

Chapter 13

DRIFT-FLUX MODEL

The basic concept of the drift-flux model is to consider the mixture as a whole, rather than two phases separately. It is clear that the drift-flux model formulation will be simpler than the two-fluid model, however it requires some drastic constitutive assumptions causing some of the important characteristics of two-phase flow to be lost. However, it is exactly this simplicity of the drift-flux model that makes it very useful in many engineering applications. As it is the case with the analyses of two-phase flow *system dynamics*, information required in engineering problems is often the response of the total mixture and not of each constituent phase (Tong, 1965). Furthermore, detailed analyses on the local behavior of each phase can be carried out with less difficulty, *if* these mixtures responses are known.

Another important aspect of the drift-flux model is concerned with the scaling of systems that has direct applications in the planning and designing of two-phase flow experimental and engineering systems. The similarities of two different systems can be studied effectively by using the drift-flux model formulation and mixture properties. The most important aspect of the drift-flux model is the reduction in the total number of field and constitutive equations required in the formulation in comparison with the two-fluid model. The drift-flux model is expressed in terms of four field equations: the mixture continuity; momentum; energy equations; and the gas continuity equation.

It can be seen, therefore, that the drift-flux model follows the standard approach used to analyze the dynamics of a mixture of gases or of miscible liquids. It is generally accepted that the drift-flux model is appropriate to the mixture where the dynamics of two components are closely coupled. This suggests that the same argument may be used for the macroscopic two-phase flows. The usefulness of the drift-flux model in many practical engineering systems comes from the fact that even two-phase mixtures that are weakly coupled locally can be considered, because the relatively large axial

dimension of the systems usually gives sufficient interaction times. The advantages of using the drift-flux model for the studies of system dynamics and instabilities caused by the *low velocity* wave propagation, namely, the void propagation, are demonstrated by Zuber (1967) and Ishii and Zuber (1970). However, there are some questions in applying the drift-flux model to the problems of acoustic wave propagations, choking phenomena and high frequency instabilities, as it has been discussed in detail by Bouré and Réocreux (1972), Bouré (1973) and Réocreux et al. (1973).

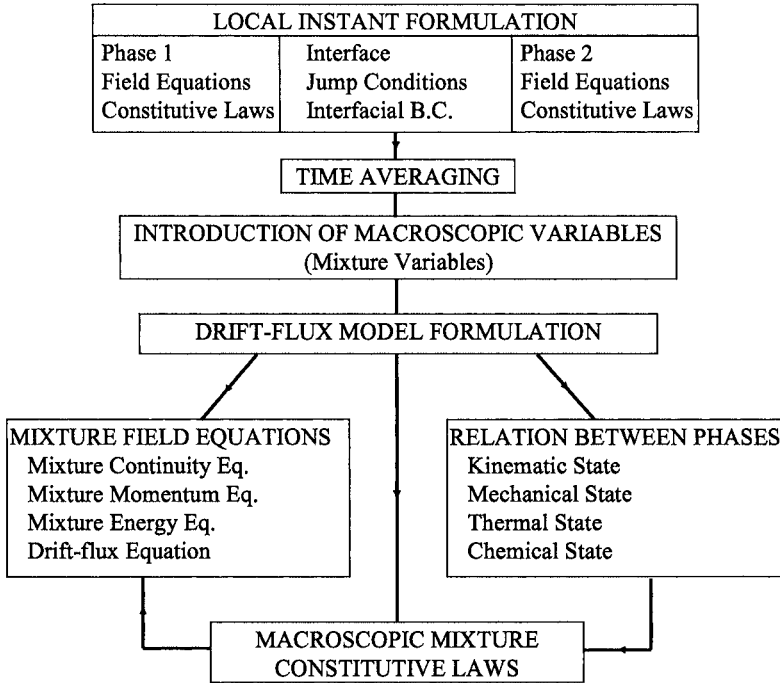
In the drift-flux model formulation we have only four field equations and, thus, one energy and one momentum equation have been eliminated from the original six field equations. Then, the relative motion and energy difference should be expressed by additional constitutive equations. In other words, the dynamic interaction relations are replaced by the constitutive laws. Furthermore, it is important to formulate the model based on the mixture center of mass in order to preserve the additive characteristic of the extensive variables, as explained in Chapter 4.

In this chapter, we develop a general formulation of the mixture model (Ishii, 1975) then discuss various special cases (Ishii, 1977) that are important in practical applications. Since we have carried out the detailed analysis on the field and constitutive equations for two-fluid model in Chapter 9, we recall and use these results for the establishment of the drift-flux model formulation whenever it is helpful. The following diagram summarizes the establishment of the drift-flux model formulation. Here we see the special importance of the kinematic, mechanical and thermal relations between the two phases. It is evident that the elimination of one of the two momentum equations from the formulation requires the kinematic relation between the phases, therefore, the relative velocity should be given by a constitutive law. Similarly, by using only the mixture energy equation for the balance of energies in a two-phase flow, thermal relation between the phases should be given.

1.1 Drift-flux model field equations

Formulation Based on the Center of Mass and Drift-flux Velocities

The most general forms of the four basic field equations for the drift-flux model have been given in the Section 1.3 of Chapter 5. In this section, first we put these equations into more realistic form by using some of the analysis on the constitutive equations for the two-fluid model. Then we discuss some appropriate simplifications which are important for practical applications. Here, we formulate the model based on the mixture continuity, momentum and thermal energy equations plus the continuity equation for one of the phases. These equations can be reduced to the following forms



The mixture continuity equation from Eq.(5-40)

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}_m) = 0 \quad (13-1)$$

The continuity equation for phase 2 from Eq.(5-41)

$$\frac{\partial \alpha_2 \bar{\rho}_2}{\partial t} + \nabla \cdot (\alpha_2 \bar{\rho}_2 \mathbf{v}_m) = \Gamma_2 - \nabla \cdot (\alpha_2 \bar{\rho}_2 \mathbf{V}_{2m}) \quad (13-2)$$

The mixture momentum equation from Eqs.(5-42) and (5-43)

$$\begin{aligned} \frac{\partial \rho_m \mathbf{v}_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}_m \mathbf{v}_m) = & -\nabla p_m \\ & + \nabla \cdot \left[\bar{\mathcal{T}} + \bar{\mathcal{T}}^T - \sum_{k=1}^2 \alpha_k \bar{\rho}_k \bar{\mathbf{V}}_{km} \bar{\mathbf{V}}_{km} \right] + \rho_m \mathbf{g}_m + \mathbf{M}_m \end{aligned} \quad (13-3)$$

where we have from Eq.(9-127)

$$\mathbf{M}_m = 2\overline{\overline{H_{21}}} \overline{\overline{\sigma}} \nabla \alpha_2 + \mathbf{M}_m^H. \quad (13-4)$$

And in Eq.(13-3) the body force field has been taken as constant. The last term on the right-hand side of the mixture momentum equation represents the effects of the surface tension force on the mixture.

There are considerable difficulties obtaining an appropriate thermal energy equation for the mixture as we have discussed in the Section 1.3 of Chapter 5. It has been shown there that we have two different methods to obtain the equation. By adding the thermal energy equation for each phase, we obtained Eq.(5-53). Consequently, from Eqs.(9-154) and (9-150), we have

$$\begin{aligned} \frac{\partial \rho_m \dot{i}_m}{\partial t} + \nabla \cdot (\rho_m \dot{i}_m \mathbf{v}_m) &= -\nabla \cdot (\bar{\mathbf{q}} + \mathbf{q}^T) \\ -\nabla \cdot \left(\sum_k \alpha_k \overline{\overline{\rho_k}} \widehat{\mathbf{i}_k} \mathbf{V}_{km} \right) &+ \sum_{k=1}^2 \alpha_k \frac{D_k \overline{\overline{p_k}}}{Dt} + \sum_{k=1}^2 \alpha_k \overline{\overline{\mathcal{T}_k}} : \nabla \widehat{\mathbf{v}_k} \\ &+ \left(\overline{\overline{T_i}} \frac{d\sigma}{dT} \frac{D_i a_i}{Dt} + E_m^H \right) \\ &- \sum_{k=1}^2 \left(\Gamma_k \frac{\widehat{v}_k^2}{2} - \mathbf{M}_{ik} \cdot \widehat{\mathbf{v}_{ki}} + \nabla \alpha_k \cdot \overline{\overline{\mathcal{T}_{ki}}} \cdot \widehat{\mathbf{v}_{ki}} \right). \end{aligned} \quad (13-5)$$

From the definitions of the mixture properties and the interfacial momentum transfer condition, we have

$$\sum_{k=1}^2 \alpha_k \frac{D_k \overline{\overline{p_k}}}{Dt} \doteq \frac{D \overline{\overline{p_m}}}{Dt} + \sum_{k=1}^2 \alpha_k \mathbf{V}_{km} \cdot \nabla \overline{\overline{p_k}} - 2\overline{\overline{H_{21}}} \overline{\overline{\sigma}} \frac{D \alpha_2}{Dt}. \quad (13-6)$$

For simplicity we define three different effects as follows

$$\Phi_m^\mu \equiv \sum_{k=1}^2 \alpha_k \overline{\overline{\mathcal{T}_k}} : \nabla \widehat{\mathbf{v}_k} \quad (13-7)$$

$$\Phi_m^\sigma \equiv E_m^H - 2\overline{\overline{H}}_{21} \overline{\overline{\sigma}} \frac{D\alpha_2}{Dt} + \overline{\overline{T}}_i \frac{d\sigma}{dT} \frac{D_i a_i}{Dt} \quad (13-8)$$

and

$$\Phi_m^i \equiv \sum_{k=1}^2 \left(-\Gamma_k \frac{\widehat{v}_k^2}{2} - \mathbf{M}_{ik} \cdot \widehat{\mathbf{v}}_{ki} + \nabla \alpha_k \cdot \overline{\overline{\mathcal{T}}}_{ki} \cdot \widehat{\mathbf{v}}_{ki} \right). \quad (13-9)$$

Then the thermal energy equation (13-5) reduces to

$$\begin{aligned} \frac{\partial \rho_m i_m}{\partial t} + \nabla \cdot (\rho_m i_m \mathbf{v}_m) = & -\nabla \cdot (\bar{\mathbf{q}} + \mathbf{q}^T) \\ & -\nabla \cdot \left(\sum_{k=1}^2 \alpha_k \overline{\overline{\rho}}_k \widehat{v}_k \mathbf{V}_{km} \right) + \frac{Dp_m}{Dt} + \Phi_m^\mu + \Phi_m^\sigma + \Phi_m^i. \end{aligned} \quad (13-10)$$

The drift-flux model with the mixture thermal energy equation faces considerable difficulties when the last three terms in the above equation cannot be neglected. These terms arise from the viscous dissipation, work due to surface tension forces and interfacial mechanical energy transfer. This is particularly true, if the term Φ_m^i given by Eq.(13-9) has significant contributions to the thermal energy exchanges. It is evident that in this case the drift-flux model requires the constitutive equations for the relative velocity as well as for the interfacial mechanical energy transfer.

