Unit 4:
Association rules

Section 1: Association Analysis: Basic Concepts and Algorithms

What Is Frequent Pattern Analysis?

- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami (1993) in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent regularities in data
  - What products were often purchased together? Beer and diapers?
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?
- Applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Frequent Pattern Mining Important?

- Discloses an intrinsic and important property of data sets
- Forms the foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: associative classification
  - Cluster analysis: frequent pattern-based clustering
  - Data warehousing: iceberg cube and cube-gradient
  - Semantic data compression: fascicles
  - Broad applications

Association Rule Mining

➢ Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Transaction data: a set of documents

- A text document data set. Each document is treated as a "bag" of keywords
  - doc1: Student, Teach, School
  - doc2: Student, School
  - doc3: Teach, School, City, Game
  - doc4: Baseball, Basketball
  - doc5: Basketball, Player, Spectator
  - doc6: Baseball, Coach, Game, Team
  - doc7: Basketball, Team, City, Game

An example

- A text document data set
  - doc 1: Student, Teach, School: Education
  - doc 2: Student, School: Education
  - doc 3: Teach, School, City, Game: Education
  - doc 4: Baseball, Basketball: Sport
  - doc 5: Basketball, Player, Spectator: Sport
  - doc 6: Baseball, Coach, Game, Team: Sport
  - doc 7: Basketball, Team, City, Game: Sport
- Let minsup = 20% and minconf = 60%. The following are two examples of class association rules:
  - Student, School ==> Education [sup= 2/7, conf = 2/2]
  - Game ==> Sport [sup= 2/7, conf = 2/3]

Association Rules Examples

- Basket Data

  Tea ^ Milk ==> Sugar [0.3 , 0.9]

- Relational Data

  diagnosis = Heart ^ sex = Male ==> age > 50 [0.4 , 0.7]

- Object-Oriented Data

  hobbies = { Sport , Art } ==> age = Young [0.5 , 0.8]

Definition: Frequent Itemset

- Itemset
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- Support count (σ)
  - Frequency of occurrence of an itemset

  σ({Milk, Bread,Diaper}) = 2
- Support
  - Fraction of transactions that contain an itemset

  s({Milk, Bread, Diaper}) = 2/5
- Frequent Itemset
  - An itemset whose support is greater than or equal to a minsup threshold

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Definition: Association Rule

- ASSOCIATION RULE
  - An implication expression of the form X -> Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} -> {Beer}
- RULE EVALUATION METRICS
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y
      appear in transactions that contain X

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(X \cup Y)}{|T|} = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(X \cup Y)}{\sigma(X)} = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support >= *minsup* threshold
  - confidence >= *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Observations:

All the above rules are binary partitions of the same itemset: **{Milk, Diaper, Beer}**

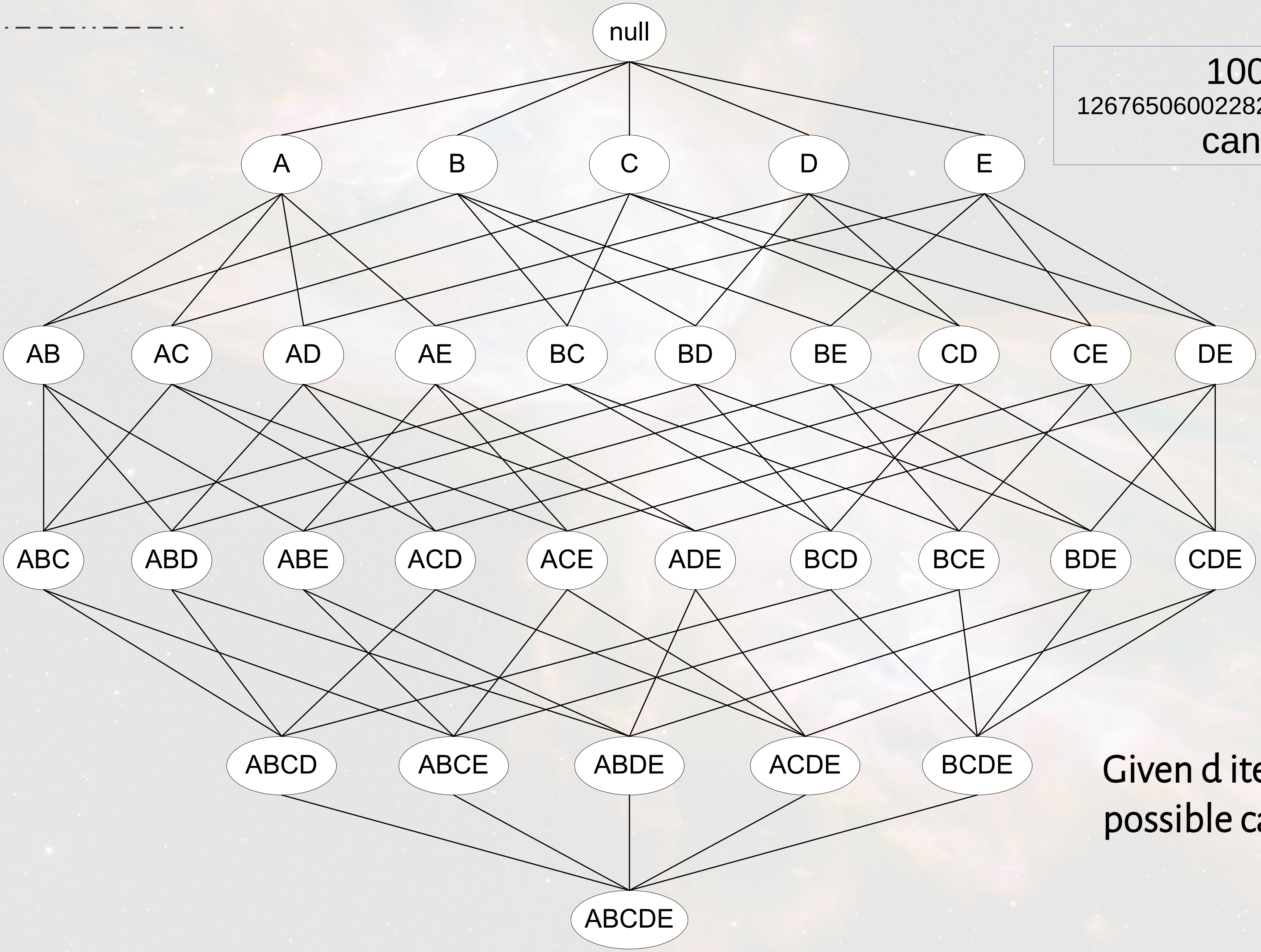
Rules originating from the same itemset have identical support but can have different confidence

Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  1) Frequent Itemset Generation
     - Generate all itemsets whose support >= minsup
  2) Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation

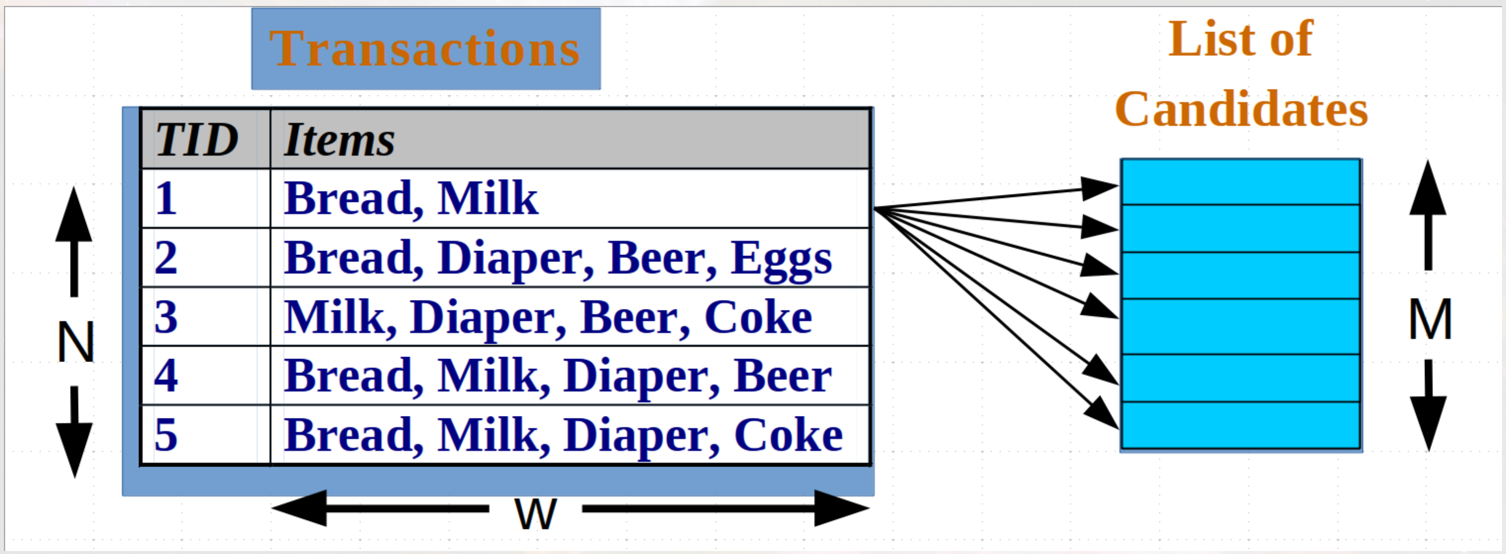


100 items:
1267650600228229401496703205376
candidates $\approx 10^{30}$

Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- ➢ BRUTE-FORCE APPROACH:
  - ➢ Each itemset in the lattice is a candidate frequent itemset
  - ➢ Count the support of each candidate by scanning the database

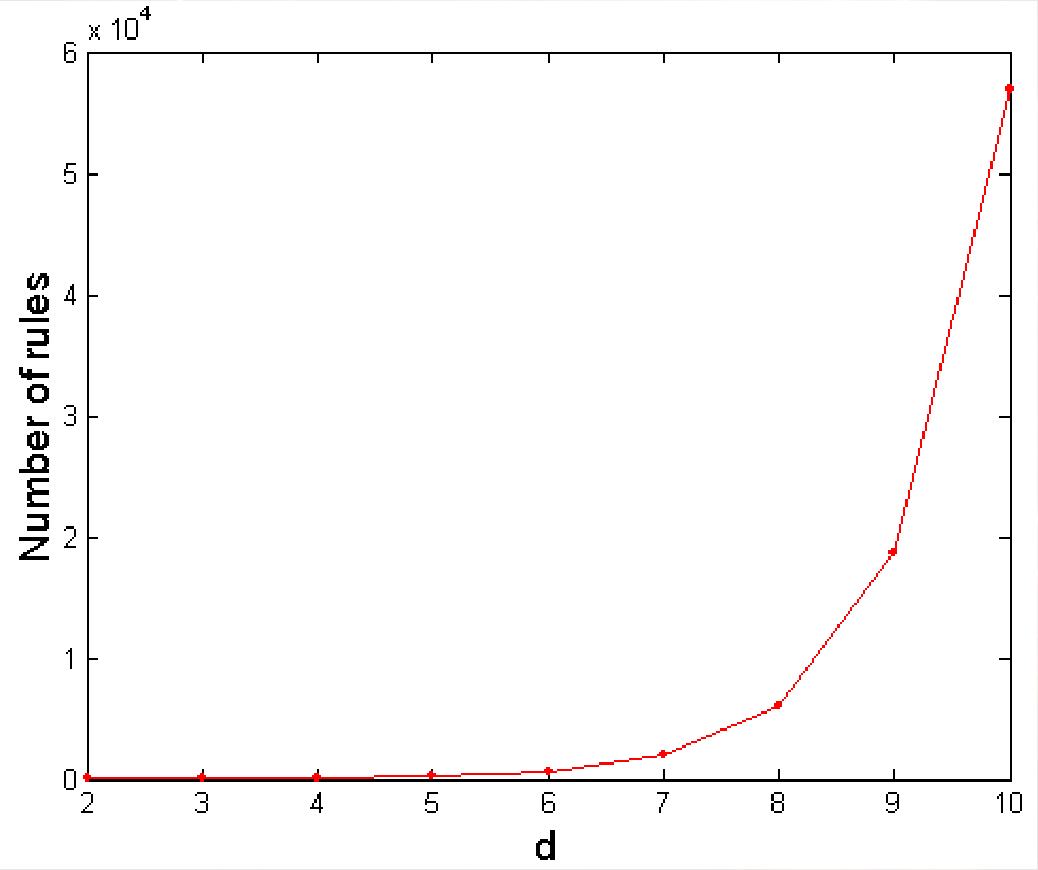


TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

  - ➢ Match each transaction against every candidate
  - ➢ Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R=\sum_{k=1}^{d-1}\left[\binom{d}{k}\times\sum_{j=1}^{d-k}\binom{d-k}{j}\right]=3^d-2^{d+1}+1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M

- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

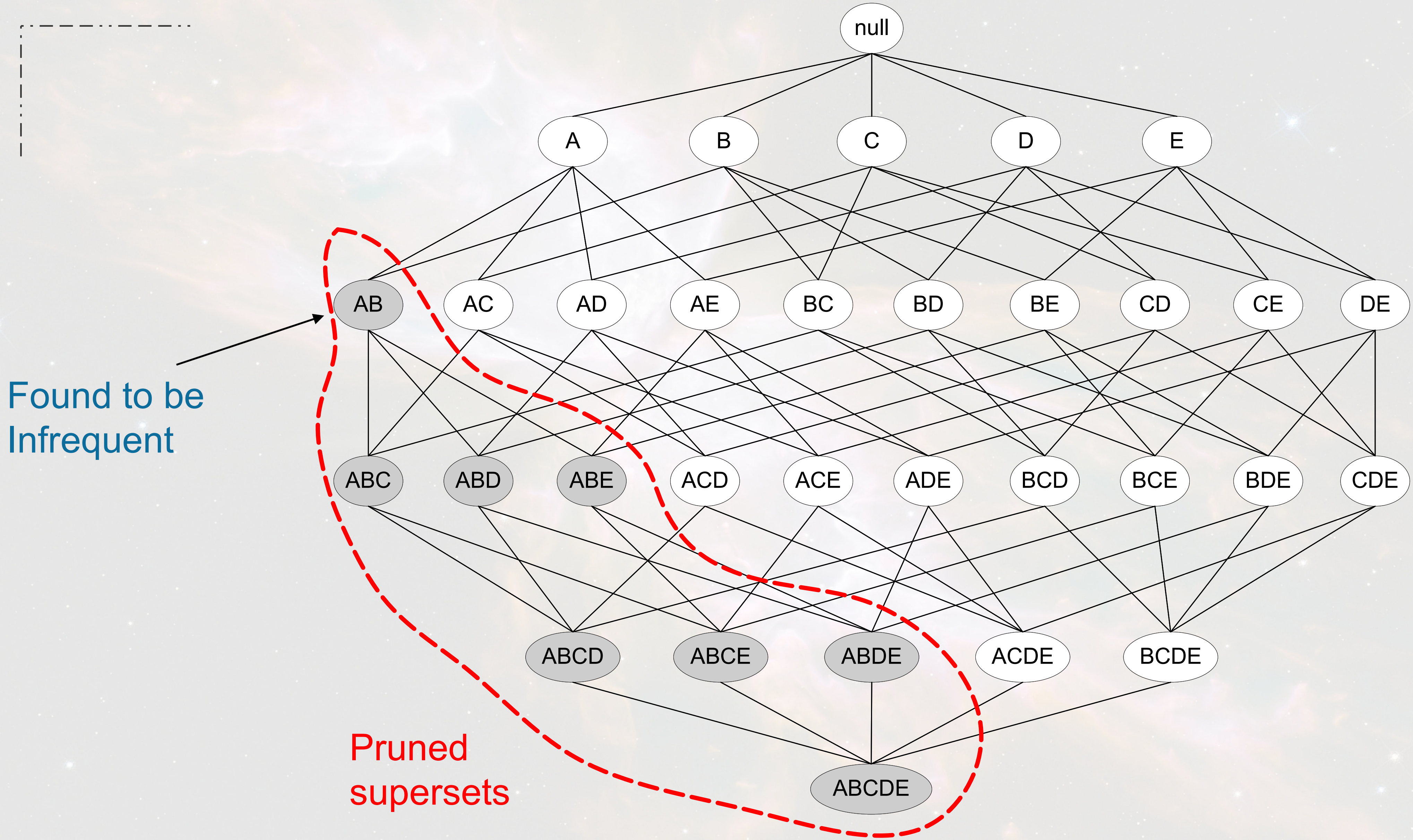
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	**3**

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
$6 + 6 + 1 = 13$

The Apriori Algorithm: Basic idea

Join Step: C_k is generated by joining L_{k-1} with itself

Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset

- Pseudo-code:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

```
L_1 = {frequent items}
for (k = 1; L_k != ∅; k++) do begin
    C_k+1 = candidates generated from L_k
    for each transaction t in database do
        increment the count of all candidates in C_k+1 that are contained in t
        L_k+1 = candidates in C_k+1 with min_support
    end
return ∪_k L_k;
```

Apriori Algorithm

- Method Fk-1 x F1:
  - Generate frequent itemsets of length 1
  - To generate frequent k-itemsets:
    - Merge frequent (k-1)-itemsets with all frequent items
  - The methods is complete: All frequent itemsets are generated
  - Many infrequent itemsets are generated
    - Heuristic pruning
  - Complexity: $O\left(\sum_k k |F_{k-1}||F_1|\right)$
  - Complexity of brute force:

$$O\left(\sum_{k=1}^{d} k \times \binom{d}{k}\right) = O\left(d \cdot 2^{d-1}\right)$$

Apriori Algorithm

- Method $F_{k-1} \times F_{k-1}$:
  - Merge two frequent (k-1)-itemsets iif their first k-2 items are common
  - Complete and generate less infrequent itemsets
  - (k-1) subsets must be frequent
  - (k-2) subsets must be test in a pruning step

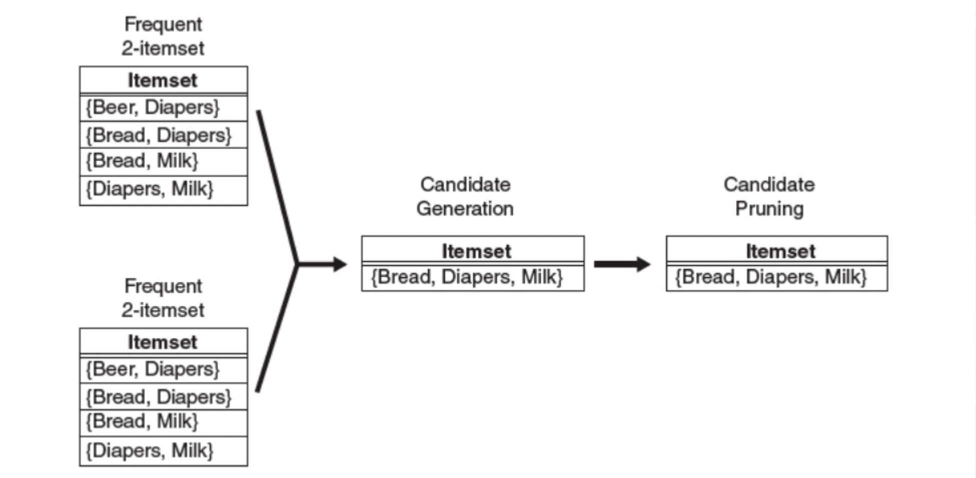


Figure 6.8. Generating and pruning candidate k-itemsets by merging pairs of frequent $(k-1)$-itemsets.

