

Numerical study of cavitation in liquid hydrogen flow

Eric Goncalves¹ and Dia Zeidan²

¹Ensm - Pprime, Department of Aeronautical Engineering, Poitiers, France

²School of Basic Sciences and Humanities, German Jordanian University, Amman, Jordan

Abstract

A compressible, two-phase, inviscid solver has been developed to investigate the behaviour of cavitation models including thermodynamic effects. The code is composed by three conservation laws for mixture variables (mass, momentum and total energy) and a supplementary transport equation for the volume fraction of gas. The mass transfer between phases is closed assuming its proportionality to the velocity divergence. A linear variation of the vapour pressure with temperature is assumed. Numerical simulations are performed on rarefaction cavitating problems in which the working fluid is hot water and liquid hydrogen in cryogenic conditions.

Keywords: two-phase flow, cavitation, homogeneous model, heat transfer

Nomenclature

P	Pressure, Pa
T	Temperature, K
V	Velocity vector, m/s
c	Speed of sound, m/s
\dot{m}	Mass transfer between phases, kg/s.m ³
B	B-factor
P _∞	Constant reference pressure, Pa
e ₀	Energy of formation, J/kg

Greek Symbols

α	Void fraction
ρ	Mass density, kg/m ³

Subscripts

l,v	Liquid, vapour values
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1. Introduction

Thermodynamic effects play an important role in the cavitation dynamics of thermosensitive fluids such as cryogenic fluids or carburant. The local cooling effect delays the cavitation phenomenon and reduces the local vapour pressure of the fluid, which leads to a lower observed cavity pressure. The temperature depression, negligible in cold water, is quite substantial.

The most used approach is based on currently used averaged two-phase fluid flow model such as the one-fluid formulation. For the averaged model, there are different approaches according to the assumptions made on the local thermodynamic equilibrium and the slip condition between phases. In this study, we use homogeneous models composed by three mixture conservation laws and an additional transport equation for the void ratio. The mass transfer between phases

appearing explicitly in the source term is closed assuming its proportionality to the mixture velocity divergence [2,3,4,8].

In the following, the good behaviour of the model is shown on rarefaction tube problems for which the running fluid is a hot water. This is followed with investigation of the thermal effects in cryogenic cavitation with liquid hydrogen at 22 K. Conclusions are presented in the last section.

2. The cavitation model

The homogeneous mixture approach is used with the assumption of thermal and mechanical local equilibrium between pure phases. The model consists in three conservation laws for mixture quantities (mass, momentum and total energy) and an additional equation for the void ratio. The expression for the void ratio equation is:

$$\frac{\partial \alpha}{\partial t} + V \nabla(\alpha) = \left(\frac{\rho_l c_l^2 - \rho_v c_v^2}{\rho_l c_l^2 + \frac{\rho_v c_v^2}{1-\alpha}} \right) \nabla \cdot (V) + \left(\frac{\frac{c_v^2}{\alpha} + \frac{c_l^2}{1-\alpha}}{\frac{\rho_l c_l^2}{1-\alpha} + \frac{\rho_v c_v^2}{\alpha}} \right) \dot{m}$$

By assuming that the mass transfer is proportional to the velocity divergence, it is possible to build a family of models in which the mass transfer is expressed as [2]

$$\dot{m} = \frac{\rho_l \rho_v}{(\rho_l - \rho_v)} \left(1 - \frac{c^2}{c_{wallis}^2} \right) \nabla \cdot (V) \quad (2)$$

where c_{wallis} is the propagation of acoustic waves without mass transfer [5]. The source term is built using a sinus equation of state (EOS) in which linear thermal variations are introduced for the vapour pressure:

$$P(\alpha, T) = P_{\text{vap}}(T) + \frac{\rho_l - \rho_v}{2} c_0^2 \text{Arcsin}(1 - 2\alpha) \quad (3)$$

$$P_{\text{vap}}(T) = P_{\text{vap}}(T_{\text{ref}}) + \frac{dP}{dT}(T - T_{\text{ref}}) \quad (4)$$

The constant quantity dP_{vap}/dT is determined with a thermodynamic table. The expression of the mixture speed of sound and the influence of parameter c_0 are given in [1]. Pure phases follow the stiffened gas EOS.

3. The numerical tool

The numerical simulations are carried out using an explicit inviscid solver based on a cell-centered finite-volume discretization. The numerical flux through the cell interface is computed with the first-order Rusanov scheme. The numerical treatment of the boundary conditions is based on the use of the characteristic relations of Euler equations. More details are given in [1].

