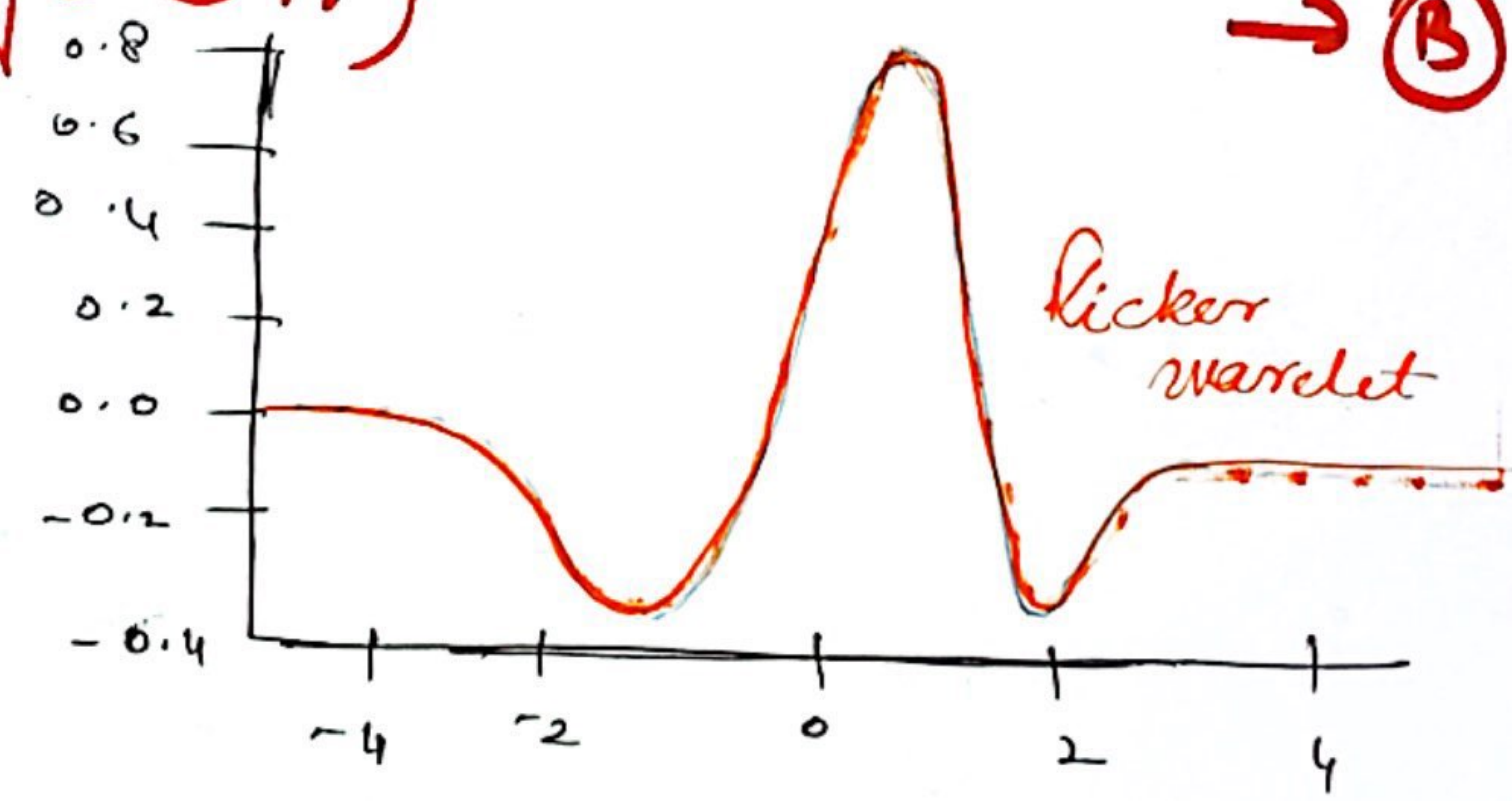
Example Problems: 29-03-2013

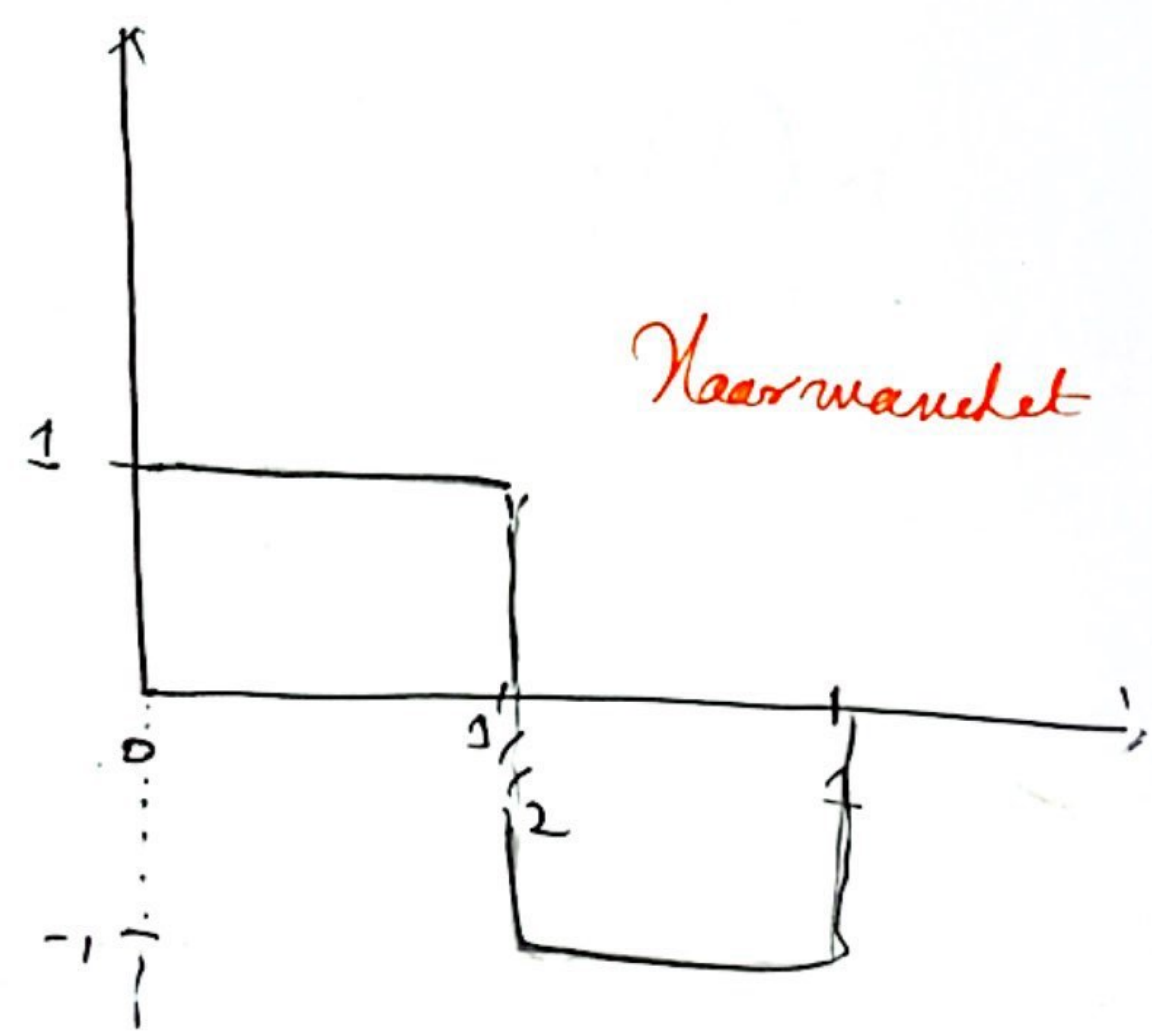
1 Given Haar Wavelet as:

$$\text{(A)}\quad \psi(t) = \begin{cases} 1 & 0 \le t < 1/2 \\ -1 & 1/2 \le t < 1 \\ 0 & \text{otherwise} \end{cases} \rightarrow \text{(A)}$$

