EE5321, Spring 2018
Homework Assignment 4: Minimum Time
Due 3/29/2018

Problem 1: **SRAM Missile** (50 points)
The Simulink model and associated scripts are posted on the class website for this problem.

The equations of motion in a 2D plane of a short range attack missile (SRAM) are

$$\dot{X} = \frac{r_s V \cos(\gamma)}{(r_s + h)}$$

$$\dot{h} = V \sin(\gamma)$$

$$\dot{V} = \frac{-\rho V^2 S C_D}{2m} - \frac{g_s r_s^2 \sin(\gamma)}{(r_s + h)^2}$$

$$\dot{\gamma} = \frac{\rho V S C_L}{2m} + \left[\frac{V}{(r_s + h)} - \frac{g_s r_s^2}{V(r_s + h)^2}\right]\cos(\gamma)$$

With initial conditions $X(0) = 0\ ft$, $h(0) = 100{,}000\ ft$, $V(0) = 11{,}000\ ft/sec$, and $\gamma(0) = 0\ rad$.

Note that $m = 15.51\ slugs$, $S = 1.5\ ft^2$, $g_s = 32.174\ \frac{ft}{s^2}$, $r_s = 20{,}908{,}092\ ft$, $\rho = 0.0023769\ exp\left(\frac{-h}{23{,}800}\right)\frac{slug}{ft^3}$, and $C_D = 0.043\left(1 + \left(\frac{C_L}{0.2888}\right)^2\right)$. The control variable is $C_L(t)$. We desire to hit a target at the final time that is located at $h(t_f) = 0\ ft$ and $X(t_f) = 72\ nm$.

Finally, to make the numerical optimization work, the final time parameter, the final condition constraints, and the velocity cost function are scaled to be about the same order of magnitude.

Start with your guess of final time of 40 seconds, but divide it by 100 and divide the velocity cost by 11,000. I suggest you divide the height constraint by 100,000 and the distance constraint by 500,000. Feel free to experiment. Also start with the initial guess of lift coefficient to be -0.1 or -0.2 for the entire time interval.

a) Determine the optimal control to <u>maximize</u> the performance index $J = V(t_f)$. Plot height versus distance, velocity as a function of time, and the flight path angle as a function of time. Also plot the control versus time. I suggest you scale the cost by dividing it by 11,000. (10 points)
b) Determine the optimal control to <u>minimize</u> the performance index $J = t_f$. Plot height versus distance, velocity as a function of time, and the flight path angle as a function of time. Also plot the control versus time. (10 points)

c) Compare the final velocity and final time determined in parts a) and b). (10 points)
d) We need to determine if atmospheric variations can potentially cause variations in the final solution. With the performance index set in part b), min time, determine the variation in optimal time, control, and states when the atmospheric density is varied by 10% (i.e. use 0.0023769*0.9 in one case and use 0.0023769*1.1 in another and compare to the baseline generated in part b). How much of a change in the optimal solutions did the variations cause? (20 points)

Problem 2: **Minimum Time Control of Zermelo's Problem.** (50 points)
The cost function is set to minimize the amount of time it takes to go from the initial condition to the origin. I suggest your initial guess for control is to make all values 255 degrees for the following (but recall that Simulink thinks in radians by default):

$$\dot{x} = V\cos\theta + Vy$$
$$\dot{y} = V\sin\theta$$
$$(x_o, y_o) = (4.9, 1.66)$$
$$V = 1$$
$$J = t_f$$
$$x_f = 0$$
$$y_f = 0$$

a) Determine the minimum final time to travel from the prescribed initial condition to the origin. Plot the state and control time histories. (25 points)
b) Modify the Simulink model to implement the first equation of motion for Zermelo's Problem, shown below (***<u>note the y-cubed term</u>***). Compare the results (states and control) of the optimization from the original solution to this new solution (with the y-squared term). How does having the y-squared term change the solution (note similarities and differences)? (25 points)

$$\dot{x} = V\cos\theta + Vy^3$$