Some Issues in the Simulation of Two-Phase Flows: the relative velocity

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Abstract. In this paper we compare numerical approximations for solving the Riemann problem for a hyperbolic two-phase flow model in two-dimensional space. The model is based on mixture parameters of state where the relative velocity between the two-phase systems is taken into account. This relative velocity appears as a main discontinuous flow variable through the complete wave structure and cannot be recovered correctly by some numerical techniques when simulating the associated Riemann problem. Simulations are validated by comparing the results of the numerical calculation qualitatively with OpenFOAM software. Simulations also indicate that OpenFOAM is unable to resolve the relative velocity associated with the Riemann problem.

Keywords: gas-liquid flow, mixture model, relative velocity, Riemann problem, finite volume, simulations
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INTRODUCTION AND GOVERNING EQUATIONS

Multiphase flows are often simulated by either two-fluid models or mixture models. These models usually have certain mathematical and numerical difficulties related to the hyperbolic and conservative nature of the governing equations. Even though hyperbolicity of such models can be provided under certain physical conditions, such as the mechanical equilibrium between phases, they are inherently non-conservative as an initial boundary-value problem for two-phase flows. In recent years, however, a variety of research articles have been published that demonstrate specific numerical treatments for two-phase flow computations of hyperbolic and non-conservative type. In most of these papers, the two-phase flux formulae are proposed in which one can employ a specific numerical technique to a real-world application of interest. (see [3, 6] and references therein). An important purpose of the current attention is always to restrain numerical oscillations and keep sharp discontinuities. Within such attention also is the fully conservative nature of the current models. In this work, we present numerical results for a hyperbolic and conservative two-phase flow model. These results are motivated by previously provided analysis for a specific model of two-fluid type with a single pressure and without velocity equilibrium between phases. This model consists of a mixture mass conservation equation, a mixture momentum balance equation and a relative velocity conservation equation between the two phase systems in two-dimension. The conservation equations of an isentropic mixture gas-liquid flows are presented as follows:

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho u)=S_1, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_x)+\frac{\partial}{\partial x}\left(\rho u_x^2+\rho c(1-c)u_{r,x}^2+P\right)+\frac{\partial}{\partial y}\left(\rho u_x u_y+\rho c(1-c)u_{r,x}u_{r,y}\right)=S_2, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho u_y)+\frac{\partial}{\partial y}\left(\rho u_y^2+\rho c(1-c)u_{r,y}^2+P\right)+\frac{\partial}{\partial x}\left(\rho u_x u_y+\rho c(1-c)u_{r,x}u_{r,y}\right)=S_3, \tag{3}$$

$$\frac{\partial}{\partial t}(u_{r,x})+\frac{\partial}{\partial x}\left(u_x u_{r,x}+(1-2c)\frac{u_{r,x}^2}{2}+\psi(P)\right)+\frac{\partial}{\partial y}\left(u_x u_{r,y}+u_y u_{r,x}+(1-2c)u_{r,x}u_{r,y}\right)=S_4. \tag{4}$$

$$\frac{\partial}{\partial t}(u_{r,y})+\frac{\partial}{\partial y}\left(u_y u_{r,y}+(1-2c)\frac{u_{r,y}^2}{2}+\psi(P)\right)+\frac{\partial}{\partial x}\left(u_x u_{r,y}+u_y u_{r,x}+(1-2c)u_{r,x}u_{r,y}\right)=S_5. \tag{5}$$

The nomenclature is as follows: ρ is the mixture density, u is the mixture velocity in x and y directions, c is the gas mass void fraction, P is the pressure which is the same between the gas and liquid, u_r is the relative velocity for the two phases in x and y directions and $\psi(P)$ is a function that recounts the gas and liquid phases through the momentum equations. Terms on the right-hand side represent the phase interaction between phases which is beyond the scope of this paper. The model presented in this paper is an extension of the one-dimensional model, which was solved analytically and numerically in [5, 6]. In the closure mixture model, equations of state must be used which is another major difficulty within two-phase flows and depends on the interest of each author. Additional relations for the void fractions, a hydrodynamical law for the two phase velocities and mixture parameters are needed to close the set of governing equations (1)-(5). The above set of governing equations is clearly conservative and gives attention to the development of different mathematical and numerical tools. In the following, we present our preliminary results on approximating the Riemann problem associated with system (1)-(5).

