Chapter 14

ONE-DIMENSIONAL DRIFT-FLUX MODEL

Two-phase flow always involves some relative motion of one phase with respect to the other; therefore, a two-phase-flow problem should be formulated in terms of two velocity fields. A general transient two-phase-flow problem can be formulated by using a two-fluid model or a drift-flux model, depending on the degree of the dynamic coupling between the phases. In the two-fluid model, each phase is considered separately; hence the model is formulated in terms of two sets of conservation equations governing the balance of mass, momentum, and energy of each phase. However, an introduction of two momentum equations in a formulation, as in the case of the two-fluid model, presents considerable difficulties due to mathematical complications and uncertainties in specifying interfacial interaction terms between two phases (Delhaye, 1968; Vernier and Delhaye, 1968; Bouré and Réocruex, 1972; Ishii, 1975). Numerical instabilities caused by improper choice of interfacial-interaction terms in the phase-momentum equations are common. Therefore, careful studies on the interfacial constitutive equations are required in the formulation of the two-fluid model. For example, it has been suggested (Réocruex, 1974) that the interaction terms should include first-order time and spatial derivatives under certain conditions.

These difficulties associated with a two-fluid model can be significantly reduced by formulating two-phase problems in terms of the drift-flux model (Zuber, 1967). In this model, the motion of the whole mixture is expressed by the mixture-momentum equation and the relative motion between phases is taken into account by a kinematic constitutive equation. Therefore, the basic concept of the drift-flux model is to consider the mixture as a whole rather than as two separated phases. The formulation of the drift-flux model based on the mixture balance equations is simpler than the two-fluid model based on the separate balance equations for each phase. The most important assumption associated with the drift-flux model is that the dynamics of two phases can be expressed by the mixture-momentum equation with the kinematic constitutive equation specifying the relative motion between phases. The use of the drift-flux model is appropriate when the motions of two phases are strongly coupled.

In the drift-flux model, the velocity fields are expressed in terms of the mixture center-of-mass velocity and the drift velocity of the vapor phase, which is the vapor velocity with respect to the volume center of the mixture. The effects of thermal non-equilibrium are accommodated in the drift-flux model by a constitutive equation for phase change that specifies the rate of mass transfer per unit volume. Since the rates of mass and momentum transfer at the interfaces depend on the structure of interface, these constitutive equations for the drift velocity and the vapor generation are functions of flow regimes (Zuber and Dougherty, 1967; Ishii et al., 1975).

The drift-flux model is an approximate formulation in comparison with the more rigorous two-fluid formulation. However, because of its simplicity and applicability to a wide range of two-phase-flow problems of practical interest, the drift-flux model is of considerable importance. Particularly, the one-dimensional drift-flux model obtained by averaging the local drift-flux formulation over the cross-sectional area is useful for complicated engineering problems involving fluid flow and heat transfer, since field equations can be reduced to quasi-one-dimensional forms. By area averaging, the information on changes of variables in the direction normal to the main flow within a channel is basically lost. Therefore, the transfer of momentum and energy between the wall and the fluid should be expressed by empirical correlations or by simplified models. In this chapter, we develop a general one-dimensional formulation of the drift-flux model, and discuss various special cases which are important in practical applications. For simplicity, mathematical symbols of time-averaging are dropped in the formulation in this chapter. The extensive review of this model is given by Ishii (1977).

1.1 Area average of three-dimensional drift-flux model

The three-dimensional form of the drift-flux model has been obtained by using the time or statistical averaging method. The result can be summarized as follows:

The mixture continuity equation from Eq.(13-70)

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \left(\rho_m \boldsymbol{v}_m\right) = 0 \tag{14-1}$$

The continuity equation for dispersed phase from Eq.(13-71)

$$\frac{\partial \alpha_d \rho_d}{\partial t} + \nabla \cdot \left(\alpha_d \rho_d \boldsymbol{v}_m\right) = \Gamma_d - \nabla \cdot \left(\frac{\alpha_d \rho_d \rho_c}{\rho_m} \boldsymbol{V}_{dj}\right) \tag{14-2}$$

The mixture momentum equation from Eq.(13-72)

$$\begin{aligned} &\frac{\partial \rho_m \boldsymbol{v}_m}{\partial t} + \nabla \cdot \left(\rho_m \boldsymbol{v}_m \boldsymbol{v}_m\right) \\ &= -\nabla p_m + \nabla \cdot \left[\overline{\mathcal{T}} + \mathcal{T}^T - \frac{\alpha_d}{1-\alpha_d} \frac{\rho_d \rho_c}{\rho_m} \boldsymbol{V}_{dj} \boldsymbol{V}_{dj}\right] + \rho_m \boldsymbol{g} \end{aligned} \tag{14-3}$$

The mixture enthalpy-energy equation from Eq.(13-73)

$$\begin{aligned} &\frac{\partial \rho_m h_m}{\partial t} + \nabla \cdot \left(\rho_m h_m \boldsymbol{v}_m\right) \\ &= -\nabla \cdot \left[\overline{\boldsymbol{q}} + \boldsymbol{q}^T + \frac{\alpha_d \rho_d \rho_c}{\rho_m} \left(h_d - h_c\right) \boldsymbol{V}_{dj}\right] \\ &+ \frac{\partial p_m}{\partial t} + \left[\boldsymbol{v}_m + \frac{\alpha_d \left(\rho_c - \rho_d\right)}{\rho_m} \boldsymbol{V}_{dj}\right] \cdot \nabla p_m + \Phi_m^\mu. \end{aligned} \tag{14-4}$$

The rational approach to obtain a one-dimensional drift-flux model is to integrate the three-dimensional drift-flux model over a cross-sectional area and then to introduce proper mean values. A simple area average over the cross-sectional area, A, is defined by

$$\langle F \rangle = \frac{1}{A} \int_A F dA \tag{14-5}$$

and the void-fraction-weighted mean value is given by

$$\langle\langle F_k \rangle\rangle = \frac{\langle \alpha_k F_k \rangle}{\langle \alpha_k \rangle}. \tag{14-6}$$

In the subsequent analysis, the density of each phase ρ_d and ρ_c within any cross-sectional area is considered to be uniform, so that $\rho_k = \langle\langle \rho_k \rangle\rangle$. For most practical two-phase flow problems, this assumption is valid since the transverse pressure gradient within a channel is relatively small. The detailed analysis without this approximation appears in a reference (Ishii, 1971). Under the above simplifying assumption, the average mixture density is given by

$$\langle \rho_m \rangle \equiv \langle \alpha_d \rangle \rho_d + \left(1 - \langle \alpha_d \rangle\right) \rho_c. \tag{14-7}$$

The axial component of the weighted mean velocity of phase k is

$$\langle\langle v_k \rangle\rangle = \frac{\langle \alpha_k v_k \rangle}{\langle \alpha_k \rangle} = \frac{\langle j_k \rangle}{\langle \alpha_k \rangle} \tag{14-8}$$

where the scalar expression of the velocity corresponds to the axial component of the vector. Then the mixture velocity is defined by

$$\overline{v_m} \equiv \frac{\langle \rho_m v_m \rangle}{\langle \rho_m \rangle} = \frac{\langle \alpha_d \rangle \rho_d \langle\langle v_d \rangle\rangle + \left(1 - \langle \alpha_d \rangle\right) \rho_c \langle\langle v_c \rangle\rangle}{\langle \rho_m \rangle} \tag{14-9}$$

and the volumetric flux is given by

$$\langle j \rangle \equiv \langle j_d \rangle + \langle j_c \rangle = \langle \alpha_d \rangle \langle\langle v_d \rangle\rangle + \left(1 - \langle \alpha_d \rangle\right) \langle\langle v_c \rangle\rangle. \tag{14-10}$$

The mean mixture enthalpy also should be weighted by the density; thus,

$$\overline{h_m} \equiv \frac{\langle \rho_m h_m \rangle}{\langle \rho_m \rangle} = \frac{\langle \alpha_d \rangle \rho_d \langle\langle h_d \rangle\rangle + \left(1 - \langle \alpha_d \rangle\right) \rho_c \langle\langle h_c \rangle\rangle}{\langle \rho_m \rangle}. \tag{14-11}$$

The appropriate mean drift velocity is defined by

$$\overline{V_{dj}} \equiv \langle\langle v_d \rangle\rangle - \langle j \rangle = \left(1 - \langle \alpha_d \rangle\right) \left(\langle\langle v_d \rangle\rangle - \langle\langle v_c \rangle\rangle \right). \tag{14-12}$$

The experimental determination of the drift velocity is possible if the volume flow rate of each phase, Q_k, and the mean void fraction $\langle \alpha_d \rangle$ are measured. This is because Eq.(14-12) can be transformed into

$$\overline{V_{dj}} = \frac{\langle j_d \rangle}{\langle \alpha_d \rangle} - \left(\langle j_d \rangle + \langle j_c \rangle \right) \tag{14-13}$$

where $\langle j_k \rangle$ is given by $\langle j_k \rangle = Q_k / A$. Furthermore, the present definition of the drift velocity can also be used for annular two-phase flows. Under the definitions of various velocity fields we obtain several important relations, such as

$$\begin{cases} \langle\langle v_d \rangle\rangle = \overline{v_m} + \dfrac{\rho_c}{\langle \rho_m \rangle} \overline{V_{dj}} \\ \langle\langle v_c \rangle\rangle = \overline{v_m} - \dfrac{\langle \alpha_d \rangle}{1 - \langle \alpha_d \rangle} \dfrac{\rho_d}{\langle \rho_m \rangle} \overline{V_{dj}} \end{cases} \tag{14-14}$$

and

$$\langle j \rangle = \overline{v_m} + \frac{\langle \alpha_d \rangle (\rho_c - \rho_d)}{\langle \rho_m \rangle} \overline{V_{dj}}. \tag{14-15}$$

In the drift-flux formulation, a problem is solved for $\langle \alpha_d \rangle$ and $\overline{v_m}$ with a given constitutive relation for $\overline{V_{dj}}$. Thus, Eq.(14-14) can be used to recover a solution for the velocity of each phase after a problem is solved.