The alternative form of the thermal energy equation can be obtained from Eq.(5-55) by substituting Eqs.(9-141) and (9-127), then using the approximation given by (9-145). These two different forms of the thermal energy equation do not give an identical drift-flux model formulation, as it has been discussed in the Section 1.3 of Chapter 5. However, we use the relation given by Eq.(13-10), which has been obtained by adding the thermal energy equations for each phase, because it has a relatively simpler form than the one based on Eq.(5-55).

The above four field equations (13-1), (13-2), (13-3) and (13-10) state the balance laws which govern the *macroscopic mixture field*. They have been obtained by the time averaging applied to the two-phase flow systems with interfacial discontinuities. We note that mixture continuity, momentum and energy equations are somewhat similar to those of a single-phase flow. Actually, the mixture continuity equation has exactly the same form as that for the continuum without internal discontinuities. This has been done by using the properly defined mixture properties. The mixture momentum

equation, however, has two additional terms that do not appear in a single-phase equation. One is the capillary force that takes into account the surface tension effects and can be considered as momentum source or sink. The other is a diffusion stress term, shown as the third stress on the right-hand side of Eq.(13-3). This term expresses the momentum diffusion due to the relative motion between two phases in addition to the molecular and turbulent diffusions that has been taken into account by the stress group $(\overline{\mathcal{T}} + \mathcal{T}^T)$. In the mixture thermal energy equation, we have three additional terms that do not appear in the single-phase flow equation. The second term on the right-hand side of Eq.(13-10) is an energy diffusion due to the transport of energy by relative motions of the phases with respect to the mixture center of mass. Recalling Eqs.(13-8) and (13-9), the terms given by Φ_m^σ and Φ_m^i represent the surface-tension effect and the contributions from the interfacial mechanical energy transfer, respectively. Under normal conditions, these two terms and energy dissipation term can be neglected almost always.

The mixture momentum and thermal energy equations given by Eqs.(13-3) and (13-10) describe the momentum and energy exchanges from the stationary observer. Thus, the convective fluxes with the mixture center of mass velocity and the additional diffusion fluxes defined with respect to the barycenter of mixture appear in the equations. These two equations can be transformed in terms of the convective derivative of Eq.(7-14) as follows

$$\begin{aligned} \rho_m \frac{D\mathbf{v}_m}{Dt} = & -\nabla p_m + \nabla \cdot (\overline{\mathcal{T}} + \mathcal{T}^T) \\ & -\nabla \cdot \left(\sum_{k=1}^2 \alpha_k \overline{\rho_k} \mathbf{V}_{km} \mathbf{V}_{km} \right) + \rho_m \mathbf{g}_m + \mathbf{M}_m \end{aligned} \quad (13-11)$$

and

$$\begin{aligned} \rho_m \frac{Di_m}{Dt} = & -\nabla \cdot (\overline{\mathbf{q}} + \mathbf{q}^T) - \nabla \cdot \left(\sum_{k=1}^2 \alpha_k \overline{\rho_k} \hat{i}_k \mathbf{V}_{km} \right) + \frac{Dp_m}{Dt} \\ & + \Phi_m^\mu + \Phi_m^\sigma + \Phi_m^i. \end{aligned} \quad (13-12)$$

The above two equations described the transfers of momentum and energy as seen from the observer moving with the velocity \mathbf{v}_m . Because of its special form, Eq.(13-11) is called the equation of motion.

Field Equations in Several Coordinate Systems

In view of practical importance, we express these four field equations in

two different coordinate systems. Since the derivatives are straightforward from the standard vector calculus (Aris, 1962; McConnell, 1957), we only list the results below.

In *rectangular coordinates* (x, y, z) we have for the conservation of mass of mixture

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial x}(\rho_m v_{xm}) + \frac{\partial}{\partial y}(\rho_m v_{ym}) + \frac{\partial}{\partial z}(\rho_m v_{zm}) = 0 \quad (13-13)$$

for the drift-flux of mass of phase 2

$$\begin{aligned} & \frac{\partial}{\partial t}(\alpha_2 \bar{\rho}_2) + \left\{ \frac{\partial}{\partial x}(\alpha_2 \bar{\rho}_2 v_{xm}) + \frac{\partial}{\partial y}(\alpha_2 \bar{\rho}_2 v_{ym}) \right. \\ & \left. + \frac{\partial}{\partial z}(\alpha_2 \bar{\rho}_2 v_{zm}) \right\} = \Gamma_2 - \left\{ \frac{\partial}{\partial x}(\alpha_2 \bar{\rho}_2 V_{xm}) \right. \\ & \left. + \frac{\partial}{\partial y}(\alpha_2 \bar{\rho}_2 V_{ym}) + \frac{\partial}{\partial z}(\alpha_2 \bar{\rho}_2 V_{zm}) \right\} \end{aligned} \quad (13-14)$$

for the conservation of mixture momentum

x -component

$$\begin{aligned} & \frac{\partial}{\partial t}(\rho_m v_{xm}) + \frac{\partial}{\partial x}(\rho_m v_{xm} v_{xm}) + \frac{\partial}{\partial y}(\rho_m v_{ym} v_{xm}) + \frac{\partial}{\partial z}(\rho_m v_{zm} v_{xm}) \\ & = -\frac{\partial p_m}{\partial x} + \rho_m g_{mx} + M_{xm} + \frac{\partial}{\partial x}(\bar{\tau}_{xx} + \tau_{xx}^T + \tau_{xx}^D) \\ & + \frac{\partial}{\partial y}(\bar{\tau}_{yx} + \tau_{yx}^T + \tau_{yx}^D) + \frac{\partial}{\partial z}(\bar{\tau}_{zx} + \tau_{zx}^T + \tau_{zx}^D) \end{aligned}$$

y -component

$$\begin{aligned} & \frac{\partial}{\partial t}(\rho_m v_{ym}) + \frac{\partial}{\partial x}(\rho_m v_{xm} v_{ym}) + \frac{\partial}{\partial y}(\rho_m v_{ym} v_{ym}) \\ & + \frac{\partial}{\partial z}(\rho_m v_{zm} v_{ym}) = -\frac{\partial p_m}{\partial y} + \rho_m g_{my} + M_{ym} \end{aligned} \quad (13-15)$$

$$\begin{aligned}
& + \frac{\partial}{\partial x} (\overline{\tau_{xy}} + \tau_{xy}^T + \tau_{xy}^D) + \frac{\partial}{\partial y} (\overline{\tau_{yy}} + \tau_{yy}^T + \tau_{yy}^D) \\
& + \frac{\partial}{\partial z} (\overline{\tau_{zy}} + \tau_{zy}^T + \tau_{zy}^D)
\end{aligned}$$

z-component

$$\begin{aligned}
& \frac{\partial}{\partial t} (\rho_m v_{zm}) + \frac{\partial}{\partial x} (\rho_m v_{xm} v_{zm}) + \frac{\partial}{\partial y} (\rho_m v_{ym} v_{zm}) \\
& + \frac{\partial}{\partial z} (\rho_m v_{zm} v_{zm}) = -\frac{\partial p_m}{\partial z} + \rho_m g_{mz} + M_{zm} \\
& + \frac{\partial}{\partial x} (\overline{\tau_{xz}} + \tau_{xz}^T + \tau_{xz}^D) + \frac{\partial}{\partial y} (\overline{\tau_{yz}} + \tau_{yz}^T + \tau_{yz}^D) \\
& + \frac{\partial}{\partial z} (\overline{\tau_{zz}} + \tau_{zz}^T + \tau_{zz}^D)
\end{aligned}$$

for the mixture thermal energy balance

$$\begin{aligned}
& \frac{\partial}{\partial t} (\rho_m i_m) + \frac{\partial}{\partial x} (\rho_m i_m v_{xm}) + \frac{\partial}{\partial y} (\rho_m i_m v_{ym}) + \frac{\partial}{\partial z} (\rho_m i_m v_{zm}) \\
& = -\frac{\partial}{\partial x} \left(\overline{q_x} + q_x^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \hat{i}_k V_{xkm} \right) \\
& - \frac{\partial}{\partial y} \left(\overline{q_y} + q_y^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \hat{i}_k V_{ykm} \right) \\
& - \frac{\partial}{\partial z} \left(\overline{q_z} + q_z^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \hat{i}_k V_{zkm} \right) \\
& + \left(\frac{\partial p_m}{\partial t} + v_{xm} \frac{\partial p_m}{\partial x} + v_{ym} \frac{\partial p_m}{\partial y} + v_{zm} \frac{\partial p_m}{\partial z} \right) \\
& + \Phi_m^\mu + \Phi_m^\sigma + \Phi_m^i.
\end{aligned} \tag{13-16}$$

If the flow is restricted to two dimensions, then it is called a plane flow. In this case the partial derivative with respect to x as well as the x -component of the momentum equation can be dropped from the formulation.