SOLUTION AND SIMULATION ISSUES

At this time we have investigated the model in hand using the Lax-Friedrich scheme and simulated rarefaction-waves propagation within an air-water mixture. The aim is to analyze the effects of choosing different lattice sizes, different Courant-Friedrichs-Lewy (CFL) numbers and to test rotational invariance. The advantage of this simple test scenario is to compare the results with the exact one dimensional solution due to translational invariance. For this purpose equations (1)-(5) are rewritten in the following form

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=\frac{\partial U}{\partial t}+J_x(U)\frac{\partial U}{\partial x}+J_y(U)\frac{\partial U}{\partial y}=S(U), \tag{6}$$

with the fluxes $F(U)$ and $G(U)$ and the Jacobien $J_x=\frac{\partial F}{\partial U}$ and $J_y=\frac{\partial G}{\partial U}$.

In two-dimension, splitting and non-splitting methods are used for the current simulations. In the spitting scheme the fields are updated first in one direction with Lax-Friedrich scheme according to

$$F^{LF}_{i+1/2}=\frac{1}{2}\left(F^n_i+F^n_{i+1}\right)+\frac{\Delta x}{2\Delta t}\left(U^n_i-U^n_{i+1}\right). \tag{7}$$

This is followed by the update step

$$U^{n+1}_i=U^n_i+\frac{\Delta t}{\Delta x}\left(F^{n,LF}_{i-1/2}-F^{n,LF}_{i+1/2}\right), \tag{8}$$

and with the new values in the second direction. In such case, the scheme converges for

$$\Delta t=C_{cfl}\cdot\min\left(\frac{\Delta x}{\lambda_x},\frac{\Delta y}{\lambda_y}\right), \tag{9}$$

with $\lambda_{x/y}$ being the largest eigenvalue of J_x and J_y respectively and $C_{cfl}\leq 1$. In the non-slitting scheme the two-dimensional Lax-Friedrichs flux are updated in one step using [2]

$$\Delta t=\frac{C_{cfl}}{\frac{\lambda_{max;x}}{\Delta x}+\frac{\lambda_{max;y}}{\Delta y}}. \tag{10}$$

During the implementation of the current numerical treatment, system (1)-(5) is found to be very sensitive to the choice of C_{cfl} within the non-splitting scheme and that the one-dimensional results could not reproduced exactly. An effort to numerically understand such sensitivity related to the model in hand is underway. At this time, however, we attempt to deepen our understanding of the relative velocity equation influence on the wave structure of the Riemann problem through numerical simulations. This is assessed against rarefaction-waves propagation within an air-water mixture test case selected from [5]. In this test case, the gas phase is governed by the ideal gas equation of state and the liquid is assumed to be incompressible with the following initial conditions for the Riemann problem of the mixture: $\rho_L=800=\rho_R$, $u_L=-100$, $u_R=100$ and $u_{rL}=0=u_{rR}$. Numerical results are displayed in figures 1-3 at time $t=0.007$. A C_{cfl} value of 0.9 is employed for each simulation presented in these figures. Results for the finite size scaling analysis for rarefaction waves moving in x-direction are shown in figure 1. The solid line appearing in this figure corresponds to the exact Riemann solver of [6]. It is observed that the numerical resolution approach the exact solution as the mesh is refined. Rotational invariance are checked on a moderate lattice size giving the rarefaction waves