By area-averaging Eqs.(14-1)-(14-4) and using the various mean values, we obtain

Mixture Continuity Equation

$$\frac{\partial \langle \rho_m \rangle}{\partial t} + \frac{\partial}{\partial z}\left(\langle \rho_m \rangle \overline{v_m} \right) = 0 \tag{14-16}$$

Continuity Equation for Dispersed Phase

$$\frac{\partial \langle \alpha_d \rangle \rho_d}{\partial t} + \frac{\partial}{\partial z}\left(\langle \alpha_d \rangle \rho_d \overline{v_m} \right) = \langle \Gamma_d \rangle - \frac{\partial}{\partial z}\left(\frac{\langle \alpha_d \rangle \rho_d \rho_c}{\langle \rho_m \rangle} \overline{V_{dj}} \right) \tag{14-17}$$

Mixture Momentum Equation

$$\begin{aligned} &\frac{\partial \langle \rho_m \rangle \overline{v_m}}{\partial t} + \frac{\partial}{\partial z}\left(\langle \rho_m \rangle \overline{v_m}^2 \right) = -\frac{\partial}{\partial z} \langle p_m \rangle \\ &+ \frac{\partial}{\partial z} \langle \tau_{zz} + \tau_{zz}^T \rangle - \langle \rho_m \rangle g_z \end{aligned} \tag{14-18}$$

$$-\frac{f_m}{2D}\langle\rho_m\rangle\overline{\overline{v_m}}\left|\overline{\overline{v_m}}\right|-\frac{\partial}{\partial z}\left[\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\left(1-\langle\alpha_d\rangle\right)\langle\rho_m\rangle}\overline{\overline{V_{dj}}}^2\right]$$
$$-\frac{\partial}{\partial z}\sum_{k=1}^{2}COV\left(\alpha_k\rho_k v_k v_k\right)$$

Mixture Enthalpy-energy Equation

$$\begin{aligned}
&\frac{\partial\langle\rho_m\rangle\overline{\overline{h_m}}}{\partial t}+\frac{\partial}{\partial z}\left(\langle\rho_m\rangle\overline{\overline{h_m}}\,\overline{\overline{v_m}}\right)=-\frac{\partial}{\partial z}\left\langle\overline{q}+q^T\right\rangle+\frac{q_W''\xi_h}{A}\\
&-\frac{\partial}{\partial z}\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\Delta h_{dc}\overline{\overline{V_{dj}}}-\frac{\partial}{\partial z}\left[\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\Delta h_{dc}\overline{\overline{V_{dj}}}\right]\\
&-\frac{\partial}{\partial z}\sum_{k=1}^{2}COV\left(\alpha_k\rho_k h_k v_k\right)+\frac{\partial\langle p_m\rangle}{\partial t}\\
&+\left[\overline{\overline{v_m}}+\frac{\langle\alpha_d\rangle\left(\rho_c-\rho_d\right)}{\langle\rho_m\rangle}\overline{\overline{V_{dj}}}\right]\frac{\partial\langle p_m\rangle}{\partial z}+\left\langle\Phi_m^\mu\right\rangle.
\end{aligned}\tag{14-19}$$

Here, $\tau_{zz}+\tau_{zz}^T$ denotes the normal components of the stress tensor in the axial direction and Δh_{dc} is the enthalpy difference between phases; thus, $\Delta h_{dc}=\langle\langle h_d\rangle\rangle-\langle\langle h_c\rangle\rangle$. The covariance terms represent the difference between the average of a product and the product of the average of two variables such that $COV\left(\alpha_k\rho_k\psi_k v_k\right)\equiv\left\langle\alpha_k\rho_k\psi_k\left(v_k-\langle\langle v_k\rangle\rangle\right)\right\rangle$. If the profile of either ψ_k or v_k is flat, then the covariance term reduces to zero. The term represented by $f_m\langle\rho_m\rangle\overline{\overline{v_m}}\left|\overline{\overline{v_m}}\right|/(2D)$ in Eq.(14-18) is the two-phase frictional pressure drop. We note here that the effects of the mass, momentum, and energy diffusion associated with the relative motion between phases appear explicitly in the drift-flux formulation, since the convective terms on the left-hand side of the field equations are expressed in terms of the mixture velocity. These effects of diffusions in the present formulation are expressed in terms of the drift velocity of the dispersed phase V_{dj}. This may be formulated in a functional form as

$$\overline{\overline{V_{dj}}}=\overline{\overline{V_{dj}}}\left(\langle\alpha_d\rangle,\langle p_m\rangle,g_z,\overline{\overline{v_m}},\text{ etc.}\right).\tag{14-20}$$

To take into account the mass transfer across the interfaces, a constitutive equation for $\langle\Gamma_d\rangle$ should also be given. In a functional form, this phase-change constitutive equation may be written as

$$\langle \Gamma_d \rangle = \langle \Gamma_d \rangle \left(\langle \alpha_d \rangle, \langle p_m \rangle, \overline{\overline{v}}_m, \frac{\partial \langle p_m \rangle}{\partial t}, \text{etc.} \right). \tag{14-21}$$

The above formulation can be extended to non-dispersed two-phase flows, such as an annular flow, provided a proper constitutive relation for a drift velocity of one of the phases is given.

1.2 One-dimensional drift velocity

1.2.1 Dispersed two-phase flow

To obtain a kinematic constitutive equation for the one-dimensional drift-flux model, we must average the local drift velocity over the channel cross section. The constitutive relation for the local drift velocity V_{dj} in a confined channel was developed in the Section 1.2 of Chapter 13. Now we relate this to the mean drift velocity $\overline{\overline{V}}_{dj}$ defined by Eq.(14-12).

From Eqs.(14-6) and (14-12),

$$\overline{\overline{V}}_{dj} = \left\langle \frac{\alpha_d (j + V_{dj})}{\langle \alpha_d \rangle} - j \right\rangle = \langle\langle V_{dj} \rangle\rangle + (C_0 - 1)\langle j \rangle \tag{14-22}$$

where

$$\langle\langle V_{dj} \rangle\rangle \equiv \frac{\langle \alpha_d V_{dj} \rangle}{\langle \alpha_d \rangle} \tag{14-23}$$

and

$$C_0 \equiv \frac{\langle \alpha_d j \rangle}{\langle \alpha_d \rangle \langle j \rangle}. \tag{14-24}$$

The second term on the right-hand side of Eq.(14-22) is a covariance between the concentration profile and the volumetric flux profile; thus it can also be expressed as $COV(\alpha_d j)/\langle \alpha_d \rangle$. The factor C_0, which has been used for bubbly or slug flows by several authors (Zuber and Findlay, 1965; Nicklin et al., 1962; Neal, 1963) is known as a distribution parameter. The inverse of this parameter was also used in the early work of Bankoff (1960). Physically, this effect arises from the fact that the dispersed phase is locally transported with the drift velocity V_{dj} with respect to local volumetric flux j and not to the average volumetric flux $\langle j \rangle$. For example, if the dispersed phase is more concentrated in the higher-flux region, then the mean transport of the dispersed phase is promoted by higher local j.

The value of C_0 can be determined from assumed profiles of the void fraction α_d and total volumetric flux j (Zuber and Findlay, 1965), or from experimental data (Zuber et al., 1967). By assuming power-law profiles in a pipe for j and α_d, we have

$$\begin{cases} \dfrac{j}{j_0} = 1 - \left(\dfrac{r}{R_W}\right)^m \\ \dfrac{\alpha_d - \alpha_{dW}}{\alpha_{d0} - \alpha_{dW}} = 1 - \left(\dfrac{r}{R_W}\right)^n \end{cases} \tag{14-25}$$

where j_0, α_{d0}, α_{dW}, r, and R_W are, respectively, the value of j and α at the center, the void fraction at the wall, radial distance, and the radius of a pipe. By substituting these profiles into the definition of C_0 given by Eq.(14-24), we obtain

$$C_0 = 1 + \frac{2}{m+n+2}\left(1 - \frac{\alpha_{dW}}{\langle \alpha_d \rangle}\right). \tag{14-26}$$

The distribution parameter based on the assumed profiles above is further discussed in a reference (Zuber et al., 1967).

Now Eq.(14-22) can be transformed to

$$\langle\langle v_d \rangle\rangle = \frac{\langle j_d \rangle}{\langle \alpha_d \rangle} = C_0 \langle j \rangle + \langle\langle V_{dj} \rangle\rangle \tag{14-27}$$

where $\langle\langle v_d \rangle\rangle$ and $\langle j \rangle$ are easily obtainable parameters in experiments, particularly under an adiabatic condition. Therefore, this equation suggests a plot of the mean velocity $\langle\langle v_d \rangle\rangle$ versus the average volumetric flux $\langle j \rangle$. If the concentration profile is uniform across the channel, then the value of the distribution parameter is equal to unity. In addition, if the effect of the local drift $\langle\langle V_{dj} \rangle\rangle$ is negligibly small, then the flow becomes essentially homogeneous. In this case, the relation between the mean velocity and flux reduces to a straight line through the origin at an angle of 45°. The deviation of the experimental data from this homogeneous flow line shows the magnitude of the drift of the dispersed phase with respect to the volume center of the mixture.

An important characteristic of such a plot is that, for two-phase flow regimes with fully developed void and velocity profiles, the data points cluster around a straight line (see Figs.14-1-14-3). This trend is particularly pronounced when the local drift velocity is constant or negligibly small. Hence, for a given flow regime, the value of the distribution parameter C_0 may be obtained from the slope of these lines, whereas the intercept of this line with the mean velocity axis can be interpreted as the weighed mean local drift velocity, $\langle\langle V_{dj}\rangle\rangle$. The extensive study by Zuber et al. (1967) shows that C_0 depends on pressure, channel geometry, and perhaps flow rate. An important effect of subcooled boiling and developing void profile on the distribution parameter has also been noted by Hancox and Nicoll (1972). Here, a simple correlation for the distribution parameter in bubbly-flow regime is presented based on study by Ishii (1977). First, by considering a fully developed bubbly flow, we assumed that C_0 depends on the density ratio ρ_g/ρ_f and on the Reynolds number based on liquid properties, GD/μ_f, where G, D, and μ_f are the total mass flow rate, hydraulic diameter, and the viscosity of the liquid, respectively. Hence,

$$C_0 = C_0\left(\frac{\rho_g}{\rho_f}, \frac{GD}{\mu_f}\right) \tag{14-28}$$

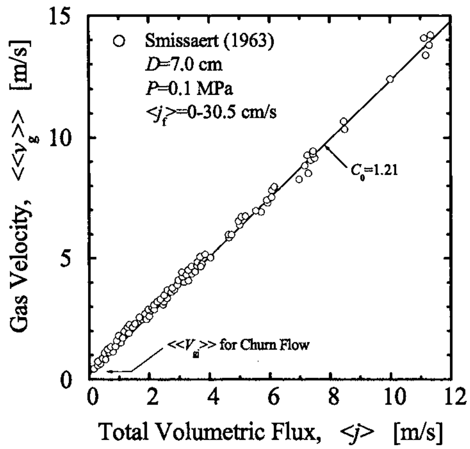

Figure 14-1. Fully developed air-water flow data (Ishii, 1977)

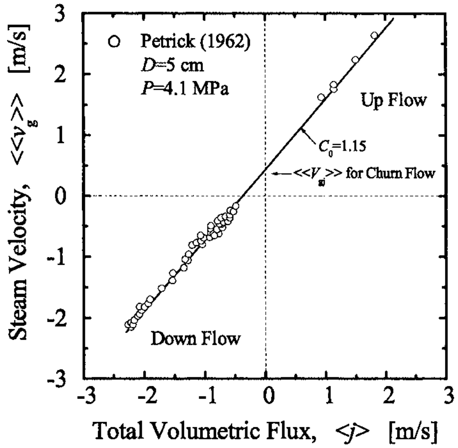

Figure 14-2. Experimental data for cocurrent upflow and cocurrent downflow of steam-water system (Ishii, 1977)

