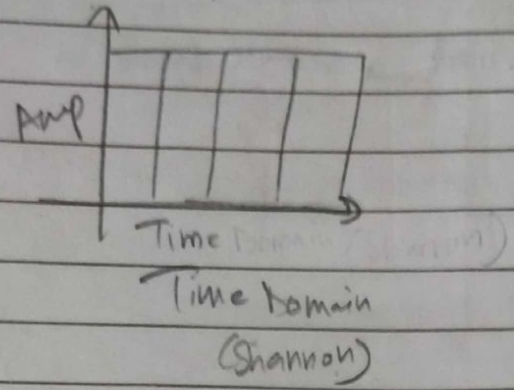
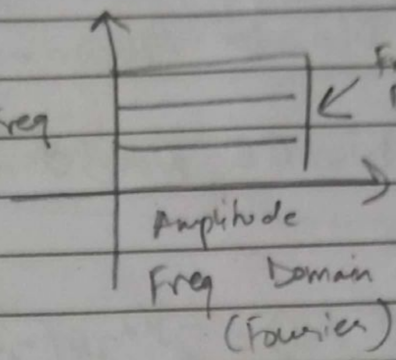
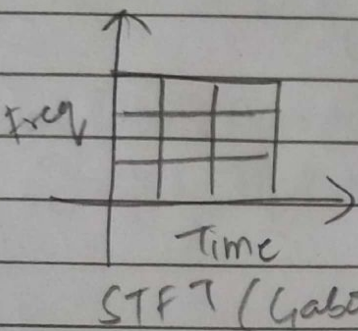
$e^{ix} = \cancel{\cos x}\ \sin x + i\cos y$
$(ux+vy)$

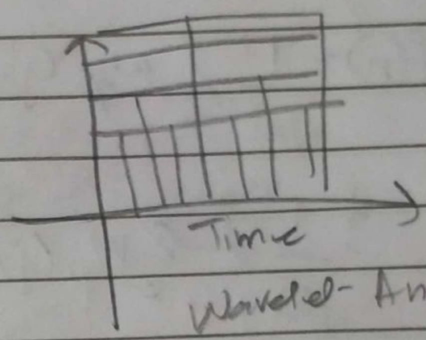
Time → "when" Fourier → "frequency"

Multiplexing → break into diff freq put in same channel









Continuous Wavelet Transform

Define fn $\psi(x)$
assume $\psi(x)$ band-limited and dc component $= 0$

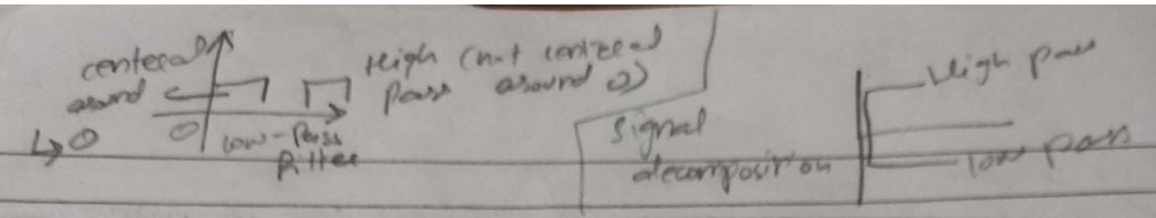
$$\psi_{s,t}(x) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{x-\tau}{s}\right)$$ → delay (shift inversion)

Wavelet taking snapshots of information

Continuous wavelet transform

$$W_\phi(s,\tau) = \int_{-\infty}^{\infty} f(x)\,\psi_{s,\tau}(x)\,dx$$

Wavelet multiplication is like convolution



Brownian Motion → collision → white noise.

Discrete Wavelet Transform

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$$

$$f(n) = \frac{1}{\sqrt{M}} \sum_k W_\phi(j_0, k)\phi_{j_0,k}(n)$$

$$+ \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j,k)\psi_{j,k}(n)$$

Scalogram → wavelet coefficient ψ

$\boxed{5+3j}$ → Magnitude & phase.

