THE UNIVERSITY OF TEXAS AT ARLINGTON, TEXAS
DEPARTMENT OF ELECTRICAL ENGINEERING

EE 5321 - 001

OPTIMAL CONTROL

HW # 3
ASSIGNMENT

by

SOUTRIK MAITI

1001569883

Presented to

Prof. Michael Niestroy

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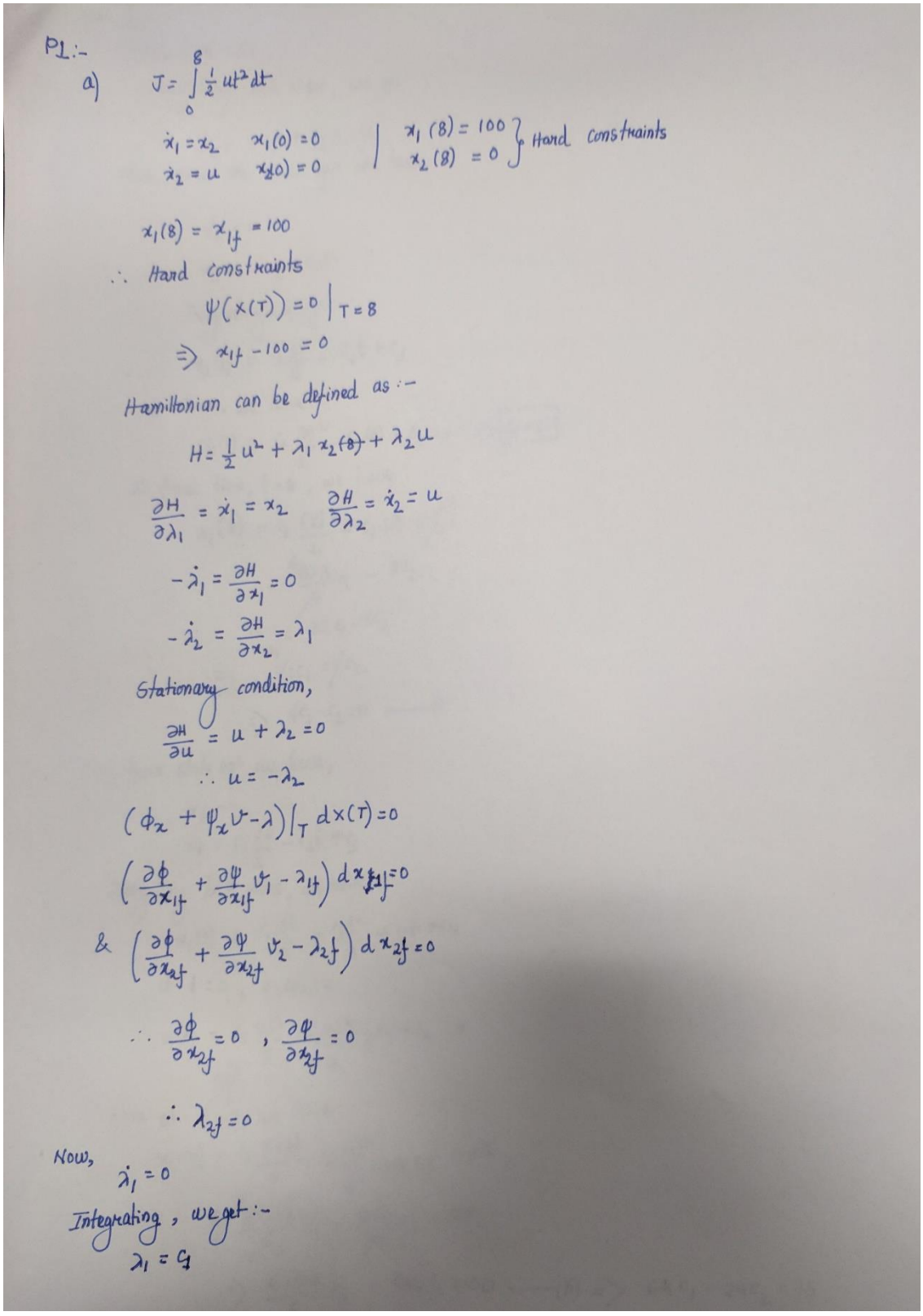
Problem 1:

a)



P1:-

a) $J = \int_0^8 \frac{1}{2} u t^2 dt$

$\dot{x}_1 = x_2 \quad x_1(0) = 0$ | $x_1(8) = 100$ } Hard constraints

$\dot{x}_2 = u \quad x_2(0) = 0$ | $x_2(8) = 0$ }

$x_1(8) = x_{1f} = 100$

∴ Hard constraints

$\psi(x(T)) = 0 \Big|_{T=8}$

$\Rightarrow x_{1f} - 100 = 0$

Hamiltonian can be defined as :-

$H = \frac{1}{2} u^2 + \lambda_1 x_2(8) + \lambda_2 u$

$\frac{\partial H}{\partial \lambda_1} = \dot{x}_1 = x_2 \quad \frac{\partial H}{\partial \lambda_2} = \dot{x}_2 = u$

$-\dot{\lambda}_1 = \frac{\partial H}{\partial x_1} = 0$

$-\dot{\lambda}_2 = \frac{\partial H}{\partial x_2} = \lambda_1$

Stationary condition,

$\frac{\partial H}{\partial u} = u + \lambda_2 = 0$

$\therefore u = -\lambda_2$

$(\phi_x + \psi_x \nu - \lambda)\Big|_T dx(T) = 0$

$\left(\frac{\partial \phi}{\partial x_{1f}} + \frac{\partial \psi}{\partial x_{1f}} \nu_1 - \lambda_{1f}\right) dx_{1f} = 0$

& $\left(\frac{\partial \phi}{\partial x_{2f}} + \frac{\partial \psi}{\partial x_{2f}} \nu_2 - \lambda_{2f}\right) dx_{2f} = 0$

$\therefore \frac{\partial \phi}{\partial x_{2f}} = 0 \;, \frac{\partial \psi}{\partial x_{2f}} = 0$

$\therefore \lambda_{2f} = 0$

Now,

$\dot{\lambda}_1 = 0$

Integrating, we get :-

$\lambda_1 = c_1$

For $\dot{\lambda}_2$:

$$-\dot{\lambda}_2 = \lambda_1 = c_1$$

Integrating both sides, we get

$$\lambda_2 = -c_1 t + c_2 = -u$$

Now, from the state eqn we have :-

$$\dot{x}_2 = u$$

$$\dot{x}_2 = -c_2 + c_1 t$$

Integrating, we get :-

$$x_2(t) = c_1 \frac{t^2}{2} - c_2 t + c_3$$

at $t = 0$, we have :-

$$x_2(0) = c_1 \frac{(0)^2}{2} - c_2(0) + c_3 = 0 \Rightarrow \boxed{c_3 = 0}$$

At final time, $t = 8$, we have

$$x_2(8) = c_1 \frac{(8)^2}{2} - c_2(8) + \cancel{c_3}^{0}$$

$$= \frac{4\cancel{8}\times 8}{\cancel{2}} c_1 - 8c_2$$

$$0 = 32 c_1 - 8c_2$$

$$\Rightarrow {}^{4}\cancel{32} c_1 = \cancel{8} c_2$$

$$\Rightarrow 4c_1 - c_2 = 0 \quad \text{——(i)}$$

Also, from state eqn we have,

$$\dot{x}_1 = x_2$$

$$\dot{x}_1 = c_1 \frac{t^2}{2} - c_2 t + c_3$$

Integrating both sides, we have :-

$$x_1(t) = c_1 \frac{t^3}{6} - c_2 \frac{t^2}{2} + c_3 t + c_4$$

at $t = 0$, $x_1(0) = 0$

$$x_1(0) = c_1 \frac{(0)^3}{6} - c_2 \frac{(0)^2}{2} + c_3(0) + c_4 = 0$$

$$\Rightarrow \boxed{c_4 = 0}$$

Now, at $t = 8$, we have

$$x_1(8) = c_1 \frac{(8)^3}{6} - c_2 \frac{(8)^2}{2} + 0 + 0 = 100$$

$$\Rightarrow \frac{{}^{4}\cancel{8}\times 8\times \cancel{8}}{\cancel{6}_{3}} c_1 - \frac{{}^{4}\cancel{8}\times 8}{\cancel{2}} c_2 = 100$$

$$\Rightarrow \frac{4\times 8\times 8}{3} c_1 - 4\times 8 c_2 = 100 \quad \text{——(ii)} \Rightarrow 64 c_1 - 24 c_2 = 75$$

——(ii)

Solving eqn (i) & (ii) simultaneously :-

$$c_1 = -\frac{75}{32}, \quad c_2 = \frac{-75}{8}$$

$$= -2.34375 \quad = -9.375$$

Now, the state eqn can be written as :-

$$x_1(t) = -\frac{2.3t^3}{6} + \frac{9.3t^2}{2}$$

$$\boxed{x_1(t) = -0.383t^3 + 4.65t^2}$$

$$x_2(t) = -\frac{2.3}{2}t^2 + 9.3t$$

~~$x_2(t) = -1.15t^2 + 9.3t$~~ $x_2(t) = -1.171875t^2 + 9.375t$

& the control i/p

~~$u(t) = -2.3t + 9.3$~~ $u(t) = -2.34375t + 9.375$

control i/p at $t = 0$ & $t = 8$

$u(0) = +9.375$ $u(8) =$ ~~9.1~~ -9.375

Now, calculating cost function

$$J = \int_0^8 \frac{1}{2} u^2 dt$$

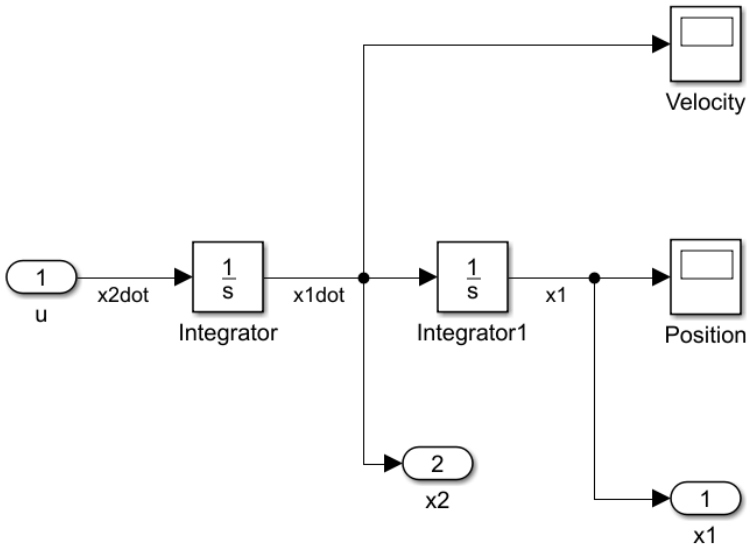
$$= \int_0^8 \frac{1}{2} [2.3t - 9.3]^2 dt$$

$$\boxed{J = 112.89}$$

$$= \int_0^8 \frac{1}{2} [-2.34375t + 9.375]^2 dt$$

$$\boxed{J = 117.1875}$$

b) *Simulink Diagram:*



MATLAB CODE:

```
clear all;
clc;

t = 0:0.1:8;
%sim with initial control
u = ones(length(t),1);
u(41:end) = -1;
[t0,y0] = sim('hw3p1b',t',[],[t' u]);
plot(t',u,'*');
title('Initial Control Input')
xlabel('Time');
ylabel('u');

%Constrained optimization

lb = ones(81,1)*(-100);
ub = ones(81,1)*(100);

options = optimset('Display','iter','PlotFcns','optimplotx');

[uf,cost] =
fmincon('hw3_cost',u,[],[],[],[],lb,ub,'hw3_constraint',options);

%sim with optimal control
```

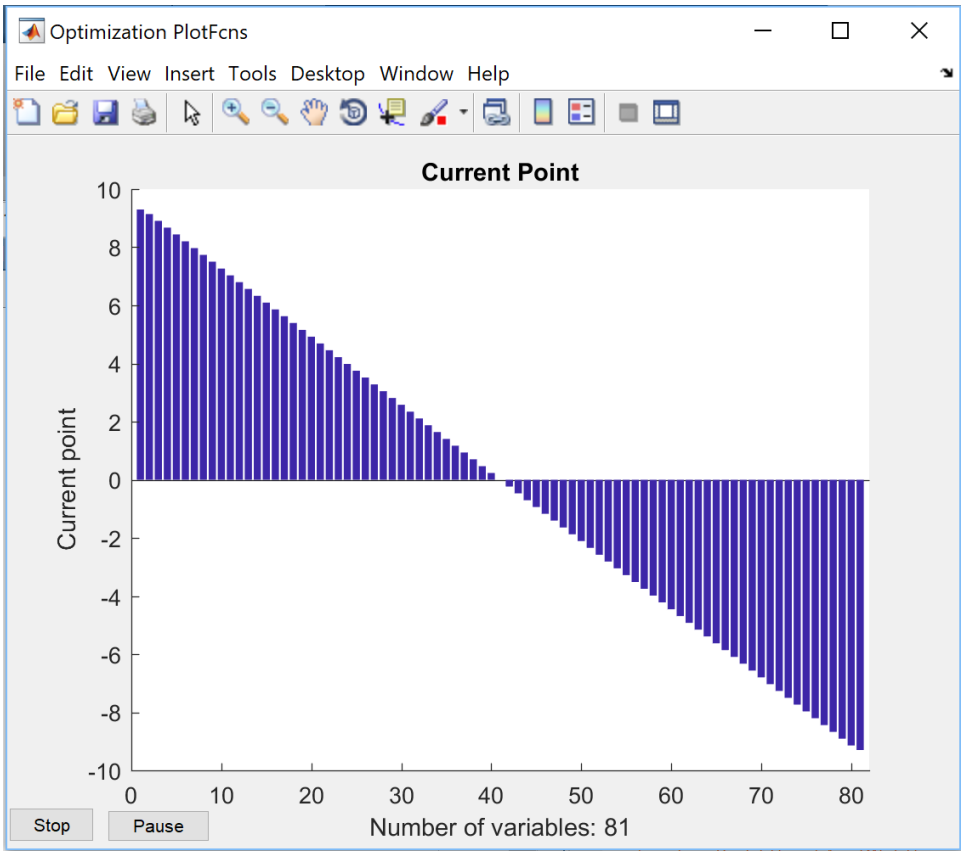
```
[tout,yout] = sim('hw3p1b',t',[],[t' uf]);
figure;
plot(tout,uf,'o');
title('Optimal Control Input')
xlabel('Time');
ylabel('u*');

%Plotting state time histories
figure;
plot(tout,yout(:,1));
grid;
xlabel('Time');
ylabel('State x1');
figure;
plot(tout,yout(:,2));
grid;
xlabel('Time');
ylabel('State x2');
```

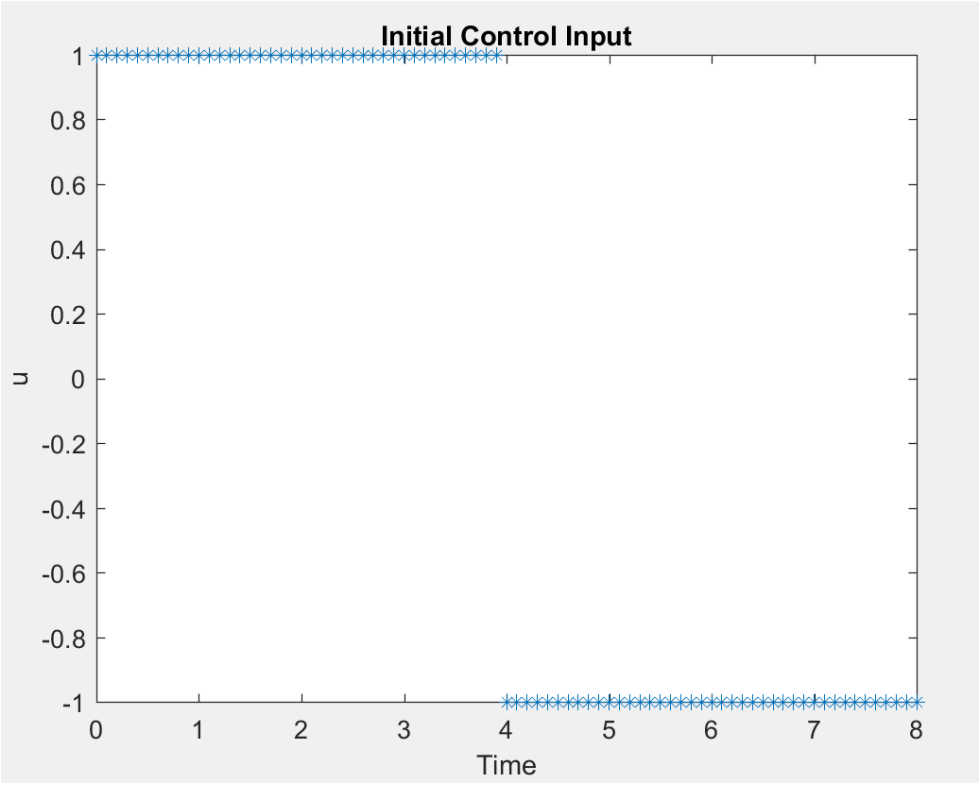
Cost function:

