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$$\frac{\partial}{\partial t}(\alpha_2\rho_2)+\frac{\partial}{\partial x}(\alpha_2\rho_2 u_2)=0 \qquad \text{——}(1)$$

$$\frac{\partial}{\partial t}(\alpha_1\rho_1)+\frac{\partial}{\partial x}(\alpha_1\rho_1 u_1)=0 \qquad \text{——}(2).$$

$$\frac{\partial}{\partial t}(\alpha_2\rho_2 u_2)+\frac{\partial}{\partial x}(\alpha_2\rho_2 u_2^2)+\alpha_2\frac{\partial}{\partial x}P=0 \qquad \text{——}(3)$$

$$\frac{\partial}{\partial t}(\alpha_1\rho_1 u_1)+\frac{\partial}{\partial x}(\alpha_1\rho_1 u_1^2)+\alpha_1\frac{\partial}{\partial x}P=0 \qquad \text{——}(4).$$

$$P_1=P_2=P$$

Consider (3)

$$u_2\frac{\partial}{\partial t}(\alpha_2\rho_2)+\alpha_2\rho_2\frac{\partial}{\partial t}(u_2)+$$

$$u_2^2\frac{\partial}{\partial x}(\alpha_2\rho_2)+\alpha_2\rho_2\, 2u_2\frac{\partial}{\partial x}u_2+\alpha_2\frac{\partial}{\partial x}P=0$$

$$\Rightarrow u_2\Big[\overbrace{\frac{\partial}{\partial t}(\alpha_2\rho_2)+u_2\frac{\partial}{\partial x}(\alpha_2\rho_2)+\alpha_2\rho_2\frac{\partial}{\partial x}u_2}^{=0\text{ by }(1)}\Big]+$$

$$\alpha_2\rho_2\frac{\partial u_2}{\partial t}+u_2\alpha_2\rho_2\frac{\partial u_2}{\partial x}+\alpha_2\frac{\partial P}{\partial x}=0$$

$$\Rightarrow \alpha_2\rho_2\frac{\partial u_2}{\partial t}+\alpha_2\rho_2\frac{\partial u_2^2}{2\,\partial x}+\alpha_2\frac{\partial P}{\partial x}=0 \qquad \text{——}(5).$$

Consider 4; do the same process to obtain

$$\alpha_1\rho_1 \frac{\partial u_1}{\partial t} + \alpha_1\rho_1 \frac{\partial}{\partial x}\frac{u_1^2}{2} + \alpha_1 \frac{\partial P}{\partial x} = 0 \quad \text{——— } (6)$$

divide 5 + 6 by $\alpha_2\rho_2$ and $\alpha_1\rho_1$ Then substract to find:

$$\frac{\partial}{\partial t}(u_2 - u_1) + \frac{\partial}{\partial x}\left(\frac{u_2^2}{2} - \frac{u_1^2}{2}\right) + \left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\frac{\partial P}{\partial x} = 0$$

i.e. $$\frac{\partial}{\partial t}(u_2 - u_1) + \frac{1}{2}\frac{\partial}{\partial x}(u_2^2 - u_1^2) + \left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\frac{\partial P}{\partial x} = 0 \leftarrow$$

USE:

\# $u_r = u_2 - u_1$ the relative velocity between gas and Liquid or solid

\# $u_1 = \frac{\partial u_1}{\partial x} = \frac{1}{2}\frac{\partial u_1^2}{\partial x}$; $u_2 = \frac{1}{2}\frac{\partial u_2^2}{\partial x}$

\# $c = \frac{\alpha_2 \rho_2}{\rho}$ for the gas phase with; $\alpha_2 + \alpha_1 = 1$.

\# $u_2 = u + (1-c)u_r$ and $u_1 = u - c u_r$

$\rho u = \alpha_1\rho_1 u_1 + \alpha_2\rho_2 u_2 = (1-c)u_1 + c u_2$

⇒

$$u_2^2 - u_1^2 = \left(u + (1-c)u_r\right)^2 - \left(u - c\,u_r\right)^2$$

$$= 2uu_r + (1-2c)\,u_r^2$$

$$\Rightarrow \frac{1}{2}\frac{\partial}{\partial x}\left(u_2^2 - u_1^2\right) = \frac{\partial}{\partial x}\left(uu_r + (1-2c)\,u_r^2\right)$$

Next:

$$\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\frac{\partial P}{\partial x} = \boxed{\frac{\partial \psi}{\partial P}}\ \boxed{\frac{\partial P}{\partial x}} = \frac{\partial \psi}{\partial x} = \psi(P)$$

$\rho_1 \equiv$ Constant i.e incompressible Liquid.

$\rho_2 =$ gaz. EOS: $P = K_2\,\rho_2^{\gamma}$

⇒

$$\frac{\partial \psi}{\partial x} = \frac{K_2^{\frac{1}{\gamma}}}{P^{1/\gamma}} - \frac{1}{\rho_1}$$

⇒

$$\psi = \frac{\gamma}{\gamma-1}\,K_2^{\frac{1}{\gamma}}\,P^{-\frac{1}{\gamma}+1} - \frac{P}{\rho_1}$$

$$\psi = \frac{\gamma}{\gamma-1}\,\rho_2^{-1}\,P - \frac{P}{\rho_1}$$

$$\Rightarrow \quad \frac{\partial}{\partial t}u_r + \frac{\partial}{\partial x}\left(u u_r + \frac{1}{2}(1-2c)u_r^2\right) + \frac{\partial \varphi}{\partial x} = 0.$$