Boundaries of Objects & Edges

H - Transform, (we've already done wavelet (w) & frequency (f) - transform)

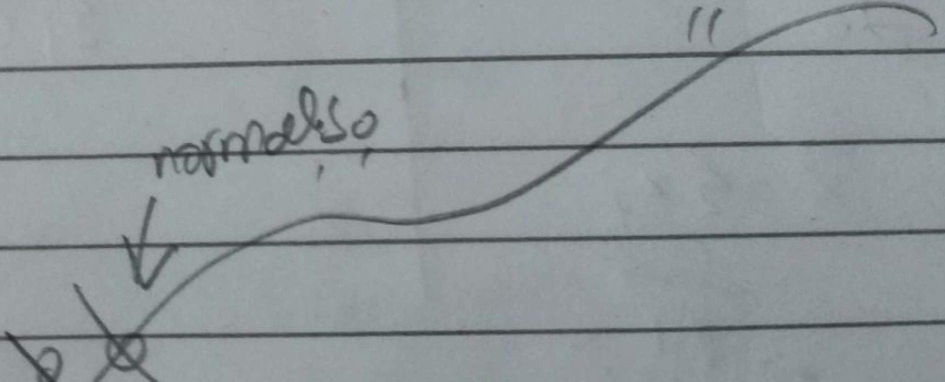
H-Tranform → Half Transform
↳ To detect boundaries

Img → (Edge detection, Thresholding) → edge image (Incomplete boundaries) → (Shrink & Expand) → Thinning

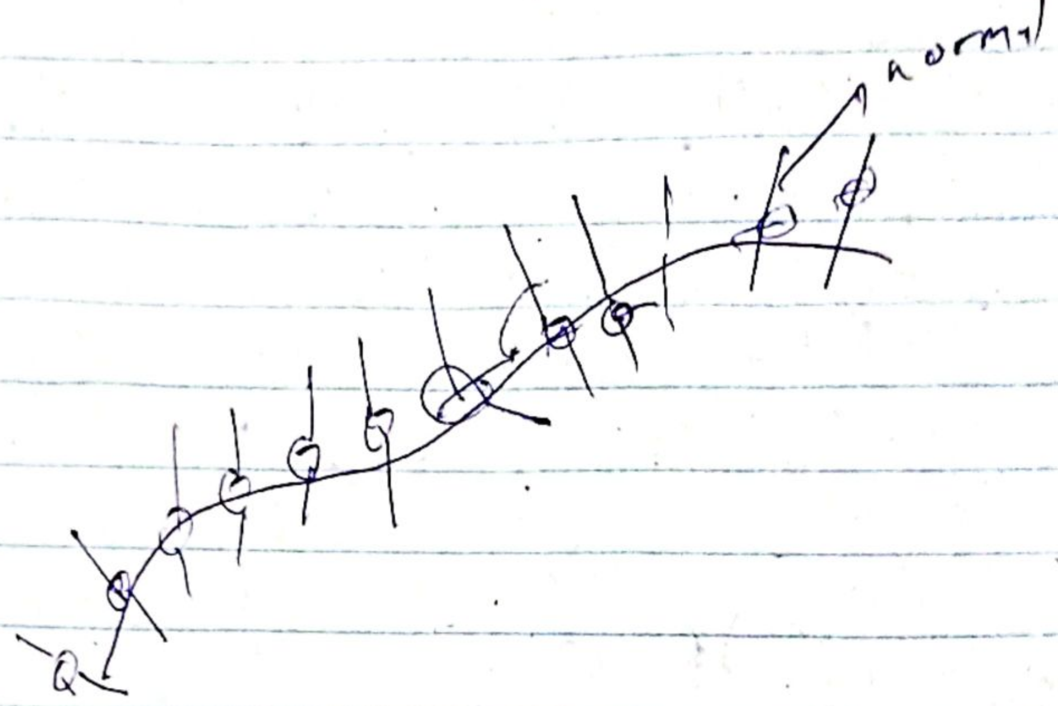
Edge Tracking Methods

Adjust a-priori boundaries

Given : Approximate Location of Boundary
Task : Find accurate Location of Boundary