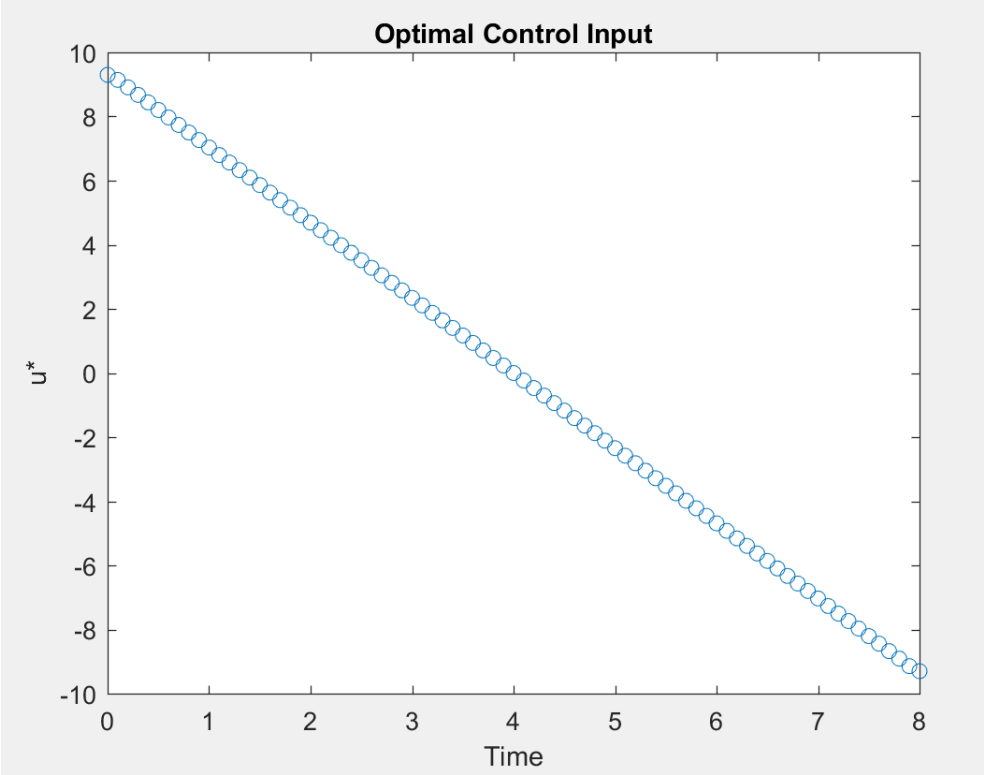
```
function cost = hw3_cost(u)
cost = 0.5*0.1*trapz(u.*u);
end
```

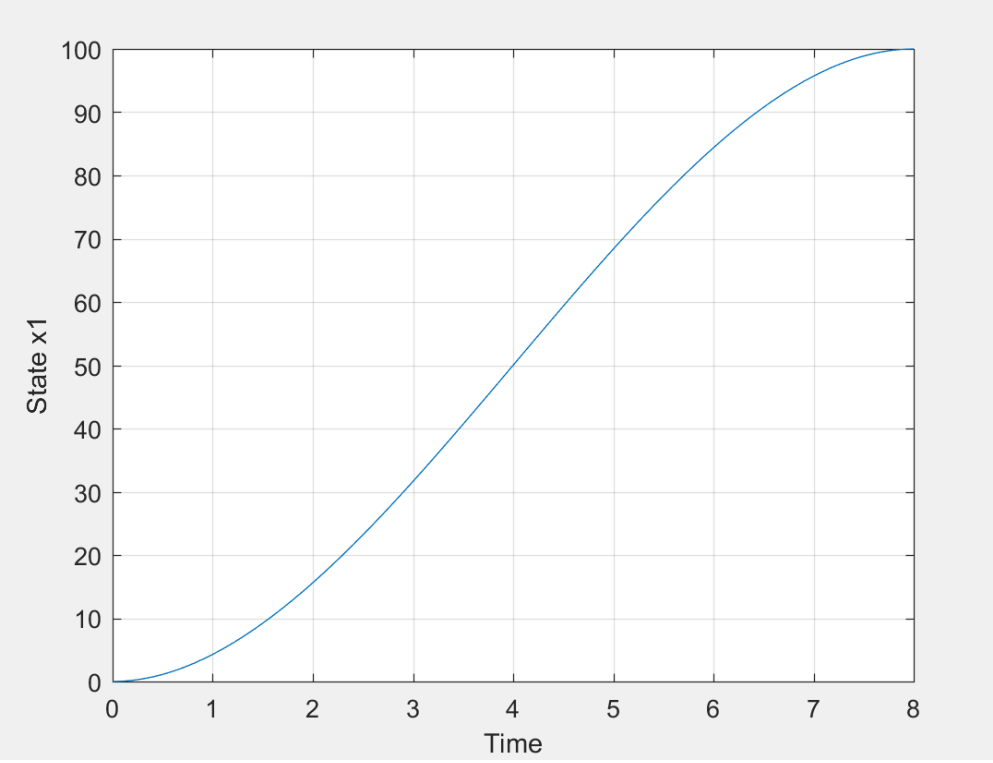
Constraint function:

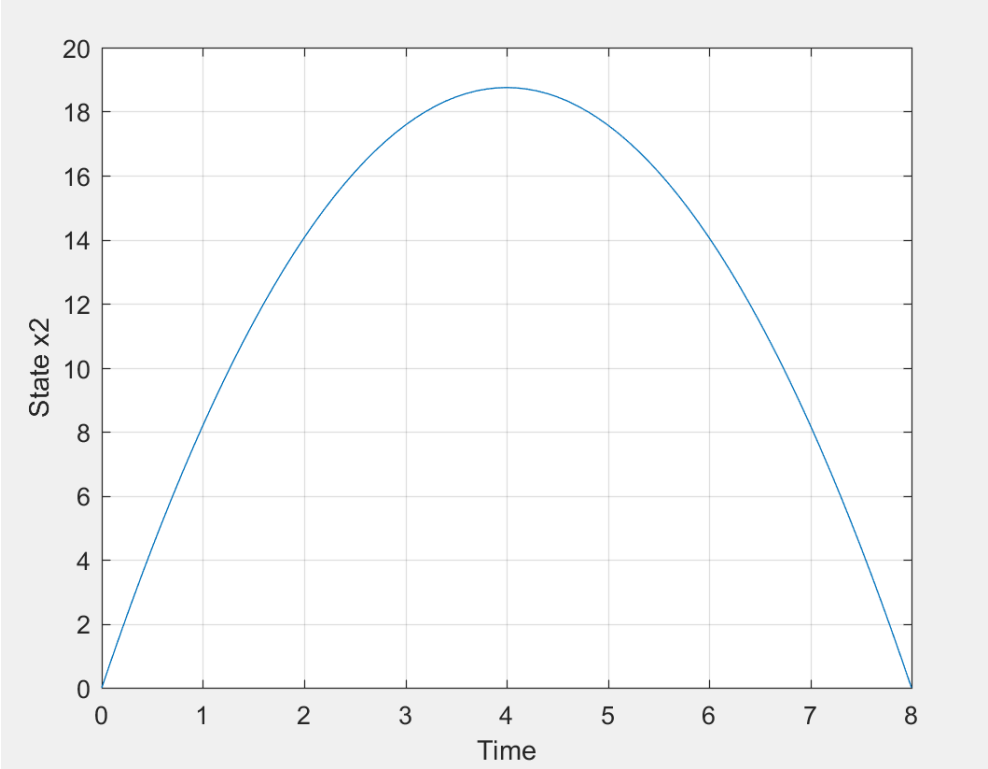
```
function [cineq,ceq] = hw3_constraint(u)

cineq = [];
t = 0:0.1:8;
[tout,yout] = sim('hw3p1b',t',[],[t' u]);

%Hard Constraints
ceq(1) = 100 - yout(end,1);
ceq(2) = yout(end,2);
end
```


Optimization PlotFcns
File Edit View Insert Tools Desktop Window Help
Current Point
Current point
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
10
20
30
40
50
60
70
80
Stop
Pause
Number of variables: 81


Initial Control Input
u
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
Time


Optimal Control Input
u*
Time
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8


State x1
Time
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8



Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	2730	1.172238e+02	1.221e-15	4.517e-06	1.713e-10

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than the default value of the step size tolerance and constraints are satisfied to within the default value of the constraint tolerance.

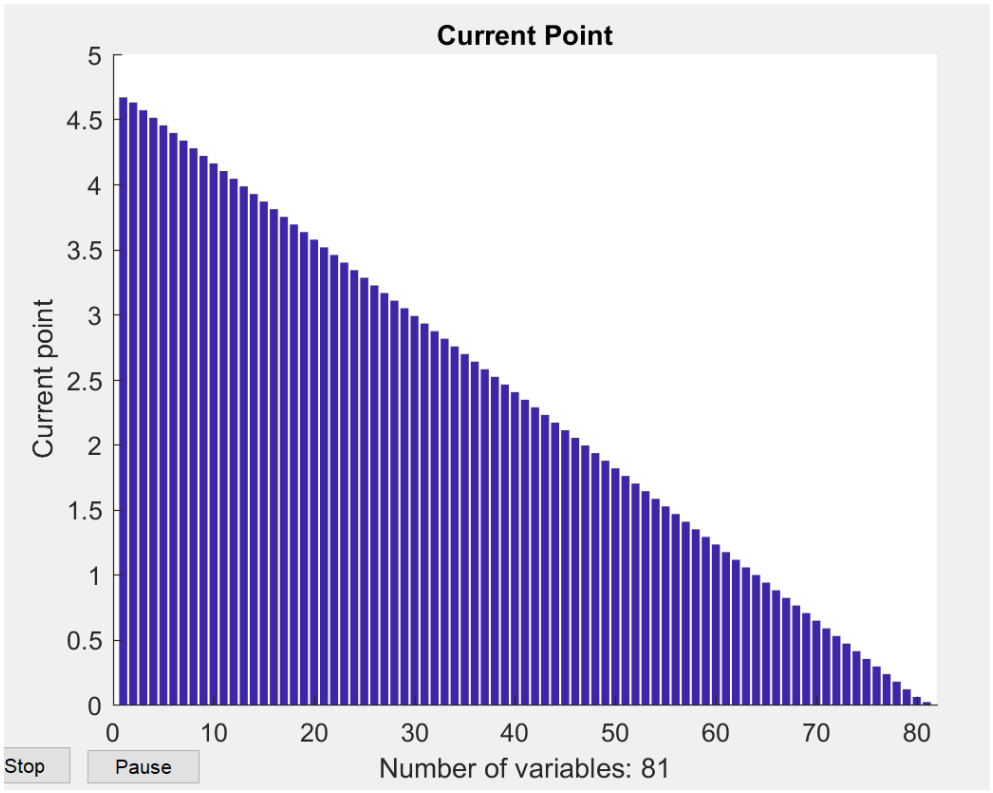
<stopping criteria details>

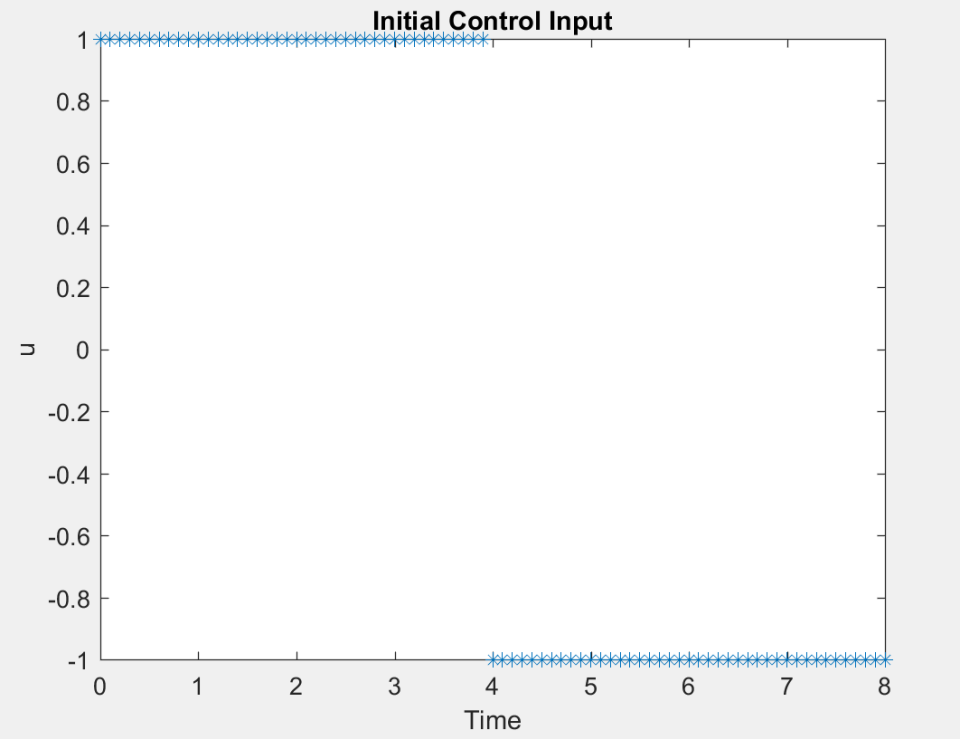
The value of the cost calculated analytically is 117.1875 which is approximately equal to 117.2238 as calculated from MATLAB.

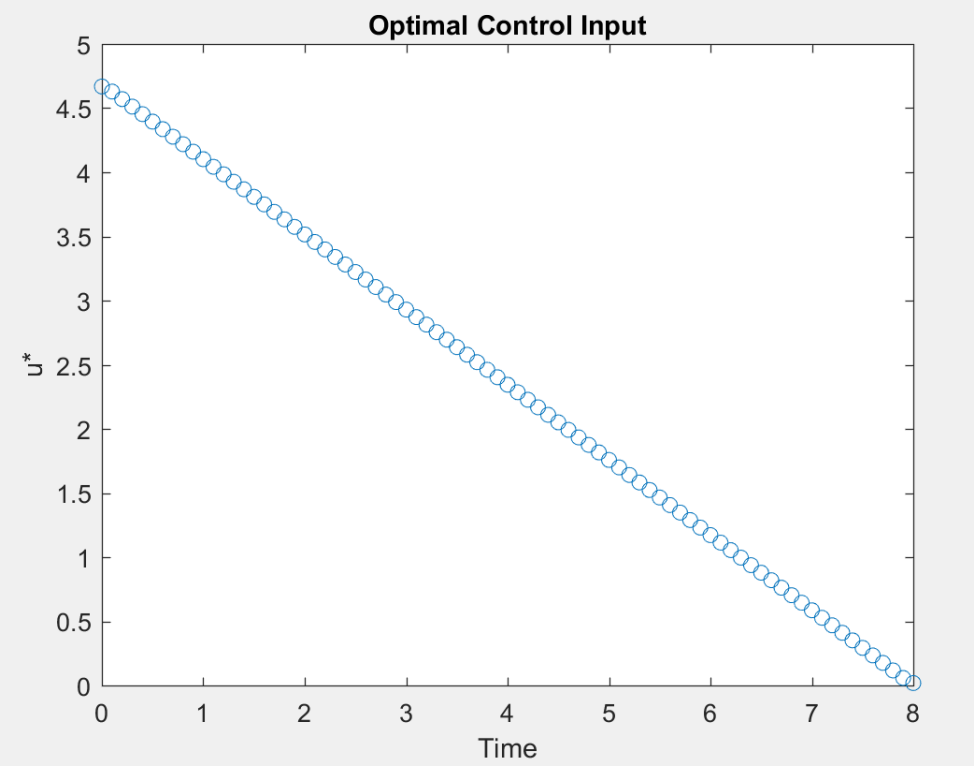
c) According to problem the constraint function can be modified as :

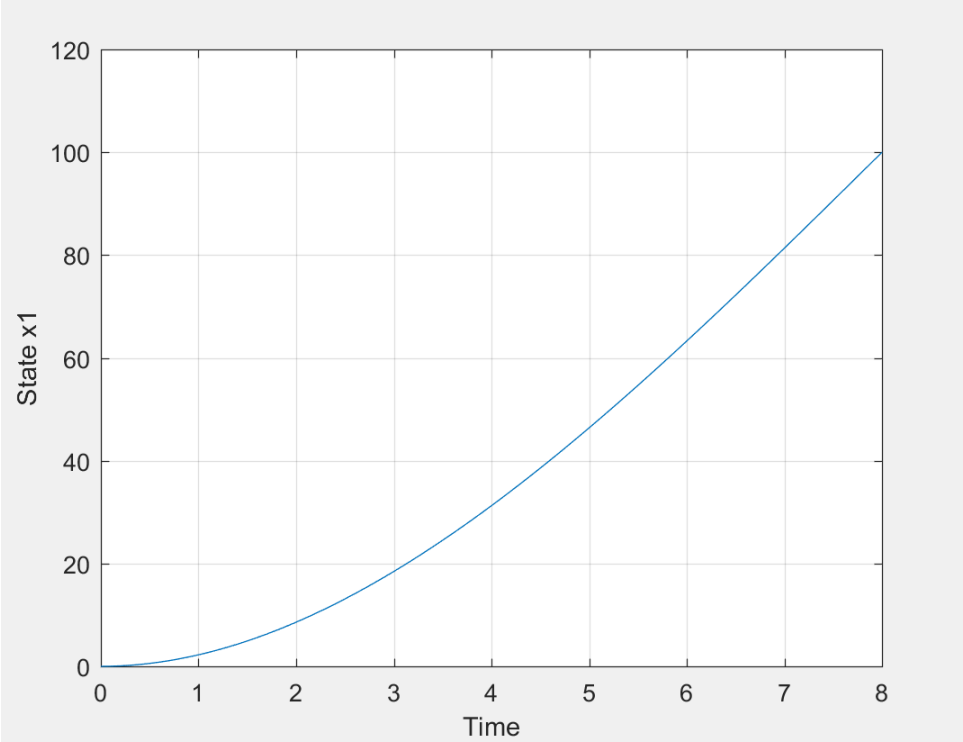
```
function [cineq,ceq] = hw3pc_constraint(u)

cineq = [];
t = 0:0.1:8;
[tout,yout] = sim('hw3p1b',t',[],[t' u]);

%Hard Constraints
cineq = 100-yout(end,1);
ceq = cineq;
end
```