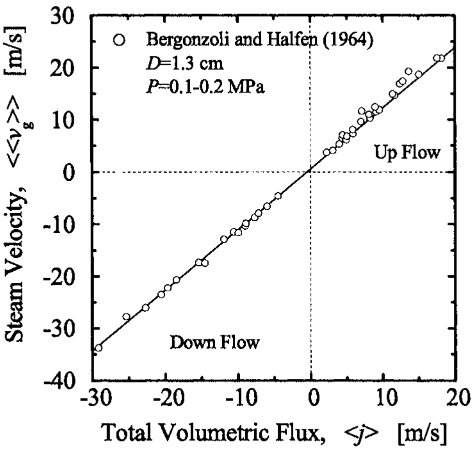

Figure 14-3. Experimental data for cocurrent upflow and cocurrent downflow of heated Santowax-R system (Ishii, 1977)

A single-phase turbulent-flow profile and the ratio of the maximum velocity to mean velocity give a theoretical limiting value of C_0 at $\alpha_d \to 0$ and $\rho_g/\rho_f \to 0$, since in this case all the bubbles should be concentrated at the central region. Thus from the experimental data of Nukuradse (1932) for a round tube, which gives the ratio of the maximum to mean velocity, we have

$$C_\infty = \lim \frac{\langle \alpha_d j \rangle}{\langle \alpha_d \rangle \langle j \rangle} = \frac{\langle \alpha_d \rangle j_0}{\langle \alpha_d \rangle \langle j \rangle} = 1.393 - 0.0155 \ln\left(\frac{GD}{\mu_f}\right) \quad (14\text{-}29)$$

as $\alpha_d \to 0$ and $\rho_g/\rho_f \to 0$. Furthermore, as the density ratio approaches unity, the distribution parameter C_0 should also become unity. Thus,

$$C_0 \to 1 \quad (14\text{-}30)$$

as $\rho_g/\rho_f \to 1$. Based on these limits and various experimental data in a fully developed flow, the distribution parameter can be given approximately by

$$C_0 = C_\infty - (C_\infty - 1)\sqrt{\rho_g/\rho_f} \quad (14\text{-}31)$$

where the density group scales the inertia effects of each phase in a transverse void distribution. Physically, Eq.(14-31) models the tendency of the lighter phase to migrate into a higher-velocity region, thus resulting in a higher void concentration in the central region (Bankoff, 1960). For a laminar flow, C_∞ is 2, but, due to the large velocity gradient, C_0 is very sensitive to $\langle \alpha_d \rangle$ at low void fractions.

Over a wide range of Reynolds number, GD/μ_f, Eq.(14-29) can be approximated by $C_\infty \cong 1.2$ for a flow in a round tube. Furthermore, for a rectangular channel, the experimental data show this value to be approximately 1.35. Thus, for a fully developed turbulent bubbly flow,

$$C_0 \cong \begin{cases} 1.2 - 0.2\sqrt{\rho_g/\rho_f}: & \text{round tube} \\ 1.35 - 0.35\sqrt{\rho_g/\rho_f}: & \text{rectangular channel.} \end{cases} \quad (14\text{-}32)$$

Figures 14-4 and 14-5 compare the above correlation with various experimental data. Each point in the figures represents anywhere from five to 150 data points. For example, the original experimental data of Smissaert (1963), shown in Fig.14-1, are presented by a single plot in Fig. 14-4. Each

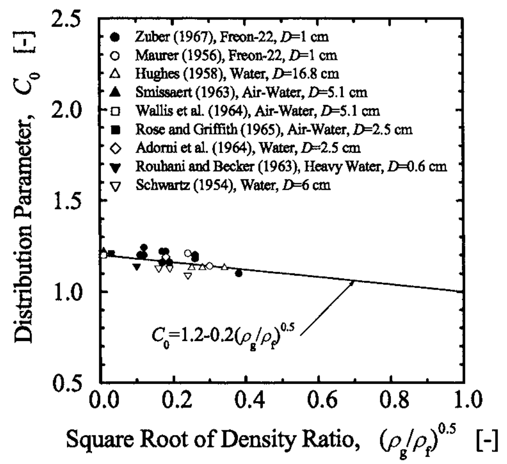

Figure 14-4. Distribution parameter for fully developed flow in a round tube (Ishii, 1977)

point in Fig.14-1 can be used to obtain a corresponding value for C_0 by using the existing correlation for the mean local drift velocity $\langle\langle V_{dj}\rangle\rangle$. However, in view of the strong linear relation between the mean velocity of the dispersed phase and the total flux, the average value of C_0 obtained by linear fitting has been used in Figs.14-4 and 14-5.

In the velocity-flux plane (see Figs.14-2 and 14-3), three operational modes can be easily identified. In the first quadrant, the flow is basically cocurrent upward; therefore both the liquid and vapor phases flow in an upward direction. In the second quadrant, the vapor phase is moving upward; however, there is a net downward flow of mixture. Consequently, the flow is countercurrent. The cocurrent downflow operation should appear in the third quadrant of the velocity-flux plane, as shown in Figs.14-2 and 14-3. These data indicate that the basic characteristic described by Eq.(14-27) is valid for both cocurrent up and down flows with an identical value for the distribution parameter, C_0. This fact demonstrates the usefulness of correlating the drift velocity in terms of the mean local drift velocity $\langle\langle V_{dj}\rangle\rangle$ and C_0.

In two-phase systems with heat addition, the change of void profiles from concave to convex can occur. The concave void-fraction profile is caused by the wall nucleation and delayed transverse migration of bubbles toward the center of a channel. Under these conditions, most of the bubbles are initially located near the nucleating wall, although even in adiabatic flow, small bubbles tend to accumulate near the wall region at low void fraction. The concave profile is particularly pronounced in the subcooled boiling regime,

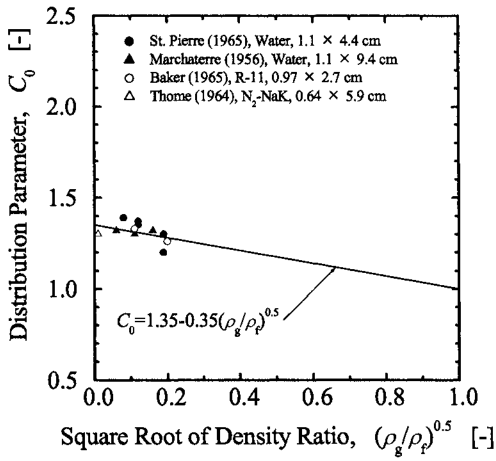

Figure 14-5. Distribution parameter for fully developed flow in a rectangular channel (Ishii, 1977)

because here only the wall-boundary layer is heated above the saturation temperature and the core liquid is subcooled. This temperature profile will induce collapses of migrating bubbles in the core region and resultant latent heat transport from the wall to the subcooled liquid. A similar concave profile can also be obtained by injecting gas into flowing liquid through a porous tube wall (Rose and Griffith, 1965).

In the region in which voids are still concentrated close to the wall, the mean velocity of vapor can be less than the mean velocity of liquid because the bulk of liquid moves with the high core velocity. However, as more and more vapor is generated along the channel, the void-fraction profile changes from concave to convex and becomes fully developed. For a flow with generation of void at the wall due to either nucleation or gas injection, the distribution parameter C_0 should have a near-zero value at the beginning of the two-phase flow region. This can be also seen from the definition of C_0 in Eq.(14-24). Hence, we have

$$\lim_{\langle\alpha_d\rangle\to 0} C_0 = \lim \frac{\langle\alpha_d j\rangle}{\langle\alpha_d\rangle\langle j\rangle} = \frac{\langle\alpha_d\rangle j_W}{\langle\alpha_d\rangle\langle j\rangle} = 0 \quad \text{for } \Gamma_g > 0. \tag{14-33}$$

With the increase in the cross-sectional mean void fraction, the peak of the local void fraction moves from the near-wall region to the central region. This will lead to the increase in the value of C_0 as the void profile develops.

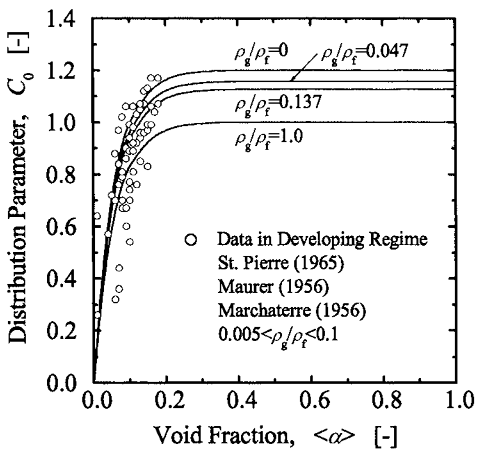

Figure 14-6. Distribution parameter in developing flow due to boiling (Data for the rectangular duct have been modified by a factor of 1.2/1.35 to obtain corresponding data for a round tube.) (Ishii, 1977)

In view of the basic characteristic described above and various experimental data (Zuber et al., 1967; Maurer, 1956; Pierre, 1965; Marchaterre, 1956) the following simple correlation is proposed (Ishii, 1977)

$$C_0 = \left[C_\infty - \left(C_\infty - 1\right)\sqrt{\rho_g/\rho_f}\right]\left(1 - e^{-18\langle\alpha_d\rangle}\right). \tag{14-34}$$

This expression indicates the significance of the developing void profile in the region given by $0 < \langle\alpha_d\rangle < 0.25$; beyond this region, the value of C_0 approaches rapidly to that for a fully developed flow (see Fig.14-6). Hence, for $\Gamma_g > 0$, we obtain

$$C_0 = \begin{cases} \left(1.2 - 0.2\sqrt{\rho_g/\rho_f}\right)\left(1\text{-}e^{-18\langle\alpha_d\rangle}\right): & \text{round tube} \\ \left(1.35 - 0.35\sqrt{\rho_g/\rho_f}\right)\left(1\text{-}e^{-18\langle\alpha_d\rangle}\right): & \text{rectangular channel.} \end{cases} \tag{14-35}$$

For most droplet or particulate flows in the turbulent regime, the volumetric flux profile is quite flat due to the turbulent mixing and particle slips near the wall, which increases the volumetric flux. The concentration of dispersed phase also tends to be uniform, except for weak peaking near the core of the flow. Because of these profiles for j and α_d, the value of the distribution parameter C_0 is expected to be close to unity $(1.0 \leq C_0 \leq 1.1)$. Thus, by assuming that the covariance terms are negligibly small for droplet or particulate flows, we have

$$\overline{V_{dj}} \cong \langle\langle V_{dj} \rangle\rangle \tag{14-36}$$

in which case the local slip becomes important.