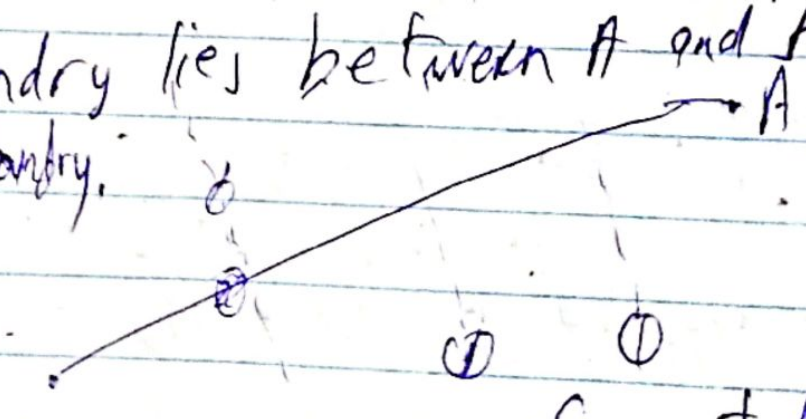
Given:- Approximate Location of bound
task:- Find Accurate location of boundry.



- search for strong edges along normals to approximate boundry

- Fit curve (eg., polynomial) to strong edges.

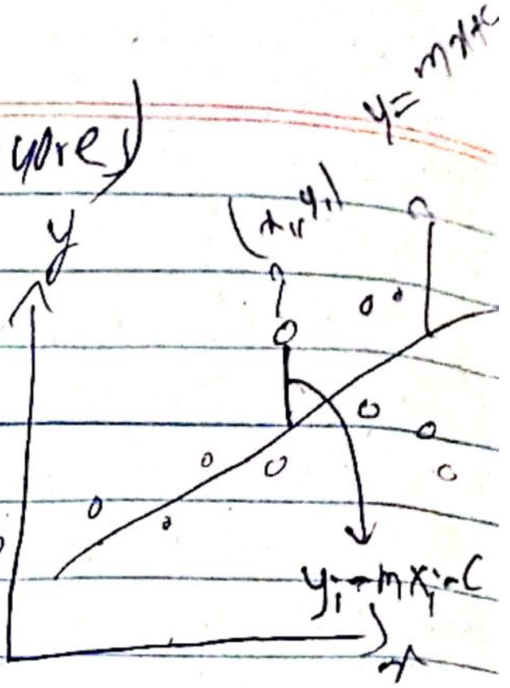
Given: Boundry lies between A and B.
Task: Find boundry.



use edges as break point

- Connect A and B with Line
- Use edge point as break point
- repeat.

Fitting Lines to edges (Least Square)



Given: Many (x_i, y_i) pairs
To find: Parameter (m, c)

Minimize: Average square distance
Error

$$E = \frac{\sum_i (y_i - mx_i - c)^2}{N} \rightarrow (1)$$

Using

$$\frac{\partial E}{\partial m} = 0 \; (2) \quad \& \quad \frac{\partial E}{\partial c} = 0 \; (3)$$

Note:

$$\bar{y} = \frac{\sum_i y_i}{N} \; (4), \quad \bar{x} = \frac{\sum_i x_i}{N} \rightarrow (5)$$