Reducing Number of Comparisons

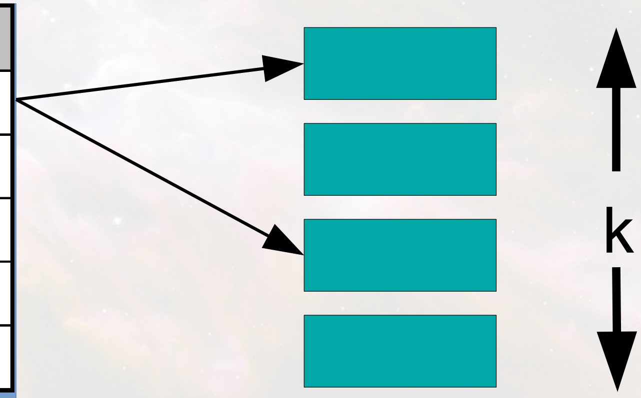
- CANDIDATE COUNTING:
  - Scan the database of transactions to determine the support of each candidate itemset
  - To reduce the number of comparisons, store the candidates in a hash structure
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Hash Structure

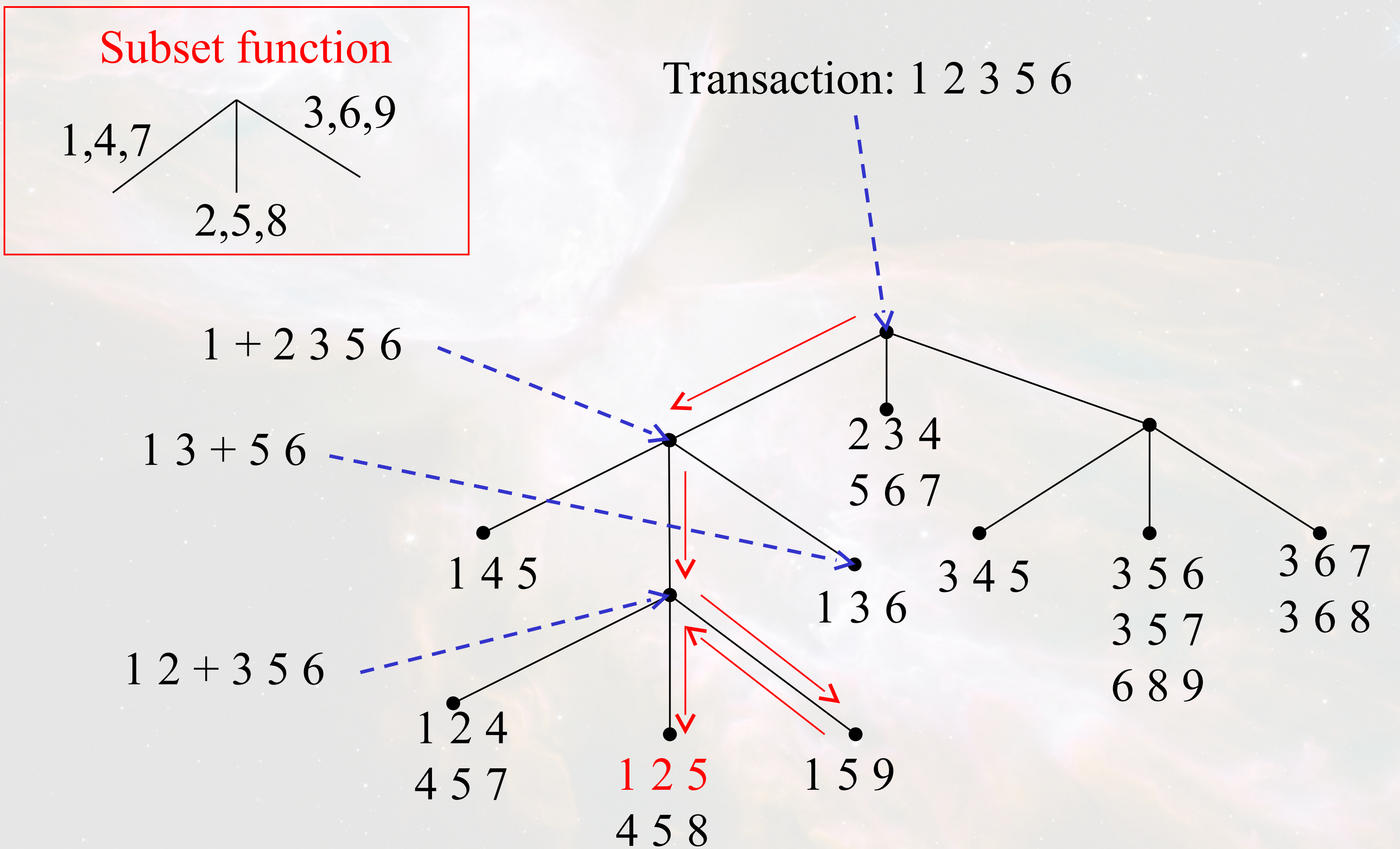


Buckets

How to Count Supports of Candidates?

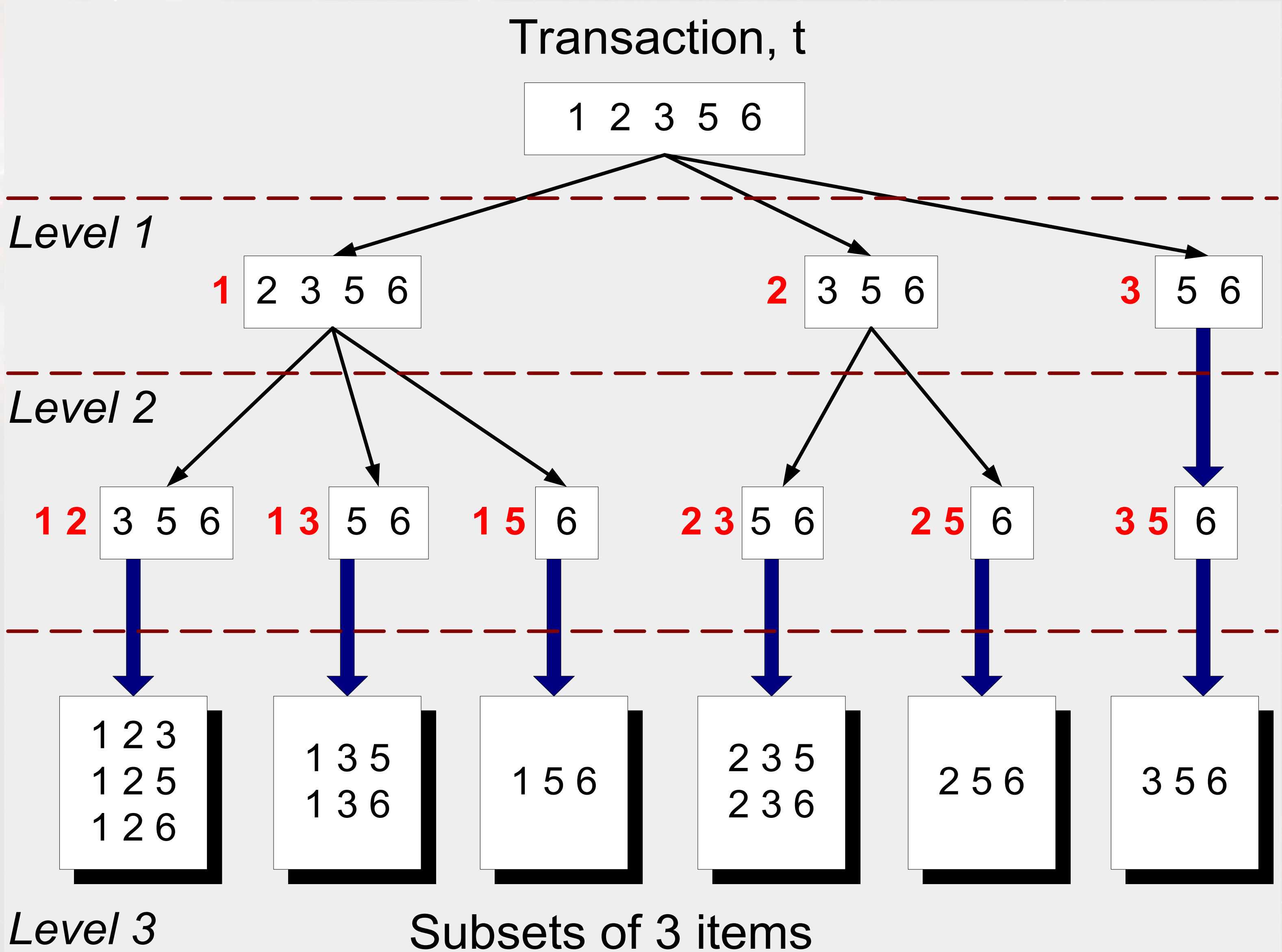
- WHY COUNTING SUPPORTS OF CANDIDATES A PROBLEM?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- METHOD:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Example: Counting Supports of Candidates



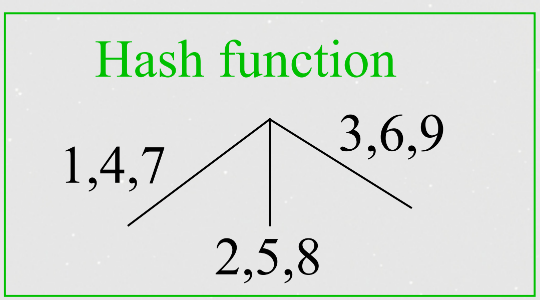
Subset Operation

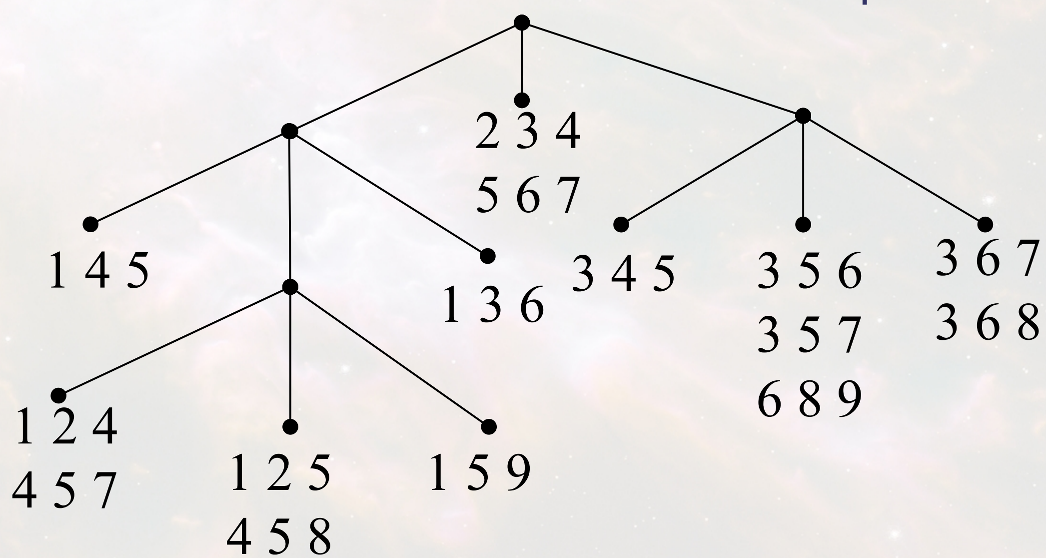
Given a transaction t, what are the possible subsets of size 3?



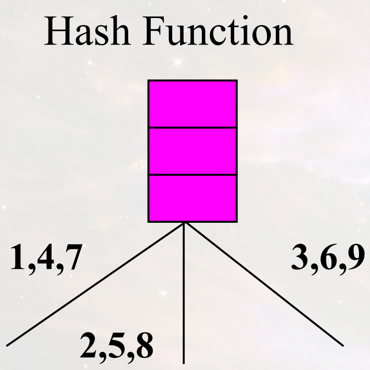
Generate Hash Tree

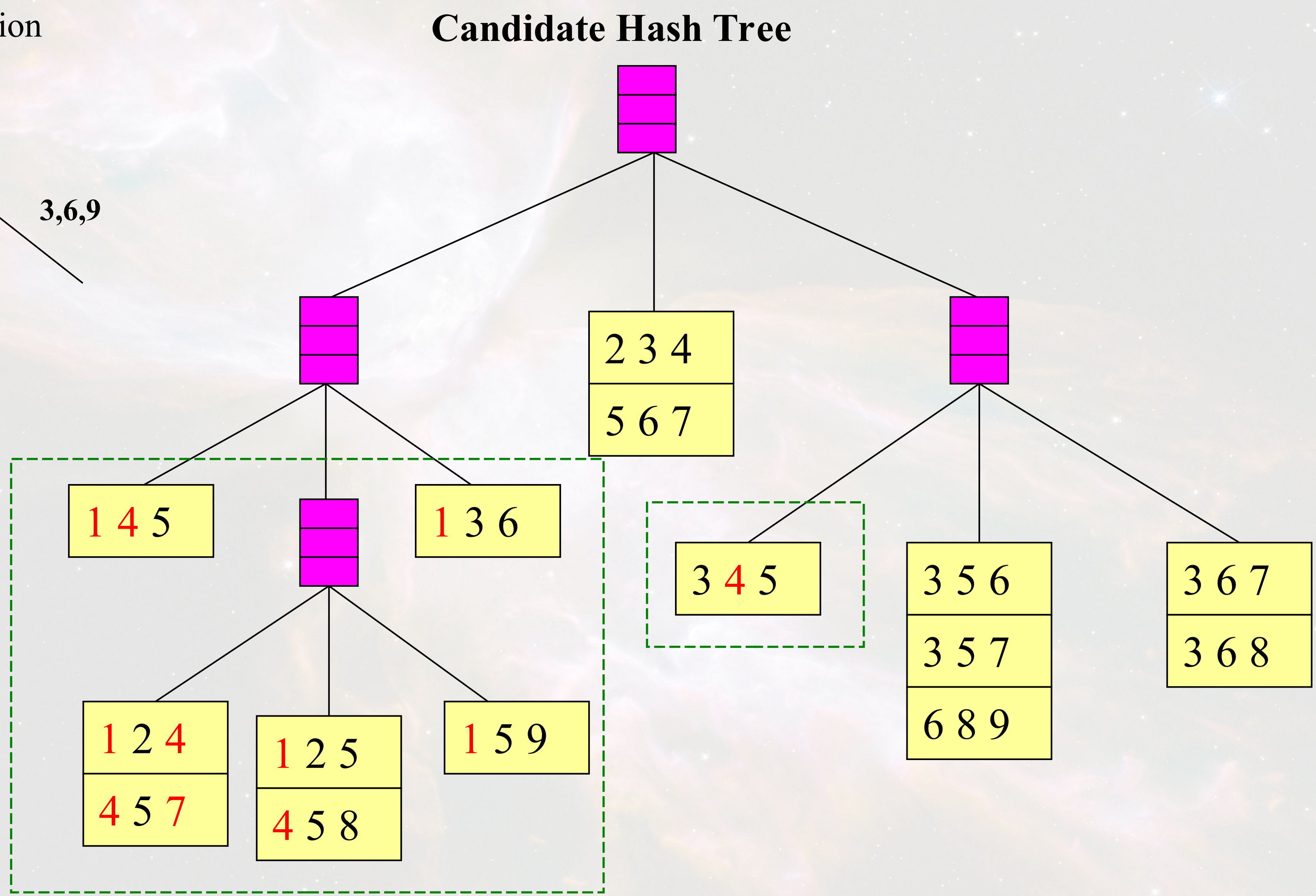
- Suppose you have 15 candidate itemsets of length 3:
  - {1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}
- You need:
  - Hash function
  - Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node).
  - In the example: 3