Ricker wavelet:—

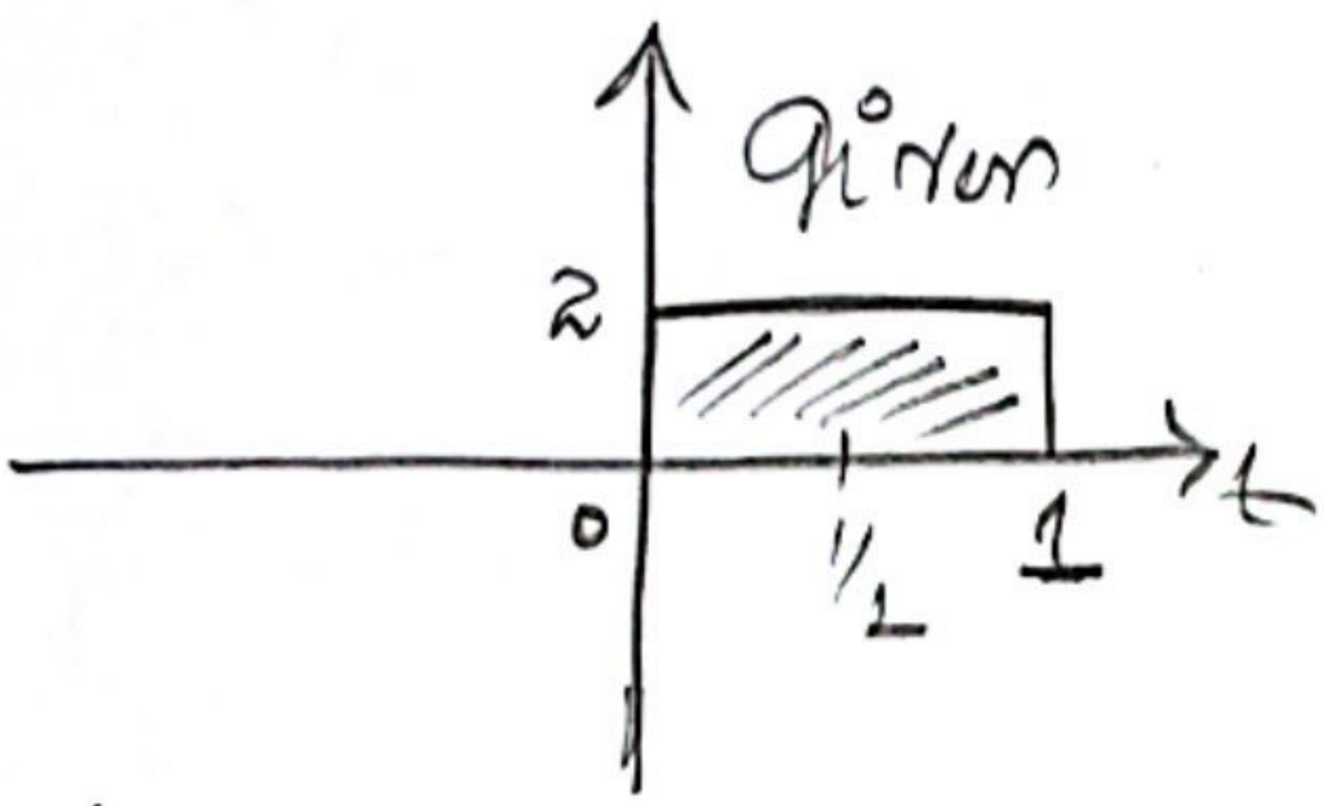
$$\text{(B)}\quad \psi(t) = \frac{2}{\left(\sqrt{3\sigma\pi}\right)^{1/4}}\left(1-\left(\frac{t}{\sigma}\right)^2\right)e^{-\frac{t^2}{2\sigma^2}} \rightarrow \text{(B)}$$





PROBLEM 1.1:.

Consider a rectangular pulse signal as shown below that shows 2-D time-freq grid of an image. Use Haar wavelet to obtain wavelet transform with in the given limits.



Solution:- Haar wavelet is given in eq (A)

$$\psi(t) = \begin{cases} 1 & 0 \le t < 1/2 \\ -1 & 1/2 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

The wavelet Transform:-

$$W_\phi(s,\tau) = \int_{\pm\infty} f(t)\, \psi_{s,\tau}(t)\, dt$$

$$= \int_0^{1/2} \underbrace{(2)}_{f(t)}\underbrace{(1)}_{\psi(t)}\, dt + \int_{1/2}^{1} (2)(-1)\, dt + 0$$

$$= 2\int_0^{1/2} dt - 2\int_{1/2}^{1} dt$$

$$= 2\, t\Big|_0^{1/2} - 2\, t\Big|_{1/2}^{1}$$

PROBLEM 1.2:

Given the Ricker wavelet, plot the wavelet with the following values of σ and t.

