# Formula lui Binet

**Formula lui Binet** ne permite să calculăm $F_n$ fără a cunoşte a priori valorile $F_{n-1}$ şi $F_{n-2}$. Putem deduce inductiv formula

$$F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right]$$

Verificăm pentru $n = 1$ şi $n = 2$.

$$F_1 = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^1 - \left( \frac{1-\sqrt{5}}{2} \right)^1 \right] = \frac{2\sqrt{5}}{2\sqrt{5}} = 1$$

$$F_2 = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^2 - \left( \frac{1-\sqrt{5}}{2} \right)^2 \right] = \frac{1}{\sqrt{5}} \frac{1+2\sqrt{5}+5-1-\sqrt{5}-5}{4} = \frac{4\sqrt{5}}{4\sqrt{5}} = 1$$

Presupunem acum că formula este adevărată pentru $n = k-2$ şi $n = k-1$:

$$F_{k-2} = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \right] \text{ şi } F_{k-1} = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-1} - \left( \frac{1-\sqrt{5}}{2} \right)^{k-1} \right]$$

Demonstrăm că $F_k = F_{k-2} + F_{k-1}$.

$$\begin{aligned}
F_{k-2} + F_{k-1} &= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} + \left( \frac{1+\sqrt{5}}{2} \right)^{k-1} - \left( \frac{1-\sqrt{5}}{2} \right)^{k-1} \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} \left( 1 + \frac{1+\sqrt{5}}{2} \right) - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \left( 1 + \frac{1-\sqrt{5}}{2} \right) \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} \left( \frac{3+\sqrt{5}}{2} \right) - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \left( \frac{3-\sqrt{5}}{2} \right) \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} \left( \frac{6+2\sqrt{5}}{4} \right) - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \left( \frac{6-2\sqrt{5}}{4} \right) \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} \left( \frac{1+2\sqrt{5}+5}{4} \right) - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \left( \frac{1-2\sqrt{5}+5}{4} \right) \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k-2} \left( \frac{1+\sqrt{5}}{2} \right)^2 - \left( \frac{1-\sqrt{5}}{2} \right)^{k-2} \left( \frac{1-\sqrt{5}}{2} \right)^2 \right] \\
&= \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k} - \left( \frac{1-\sqrt{5}}{2} \right)^{k} \right] = F_k \blacksquare
\end{aligned}$$