Chapter 15

ONE-DIMENSIONAL TWO-FLUID MODEL

The two-fluid model is the most detailed and accurate macroscopic formulation of the thermo-fluid dynamics of two-phase systems. In the two-fluid model, the field equations are expressed by the six conservation equations consisting of mass, momentum and energy equations for each phase. Since these field equations are obtained from an appropriate averaging of local instantaneous balance equations, the phasic interaction term appears in each of the averaged balance equations. These terms represent the mass, momentum and energy transfers through the interface between the phases. The existence of the interfacial transfer terms is one of the most important characteristics of the two-fluid model formulation. These terms determine the rate of phase changes and the degree of mechanical and thermal non-equilibrium between phases, thus they are the essential closure relations that should be modeled accurately. However, because of considerable difficulties in terms of measurements and modeling, reliable and accurate closure relations for the interfacial transfer terms are not fully developed. In spite of these shortcomings of two-fluid models, there is, however, no substitute available for modeling accurately two-phase phenomena where two phases are weakly coupled. Examples of these are:

- Sudden mixing of two phases;
- Transient flooding and flow reversal;
- Transient countercurrent flow;
- Two-phase flow with sudden acceleration.

A three-dimensional, two-fluid model has been obtained by using temporal or statistical averaging. In view of practical engineering problems, a one-dimensional, two-fluid model obtained by averaging local two-fluid formulation over the cross-sectional area is useful for complicated engineering problems involving fluid flow and heat transfer. This is due to the fact that field equations can be reduced to quasi-one-dimensional forms. By area averaging, the information on changes of variables in the direction normal to the main flow within a channel is basically lost. Therefore, the transfer of momentum and energy between the wall and the fluid should be expressed by empirical correlations or by simplified models. In this chapter, we develop a general one-dimensional formulation of the two-fluid model, and discuss various special cases that are important in practical applications. For simplicity, mathematical symbols of time-averaging in one-dimensional formulation are dropped in the formulation in this chapter.

1.1 Area average of three-dimensional two-fluid model

The three-dimensional form of the two-fluid model has been obtained by the temporal or statistical averaging method. For most practical applications, the model developed by Ishii (1975) can be simplified to the following forms:

Continuity equation

$$\frac{\partial \alpha_k \overline{\overline{\rho_k}}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k} \right) = \Gamma_k \tag{15-1}$$

Momentum equation

$$\begin{aligned} &\frac{\partial \alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{v}_k} \widehat{\boldsymbol{v}_k} \right) = -\alpha_k \nabla \overline{\overline{p_k}} \\ &+ \nabla \cdot \left[\alpha_k \left(\overline{\overline{\boldsymbol{\mathcal{T}}_k}} + \boldsymbol{\mathcal{T}}_k^T \right) \right] + \alpha_k \overline{\overline{\rho_k}} \widehat{\boldsymbol{g}_k} + \widehat{\boldsymbol{v}_{ki}} \Gamma_k + \boldsymbol{M}_{ik} \\ &- \nabla \alpha_k \cdot \overline{\overline{\boldsymbol{\mathcal{T}}_{ki}}} + \left(\overline{\overline{p_{ki}}} - \overline{\overline{p_k}} \right) \nabla \alpha_k \end{aligned} \tag{15-2}$$

Enthalpy energy equation

$$\begin{aligned} &\frac{\partial \alpha_k \overline{\overline{\rho_k}} \widehat{h_k}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k}} \widehat{h_k} \widehat{\boldsymbol{v}_k} \right) = -\nabla \cdot \alpha_k \left(\overline{\overline{\boldsymbol{q}_k}} + \boldsymbol{q}_k^T \right) \\ &+ \alpha_k \frac{D_k}{Dt} \overline{\overline{p_k}} + \widehat{h_{ki}} \Gamma_k + a_i q''_{ki} + \Phi_k \end{aligned} \tag{15-3}$$