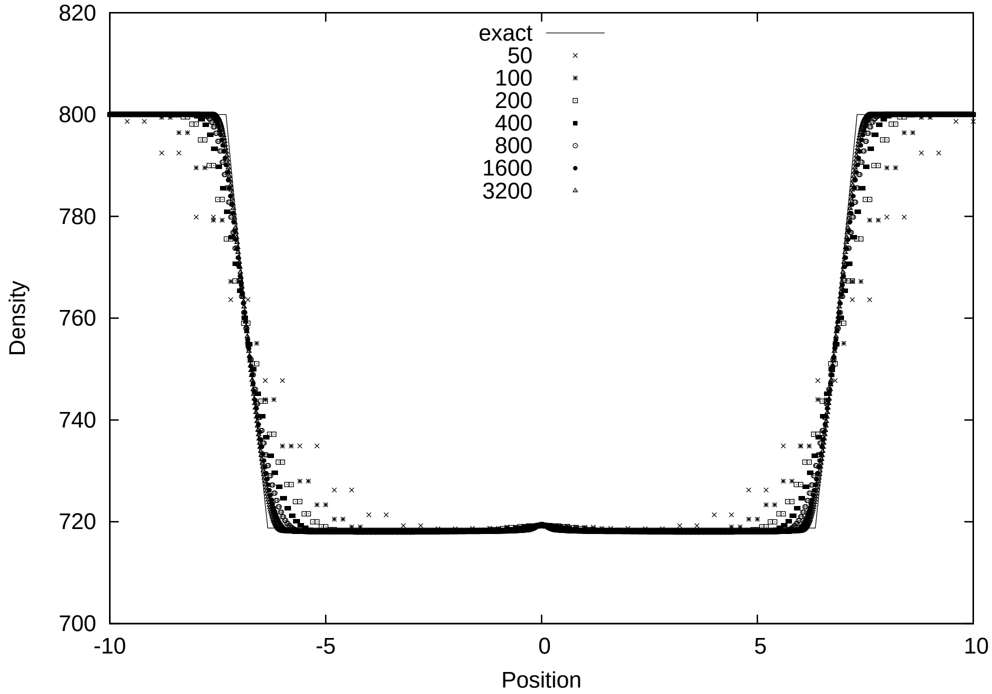

FIGURE 1. Rarefaction-waves propagation within an air-water mixture: mixture density for different lattice sizes with $C_{cfl} = 0.9$, compared the the exact results in one-dimension.

different directions as displayed in figure 2. In figure 2 one can see that the rotational symmetry is fulfilled quite well by comparing the results of the two rarefaction waves. Interestingly the relative velocity is changing discontinuously across the middle wave as observed in earlier investigation [6]. To get further insight the current simulations, we validate the current results using OpenFOAM software on the basis of finite volume discretization [1]. The numerical tool employed is Kurganov and Tadmor central-upwind scheme [4]. Results are displayed on figure 3. As clearly visible, the left and right rarefaction waves were reproduced accurately using OpenFOAM. The contact discontinuity is also clearly reproduced without any spurious oscillations. It is worth noting that the current OpenFOAM tool could not reproduce the relative velocity profile by using the relative velocity equations of the previous section which is a key limitation of the current investigation. As a result, current understanding of this relative velocity effects on rarefaction wave propagation is limited using OpenFOAM. Further, the consequence of this relative velocity equation within OpenFOAM is not clear yet and further computational modeling research is needed here.

CONCLUDING REMARKS

In this preliminary study, we have examined the numerical treatments of the Riemann problem related to two-phase flows of a gas-liquid mixture. Model equations have been adapted from [5] in which hyperbolicity and conservativity have been developed. Numerical verification indicates that the proposed method is computationally stable in practice. We also have looked at the behavior of the current results which show that the proposed method performs well even when the number of computational cells is relatively high. The results have been compared with other simulations provided from OpenFOAM with significantly inherent limitation, yet similar mixture parameters.

Our primary motivation in undertaking this work was to explore deficiencies in the performance of numerical simulations related to non-equilibrium mixture models of two-phase flows. Model and computational developments are ongoing, and improvements will be added to include mass transfer effects and OpenFOAM validations.

