \{ (P_{j,t|t}^{(v-1)})^{-1} +$
    $+ tr((R_{t|t}^{(v-1)})^{-1} [V_{j,t|t}^{(v-1)} + X_{j-col,t|t}^{(v-1)} (X_{j-col,t|t}^{(v-1)})^T]) \}^{-1}$
  - $f_{j,t|t}^{(v)} = Q_{j,t|t}^{(v)} \{ (P_{j,t|t}^{(v-1)})^{-1} f_{j,t|t}^{(0)} +$
    $+ tr((R_{t|t}^{(v-1)})^{-1} [y_t - \sum_{k \neq j} f_{k,t|t}^{(v-1)} X_{k-col,t|t}^{(v-1)}] (X_{j-col,t|t}^{(v-1)})^T) \}$
  - $P_{j,t|t}^{(v)} = (T_0 + 1)^{-1} [T_0 P_{j,t|t}^{(0)} + Q_{j,t|t}^{(v)} + (f_{j,t|t}^{(v)} - f_{j,t|t}^{(0)})^2]$

VSBFA Algorithm (IV)

- At each VBA iterate $v \geq 1$ at time t (continue)
  - The second group of statements: updating loadings one by one, for $j = 1, 2, \cdots, m$,
  - $V_{j,t|t}^{(v)} = U_{j,t|t}^{(v-1)} -$

    $$- U_{j,t|t}^{(v-1)} \{ U_{j,t|t}^{(v-1)} + R_{t|t}^{(v-1)} / [Q_{j,t|t}^{(v)} + (f_{j,t|t}^{(v)})^2] \}^{-1} U_{j,t|t}^{(v-1)}$$
  - $X_{j-col,t|t}^{(v)} = V_{j,t|t}^{(v)} \{ (U_{j,t|t}^{(v-1)})^{-1} X_{j-col,t|t}^{(0)} +$

    $$+ (R_{t|t}^{(v-1)})^{-1} f_{j,t|t}^{(v)} [y_t - \sum_{k \neq j} f_{k,t|t}^{(v)} X_{k-col,t|t}^{(v-1)}] \}$$
  - $U_{j,t|t}^{(v)} = (T_0 + 1)^{-1} diag(T_0 U_{j,t|t}^{(0)} + V_{j,t|t}^{(v)} +$

    $$+ (X_{j-col,t|t}^{(v)} - X_{j-col,t|t}^{(0)}) (X_{j-col,t|t}^{(v)} - X_{j-col,t|t}^{(0)})^T)$$

VSBFA Algorithm (V)

- At each VBA iterate $v \geq 1$ at time t (continue)
  - The last statement: updating variance of errors
  - $R_{t|t}^{(v)} = (T_0 + 1)^{-1}\, diag(\, T_0\, R_{t|t}^{(0)} +$
    $+ \sum_{j=1}^{m} [Q_{j,t|t}^{(v)} V_{j,t|t}^{(v)} + Q_{j,t|t}^{(v)} X_{j-col,t|t}^{(v)} (X_{j-col,t|t}^{(v)})^T + (f_{j,t|t}^{(v)})^2 V_{j,t|t}^{(v)}] +$
    $+ (\, y_t - X_{t|t}^{(v)} f_{t|t}^{(v)}\,)\,(\, y_t - X_{t|t}^{(v)} f_{t|t}^{(v)}\,)^T\,)$
- Making next VBA iterate: $v \Leftarrow v + 1 \leq L$
- VSBFA estimates as of time t
  - factors: $f_{t|t} = f_{t|t}^{(L)}$; $P_{t|t} = P_{t|t}^{(L)}$
  - loadings: $X_{t|t} = X_{t|t}^{(L)}$; $U_{j,t|t} = U_{j,t|t}^{(L)}$, $j = 1, 2, \cdots, m$
  - errors: $R_{t|t} = R_{t|t}^{(L)}$

VSBFA

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Disclaimer

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