The calculation of $\langle\langle V_{dj} \rangle\rangle$ based on the local constitutive equations is the integral transformation, Eq.(14-23); thus, it will require additional information on the void profile (Ishii, 1976). Since this profile is not known in general, we make the following simplifying approximations. The average drift velocity $\langle\langle V_{dj} \rangle\rangle$ due to the local slip can be predicted by the same expression as the local constitutive relations given in a reference (Ishii, 1976), provided the local void fraction α_d and the non-dimensional difference of the stress gradient are replaced by average values. These approximations are good for flows with a relatively flat void-fraction profile; also, they can be considered acceptable from the overall simplicity of the one-dimensional model.

For a fully developed vertical flow, the stress distribution in the fluid and in the dispersed phase should be similar; thus, the effect of shear gradient on the mean local drift velocity can be neglected. Under these conditions we obtain the following results.

Undistorted-particle Regime

$$\begin{aligned}\overline{V_{dj}} &= (C_0 - 1)\langle j \rangle + \frac{10.8\mu_c}{\rho_c r_d}\frac{\mu_c}{\langle \mu_m \rangle}\left(1 - \langle \alpha_d \rangle\right)^2 \\ &\times \frac{\psi^{4/3}(1+\psi)}{1 + \psi\left[\frac{\mu_c}{\langle \mu_m \rangle}\left(1 - \langle \alpha_d \rangle\right)^{0.5}\right]^{6/7}}\frac{\rho_c - \rho_d}{\Delta\rho}\end{aligned} \tag{14-37}$$

where $\psi(r_d^*) = 0.55\left[\left(1 + 0.08 r_d^{*3}\right)^{4/7} - 1\right]^{0.75}$ for $r_d^* < 34.65$ and $\psi(r_d^*) = 17.67$ for $r_d^* \geq 34.65$. The limiting case of the undistorted-particle regime is the Stokes regime in which the mean drift velocity reduces to

$$\overline{V_{dj}} = \left(C_0 - 1\right)\langle j\rangle + \frac{2}{9} r_d^2 \frac{g\Delta\rho}{\mu_c}\left(1 - \langle\alpha_d\rangle\right)^2 \frac{\mu_c}{\langle\mu_m\rangle}\frac{\rho_c - \rho_d}{\Delta\rho} \tag{14-38}$$

Distorted-particle Regime $(1.75 \le n \le 2.25)$

$$\overline{V_{dj}} = \left(C_0 - 1\right)\langle j\rangle + \sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4}\left(1 - \langle\alpha_d\rangle\right)^n \frac{\rho_c - \rho_d}{\Delta\rho}. \tag{14-39}$$

Here the value of n depends on the viscosities as

$$\begin{aligned} n &= 1.75 \quad ; \quad \mu_d << \mu_c \\ n &= 2 \quad ; \quad \mu_d \cong \mu_c \\ n &= 2.25 \quad ; \quad \mu_d >> \mu_c \end{aligned} \tag{14-40}$$

Churn-turbulent-flow Regime

$$\overline{V_{dj}} = \left(C_0 - 1\right)\langle j\rangle + \sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4}\frac{\rho_c - \rho_d}{\Delta\rho}. \tag{14-41}$$

Here the mean mixture viscosity (Ishii, 1976) is given by

$$\frac{\langle\mu_m\rangle}{\mu_c} = \left(1 - \frac{\langle\alpha_d\rangle}{\alpha_{dm}}\right)^{-2.5\alpha_{dm}(\mu_d + 0.4\mu_c)/(\mu_d + \mu_c)}. \tag{14-42}$$

The value of maximum packing, $\alpha_{dm} = 0.62$, is recommended for solid particle-fluid systems, although it can range from 0.5 to 0.74. However, for a bubbly flow, the theoretical value of α_{dm} can be much higher. If we consider the standard range of interest of void fraction in bubbly flow, α_{dm} may be approximated by $\alpha_{dm} = 1$. Hence, for a bubbly flow, the mixture viscosity becomes

$$\frac{\langle\mu_m\rangle}{\mu_c} = \frac{1}{1 - \langle\alpha_d\rangle}. \tag{14-43}$$

However, for a particulate flow with a low particle concentration, namely, $\langle\alpha_d\rangle << 1$, $\langle\mu_m\rangle$ can be approximated by

$$\frac{\langle \mu_m \rangle}{\mu_c} = \left(1 - \langle \alpha_d \rangle\right)^{-2.6}. \tag{14-44}$$

In a horizontal flow with a complete suspension of the dispersed phase, the transverse mixing, which keeps the particles suspended, can significantly influence the stress gradient of each phase; thus, the stress gradient effect may not be neglected. However, in view of the present state of the art, the assumption $\langle\langle V_{dj} \rangle\rangle \cong 0$ may be used as a first-order approximation, particularly in high-flux flows. As explained at the end of this chapter, the actual local drift velocity depends also upon the pressure gradient due to friction and, therefore, in strict sense it is not zero even in horizontal flow.

For high-flux flows, the effect of the local drift $\langle\langle V_{dj} \rangle\rangle$ on the mean drift velocity is small in comparison with the covariance term $(C_0 - 1)\langle j \rangle$. Thus, by neglecting the former, we have

$$\overline{V_{dj}} = \frac{(C_0 - 1)\langle \rho_m \rangle \overline{v_m}}{\langle \rho_m \rangle - (C_0 - 1)\langle \alpha_d \rangle (\rho_c - \rho_d)}. \tag{14-45}$$

For bubbly flows, the above equation imposes a condition on applicable void-fraction ranges; thus, we should have $\langle \rho_m \rangle > (C_0 - 1)\langle \alpha_d \rangle (\rho_d - \rho_c)$. Here, a simple criterion for the boundary between the high- and low-flux flow can be obtained by taking the ratio of the total volumetric flux and the terminal velocity. If this ratio is more than 10, the flow can be considered a high-flux flow.

The other limiting case of the dispersed two-phase flow in a confined channel is slug flow. When the volume of a bubble is very large, the shape of the bubble is significantly deformed to fit the channel geometry. The diameters of the bubbles become approximately that of the pipe with a thin liquid film separating the bubbles from the wall. The bubbles have the bullet form with a cap-shaped nose. The motion of these bubbles in relatively inviscid fluids can be studied by using a potential flow analysis around a sphere (Dumitrescu, 1943), and the result is shown to agree with experimental data. Thus,

$$\overline{V_{dj}} = 0.2\langle j \rangle + 0.35\left(\frac{gD\Delta\rho}{\rho_c}\right)^{1/2} \tag{14-46}$$

which was originally proposed by Nicklin et al. (1962) and Neal (1963).

1.2.2 Annular two-phase flow

In annular two-phase flow, the relative motions between phases are governed by the interfacial geometry, the body-force field, and the interfacial momentum transfer. The constitutive equation for the vapor-drift velocity in annular two-phase flow has been developed by taking into account those macroscopic effects of the structured two-phase flows (Ishii et al., 1976). Assuming steady-state adiabatic two-phase annular flow with constant single-phase properties, we have the following one-dimensional momentum equations for each phase.

$$-\left(\frac{dp_m}{dz}+\rho_g g_z\right)=\frac{\tau_i P_i}{\langle\alpha_g\rangle A} \tag{14-47}$$

and

$$-\left(\frac{dp_m}{dz}+\rho_f g_z\right)=\frac{\tau_{wf} P_{wf}}{\left(1-\langle\alpha_g\rangle\right)A}-\frac{\tau_i P_i}{\left(1-\langle\alpha_g\rangle\right)A} \tag{14-48}$$

where τ_i, τ_{wf}, P_i, and P_{wf} are the interfacial shear, wall shear, interfacial wetted perimeter, and wall wetted perimeter, respectively. The hydraulic diameter and the ratio of wetted perimeters are defined by $D\equiv 4A/P_{wf}$ and $\xi\equiv P_i/P_{wf}$. By assuming that the film thickness δ is small compared with D, we have $4\delta/D\cong 1-\langle\alpha_g\rangle$. Furethermore, for an annular flow in a pipe, ξ reduces to $\sqrt{\alpha_g}$.

The wall shear can be expressed through the friction factor with a gravity-correction term by $\tau_{wf}=f_{wf}\rho_f\langle\langle v_f\rangle\rangle\left|\langle\langle v_f\rangle\rangle\right|/2-\Delta\rho g_z\delta/3$, where f_{wf} can be given by the standard friction-factor correlation: $f_{wf}=16/Re_f$ for laminar film flows and $f_{wf}=0.0791Re_f^{-0.25}$ for turbulent flows. Here the liquid-film Reynolds number is given by $Re_f=\rho_f\left|\langle j_f\rangle\right|D/\mu_f$. Similarly, the interfacial shear can be expressed as $\tau_i=f_i\rho_g\left|\overline{v_r}\right|\overline{v_r}/2$ with the interfacial friction factor given by $f_i=0.005\left[1+75\left(1-\langle\alpha_g\rangle\right)\right]$ for rough wavy films (Wallis, 1969).

By definition, the vapor-drift velocity is related to v_r, namely, $\overline{V_{gj}}=\left(1-\langle\alpha_g\rangle\right)\overline{v_r}$. Hence, by eliminating the pressure gradient from the momentum equations, we obtain for a laminar film

$$\overline{V_{gj}} = \pm \left[\frac{16\langle\alpha_g\rangle}{\rho_g f_i \xi} \left| \frac{\mu_f \langle j_f\rangle}{D} + \frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)^3}{48} \right| \right]^{1/2} \tag{14-49}$$

and for a turbulent film

$$\overline{V_{gj}} = \pm \left[\frac{\langle\alpha_g\rangle\left(1-\langle\alpha_g\rangle\right)^3 D}{\rho_g f_i \xi} \times \left| \frac{0.005\rho_f \langle j_f\rangle \left|\langle j_f\rangle\right|}{D\left(1-\langle\alpha_g\rangle\right)^3} + \frac{1}{3}\Delta\rho g_z \right| \right]^{1/2}. \tag{14-50}$$

Here, the negative root is taken when the term within the absolute signs becomes negative. The drift velocity in the form expressed by Eqs.(14-49) and (14-50) is convenient for use in analyzing steady-state adiabatic or thermal-equilibrium flows since, in these cases, the value of $\langle j_f\rangle$ can be easily obtained.