Here Γ_k, $\boldsymbol{M}_{ik}$, $\overline{\overline{\boldsymbol{\mathcal{T}}_{ki}}}$, q''_{ki} and Φ_k are the mass generation, generalized interfacial drag, interfacial shear stress, interfacial heat flux and dissipation, respectively. The subscript k denotes k-phase and i stands for the value at the interface. $1/a_i$ denotes the length scale at the interface and a_i has the physical meaning of the interfacial area per unit volume (Ishii, 1975; Ishii and Mishima, 1981). Thus,

$$a_i = \frac{\text{interfacial area}}{\text{mixture volume}}. \tag{15-4}$$

From the above field equations it can be seen that several interfacial transfer terms appear on the right-hand side of the equations. Since these interfacial transfer terms also should obey the balance laws at the interface, interfacial transfer conditions could be obtained from an average of the local jump conditions (Ishii, 1975). They are given by

$$\begin{cases} \sum_{k=1}^{2} \Gamma_k = 0 \\ \sum_{k=1}^{2} \boldsymbol{M}_{ik} = 0 \\ \sum_{k=1}^{2} \left(\Gamma_k \widehat{h_{ki}} + a_i q''_{ki} \right) = 0. \end{cases} \tag{15-5}$$

Therefore, constitutive equations for $\boldsymbol{M}_{ik}$, $a_i q''_{fi}$ and $a_i q''_{gi}$ are necessary for the interfacial transfer terms. The enthalpy interfacial transfer condition indicates that specifying the heat flux at the interface for both phases is equivalent to the constitutive relation for Γ_k if the mechanical energy transfer terms can be neglected (Ishii, 1975). This aspect greatly simplifies the development of the constitutive relations for interfacial transfer terms.

The rational approach to obtain a one-dimensional model is to integrate the three-dimensional model over a cross-sectional area and then to introduce proper mean values. A simple area average over the cross-sectional area A is defined by Eq.(14-5) and the void-fraction-weighted mean value is given by Eq.(14-6). In the subsequent analysis, the density of each phase within any cross-sectional area is considered to be uniform so that $\rho_k = \langle\langle \rho_k \rangle\rangle$. For most practical two-phase flow problems this assumption is valid since the transverse pressure gradient within a channel is relatively small. The axial component of the weighted mean velocity of phase k is

$$\langle\langle v_k \rangle\rangle = \frac{\langle \alpha_k v_k \rangle}{\langle \alpha_k \rangle} = \frac{\langle j_k \rangle}{\langle \alpha_k \rangle} \tag{15-6}$$

where the scalar expression of the velocity corresponds to the axial component of the vector. By area averaging Eqs.(15-1) to (15-3), and making some simplifications which are applicable to most practical problems, the following field equations can be obtained

Continuity equation

$$\frac{\partial \langle \alpha_k \rangle \rho_k}{\partial t} + \frac{\partial}{\partial z} \langle \alpha_k \rangle \rho_k \langle\langle v_k \rangle\rangle = \langle \Gamma_k \rangle \tag{15-7}$$

Momentum equation

$$\begin{aligned}
&\frac{\partial}{\partial t} \langle \alpha_k \rangle \rho_k \langle\langle v_k \rangle\rangle + \frac{\partial}{\partial z} C_{vk} \langle \alpha_k \rangle \rho_k \langle\langle v_k \rangle\rangle^2 \\
&= -\langle \alpha_k \rangle \frac{\partial}{\partial z} \langle\langle p_k \rangle\rangle + \frac{\partial}{\partial z} \langle \alpha_k \rangle \langle\langle \tau_{kzz} + \tau_{kzz}^T \rangle\rangle \\
&- \frac{4\alpha_{kw}\tau_{kw}}{D} - \langle \alpha_k \rangle \rho_k g_z + \langle \Gamma_k \rangle \langle\langle v_{ki} \rangle\rangle + \langle M_k^d \rangle \\
&+ \left\langle (p_{ki} - p_k) \frac{\partial \alpha_k}{\partial z} \right\rangle
\end{aligned} \tag{15-8}$$

where α_{kw} and τ_{kw} are the mean void fraction at the wall and wall shear stress, respectively. The pressure difference and void gradient term can be important for horizontal stratified flow. Except for this case, this term may be neglected. $\langle M_k^d \rangle$ is the total interfacial shear force given by

$$\langle M_k^d \rangle = \langle \boldsymbol{M}_{ik} - \nabla \alpha_k \cdot \tau_i \rangle_z . \tag{15-9}$$

The first term on the right-hand side is the generalized particle drag which is important for a dispersed flow. The second term is the effect of the interfacial shear and the void gradient. This term is particularly important for a separated flow. In the convective term, the distribution parameter for the k-phase momentum, C_{vk}, appears due to the difference between the average of a product of variables and the product of averaged variables.