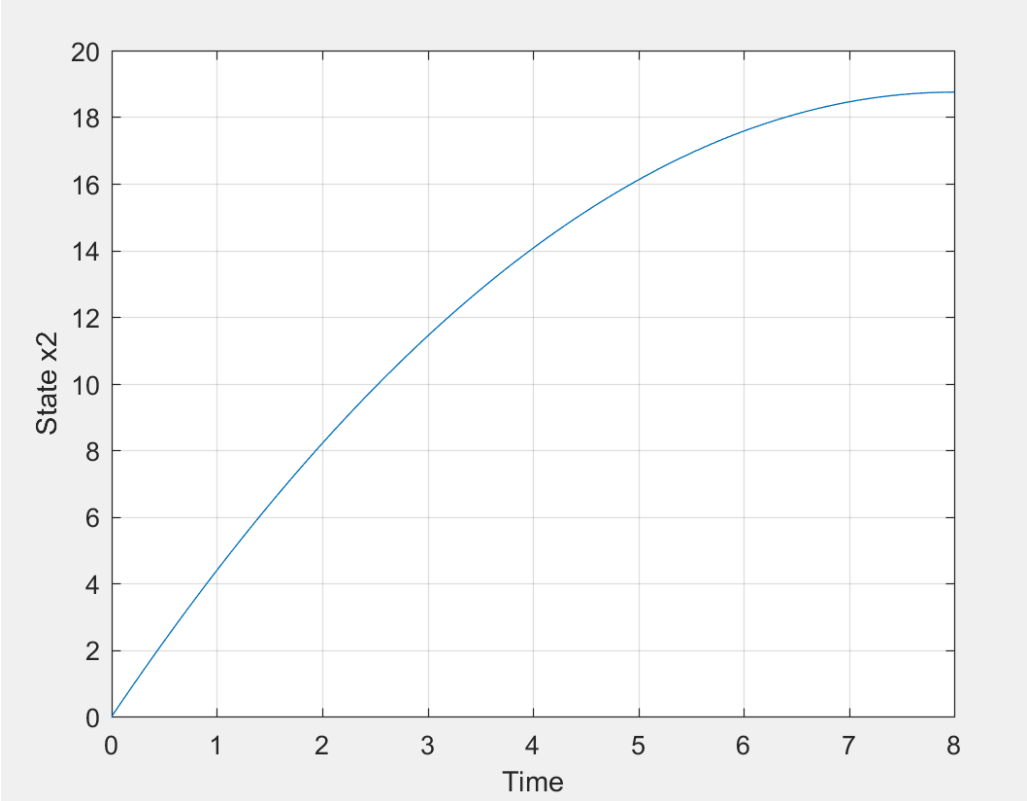
Results:




Initial Control Input
u
Time
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8


Optimal Control Input
u*
Time
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8

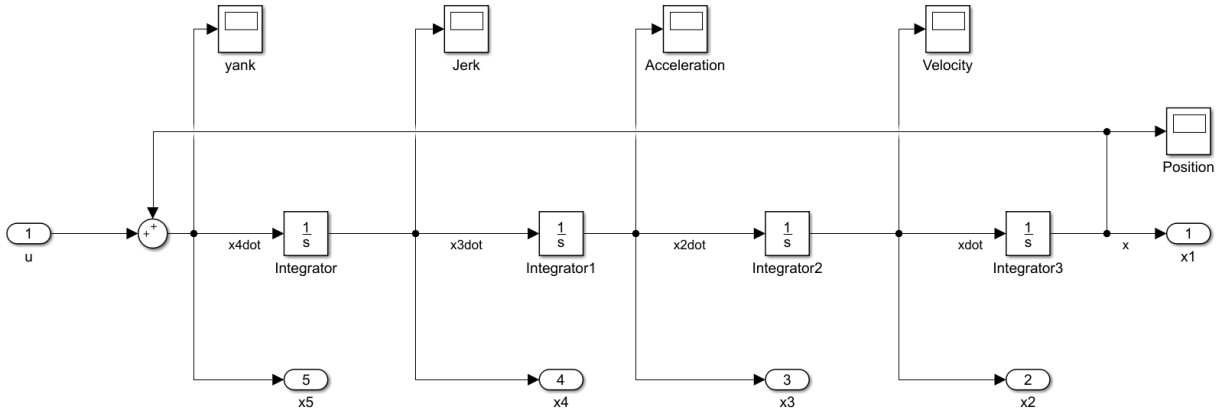




The time taken to reach the solution increased as we changed the constraints.

Problem 2:

Simulink diagram:



a) *MATLAB code:*

```
clear all;
clc;

t = 0:0.1:5;
% Sim with Initial control

u = 0.1 * ones(length(t),1);
u(26:end) = -0.1;

[t0,y0] = sim('hw3p2',t',[],[t' u]);
plot(t',u,'*');
title('Initial Control Input')
xlabel('Time');ylabel('u');

% Constrianed Optimization

lb = ones(51,1)*(-100);
ub = ones(51,1)*100;

options = optimset('Display','iter','PlotFcns','optimplotx');

[uf,cost] =
fmincon('hw3p2cost',u,[],[],[],[],lb,ub,'hw3p2constraint',options);
% Sim with Optimal Control
```

```
[tout,xout,yout] = sim('hw3p2',t',[],[t' uf]);
figure;

plot(tout,uf,'o');
title('Optimal Control input')
xlabel('Time');ylabel('u*');

% Plotting State time histories
figure(2);
plot(tout,yout(:,1));
grid; xlabel('Time'); ylabel('Position x1');title('Position');
figure(3);
plot(tout,yout(:,2));
grid; xlabel('Time'); ylabel('Velocity x2');title('Velocity');
figure(4);
plot(tout,yout(:,3));
grid; xlabel('Time'); ylabel('Acceleration x3');title('Acceleration');
figure(5);
plot(tout,yout(:,4));
grid; xlabel('Time'); ylabel('Jerk x4');title('Jerk');
figure(6);
plot(tout,yout(:,5));
grid; xlabel('Time'); ylabel('Yank x5');title('Yank');
```

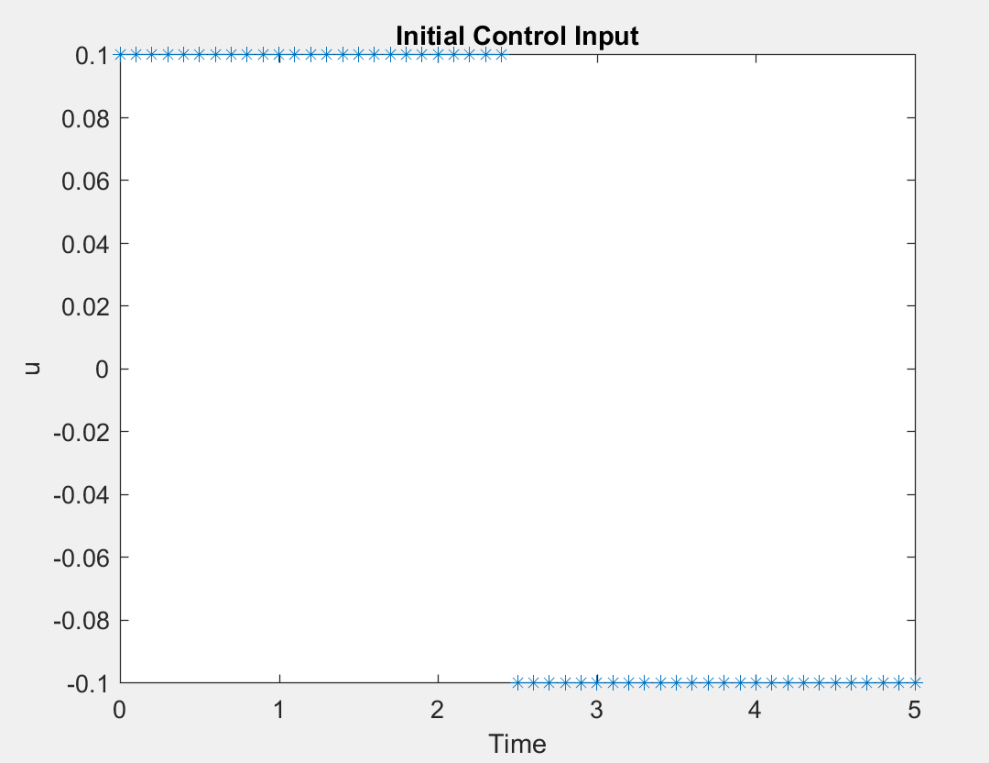
Cost function:

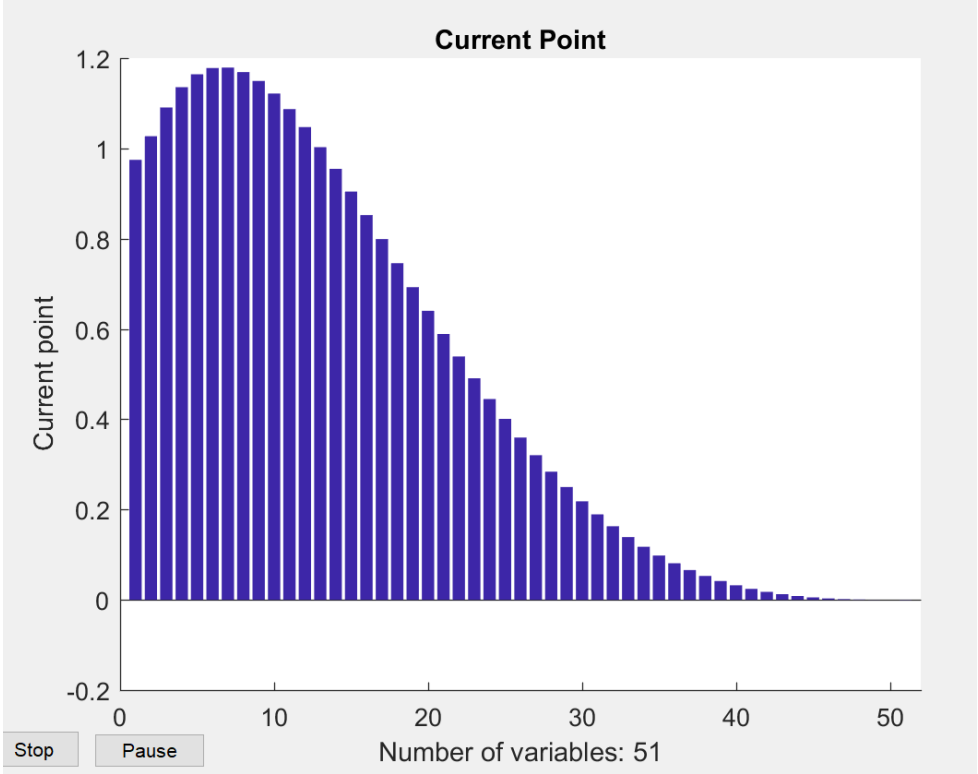
```
function cost = hw3p2cost(u)
    cost = 0.5 * 0.1 * trapz(u.*u);
end
```

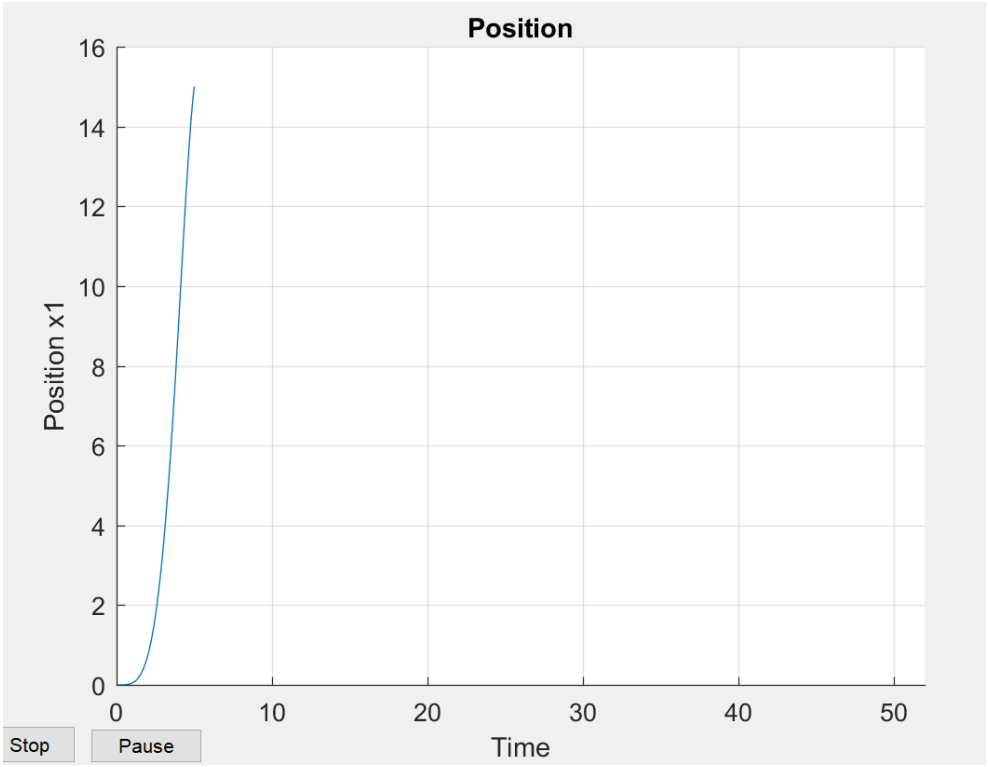
Constraint function with only Position as a hard constraint:

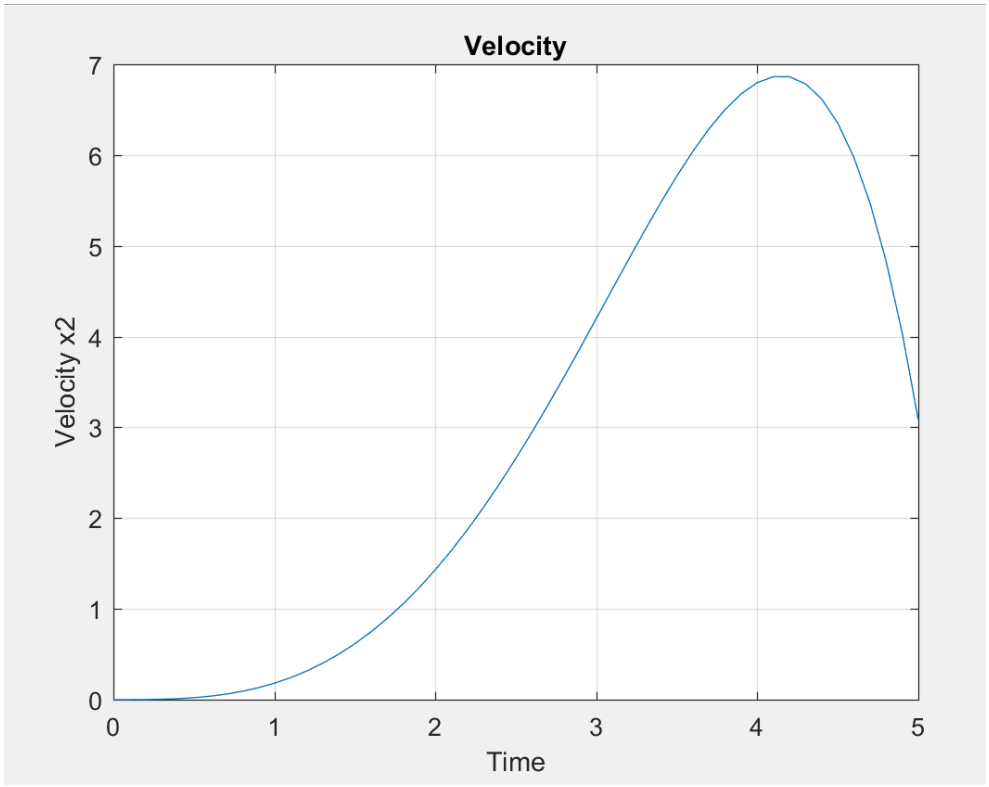
```
function [cineq,ceq] = hw3p2constraint(u)
    cineq = [];
    t = 0:0.1:5;

    [tout,xout,yout] = sim('hw3p2',t',[],[t' u]);
    % Position Hard Contraints
    ceq(1) = 15 - yout(end,1);
```