Similarly in *cylindrical coordinates* (r, θ, z) we have for the conservation of mass of mixture

$$\begin{aligned}
& \frac{\partial}{\partial t} \rho_m + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_m v_{rm}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_m v_{\theta m}) \\
& + \frac{\partial}{\partial z} (\rho_m v_{zm}) = 0
\end{aligned} \tag{13-17}$$

for the drift-flux of mass of phase 2

$$\begin{aligned}
& \frac{\partial}{\partial t} (\alpha_2 \overline{\rho_2}) + \frac{1}{r} \frac{\partial}{\partial r} (r \alpha_2 \overline{\rho_2} v_{r2}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\alpha_2 \overline{\rho_2} v_{\theta 2}) \\
& + \frac{\partial}{\partial z} (\alpha_2 \overline{\rho_2} v_{z2}) = \Gamma_2 - \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \alpha_2 \overline{\rho_2} V_{r2m}) \right. \\
& \left. + \frac{1}{r} \frac{\partial}{\partial \theta} (\alpha_2 \overline{\rho_2} V_{\theta 2m}) + \frac{\partial}{\partial z} (\alpha_2 \overline{\rho_2} V_{z2m}) \right\}
\end{aligned} \tag{13-18}$$

for the conservation of mixture momentum

r-component

$$\begin{aligned}
& \frac{\partial}{\partial t} (\rho_m v_{rm}) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_m v_{rm} v_{rm}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_m v_{rm} v_{\theta m}) \\
& - \frac{\rho_m v_{\theta m}^2}{r} + \frac{\partial}{\partial z} (\rho_m v_{rm} v_{zm}) = -\frac{\partial p_m}{\partial r} + \rho_m g_{mr} + M_{rm} \\
& + \frac{1}{r} \frac{\partial}{\partial r} \left\{ r (\overline{\tau_{rr}} + \tau_{rr}^T + \tau_{rr}^D) \right\} + \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{\tau_{r\theta}} + \tau_{r\theta}^T + \tau_{r\theta}^D) \\
& - \frac{1}{r} (\overline{\tau_{\theta\theta}} + \tau_{\theta\theta}^T + \tau_{\theta\theta}^D) + \frac{\partial}{\partial z} (\overline{\tau_{rz}} + \tau_{rz}^T + \tau_{rz}^D)
\end{aligned}$$

θ-component

$$\begin{aligned}
& \frac{\partial}{\partial t} (\rho_m v_{\theta m}) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho_m v_{rm} v_{\theta m}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_m v_{\theta m} v_{\theta m}) \\
& + \frac{\partial}{\partial z} (\rho_m v_{\theta m} v_{zm}) = -\frac{\partial p_m}{\partial \theta} + \rho_m g_{m\theta} + M_{\theta m} \\
& + \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 (\overline{\tau_{r\theta}} + \tau_{r\theta}^T + \tau_{r\theta}^D) \right\} + \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{\tau_{\theta\theta}} + \tau_{\theta\theta}^T + \tau_{\theta\theta}^D)
\end{aligned} \tag{13-19}$$

$$+ \frac{\partial}{\partial z} (\overline{\tau_{\theta z}} + \tau_{\theta z}^T + \tau_{\theta z}^D)$$

z-component

$$\begin{aligned} & \frac{\partial}{\partial t} (\rho_m v_{zm}) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_m v_{rm} v_{zm}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_m v_{\theta m} v_{zm}) \\ & + \frac{\partial}{\partial z} (\rho_m v_{zm} v_{zm}) = - \frac{\partial p_m}{\partial z} + \rho_m g_{mz} + M_{zm} \\ & + \frac{1}{r} \frac{\partial}{\partial r} \left\{ r (\overline{\tau_{rz}} + \tau_{rz}^T + \tau_{rz}^D) \right\} + \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{\tau_{\theta z}} + \tau_{\theta z}^T + \tau_{\theta z}^D) \\ & + \frac{\partial}{\partial z} (\overline{\tau_{zz}} + \tau_{zz}^T + \tau_{zz}^D) \end{aligned}$$

for the mixture thermal energy balance

$$\begin{aligned} & \frac{\partial}{\partial t} (\rho_m i_m) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_m i_m v_{rm}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_m i_m v_{\theta m}) \\ & + \frac{\partial}{\partial z} (\rho_m i_m v_{zm}) = - \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\overline{q_r} + q_r^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \widehat{i_k} V_{rkm} \right) \right] \right. \\ & + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\overline{q_\theta} + q_\theta^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \widehat{i_k} V_{\theta km} \right) \\ & + \left. \frac{\partial}{\partial z} \left(\overline{q_z} + q_z^T + \sum_{k=1}^2 \alpha_k \overline{\rho_k} \widehat{i_k} V_{zkm} \right) \right\} + \Phi_m^\mu + \Phi_m^\sigma + \Phi_m^i \\ & + \left(\frac{\partial p_m}{\partial t} + v_{rm} \frac{\partial p_m}{\partial r} + \frac{v_{\theta m}}{r} \frac{\partial p_m}{\partial \theta} + v_{zm} \frac{\partial p_m}{\partial z} \right). \end{aligned} \tag{13-20}$$

If the flow is axisymmetric, the partial derivative with respect to θ drops from the equations. Furthermore if the flow is free from the circulatory motion around the z -axis, namely, the flow is restricted to two directions r and z , then $v_{\theta m}$ is zero, thus the momentum equation for θ direction can be eliminated.

1.2 Drift-flux (or mixture) model constitutive laws

It is evident that the drift-flux model based on the four field equations is an approximate theory of the two-fluid model. In order to complete the drift-flux model it is necessary to supply several constitutive laws for mixtures. We can consider two distinct approaches to accomplish our purpose. The first method is to start our analysis on the constitutive equations from the mixture field equations and the mixture entropy inequality, then to apply various constitutive axioms directly to the mixtures and independently of the two-fluid model. The second method is to obtain the necessary constitutive equations by the reduction from the two-fluid model formulation.

At first it seems to be more logical to follow the former approach, because it is a self-sufficient and independent formulation of the model for the mixtures. However, in reality it is confronted by great difficulties which cannot be overlooked lightly. The main problems arise from the fact that in general two phases are not in thermal equilibrium, thus it is not possible to introduce a mixture temperature. This suggests that we cannot expect the existence of a simple equation of state in terms of the macroscopic mixture properties.

It can be seen that the thermal non-equilibrium condition and the structures of the interfaces are the governing factors of the changes of phases. Furthermore, the kinematic and mechanical state between two phases is greatly influenced by the interfacial properties and structures. In order to bring these important effects into the drift-flux model formulation, it is simpler and more realistic to use reductions from the two-fluid model than the former approach. Consequently, in this section we develop our analysis on the mixture constitutive equations in parallel with the Section 1.2 of Chapter 9.

Principle of Determinism

The drift-flux model field equations have been given by Eqs.(13-1), (13-2), (13-3) and (13-10) which are not sufficient to describe the system completely. From the principle of determinism, it is necessary to supply additional constitutive equations that specify the response characteristics of certain group of macroscopic two-phase mixtures. In order to keep the thermal non-equilibrium effects in the drift-flux (or mixture) model formulation, we introduce a fundamental equation of state *for each phase*. Furthermore various mixture properties can be related to the properties of each phase through definitions of the Section 1.5 of Chapter 4.

By taking into account the above considerations, we have following variables appearing in the drift-flux model formulation.

- | | |
|------------------------------|--|
| 1. Equation of State: | $\overline{\rho}_k, \overline{p}_k, \overline{T}_k, \hat{i}_k, \overline{T}_i, \overline{\sigma};$ |
| 2. Conservation of Mass: | $\rho_m, v_m;$ |
| 3. Conservation of Momentum: | $p_m, V_{km}, \overline{\mathcal{T}}, \overline{\mathcal{T}}^T, M_m, \alpha_k;$ |
| 4. Conservation of Energy: | $i_m, \overline{q}, \overline{q}^T, \Phi_m^\mu, \Phi_m^\sigma, \Phi_m^i;$ |
| 5. Drift-flux Equation: | $\Gamma_2;$ |

where $k=1$ and 2. Hence the total number of the variables is twenty seven. For a properly formulated drift-flux model we should have also the same number of equations. These can be classified into following groups.

<i>Equations</i>		<i>Number of Equations</i>
1) Field equations		
mixture mass	Eq.(13-1)	1
mixture momentum	Eq.(13-3)	1
mixture energy	Eq.(13-10)	1
drift-flux	Eq.(13-2)	1
2) Axiom of continuity		
$\alpha_1 = 1 - \alpha_2$	Eq.(4-13)	1
3) Equation of state for phase		
thermal equation of state	Eq.(9-56)	2
caloric equation of state	Eq.(9-57)	2
definition of ρ_m	Eq.(4-66)	1
definition of i_m	Eq.(4-74)	1
definition of p_m	Eq.(4-72)	1
4) Equation of state for interfaces		
$\overline{\sigma} = \overline{\sigma}(\overline{T}_i)$	Eq.(9-130)	1
5) Identity on the drift-flux velocities		
$\sum_{k=1}^2 \alpha_k \overline{\rho}_k V_{km} = 0$	Eq.(4-90)	1
6) Kinematic constitutive equation for V_{2m}		1
7) Mechanical state between two phases		
$\overline{p}_1 - \overline{p}_2 = -2\overline{H}_{21} \overline{\sigma}$	Eq.(9-128)	1
8) Thermal state between two phases		
$\overline{T}_1 - \overline{T}_2 = 0$		1

9) Phase change condition

$$\overline{\overline{p_2}} - p^{sat}(\overline{\overline{T_i}}) = 2\overline{\overline{H_{21}}} \overline{\overline{\sigma}} \frac{\overline{\overline{\rho_2}}}{\overline{\overline{\rho_2}} - \overline{\overline{\rho_1}}} \quad \text{Eq.(9-163)} \quad 1$$

10) Mechanical constitutive equations

viscous stress $\overline{\mathcal{T}}$	1
turbulent stress $\overline{\mathcal{T}^T}$	1
mixture momentum source $\overline{M_m}$	1

11) Energetic constitutive equations

conduction heat flux $\overline{\mathbf{q}}$	1
turbulent heat flux $\overline{\mathbf{q}^T}$	1
dissipation term $\overline{\Phi_m^\mu}$	1
surface tension effect $\overline{\Phi_m^\sigma}$	1
mechanical energy effect $\overline{\Phi_m^i}$	1

12) Constitutive equation for phase change

mass generation $\overline{\Gamma_2}$	1
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This shows that we have also twenty seven equations, thus the present formulation is consistent. We note here that these field and constitutive equations are required from the principle of determinism, however they do not ensure the existence of a solution. It is very difficult to prove for our system that the problem is properly set, namely, the just setting (Truesdell and Toupin, 1960) because it concerns with the existence, uniqueness as well as the proper initial and boundary conditions. Usually, it can be checked only for very simplified classes of problems. Now we proceed to the detailed discussion of the above constitutive equations.

