$u_2 - u_1 = u_r$ $\quad c = \frac{\alpha_2 \rho_2}{\rho}$

$u_r \neq 0$

2/3 - Dim

$\boxed{\text{Keep it}}$

G

$u_r \cdot u_r$

$\boxed{\vec{u}_r = \vec{V}_1 - \vec{V}_1}$

- equations of mass, momentum and energy for a mixture of gas-liquid

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$V_1 V$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + \boxed{P + \rho c(1-c)u_r^2}\right) + \frac{\partial}{\partial y}\left(\rho uv\right) + \frac{\partial}{\partial z}\left(\rho uw\right) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}\left(\rho uv\right) + \frac{\partial}{\partial y}\left(\rho v^2 + \left[P + \rho c(1-c)u_r^2\right]\right) + \frac{\partial}{\partial z}\left(\rho vw\right) = 0$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}\left(\rho uw\right) + \frac{\partial}{\partial y}\left(\rho vw\right) + \frac{\partial}{\partial z}\left(\rho w^2 + \left(P + \rho c(1-c)u_r^2\right)\right) = 0$$

$(V_1 - V_1) \cdot U$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho E\right) + \frac{\partial}{\partial x}\left(\rho(E+P)u + \rho c(1-c)u_r\left(uu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &+ \\ \frac{\partial}{\partial y}\left(\rho(E+P)v + \rho c(1-c)u_r\left(vu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &+ \\ \frac{\partial}{\partial z}\left(\rho(E+P)w + \rho c(1-c)u_r\left(wu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &= 0\end{aligned}$$

$\frac{\rho_1}{\rho}$

$E + \frac{1}{2}(V \cdot V)$

$\rho u w E$

$\alpha_1 \rho$

$$\frac{\partial \alpha_1 \rho_1}{\partial t} + \nabla \cdot \rho \alpha_1 V_1 = 0$$

$\alpha_1 \rho$

ρ

$$\frac{\partial \alpha_2 \rho_2}{\partial t} + \nabla \cdot \rho \alpha_2 V_2 = 0$$

$$\left(\frac{\rho_1}{\rho}\frac{V_1 V_1}{2} + \frac{\rho_2 V_2 V_2}{\rho}\right) - V_1 V_1$$

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \left(\alpha_1 \rho_1 V_1 + \alpha_2 \rho_2 V_2\right)$$

$$+ \nabla \cdot \bar{\rho} \vec{V}$$

$$\bar{\rho} = \alpha_1 \rho_1 + \alpha_2 \rho_1 \qquad \bar{V} = \frac{\alpha_1 \rho_1 V_1 + \alpha_2 \rho_2 V_2}{\bar{\rho}}$$

$\cancel{V = \alpha_1 V_1 + \alpha_2 V_2}$

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0.$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + P) + \frac{\partial}{\partial y}(\rho u v) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v^2 + P) = 0.$$

$$\frac{\partial}{\partial t}(\rho s) + \frac{\partial}{\partial x}(\rho u s) + \frac{\partial}{\partial y}(\rho v s) = 0.$$

$$\frac{\partial}{\partial t}(\rho \alpha) + \frac{\partial}{\partial x}(\rho u \alpha) + \frac{\partial}{\partial y}(\rho v \alpha) = 0$$

$$\frac{\partial}{\partial t}(\rho c) + \frac{\partial}{\partial x}(\rho u c) + \frac{\partial}{\partial y}(\rho v c) = 0.$$

$$\frac{\partial}{\partial t}(\rho \chi) + \frac{\partial}{\partial x}(\rho u \chi) + \frac{\partial}{\partial y}(\rho v \chi) = 0$$

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ — direction indi

1

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

2

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + \rho c(1-c)u_r^2 + P\right) + \frac{\partial}{\partial y}(\rho u v) = 0$$

3

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}\left(\rho v^2 + \rho c(1-c)u_r^2 + P\right) = 0$$

$u u_r \quad v u_r$

4

$$\frac{\partial}{\partial t}\left(u_r^x\right) + \frac{\partial}{\partial x}\left[\left(u u_r^x + \cancel{v u_r^y}\right) + \overbrace{\boxed{\frac{1-2c}{2} u_r^2 + e_c}}^{E_c}\right]$$

$$+ \frac{\partial}{\partial y}\left[\left(v u_r^x + \cancel{v u_r^y}\right) + \overbrace{\cancel{\frac{1-2c}{2} u_r^2} + e_c}^{E_c}\right] = 0$$

5

$$\frac{\partial}{\partial t}\left(u_r^y\right) + \frac{\partial}{\partial x}\left[\left(u u_r^x + \cancel{u u_r^y}\right) + \overbrace{\frac{1-2c}{2} u_r^2 + e_c}^{E_c}\right]$$

$$+ \frac{\partial}{\partial y}\left[\left(v u_r^x + \cancel{v u_r^y}\right) + \overbrace{\frac{1-2c}{2} u_r^2 + e_c}^{E_c}\right] = 0$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x}\left[\rho u(E+P) + \rho u\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c\cdot(1-c)u_{r_x}\right]$$

$$+\frac{\partial}{\partial y}\left[\rho v(E+P) + \rho v\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c(1-c)u_{r_y}\right]$$

$$+\frac{\partial}{\partial z}\left[\rho w(E+P) + \rho w\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c(1-c)u_{r_z}\right] = 0$$

$$\vec{V}_r = \vec{V}_1 - \vec{V}_a$$
$$= (u_1 - u_a)i$$
$$+ (v_1 - v_a)j$$
$$+ (w_1 - w_a)k$$