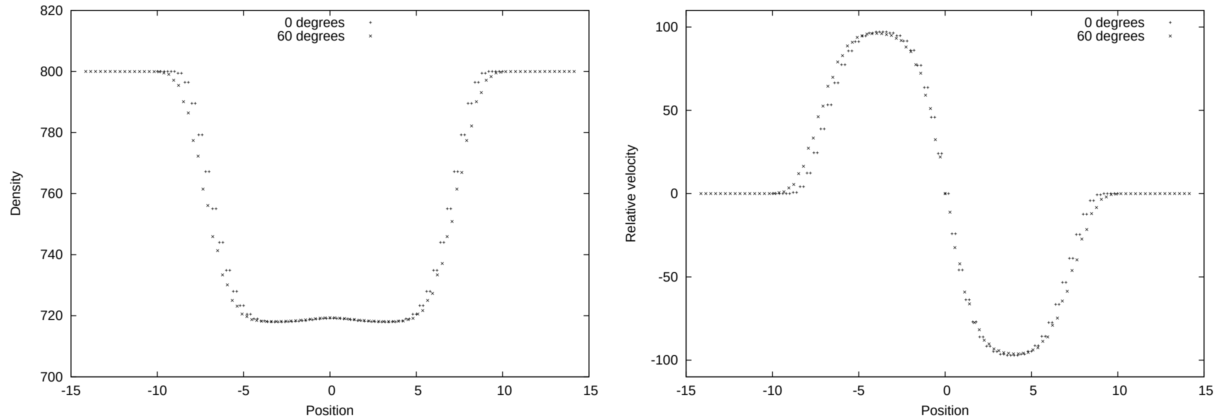

FIGURE 2. Rarefaction-waves propagation within an air-water mixture: mixture density and relative velocity profiles for a rarefaction wave going in *x*-direction and with an angle of 60 degrees between the wave and the *x*-direction.

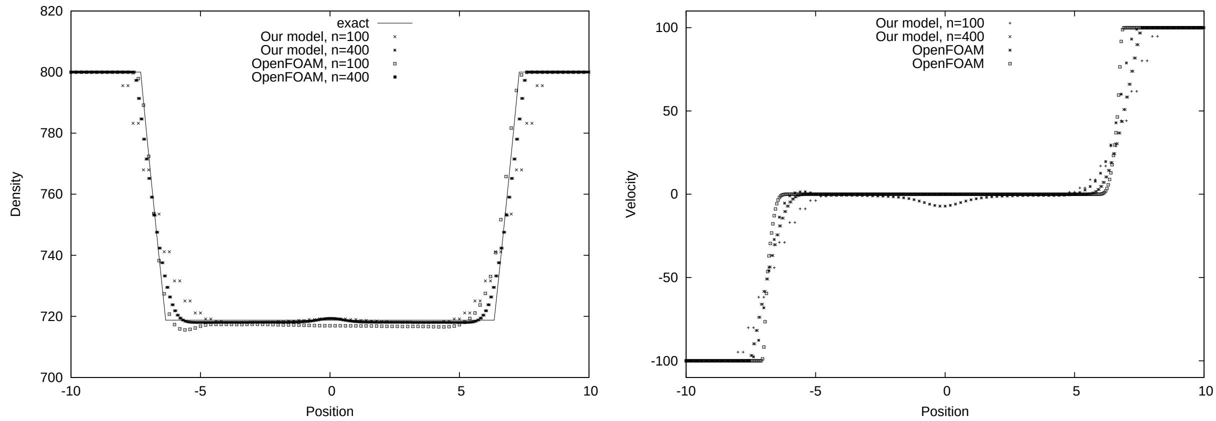

FIGURE 3. Rarefaction-waves propagation within an air-water mixture: mixture density and mixture velocity profiles computed with the mixture model of OpenFOAM and with the current model.

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REFERENCES

1. OpenFOAM Foundation, OpenFOAM®Documentation, see `www.openfoam.org/docs`.
2. E.F. Toro: *Rieman Solvers and Numerical Methods for Fluid Dynamics* Springer Verlag,3rd Edition, 2010.
3. S. Kuila,T.R. Sekhar and D. Zeidan*A Robust and accurate Riemann solver for a compressible two-phase flow model*, Applied Mathematics and Computation 265 (2015) 681-695
4. A. Kurganov and E. Tadmor: *New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection-Diffusion Equtions*, Journal of Computational Physics 160, (2000) 241 - 282.
5. D. Zeidan and A. Slaouti: *Validation of hyperbolic model for two-phase flow in conservative form*, International Journal of Computational Fluid Dynamics, Vol 23, No 9, (2009) 623-641
6. D. Zeidan: *The Riemann problem for a hyperbolic model of two-phase flow in conservative form*, International Journal of Computational Fluid Dynamics, Vol 25, No 6, (2011) 299-318