Equations of State and Mixture Properties

The axiom of continuity requires that an interface does not stay at a point for a finite time interval (see Section 1.4 of Chapter 5), thus we have

$$\alpha_1 + \alpha_2 = 1. \quad (13-21)$$

The mixture density ρ_m is defined by

$$\rho_m = \alpha_1 \overline{\overline{\rho_1}} + \alpha_2 \overline{\overline{\rho_2}} \quad (13-22)$$

with the thermal equation of state for each phase given by

$$\overline{\overline{\rho}}_1 = \overline{\overline{\rho}}_1(\overline{\overline{T}}_1, \overline{\overline{p}}_1) \quad (13-23)$$

$$\overline{\overline{\rho}}_2 = \overline{\overline{\rho}}_2(\overline{\overline{T}}_2, \overline{\overline{p}}_2). \quad (13-24)$$

And the mixture pressure is related to the phase pressures by

$$p_m = \alpha_1 \overline{\overline{p}}_1 + \alpha_2 \overline{\overline{p}}_2. \quad (13-25)$$

The mixture density given by Eq.(13-22) with Eqs.(13-23) and (13-24) can be considered as the mixture thermal equation of state which has constraints imposed by the thermal, mechanical and the chemical relations between two phases.

The mixture enthalpy is defined by

$$i_m = \frac{\alpha_1 \overline{\overline{\rho}}_1 \hat{i}_1 + \alpha_2 \overline{\overline{\rho}}_2 \hat{i}_2}{\rho_m} \quad (13-26)$$

with the caloric equation of state for each phase

$$\hat{i}_1 = \hat{i}_1(\overline{\overline{T}}_1, \overline{\overline{p}}_1) \quad (13-27)$$

$$\hat{i}_2 = \hat{i}_2(\overline{\overline{T}}_2, \overline{\overline{p}}_2). \quad (13-28)$$

By substituting Eqs.(13-27) and (13-28) into Eq.(13-26) we obtain the mixture caloric equation of state which shows the dependence of i_m on the temperatures, pressures and local void fractions. As in the case of mixture thermal equations of state, however, it has constraints imposed by the constitutive equations for the temperature difference and the pressure difference between two phases as well as by the phase change condition, Eq.(9-163).

The thermal equation of state for the interfaces in the macroscopic field can be approximated by

$$\overline{\overline{\sigma}} = \overline{\overline{\sigma}}(\overline{\overline{T}}_i) \quad (13-29)$$

which can be considered as the fundamental equation of state for the interfaces without surface mass.

Kinematic Constitutive Equation

As it has been explained in the previous section, we should supply a constitutive equation for the relative motion of phases. Since in the drift-flux model formulation we have eliminated one momentum equation, the kinematic constitutive equation stands as a relative equation of motion. It can be expected, however, that the *dynamic interactions* between two phases will be lost by replacing the momentum equation by a kinematic relation.

The diffusion velocity of each phase is related by an identity

$$\alpha_1 \overline{\rho_1} \mathbf{V}_{1m} + \alpha_2 \overline{\rho_2} \mathbf{V}_{2m} = 0. \quad (13-30)$$

Thus, we should supply only one of the diffusion velocities by the kinematic constitutive equation. However, since the diffusion velocity \mathbf{V}_{km} can be related to the relative velocity between phases or the drift velocities by the definitions in the Section 1.6 of Chapter 4 as

$$\begin{aligned} \mathbf{V}_{2m} &= -\frac{\alpha_1 \overline{\rho_1}}{\alpha_2 \overline{\rho_2}} \mathbf{V}_{1m} = -\frac{\alpha_1 \overline{\rho_1}}{\rho_m} (\widehat{\mathbf{v}}_1 - \widehat{\mathbf{v}}_2) = \frac{\overline{\rho_1}}{\rho_m} \mathbf{V}_{2j} \\ &= -\frac{\alpha_1 \overline{\rho_1}}{\alpha_2 \rho_m} \mathbf{V}_{1j} \end{aligned} \quad (13-31)$$

the constitutive equation can be given in terms of any of the above velocities.

The relative velocity between two phases depends upon the drag force acting at the interfaces as well as the interfacial geometry. Thus it can be expected that relative velocity will vary whenever the interfacial structure of the mixture changes. It has been shown in the Section 1.4 of Chapter 9 that, in a dispersed two-phase flow, the drag correlation should be expressed in terms of the drift velocity $(\mathbf{j} - \widehat{\mathbf{v}}_d)$ and the Reynolds number based on that velocity, Eqs.(9-223) and (9-224). This suggests that the kinematic constitutive equation on the relative motion between phases is best studied in terms of the drift velocity of the dispersed phase as it has been proposed by Zuber (1964b), Zuber et al. (1964) and Zuber and Staub (1966).

In view of the results obtained in the above references, it can be said that the drift velocity is a function of a terminal velocity \mathbf{v}_∞ of a single particle in an infinite medium and the void fraction of a continuous phase. In order to take into account the drift effect due to the concentration gradient, we propose a linear constitutive law in the following form

$$\mathbf{V}_{dj} = \mathbf{j} - \widehat{\mathbf{v}}_d = \mathbf{v}_\infty (1 - \alpha_d)^n - \frac{D_d^\alpha}{\alpha_d} \nabla \alpha_d \quad (13-32)$$

where D_d^α is the drift coefficient based on the void fraction α_d . The first term on the right-hand side takes into account for the *effect of gravity and forces which is usually the dominant part of the drift velocity*. A detailed analysis on this term in the bubbly flow regime has been made in Zuber et al. (1964), and Ishii (1977) also demonstrated that the constitutive equation of the drift velocity can be derived from the two-fluid model for various flow regimes. In the absence of the wall and under a steady-state condition without phase change, the multiparticle system in an infinite medium essentially reduces to a gravity and drag dominated one-dimensional flow, since the averaged void and velocity profiles become flat. Solving the momentum equations for each phase yields the relative velocity law. Thus, we use the results for drag correlations in Chapter 12.

For a viscous regime, the drift velocity can be given by

$$\begin{aligned} V_{dj} \simeq & 10.8 \left(\frac{\mu_c g \Delta \rho}{\rho_c^2} \right)^{1/3} \frac{(1 - \alpha_d)^{1.5} f(\alpha_d)}{r_d^*} \\ & \times \frac{\psi(r_d^*)^{4/3} \{1 + \psi(r_d^*)\}}{1 + \psi(r_d^*) \{f(\alpha_d)\}^{6/7}} \end{aligned} \quad (13-33)$$

where

$$f(\alpha_d) = (1 - \alpha_d)^{1/2} \frac{\mu_c}{\mu_m} \quad (13-34)$$

and

$$\psi(r_d^*) = 0.55 \left\{ (1 + 0.08 r_d^{*3})^{4/7} - 1 \right\}^{0.75} \quad (13-35)$$

for the viscous regime. The non-dimensional radius, r_d^* , is defined by

$$r_d^* \equiv r_d \left(\frac{\rho_c g \Delta \rho}{\mu_c^2} \right)^{1/3}. \quad (13-36)$$

For a Newton's regime ($\tau_d^* \geq 34.65$), the drift velocity can be given by

$$V_{dj} = 2.43 \left(\frac{r_d g \Delta \rho}{\rho_c} \right)^{1/2} (1 - \alpha_d)^{1.5} f(\alpha_d) \quad (13-37)$$

$$\times \frac{18.67}{1 + 17.67 \{f(\alpha_d)\}^{6/7}}.$$

For a distorted-fluid-particle regime, the drift velocity can be given by

$$V_{dj} \simeq \sqrt{2} \left(\frac{\sigma g \Delta \rho}{\rho_c^2} \right)^{1/4} \times \begin{cases} (1 - \alpha_d)^{1.75} & \mu_c \gg \mu_d \\ (1 - \alpha_d)^2 & \mu_c \simeq \mu_d \\ (1 - \alpha_d)^{2.25} & \mu_d \gg \mu_c. \end{cases} \quad (13-38)$$

The above criterion is applicable for $N_\mu \geq 0.11(1 + \psi)/\psi^{8/3}$ where N_μ is the viscosity number given by Eq.(12-28).

For a churn-turbulent flow regime, the drift velocity can be given by

$$V_{dj} = \left\{ \begin{array}{l} \sqrt{2} \\ \text{or } 1.57 \end{array} \right\} \left(\frac{\sigma g \Delta \rho}{\rho_c^2} \right)^{1/4} \frac{\rho_c - \rho_d}{\Delta \rho} (1 - \alpha_d)^{1/4} \quad (13-39)$$

$$\simeq \sqrt{2} \left(\frac{\sigma g \Delta \rho}{\rho_c^2} \right)^{1/4} \frac{\rho_c - \rho_d}{\Delta \rho}.$$

In the exact expression for V_{dj} , the proportionality constant $\sqrt{2}$ is applicable for bubbly flows and 1.57 for droplet flows. However, in view of the uncertainty in predicting the drag coefficient, this difference as well as the effect of the void fraction may be neglected.

For a slug flow regime, the drift velocity can be given by

$$V_{dj} = 0.35 \left(\frac{g \Delta \rho D}{\rho_c} \right)^{1/2} \quad (13-40)$$

where D is the diameter of the tube.