Enthalpy energy equation

$$\begin{aligned}
&\frac{\partial}{\partial t}\langle\alpha_k\rangle\rho_k\langle\langle h_k\rangle\rangle+\frac{\partial}{\partial z}C_{hk}\langle\alpha_k\rangle\rho_k\langle\langle h_k\rangle\rangle\langle\langle v_k\rangle\rangle\\
&=-\frac{\partial}{\partial z}\langle\alpha_k\rangle\left\langle\left\langle q_k+q_k^T\right\rangle\right\rangle_z+\langle\alpha_k\rangle\frac{D_k}{Dt}\langle\langle p_k\rangle\rangle\\
&+\frac{\xi_h}{A}\alpha_{kw}q''_{kw}+\langle\Gamma_k\rangle\langle\langle h_{ki}\rangle\rangle+\langle a_i q''_{ki}\rangle+\langle\Phi_k\rangle
\end{aligned}\tag{15-10}$$

where ξ_h and q''_{kw} are the heated perimeter and wall heat flux, respectively. C_{hk} is the distribution parameter for the k-phase enthalpy. From the macroscopic jump conditions at the interface the following relations between the interfacial transfer terms hold,

$$\begin{cases}
\sum_{k=1}^{2}\langle\Gamma_k\rangle=0\\
\sum_{k=1}^{2}\langle M_k^d\rangle=\sum_{k=1}^{2}\langle \boldsymbol{M}_{ik}-\nabla\alpha_k\cdot\tau_i\rangle_z=0\\
\sum_{k=1}^{2}\left\{\langle\langle\Gamma_k\rangle\rangle\langle\langle h_{ki}\rangle\rangle+\langle a_i q''_{ki}\rangle\right\}=0.
\end{cases}\tag{15-11}$$

1.2 Special consideration for one-dimensional constitutive relations

1.2.1 Covariance effect in field equations

In a one-dimensional model, a very careful examination of transverse distributions of various variables and their effects on the balance and constitutive equations is essential. If this is not done properly, the resulting two-phase flow formulation can be inconsistent. Improper modeling, or disregard of the distribution effects, may lead not only to a grossly inaccurate model, but also to various numerical instabilities. The distribution effects can be divided into two groups. The first one is the covariance effect which directly affects the form of the convective term in the field equation. The second effect appears in the averaging of the various local constitutive relations. These two effects are discussed separately below.

The covariance of the convective terms is defined by

$$COV\left(\rho_k \alpha_k \psi_k v_k\right) \equiv \left\langle \alpha_k \rho_k \psi_k \left(v_k - \langle\langle v_k \rangle\rangle\right)\right\rangle \tag{15-12}$$

To close the set of the governing equations we must specify relations for these covariance terms. This can be done by introducing distribution parameters for the momentum and energy fluxes. If we define a distribution parameter for a flux as

$$C_{\psi k} \equiv \frac{\left\langle \alpha_k \psi_k v_k \right\rangle}{\left\langle \alpha_k \right\rangle \langle\langle \psi_k \rangle\rangle \langle\langle v_k \rangle\rangle} \tag{15-13}$$

the covariance term becomes

$$\begin{aligned} COV\left(\rho_k \alpha_k \psi_k v_k\right) &= \rho_k \left\langle \alpha_k \psi_k \left(v_k - \langle\langle v_k \rangle\rangle\right)\right\rangle \\ &= \left(C_{\psi k} - 1\right) \rho_k \left\langle \alpha_k \right\rangle \langle\langle \psi_k \rangle\rangle \langle\langle v_k \rangle\rangle. \end{aligned} \tag{15-14}$$