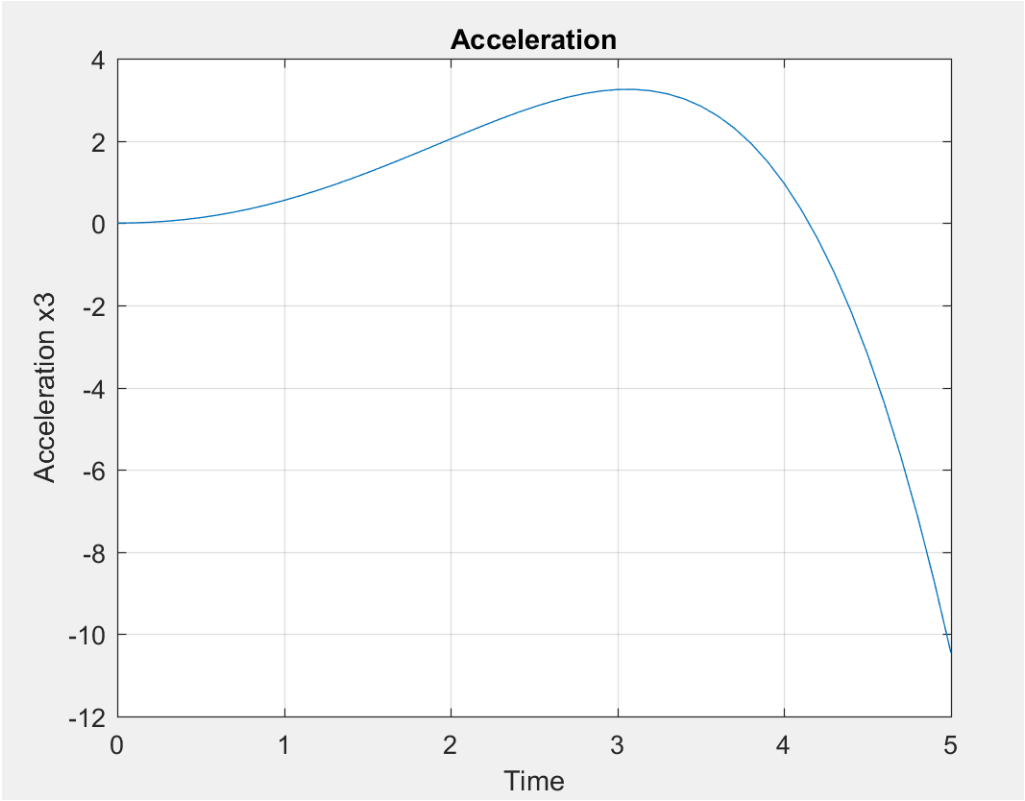
Result:

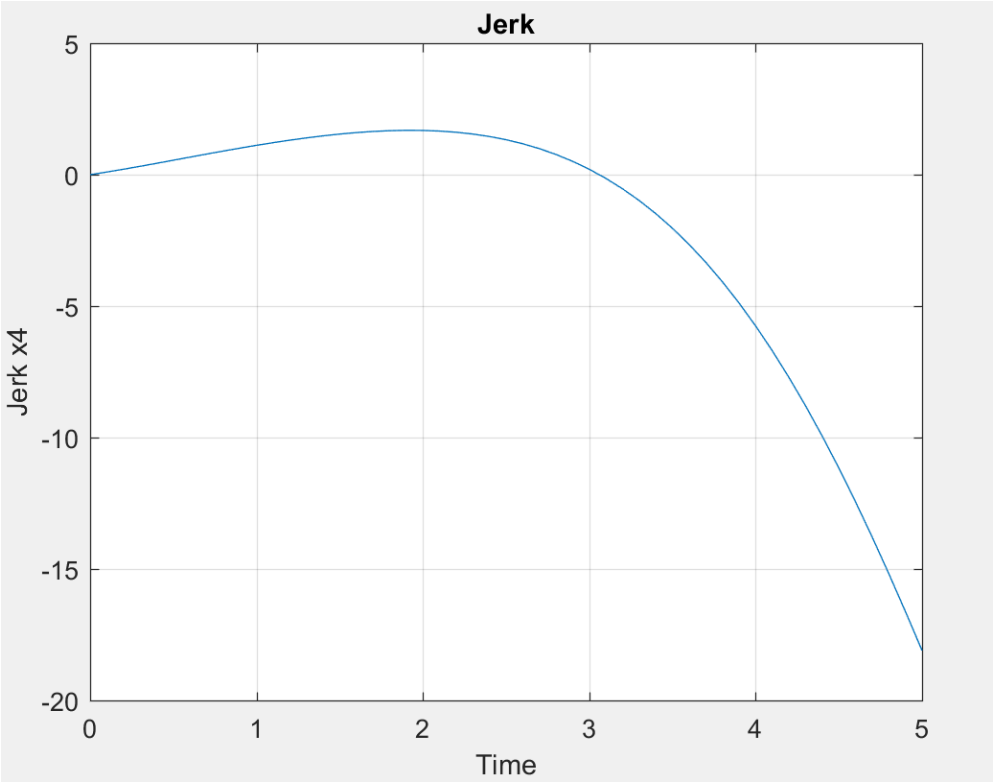

Initial Control Input
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
u
0
1
2
3
4
5
Time

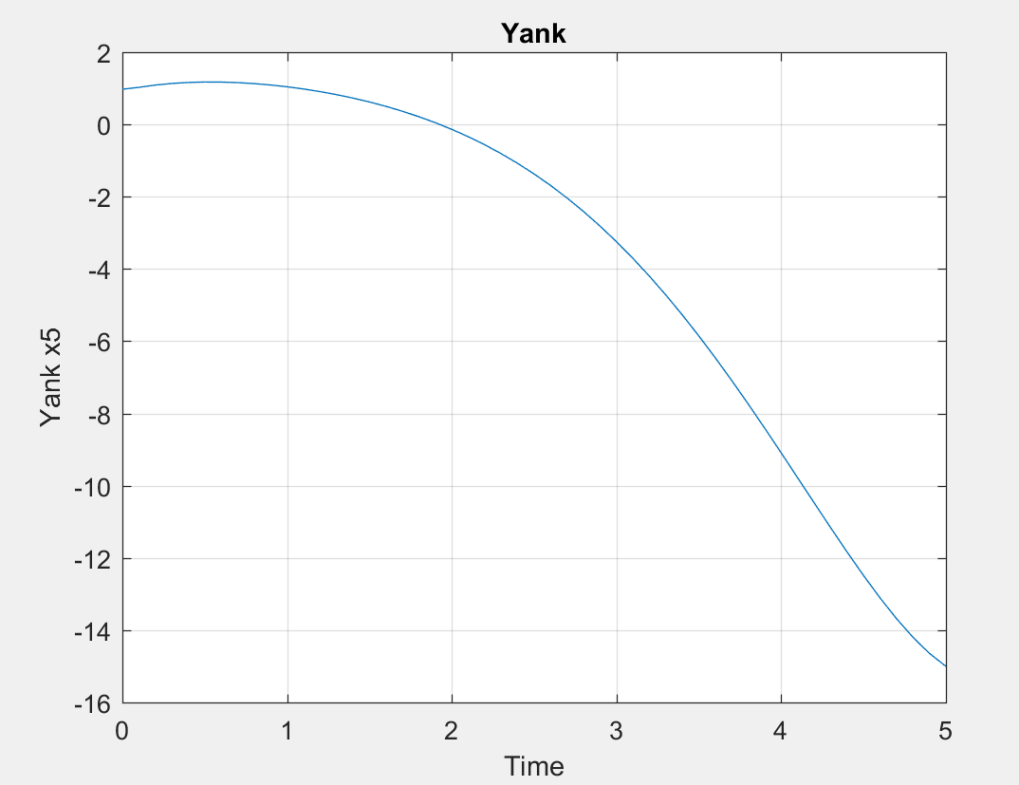

Current Point
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
Current point
0
10
20
30
40
50
Number of variables: 51
Stop
Pause


Position
Position x1
Time
Stop
Pause
0
2
4
6
8
10
12
14
16
0
10
20
30
40
50


Velocity
Velocity x2
Time
0
1
2
3
4
5
6
7
0
1
2
3
4
5


Acceleration
Acceleration x3
4
2
0
-2
-4
-6
-8
-10
-12
0
1
2
3
4
5
Time


Jerk
Jerk x4
5
0
-5
-10
-15
-20
0
1
2
3
4
5
Time



Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	52	2.500000e-02	1.366e+01	1.321e-02	
1	104	1.102611e+00	1.496e-06	1.081e-01	4.265e+00
2	156	1.099656e+00	1.137e-08	2.042e-02	5.566e-02
3	208	1.089796e+00	9.553e-09	1.527e-02	2.531e-01
4	260	1.085584e+00	5.305e-08	8.655e-03	3.021e-01
5	312	1.085489e+00	1.245e-09	8.095e-03	1.146e-02
6	364	1.085101e+00	4.172e-10	5.698e-03	5.015e-02
7	416	1.084715e+00	1.282e-08	1.924e-05	1.200e-01
8	468	1.084715e+00	2.158e-11	1.884e-05	1.425e-04
9	520	1.084715e+00	7.223e-12	1.763e-05	4.581e-05
10	572	1.084715e+00	6.304e-11	1.119e-05	1.544e-04
11	624	1.084715e+00	2.945e-12	4.531e-06	1.356e-04
12	676	1.084715e+00	7.478e-13	6.687e-07	1.059e-04

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the optimality tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>

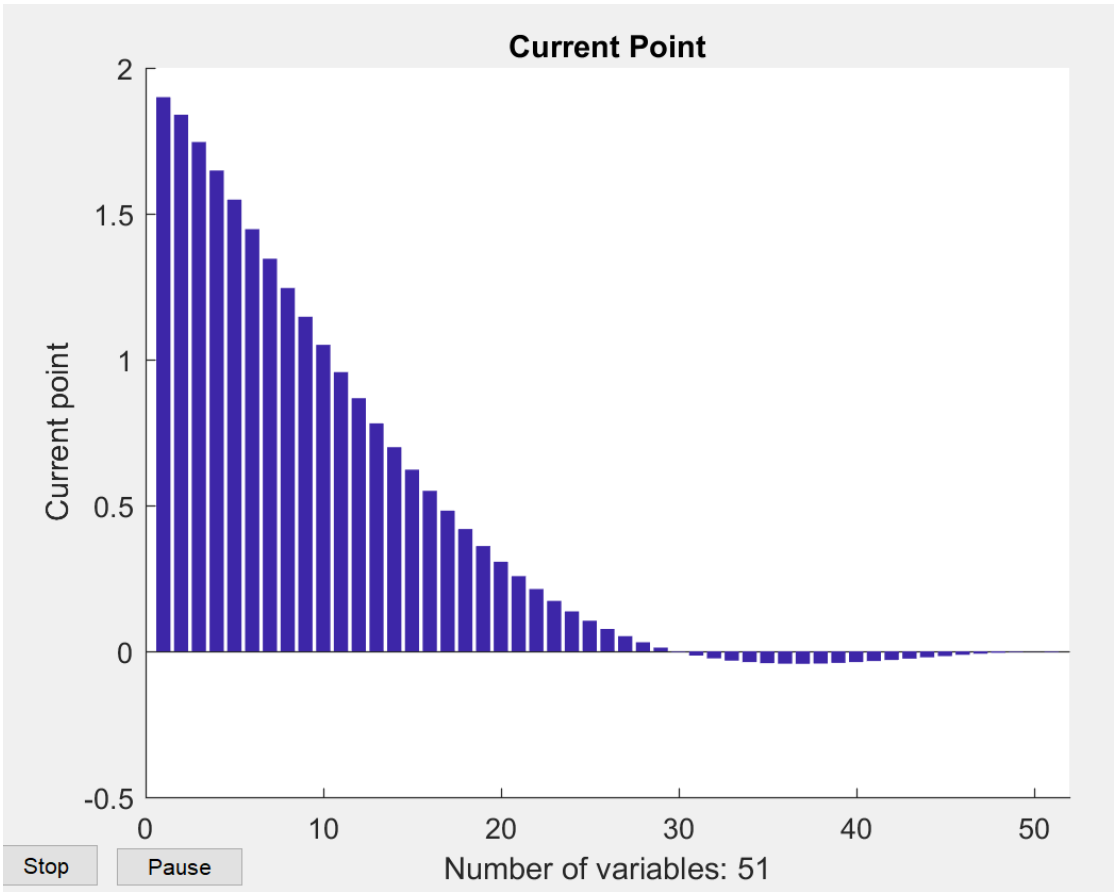
The value of cost function is 1.084715 as per the above result.

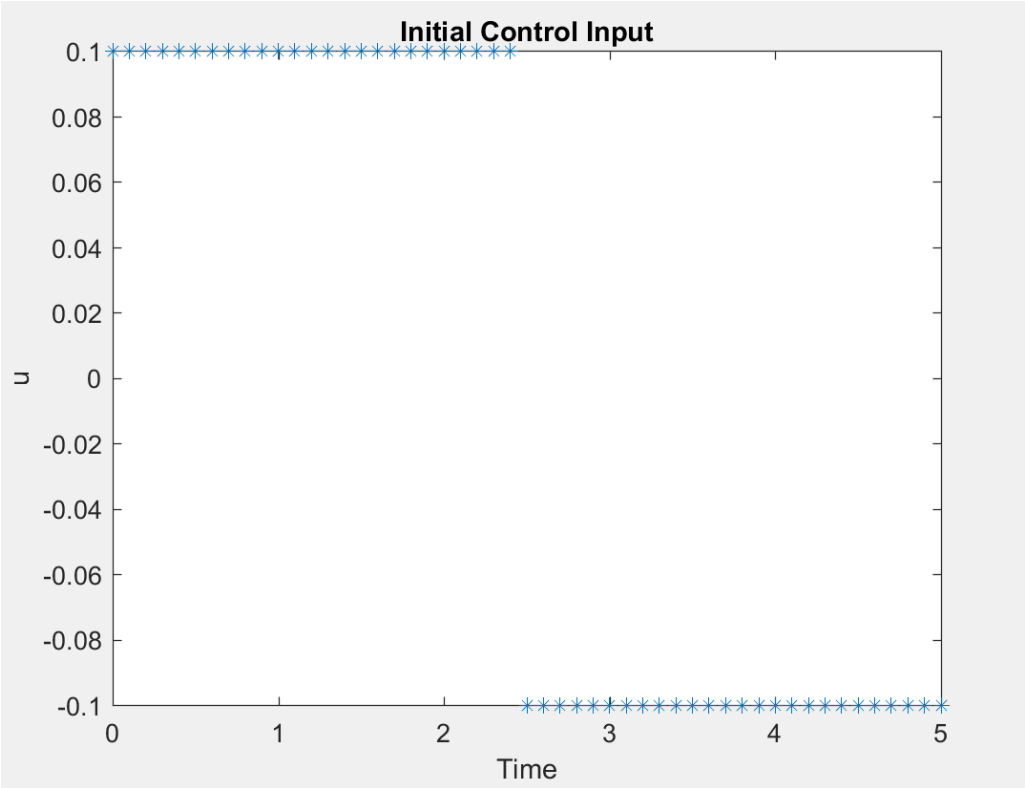
b)

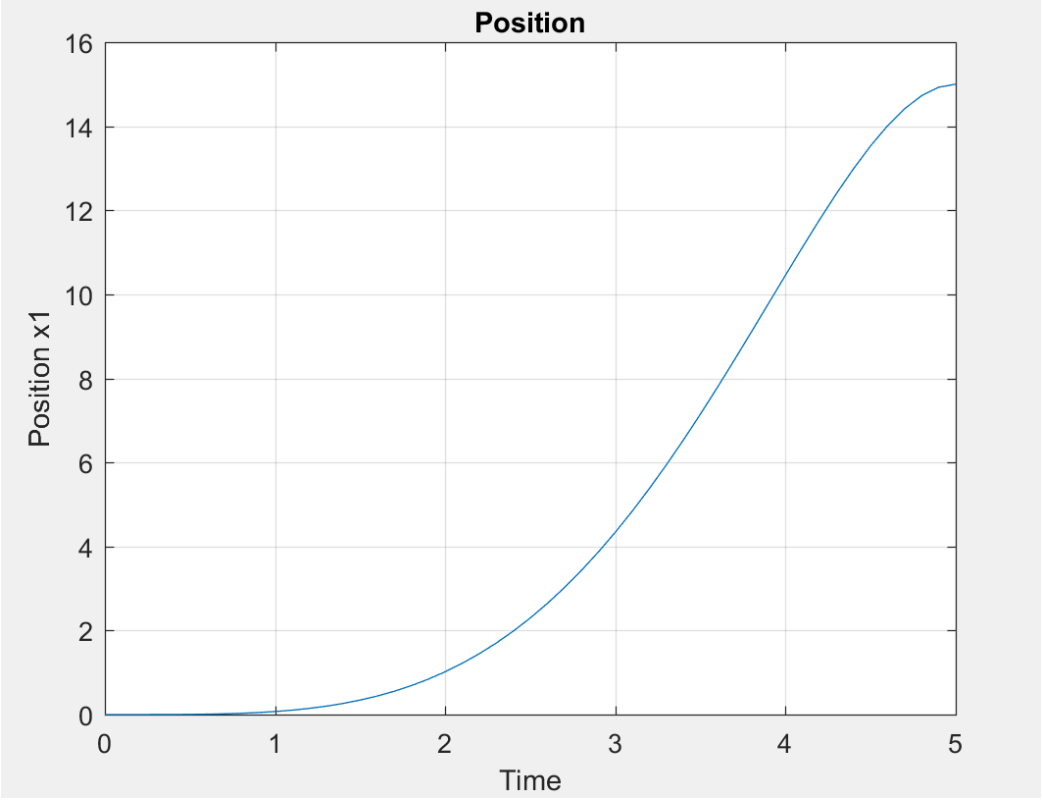
With position and velocity as hard constraints