Figure 13-1 compares the present analytical result, Eq.(13-37), with the empirical correlation for solid-particles flow systems (Richardson and Zaki,

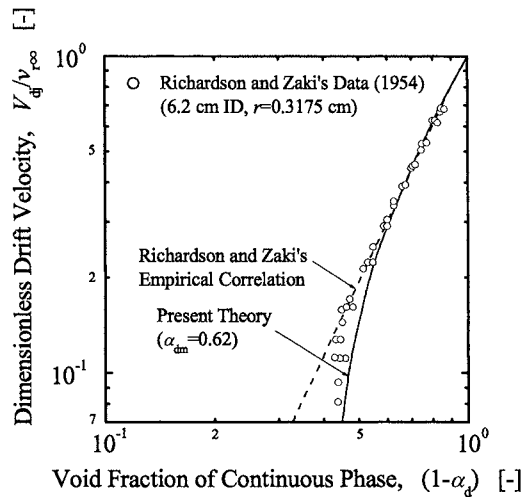


Figure 13-1. Comparison with experimental data for solid-particle system at high Reynolds number (Ishii, 1977)

1954). An agreement at relatively high void fraction of continuous phase is excellent. At very high values of α_d , Eq.(13-37) predicts much lower drift velocities than the Richardson-Zaki correlation. However, the original experimental data of Richardson and Zaki also indicate this trend, which is predicted by Eq.(13-37). Figure 13-2 shows the relative velocity in both the bubble and the droplet-liquid flow regimes. These data of Lackme (1973) clearly indicate the difference in the concentration dependence of the relative velocity between a bubbly flow and a droplet flow. These characteristics have been correctly predicted by the model. Figures 13-3 and 13-4 make further comparisons between the theoretical predictions and experimental data in both the batch and countercurrent bubbly flows and in a liquid-liquid dispersion system, respectively. The theoretical predictions agree with the data very well. Figure 13-5 compares the prediction for churn-turbulent flow and the experimental data of Yoshida and Akita (1965). The data were taken for an air-aqueous sodium sulfite solution system with various column diameters ranging from 7.7 to 60 cm. As can be seen from the figure, the theory underestimates the gas flux for smaller-column-diameter experiments. However, for larger column diameters, the agreement between the prediction and the data becomes increasingly satisfactory. This tendency can be easily explained by the two-dimensional effect due to the void and velocity profiles. The dispersed phase is locally transported with the local drift velocity with respect to local volumetric flux. Therefore, if more particles are

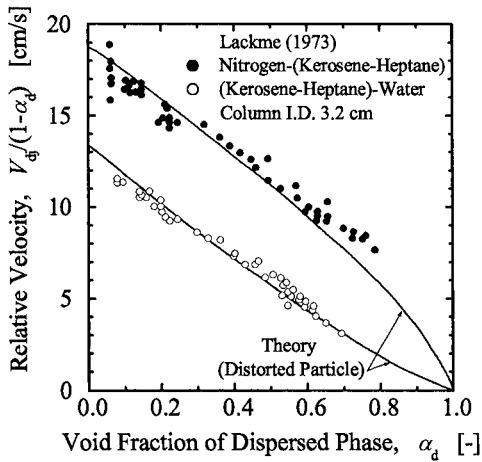


Figure 13-2. Difference between bubble system and droplet-dispersion system in distorted-particle regime (Ishii and Chawla, 1979)

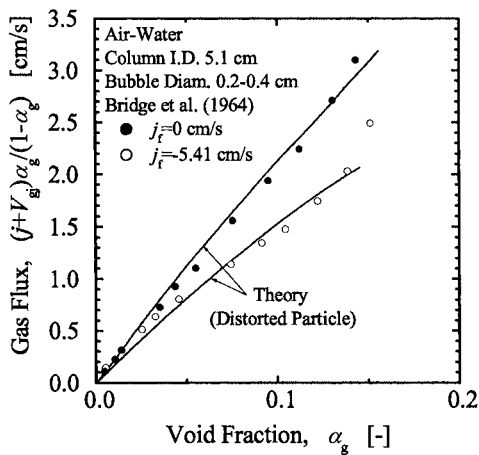


Figure 13-3. Comparison of gas volumetric flux with distorted-bubble-regime data in a flowing system (Ishii and Chawla, 1979)

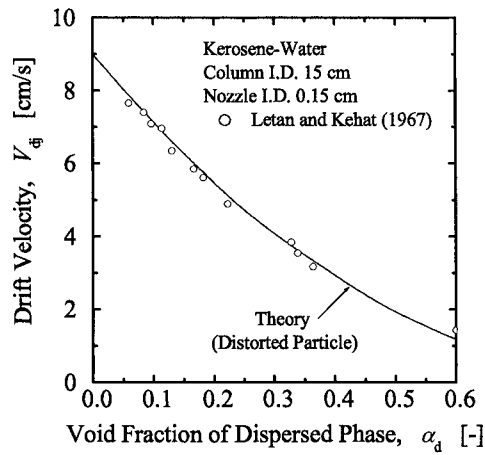


Figure 13-4. Comparison of predicted distorted-particle-regime drift velocity to data in a Kerosene-water system (Ishii and Chawla, 1979)

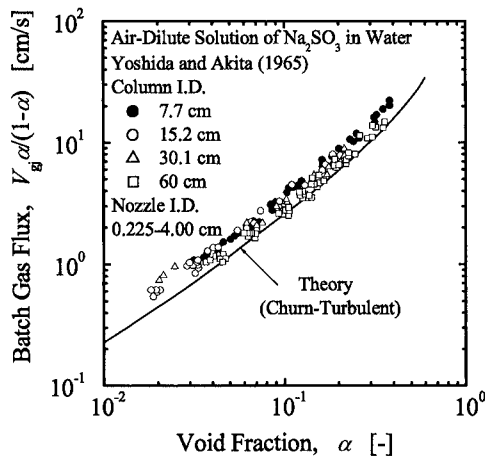


Figure 13-5. Comparison of predicted gas volumetric flux based on churn-flow-regime drift velocity to data (Ishii and Chawla, 1979)

concentrated in higher-flux regions, this will give a higher dispersed-phase flux than the case with uniform profiles. Then the mean gas volumetric flux should be somewhat higher than the prediction. For more detailed discussions on this point, see Zuber and Findlay (1965), Ishii (1977) and Werther (1974).

Thermodynamic State between Two Phases

As we have discussed in the preceding section it is necessary to specify the mechanical, thermal and chemical states between two phases. The simplified normal momentum jump condition gives the mechanical relation between two phases, thus we have

$$\overline{\overline{p_1}} - \overline{\overline{p_2}} = -2 \overline{\overline{H_{21}}} \overline{\overline{\sigma}}. \quad (13-41)$$

As we see from the above equation, the pressure difference can be important only if the mean curvature $\overline{\overline{H_{21}}}$ is large, namely, for a bubbly or droplet flow with small fluid particle diameters. Consequently, in many practical engineering problems where the drift-flux model can be applied, the pressure difference between two phases can be neglected. Then we have

$$\overline{\overline{p_1}} \approx \overline{\overline{p_2}}. \quad (13-42)$$

The chemical state between phases decides the condition of phase changes and it is given from Eq.(9-163) as

$$\overline{\overline{p_2}} - p^{sat}(\overline{\overline{T_i}}) = 2 \overline{\overline{H_{21}}} \overline{\overline{\sigma}} \left(\frac{\overline{\overline{\rho_2}}}{\overline{\overline{\rho_2}} - \overline{\overline{\rho_1}}} \right). \quad (13-43)$$

However, this can be approximated by

$$\overline{\overline{p_g}} \approx p^{sat}(\overline{\overline{T_i}}) \quad (13-44)$$

where $\overline{\overline{p_g}}$ denotes the vapor phase pressure. If the mechanical equilibrium condition (13-42) can be applied, then we have

$$\overline{\overline{p_1}} = \overline{\overline{p_2}} = p^{sat}(\overline{\overline{T_i}}). \quad (13-45)$$

The constitutive equation for the thermal state between two phases is one

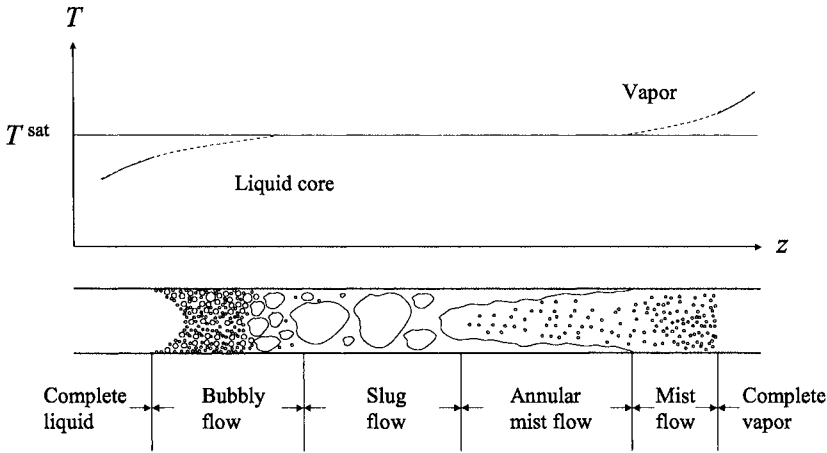


Figure 13-6. Axial temperature distribution (Ishii, 1975)

of the difficult constitutive equations to obtain, since this specifies the degree of thermal non-equilibrium. First we note that it can be replaced by the relation for $\overline{T}_1 - \overline{T}_i$ or $\overline{T}_2 - \overline{T}_i$. And if both relations are given, then the constitutive equation for the mass transfer Γ_2 becomes redundant. It is very important to realize that in many practical problems one of the phases is approximately in thermal equilibrium with the interfaces thus

$$\overline{T}_k - \overline{T}_i \approx 0 \quad \text{for } k = 1 \text{ or } 2 \quad (13-46)$$

For example in a boiling system we may assume that

$$\begin{aligned} \overline{T}_g &= \overline{T}_i; & \text{for bubbly and mixed flow} \\ \overline{T}_f &= \overline{T}_i; & \text{for droplet flow} \end{aligned} \quad (13-47)$$

as shown in the Fig.13-6.

