
THERMO-FLUID DYNAMICS OF TWO-PHASE FLOW

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Dedication

This book is dedicated to our parents.

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Preface

This book is intended to be an introduction to the theory of thermo-fluid dynamics of two-phase flow for graduate students, scientists and practicing engineers seriously involved in the subject. It can be used as a text book at the graduate level courses focused on the two-phase flow in Nuclear Engineering, Mechanical Engineering and Chemical Engineering, as well as a basic reference book for two-phase flow formulations for researchers and engineers involved in solving multiphase flow problems in various technological fields.

The principles of single-phase flow fluid dynamics and heat transfer are relatively well understood, however two-phase flow thermo-fluid dynamics is an order of magnitude more complicated subject than that of the single-phase flow due to the existence of moving and deformable interface and its interactions with the two phases. However, in view of the practical importance of two-phase flow in various modern engineering technologies related to nuclear energy, chemical engineering processes and advanced heat transfer systems, significant efforts have been made in recent years to develop accurate general two-phase formulations, mechanistic models for interfacial transfer and interfacial structures, and computational methods to solve these predictive models.

A strong emphasis has been put on the rational approach to the derivation of the two-phase flow formulations which represent the fundamental physical principles such as the conservations laws and constitutive modeling for various transfer mechanisms both in bulk fluids and at interface. Several models such as the local instant formulation based on the single-phase flow model with explicit treatment of interface and the macroscopic continuum formulations based on various averaging methods are presented and

discussed in detail. The macroscopic formulations are presented in terms of the two-fluid model and drift-flux model which are two of the most accurate and useful formulations for practical engineering problems.

The change of the interfacial structures in two-phase flow is dynamically modeled through the interfacial area transport equation. This is a new approach which can replace the static and inaccurate approach based on the flow regime transition criteria. The interfacial momentum transfer models are discussed in great detail, because for most two-phase flow, thermo-fluid dynamics are dominated by the interfacial structures and interfacial momentum transfer. Some other necessary constitutive relations such as the turbulence modeling, transient forces and lift forces are also discussed.

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Foreword

Thermo-Fluid Dynamics of Two-Phase Flow takes a major step forward in our quest for understanding fluids as they metamorphose through change of phase, properties and structure. Like Janus, the mythical Roman God with two faces, fluids separating into liquid and gas, each state sufficiently understood on its own, present a major challenge to the most astute and insightful scientific minds when it comes to deciphering their dynamic entanglement.

The challenge stems in part from the vastness of scale where two phase phenomena can be encountered. Between the microscopic *nano*-scale of molecular dynamics and deeply submerged modeling assumptions and the *macro*-scale of measurements, there is a *meso*-scale as broad as it is nebulous and elusive. This is the scale where everything is in a permanent state of exchange, a Heraclitean state of flux, where nothing ever stays the same and where knowledge can only be achieved by firmly grasping the underlying principles of things.

The subject matter has sprung from the authors' own firm grasp of fundamentals. Their bibliographical contributions on two-phase principles reflect a scientific tradition that considers theory and experiment a duality as fundamental as that of appearance and reality. In this it differs from other topical works in the science of fluids. For example, the leading notion that runs through two-phase flow is that of interfacial velocity. It is a concept that requires, amongst other things, continuous improvements in both modeling and measurement. In the *meso*-scale, this gives rise to new science of the interface which, besides the complexity of its problems and the fuzziness of its structure, affords ample scope for the creation of elegant, parsimonious formulations, as well as promising engineering applications.

The two-phase flow theoretical discourse and experimental inquiry are closely linked. The synthesis that arises from this connection generates immense technological potential for measurements informing and validating dynamic models and conversely. The resulting technology finds growing utility in a broad spectrum of applications, ranging from next generation nuclear machinery and space engines to pharmaceutical manufacturing, food technology, energy and environmental remediation.

This is an intriguing subject and its proper understanding calls for exercising the rigorous tools of advanced mathematics. The authors, with enormous care and intellectual affection for the subject reach out and invite an inclusive audience of scientists, engineers, technologists, professors and students.

It is a great privilege to include the *Thermo-Fluid Dynamics of Two-Phase Flow* in the series ***Smart Energy Systems: Nanowatts to Terawatts***. This is work that will stand the test of time for its scientific value as well as its elegance and aesthetic character.

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