① $\sigma = 2$ $t = 3$

② $\sigma = 3$ $t = 4$

③ $\sigma = 5$ $t = 6$

Solution:-

Use equation (15)

$$\psi(t) = \frac{2}{(\sqrt{3\sigma\pi})^{1/4}}\left(1 - \left(\frac{t}{\sigma}\right)^2\right)e^{-t^2/2\sigma^2}$$

Put $\sigma = 2$ $t = 3$

$$\psi(3) = \frac{2}{(\sqrt{3 \cdot 2\pi})^{1/4}}\left(1 - \left(\frac{3}{2}\right)^2\right)e^{-\frac{3^2}{2\times 2^2}} =$$

Similarly put ② and ③ and get results

Problem 1.3:

Given $f(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ 0 & \text{Otherwise.} \end{cases}$

Obtain wavelet transform using Haar wavelet.

Solution:

$$\psi(t) = \begin{cases} 1 & 0 \le t < 1/2 \\ -1 & 1/2 \le t < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$W_f(s, \tau) = \int_{-\infty}^{+\infty} f(t)\, \psi_{s,\tau}(t)\, dt$$

$$= \int_0^{1/2} t^2 \cdot 1 \, dt + \int_{1/2}^{1} t^2(-1)\, dt$$

$$= t^3/3 \Big|_0^{1/2} - 1 \, \frac{t^3}{3} \Big|_{1/2}^{1}$$

Scanned with CamScanner

Hough transform:

Problem:-

Using hough transform show that the points (1, 4), (2, 3) and (3, 4) are collinear?

Solution:

$$y = mx + c \rightarrow ①$$

$$c = -mx + y \rightarrow ②$$

Take 1st point (1, 4) in x-y plane

eq 2

$$c = -mx + y \rightarrow ②$$

Put $c = 0$ $x = 1, y = 4$

$$0 = -1m + 4$$

$$\boxed{m = 4}$$

Put $m = 0$ $x = 1, y = 4$

$$c = 0 + 4$$

$$\boxed{c = 4}$$

(c, m)
(4, 4)

Take 2ND point:-
(2, 3)

$$c = -mx + y$$

$c = 0$

$$0 = -m \times 2 + 3$$

$$2m = 3$$

$$\boxed{m = 3/2}$$

$m = 0$

$$c = 0 + 3$$

$$\boxed{c = 3}$$

$(3, 3/2)$ ③

Take 3RD point
(3, 4)