Mechanical Constitutive Equations: $\overline{\mathcal{T}}$ and $\overline{\mathcal{T}}^T$

The average viscous stress for each phase has been obtained in the Section 1.2 of Chapter 9, thus we have

$$\overline{\mathcal{T}} = \sum_{k=1}^2 \left\{ \alpha_k \overline{\mu}_k \left[\nabla \widehat{\mathbf{v}}_k + (\nabla \widehat{\mathbf{v}}_k)^+ \right] + 2\alpha_k \overline{\mu}_k D_{ki} \right\} \quad (13-48)$$

where the first term on the right-hand side has the standard form of the viscous stress based on the deformation tensor. The second term takes into account the effects of the interfaces, and the interfacial extra deformation tensor D_{ki} is defined by Eq.(9-73). By substituting the definition of the drift velocity we obtain

$$\begin{aligned} \overline{\mathcal{T}} = & \left(\sum_{k=1}^2 \alpha_k \overline{\mu_k} \right) \left[\nabla \mathbf{v}_m + (\nabla \mathbf{v}_m)^+ \right] \\ & + \sum_{k=1}^2 \alpha_k \overline{\mu_k} \left\{ \left[\nabla \mathbf{V}_{km} + (\nabla \mathbf{V}_{km})^+ \right] + 2D_{ki} \right\}. \end{aligned} \quad (13-49)$$

This shows that if the effects of the relative velocity and the interfacial deformation are small then the mixture viscosity can be given by $\sum_{k=1}^2 \alpha_k \overline{\mu_k}$.

For a dispersed two-phase flow, the interfacial extra deformation stress tensor can be put into a simple form, Eq.(9-76), if the mass transfer effects are not significant. In this case Eq.(13-48) can be reduced to the following form

$$\begin{aligned} \overline{\mathcal{T}} = & \sum_{k=1}^2 \alpha_k \overline{\mu_k} \left[\nabla \mathbf{j} + (\nabla \mathbf{j})^+ \right] \\ & + \alpha_d (\overline{\mu_d} - \overline{\mu_c}) \left[\nabla \mathbf{V}_{dj} + (\nabla \mathbf{V}_{dj})^+ \right] \end{aligned} \quad (13-50)$$

where \mathbf{j} and \mathbf{V}_{dj} are the volumetric flux of the mixture and the drift velocity of the dispersed phase, respectively. Furthermore we note that \mathbf{j} can be related to \mathbf{v}_m and \mathbf{V}_{dj} by Eq.(4-93). The above result is important, since it shows that in a dispersed flow the mixture-deformation tensor should be based on the velocity of the volume center rather than that of the mass center. The mixture viscosity is then given by

$$\mu_m = \alpha_1 \overline{\mu_1} + \alpha_2 \overline{\mu_2}. \quad (13-51)$$

As we can see from Eq.(13-50), the mixture viscous stress has an additional term from the relative motion. The kinematic constitutive equation for \mathbf{V}_{dj} shows that, in many cases, this term can be expressed as a function of the void fraction only.

Now let us consider the mixture turbulent stress \mathcal{T}^T which appears in the mixture momentum equation (13-3). It is evident from the definition that \mathcal{T}^T is the addition of the turbulent stresses for each phase $\alpha_k \mathcal{T}_k^T$. In the

Section 1.2 of Chapter 9 we applied the mixing-length theory to the two-fluid model formulation, thus we obtained the constitutive equation for \mathcal{T}_k^T . These expressions given by Eq.(9-89) are not appropriate for the drift-flux model formulation, however, since they are written in terms of the variables of each phase and not of the mixture.

We can consider two methods to obtain the mixture turbulent flux \mathcal{T}^T , namely the derivation of \mathcal{T}^T from the ones for each phase by using the definitions of the mixture velocity and the drift velocity, or the establishment of the mixing length model in terms of the mixture properties in analogy with single-phase flows. If the two-phases are strongly coupled or the sizes of eddies are large in comparison with the characteristic dimension of a dispersed phase, the latter approach is justified. Then we have

$$\mathcal{T}^T = 2\mu_m^{T*} \rho_m \ell^2 \sqrt{2D_m:D_m} D_m \quad (13-52)$$

where

$$D_m = \frac{1}{2} \left[\nabla \widehat{\mathbf{v}}_m + (\nabla \widehat{\mathbf{v}}_m)^+ \right]. \quad (13-53)$$

And the non-dimensional coefficient (μ_m^{T*}) corresponds to a mixing length constant.

For a dispersed flow, we may obtain a different expression for \mathcal{T}^T . If the dimension of a dispersed phase is comparative to that of turbulent eddies, the dominant part of the mixture stress is given by that of the *continuous phase*. Thus, we have

$$\mathcal{T}^T \doteq \alpha_c \mathcal{T}_c^T = 2\alpha_c \mu_c^{T*} \rho_c \ell^2 \sqrt{2D_c:D_c} D_c \quad (13-54)$$

where we have used Eq.(9-89). The total deformation tensor of the continuous phase D_c is given by Eqs.(9-82), (9-74) and (9-76). Thus, we have

$$D_c = \frac{1}{2} [\nabla \mathbf{j} + (\nabla \mathbf{j})^+] - \frac{1}{2} \left(\frac{\alpha_d}{1 - \alpha_d} \right) [\nabla \mathbf{V}_{dj} + (\nabla \mathbf{V}_{dj})^+]. \quad (13-55)$$

Here we used the definition (4-91) and the identity (4-95). For many practical cases the drift velocity of a dispersed phase can be taken as a function of α_d only, as we have discussed in connection with the kinematic constitutive equation. Further simplification can be made if we assume that

the drift velocity is constant and, then, by combining Eqs.(13-50) and (13-54) the total mixture stress can be approximated by

$$\begin{aligned} & \overline{\mathcal{T}} + \mathcal{T}^T + \mathcal{T}^D \\ & \doteq \left(\sum_{k=1}^2 \alpha_k \overline{\mu_k} + \alpha_c \mu_c^{T*} \overline{\rho_c} \ell^2 \sqrt{2 \mathcal{D}_c : \mathcal{D}_c} \right) [\nabla \mathbf{j} + (\nabla \mathbf{j})^+] \\ & - \frac{\alpha_d}{1 - \alpha_d} \frac{\overline{\rho_c} \overline{\rho_d}}{\rho_m} \mathbf{V}_{dj} \mathbf{V}_{dj}. \end{aligned} \quad (13-56)$$

For a fully developed pipe flow, we have

$$\begin{aligned} & \left(\overline{\tau} + \tau^T + \tau^D \right)_{rz} \\ & = \left[\sum_{k=1}^2 \alpha_k \overline{\mu_k} + \alpha_c \mu_c^{T*} \overline{\rho_c} (R - r)^2 \left| \frac{dj_z}{dr} \right| \right] \frac{dj_z}{dr}. \end{aligned} \quad (13-57)$$

The mixture momentum source \mathbf{M}_m for a dispersed flow can be given by

$$\mathbf{M}_m = \nabla \left(2 \overline{H_{dc}} \overline{\sigma} \alpha_d \right). \quad (13-58)$$

And for a transitional flow we may take

$$\mathbf{M}_m \approx 0. \quad (13-59)$$

Energetic Constitutive Equations

The average conduction heat flux for each phase has been obtained in the Section 1.2 of Chapter 9, thus we have

$$\overline{\mathbf{q}} = - \sum_{k=1}^2 \alpha_k \overline{K_k} \left\{ \nabla \overline{T_k} + \frac{1}{\alpha_k} \left(\overline{T_i} - \overline{T_k} \right) (-\nabla \alpha_k) \right\} \quad (13-60)$$

where the first term on the right-hand side has the standard significance of the conduction heat transfer due to the temperature gradient. The second term takes into account the effects of the interfaces. By rearranging the terms, the above equation reduces to

$$\bar{\mathbf{q}} = - \left(\sum_{k=1}^2 \alpha_k \bar{K}_k \right) \nabla \bar{T}_i - \sum_{k=1}^2 \bar{K}_k \nabla \left[\alpha_k \left(\bar{T}_k - \bar{T}_i \right) \right]. \quad (13-61)$$

This form of the average conduction heat flux suggests that the concept of the mixture temperature can be represented by \bar{T}_i with the mixture conductivity given by $\sum_{k=1}^2 \alpha_k \bar{K}_k$. The second term on the right-hand side represents the effect of thermal non-equilibrium.

For many practical systems the effect of the pressure drop on the thermodynamic properties can be neglected, and as we have already discussed, the temperature of the dispersed phase can be approximated by the interfacial temperature. Thus by taking

$$\begin{aligned} \nabla \bar{T}_i &\approx 0 \\ \bar{T}_d &\approx \bar{T}_i \end{aligned} \quad (13-62)$$

the constitutive equation for the heat flux becomes

$$\bar{\mathbf{q}} \doteq -\bar{K}_c \nabla \left[\alpha_c \left(\bar{T}_c - \bar{T}_i \right) \right]. \quad (13-63)$$

Furthermore, the turbulent heat flux can be developed in parallel with the turbulent stress tensor. However, a special care should be taken here because the mixture temperature is not well defined. In view of the constitutive equation for the turbulent energy transfer for each phase, Eq.(9-92), we have

$$\mathbf{q}^T = \sum_{k=1}^2 \alpha_k \mathbf{q}_k^T = - \sum_{k=1}^2 \alpha_k K_k^T \left\{ \nabla \bar{T}_k - \frac{\nabla \alpha_k}{\alpha_k} \left(\bar{T}_i - \bar{T}_k \right) \right\} \quad (13-64)$$

where K_k^T is given by Eqs.(9-94) and (9-95).

For a dispersed flow, we may use the approximation given by Eq.(13-62), then we have

$$\mathbf{q}^T \doteq -K_c^T \nabla \left[\alpha_c \left(\bar{T}_c - \bar{T}_i \right) \right]. \quad (13-65)$$

In view of Eq.(9-94) we obtain

$$K_c^{T*} = \frac{K_c^T}{\overline{\rho_c c_{pc}} \ell^2 \sqrt{2 \overline{D_c} : \overline{D_c}}}. \quad (13-66)$$

Consequently we get

$$\mathbf{q}^T = -K_c^{T*} \overline{\rho_c c_{pc}} \ell^2 \sqrt{2 \overline{D_c} : \overline{D_c}} \nabla \left[\alpha_c \left(\overline{\overline{T_c}} - \overline{\overline{T_i}} \right) \right]. \quad (13-67)$$

Here the non-dimensional coefficient K_c^{T*} corresponds to a thermal mixing-length constant. We expect that it depends on the conductivity, surface area concentration, mean curvature and the void fraction as shown by Eq.(9-95).