For the momentum flux, the distribution parameter is defined by

$$C_{vk} \equiv \frac{\left\langle \alpha_k v_k^2 \right\rangle}{\left\langle \alpha_k \right\rangle \langle\langle v_k \rangle\rangle^2}. \tag{15-15}$$

Physically, C_{vk} represents the effect of the void and momentum-flux profiles on the cross-sectional-area-averaged momentum flux of k-phase. A quantitative study of C_{vk} can be made by considering a symmetric flow in a circular duct and introducing the power-law expressions in parallel with the analysis of the drift-flux modeling (Zuber and Findlay, 1965; Ishii, 1977). The following is the summary obtained by Ishii (1977).

Hence, for bubbly, slug and churn-turbulent flow, it is postulated that

$$\frac{\alpha_k - \alpha_{kw}}{\alpha_{k0} - \alpha_{kw}} = 1 - \left(\frac{r}{R_w}\right)^n \tag{15-16}$$

and

$$\frac{v_k}{v_{k0}} = 1 - \left(\frac{r}{R_w}\right)^m \tag{15-17}$$

where the subscripts 0 and w refer to the value at the centerline and at the wall of a tube. For simplicity it is assumed that the void and velocity profiles are similar; namely, $n = m$. This assumption is widely used in mass-transfer problems, and it may not be unreasonable for fully developed two-phase flows if one considers that the vapor flux and, hence, the void concentration greatly influence the velocity distributions. Under this assumption, it can be shown that

$$C_{vk} = \frac{\frac{n+2}{n+1}\left(\alpha_{kw} + \Delta\alpha_k \frac{3n}{3n+2}\right)\left(\alpha_{kw} + \Delta\alpha_k \frac{n}{n+2}\right)}{\left(\alpha_{kw} + \Delta\alpha_k \frac{n}{n+1}\right)^2} \tag{15-18}$$

where $\Delta\alpha_k = \alpha_{k0} - \alpha_{kw}$. The volumetric-flux-distribution parameter C_0 of the drift-flux model is given by

$$C_0 \simeq \frac{(n+2)}{(n+1)} \tag{15-19}$$

where C_0 can be given by the following empirical correlation (Ishii, 1977)

$$C_0 = 1.2 - 0.2\sqrt{\rho_g/\rho_f} \tag{15-20}$$

for a fully developed flow in a round tube. For a subcooled boiling or flow in a rectangular channel, see Eq.(14-35) or Eq.(14-32), respectively. Therefore, in the standard range of n, the parameter C_{vg} can be given approximately by

$$C_{vg} \simeq 1 + 0.5(C_0 - 1). \tag{15-21}$$

For a liquid phase in a vapor-dispersed-flow regime, $\alpha_{fw} \simeq 1$ and $\alpha_{f0} < 1$. Then from Eq.(15-18) it can be shown that, for a standard range of α_{f0} in the bubbly- and churn-flow regimes, C_{vf} can be approximated by

$$C_{vf} = 1 + 1.5(C_0 - 1). \tag{15-22}$$

For an annular flow, the momentum covariance term can also be calculated by using the standard velocity profiles for the vapor and liquid flows. Thus we obtain

$$C_{vk} \cong \begin{cases} 1.02 & \text{(turbulent flow)} \\ 1.33 & \text{(laminar flow).} \end{cases} \quad (15\text{-}23)$$

Similarly, the distribution parameter for the enthalpy flux can be defined by

$$C_{hk} \equiv \frac{\langle \alpha_k h_k v_k \rangle}{\langle \alpha_k \rangle \langle\langle h_k \rangle\rangle \langle\langle v_k \rangle\rangle}. \quad (15\text{-}24)$$

For a thermal-equilibrium flow, $h_g = h_{gs}$ and $h_f = h_{fs}$, where h_{gs} and H_{fs} are the saturation enthalpies of vapor and liquid. Since, in this case, the enthalpy profile is completely flat for each phase, the distribution parameters become unity; namely, $C_{hg} = C_{hf} = 1$. It is also evident that if one of the phases is in the saturated condition, then C_{hk} for that phase becomes unity.