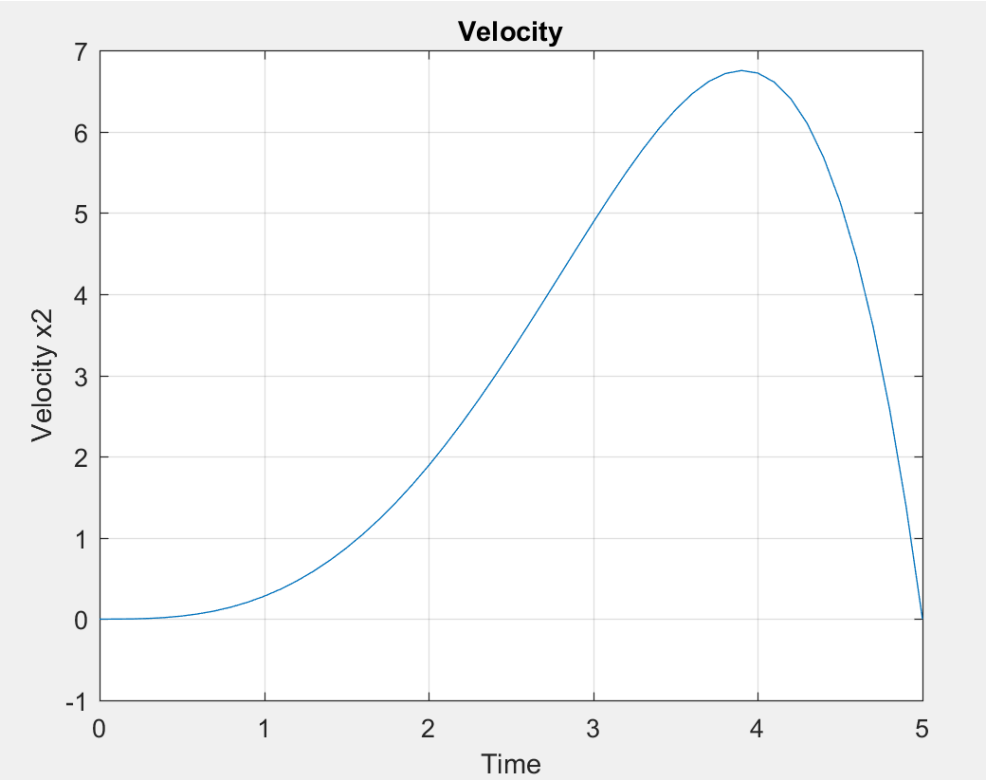
```
function [cineq,ceq] = hw3p2constraint2(u)
    cineq = [];
    t = 0:0.1:5;

    [tout,xout,yout] = sim('hw3p2',t',[],[t' u]);
    % Position and velocity hard constraints
    ceq(1) = 15 - yout(end,1);
    ceq(2) = yout(end,2);
```

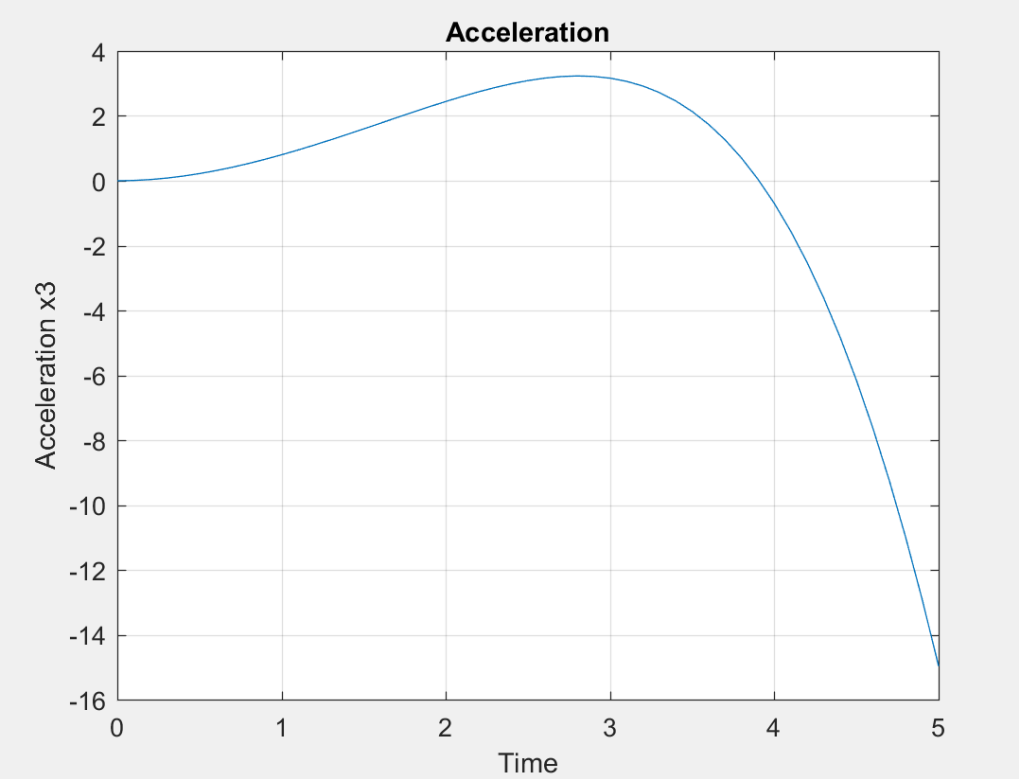
Result:

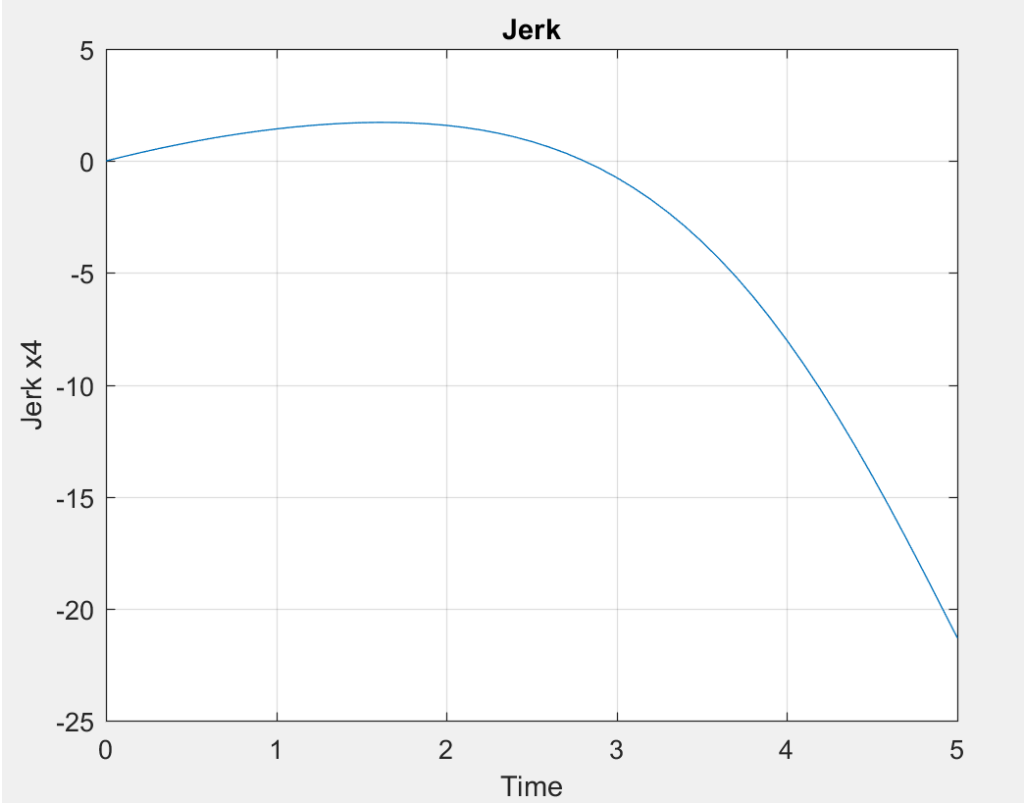


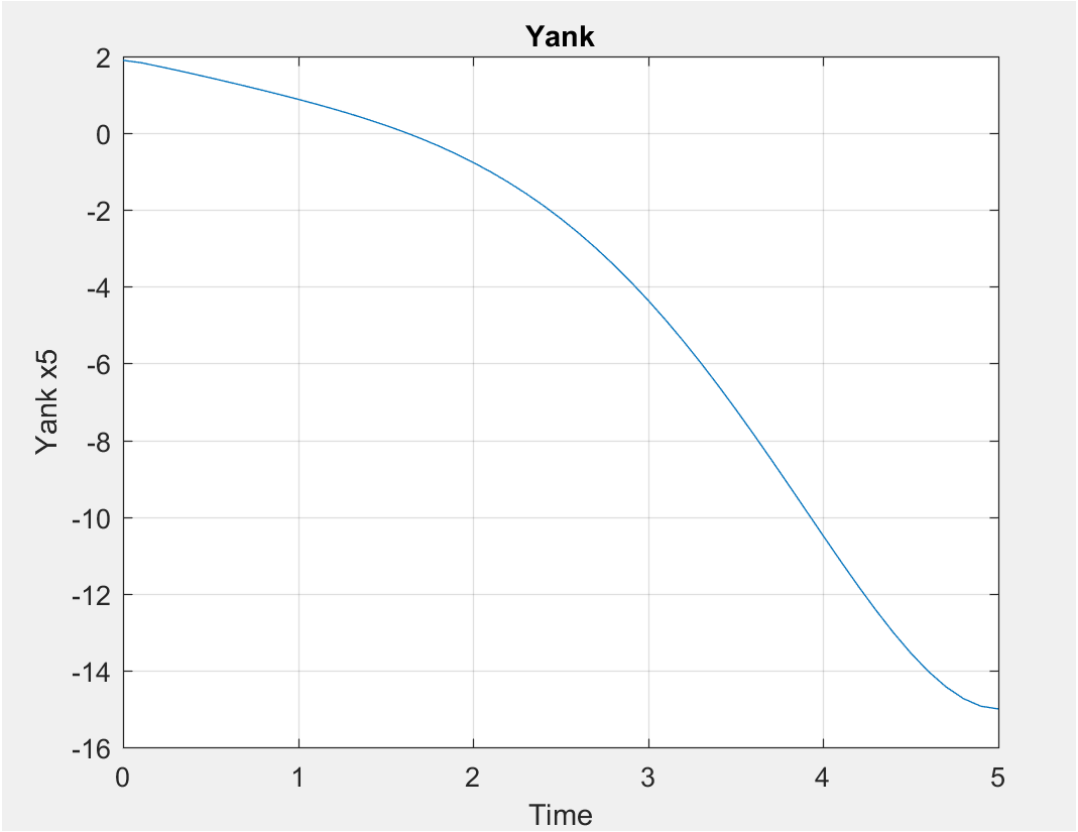

Initial Control Input
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
u
0
1
2
3
4
5
Time


Position
16
14
12
10
8
6
4
2
0
Position x1
0
1
2
3
4
5
Time


Velocity
Velocity x2
Time
7
6
5
4
3
2
1
0
-1
0
1
2
3
4
5


Acceleration
Acceleration x3
Time
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
0
1
2
3
4
5


Jerk
5
0
-5
-10
-15
-20
-25
Jerk x4
0
1
2
3
4
5
Time


Yank
2
0
-2
-4
-6
-8
-10
-12
-14
-16
Yank x5
0
1
2
3
4
5
Time

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	52	2.500000e-02	1.366e+01	1.182e-02	
1	104	1.309732e+00	1.726e-06	1.794e-01	4.722e+00
2	156	1.305132e+00	3.593e-08	4.311e-02	6.920e-02
3	208	1.288658e+00	1.936e-07	3.156e-02	3.198e-01
4	260	1.278309e+00	1.548e-07	1.226e-02	5.158e-01
5	312	1.277991e+00	2.919e-09	1.110e-02	2.308e-02
6	364	1.276766e+00	3.431e-08	6.813e-03	9.470e-02
7	416	1.275994e+00	3.103e-08	1.254e-04	1.534e-01
8	468	1.275994e+00	1.248e-10	1.208e-04	5.050e-04
9	520	1.275994e+00	2.427e-10	1.065e-04	5.041e-04
10	572	1.275994e+00	2.786e-11	6.965e-05	8.386e-04
11	624	1.275994e+00	9.507e-12	2.350e-05	9.254e-04
12	676	1.275994e+00	8.142e-12	7.598e-06	5.998e-04
13	728	1.275994e+00	3.926e-11	1.888e-06	2.056e-04
14	780	1.275994e+00	2.577e-12	2.771e-07	4.605e-05

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the optimality tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

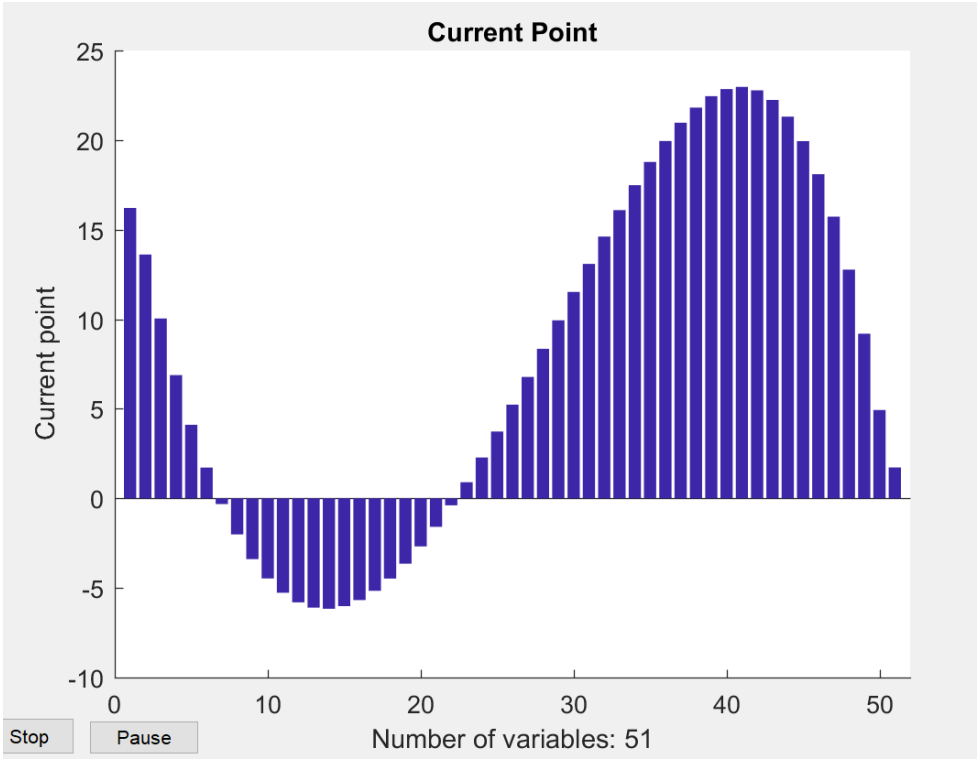
<stopping criteria details>

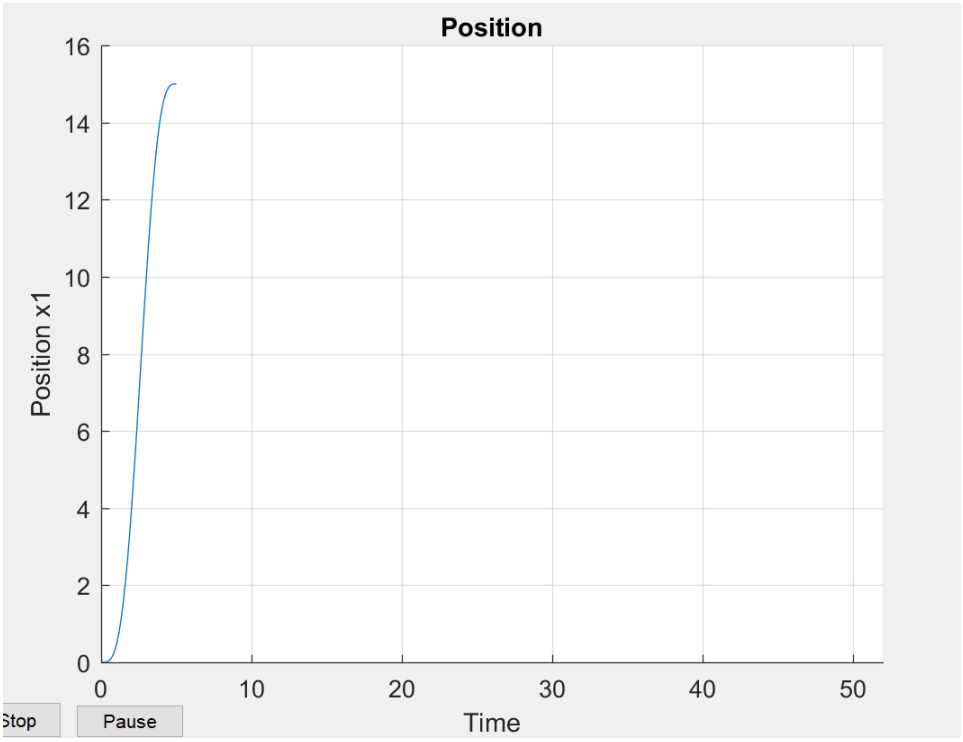
The value of cost function as calculated from the above is1.275994.