For the terms represented by Φ_m^μ , Φ_m^σ , and Φ_m^i , we only note that if these effects due to the viscous dissipation, surface tension and mechanical energy interaction have to be included in the analyses, then they should be specified by three constitutive equations. It is evident from the definitions of these terms, Eqs.(13-7), (13-8) and (13-9), that such constitutive relations are expected to be quite complicated. This means that most of the advantages of using the thermal energy equation diminishes if these effects cannot be neglected.

Constitutive Equation for Phase Change

The constitutive equation for the mass transfer at the interface has been given by Eq.(9-111). Furthermore, we have noted that for a drift-flux model it is necessary to supply information on the thermal state between two phases. In a simplified form it can be given by Eq.(13-47) which is useful for most of the practical problems. Thus for a dispersed flow regime we have

$$\Gamma_c \doteq b_c^{I*} \frac{\left(\overline{\overline{K_c}} + K_c^T \right) a_i^2}{\left| \widehat{i_{di}} - \widehat{i_{ci}} \right|} \left(\overline{\overline{T_i}} - \overline{\overline{T_c}} \right) \quad (13-68)$$

where non-dimensional coefficient b_c^{I*} is expected to depend on the following groups.

$$b_c^{I*} = b_c^{I*} \left(\frac{\overline{\overline{\rho_c}}}{\overline{\overline{\rho_d}}}, N_{Jc}, \frac{\overline{\overline{H_{dc}}}}{a_i}, \alpha_c \right) \quad (13-69)$$

Geometrical Constitutive Equations

If the effect of the mean curvature $\overline{\overline{H_{21}}}$ and the surface area

concentration have to be included explicitly in the formulation, we should give two additional geometrical constitutive equations. In general these are given by Eqs.(9-137) and (9-138), however for a dispersed two-phase flow they can be simplified to Eqs.(9-213) and (9-215).

1.3 Drift-flux (or mixture) model formulation

The general case of the field and constitutive equations for the drift-flux model formulation has been discussed in the Sections 1.1 and 1.2 of Chapter 13. We have noted the importance of the mixture center of mass velocity as well as of the drift velocities in the formulation. It may be appropriate to call our present model the *drift-flux model* in order to emphasize that the effects of relative motions between two phases are taken into account by the drift velocities \mathbf{V}_{kj} .

1.3.1 Drift-flux model

In view of the definitions for \mathbf{V}_{ki} and \mathbf{V}_{km} given by Eqs.(4-91) and (4-89), respectively, the field equations for the drift-flux model can be given as follows:

The mixture continuity equation from Eq.(13-1)

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}_m) = 0 \quad (13-70)$$

The continuity equation for phase 2 from Eq.(13-2)

$$\frac{\partial \alpha_2 \bar{\bar{\rho}}_2}{\partial t} + \nabla \cdot (\alpha_2 \bar{\bar{\rho}}_2 \mathbf{v}_m) = \Gamma_2 - \nabla \cdot \left(\frac{\alpha_2 \bar{\bar{\rho}}_1 \bar{\bar{\rho}}_2}{\rho_m} \mathbf{V}_{2j} \right) \quad (13-71)$$

The mixture momentum equation from Eq.(13-3)

$$\begin{aligned} \frac{\partial}{\partial t} \rho_m \mathbf{v}_m + \nabla \cdot (\rho_m \mathbf{v}_m \mathbf{v}_m) = & -\nabla p_m + \nabla \cdot (\bar{\mathcal{T}} + \mathcal{T}^T) \\ & - \nabla \cdot \left(\frac{\alpha_2}{1 - \alpha_2} \frac{\bar{\bar{\rho}}_1 \bar{\bar{\rho}}_2}{\rho_m} \mathbf{V}_{2j} \mathbf{V}_{2j} \right) + \rho_m \mathbf{g}_m + \mathbf{M}_m \end{aligned} \quad (13-72)$$

The mixture thermal energy equation from Eq.(13-10)

$$\begin{aligned} \frac{\partial}{\partial t} \rho_m i_m + \nabla \cdot (\rho_m i_m \mathbf{v}_m) = & -\nabla \cdot (\bar{\mathbf{q}} + \mathbf{q}^T) \\ & -\nabla \cdot \left\{ \alpha_2 \frac{\overline{\rho_1 \rho_2}}{\rho_m} \mathbf{V}_{2j} (\hat{i}_2 - \hat{i}_1) \right\} + \frac{Dp_m}{Dt} + \Phi_m^\mu + \Phi_m^o + \Phi_m^i. \end{aligned} \quad (13-73)$$

Here we have formulated the model in terms of the mixture properties, void fraction α_2 and the drift velocity \mathbf{V}_{2j} . The model is most effective for a dispersed two-phase flow, since for this case the constitutive equations can be reduced to realistic forms as discussed in the Section 1.2 of Chapter 13.

It should be emphasized again that the drift-flux model is useful for the two-phase flow *system analyses*. This is particularly true if the motions of two phases are strongly coupled. Because of its simplicity the model can be used to make realistic similarity analyses as well as to solve many important engineering problems.

1.3.2 Scaling parameters

Let us denote the reference parameters by the subscript o. The characteristic length scale is L_o and the time scale is taken as the ratio of L_o to the velocity scale. Then we define the non-dimensional parameters whose order of magnitude is considered to be 1 as follows:

$$\begin{aligned} \rho_m^* &= \frac{\rho_m}{\rho_{mo}}, \mathbf{v}_m^* = \frac{\mathbf{v}_m}{v_{mo}}, t^* = \frac{t}{\tau_o} = \frac{t}{(L_o/v_{mo})}, \nabla^* = L_o \nabla \\ \rho_1^* &= \frac{\overline{\rho_1}}{\rho_{1o}}, \rho_2^* = \frac{\overline{\rho_2}}{\rho_{2o}}, \mathbf{V}_{2j}^* = \frac{\mathbf{V}_{2j}}{V_{2jo}}, \Gamma_2^* = \frac{\Gamma_2}{\Gamma_{2o}}, p_m^* = \frac{p_m}{\rho_{mo} v_{mo}^2} \\ i_m^* &= \frac{i_m - i_{1o}}{i_{2o} - i_{1o}}, \Delta i_{12}^* = \frac{\hat{i}_2 - \hat{i}_1}{i_{2o} - i_{1o}} \\ (\bar{\mathcal{C}} + \mathcal{C}^T)^* &= \frac{\bar{\mathcal{C}} + \mathcal{C}^T}{\mu_{mo} v_{mo} / L_o}, M_m^* = \frac{M_m}{2 H_{21o} \sigma_o / L_o} \\ (\bar{\mathbf{q}} + \mathbf{q}^T)^* &= \frac{\bar{\mathbf{q}} + \mathbf{q}^T}{K_{mo} \Delta T_o / L_o}, \Phi_m^{\mu*} = \frac{\Phi_m^\mu}{(\mu_{mo} v_{mo} / L_o)(v_{mo} / L_o)} \end{aligned} \quad (13-74)$$

$$\Phi_m^{\sigma*} = \frac{\Phi_m^{\sigma}}{2H_{21}\sigma_o v_{mo}/L_o}, \Phi_m^{i*} = \frac{\Phi_m^i}{(\rho_{1o} - \rho_{2o})v_{mo}^2 V_{2jo}/L_o}.$$

Substituting these new parameters into the field equations we obtain the following results:

Non-dimensional mixture continuity equation

$$\frac{\partial}{\partial t^*} \rho_m^* + \nabla^* \cdot (\rho_m^* \mathbf{v}_m^*) = 0 \quad (13-75)$$

Non-dimensional continuity equation for phase 2

$$\frac{\partial \alpha_2 \rho_2^*}{\partial t^*} + \nabla^* \cdot (\alpha_2 \rho_2^* \mathbf{v}_m^*) = N_{pch} \Gamma_2^* - N_D \nabla^* \cdot \left(\frac{\alpha_2 \rho_1^* \rho_2^*}{\rho_m^*} \mathbf{V}_{2j}^* \right) \quad (13-76)$$

Non-dimensional momentum equation for mixture

$$\begin{aligned} \frac{\partial}{\partial t^*} (\rho_m^* \mathbf{v}_m^*) + \nabla^* \cdot (\rho_m^* \mathbf{v}_m^* \mathbf{v}_m^*) &= -\nabla^* p_m^* \\ &+ \frac{1}{N_{Re}} \nabla^* \cdot (\overline{\mathcal{T}} + \mathcal{T}^T)^* + \frac{1}{N_{Fr}} \rho_m^* \frac{\mathbf{g}_m}{|\mathbf{g}_m|} \\ &- N_{\rho} N_D^2 \nabla^* \cdot \left\{ \left(\frac{\alpha_2}{1 - \alpha_2} \right) \frac{\rho_1^* \rho_2^*}{\rho_m^*} \mathbf{V}_{2j}^* \mathbf{V}_{2j}^* \right\} + N_{\sigma} \mathbf{M}_m^* \end{aligned} \quad (13-77)$$

Non-dimensional thermal energy equation for mixture

$$\begin{aligned} \frac{\partial}{\partial t^*} (\rho_m^* i_m^*) + \nabla^* \cdot (\rho_m^* i_m^* \mathbf{v}_m^*) &= -\frac{1}{N_{Pe}} \nabla^* \cdot (\bar{\mathbf{q}} + \mathbf{q}^T)^* \\ &- \frac{N_{\rho}}{(1 - \alpha_2) + \alpha_2 N_{\rho}} N_D \nabla^* \cdot \left\{ \frac{\alpha_2 \rho_1^* \rho_2^*}{\rho_m^*} \mathbf{V}_{2j}^* \Delta i_{12}^* \right\} \\ &+ N_{Ec} \left\{ \frac{Dp_m^*}{Dt^*} + \frac{1}{N_{Re}} \Phi_m^{\mu*} + N_{\sigma} \Phi_m^{\sigma*} + (N_{\rho} - 1) N_D \Phi_m^{i*} \right\}. \end{aligned} \quad (13-78)$$

Here we have defined

$$\begin{aligned}
\text{Phase change number } N_{pch} &\equiv \frac{\Gamma_{2o} L_o}{\rho_{2o} v_{mo}} \\
\text{Drift number } N_D &\equiv \frac{\rho_{1o} V_{2jo}}{\rho_{mo} v_{mo}} \\
\text{Density ratio } N_\rho &\equiv \frac{\rho_{2o}}{\rho_{1o}} \\
\text{Reynolds number } N_{Re} &\equiv \frac{\rho_{mo} v_{mo} L_o}{\mu_{mo}} \\
\text{Froude number } N_{Fr} &\equiv \frac{v_{mo}^2}{|g_m| L_o} \\
\text{Surface number } N_\sigma &\equiv \frac{2 H_{21o} \sigma_o}{\rho_{mo} v_{mo}^2} \\
\text{Peclet number } N_{Pe} &\equiv \frac{\rho_{mo} v_{mo} \Delta i_{12o}}{K_{mo} \Delta T_o} \\
\text{Eckert number } N_{Ec} &\equiv \frac{v_{mo}^2}{\Delta i_{12o}}.
\end{aligned} \tag{13-79}$$

We note that these eight groups are *the scaling parameters for the mixtures* based on the drift-flux model formulation. These groups are analogous to the ones obtained for the one-dimensional, two-phase flow model in Ishii (1971). An exception is the Reynolds and Peclet numbers, however, since for the latter model they are replaced by the groups from the boundary conditions, namely, the friction and Stanton numbers.