4. Hot water simulations

A double rarefaction tube problem is considered with an initial velocity discontinuity located at the middle of the tube. This test consists in a one meter long tube filled with liquid water at atmospheric pressure and temperature $T_{\text{ref}} = 355$ K. A weak volume fraction of vapor $\alpha = 0.01$ is initially added to the liquid. The left velocity and the right velocity have the same value but an opposite sign: $|u| = 100$ m/s. The vapour pressure $P_{\text{vap}}(T_{\text{ref}})$ is set to 51000 Pa at the initial temperature. The mesh contains 5000 cells. The time step is set to 10^{-7} s. The strong rarefaction leads to two evaporation fronts resulting in a large cavitation pocket where the gas volume fraction is

close to 1. However, this pocket does not contain pure gas but a mixture at thermodynamic equilibrium.

The volume fraction, pressure, velocity and temperature profiles are plotted in Figure 1 at time $t = 1.5$ ms. Results are compared with the two-fluid solution computed by Zein et al. [6]. No differences appear on the void ratio between models. For the pressure profiles, the pressure drop under $P_{\text{vap}}(T_{\text{ref}})$ is around 0.3 bar in close agreement with the Zein solution. Discrepancies appear for the velocity profile in comparison with the two-fluid solution: variations across the evaporation front are stiffer with our model. The temperature drop in the cavitation pocket due to the phase transition is very large and reaches more than 90 K. However, we have no estimation of the temperature depression and further research is needed in this direction.

5. Liquid hydrogen simulations

A similar rarefaction case is proposed for which the running fluid is liquid hydrogen in cryogenic conditions. Liquid hydrogen is initially at the pressure 2 bar and at the reference temperature $T_{\text{ref}} = 22.1$ K. The vapour pressure is set to 1.63 bar. Parameters of the stiffened gas are given in Table 1. The initial discontinuity velocity is $|u| = 100$ m/s. A weak volume fraction of vapor $\alpha = 0.01$ is initially added to the liquid. The mesh contains 5000 cells. The time step is set to 10^{-7} s. The quantity dP_{vap}/dT is determined with a thermodynamic table and is set to 20000 Pa/K.

Figure 2 illustrates the volume fraction, pressure, velocity and temperature profiles at time $t = 0.8$ ms. The pressure drop under the initial value of the vapour pressure is large and reaches 1.2 bar. Thermal effects in LH2 are more pronounced in comparison with water due to the large value of the slope dP_{vap}/dT . As observed previously on velocity profiles in water, variations across the evaporation fronts are very stiff.

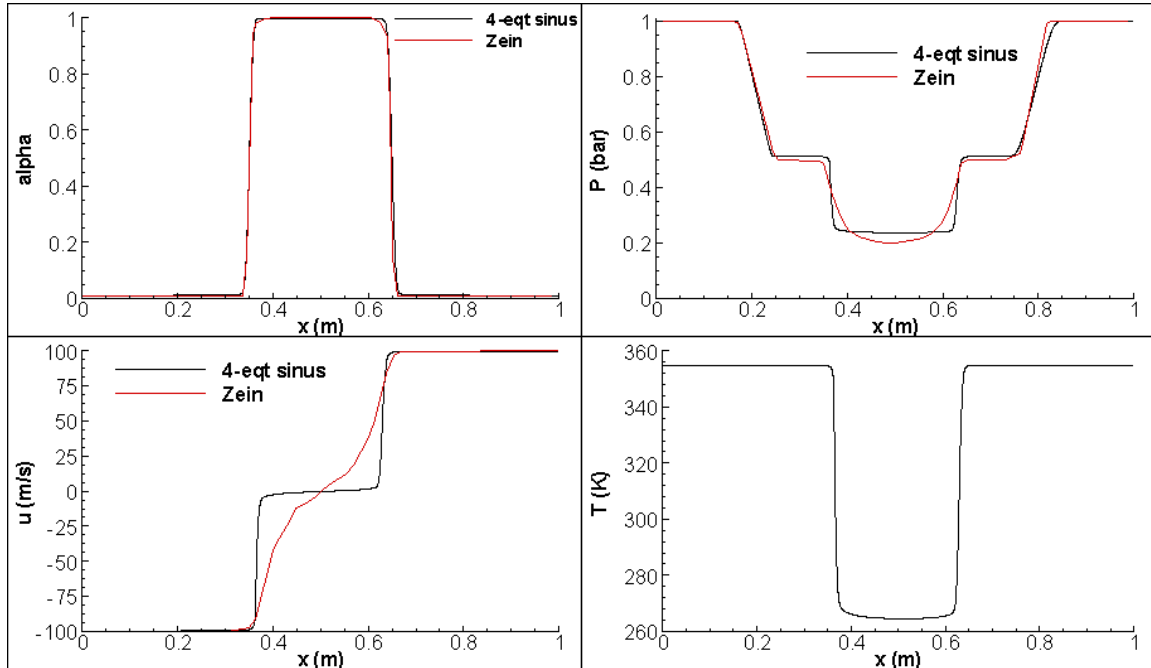


Fig. 1. Water double rarefaction with cavitation, $|u|=100$ m/s, $t=1.5$ ms. Void ratio, pressure, velocity and temperature profiles.
Table 1. Parameters of the stiffened gas EOS for liquid hydrogen at the reference temperature 22.1 K.