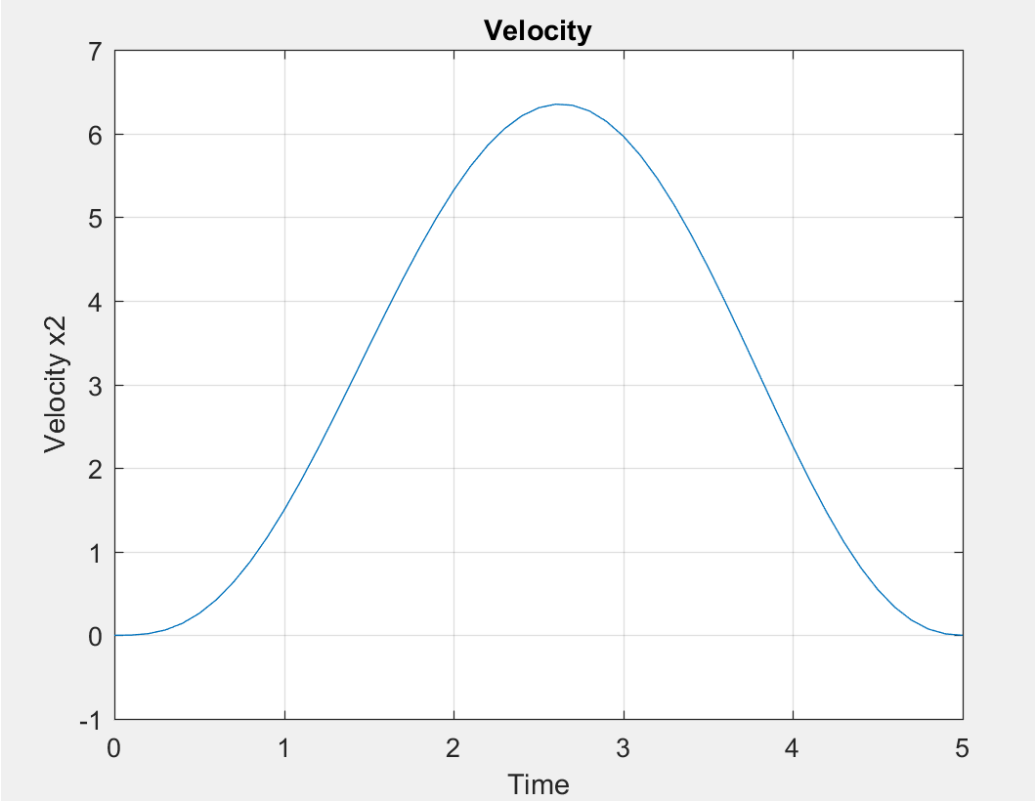
c) With position, velocity and acceleration as hard constraints

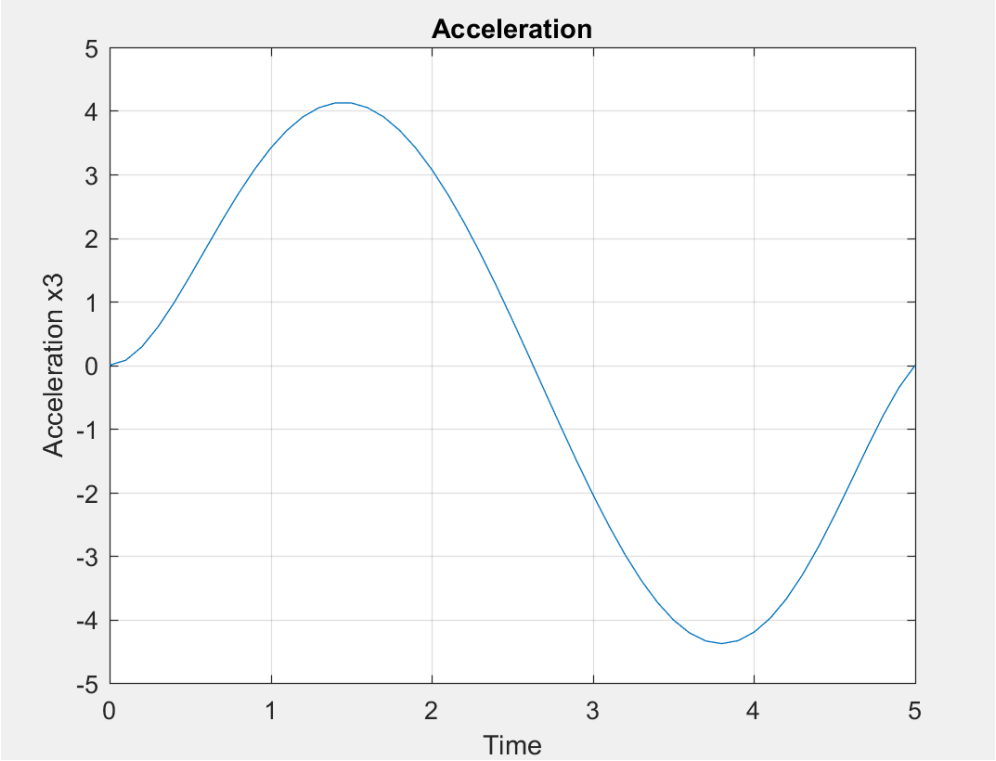
```
function [cineq,ceq] = hw3p2constraint3(u)
    cineq = [];
    t = 0:0.1:5;

    [tout,xout,yout] = sim('hw3p2',t',[],[t' u]);
    % Position, velocity and acceleration hard constraints
    ceq(1) = 15 - yout(end,1);
    ceq(2) = yout(end,2);
    ceq(3) = yout(end,3);
end
```

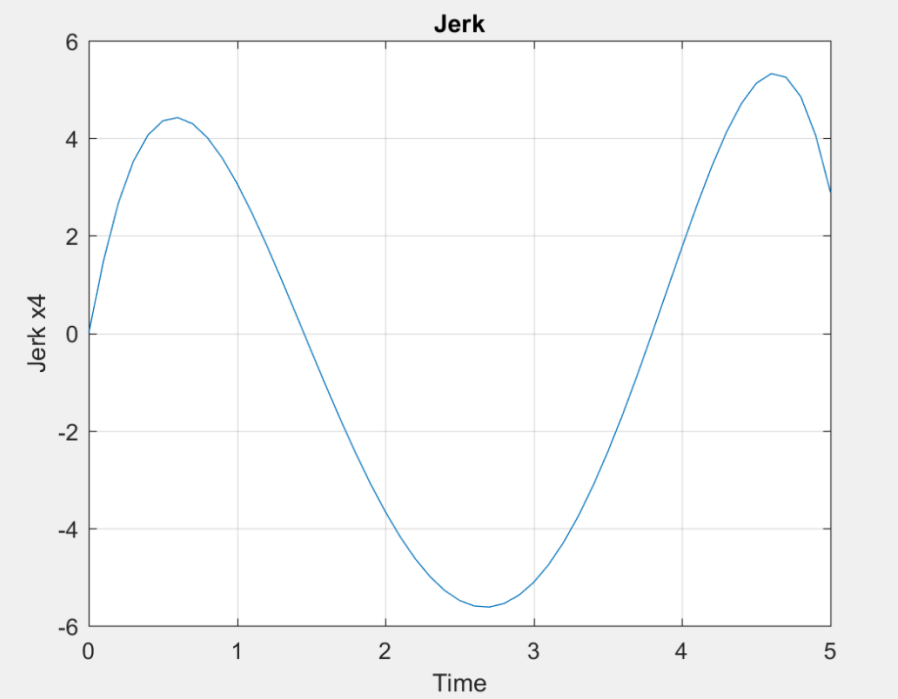
Result:

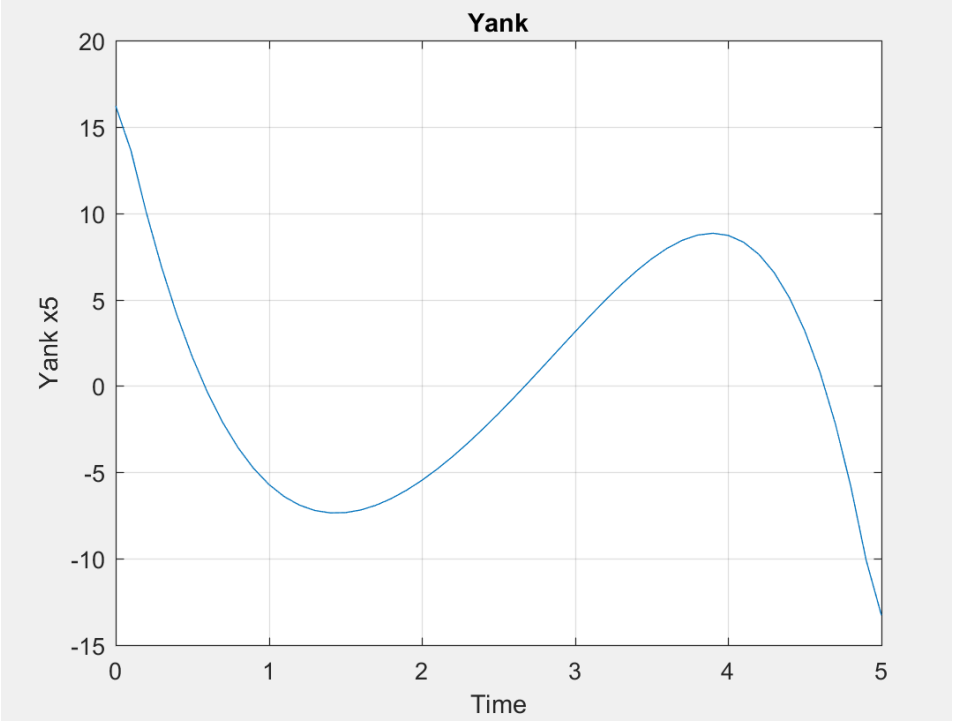

Current Point
Current point
25
20
15
10
5
0
-5
-10
0
10
20
30
40
50
Number of variables: 51
Stop
Pause


Position
Position x1
16
14
12
10
8
6
4
2
0
0
10
20
30
40
50
Time
Stop
Pause


Velocity
Velocity x2
Time
7
6
5
4
3
2
1
0
-1
0
1
2
3
4
5


Acceleration
Acceleration x3
Time
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5


Jerk
Jerk x4
Time
6
4
2
0
-2
-4
-6
0
1
2
3
4
5


Yank
Yank x5
Time
20
15
10
5
0
-5
-10
-15
0
1
2
3
4
5

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	52	2.500000e-02	1.366e+01	9.724e-03	
1	104	4.052469e+02	1.647e-05	2.324e+00	9.070e+01
2	156	4.050752e+02	2.878e-08	3.788e-01	4.191e-01
3	208	4.043949e+02	2.596e-07	2.767e-01	1.983e+00
4	260	4.036195e+02	9.042e-07	5.997e-03	5.353e+00
5	312	4.036192e+02	5.002e-09	5.390e-03	1.817e-02
6	364	4.036183e+02	1.322e-08	2.675e-03	8.044e-02
7	416	4.036179e+02	3.471e-09	1.258e-03	8.199e-02
8	468	4.036179e+02	9.699e-12	1.195e-03	1.319e-03
9	520	4.036179e+02	4.718e-11	8.844e-04	6.197e-03
10	572	4.036179e+02	4.755e-10	1.770e-05	1.764e-02
11	624	4.036179e+02	1.030e-11	5.134e-06	3.645e-04
12	682	4.036179e+02	3.187e-12	1.290e-05	5.206e-06
13	738	4.036179e+02	3.568e-12	7.049e-06	5.206e-06
14	791	4.036179e+02	3.158e-12	9.449e-06	5.206e-06
15	844	4.036179e+02	9.225e-15	6.250e-06	3.165e-05
16	897	4.036179e+02	7.803e-15	5.338e-06	4.263e-05
17	969	4.036179e+02	4.774e-15	1.921e-05	1.988e-07
18	1026	4.036179e+02	1.421e-14	2.524e-05	1.988e-07
19	1079	4.036179e+02	3.286e-14	1.347e-05	1.988e-07
20	1132	4.036179e+02	2.609e-15	1.868e-05	8.784e-05
21	1195	4.036179e+02	3.053e-14	1.347e-05	2.196e-05
22	1253	4.036179e+02	1.016e-14	1.202e-05	2.196e-05
23	1306	4.036179e+02	2.043e-14	2.708e-05	6.051e-05
24	1368	4.036179e+02	2.809e-14	1.686e-05	1.970e-05
25	1426	4.036179e+02	6.144e-15	1.016e-05	1.383e-05
26	1479	4.036179e+02	1.776e-15	1.337e-05	1.552e-05
27	1563	4.036179e+02	2.498e-15	2.235e-05	1.894e-09

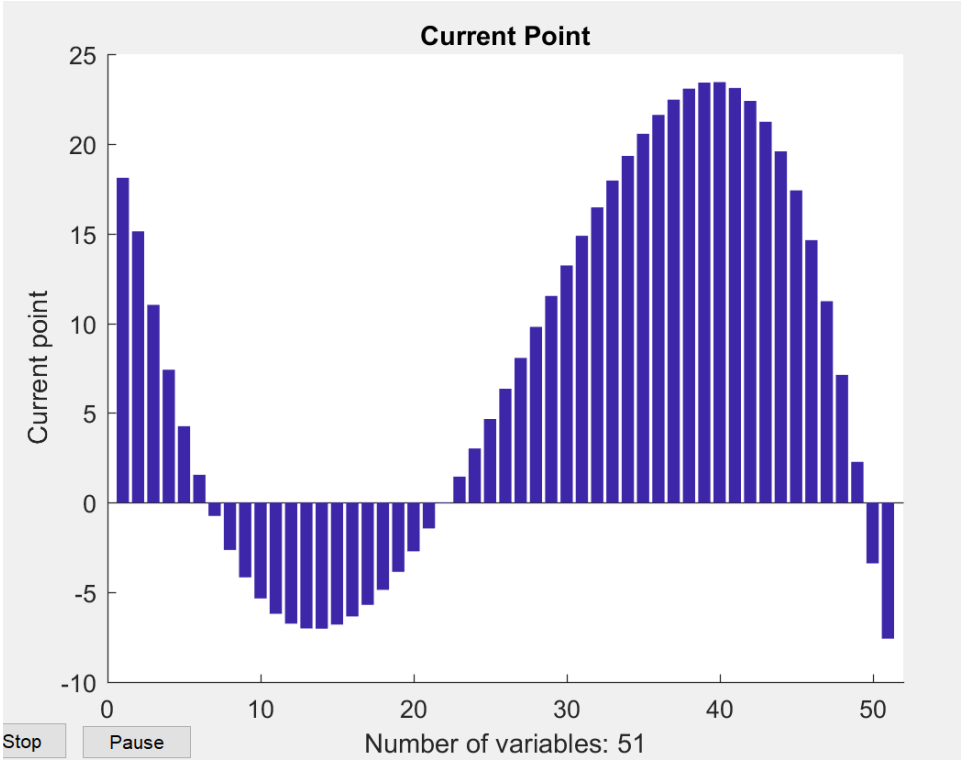
The value of cost function as calculated from the above is 4.036179e+02.