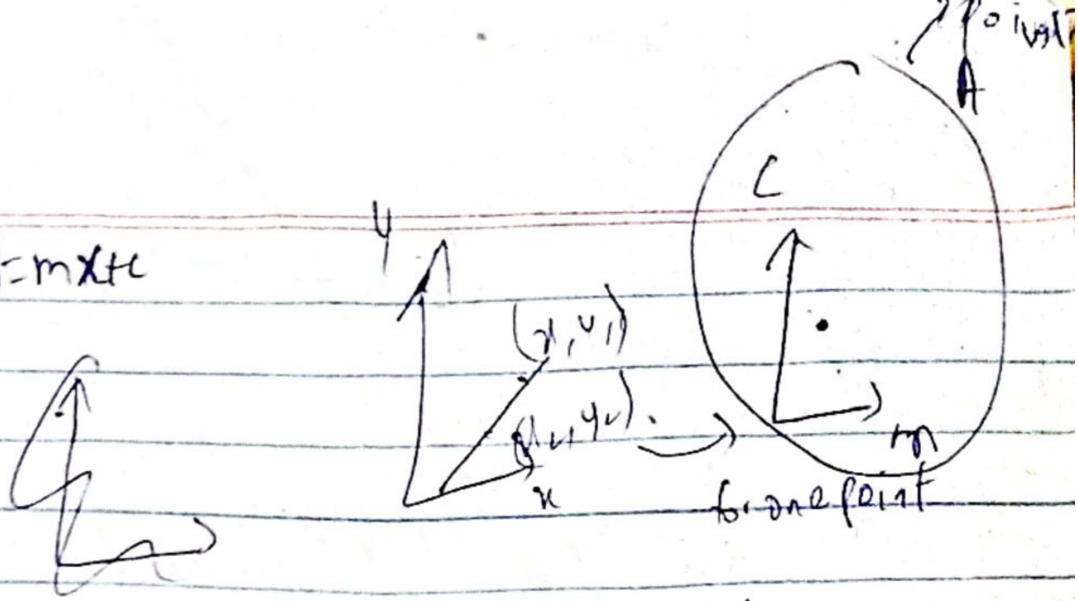
intercept

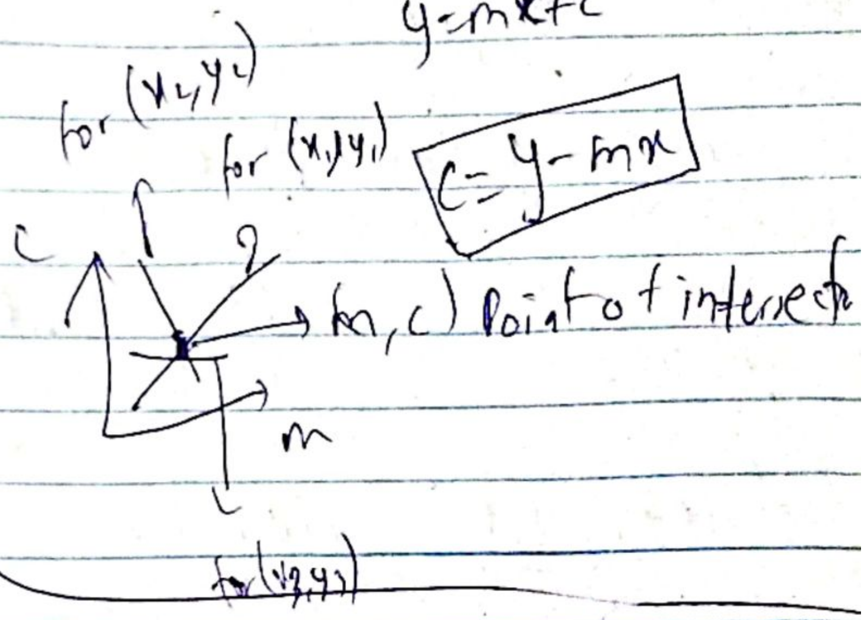
$$c = \bar{y} - m\bar{x} \rightarrow (A)$$

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} \; - (B)$$

slope

$y = mx + c$





Algorithm:-

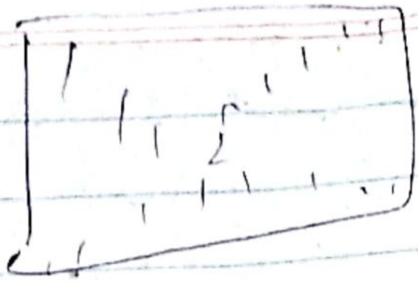
- Quantize Parameter Scope
- Create Accumulator Array $A(m, c)$
- Set $A(m, c) = 0 \ \forall \ m, c$
- For each image edge (x_i, y_i) increment:

$$A(m, c) = A(m, c) + 1$$

If (m, c) lies on line:

$$c = -x_i m + y_i$$

Find ~~the~~ local maxima in $A(m, c)$



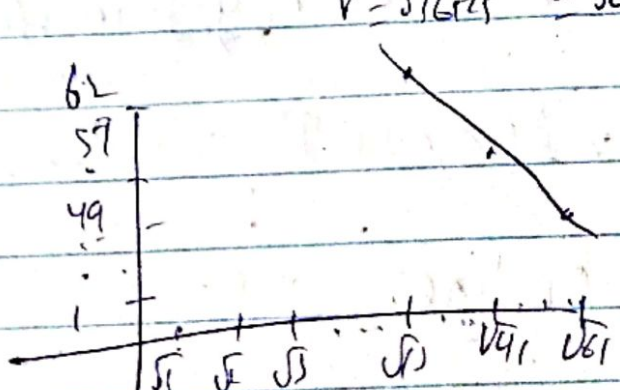
we convert linear to polar cordinate system.

because in x,y plane

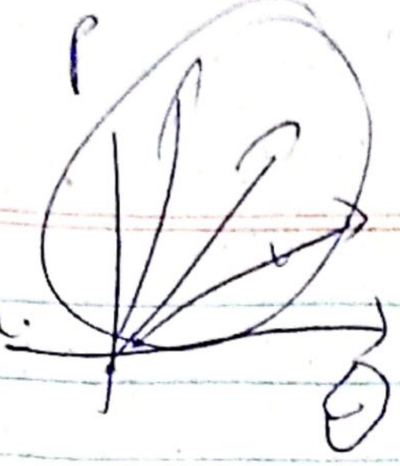
$$\left.\begin{aligned} m &\to -\infty, \infty \\ c &\to -\infty, \infty \end{aligned}\right\}$$ too many points to be made.

in rectangular coordinate.

A(2,3)	$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{13}$	$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 62$
B(4,5)	$r = \sqrt{16+25} = \sqrt{41}$	$\theta = \tan^{-1}\left(\frac{5}{4}\right) = 57$
C(6,5)	$r = \sqrt{36+25} = \sqrt{61}$	$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 49$



Huff transform



d
For boundary detection.

→ Fourier 1d and 2d
F
W → 1d and 2d
H
↓
Hough → transformation within plane.

Difficulties
- How many lines should pass.

SIFT. → scale invariant feature transform

. ~~SWIWS~~ Transform.

. Wavelet transform.

└ It is used to detect salient ~~using~~ features.

Wavelet transform 1 and wavelet transform

. what is correspondence of features
↓
relevancy.

. identify (distinct feature)

. Local feature (colour, size)

$I(x_i, y_i)$
↳ Intensity of pixel

Locally stable
If you change/rotate picture is the feature doesn't change

. direction of edges

Problem:-

Using H-Transform show that the points (1,1), (2,2) and (3,3) are collinear

Solution:-

$y = mx + c \longrightarrow (1)$

$c = y - mx \longrightarrow (2)$

For (1,1)

put $c=0$ in eq (2)
$m=0$ in eq (2)

$(1,1) \rightarrow (1,1)$
(c,m)
$(2,2) \rightarrow (2,1)$
$(3,3) \rightarrow (3,1)$

(1,1)

$c = y - mx$
for $c=0$

$0 = 1 - 1m$

$1m = 1$

$\boxed{m=1}$

for $m=0$

$c = y - 0(x)$

$c = 1$

(2,2)