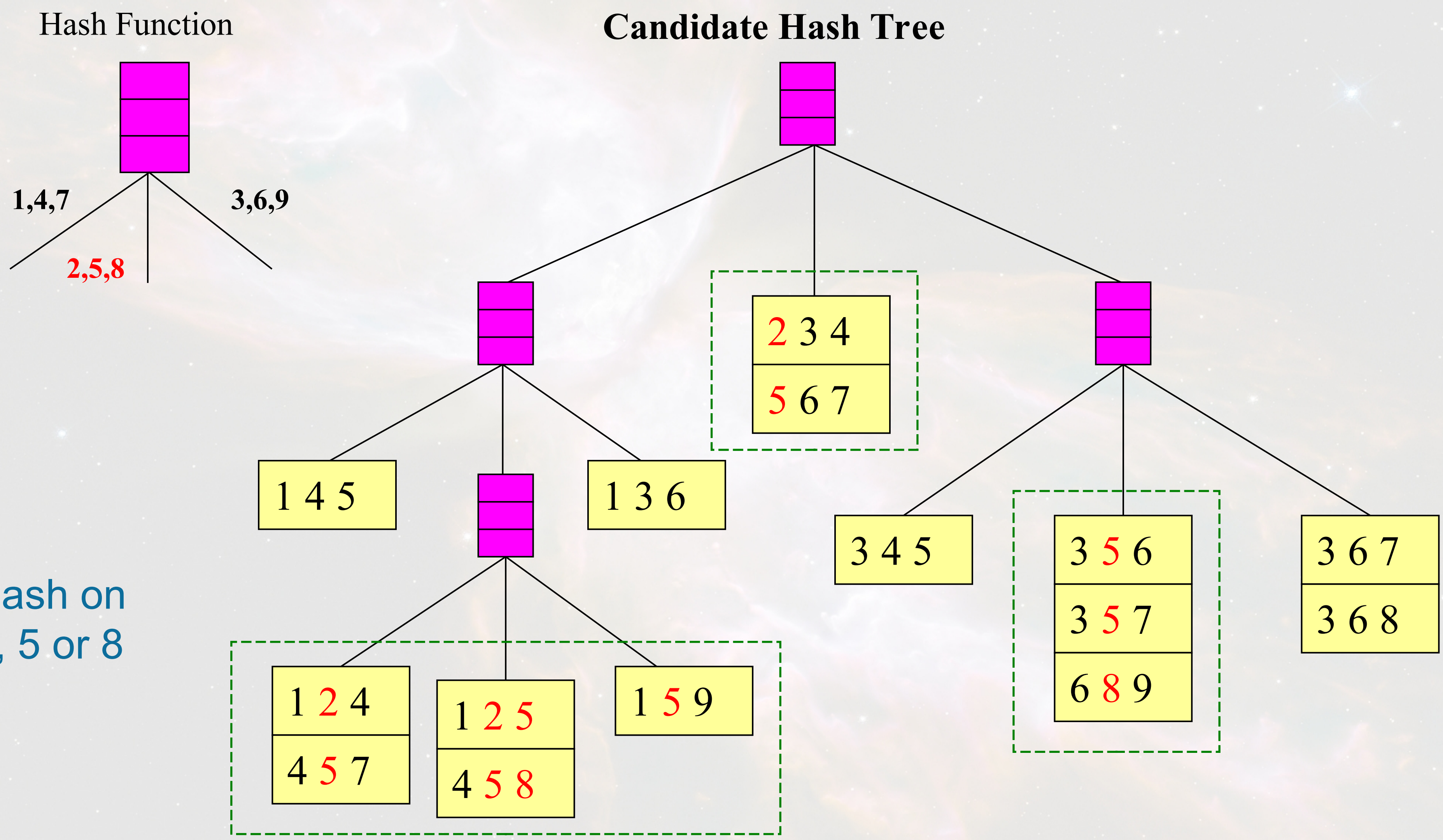
Association Rule Discovery: Hash tree





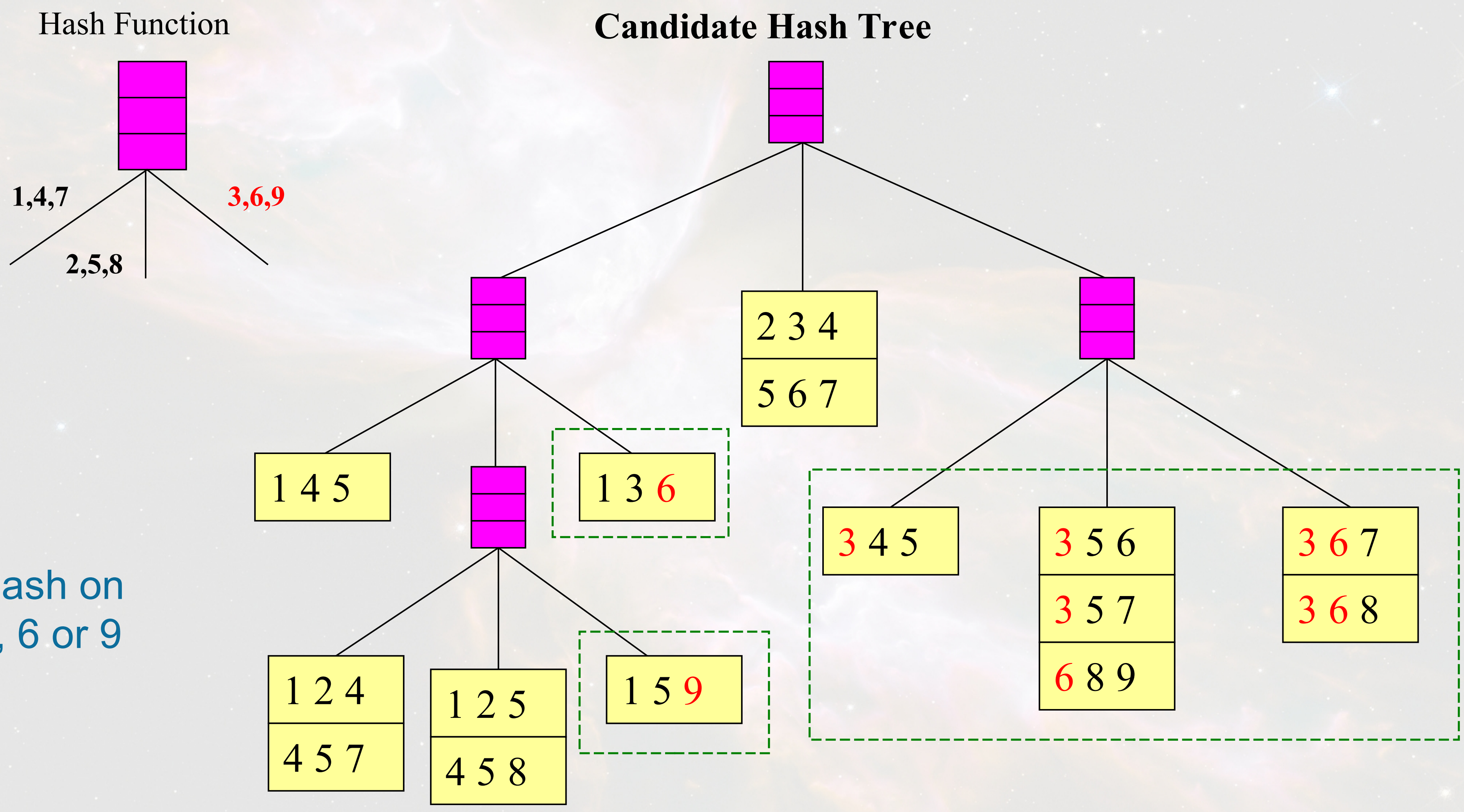
Hash on
1, 4 or 7

Association Rule Discovery: Hash tree



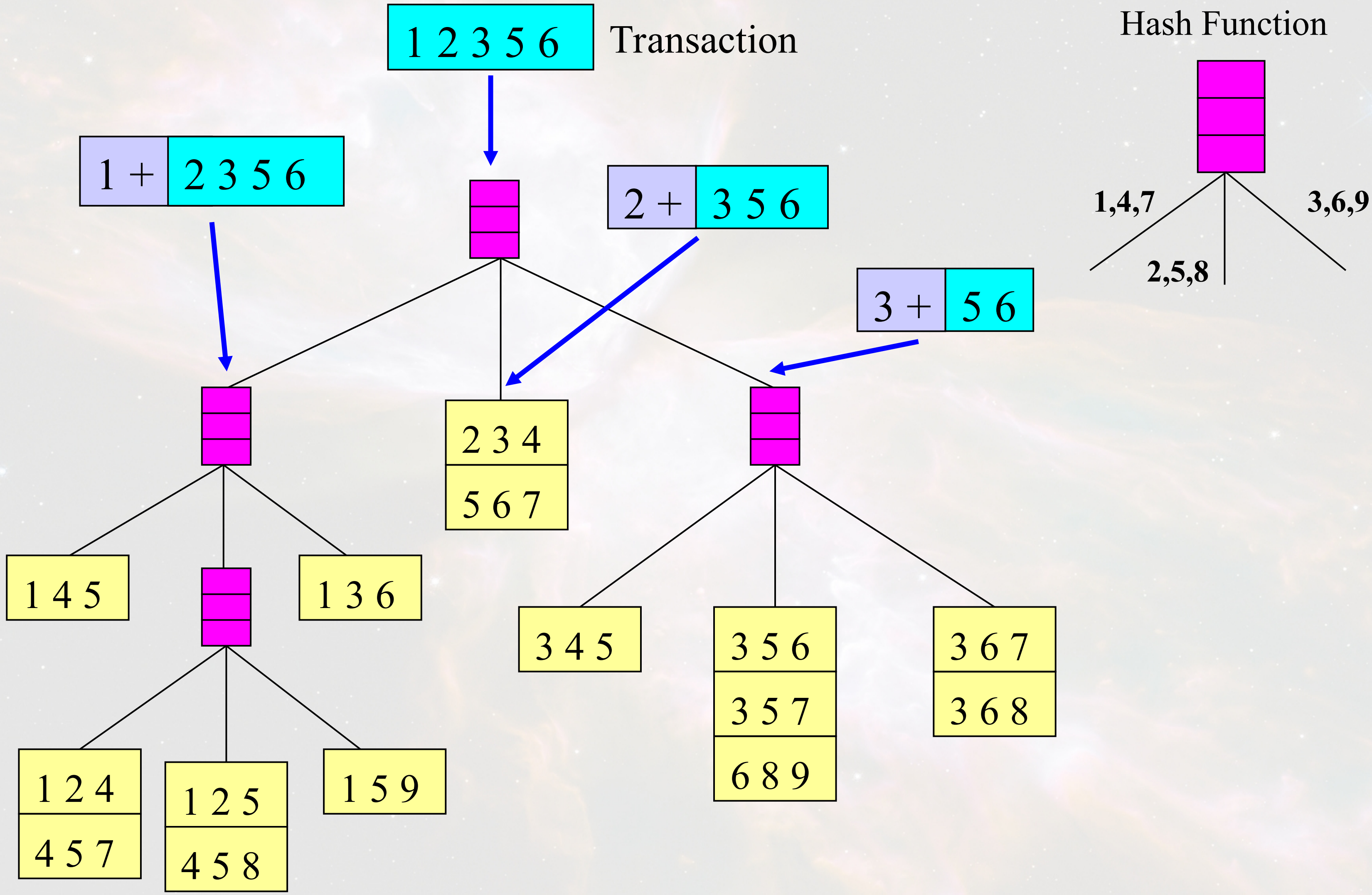
Hash on
2, 5 or 8

Association Rule Discovery: Hash tree

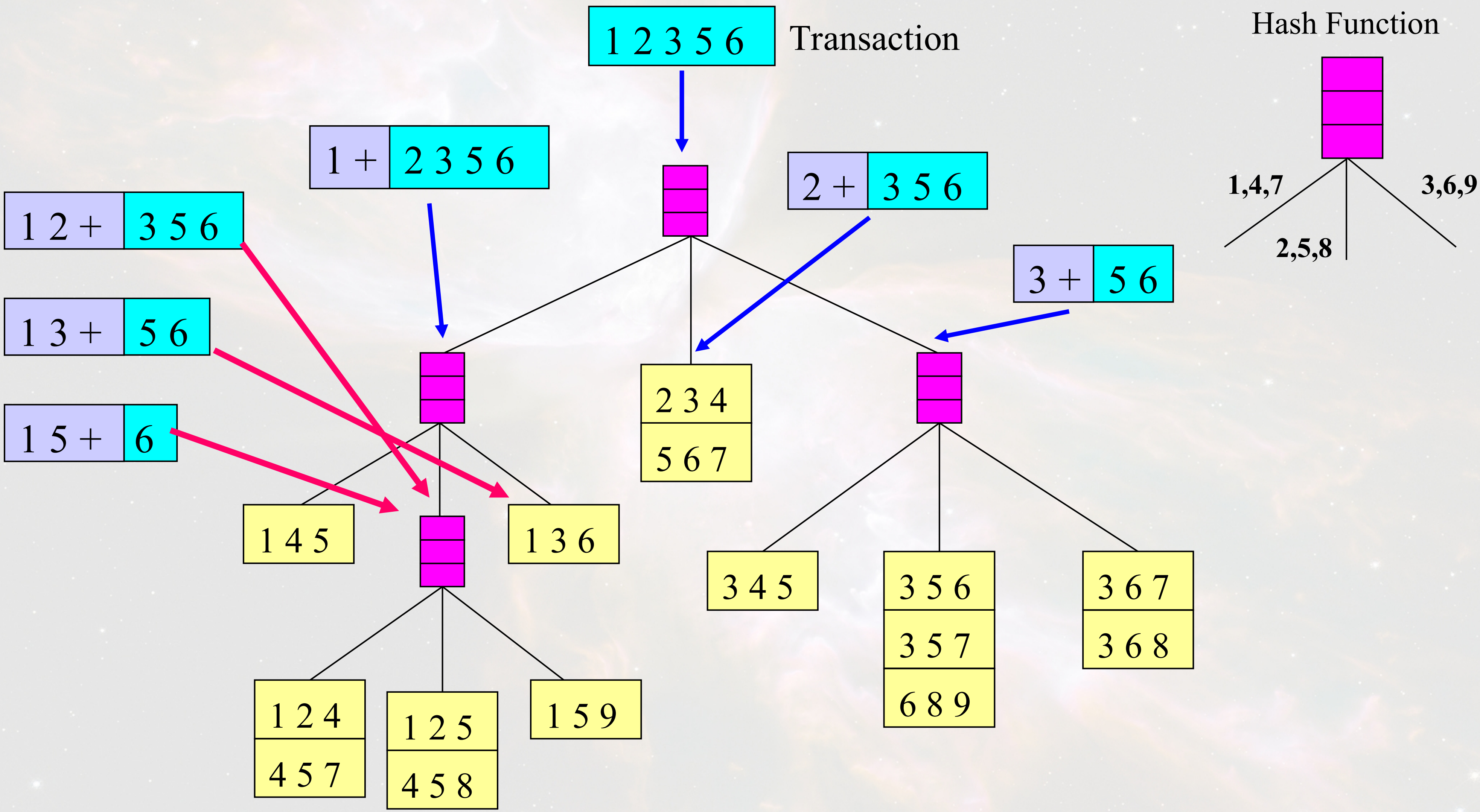


Hash on
3, 6 or 9

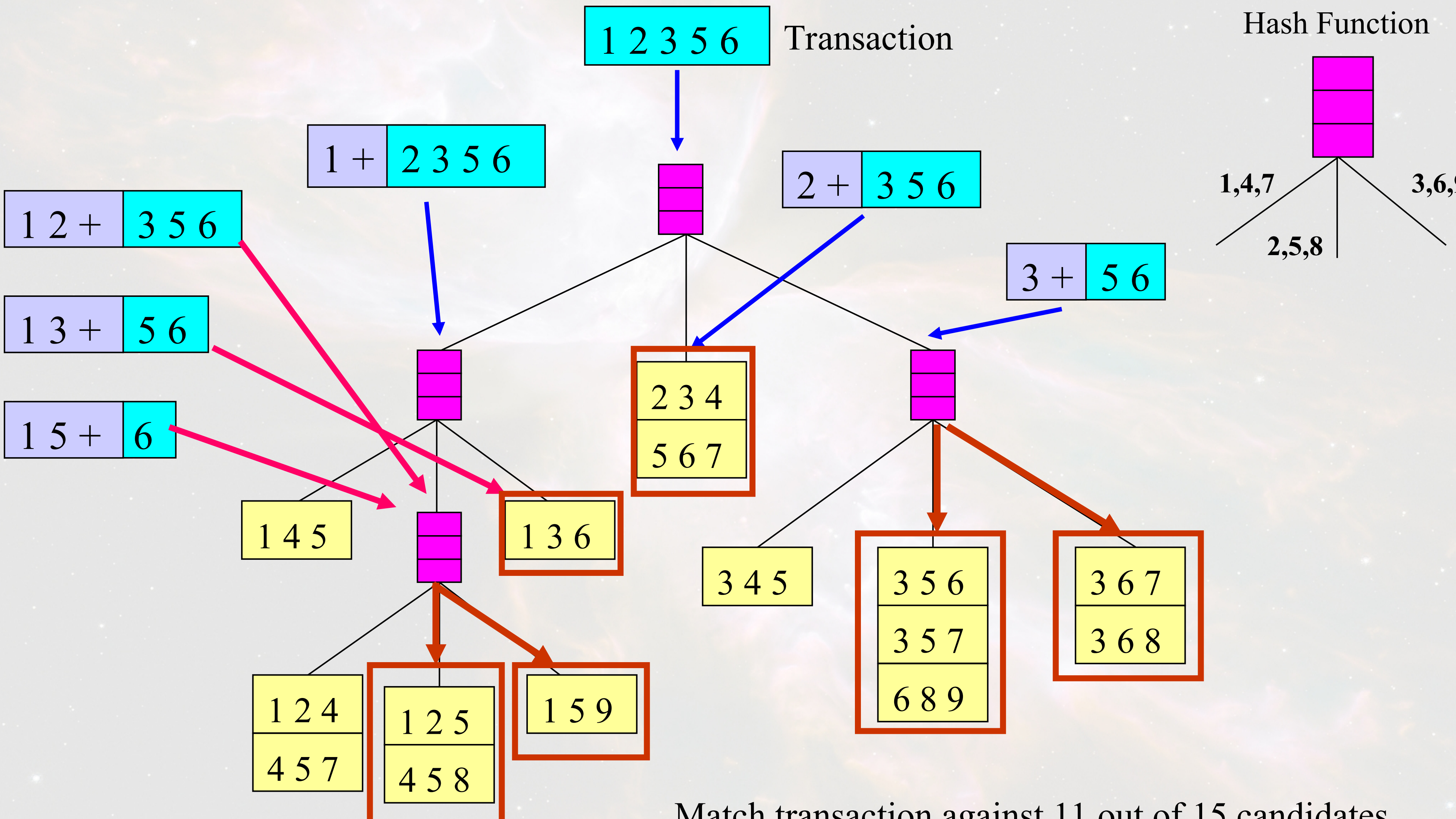
Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Match transaction against 11 out of 15 candidates

Factors Affecting Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Compact Representation of Frequent Itemsets

- Some itemsets are redundant because they have identical support as their supersets

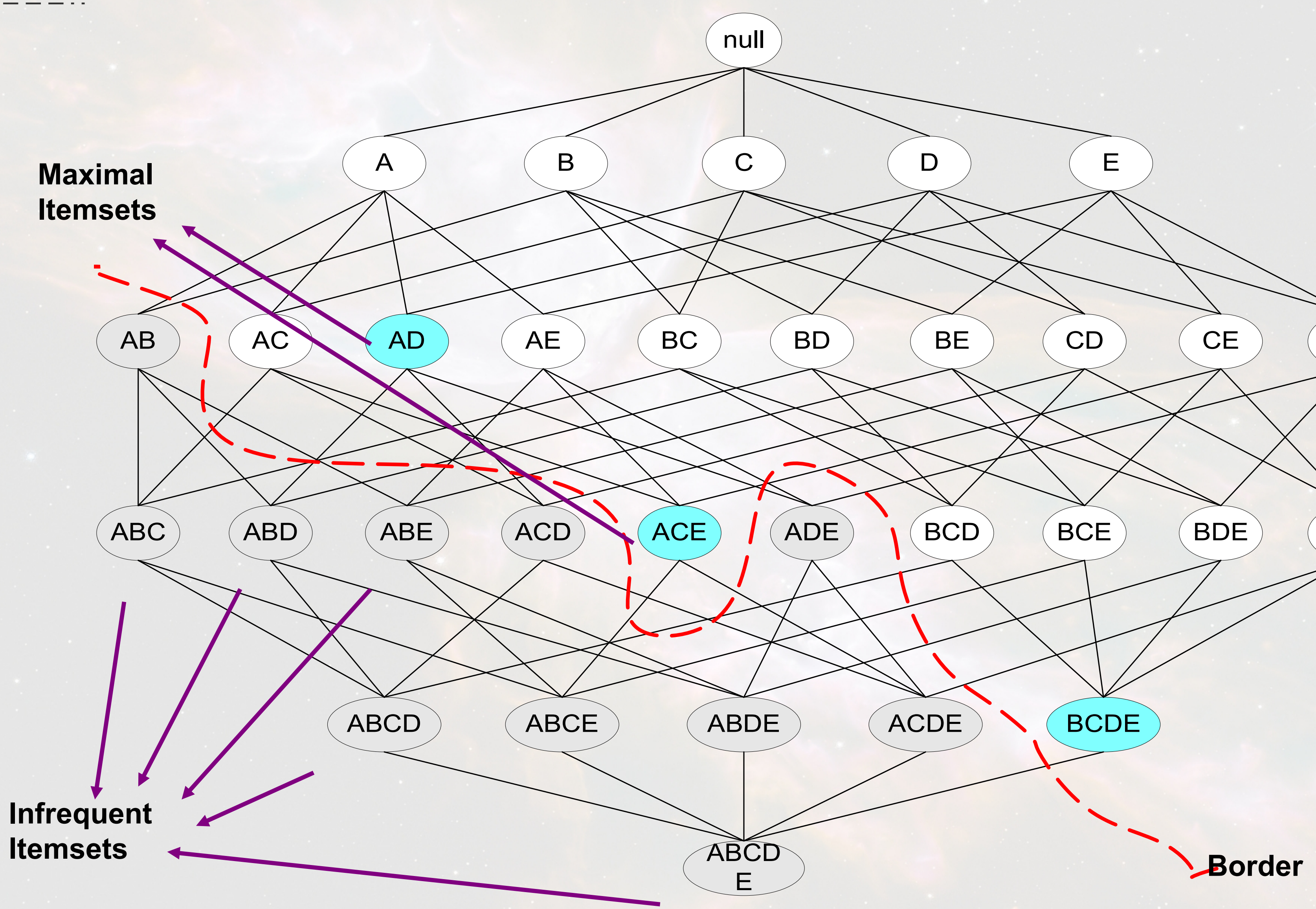
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

- Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$
- Need a compact representation

Maximal frequent itemsets

- An itemset is maximal frequent if none of its immediate supersets is frequent
  - Maximal frequent intemsets are a compact representation of all frequent intemsets
  - All frequents itemsets are either:
    - Maximal frequent itemsets
    - Subsets of maximal frequent itemsets

Maximal Frequent Itemset



Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

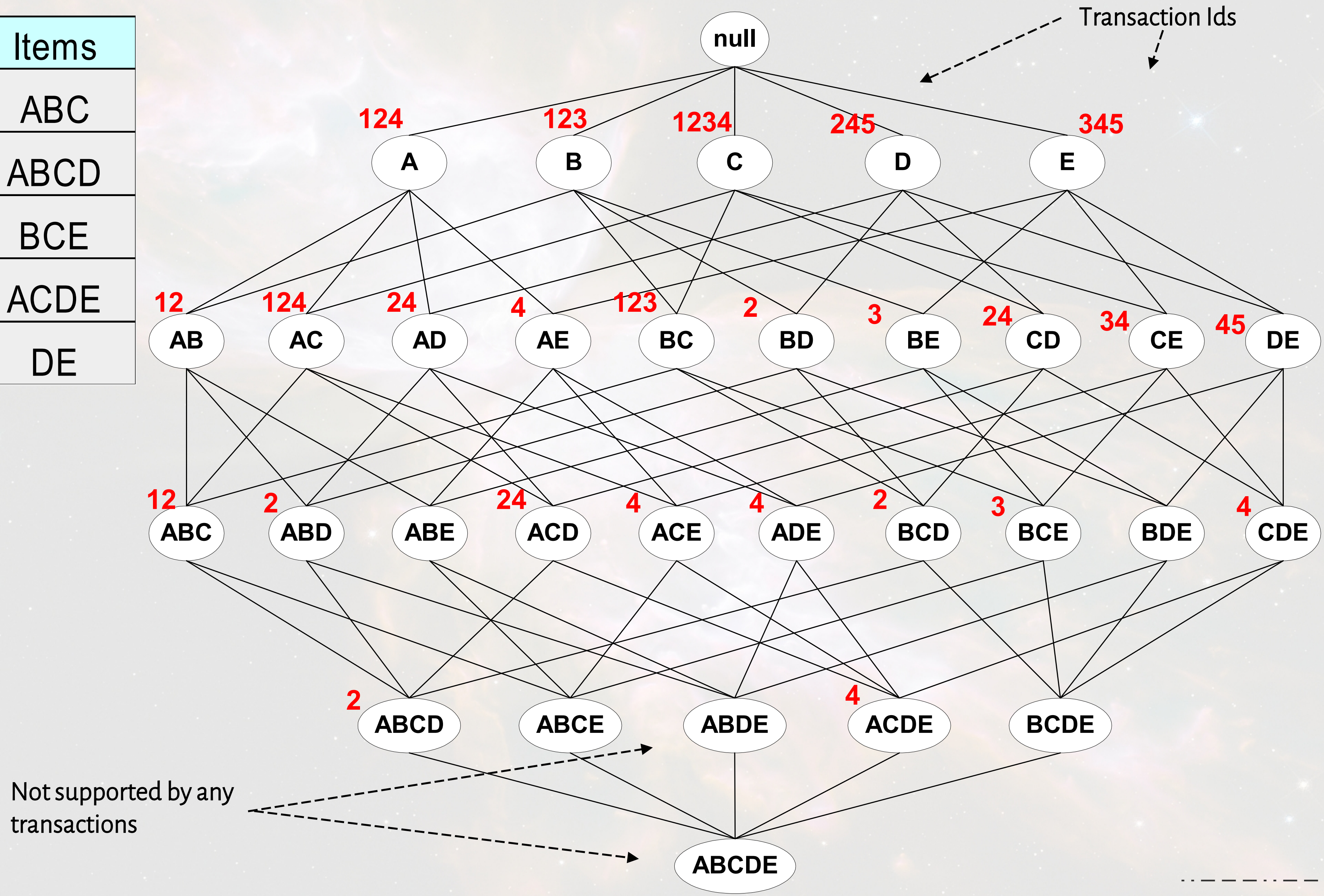
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

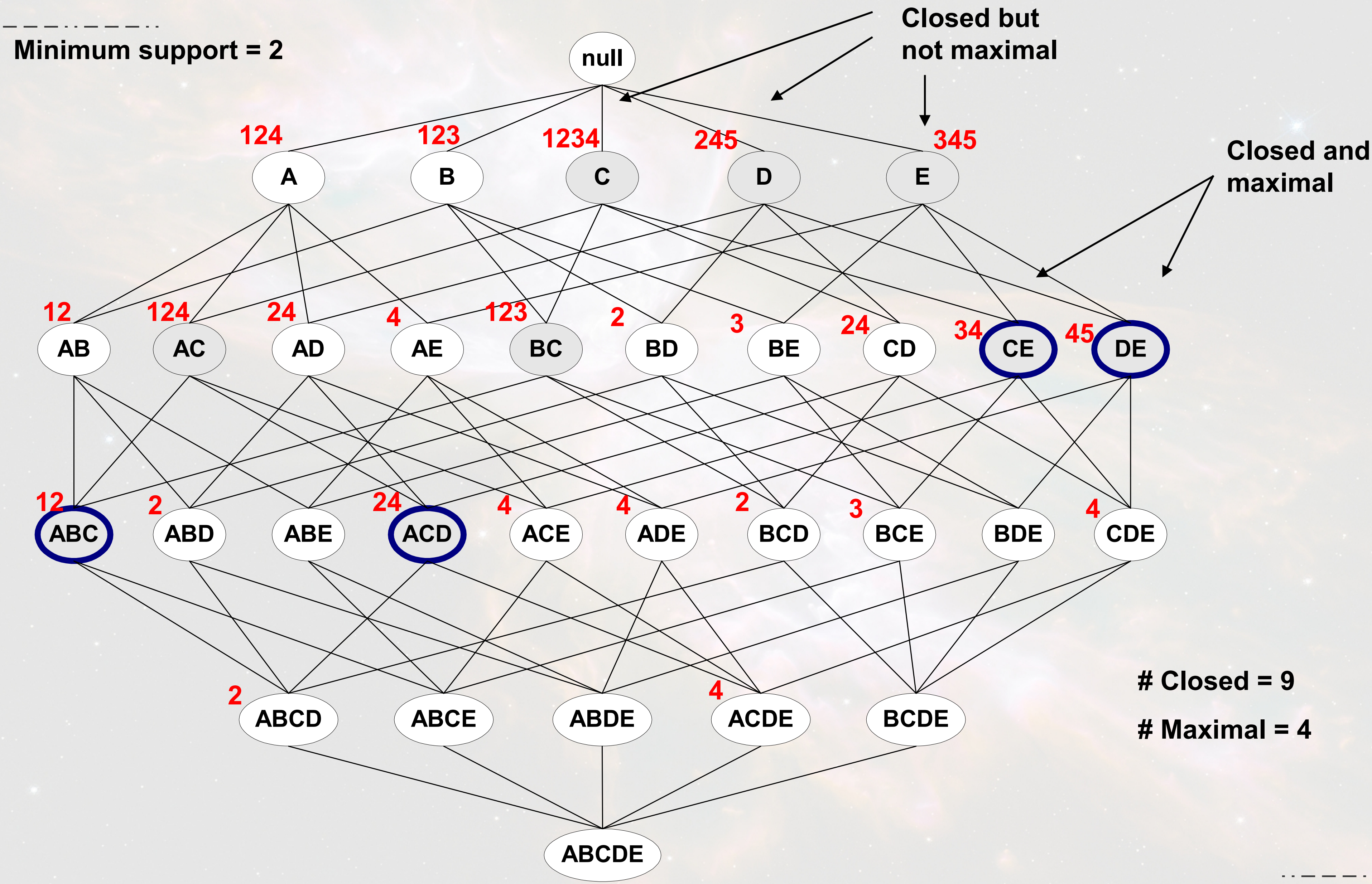
Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Maximal vs Closed Itemsets

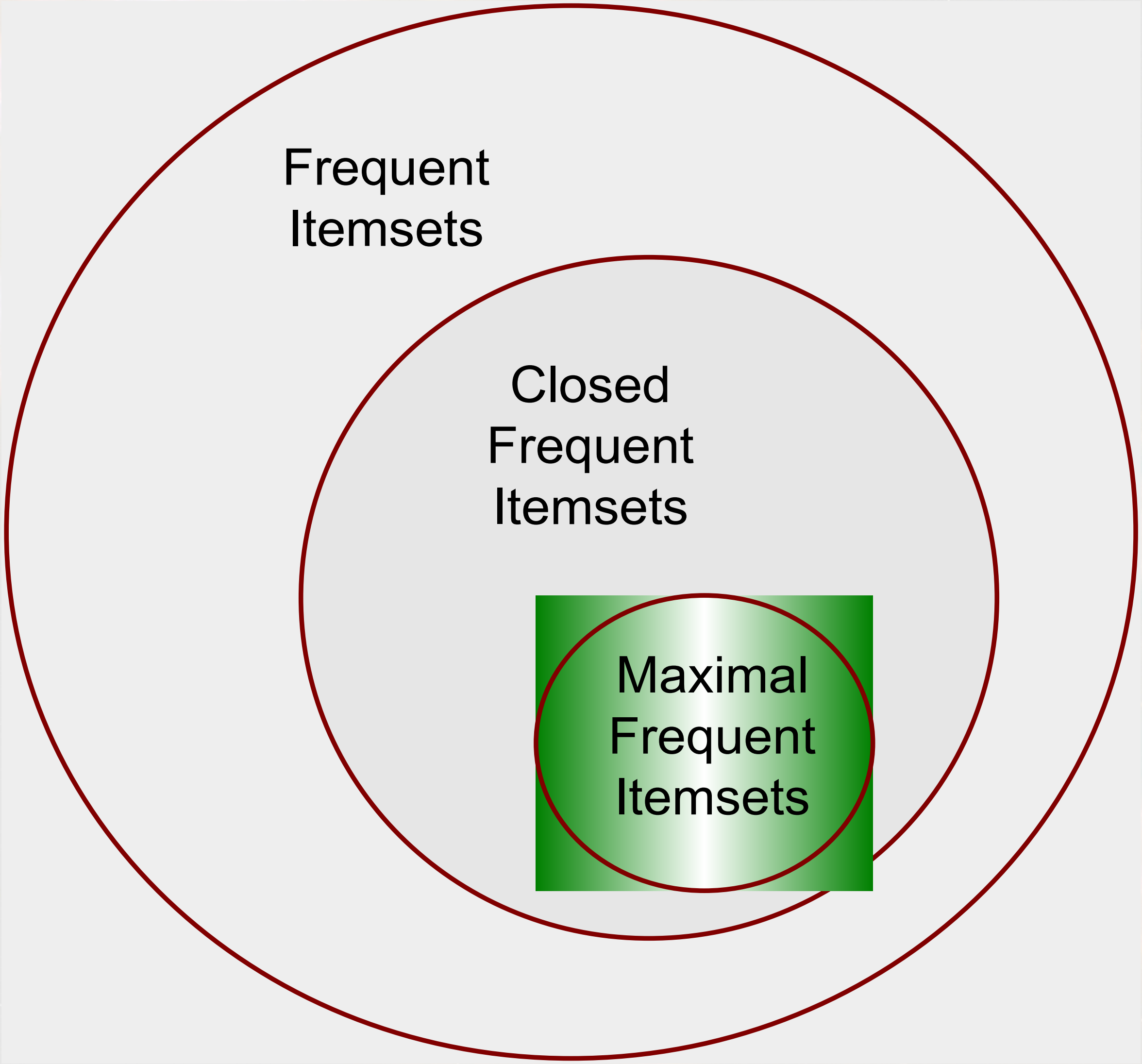
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets



Maximal vs Closed Itemsets



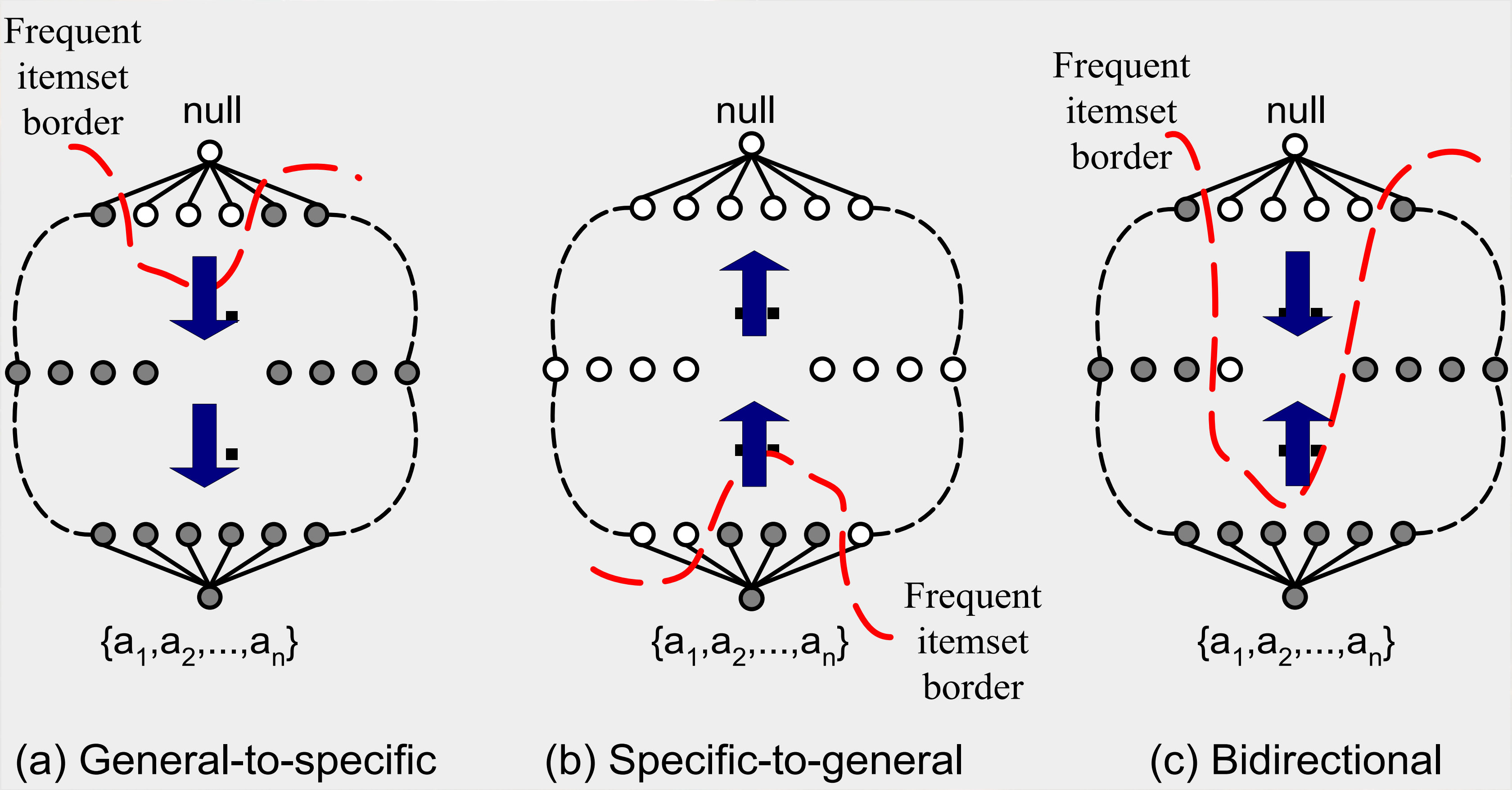
Redundant association rules

- An association rule X ==> Y is redundant if:
  - Exists another association rule X' ==> Y' with, at least, the same support and confidence
  - $X \subseteq X'$ and $Y' \subseteq Y$
- Example:
  - {a} ==> {c, f} is redundant if **{a} ==> {c, e, f}** has the same support and confidence
  - {a, b} ==> {e, f} is redundant if **{a} ==> {e, f}** has the same support and confidence
- Using only closed itemsets redundant rules are not considered

Non redundant rules

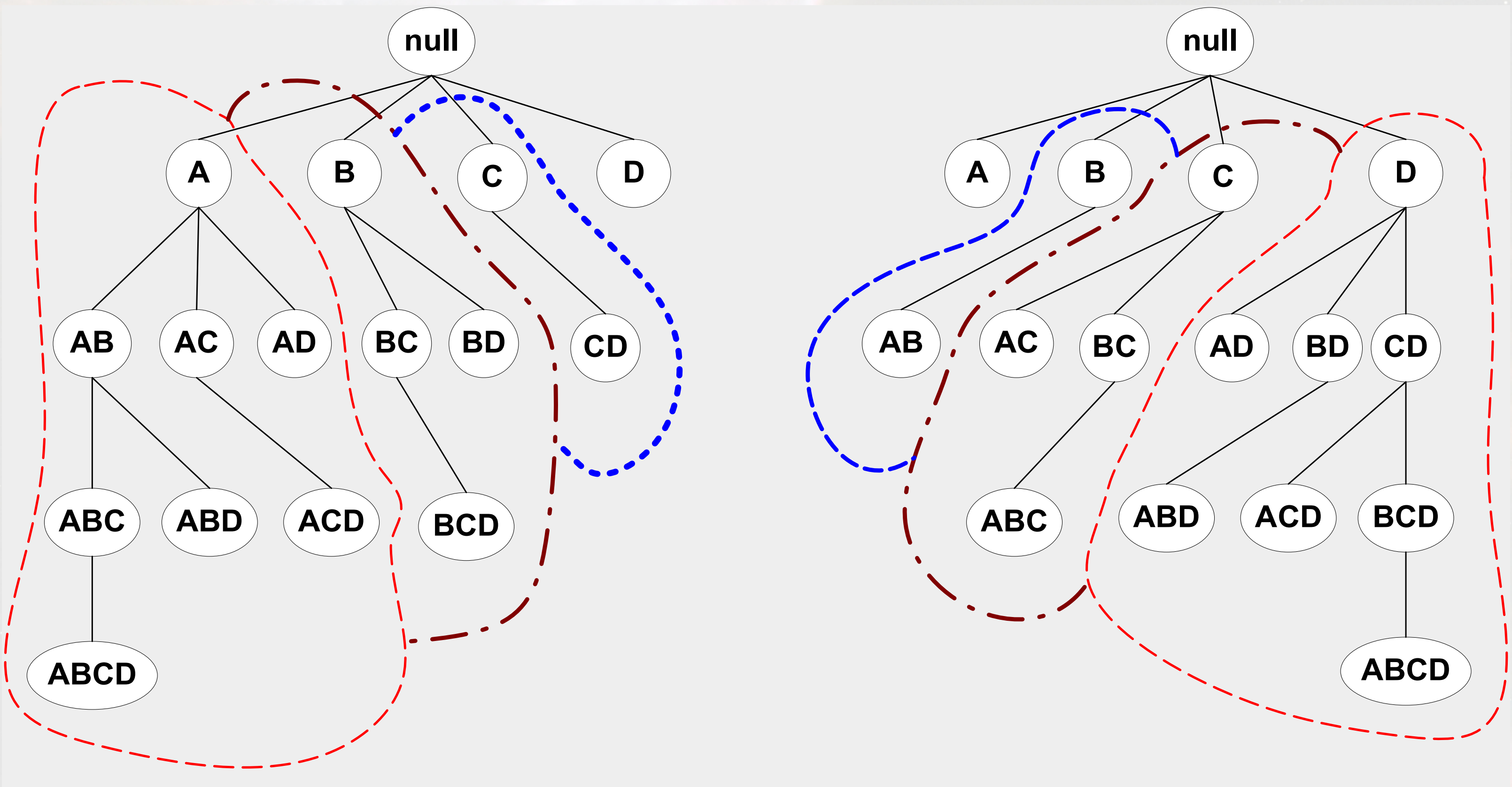
Alternative Methods for Frequent Itemset Generation