$$c = -mx + y$$

$c = 0$

$$0 = -m \times 3 + 4$$

$$3m = 4$$

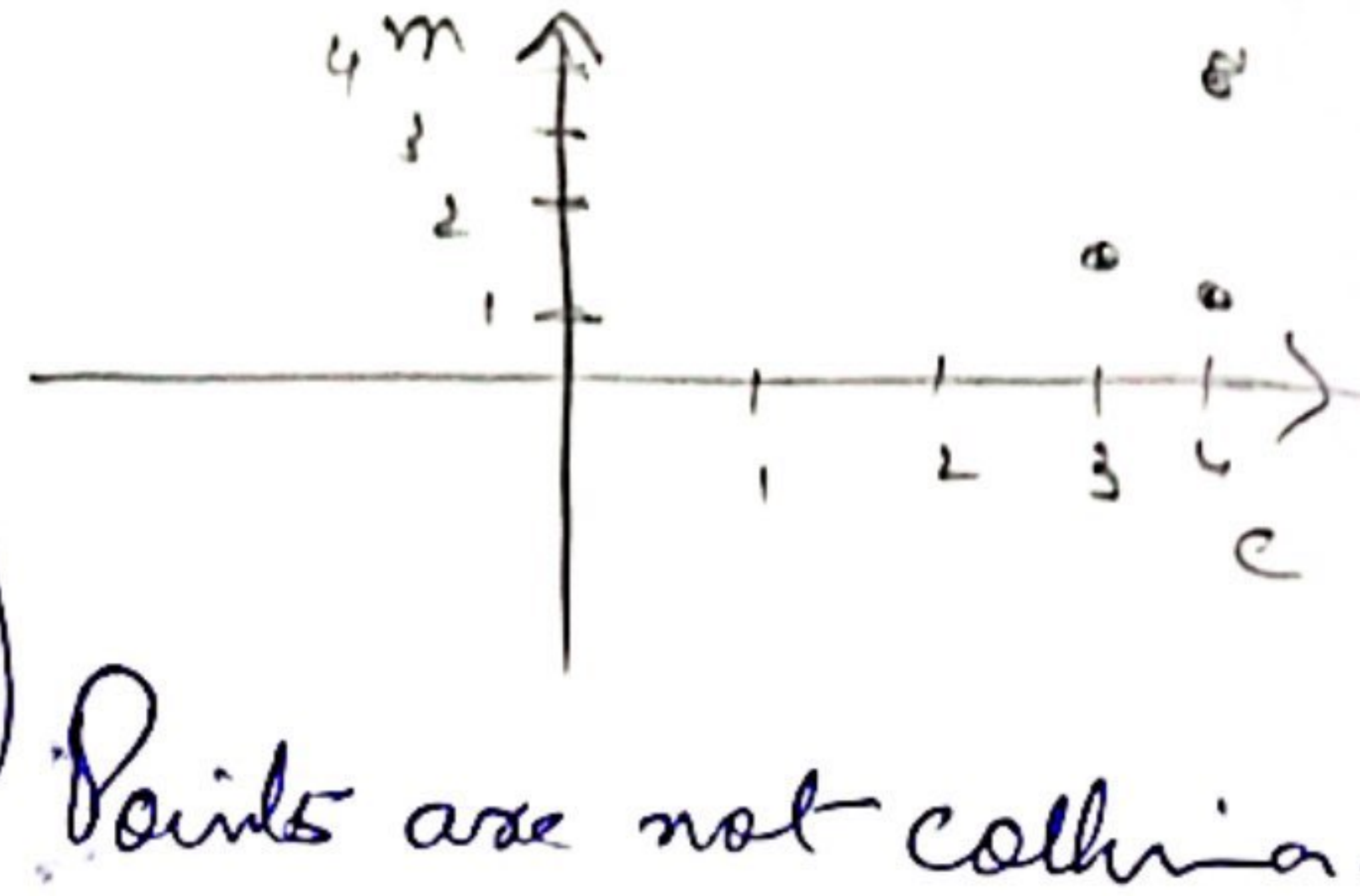
$$\boxed{m = 4/3}$$

$m = 0$

$$c = 0 + 4$$

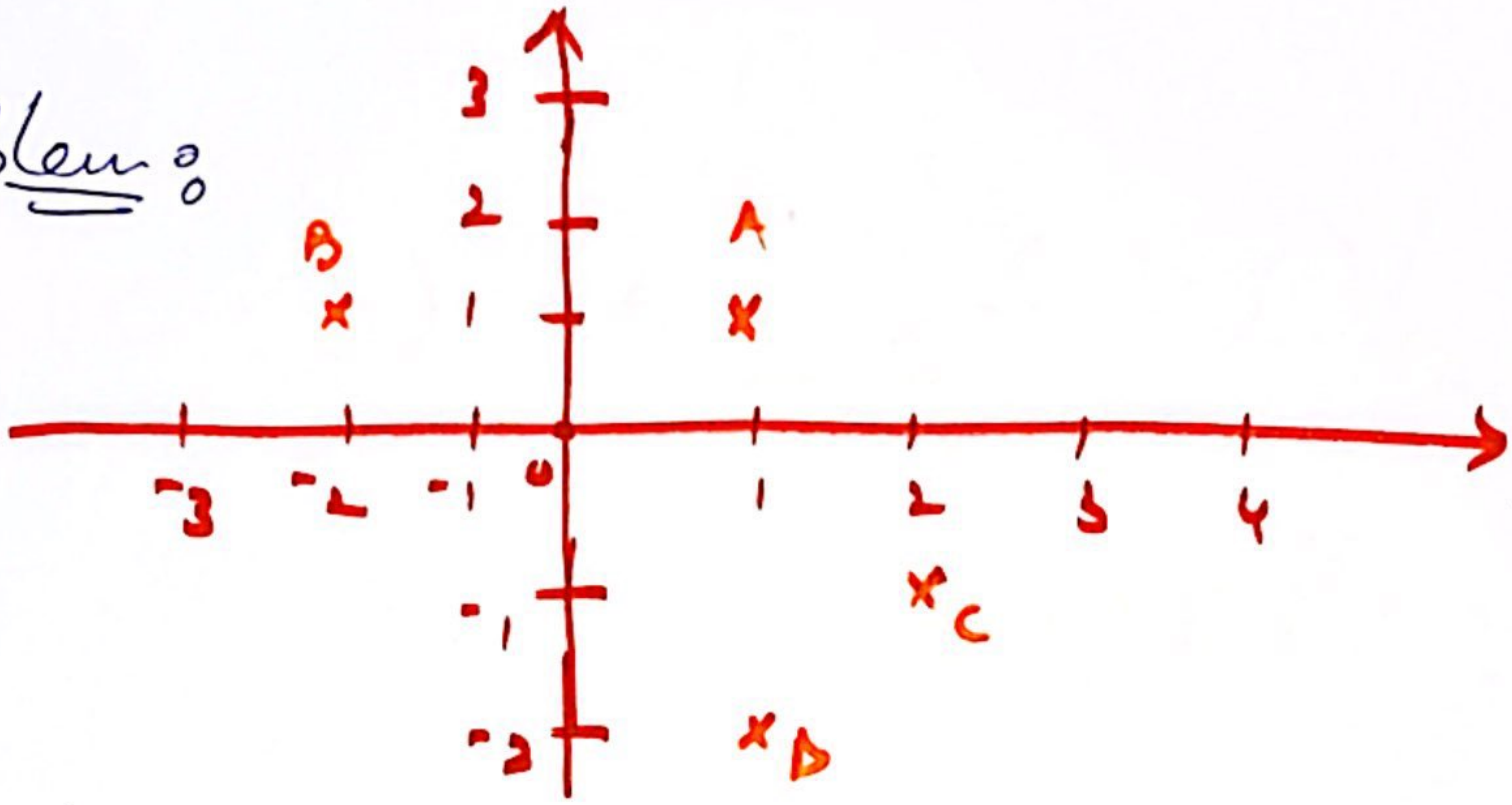
$$\boxed{c = 4}$$

$(4, 4/3)$



Points are not collinear.

Problem:



Solu →

Given the above points, use Hough-Transform to show that points are collinear.

A → (1, 1) B (-2, 1) and C (2, -1)

D (1, -2)

Do the same procedure as done in the last example using equation

$$C = -mX + c$$

SIFT

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

Given some pixels value with the following data

$$\overset{x}{(2,} \overset{y}{1)} = 3.5 \quad \leftarrow \text{Intensity value.}$$

$(4, 6) = 4.6$

$(20, 30) = 5$

$(2, 4) = 6$

Find $L(x, y, 1)$ for all the points mentioned above.

$$G(2, 1, 1) = \frac{1}{2\pi(1)^2} e^{-(2^2+1^2)/2\times 1^2} =$$

$$G(2, 1, 1) = \frac{1}{2\pi} e^{-(5/2)} = 0.01307$$