$0 = 2 - 2m$

$2m = 2$

$m = 1$

$c = y - 0(x)$

$c = 2$

(3, 3) For $m = 0$

$C = y - mx$

$C = 3 - (0)x$

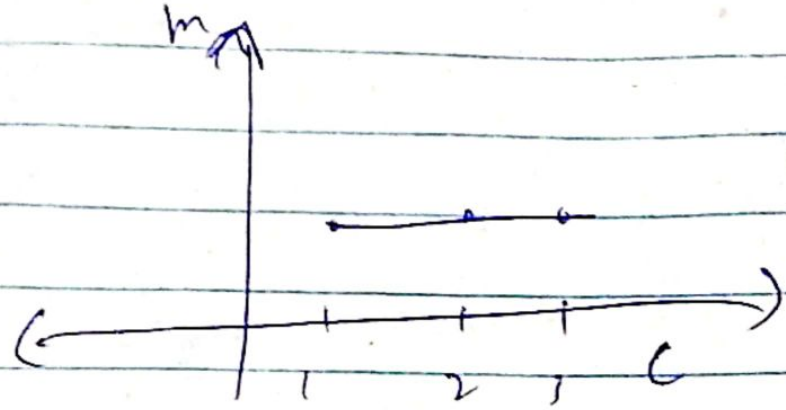
$\boxed{C = 3}$

For $c = 0$

$0 = 3 - 3m$

$\cancel{3}m = \cancel{3}$

$\boxed{m = 1}$



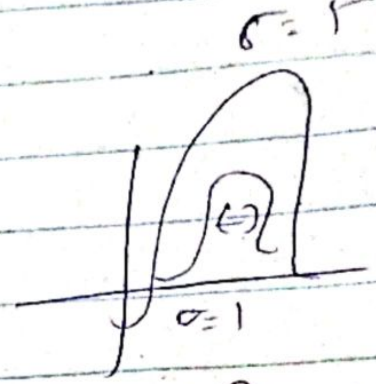
Steps for SIFT

- Approximate location and salient feature point
- Refine their location and scale
- Determine orientations for each keypoints.
- Define descriptor foreach keypoint.

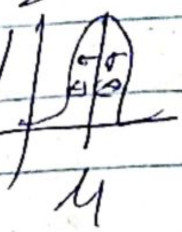
filter:

It is a

$$I(x,y) * Filter(x,y)$$



Gaussian Filter



$$G(x,y,\sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x,y,\sigma) = G(x,y,\sigma) * I(x,y) \quad \text{— (A)}$$

scaling factor

$$D(x,y,\sigma) = \left(G(x,y,k\sigma) - G(x,y,\sigma)\right) * I(x,y) \quad \text{— (D)}$$

DoG

Difference of Gaussian.

If Gaussian feature is high if DoG is high.

Or there are no salient feature.

high scale par distinct features nhi milte. → filter size

Taylor series:-
1^{st} order, 2^{nd}, 3^{rd} derivative.

Hessian matrix:-

It defines derivatives in a matrix.

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

- The eigen values of H give alot of information about the local ~~feature~~ structure around the key point.

give more info about feature

- In fact, the eigenvalues are the maximal and minimal principal curvatures of the surface $D(x,y)$ i.e. of the DoG function at that point.

give less info about feature

Wavelet
Heff
SIFT

$$L(x,y,\sigma) = G(x,y,\sigma) * I(x,y)$$

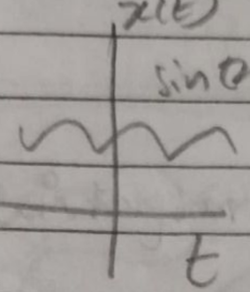
$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

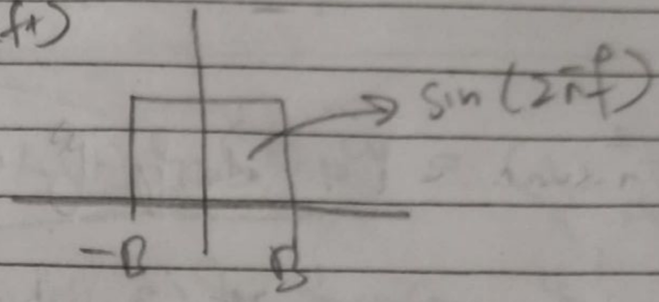
$$\theta(x,y) = \tan^{-1}((L(x,y+1) -$$

$$\mu_1(t) = \frac{\sum_{i=1}^{t-1} i\,p(i)}{w_1 t}$$

$$\mu_T = \sum_{i=0}^{L-1} i\,p(i)$$

March 15th, 2023





STFT
↓
Short Time Fourier Transform

T-f analysis