- TRAVERSAL OF ITEMSET LATTICE
  - General-to-specific vs Specific-to-general



Alternative Methods for Frequent Itemset Generation

- TRAVERSAL OF ITEMSET LATTICE
  - Equivalent Classes

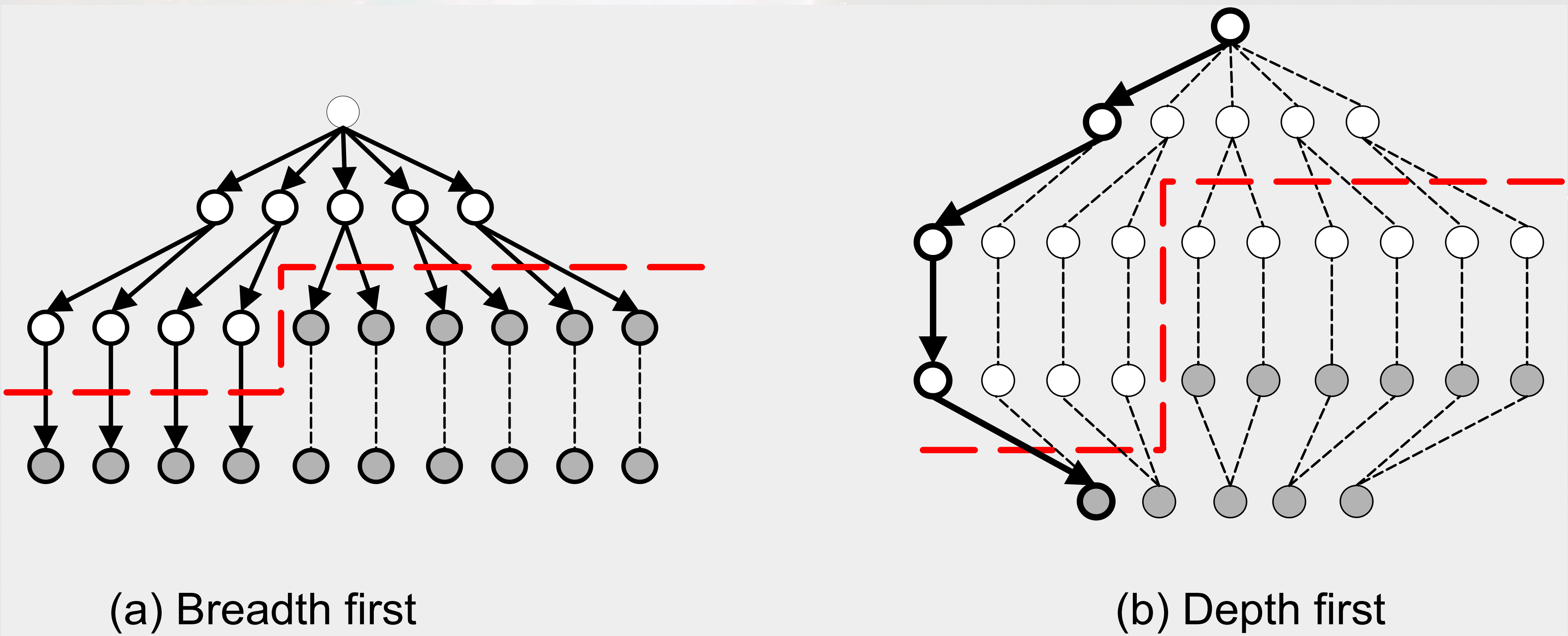


(a) Prefix tree

(b) Suffix tree

Alternative Methods for Frequent Itemset Generation

- Traversal of Itemset Lattice
  - Breadth-first vs Depth-first



Alternative Methods for Frequent Itemset Generation

- Representation of Database
  - horizontal vs vertical data layout

Horizontal
Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

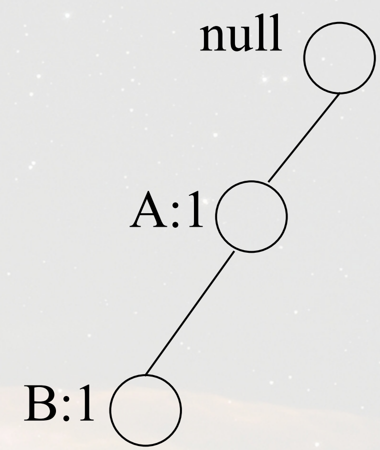
FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree
- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

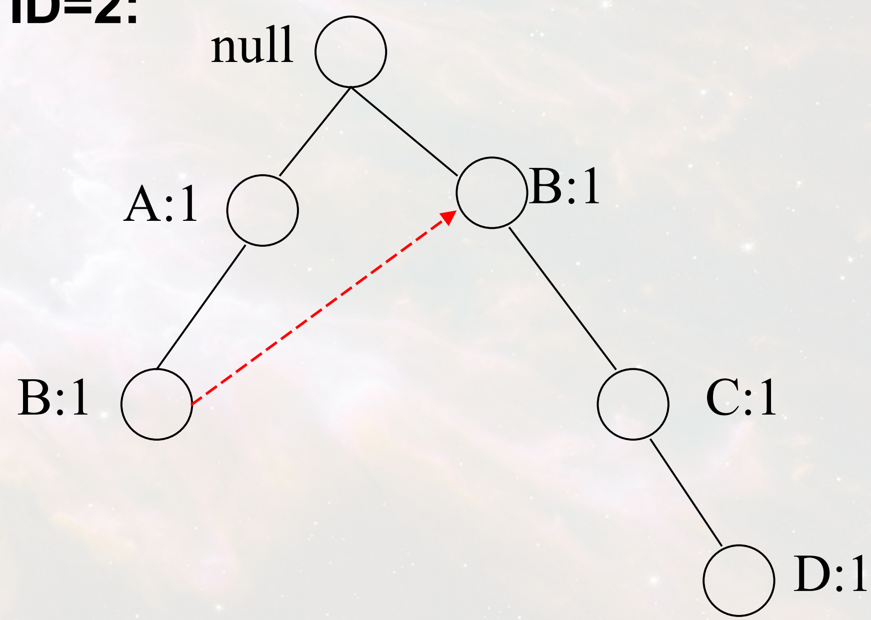
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:



After reading TID=2:



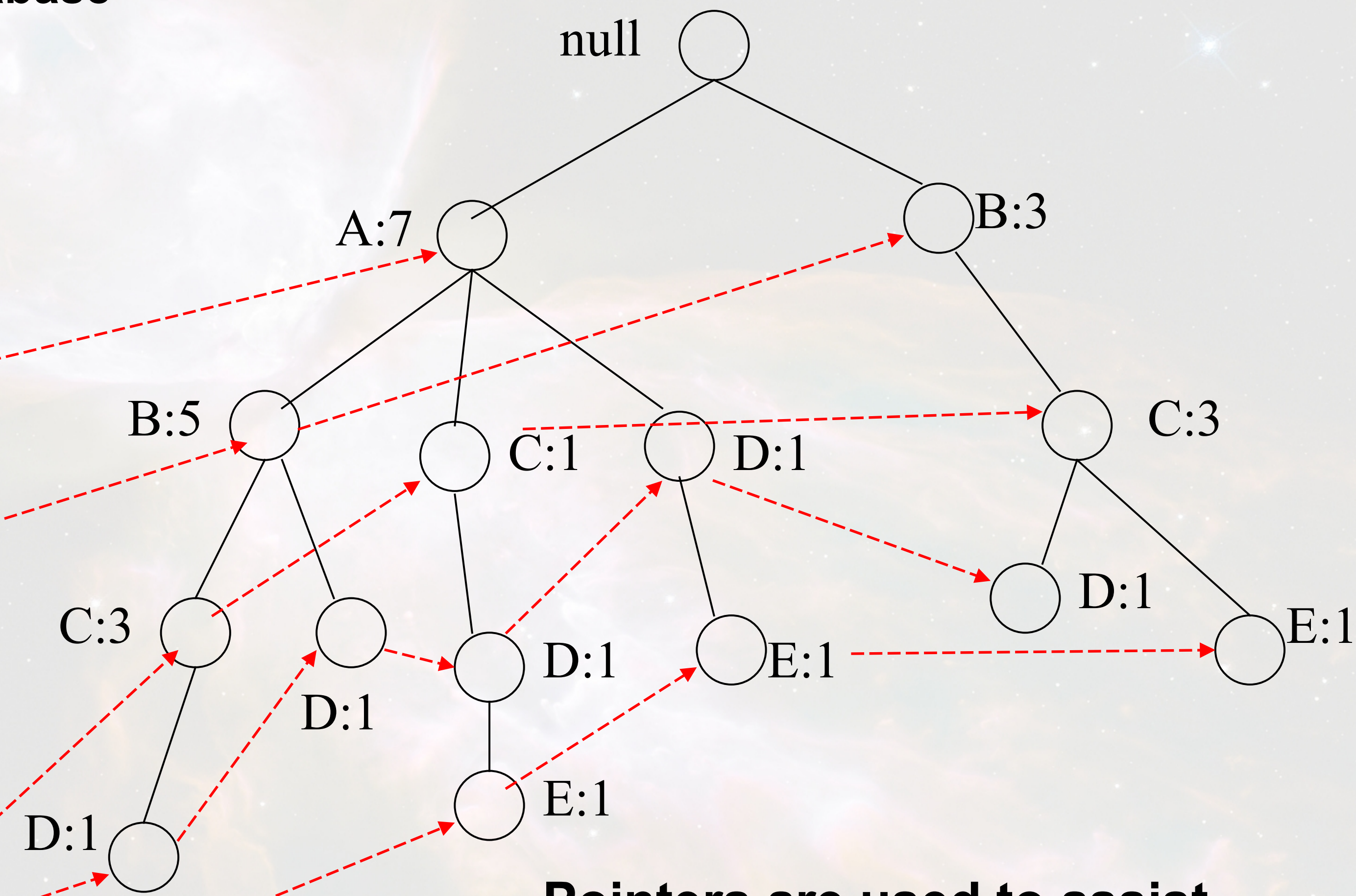
FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database

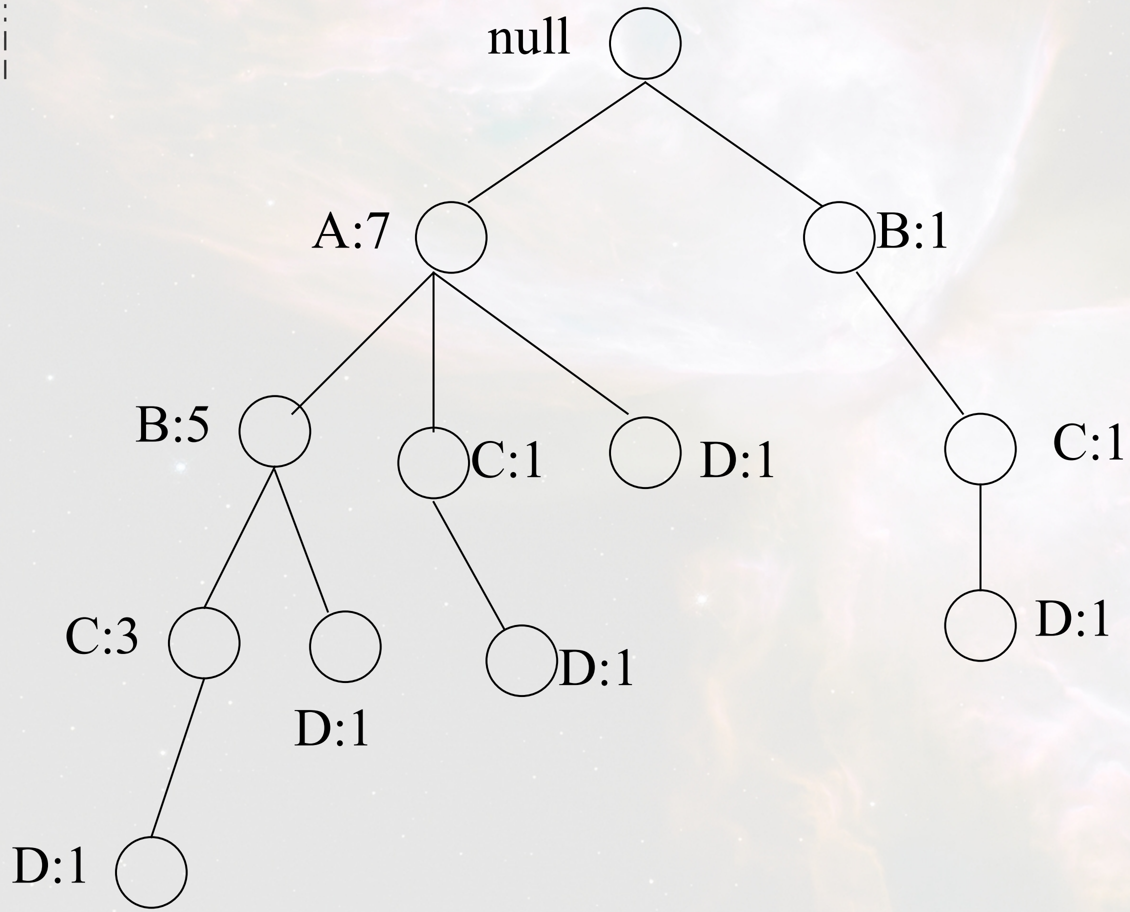
Header table

Item	Pointer
A	
B	
C	
D	
E	



Pointers are used to assist frequent itemset generation

FP-growth



Conditional Pattern base for D:

P = {(A:1,B:1,C:1),
(A:1,B:1),
(A:1,C:1),
(A:1),
(B:1,C:1)}

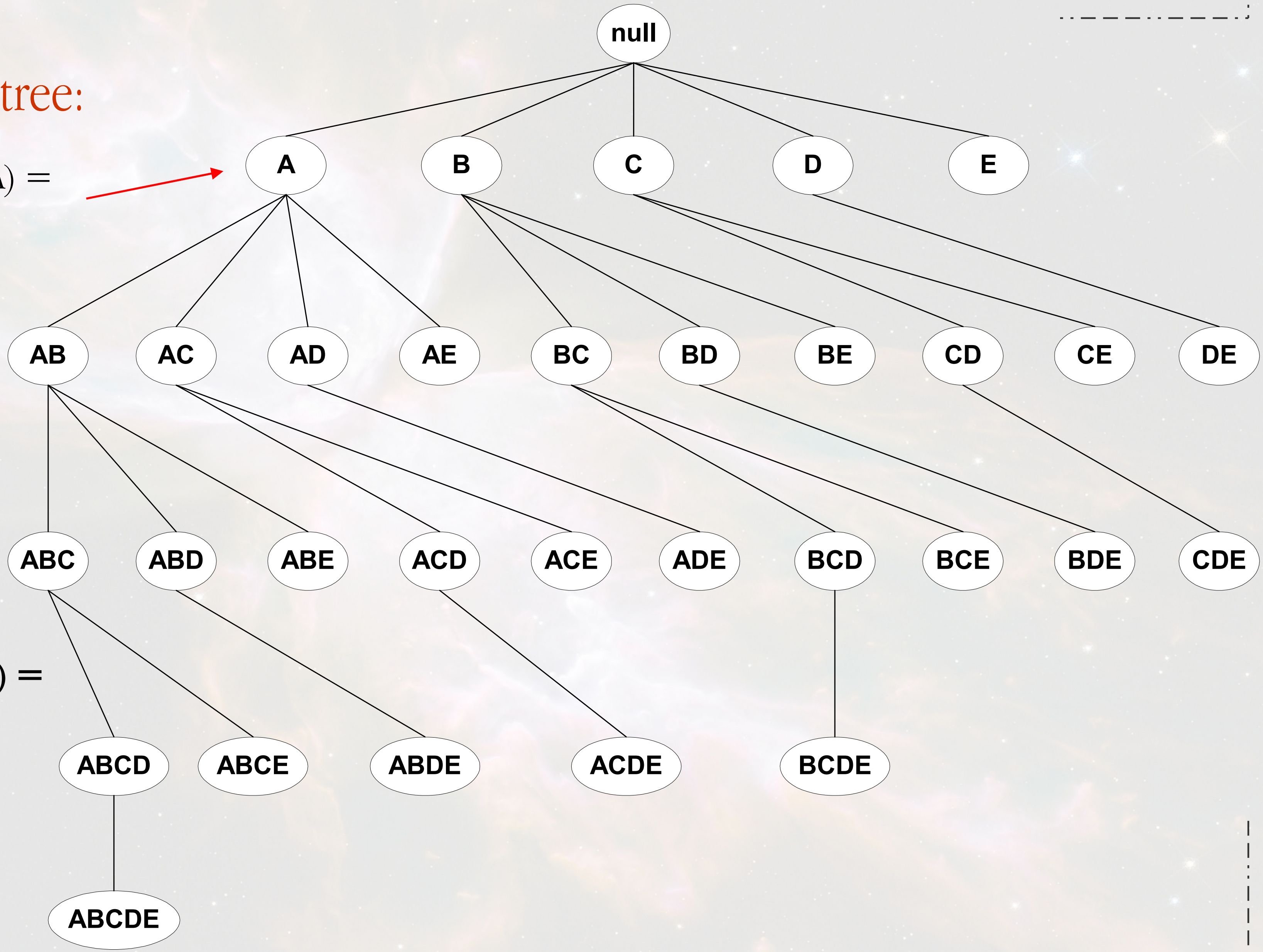
Recursively apply FP-growth on P

Frequent Itemsets found (with sup > 1):
AD, BD, CD, ACD, BCD

Tree Projection

Set enumeration tree:

Possible Extension: E(A) = {B,C,D,E}



Possible Extension: E(ABC) = {D,E}

Tree Projection

- Items are listed in lexicographic order
- Each node P stores the following information:
  - Itemset for node P
  - List of possible lexicographic extensions of P: E(P)
  - Pointer to projected database of its ancestor node
  - Bitvector containing information about which transactions in the projected database contain the itemset

Projected Database

Original Database:

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Projected Database for node A:

TID	Items
1	{B}
2	{}
3	{C,D,E}
4	{D,E}
5	{B,C}
6	{B,C,D}
7	{}
8	{B,C}
9	{B,D}
10	{}

For each transaction T, projected transaction at node A is $T \cap E(A)$

ECLAT

- For each item, store a list of transaction ids (tids)

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

TID-list

ECLAT

- Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.