	γ	P_∞ (Pa)	e_0 (J/kg)	C_p (J/K.kg)	ρ_{sat} (kg/m ³)
liquid	2.8	2.10^5	$-2.21 \cdot 10^5$	10875	68.55
vapor	1.38	0	$1.66 \cdot 10^5$	13090	2.07

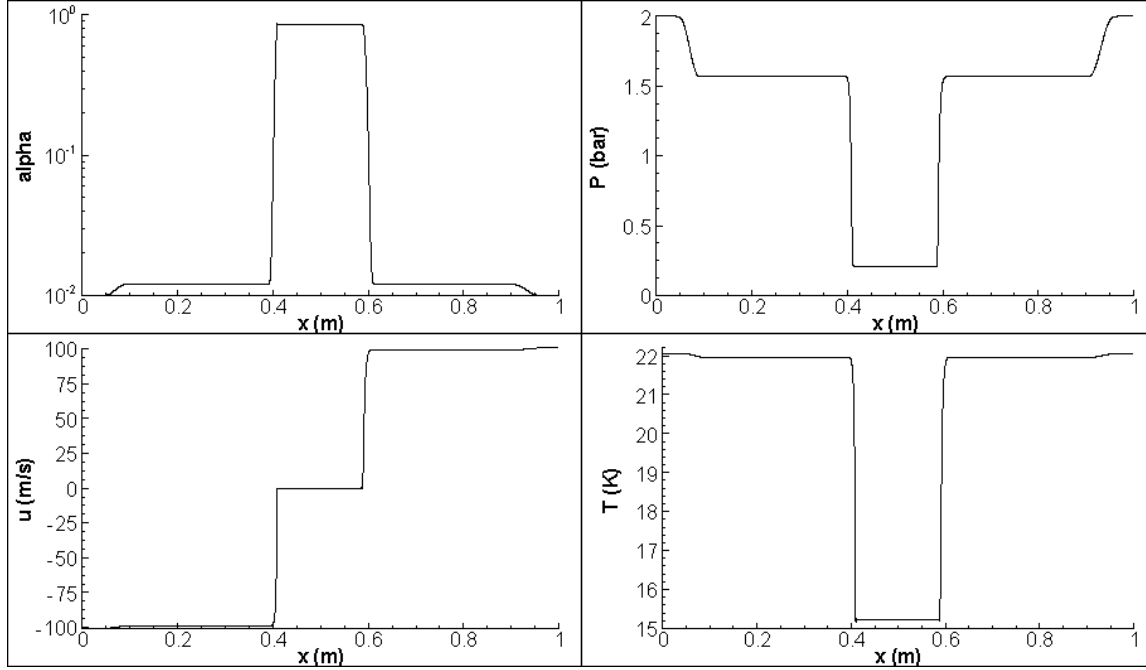


Fig. 2. LH2 double rarefaction with cavitation, $|u|=100$ m/s, $t=0.8$ ms. Void ratio, pressure, velocity and temperature profiles.

Using the B-factor theory for an ideal gas-liquid mixture, we have the relation [7]:

$$B = \frac{\Delta T}{\Delta T^*} = \frac{\alpha}{1 - \alpha} \quad (5)$$

where B is the ratio between the actual temperature drop ΔT and the temperature difference ΔT^* caused by thermal effects. With the maximum void ratio value reached during the computation $\alpha=0.86$, the ideal temperature depression is around 6 K. The computed temperature drop is around 6.5 K, in good agreement with the theoretical value.

References

- [1] Goncalves E., Fortes Patella R. Numerical study of cavitating flows with thermodynamic effect. *Computers & Fluids* 2010; 31:99-113.
- [2] Goncalves E. Numerical study of expansion tube problems: Toward the simulation of cavitation. *Computers & Fluids* 2013; 72:1-19.
- [3] Goncalves E., Charrière B. Modelling for isothermal cavitation with a four-equation model. *Int. J. Multiphase Flow* 2014; 59:54-72.
- [4] Goncalves E. Modelling for non isothermal cavitation using four-equation models. *Int. J. Heat and Mass Transfer* 2014; 76:247-262.
- [5] Wallis, G.: One-dimensional two-phase flow. New York Mc Graw-Hill, 1967.
- [6] Zein A., Hantke M., Warnecke G. Modeling phase transition for compressible two-phase flows applied to metastable liquids. *J. Computational Physics* 2010; 229:2964-2998.
- [7] Franc J-P., Michel J-M. Fundamentals of cavitation. Springer, 2004.
- [8] Zeidan, D. Validation of hyperbolic model for two-phase flow in conservative form. *International Journal of Computational Fluid Dynamics* 2009; 23: 623-641.