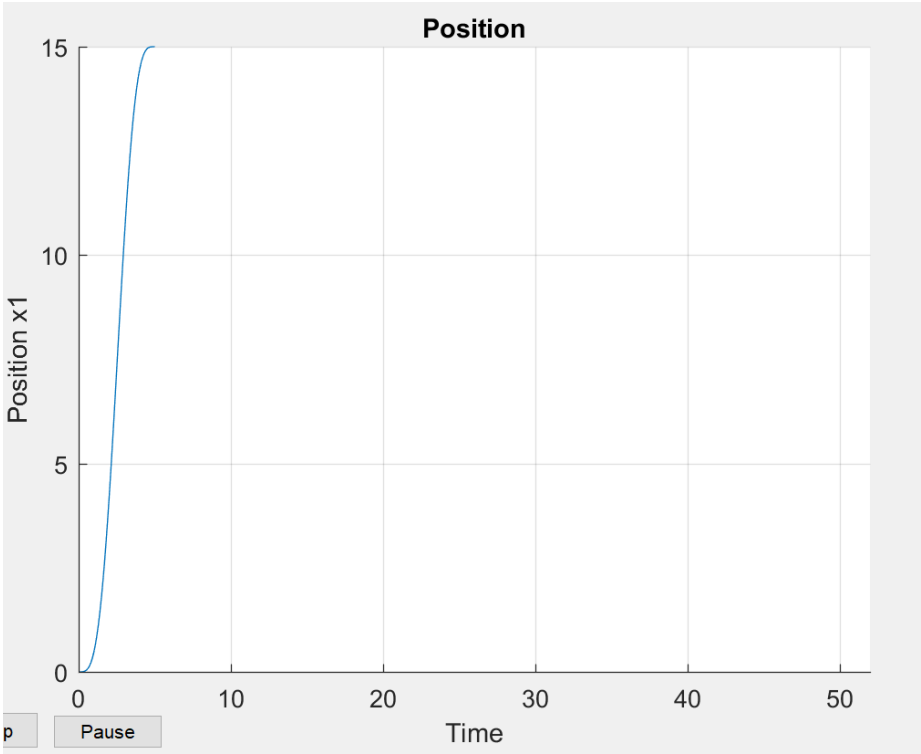
d)

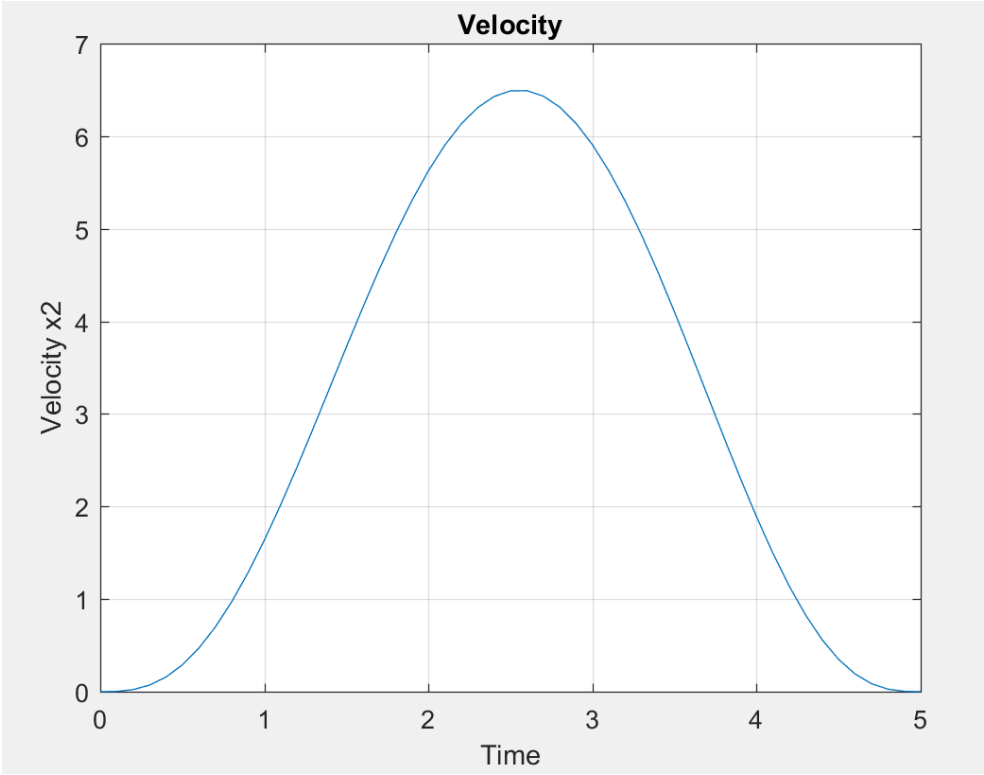
When position, velocity, acceleration and jerk are the hard constraints.

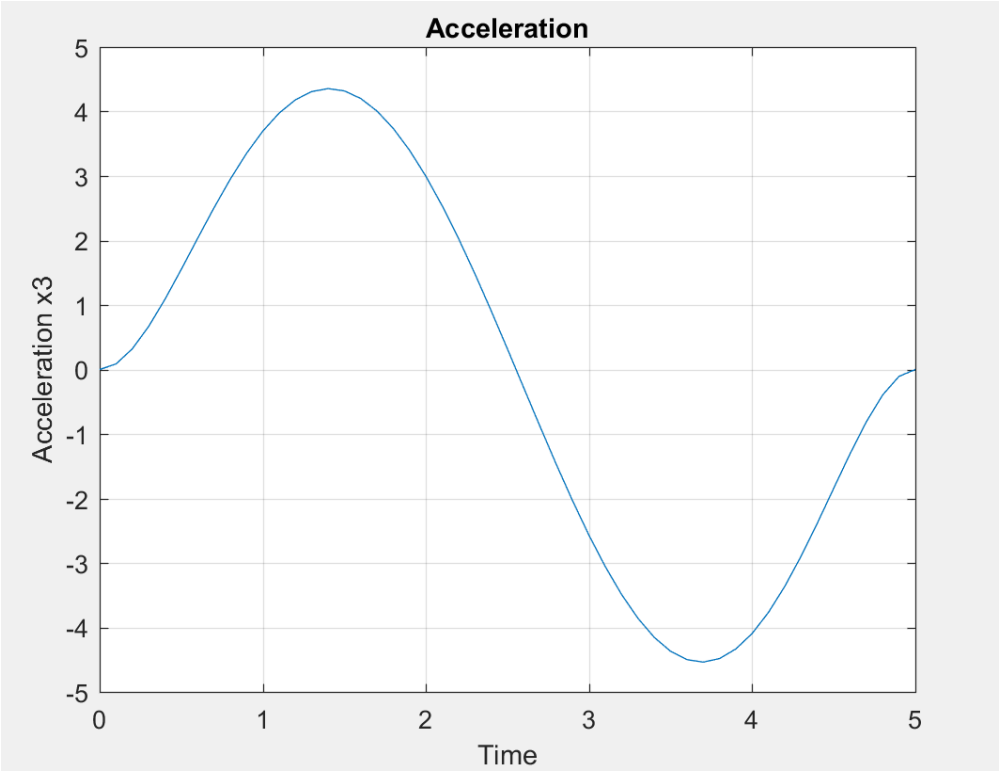
```
function [cineq,ceq] = hw3p2constraint4(u)
    cineq = [];
    t = 0:0.1:5;

    [tout,xout,yout] = sim('hw3p2',t',[],[t' u]);
    % Position, velocity, acceleration and jerk hard constraints
    ceq(1) = 15 - yout(end,1);
    ceq(2) = yout(end,2);
    ceq(3) = yout(end,3);
    ceq(4) = yout(end,4);
end
```

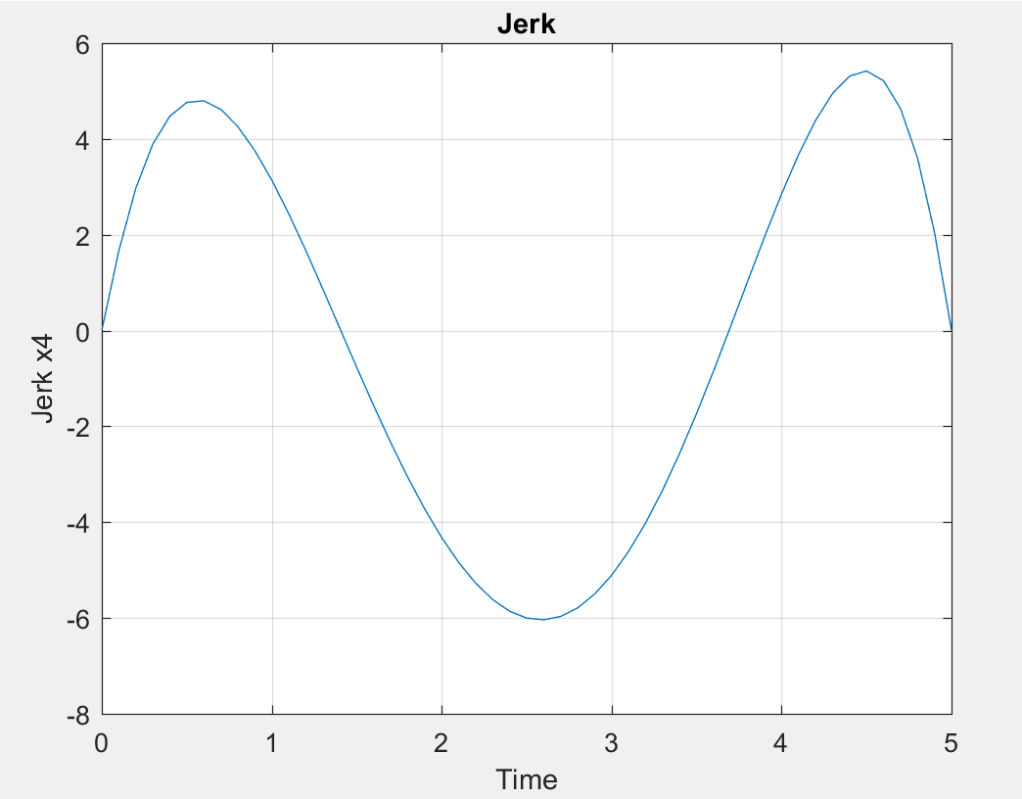
Result:

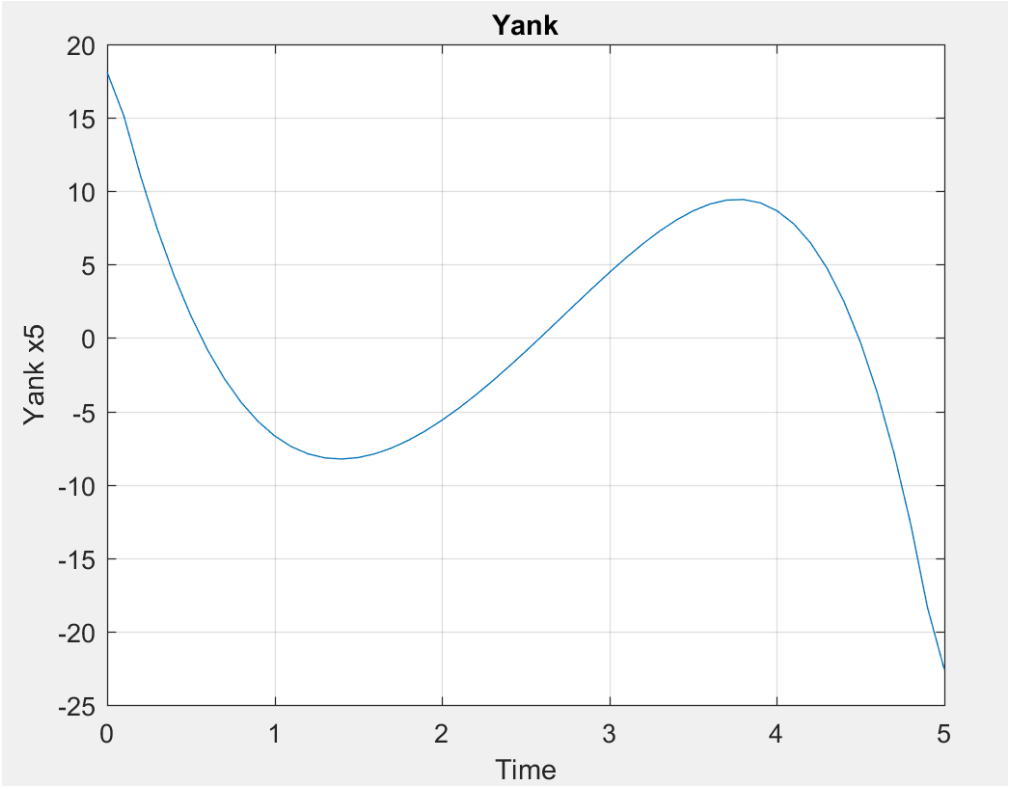

Current Point
Current point
25
20
15
10
5
0
-5
-10
0
10
20
30
40
50
Number of variables: 51
Stop
Pause


Position
Position x1
15
10
5
0
0
10
20
30
40
50
Time
Pause


Velocity
Velocity x2
Time
0
1
2
3
4
5
6
7


Acceleration
Acceleration x3
Time
-5
-4
-3
-2
-1
0
1
2
3
4
5


Jerk
Jerk x4
Time
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5


Yank
Yank x5
Time
20
15
10
5
0
-5
-10
-15
-20
-25
0
1
2
3
4
5

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	52	2.500000e-02	1.366e+01	9.647e-03	
1	104	4.201614e+02	1.589e-05	2.375e+00	9.242e+01
2	156	4.199108e+02	2.136e-07	4.228e-01	5.068e-01
3	208	4.189201e+02	7.820e-08	3.077e-01	2.396e+00
4	260	4.178173e+02	6.220e-07	5.692e-03	6.337e+00
5	312	4.178171e+02	5.369e-09	5.117e-03	1.769e-02
6	364	4.178161e+02	7.781e-09	2.987e-03	7.861e-02
7	416	4.178158e+02	2.719e-09	1.782e-03	8.466e-02
8	468	4.178158e+02	5.582e-11	1.673e-03	2.278e-03
9	520	4.178158e+02	2.878e-10	1.192e-03	1.030e-02
10	572	4.178157e+02	6.204e-10	2.484e-05	2.516e-02
11	624	4.178157e+02	7.018e-11	2.809e-05	3.023e-04
12	676	4.178157e+02	2.919e-11	1.605e-05	5.324e-05
13	736	4.178157e+02	5.419e-12	1.156e-05	4.389e-06
14	789	4.178157e+02	3.747e-15	2.274e-05	4.427e-05
15	846	4.178157e+02	1.160e-14	7.316e-06	8.854e-05
16	899	4.178157e+02	7.105e-15	1.394e-05	5.603e-05
17	959	4.178157e+02	3.908e-14	1.182e-05	3.942e-05
18	1017	4.178157e+02	2.354e-14	8.879e-06	7.885e-05
19	1070	4.178157e+02	6.314e-15	1.168e-05	7.266e-05
20	1130	4.178157e+02	1.520e-14	1.362e-05	1.771e-05
21	1188	4.178157e+02	1.910e-14	1.131e-05	1.771e-05
22	1241	4.178157e+02	1.210e-14	1.313e-05	2.363e-05
23	1294	4.178157e+02	1.693e-14	1.278e-05	3.157e-05
24	1355	4.178157e+02	2.220e-14	1.311e-05	1.579e-05
25	1413	4.178157e+02	8.045e-15	8.043e-06	1.579e-05
26	1493	4.178157e+02	5.329e-15	7.800e-06	3.854e-09
27	1546	4.178157e+02	2.623e-15	1.928e-05	7.709e-09
28	1606	4.178157e+02	1.332e-15	6.111e-06	3.854e-09

The value of cost function calculated is 4.178157e+02 as calculated above.