A
1
4
5
6
7
8
9

∧

B
1
2
5
7
8
10

→

AB
1
5
7
8

- 3 traversal approaches:
  - top-down, bottom-up and hybrid
- Advantage: very fast support counting
- Disadvantage: intermediate tid-lists may become too large for memory

Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    ABC →D, ABD →C, ACD →B, BCD →A,
    A →BCD, B →ACD, C →ABD, D →ABC
    AB →CD, AC → BD, AD → BC, BC →AD,
    BD →AC, CD →AB,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    c(ABC →D) can be larger or smaller than c(AB →D)

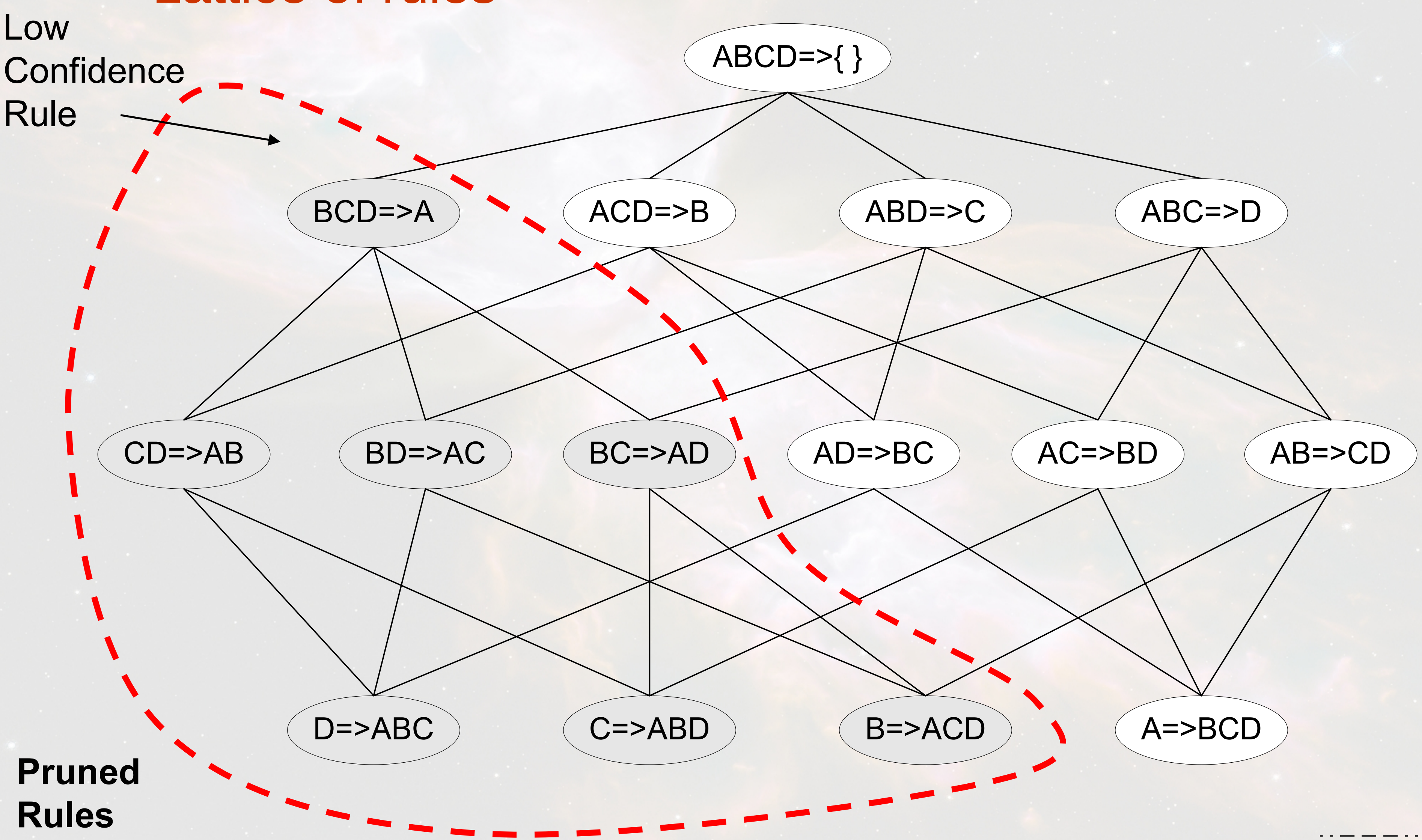
  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

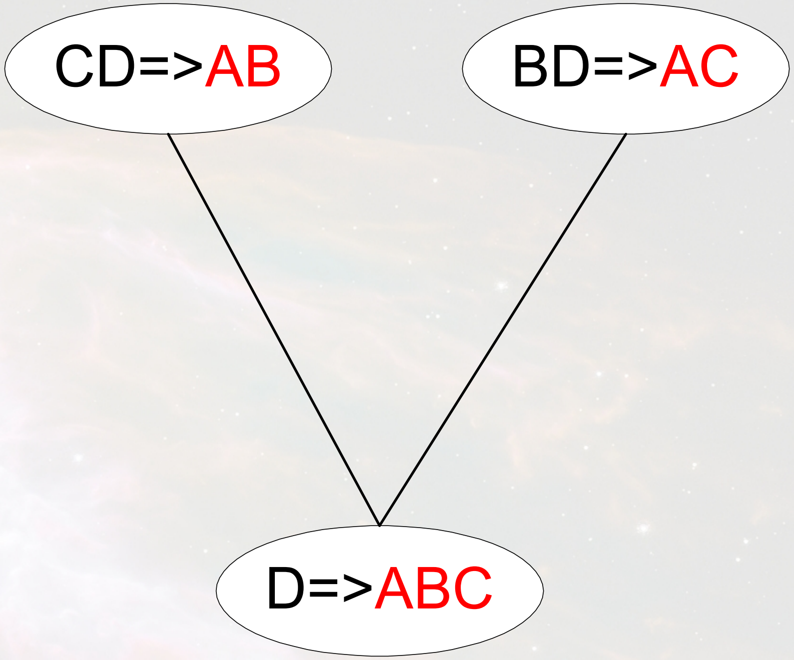
Rule Generation for Apriori Algorithm

Lattice of rules



Rule Generation for Apriori Algorithm

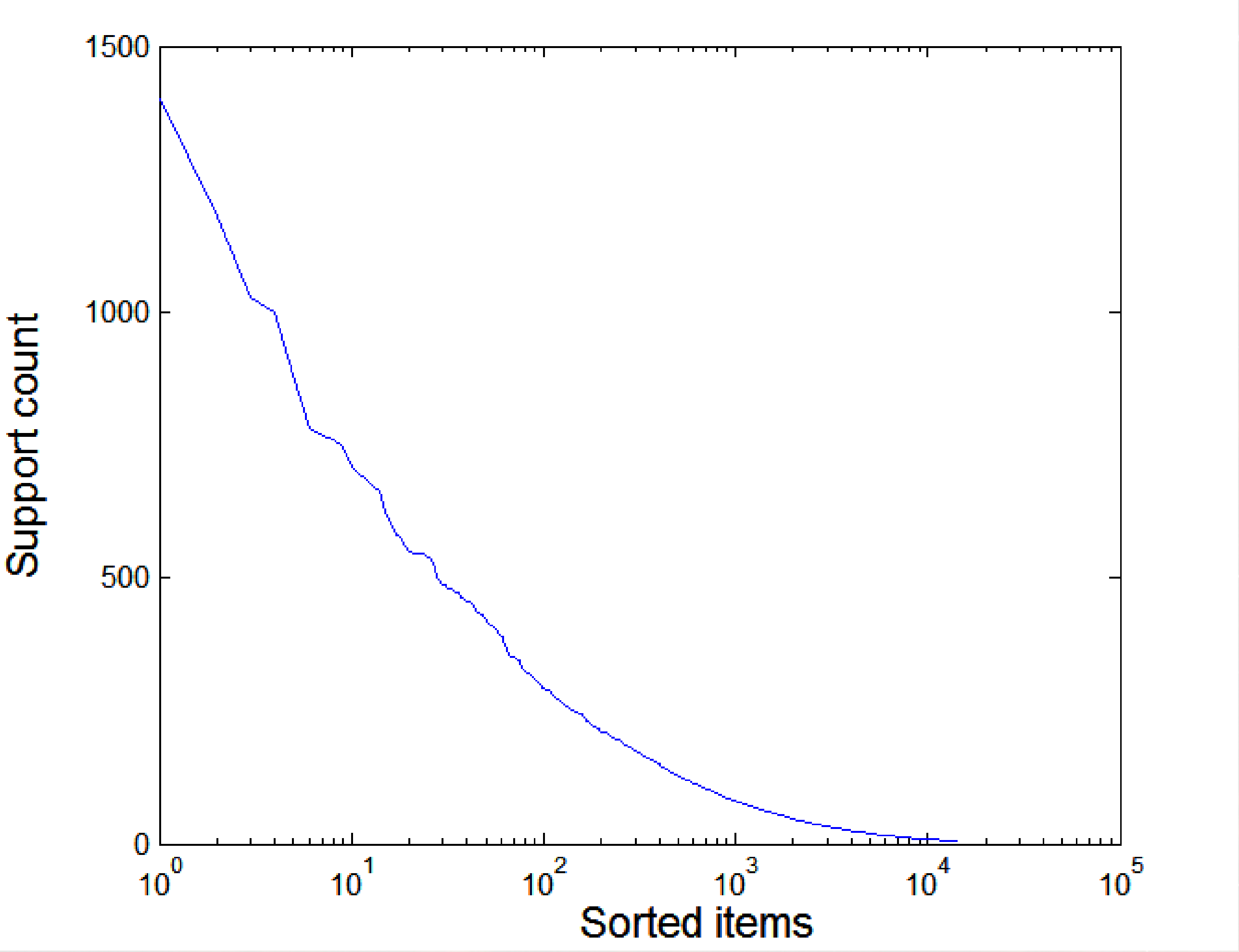
- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent
- join(CD=>AB,BD=>AC) would produce the candidate rule D => ABC
- Prune rule D=>ABC if its subset AD=>BC does not have high confidence



Effect of Support Distribution

- Many real data sets have skewed support distribution

Support distribution of a retail data set



Effect of Support Distribution

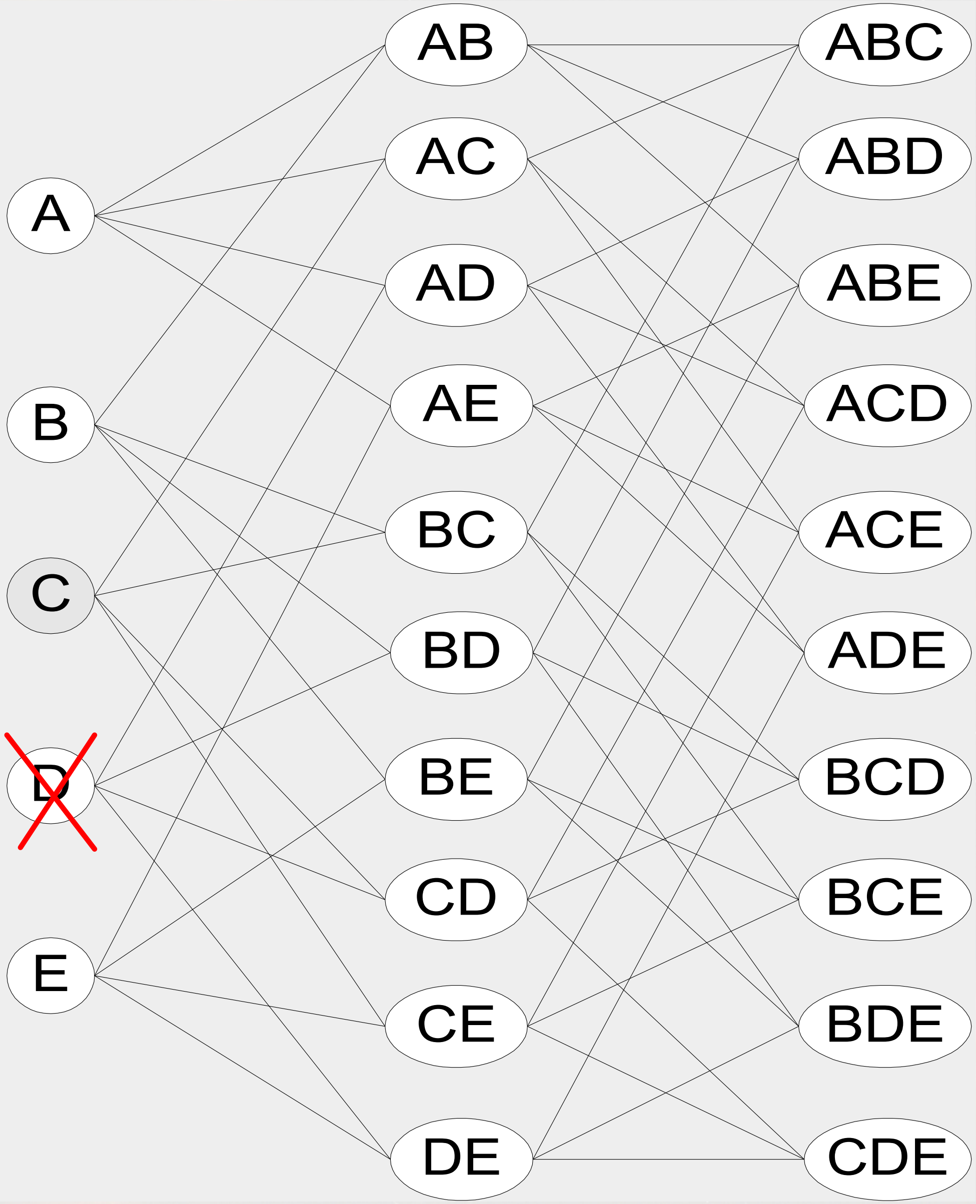
- How to set the appropriate *minsup* threshold?
  - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Multiple Minimum Support

- How to apply multiple minimum supports?
  - MS(i): minimum support for item i
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%,
    MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - MS({Milk, Broccoli}) = min (MS(Milk), MS(Broccoli))
    = 0.1%
  - Challenge: **Support is no longer anti-monotone**
    - Suppose: Support(Milk, Coke) = 1.5% and
      Support(Milk, Coke, Broccoli) = 0.5%
    - {Milk,Coke} is infrequent but {Milk,Coke,Broccoli} is frequent

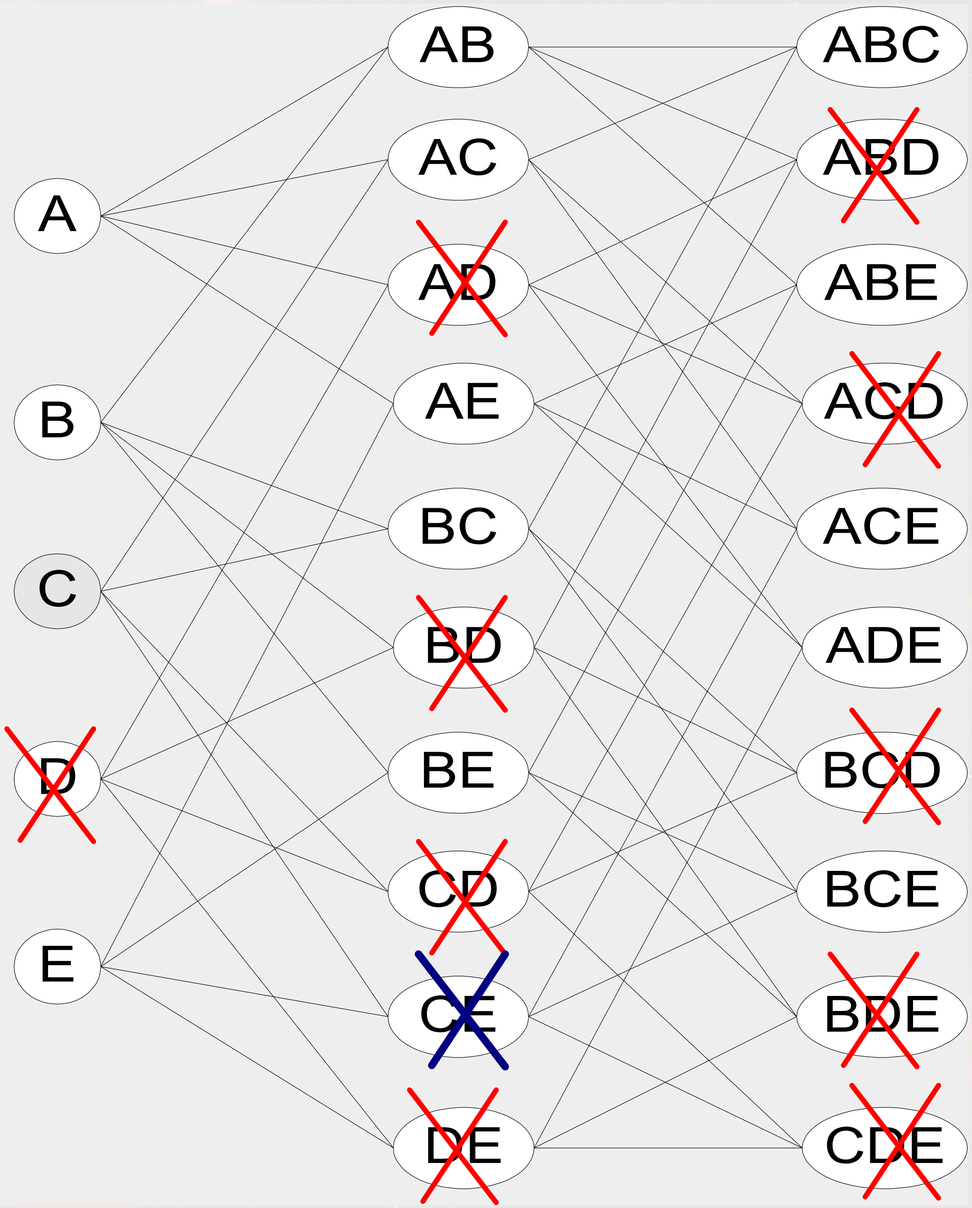
Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support (Liu 1999)

- ORDER THE ITEMS ACCORDING TO THEIR MINIMUM SUPPORT (IN ASCENDING ORDER)
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%, MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - Ordering: Broccoli, Salmon, Coke, Milk
- NEED TO MODIFY APRIORI SUCH THAT:
  - L_1 : set of frequent items
  - F_1 : set of items whose support is $\geq$ MS(1) where MS(1) is $\min_i$(MS(i))
  - C_2 : candidate itemsets of size 2 is generated from F_1 instead of L_1

Multiple Minimum Support (Liu 1999)

- Modifications to Apriori:
  - In traditional Apriori,
    - A candidate (k+1)-itemset is generated by merging two frequent itemsets of size k
    - The candidate is pruned if it contains any infrequent subsets of size k
  - Pruning step has to be modified:
    - Prune only if subset contains the first item
    - e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)
    - {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent
      - Candidate is not pruned because {Coke,Milk} does not contain the first item, i.e., Broccoli.