In a general drift-flux-model formulation, $\overline{V_{gj}}$ should be expressed in terms of the mixture velocity $\overline{v_m}$ rather than $\langle j_f\rangle$, as $\overline{v_m}$ is the velocity used in the formulation. From the definition, we have

$$\langle j_f\rangle = \left(1-\langle\alpha_g\rangle\right)\overline{v_m} - \frac{\langle\alpha_g\rangle\rho_g}{\langle\rho_m\rangle}\overline{V_{gj}}. \tag{14-51}$$

By substituting Eq.(14-51) into Eq.(14-49), we obtain for a laminar film

$$\overline{V_{gj}} = \pm \frac{8\mu_f\langle\alpha_g\rangle^2}{\langle\rho_m\rangle D f_i \xi} \left[-1 + \left\{ 1 + \frac{f_i D \langle\rho_m\rangle^2 \left(1-\langle\alpha_g\rangle\right)\xi}{4\mu_f\langle\alpha_g\rangle^3 \rho_g} \times \left| \overline{v_m} + \frac{\Delta\rho g_z D^2\left(1-\langle\alpha_g\rangle\right)^2}{48\mu_f} \right| \right\}^{1/2} \right] \tag{14-52}$$

which is valid for the laminar range given by

$$\frac{\left(1-\langle\alpha_g\rangle\right)\langle\rho_m\rangle\overline{v_m}-\langle\rho_m\rangle\langle j_f\rangle_{tr}}{\langle\alpha_g\rangle\rho_g}\leq\overline{V_{gj}}$$
$$\leq\frac{\left(1-\langle\alpha_g\rangle\right)\langle\rho_m\rangle\overline{v_m}+\langle\rho_m\rangle\langle j_f\rangle_{tr}}{\langle\alpha_g\rangle\rho_g}. \tag{14-53}$$

Here the laminar turbulent-transition volumetric flow is defined by $\langle j_f\rangle_{tr} = 3200\mu_f/\rho_f D$. The negative root of Eq.(14-52) applies when the term within the absolute signs becomes negative. It is easy to show that, for $\overline{V_{gj}} \leq \left(1-\langle\alpha_g\rangle\right)\langle\rho_m\rangle\overline{v_m}/\left(\langle\alpha_g\rangle\rho_g\right)$, the flow is cocurrent upward, whereas, for $\overline{V_{gj}}$ larger than the above limit, the liquid flow is downward. The solution for the case of turbulent film flow is somewhat more complicated. For convenience, let us introduce the following parameters.

$$\begin{cases} a\equiv\dfrac{f_i\xi\rho_g}{0.005\langle\alpha_g\rangle\rho_f\left(1-\langle\alpha_g\rangle\right)^2} \\ b\equiv\dfrac{\langle\alpha_g\rangle\rho_g}{\langle\rho_m\rangle\left(1-\langle\alpha_g\rangle\right)} \\ c\equiv\dfrac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)}{0.015\rho_f} \end{cases} \tag{14-54}$$

Then, for upward liquid flow, we have

$$\overline{V_{gj}}=\begin{cases}\dfrac{-b\overline{v_m}+\left[a\overline{v_m}^2+\left(a-b^2\right)c\right]^{1/2}}{\left(a-b^2\right)} & \text{if } a-b^2\neq 0\\ \dfrac{\overline{v_m}^2+c}{2b\overline{v_m}} & \text{if } a-b^2=0\end{cases} \tag{14-55}$$

which applies under the condition $\overline{v_m}\geq\sqrt{cb^2/a}$. However, in the transition regime given by $-\sqrt{c}\leq\overline{v_m}<\sqrt{cb^2/a}$, where the liquid film flow is downward with upward interfacial shear forces on the film, the vapor-drift velocity becomes

$$\overline{V_{gj}} = \frac{b\overline{v_m} + \left[-a\overline{v_m}^2 + \left(a + b^2\right)c\right]^{1/2}}{a + b^2}. \tag{14-56}$$

In the range of $\overline{v_m}$ given by $\overline{v_m} \leq -\sqrt{c}$,

$$\overline{V_{gj}} = \frac{-b\overline{v_m} - \left[a\overline{v_m}^2 - \left(a - b^2\right)c\right]^{1/2}}{a - b^2} \tag{14-57}$$

which applies to the cocurrent downward flow.

The above solution can be applied only if the following turbulent-flow criterion is satisfied.

$$\begin{cases} \overline{V_{gj}} \leq \dfrac{\left(1 - \langle\alpha_g\rangle\right)\langle\rho_m\rangle\overline{v_m} - \langle\rho_m\rangle\langle j_f\rangle_{tr}}{\langle\alpha_g\rangle\rho_g} \\ \text{or} \\ \overline{V_{gj}} \geq \dfrac{\left(1 - \langle\alpha_g\rangle\right)\langle\rho_m\rangle\overline{v_m} + \langle\rho_m\rangle\langle j_f\rangle_{tr}}{\langle\alpha_g\rangle\rho_g} \end{cases} \tag{14-58}$$

These results do not have a very simple form for a turbulent film. However, if the absolute value of the mixture velocity is large, so that the flow is essentially cocurrent and the gravity effect is small, then the turbulent solution can be approximated by the simple form

$$\overline{V_{gj}} = \frac{\left(1 - \langle\alpha_g\rangle\right)\overline{v_m}}{\dfrac{\langle\alpha_g\rangle\rho_g}{\langle\rho_m\rangle} + \left\{\dfrac{\xi\rho_g\left[1 + 75\left(1 - \langle\alpha_g\rangle\right)\right]}{\langle\alpha_g\rangle\rho_f}\right\}^{1/2}}. \tag{14-59}$$

Equation (14-59) for the drift velocity can be transformed to obtain the slip ratio v_g/v_f under the simplifying assumption that the average liquid velocity is much smaller than the vapor velocity. Then we have

$$\frac{\langle\langle v_g\rangle\rangle}{\langle\langle v_f\rangle\rangle}=\sqrt{\frac{\rho_f}{\rho_g}}\left[\frac{\sqrt{\langle\alpha_g\rangle}}{1+75\left(1-\langle\alpha_g\rangle\right)}\right]^{1/2} \tag{14-60}$$

for an annular flow in a pipe for which $\xi=\sqrt{\langle\alpha_g\rangle}$. The above expression for slip ratio is similar to that obtained by Fauske (1962), namely $\langle\langle v_g\rangle\rangle/\langle\langle v_f\rangle\rangle=\sqrt{\rho_f/\rho_g}$, which has no dependence on the void fraction. The factor that takes the void fraction into account in Eq.(14-60) varies roughly from 0.24 to 1 for the range $0.8<\langle\alpha_g\rangle<1$. Therefore, for a turbulent film, the Fauske correlation should give reasonably accurate results at high void fractions.

The drift-velocity correlation for the annular flow has been expressed in terms of the mixture velocity, since $\overline{v_m}$ is the basic variable in the formulation of the general drift-flux model. However, it is also interesting and important to resolve the expression for $\overline{V_{gj}}$ in terms of the total volumetric flux $\langle j\rangle$, since $\langle j\rangle$ was the variable used to correlate $\overline{V_{gj}}$ in dispersed two-phase flow regimes.

By considering the turbulent film-flow regime and using the definition $\langle j_f\rangle=\left(1-\langle\alpha_g\rangle\right)\langle j\rangle-\langle\alpha_g\rangle\overline{V_{gj}}$, we can resolve Eq.(14-50) for the mean drift velocity $\overline{V_{gj}}$. The result does not have a simple form; however, for most practical cases, it can be approximated by a linear function of $\langle j\rangle$.

$$\overline{V_{gj}}\cong\frac{1-\langle\alpha_g\rangle}{\langle\alpha_g\rangle+\left[\frac{1+75\left(1-\langle\alpha_g\rangle\right)}{\sqrt{\langle\alpha_g\rangle}}\frac{\rho_g}{\rho_f}\right]^{1/2}} \tag{14-61}$$

$$\times\left[\langle j\rangle+\sqrt{\frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)}{0.015\rho_f}}\right]$$

This expression may be further simplified for $\rho_g/\rho_f<<1$ as

$$\overline{V_{gj}}\cong\frac{1-\langle\alpha_g\rangle}{\langle\alpha_g\rangle+4\sqrt{\rho_g/\rho_f}}\left[\langle j\rangle+\sqrt{\frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)}{0.015\rho_f}}\right]. \tag{14-62}$$

From the comparison of Eq.(14-62) to Eq.(14-22), the apparent distribution parameter for annular flow becomes

$$C_0 \simeq 1+\frac{1-\langle\alpha_g\rangle}{\langle\alpha_g\rangle+4\sqrt{\rho_g/\rho_f}}; \quad \left(\rho_g/\rho_f << 1\right). \tag{14-63}$$

This indicates that the apparent C_0 in annular flow should be close to unity.

1.2.3 Annular mist flow

As the gas velocity increases in the annular flow, the entrainment of liquid from the film to the gas-core flow takes place. Based on criteria developed for an onset of entrainment (Ishii and Grolmes, 1975), the critical gas velocity for a rough turbulent film flow can be given by

$$\left|\langle j_g\rangle\right| > \frac{\sigma}{\mu_f}\sqrt{\frac{\rho_f}{\rho_g}}\times\begin{cases} N_{\mu f}^{0.8} & \text{for } N_{\mu f}\le\dfrac{1}{15} \\ 0.1146 & \text{for } N_{\mu f}>\dfrac{1}{15}\end{cases} \tag{14-64}$$

where $N_{\mu f}\equiv\mu_f\Big/\left[\rho_f\sigma\sqrt{\sigma/g\Delta\rho}\right]^{1/2}$. However, in general, the vapor flux is much larger than the liquid flux in the annular-mist-flow regime. Then, for a weakly viscous fluid such as water or sodium, the above correlation may be replaced by

$$\left|\langle j_g\rangle\right| \simeq \left|\langle j\rangle\right| > \left(\frac{\Delta\rho g}{\rho_g^2}\right)^{1/4} N_{\mu f}^{-0.2}. \tag{14-65}$$

If Inequality (14-65) is satisfied, then the droplet entrainment into the gas-core flow should be considered; otherwise the correlation for annular flow, Eq.(14-62) can be applied.

The correlation for $\overline{V_{gj}}$ in annular mist flow can be readily developed by combining the previous results for a dispersed flow and pure annular flow. The area fraction of liquid entrained in the gas core from total liquid area at any cross section is denoted by E_d, and the cross-sectional-area-averaged void fraction by $\langle\alpha_g\rangle$. Then the film-area fraction is given by

$$1-\alpha_{core}=\frac{\text{liquid-film cross-sectional area}}{\text{total cross-sectional area}} \qquad (14\text{-}66)$$
$$=\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)$$

and the mean liquid-droplet fraction in the gas core alone is given by

$$\alpha_{drop}=\frac{\text{cross-sectional area of drops}}{\text{cross-sectional area of core}} \qquad (14\text{-}67)$$
$$=\frac{\left(1-\langle\alpha_g\rangle\right)E_d}{1-\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)}.$$

Consequently, α_{core} should be used in the annular-flow correlation, Eq.(14-62), to obtain the relative motion between the core and the film, whereas α_{drop} should be used in the dispersed-flow correlation to obtain a slip between droplets and gas-core flow.