In the single-phase region, the distribution parameter can be calculated from the assumed profiles for the velocity and enthalpy. Using the standard power-law profiles for a turbulent flow; namely, $v/v_0 = (y/R)^{1/n}$ and $(h - h_w)/(h_0 - h_w) = (y/R)^{1/m}$, where y is the distance from the wall, we can show that the covariance term is negligibly small both for developing and fully developed flows under normal conditions. Then

$$C_{hk} \simeq 1.0. \quad (15\text{-}25)$$

Therefore, except for highly transient cases, the enthalpy covariance may be neglected.

1.2.2 Effect of phase distribution on constitutive relations

The greatest shortcoming of the conventional two-fluid model is in the modeling of the constitutive equation for the interfacial shear $\langle M_k^d \rangle$ defined by Eq.(15-9). This is particularly true when the two-fluid model was applied to other than a separated flow. The problem is twofold:

1. Modeling of the averaged drag $\langle \boldsymbol{M}_{ik} \rangle_z$;
2. Modeling of the effect of interfacial shear $\langle -\nabla \alpha_k \cdot \tau_i \rangle_z$.

These will be discussed separately below.

For a dispersed two-phase flow the averaged interfacial drag term could be given approximately by

$$\langle \boldsymbol{M}_{id} \rangle_z \doteq -\frac{3}{8}\frac{C_D}{r_d}\langle \alpha_d \rangle \rho_c \langle v_r \rangle |\langle v_r \rangle|. \tag{15-26}$$

Here, only the steady-state drag force part of $\boldsymbol{M}_{ik}$ is considered because it is the most important term. The above approximate form is obtained based on the experimental observation that the local relative velocity v_r is comparatively uniform across a flow channel (Serizawa et al., 1975; Hibiki and Ishii, 1999; Hibiki et al., 2001a) and the fact that the local relative velocity is much smaller than the phase velocities in most two-phase flow.

The important point, however, is that the averaged drag force should be related to the averaged local relative velocity $\langle v_r \rangle$ given by

$$\langle v_r \rangle \equiv \frac{1}{A}\int v_r dA \tag{15-27}$$

and not to the difference between the area averaged mean velocities of phases given by

$$\overline{v_r} \equiv \langle\langle v_d \rangle\rangle - \langle\langle v_c \rangle\rangle. \tag{15-28}$$

In general,

$$\langle v_r \rangle \neq \overline{v_r}. \tag{15-29}$$

The difference between these two relative velocities can be very large. The reason is that in one-dimensional formulation, the slip, $\overline{v_r}$, between two phases is caused by two completely different effects; namely, the local relative motion and integral effect of the phase and velocity distributions. The existence of these two effects is already well-known (Zuber and Findlay, 1965; Ishii, 1977; Bankoff, 1960). The first effect is the true relative motion between two phases at a local point and does not require any further explanation. The second effect of the distribution arises due to the area averaging. For example, if the dispersed phase is more concentrated in the high velocity core region, then the mean velocity of the dispersed phase should be much higher than that of the continuous phase which is concentrated near the low velocity wall region. This is true even when the two phases are locally moving with the same velocity.

Based on the drift-flux model formulation it can be shown that the approximate expression for $\langle v_r \rangle$ is given by

$$\langle v_r \rangle \simeq \frac{1 - C_0 \langle \alpha_g \rangle}{1 - \langle \alpha_g \rangle} \langle\langle v_g \rangle\rangle - C_0 \langle\langle v_r \rangle\rangle \tag{15-30}$$

for bubbly, slug and churn turbulent flow. Therefore, from the flow regime criterion (Ishii and Mishima, 1981; Ishii, 1977), it is applicable under the following conditions.

$$\begin{cases} \langle \alpha_g \rangle < \dfrac{1}{C_0}, \text{ and} \\ \langle j_g \rangle \sqrt{\dfrac{\rho_g}{\Delta \rho g D}} > \langle \alpha_g \rangle - 0.1 \end{cases} \tag{15-31}$$

This criterion is valid when the tube diameter is relatively small. For more general conditions, see Ishii and Mishima (1981). The constitutive equation for C_0 for a simple case is given by Eq.(15-20).