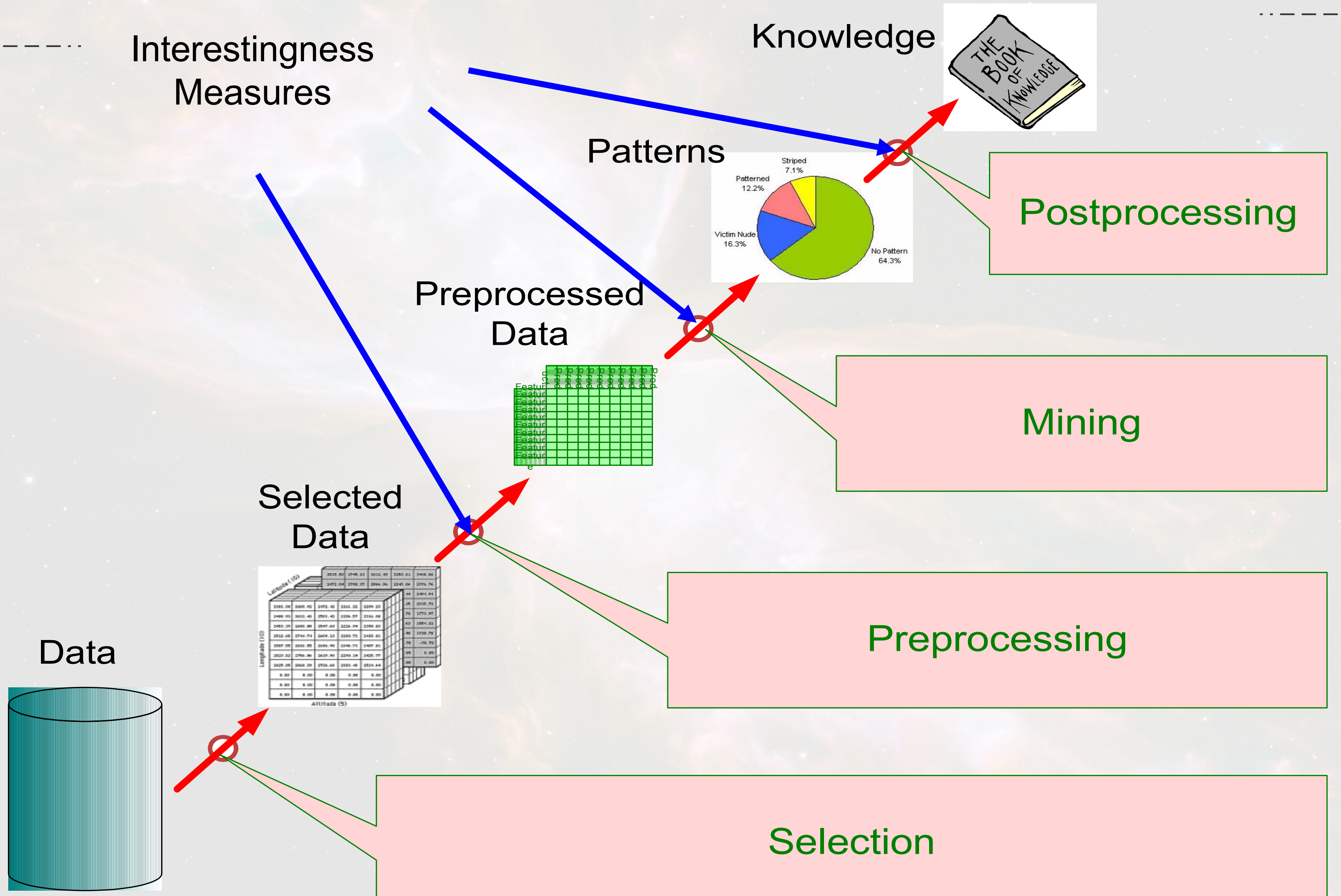
Pattern Evaluation

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence

- Interestingness measures can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

Application of Interestingness Measure



Computing Interestingness Measure

➢ Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y

f_{10}: support of X and $\overline{Y}$

f_{01}: support of $\overline{X}$ and Y

f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

- HIDDEN VARIABLES
  - Spurious rules due to unconsidered variables

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

$\Rightarrow$ Although confidence is high, rule is misleading

$\Rightarrow P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

Problem with confidence

- Confidence of X -> Y:

$$c = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

- The support of the consequent $\sigma(Y)$ is not considered in the formula
- What happens if: $\sigma(Y)$ *is high ?*

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

  - $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
  - $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
  - $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y|X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\varphi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Example: Lift/Interest

- For binary variables lift & interest are equivalent
- Example:
  - Association Rule: Tea →Coffee
  - Confidence= P(Coffee|Tea) = 0.75
  - but P(Coffee) = 0.9
  - Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

$$I(A,B)\begin{cases} = 1, & \text{if } A \text{ and } B \text{ are independent;} \\ > 1, & \text{if } A \text{ and } B \text{ are positively correlated;} \\ < 1, & \text{if } A \text{ and } B \text{ are negatively correlated.} \end{cases}$$

Drawback of Lift & Interest

	Y	$\overline{Y}$	
X	10	0	10
$\overline{X}$	0	90	90
	10	90	100

	Y	$\overline{Y}$	
X	90	0	90
$\overline{X}$	0	10	10
	90	10	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\mid A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\mid A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\mid B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\mid B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\mid A)^2+P(\overline{B}\mid A)^2]+P(\overline{A})[P(B\mid \overline{A})^2+P(\overline{B}\mid \overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\mid B)^2+P(\overline{A}\mid B)^2]+P(\overline{B})[P(A\mid \overline{B})^2+P(\overline{A}\mid \overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\mid A), P(A\mid B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\mid A)-P(B)}{1-P(B)}, \frac{P(A\mid B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$

Properties of A Good Measure

- Piatetsky-Shapiro:

3 properties a good measure M must satisfy:

  - M(A,B) = 0 if A and B are statistically independent
  - M(A,B) increase monotonically with P(A,B) when P(A) and P(B) remain unchanged
  - M(A,B) decreases monotonically with P(A) [or P(B)] when P(A,B) and P(B) [or P(A)] remain unchanged

Comparing Different Measures

10 examples of contingency tables:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	10	7

Property under Variable Permutation

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

	A	$\overline{A}$
B	p	r
$\overline{B}$	q	s

Does M(A,B) = M(B,A)?

Symmetric measures:

- support, lift, collective strength, cosine, Jaccard, etc

Asymmetric measures:

- confidence, conviction, Laplace, J-measure, etc

Property under Row/Column Scaling

Grade-Gender Example (Mosteller, 1968):

	Male	Female	
High	2	3	5
Low	1	4	5
	3	7	10

	Male	Female	
High	4	30	34
Low	2	40	42
	6	70	76

2x 10x

Mosteller:

Underlying association should be independent of the relative number of male and female students in the samples

Property under Inversion Operation

	A	B
Transaction 1	1	0
■	0	0
	0	0
■	0	0
■	0	1
	0	0
■	0	0
■	0	0
	0	0
Transaction N	1	0

(a)

C	D
0	1
1	1
1	1
1	1
1	0
1	1
1	1
1	1
1	1
0	1

(b)

E	F
0	0
1	0
1	0
1	0
1	1
1	0
1	0
1	0
1	0
0	0

(c)

Example: ϕ-Coefficient

- ϕ-COEFFICIENT IS ANALOGOUS TO CORRELATION COEFFICIENT FOR CONTINUOUS VARIABLES

	Y	$\overline{Y}$	
X	60	10	70
$\overline{X}$	10	20	30
	70	30	100

	Y	$\overline{Y}$	
X	20	10	30
$\overline{X}$	10	60	70
	30	70	100

$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

ϕ Coefficient is the same for both tables

Property under Null Addition

	B	$\overline{\mathbf{B}}$
A	p	q
$\overline{\mathbf{A}}$	r	s

$\Rightarrow$

	B	$\overline{\mathbf{B}}$
A	p	q
$\overline{\mathbf{A}}$	r	s + k

Invariant measures:

- support, cosine, Jaccard, etc

Non-invariant measures:

- correlation, Gini, mutual information, odds ratio, etc

Different Measures have Different Properties

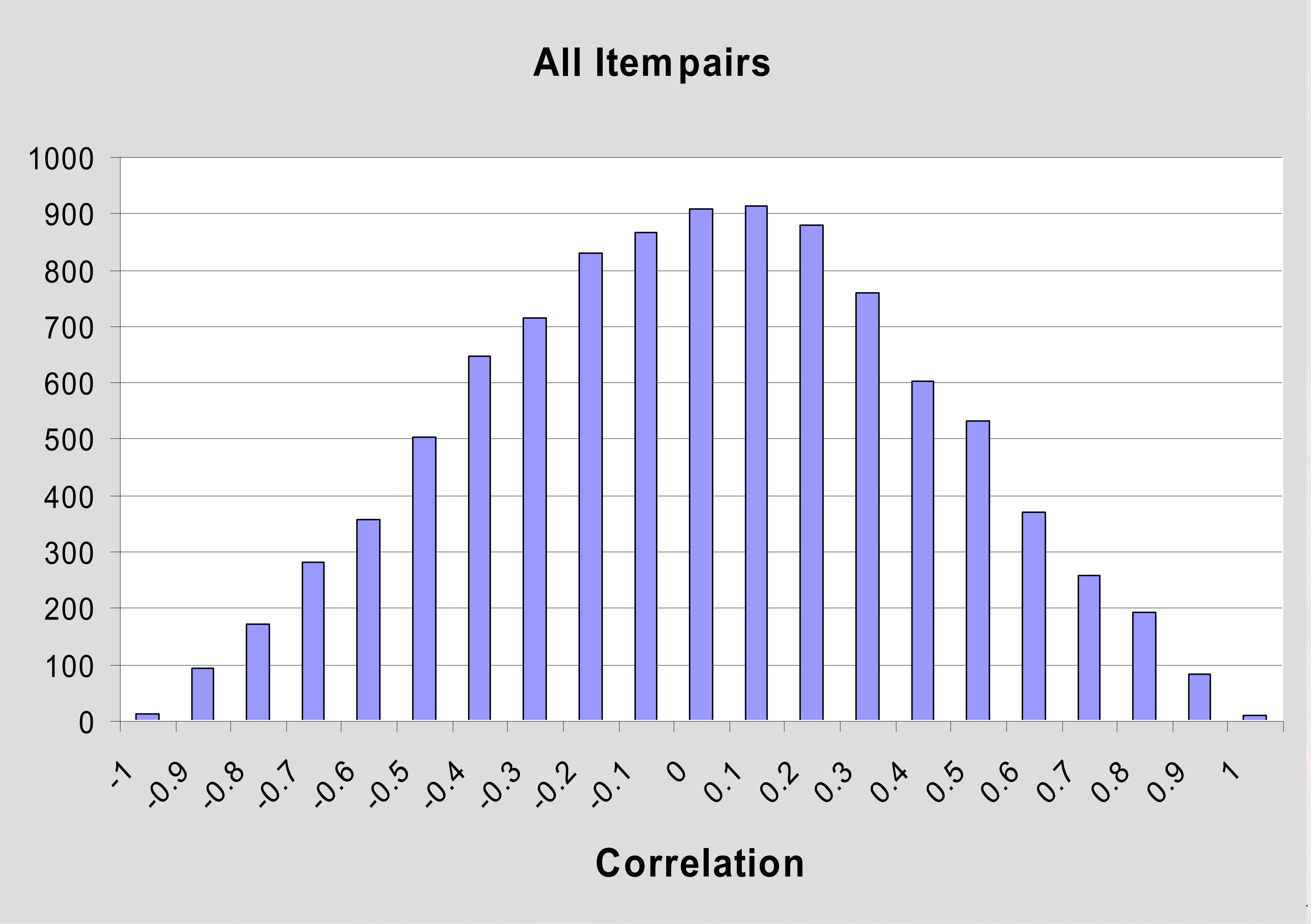
Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
Φ	Correlation	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Lambda	0 … 1	Yes	No	No	Yes	No	No*	Yes	No
α	Odds ratio	0 … 1 … ∞	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	0 … 1	Yes	Yes	Yes	Yes	No	No*	Yes	No
J	J-Measure	0 … 1	Yes	No	No	No	No	No	No	No
G	Gini Index	0 … 1	Yes	No	No	No	No	No*	Yes	No
s	Support	0 … 1	No	Yes	No	Yes	No	No	No	No
c	Confidence	0 … 1	No	Yes	No	Yes	No	No	No	Yes
L	Laplace	0 … 1	No	Yes	No	Yes	No	No	No	No
V	Conviction	0.5 … 1 … ∞	No	Yes	No	Yes**	No	No	Yes	No
I	Interest	0 … 1 … ∞	Yes*	Yes	Yes	Yes	No	No	No	No
IS	IS (cosine)	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	-0.25 … 0 … 0.25	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	-1 … 0 … 1	Yes	Yes	Yes	No	No	No	Yes	No
AV	Added value	0.5 … 1 … 1	Yes	Yes	Yes	No	No	No	No	No
S	Collective strength	0 … 1 … ∞	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's		Yes	Yes	Yes	No	No	No	No	No

Support-based Pruning

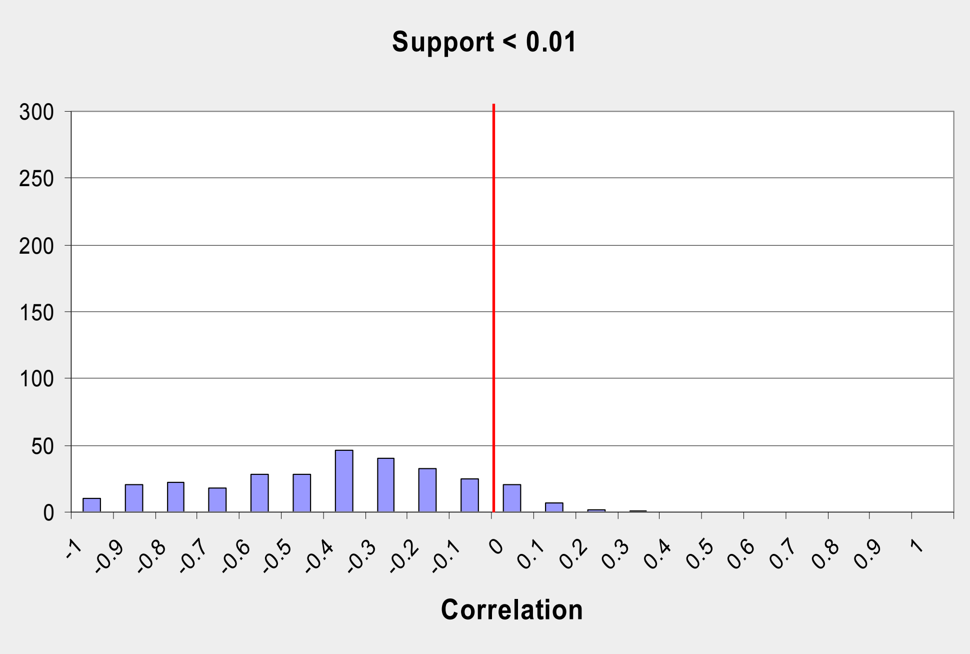
- Most of the association rule mining algorithms use support measure to prune rules and itemsets

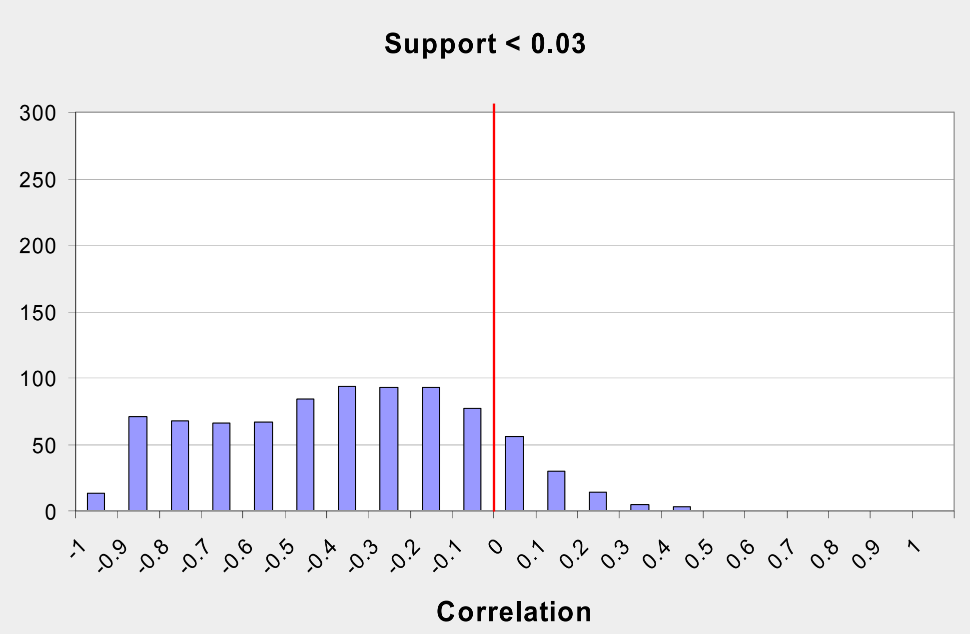
- Study effect of support pruning on correlation of itemsets
  - Generate 10000 random contingency tables
  - Compute support and pairwise correlation for each table
  - Apply support-based pruning and examine the tables that are removed

