EE5321, Spring 2018
Homework 3: Continuous Time Optimal Control – Due Feb 22

Problem 1) 40 points

Given the following performance index $J = \int_0^8 \frac{1}{2}u^2\, dt$ and the state constraints $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$, with $x_1(0) = 0$ and $x_2(0) = 0$.

a) Implement the requirements $x_1(8) = 100$ and $x_2(8) = 0$ as hard constraints and analytically determine the state and control time histories and the associated optimal cost J.
b) Implement the state dynamics in Simulink using a fixed step solver size of 0.1 seconds and use *fmincon* to determine the optimal state and control time histories for the system and constraints described above, breaking the control into 81 parameters that you optimize. See the PowerPoint handout dated February 1 to help get started.
c) In your constraint function, change the $x_1(8) = 100$ to $x_1(8) \geq 100$ and rerun. Note if the solution changes and comment on the amount of time it took to reach an answer compared to part b).

Problem 2) 60 points

With a fixed step solver of 0.1 seconds, implement in Simulink the following equation

$$x'''' - x = u \qquad x(0) = x'(0) = x''(0) = x'''(0) = x''''(0) = 0,\ x(5) = 15$$

I suggest you set the initial control to be 0.1 for the first half of the time interval and -0.1 for the second half of the time period. For each problem, plot the values of x, x', x'', x''' and x'''' and the optimal control time history and the final cost. Note that x is position, x' is velocity, x'' is acceleration, x''' is jerk and x'''' is jounce (or yank).

a) With the performance index $J = \int_0^5 \frac{1}{2}u^2\, dt$, use *fmincon* to determine the optimal control time history and cost with the only constraint being $x(5) = 15$.
b) Repeat the optimization after adding the additional hard constraint $x'(5) = 0$.
c) Repeat the optimization after adding the additional hard constraint $x''(5) = 0$.
d) Repeat the optimization after adding the additional hard constraint $x'''(5) = 0$.
e) Repeat the optimization after adding the additional hard constraint $x''''(5) = 0$.
f) Plot the final cost values as each constraint is added, showing whether the final cost increases or decreases as more hard constraints are added.