By denoting the gas-core velocity, liquid-drop velocity, and film velocity by v_{gc}, v_{fc}, and v_{ff}, respectively, the total volumetric flux is given by

$$\langle j\rangle=\left[v_{gc}\left(1-\alpha_{drop}\right)+\alpha_{drop}v_{fc}\right]\alpha_{core}+v_{ff}\left(1-\alpha_{core}\right). \qquad (14\text{-}68)$$

Furthermore, by denoting the total volumetric flux in the core based on the core area by j_{core}, we have from the annular correlation, Eq.(14-62),

$$j_{core}-\langle j\rangle\cong\frac{\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)}{\langle\alpha_g\rangle+4\sqrt{\rho_g/\rho_f}} \qquad (14\text{-}69)$$

$$\times\left[\langle j\rangle+\sqrt{\frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)}{0.015\rho_f}}\right].$$

From the dispersed-flow correlations, it can be shown that, for a distorted-droplet or churn-droplet flow regime, the drift velocity can be given approximately by

$$\langle\langle v_g \rangle\rangle - j_{core} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_g^2}\right)^{1/4} \frac{E_d\left(1-\langle\alpha_g\rangle\right)}{\langle\alpha_g\rangle + E_d\left(1-\langle\alpha_g\rangle\right)}. \tag{14-70}$$

Here we have used an approximation based on $\left(1-\langle\alpha_g\rangle\right) << 1$. However, depending on the core-gas velocity, the dispersed-flow drift-velocity correlation for a much smaller particle should be used. When the droplets are generated by the entrainment of liquid film, the following approximate form is suggested for an undistorted-particle regime outside the Stokes regime (Ishii, 1976).

$$\langle\langle v_g \rangle\rangle - j_{core} = 0.5 r_d \left[\frac{(g\Delta\rho)^2}{\mu_g \rho_g}\right]^{1/3} \frac{E_d\left(1-\langle\alpha_g\rangle\right)}{\langle\alpha_g\rangle + E_d\left(1-\langle\alpha_g\rangle\right)} \tag{14-71}$$

where the particle radius may be approximated from the Weber-number criterion at the shearing-off wave crests. Thus,

$$r_d \cong \frac{6\sigma}{\rho_g} \frac{1}{\langle j \rangle^2}. \tag{14-72}$$

The above relations apply only when the total volumetric flux is sufficiently high to induce fragmentations of the wave crests. Hence, Eq.(14-71) should be used when

$$|\langle j \rangle| > 1.456\left(\frac{\sigma g \Delta\rho}{\rho_g^2}\right)^{1/4} \left[\frac{\mu_g^2}{\rho_g \sigma \sqrt{\sigma / g\Delta\rho}}\right]^{-1/12}. \tag{14-73}$$

By combining the above results, we obtain

$$\overline{V_{gj}} = \frac{\left(1-\langle\alpha_g\rangle\right)(1-E_d)}{\langle\alpha_g\rangle + 4\sqrt{\rho_g/\rho_f}} \times\left[\langle j \rangle + \sqrt{\frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)(1-E_d)}{0.015\rho_f}}\right] \tag{14-74}$$

$$
+\frac{E_d\left(1-\langle\alpha_g\rangle\right)}{\langle\alpha_g\rangle+E_d\left(1-\langle\alpha_g\rangle\right)}\times\begin{cases}\sqrt{2}\left(\dfrac{\sigma g\Delta\rho}{\rho_g^2}\right)^{1/4}\\ \text{or}\\ \dfrac{3\sigma}{\rho_g}\left[\dfrac{(g\Delta\rho)^2}{\mu_g\rho_g}\right]^{1/3}\dfrac{1}{\langle j\rangle^2}\end{cases}
$$

where the latter expression applies under the condition given by Eq.(14-73). If the radius of the particle is very small, then the essential contribution to the relative motion between phases comes from the first term of Eq.(14-74), and the core flow may be considered as a homogeneous dispersed flow. In such a case, Eq.(14-74) reduces to

$$
\overline{\overline{V_{dj}}}\cong\frac{\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)}{\langle\alpha_g\rangle+4\sqrt{\rho_g/\rho_f}}\times\left(\langle j\rangle+\sqrt{\frac{\Delta\rho g_z D\left(1-\langle\alpha_g\rangle\right)\left(1-E_d\right)}{0.015\rho_f}}\right). \tag{14-75}
$$

This expression shows a linear decrease of drift velocity in terms of entrained liquid fraction, which can be observed in various experimental data (Alia et al., 1965; Cravarolo et al., 1964).

1.3 Covariance of convective flux

In the one-dimensional drift-flux model, the momentum and energy convective fluxes have been divided into three terms: the mixture convective flux; the drift convective flux; and the covariance term, as can be seen from Eqs.(14-18) and (14-19). In other words, the convective flux of quantity ψ for the mixture can be written as

$$
\frac{\partial}{\partial z}\left(\sum_{k=1}^{2}\langle\alpha_k\rho_k\psi_k v_k\rangle\right)=\frac{\partial}{\partial z}\left(\rho_m\overline{\overline{\psi_m}}\,\overline{\overline{v_m}}\right)
+\frac{\partial}{\partial z}\left(\frac{\langle\alpha_d\rangle\rho_d\rho_c}{\langle\rho_m\rangle}\Delta\psi_{dc}\overline{\overline{V_{dj}}}\right)+\frac{\partial}{\partial z}\sum_{k=1}^{2}COV\left(\alpha_k\rho_k\psi_k v_k\right) \tag{14-76}
$$

where $\Delta\psi_{dc} \equiv \langle\langle\psi_d\rangle\rangle - \langle\langle\psi_c\rangle\rangle$ and $COV(\alpha_k\rho_k\psi_k v_k) \equiv \langle\alpha_k\rho_k\psi_k(v_k - \langle\langle v_k\rangle\rangle)\rangle$. Therefore, for the momentum flux, we have $\psi_k = v_k$ and $\Delta\psi_{dc} = \overline{V}_{dj}/(1-\langle\alpha_g\rangle)$. For the enthalpy flux, we have $\psi_k = h_k$ and $\Delta\psi_{dc} = \langle\langle h_d\rangle\rangle - \langle\langle h_c\rangle\rangle$, which is equivalent to the latent heat if phases are in thermal equilibrium.

To close the set of the governing equations, we must specify relations for these covariance terms. This can be done by introducing distribution parameters for the momentum and energy fluxes. If we define a distribution parameter for a flux as

$$C_{\psi k} \equiv \frac{\langle\alpha_k\psi_k v_k\rangle}{\langle\alpha_k\rangle\langle\langle\psi_k\rangle\rangle\langle\langle v_k\rangle\rangle} \tag{14-77}$$

the covariance term becomes

$$\begin{aligned} &COV(\rho_k\alpha_k\psi_k v_k) = \rho_k\langle\alpha_k\psi_k(v_k - \langle\langle v_k\rangle\rangle)\rangle \\ &= (C_{\psi k} - 1)\rho_k\langle\alpha_k\rangle\langle\langle\psi_k\rangle\rangle\langle\langle v_k\rangle\rangle. \end{aligned} \tag{14-78}$$

For the momentum flux, the distribution parameter is defined by

$$C_{vk} \equiv \frac{\langle\alpha_k v_k^2\rangle}{\langle\alpha_k\rangle\langle\langle v_k\rangle\rangle^2}. \tag{14-79}$$

Physically, C_{vk} represents the effect of the void and momentum-flux profiles on the cross-sectional-area-averaged momentum flux of k phase. A quantitative study of C_{vk} can be made by considering a symmetric flow in a circular duct and introducing the power-law expressions in parallel with the analysis of C_0 in the Section 1.2.1 of Chapter 14. Hence we postulate that

$$\frac{\alpha_k - \alpha_{kw}}{\alpha_{k0} - \alpha_{kw}} = 1 - \left(\frac{r}{R_w}\right)^n \tag{14-80}$$

and

$$\frac{v_k}{v_{k0}} = 1 - \left(\frac{r}{R_w}\right)^m \tag{14-81}$$

where the subscripts 0 and w refer to the value at the centerline and at the wall of a tube.

For simplicity, it is assumed that the void and velocity profiles are similar; namely, $n = m$. This assumption is widely used in mass-transfer problems. It may also be reasonable for fully developed two-phase flows if one considers that the vapor flux and, hence, the void concentration greatly influence the velocity distributions. Under this assumption, it can be shown that

$$C_{vk} = \frac{\frac{n+2}{n+1}\left(\alpha_{kw} + \Delta\alpha_k \frac{3n}{3n+2}\right)\left(\alpha_{kw} + \Delta\alpha_k \frac{n}{n+2}\right)}{\left(\alpha_{kw} + \Delta\alpha_k \frac{n}{n+1}\right)^2} \tag{14-82}$$

where $\Delta\alpha_k = \alpha_{k0} - \alpha_{kw}$.

For a dispersed vapor phase, $\alpha_{gw} << \Delta\alpha_g$; hence,

$$C_{vg} \cong \frac{3n+3}{3n+2}. \tag{14-83}$$

However, from Eq.(14-26), the volumetric-flux-distribution parameter C_0 becomes

$$C_0 \cong \frac{n+2}{n+1}. \tag{14-84}$$

Therefore, in the standard range of n, the parameter C_{vg} can be given approximately by

$$C_{vg} \simeq 1 + 0.5\left(C_0 - 1\right). \tag{14-85}$$

For a liquid phase in a vapor-dispersed-flow regime, $\alpha_{fw} \cong 1$ and $\alpha_{f0} < 1$. Then from Eq.(14-82) it can be shown that, for a standard range of α_{f0} in the bubbly- and churn-flow regimes, C_{vf} can be approximated by

$$C_{vf} = 1 + 1.5\left(C_0 - 1\right). \tag{14-86}$$

For an annular flow, the momentum covariance term can also be calculated by using the standard velocity profiles for the vapor and liquid flows. Thus we obtain

$$C_{vk} \simeq \begin{cases} 1.02 & \text{(turbulent flow)} \\ 1.33 & \text{(laminar flow).} \end{cases} \tag{14-87}$$

The above result for the individual phases can now be used to study the mixture covariance term. By defining the mixture-momentum-distribution parameter as

$$C_{vm} \equiv \frac{C_{vd}\rho_d\langle\alpha_d\rangle + C_{vc}\rho_c\langle\alpha_c\rangle}{\langle\rho_m\rangle} \tag{14-88}$$

the covariance term becomes

$$\begin{aligned} &\sum_{k=1}^{2} COV\left(\alpha_k\rho_k v_k^2\right) \\ &= \left(C_{vm}-1\right)\left[\langle\rho_m\rangle\overline{\overline{v}}_m^2 + \frac{\rho_c\rho_d\langle\alpha_d\rangle}{\left(1-\langle\alpha_d\rangle\right)\langle\rho_m\rangle}\overline{\overline{V}}_{dj}^2\right] \\ &+\frac{2\rho_c\rho_d\langle\alpha_d\rangle}{\langle\rho_m\rangle}\left(C_{vd}-C_{vc}\right)\overline{\overline{v}}_m\overline{\overline{V}}_{dj}. \end{aligned} \tag{14-89}$$