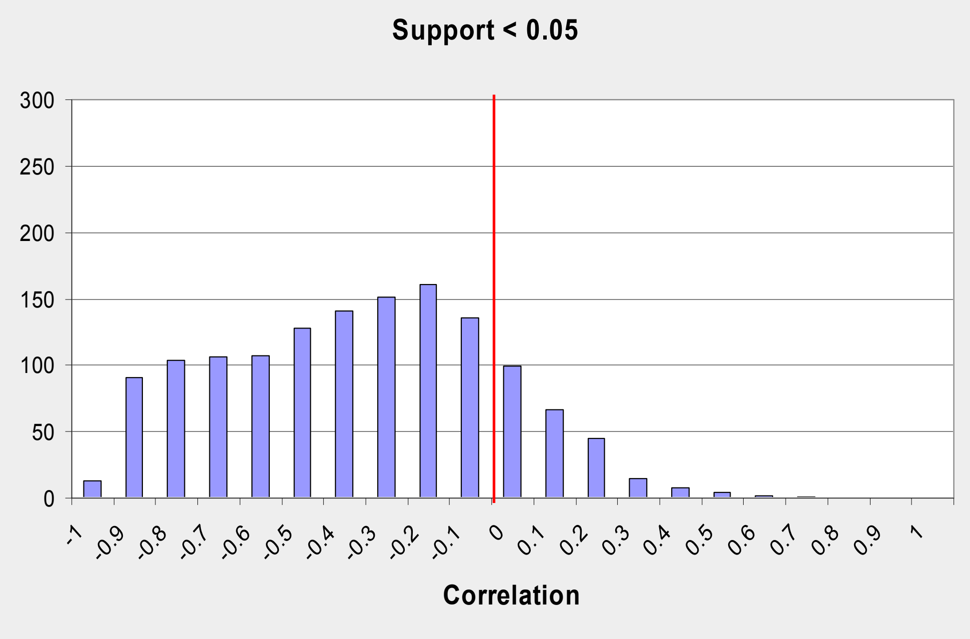
Effect of Support-based Pruning



Effect of Support-based Pruning







Support-based pruning eliminates mostly negatively correlated itemsets

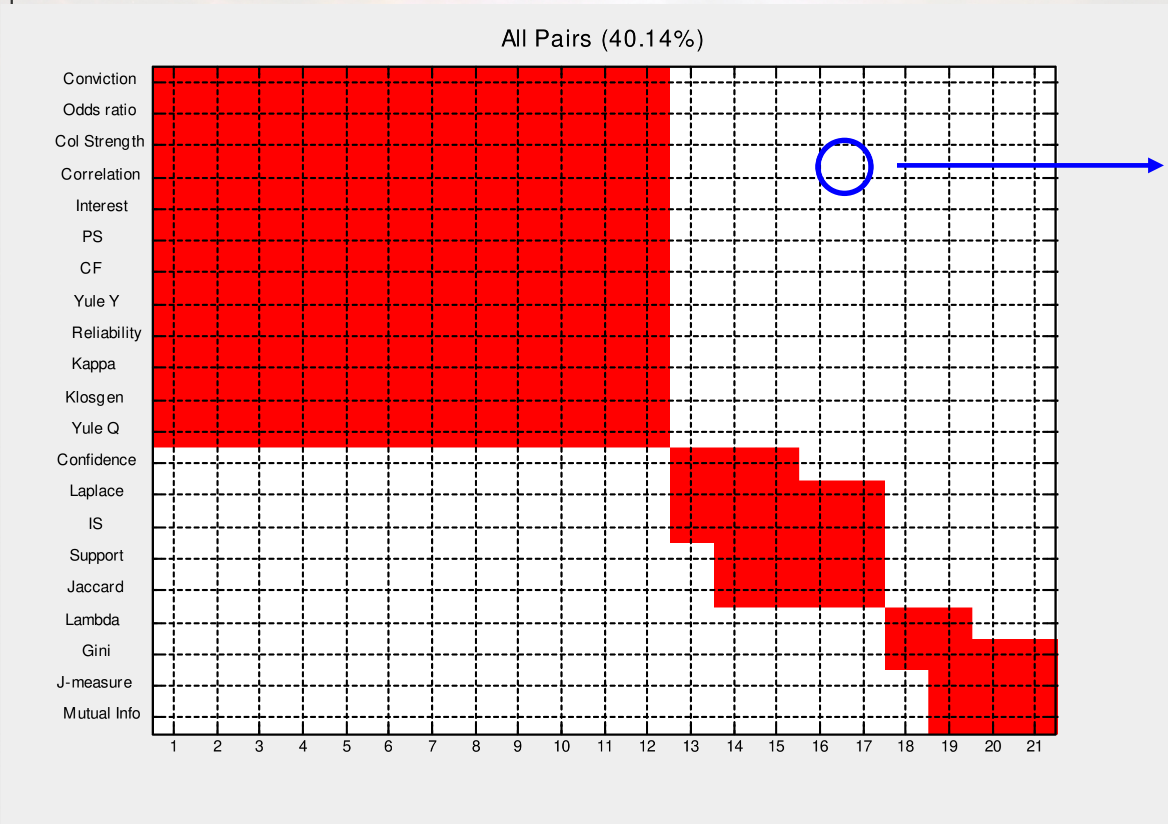
Effect of Support-based Pruning

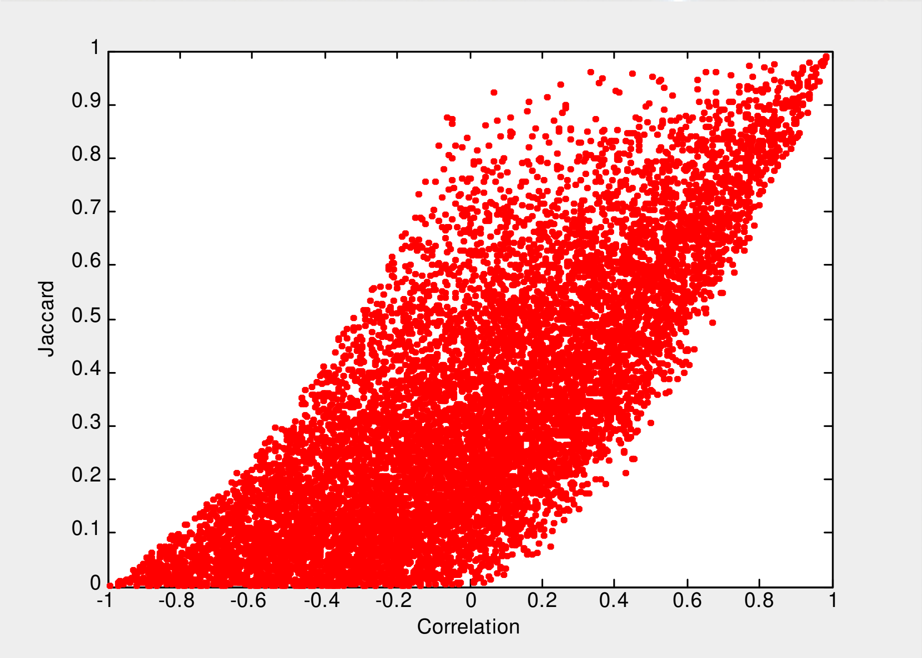
- Investigate how support-based pruning affects other measures

- Steps:
  - Generate 10000 contingency tables
  - Rank each table according to the different measures
  - Compute the pair-wise correlation between the measures

Effect of Support-based Pruning

- Without Support Pruning (All Pairs)



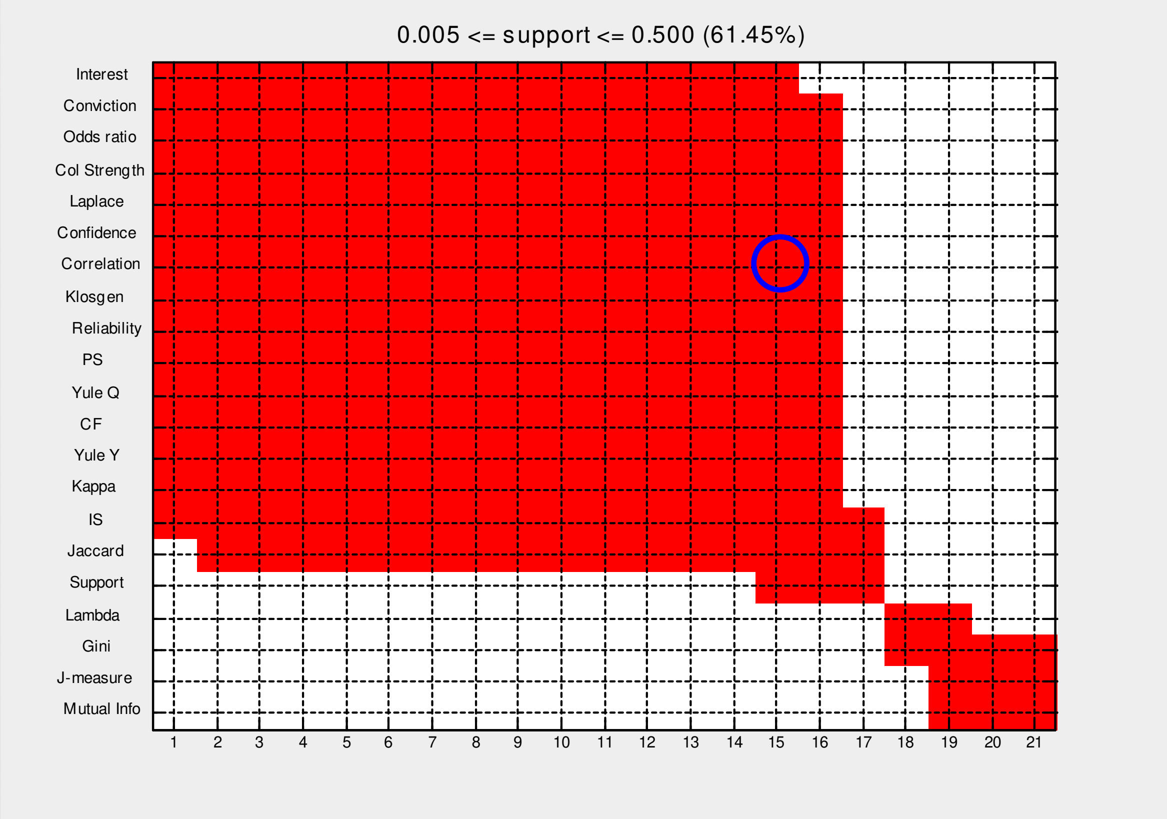


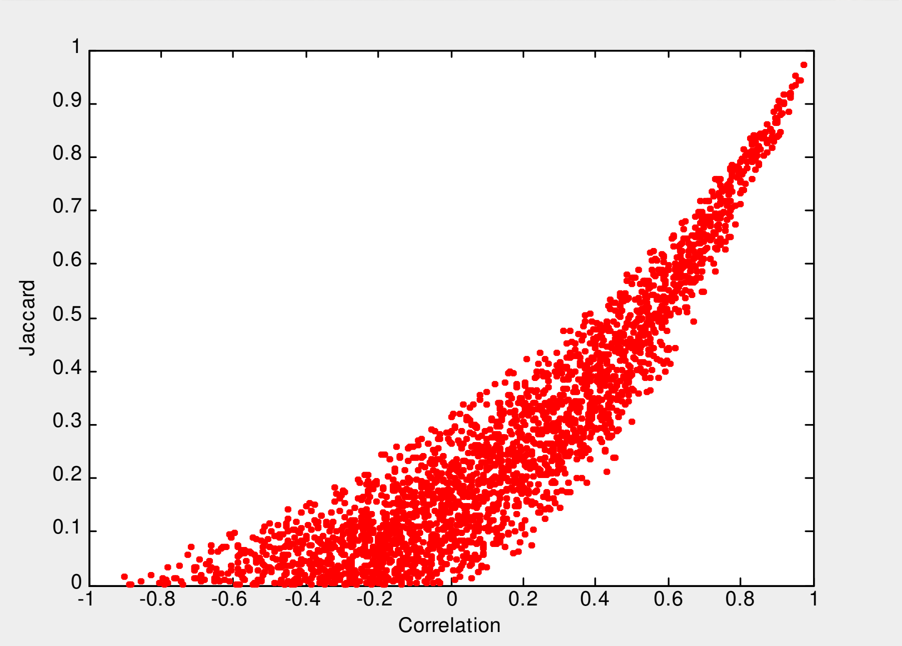
Scatter Plot between Correlation & Jaccard Measure

- Red cells indicate correlation between the pair of measures > 0.85
- 40.14% pairs have correlation > 0.85

Effect of Support-based Pruning

- 0.5% ≤ support ≤ 50%



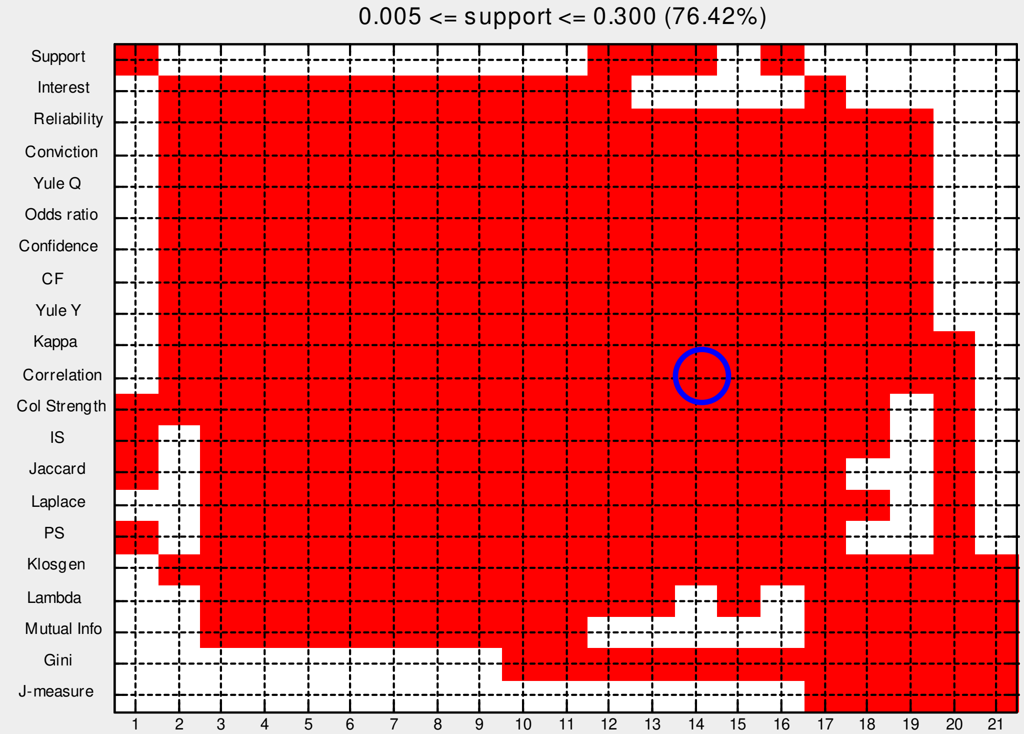


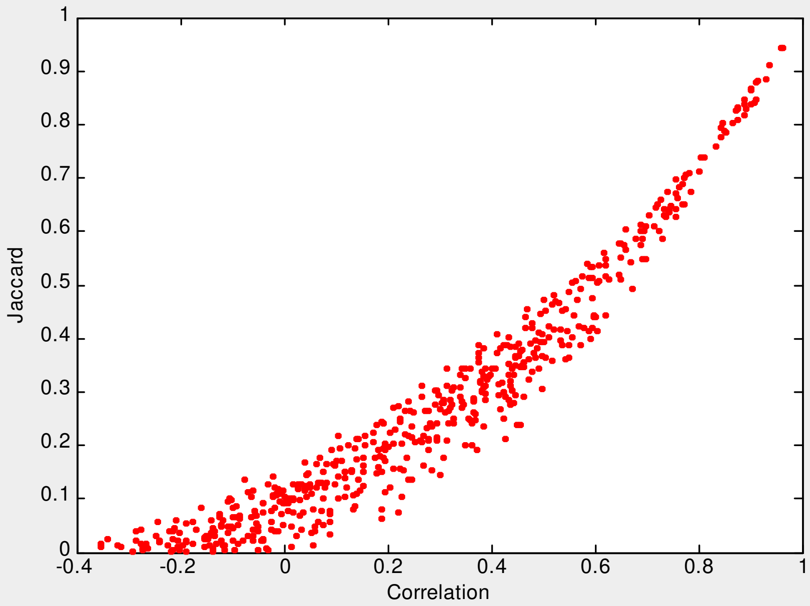
Scatter Plot between Correlation & Jaccard Measure:

- 61.45% pairs have correlation > 0.85

Effect of Support-based Pruning

- $0.5\% \leq \text{support} \leq 30\%$





Scatter Plot between Correlation & Jaccard Measure

- 76.42% pairs have correlation > 0.85

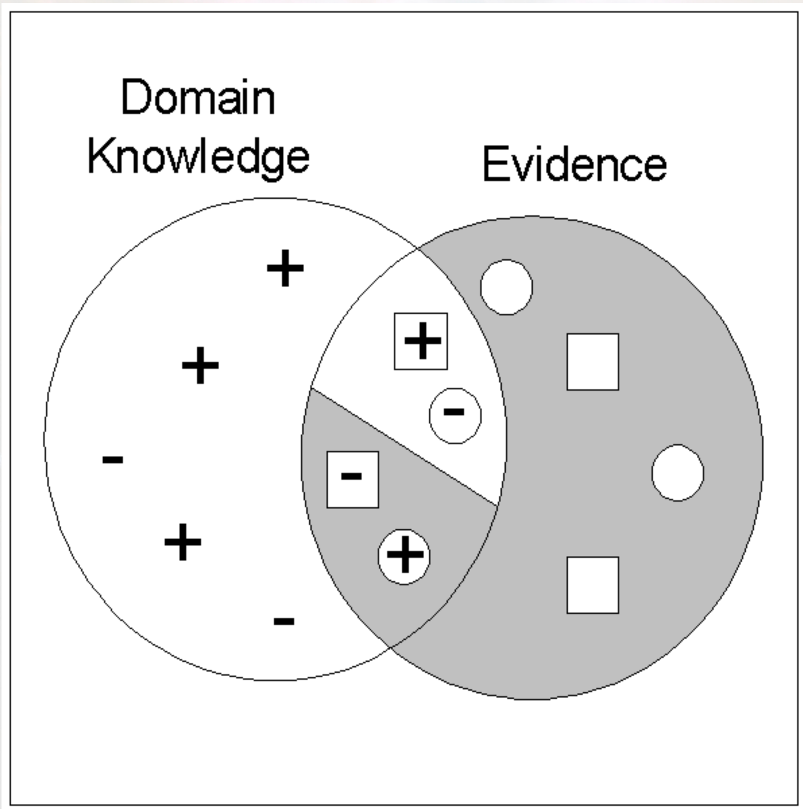
Subjective Interestingness Measure

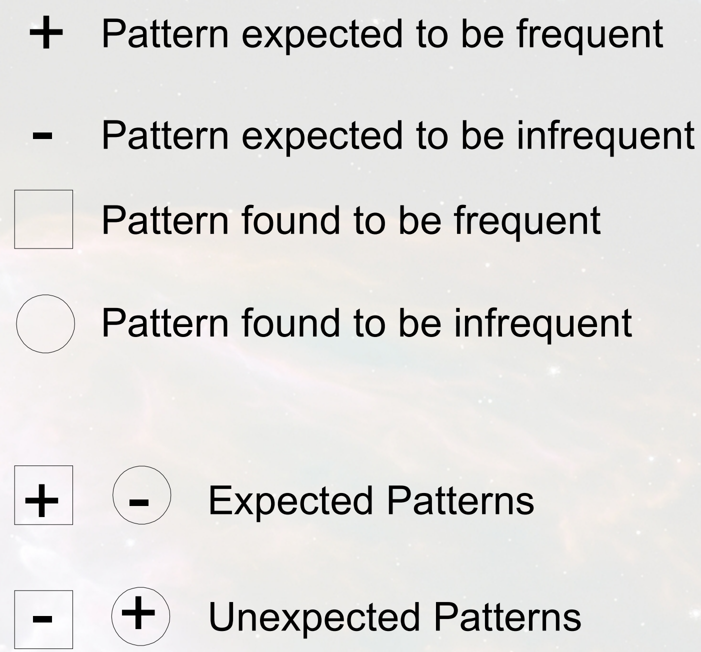
- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

- Need to model expectation of users (domain knowledge)





- Nee… ers with evidence from data (i.e., extr…

Interestingness via Unexpectedness

- Web Data (Cooley et al 2001)
  - Domain knowledge in the form of site structure
  - Given an itemset $F = \{X_1, X_2, \ldots, X_k\}$ (X_i : Web pages)
    - L: number of links connecting the pages
    - $\text{lfactor} = L / (k \times k-1)$
    - cfactor = 1 (if graph is connected), 0 (disconnected graph)
  - Structure evidence = cfactor × lfactor
  - Usage evidence $= \dfrac{P(X_1 \cap X_2 \cap \ldots \cap X_k)}{P(X_1 \cup X_2 \cup \ldots \cup X_k)}$
  - Use Dempster-Shafer theory to combine domain knowledge and evidence from data