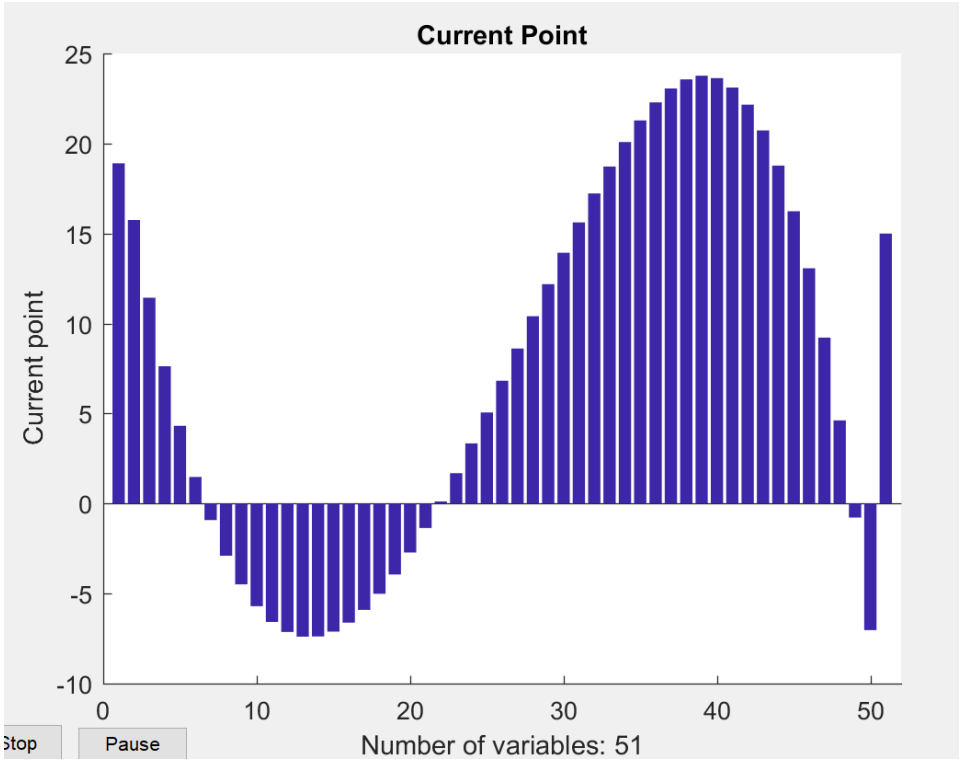
e)

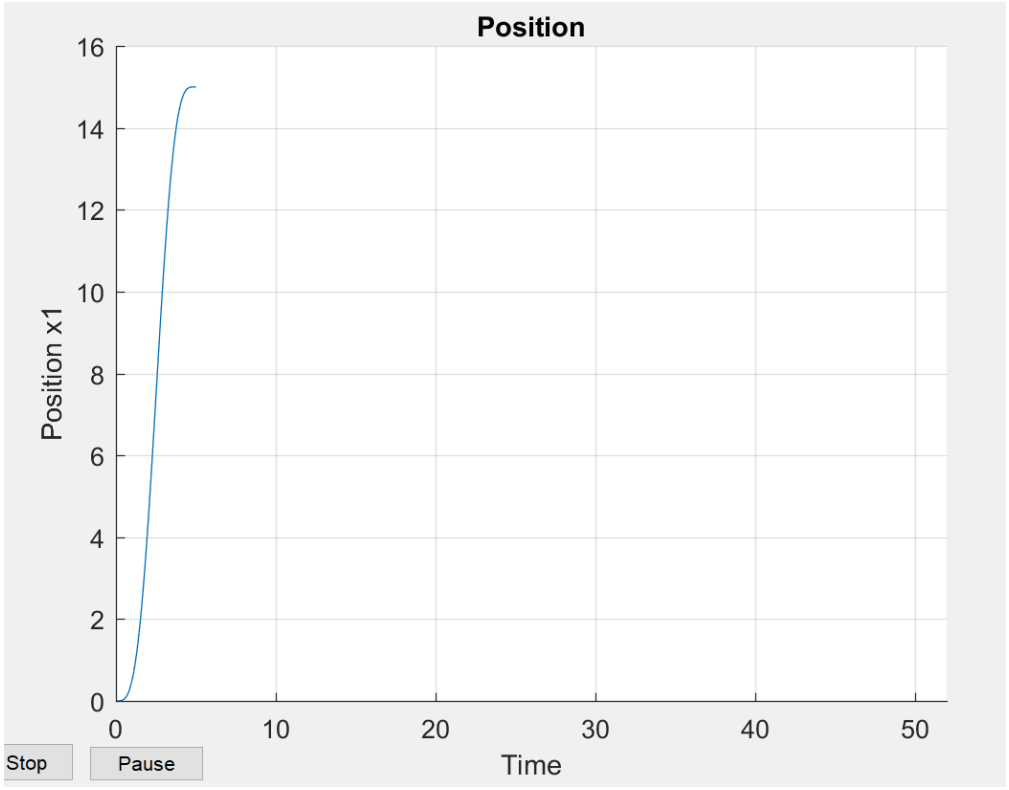
With position, velocity, acceleration, jerk and yank as the hard constraints

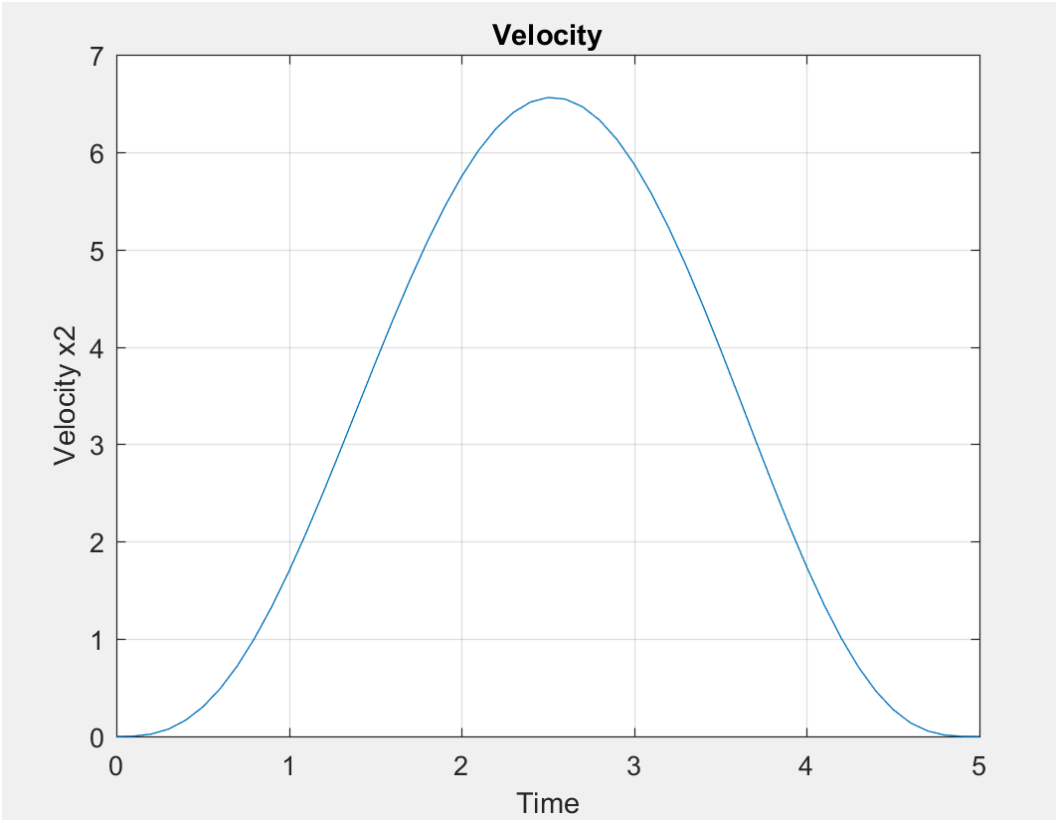
```
function [cineq,ceq] = hw3p2constraint5(u)
    cineq = [];
    t = 0:0.1:5;
```

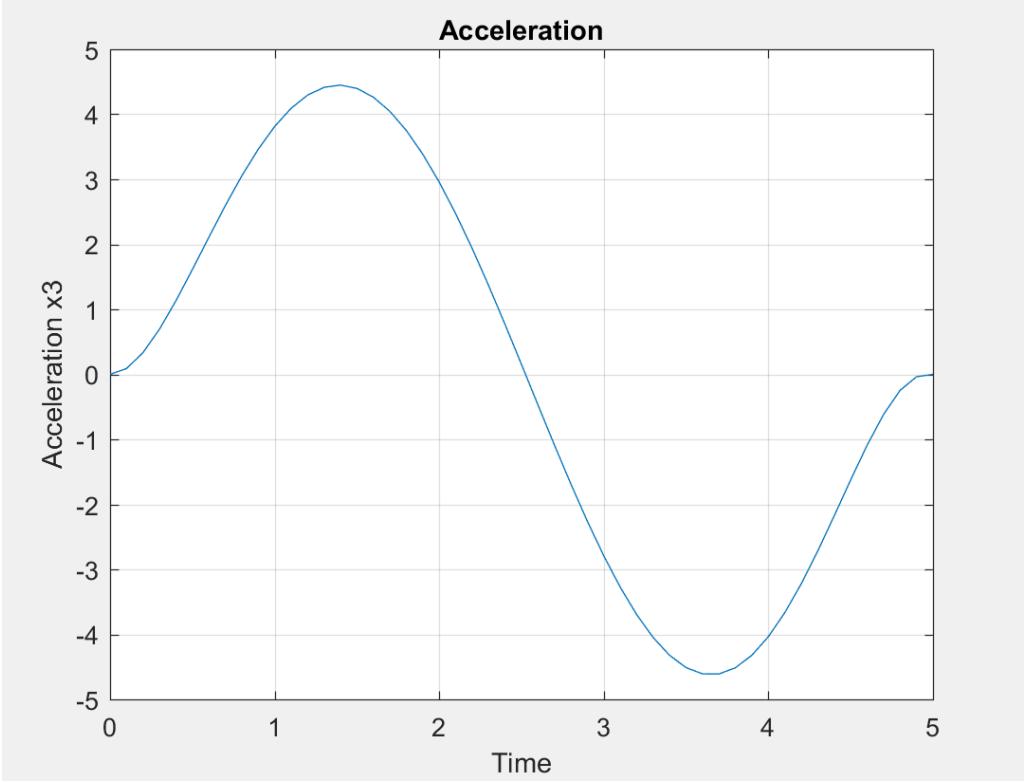
```
    [tout,xout,yout] = sim('hw3p2',t',[],[t' u]);
    % Position, velocity, acceleration and jerk hard constraints
    ceq(1) = 15 - yout(end,1);
    ceq(2) = yout(end,2);
    ceq(3) = yout(end,3);
    ceq(4) = yout(end,4);
    ceq(5) = yout(end,5);
end
```

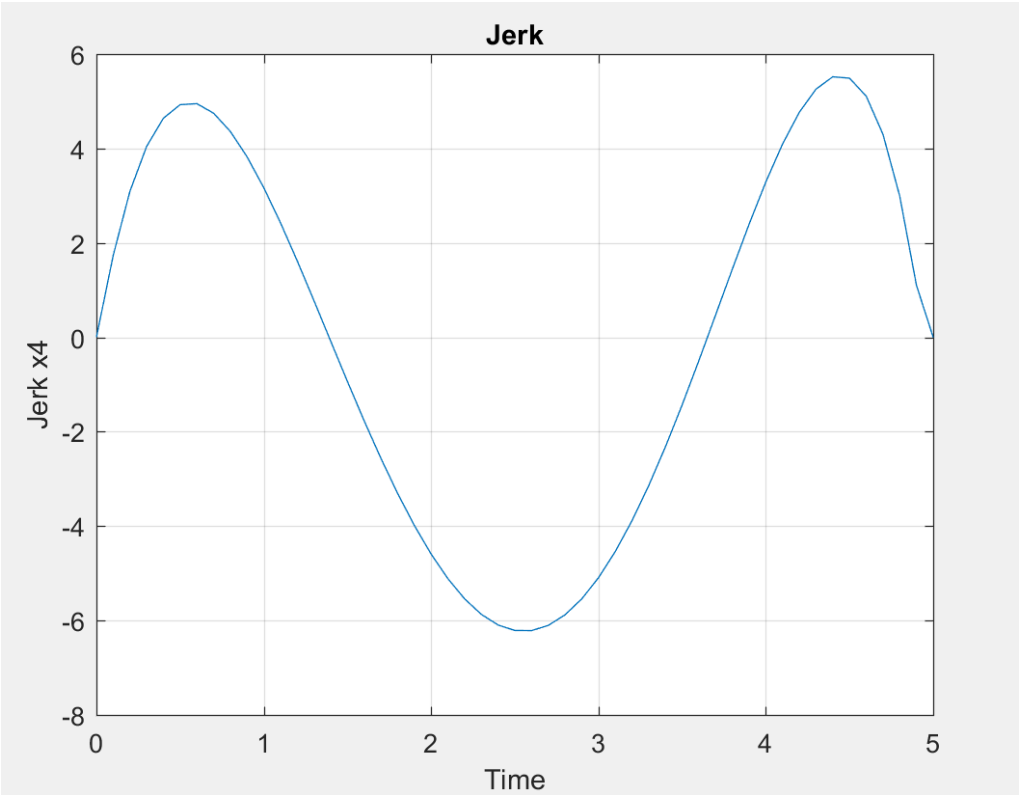
Result:

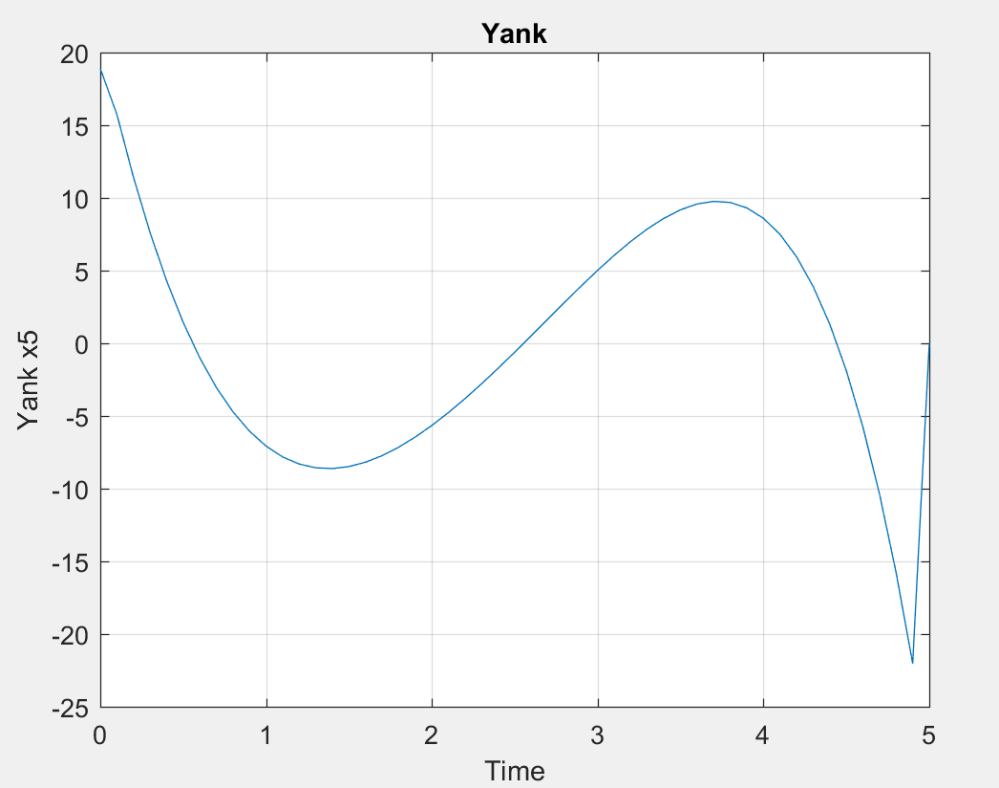



Position
16
14
12
10
8
6
4
2
0
Position x1
0
10
20
30
40
50
Time
Stop
Pause


Velocity
7
6
5
4
3
2
1
0
Velocity x2
0
1
2
3
4
5
Time


Acceleration
Acceleration x3
Time
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5


Jerk
Jerk x4
Time
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5



Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	52	2.500000e-02	1.366e+01	9.598e-03	
1	104	4.350495e+02	1.563e-05	2.405e+00	9.460e+01
2	156	4.348182e+02	1.429e-07	4.420e-01	4.868e-01
3	208	4.339013e+02	3.469e-07	3.224e-01	2.303e+00
4	260	4.328629e+02	8.778e-07	5.918e-03	6.182e+00
5	312	4.328627e+02	1.942e-09	5.333e-03	1.735e-02
6	364	4.328618e+02	1.566e-08	2.654e-03	7.657e-02
7	416	4.328615e+02	1.007e-08	1.693e-05	7.522e-02
8	468	4.328615e+02	8.578e-12	3.981e-05	5.678e-05
9	520	4.328615e+02	2.295e-10	2.678e-05	3.340e-04
10	572	4.328615e+02	3.240e-11	1.615e-05	2.231e-04
11	626	4.328615e+02	8.397e-11	1.001e-05	1.508e-04
12	679	4.328615e+02	2.758e-11	2.070e-05	3.497e-05
13	731	4.328615e+02	1.121e-11	8.846e-06	3.905e-05
14	784	4.328615e+02	6.807e-12	1.140e-05	2.443e-05
15	839	4.328615e+02	7.815e-12	1.404e-05	1.779e-05
16	895	4.328615e+02	1.954e-12	8.636e-06	3.558e-05
17	955	4.328615e+02	1.465e-14	1.511e-05	3.113e-05
18	1012	4.328615e+02	1.510e-14	1.679e-05	6.227e-05
19	1073	4.328615e+02	1.745e-14	1.192e-05	3.113e-05
20	1127	4.328615e+02	1.558e-12	9.188e-06	1.557e-05
21	1180	4.328615e+02	1.533e-12	3.227e-05	4.663e-05
22	1240	4.328615e+02	9.213e-15	2.111e-05	2.332e-05
23	1295	4.328615e+02	9.909e-13	1.947e-05	1.166e-05
24	1352	4.328615e+02	1.123e-14	7.096e-06	2.332e-05
25	1409	4.328615e+02	1.044e-14	7.634e-06	2.332e-05
26	1464	4.328615e+02	5.598e-13	1.578e-05	5.829e-06
27	1520	4.328615e+02	3.055e-12	1.146e-05	1.166e-05
28	1580	4.328615e+02	4.245e-13	2.062e-05	1.457e-06
29	1637	4.328615e+02	5.496e-15	1.299e-05	2.915e-06
30	1694	4.328615e+02	1.155e-14	7.758e-06	5.829e-06

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	1750	4.328615e+02	4.696e-13	1.698e-05	5.829e-06
32	1814	4.328615e+02	1.558e-13	2.666e-05	1.822e-07
33	1870	4.328615e+02	7.172e-14	1.515e-05	1.822e-07
34	1930	4.328615e+02	5.407e-14	5.339e-06	2.277e-08
35	1986	4.328615e+02	1.421e-14	9.608e-06	2.277e-08
36	2039	4.328615e+02	1.033e-14	9.441e-06	4.554e-08

The value of cost is 4.328615e+02 according to the above results.

f) Thus, it can be observed that the value of cost increases as more constraints are added to the constraint function.

Constraint	Cost
Position	*1.084715*
Position, velocity	*1.275994*
Position, velocity, acceleration	*403.6179*
Position, velocity, acceleration, jerk	*417.8157*
Position, velocity, acceleration, jerk, yank	*432.8615*