In view of the above analysis, the order of magnitude of $\left(C_{vd}-C_{vc}\right)$ is the same as that of $\left(C_{vm}-1\right)$ or less; therefore, the last term on the right-hand side of Eq.(14-89) can be neglected for almost all cases. This term may be important only in the near critical regime and if $\overline{\overline{v}}_m \cong \overline{\overline{V}}_{dj}$. However, in general, $\overline{\overline{V}}_{dj}$ becomes insignificant as the density ratio approaches unity. Hence, under the above conditions, the convective term itself becomes relatively small. Consequently, even for this case, the term may be dropped. Thus we have

$$\begin{aligned} &\sum_{k=1}^{2} COV\left(\alpha_k\rho_k v_k^2\right) \\ &\cong \left(C_{vm}-1\right)\left[\langle\rho_m\rangle\overline{\overline{v}}_m^2 + \frac{\rho_c\rho_d\langle\alpha_d\rangle}{\left(1-\langle\alpha_d\rangle\right)\langle\rho_m\rangle}\overline{\overline{V}}_{dj}^2\right]. \end{aligned} \tag{14-90}$$

The value of C_{vm} can be evaluated from Eq.(14-88) by using Eqs.(14-85) and (14-86) or Eq.(14-87). In the bubbly- and churn-flow regimes of practical importance, C_{vm} can be given approximately by

$$C_{vm} \cong 1 + 1.5(C_0 - 1). \tag{14-91}$$

However, in the near critical regime, C_{vm} depends also on the void fraction and the density ratio. Furthermore, at very low void fractions in a fully developed flow or in a developing flow, the value of C_{vm} should be reduced to the one for the single-phase flow given by Eq.(14-87). The effect of the development of the void profile into that given by the power law may be taken into account by a similar void-fraction correction term used in the correlation for C_0 in Eq.(14-34). By recalling that for a turbulent flow $C_{vm} = 1.0$ at $\alpha_d \to 0$, we obtain for a round tube

$$C_{vm} \cong 1 + 0.3\left(1 - \sqrt{\rho_g/\rho_f}\right)\left(1 - e^{-18\langle\alpha_d\rangle}\right) \tag{14-92}$$

which may be used both for a fully developed flow and for a developing flow.

For a turbulent-annular-flow regime, we have, from Eqs.(14-87) and (14-88), $C_{vm} \cong 1.02$. For all practical purposes, this may be further approximated by

$$C_{vm} \cong 1. \tag{14-93}$$

In reality, the transition from the value given by Eq.(14-92) to that given by Eq.(14-93) is a gradual one through the churn-annular (or slug-annular)-flow regime in which characteristics of churn and annular flows alternate. If a single correlation for C_{vm} is preferred, regardless of the flow-regime transitions, then Eq.(14-92) may be safely extrapolated into higher-void-fraction regime by a simple modification given by

$$C_{vm} \cong 1 + 0.3\left(1 - \sqrt{\rho_g/\rho_f}\right)\left\{1 - e^{-18\langle\alpha_d\rangle(1-\langle\alpha_d\rangle)}\right\}. \tag{14-94}$$

A similar analysis can be carried out for the enthalpy-covariance term by assuming the void, velocity, and enthalpy profiles. In general,

$$\begin{aligned}\sum_{k=1}^{2} COV\left(\alpha_k\rho_k h_k v_k\right) &= \left(C_{hm} - 1\right)\langle\rho_m\rangle\overline{h_m}\,\overline{v_m}\\ &+\frac{\rho_c\rho_d\langle\alpha_d\rangle}{\langle\rho_m\rangle}\left[-\left(C_{hc} - 1\right)\langle\langle h_c\rangle\rangle + \left(C_{hd} - 1\right)\langle\langle h_d\rangle\rangle\right]\overline{V_{dj}}\end{aligned} \tag{14-95}$$

where

$$\begin{cases} C_{hm} \equiv \sum_{k=1}^{2} C_{hk}\rho_k \langle\alpha_k\rangle\langle\langle h_k\rangle\rangle / \left(\langle\rho_m\rangle \overline{h_m}\right) \\ C_{hk} \equiv \langle\alpha_k h_k v_k\rangle / \left(\langle\alpha_k\rangle\langle\langle h_k\rangle\rangle\langle\langle v_k\rangle\rangle\right). \end{cases} \tag{14-96}$$

For a thermal-equilibrium flow, $h_g = h_{gs}$ and $h_f = h_{fs}$, where h_{gs} and h_{fs} are the saturation enthalpies of vapor and liquid. Since, in this case, the enthalpy profile is completely that for each phase, the distribution parameters become unity; namely, $C_{hg} = C_{hf} = C_{hm} = 1$. It is also evident that if one of the phases is in the saturated condition, then C_{hk} for that phase becomes unity.

In the single-phase region, the distribution parameter can be calculated from the assumed profiles for the velocity and enthalpy. Using the standard power-law profiles for a turbulent flow, namely, $v/v_0 = \left(y/R_w\right)^{1/n}$ and $\left(h - h_w\right)/\left(h_0 - h_w\right) = \left(y/R_w\right)^{1/m}$, where y is the distance from the wall, we can show that the covariance term is negligibly small both for developing and fully developed flows.

From the above two limiting cases, we can conclude that the enthalpy covariant term may become important only in highly non-equilibrium flow. Even in that case, the energy associated with phase change is considerably larger than that associated with changes in transverse temperature profiles. Therefore, except for highly transient cases, the enthalpy covariance can be neglected. Hence,

$$\frac{\partial}{\partial z}\sum_{k=1}^{2} COV\left(\alpha_k\rho_k h_k v_k\right) \cong 0 \tag{14-97}$$

1.4 One-dimensional drift-flux correlations for various flow conditions

In this section, constitutive equations of the one-dimensional drift-flux model for various flow conditions, which are of practically importance, are summarized.

1.4.1 Constitutive equations for upward bubbly flow

The constitutive equation of the distribution parameter of upward adiabatic bubbly flow in a round tube, Eq.(14-32), has been improved by considering the bubble lateral migration characteristics as (Hibiki and Ishii, 2002b; 2003b)

$$C_0 = 2.0e^{-0.000584 Re_f} + 1.2\left(1 - e^{-22\langle D_{Sm}\rangle / D}\right) \times\left(1 - e^{-0.000584 Re_f}\right) - \left[2.0e^{-0.000584 Re_f} + 1.2\left(1 - e^{-22\langle D_{Sm}\rangle / D}\right)\left(1 - e^{-0.000584 Re_f}\right) - 1\right]\sqrt{\frac{\rho_g}{\rho_f}} \quad (14\text{-}98)$$

where Re_f is defined by $\langle j_f \rangle D / \nu_f$. The bubble diameter D_{Sm} in Eq.(14-98) can be predicted by the bubble diameter correlation (Hibiki and Ishii, 2002a). As can be seen from Eq.(14-98), as the liquid Reynolds number increases, the distribution parameter predicted by Eq.(14-98) asymptotically approaches Eq.(14-32). The constitutive equation of the drift-velocity in gas-liquid bubbly flow is given by

$$\langle\langle V_{gj} \rangle\rangle = \sqrt{2}\left(\frac{\Delta\rho g \sigma}{\rho_f^2}\right)^{1/4}\left(1 - \langle\alpha_g\rangle\right)^{1.75} \quad \left(\mu_g << \mu_f\right). \quad (14\text{-}99)$$

1.4.2 Constitutive equations for upward adiabatic annulus and internally heated annulus

The applicability of Eqs.(14-98) and (14-99) has been confirmed by upward air-water turbulent bubbly flow data taken in a vertical concentric annulus at atmospheric pressure (Hibiki et al., 2003b). The constitutive equation of the distribution parameter of boiling bubbly flow in an internally heated annulus has been derived from Eq.(14-35) by considering the channel geometry difference as (Hibiki et al., 2003a)

$$C_0 = \left(1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_f}}\right)\left(1 - e^{-3.12\langle\alpha_g\rangle^{0.212}}\right). \quad (14\text{-}100)$$

The drift-flux model with Eqs.(14-99) and (14-100) can predict the data taken in an internally heated annulus well (Hibiki et al., 2003a).

1.4.3 Constitutive equations for downward two-phase flow

The constitutive equation of the distribution parameter of downward two-phase flow for all flow regimes is given by (Goda et al., 2003)

$$\begin{aligned} &C_0 = \left(-0.0214\left\langle j^*\right\rangle + 0.772\right) + \left(0.0214\left\langle j^*\right\rangle + 0.228\right)\sqrt{\frac{\rho_g}{\rho_f}} \\ &\text{for } -20 \le \left\langle j^*\right\rangle < 0, \\ &C_0 = \left(0.2e^{0.00848\left(\left\langle j^*\right\rangle + 20\right)} + 1.0\right) - 0.2e^{0.00848\left(\left\langle j^*\right\rangle + 20\right)}\sqrt{\frac{\rho_g}{\rho_f}} \\ &\text{for } \left\langle j^*\right\rangle < -20 \end{aligned} \tag{14-101}$$

where $\left\langle j^*\right\rangle \equiv \langle j\rangle / \left\langle\left\langle V_{gj}\right\rangle\right\rangle$. The constitutive equation of the drift velocity of downward two-phase flow for all flow regimes is approximated by

$$\left\langle\left\langle V_{gj}\right\rangle\right\rangle = \sqrt{2}\left(\frac{\Delta\rho g\sigma}{\rho_f^2}\right)^{1/4}. \tag{14-102}$$

These constitutive equations for distribution parameter and drift velocity were developed by one-dimensional data, and they have not been validated separately by detailed local flow data. Thus, they should not be used individually.

1.4.4 Constitutive equations for bubbling or boiling pool systems

In bubbling or pool boiling systems, the ratio of the vessel diameter to the length is often large in comparison with forced convection systems. It is noted that a recirculation flow pattern may develop in a large vessel at low flow. This may significantly affect the transverse velocity and void fraction profiles. The constitutive equation of the drift velocity for bubbling or boiling pool systems ($\left\langle j_f\right\rangle = 0$) is given by (Kataoka and Ishii, 1987)

Low viscous case: $N_{\mu f} \leq 2.25\times10^{-3}$

$$\left\langle\left\langle V_{gj}^{+}\right\rangle\right\rangle = 0.0019 D_H^{*\,0.809}\left(\frac{\rho_g}{\rho_f}\right)^{-0.157} N_{\mu f}^{-0.562} \text{ for } D_H^{*} \leq 30$$

$$\left\langle\left\langle V_{gj}^{+}\right\rangle\right\rangle = 0.030\left(\frac{\rho_g}{\rho_f}\right)^{-0.157} N_{\mu f}^{-0.562} \text{ for } D_H^{*} \geq 30 \tag{14-103}$$

Higher viscous case: $N_{\mu f} > 2.25\times10^{-3}$

$$\left\langle\left\langle V_{gj}^{+}\right\rangle\right\rangle = 0.92\left(\frac{\rho_g}{\rho_f}\right)^{-0.157} \text{ for } D_H^{*} \geq 30 \tag{14-104}$$

where $\left\langle\left\langle V_{gj}^{+}\right\rangle\right\rangle = \left\langle\left\langle V_{gj}\right\rangle\right\rangle \big/ \left(\sigma g \Delta\rho/\rho_f^2\right)^{1/4}$ and $D_H^{*} = D_H \big/ \sqrt{\sigma/(g\Delta\rho)}$. The constitutive equation of the distribution parameter in bubbling or pool boiling system is given in terms of channel geometry as Eq.(14-32).