The distinction between the *scaling parameters* and the *similarity groups* should be clearly made, since they are not synonymous. The similarity groups are obtainable from the field equations and the boundary and the initial conditions with all the constitutive equations specified. Consequently, the similarity criteria cannot be discussed in detail unless the system and the problem are clearly defined. The scaling parameter enables us to make various assumptions and approximations, because they give the order of magnitude of various terms appearing in the field equations.

The first three groups of Eq.(13-79), N_{pch} , N_D , and N_ρ are the kinematic groups. If the phase change number N_{pch} is much larger than the drift number N_D , the systems is controlled by the changes of phases. However, if $N_D \gg N_{pch}$, then the system is controlled by the redistributions of phases. The number denoted by N_{Re} , N_{Fr} , and N_σ are

dynamic groups, since they scale the various forces arisen in the mixture momentum equation, whereas the Peclet and Eckert numbers are the energy groups. We note the particular importance of the Eckert number N_{Ec} . It is evident that except for high speed flows, N_{Ec} is small, thus the terms given by Φ_m^μ , Φ_m^σ and Φ_m^i in the thermal energy equation can be neglected. As we have already discussed before, this is the single most important approximation which greatly simplifies the heat transfer problems.

From Eqs.(13-76), (13-77) and (13-78), we set that the drift transport of mass, momentum and energy are not weighted by the same scaling parameter. The phase drift and the enthalpy transport due to the relative motion between phases has approximately the same order N_D , thus these two terms should be treated under the same condition. However, the momentum drift are weighted by $N_\rho N_D^2$. Thus, depending on the magnitude of this scaling parameter, it is possible to neglect the drift-stress tensor independently of the other two effects.

We note that in the *scaling parameters*, we have introduced the mixture viscosity and the conductivity given by μ_m and K_m . These parameters need not to be precisely defined as long as the scaling parameters are concerned, because only their order of magnitude is important. We may choose them as the larger viscosity and conductivity among $\overline{\mu_k}$ and $\overline{K_k}$ or defined them by $\sum_{k=1}^2 \alpha_{ko} \overline{\mu_{ko}}$ and $\sum_{k=1}^2 \alpha_{ko} \overline{K_{ko}}$ where α_{ko} denotes the reference value of α_k , for example at the system boundary.

However, if the *similarity groups* are concerned, then the exact constitutive equations for the stress tensor and heat flux should be used. Thus, according to the forms and the variables appearing in the constitutive equations, the correct reference parameters should be chosen. Based on the order of magnitude analysis and the scaling parameters obtained above, we discuss some of the important special cases.

1.3.3 Homogeneous flow model

If the drift number N_D is much smaller than the phase change number N_{pch} , then the system is reaction (phase change) controlled and the drift or diffusion of mass is negligible in view of the continuity equation for phase 2. Furthermore, if the drift number N_D is much smaller than unity, then all of the drift terms and the interfacial mechanical energy transfer effect Φ_m^i can be neglected in the field equations. We may not drop the continuity equation for phase 2 from the formulation, however, since it takes into account for the thermal non-equilibrium effect as discussed by Zuber and Dougherty (1967) in connection with the one-dimensional model. Thus, we have the following four field equations for the general homogeneous flow model:

$$\begin{aligned}
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}_m) &= 0 \\
\frac{\partial \alpha_2 \bar{\rho}_2}{\partial t} + \nabla \cdot (\alpha_2 \bar{\rho}_2 \mathbf{v}_m) &= \Gamma_2 \\
\frac{\partial \rho_m \mathbf{v}_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}_m \mathbf{v}_m) &= -\nabla p_m + \nabla \cdot (\bar{\mathcal{T}} + \mathcal{T}^T) \\
&+ \rho_m \mathbf{g}_m + \mathbf{M}_m \\
\frac{\partial \rho_m i_m}{\partial t} + \nabla \cdot (\rho_m i_m \mathbf{v}_m) &= -\nabla \cdot (\bar{\mathbf{q}} + \mathbf{q}^T) + \frac{Dp_m}{Dt} \\
&+ \Phi_m^\mu + \Phi_m^\sigma.
\end{aligned} \tag{13-80}$$

In this case, the mechanical constitutive equations for the dispersed two-phase flow can be reduced to the simple forms. Thus, from Eq.(13-50), we have

$$\bar{\mathcal{T}} = (\alpha_1 \bar{\mu}_1 + \alpha_2 \bar{\mu}_2) [\nabla \mathbf{v}_m + (\nabla \mathbf{v}_m)^+]. \tag{13-81}$$

The turbulent stress is given by Eq.(13-52) as

$$\mathcal{T}^T = 2\mu_m^{T*} \rho_m \ell^2 \sqrt{2\mathcal{D}_m : \mathcal{D}_m} \mathcal{D}_m. \tag{13-82}$$

Furthermore, the constitutive equations for the heat fluxes are given by

$$\bar{\mathbf{q}} \doteq -\bar{K}_c \nabla \left[\alpha_c \left(\bar{T}_c - \bar{T}_i \right) \right] \tag{13-83}$$

and

$$\mathbf{q}^T = -K_c^{T*} \bar{\rho}_c c_{pc} \ell^2 \sqrt{2\mathcal{D}_m : \mathcal{D}_m} \nabla \left[\alpha_c \left(\bar{T}_c - \bar{T}_i \right) \right] \tag{13-84}$$

where the dispersed phase has been assumed to be in thermal equilibrium with interfaces, namely, Eq.(13-62). If the Eckert number N_{Ec} and the surface number N_σ are small, then the capillary force \mathbf{M}_m in the mixture momentum equation, and the compressibility effect Dp_m/Dt , the dissipation term Φ_m^μ , and the surface tension effect Φ_m^σ in the energy equation can be dropped.

1.3.4 Density propagation model

For a number of systems of practical interest, particularly for systems at high-reduced pressures without large acoustic interactions, it is reasonable to assume that each phase is essentially incompressible. Then we have

$$\overline{\overline{\rho_k}} = \text{constant.} \quad (13-85)$$

Furthermore, we consider the case when the drift velocity of phase 2 is approximately a function of α_2 only. We recall that this is a valid assumption for many practical flow regimes as it has been discussed in the Section 1.3 of Chapter 7 and the Section 1.2 of Chapter 13. Thus we have

$$\mathbf{V}_{2j} \approx \mathbf{V}_{2j}(\alpha_2). \quad (13-86)$$

For simplicity, we also assume that the surface number N_σ and the Eckert number N_{Ec} are small such that

$$N_\sigma \ll 1 \quad \text{and} \quad N_{Ec} \ll 1. \quad (13-87)$$

Then we obtain from Eqs.(7-31), (7-38), (13-72) and (13-73) the following field equations:

$$\begin{aligned} \nabla \cdot \mathbf{j} &= \Gamma_2 \left(\frac{1}{\overline{\overline{\rho_2}}} - \frac{1}{\overline{\overline{\rho_1}}} \right) \\ \frac{\partial \rho_m}{\partial t} + \mathbf{C}_K \cdot \nabla \rho_m &= \frac{\rho_m}{\overline{\overline{\rho_1}} \overline{\overline{\rho_2}}} \Gamma_2 (\overline{\overline{\rho_2}} - \overline{\overline{\rho_1}}) \\ \rho_m \frac{D\mathbf{v}_m}{Dt} &= -\nabla p_m + \nabla \cdot (\overline{\overline{\mathcal{T}}} + \overline{\overline{\mathcal{T}}}^T) \\ &\quad - \nabla \cdot \left(\frac{\alpha_2}{1 - \alpha_2} \frac{\overline{\overline{\rho_1}} \overline{\overline{\rho_2}}}{\rho_m} \mathbf{V}_{2j} \mathbf{V}_{2j} \right) + \rho_m \mathbf{g}_m \end{aligned} \quad (13-88)$$

$$\rho_m \frac{D\dot{i}_m}{Dt} = -\nabla \cdot (\bar{\mathbf{q}} + \mathbf{q}^T) - \nabla \cdot \left(\frac{\alpha_2 \bar{\rho}_1 \bar{\rho}_2}{\rho_m} \mathbf{V}_{2j} (\hat{i}_2 - \hat{i}_1) \right)$$

where \mathbf{C}_K and \mathbf{j} are given by

$$\mathbf{C}_K = \mathbf{j} + \frac{\partial}{\partial \alpha_2} (\alpha_2 \mathbf{V}_{2j}) \quad (13-89)$$

and

$$\mathbf{j} = \mathbf{v}_m + \frac{\alpha_2 (\bar{\rho}_1 - \bar{\rho}_2)}{\rho_m} \mathbf{V}_{2j}. \quad (13-90)$$

The present density-wave propagation model is best suited for a dispersed (or mixed) two-phase flow regime. In this case, phase 2 should be taken as the dispersed phase. Then the constitutive equations for the stresses can be given by Eqs.(13-50) and (13-54), whereas the heat fluxes are given by Eqs.(13-63) and (13-67).