$$L(2, 1, 1) = 0.01307 \times 3.5$$

$$\boxed{L(2, 1, 1) = 0.045745}$$

Similarly you can calculate for the rest of the values.

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2} \rightarrow (A)$$

and

$$\theta(x,y) = \tan^{-1}\left(\frac{(L(x,y+1) - L(x,y-1))}{(L(x+1,y) - L(x-1,y))}\right) \rightarrow (B)$$

We can calculate m and θ for (2, 1), (4, 6) (20, 30) and (2, 4)

For $\overset{x}{(2}, \overset{y}{1)}$

$$m = \sqrt{(L(2+1, 1) - L(2-1, 1))^2 + (L(2, 1+1) - L(2, 1-1))^2}$$

$$= \sqrt{(L(3, 1) - L(1, 1))^2 + (L(2, 3) - L(2, 0))^2}$$

Calculate L(3, 1)

$$L(3, 1, 1) = G(3, 1, 1) \times I(3, 1)$$ → This is not given so we can assume 1

$$= \frac{1}{2\pi(1)^2} e^{-(3^2+1^2)/2\times 1} = 5.68 \times 10^{-3}$$

$$= 5.68 \times 10^{-3} \times 1$$

$$L(3, 1, 1) = 5.68 \times 10^{-3}$$

Similarly calculate $L(1,1)$, $L(2,3)$ and $L(2,0)$ and put the values in eq (A) and (B) to get $m(x,y)$ and $\theta(x,y)$ values.