1.4.5 Constitutive equations for large diameter pipe systems

In a large diameter channel ($D_H \geq 40\sqrt{\sigma/(g\Delta\rho)}$), slug bubbles cannot be sustained due to the interfacial instability and they disintegrate to cap bubbles. A recirculation flow pattern may develop in a large diameter channel at a low-flow rate. A flow regime at a test section inlet and a flow regime transition in a developing flow may also have an influence on the liquid recirculation pattern. The liquid recirculation, inlet flow regime and flow regime transition may affect the transverse velocity and the void fraction profile significantly. The constitutive equation of the drift velocity for upward bubbly flow in large diameter pipe systems is approximated by (Hibiki and Ishii, 2003a)

$$\left\langle\left\langle V_{gj}^{+}\right\rangle\right\rangle = \left\langle\left\langle V_{gj,B}^{+}\right\rangle\right\rangle e^{-1.39\left\langle j_g^{+}\right\rangle} + \left\langle\left\langle V_{gj,P}^{+}\right\rangle\right\rangle\left(1 - e^{-1.39\left\langle j_g^{+}\right\rangle}\right) \tag{14-105}$$

where $\left\langle\left\langle V_{gj,B}^{+}\right\rangle\right\rangle$ and $\left\langle\left\langle V_{gj,P}^{+}\right\rangle\right\rangle$ are, respectively, given by Eq.(14-102) and Eqs.(14-103) and (14-104), and $\left\langle j_g^{+}\right\rangle \equiv \left\langle j_g\right\rangle \big/ \left(\sigma g\Delta\rho/\rho_f^2\right)^{1/4}$. The constitutive equation of the distribution parameter for upward bubbly flow in large diameter pipe systems is given by

Case for inlet flow regime as uniformly distributed bubbly flow

$$C_0 = e^{0.475\left(\langle j_g^+\rangle/\langle j^+\rangle\right)^{1.69}} - \left[e^{0.475\left(\langle j_g^+\rangle/\langle j^+\rangle\right)^{1.69}} - 1\right]\sqrt{\frac{\rho_g}{\rho_f}}$$

for $0 \le \langle j_g^+\rangle/\langle j^+\rangle \le 0.9$

$$C_0 = \left\{-2.88\left(\frac{\langle j_g^+\rangle}{\langle j^+\rangle}\right) + 4.08\right\} - \left\{-2.88\left(\frac{\langle j_g^+\rangle}{\langle j^+\rangle}\right) + 3.08\right\}\sqrt{\frac{\rho_g}{\rho_f}} \qquad (4\text{-}106)$$

for $0.9 \le \langle j_g^+\rangle/\langle j^+\rangle \le 1$

where $\langle j^+\rangle \equiv \langle j\rangle/\left(\sigma g \Delta\rho/\rho_f^2\right)^{1/4}$.

Case for inlet flow regime as cap bubbly or slug flow

$$C_0 = 1.2e^{0.110\langle j^+\rangle^{2.22}} - \left\{1.2e^{0.110\langle j^+\rangle^{2.22}} - 1\right\}\sqrt{\frac{\rho_g}{\rho_f}}$$

for $0 \le \langle j^+\rangle \le 1.8$

$$C_0 = \left[0.6e^{-1.2\left(\langle j^+\rangle - 1.8\right)} + 1.2\right] - \left[0.6e^{-1.2\left(\langle j^+\rangle - 1.8\right)} + 0.2\right]\sqrt{\frac{\rho_g}{\rho_f}} \qquad (4\text{-}107)$$

for $1.8 \le \langle j^+\rangle$

These constitutive equations for distribution parameter and drift velocity were developed by one-dimensional data, and they have not been validated separately by detailed local flow data. Thus, they should not be used individually. In slug, churn, and annular flow regime, the distribution parameter effect is dominant over the local slip effect, namely, $V_{gj}^+ << C_0\langle j^+\rangle$. Thus, the constitutive equations detailed in the Section 1.2 of Chapter 14 can be applicable to such flow regimes.

1.4.6 Constitutive equations at reduced gravity conditions

To extend the applicability range of the drift-flux model to reduced gravity conditions, the constitutive equations of the drift velocity detailed in the Section 1.2 of Chapter 14 have been reformulated by considering the frictional pressure loss in addition to the gravitational pressure loss as (Hibiki et al., 2004)

Bubbly Flow Regime (Distorted-fluid-particle Regime)

$$\left\langle\left\langle V_{gj}\right\rangle\right\rangle=\sqrt{2}\left\{\frac{\left(\Delta\rho g_z+M_{F\infty}\right)\sigma}{\rho_f^2}\right\}^{1/4}\left(1-\left\langle\alpha_g\right\rangle\right)\left\{F\left(\left\langle\alpha_g\right\rangle\right)\right\}^2$$
$$\times\frac{\mu_m}{\mu_f}\frac{18.67}{1+17.67\left\{F\left(\left\langle\alpha_g\right\rangle\right)\right\}^{6/7}}. \tag{14-108}$$

The frictional pressure gradients in single-particle system, $M_{F\infty}$, and in multi-particle system, M_F, are defined by

$$M_{F\infty}\equiv\frac{f}{2D}\rho_f\left\langle v_f\right\rangle^2=\frac{f}{2D}\rho_f\left\langle j_f\right\rangle^2 \text{ and } M_F\equiv\left(-\frac{dp}{dz}\right)_F \tag{14-109}$$

where f is the wall friction factor. The function, $F\left(\left\langle\alpha_g\right\rangle\right)$, is defined by

$$F\left(\left\langle\alpha_g\right\rangle\right)\equiv\left\{\frac{\Delta\rho g_z\left(1-\left\langle\alpha_g\right\rangle\right)+M_F}{\Delta\rho g_z+M_{F\infty}}\right\}^{1/2}\left(\frac{\mu_f}{\mu_m}\right). \tag{14-110}$$

Equation (14-108) holds for $N_{\mu f}\geq 0.11\left\{1+\psi\left(r_b^*\right)\right\}/\left\{\psi\left(r_b^*\right)\right\}^{8/3}$. The parameter, $\psi\left(r_b^*\right)$, is given by

$$\psi\left(r_b^*\right)=0.55\left\{\left(1+0.08r_b^*\right)^{4/7}-1\right\}^{3/4} \tag{14-111}$$

where $r_b^*\equiv r_b\left\{\rho_f\left(\Delta\rho g_z+M_{F\infty}\right)/\mu_f^2\right\}^{1/3}$.

Slug Flow Regime

$$\left\langle\left\langle V_{gj}\right\rangle\right\rangle=0.35\left\{\frac{\left(\Delta\rho g_z+M_{F\infty}\right)D}{\rho_f}\right\}^{1/2}$$
$$\times\left[\frac{\left\{\Delta\rho g_z\left(1-\left\langle\alpha_g\right\rangle\right)+M_F\right\}}{\left(\Delta\rho g_z+M_{F\infty}\right)\left(1-\left\langle\alpha_g\right\rangle\right)}\right]^{1/2} \tag{14-112}$$

Churn Flow Regime

$$\left\langle\left\langle V_{gj}\right\rangle\right\rangle = \sqrt{2}\left\{\frac{\left(\Delta\rho g_z + M_{F\infty}\right)\sigma}{\rho_f^2}\right\}^{1/4} \times\left\{\frac{\Delta\rho g_z\left(1-\left\langle\alpha_g\right\rangle\right)+M_F}{\Delta\rho g_z + M_{F\infty}}\right\}^{1/4}. \tag{14-113}$$

Annular Flow Regime

In separated flows, local relative velocity between two phases cannot be defined (Hibiki and Ishii, 2003b). If small liquid droplets are entrained in the gas core or small gas bubbles are entrained in the liquid film, local relative velocity may be approximated to be zero due to large gas and liquid velocity, resulting in $\left\langle\left\langle V_{gj}\right\rangle\right\rangle \approx 0$. This approximation may be acceptable in annular flows where the entrainment of liquid from the film to the gas-core flow is negligibly small.

The constitutive equations of the distribution parameter have been improved by considering the gravity effect on the void distribution (Hibiki et al., 2004) as

Bubbly Flow Regime

$$\begin{aligned} C_0 = {} & 2.0e^{-0.000584 Re_f} + \left\{1.2e^{-5.55\left(\frac{g}{g_N}\right)^3}\right. \\ & \left. +1.2\left(1-e^{-22\frac{\langle D_{Sm}\rangle}{D}}\right)\left(1-e^{-5.55\left(\frac{g}{g_N}\right)^3}\right)\right\}\left(1-e^{-0.000584 Re_f}\right) \\ & -\left[2.0e^{-0.000584 Re_f} + \left\{1.2e^{-5.55\left(\frac{g}{g_N}\right)^3} + 1.2\left(1-e^{-22\frac{\langle D_{Sm}\rangle}{D}}\right)\right.\right. \\ & \left.\left.\times\left(1-e^{-5.55\left(\frac{g}{g_N}\right)^3}\right)\right\}\left(1-e^{-0.000584 Re_f}\right)-1\right]\sqrt{\frac{\rho_g}{\rho_f}}. \end{aligned} \tag{14-114}$$

where g_N is the normal gravitational acceleration (=9.8 m/s^2).

Slug Flow Regime

$$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_f}} \tag{14-115}$$

Churn Flow Regime

$$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_f}} \tag{14-116}$$

Annular Flow Regime

$$C_0 \simeq \frac{\overline{V_{gj}}}{\langle j \rangle} + 1 = \frac{1-\langle \alpha_g \rangle}{\langle \alpha_g \rangle + \left\{ \frac{1+75\left(1-\langle \alpha_g \rangle\right)}{\sqrt{\langle \alpha_g \rangle}} \frac{\rho_g}{\rho_f} \right\}^{1/2}} \times \left(1 + \frac{\sqrt{\frac{\Delta\rho g_z D\left(1-\langle \alpha_g \rangle\right)}{0.015\rho_f}}}{\langle j \rangle} \right) + 1 \tag{14-117}$$