The expression for the drag force given by Eq.(15-26) with Eq.(15-30) compensates for the slip due to the distributions of phases and velocities. This difference between $\overline{v_r}$ and $\langle v_r \rangle$ has never been taken into account in the conventional two-fluid model. In most two-phase flow systems, the slip due to the distribution of phases is much greater than the local slip between phases. Therefore, neglecting the above-mentioned effect will lead to large errors in predictions of the void fraction and velocities in bubbly, slug and churn turbulent flow regimes. As a result, even the steady-state predictions from two-fluid model were not as good as those from a drift-flux model in these flow regimes. This was one of the most significant shortcomings of the conventional two-fluid model and it should be corrected in all future analyses.

1.2.3 Interfacial shear term

The total interfacial shear force denoted by $\langle M_k^d \rangle$ has two sources; namely, the generalized drag $\langle \boldsymbol{M}_{ik} \rangle_z$ and the contribution of the interfacial shear and void gradient $\langle -\nabla \alpha_k \cdot \tau_i \rangle_z$ as shown in Eq.(15-9). In a separated flow, the second term is the dominant one. For example, for an annular flow in a tube it can be shown that

$$\left\langle -\nabla\alpha_k \cdot \tau_i \right\rangle_z = -\frac{1}{A}\int_A \frac{\partial \alpha_g}{\partial r}\left(\tau_{gi}\right) 2\pi r dr$$
$$= -\frac{1}{A}\lim_{\delta\to 0}\int_\delta \frac{\partial \alpha_g}{\partial r}\tau_{gi} 2\pi r dr = -\frac{\xi_i}{A}\tau_{gi} \tag{15-32}$$

where ξ_i is the wetted perimeter of the gas core.

The constitutive relation for τ_{gi} in this case can be given in terms of the standard interfacial friction factor as

$$\tau_{gi} = \frac{f_i}{2}\rho_g \overline{v_r}\left|\overline{v_r}\right| \tag{15-33}$$

where $\overline{v_r} = \langle\langle v_g \rangle\rangle - \langle\langle v_f \rangle\rangle$. There are a number of correlations for the interfacial friction factor f_i. The Wallis correlation is given by

$$f_i = 0.005\left[1 + 75\left(1 - \left\langle \alpha_g \right\rangle\right)\right] \tag{15-34}$$

which is applicable to the case with rough wavy films.

For annular flow, this interfacial shear term has been correctly taken into account in the conventional two-fluid model. However, the effect of this term in the bubbly, slug and churn turbulent flows has been generally neglected. The inclusion of this term is important for the proper modeling of the interfacial momentum coupling between phases. In order to obtain a constitutive relation for this interfacial shear term, several assumptions are necessary since it requires information on the void and shear stress distributions. For this purpose the following power-law distribution is assumed

$$\tau_i \sim \tau_w \left(\frac{r}{R_w}\right)^m . \tag{15-35}$$

From this and the void profile of Eq.(15-16), it can be shown that

$$-\left\langle \nabla\alpha_g \cdot \tau_i \right\rangle_z = -\frac{4\tau_w}{D}\left\langle \alpha_g \right\rangle \frac{n+2}{n+1+m} \tag{15-36}$$

where α_g is the void fraction of dispersed phase. By introducing the distribution parameter C_τ given by

$$C_{\tau} = \frac{n+2}{n+1+m} \tag{15-37}$$

the interfacial shear term for a dispersed two-phase flow becomes

$$-\left\langle \nabla \alpha_g \cdot \tau_i \right\rangle_z = -\frac{4\tau_w}{D}\left\langle \alpha_g \right\rangle C_{\tau} \tag{15-38}$$

where C_{τ} is expected to be very close to one. In a horizontal channel this term will contribute to the slip between phases even under a steady state condition. The inclusion of this term does not alter the overall momentum balance of a two-phase mixture because of the macroscopic momentum jump condition. However, it indicates that the momentum interaction between phases is affected by the wall shear stress through the interfacial shear and void gradient distributions.