Some Issues in the Simulation of Two-Phase Flows: the relative velocity

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Multiphase flows

What models are available

- two-fluid models
  - VOF, Euler-Euler
  - equations are solved for each phase
  - Computational expensive
- single-fluid models
  - mixture model
  - one equation only
  - Computational much more effective
  - robust just in equilibrium states
  - loss of relative velocity between phases

Our approach

In our program

- we use the mixture model
- take the relative velocity into account
- formulate it as a relative velocity conservation equation
- implement the Lax-Friedrichs flux
- ready for 1D and 2D cases
- simple meshes only

Our system model

Mass conservation:

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho u) = S_1$$

Momentum conservation:

$$\frac{\partial}{\partial t}(\rho u_x) + \frac{\partial}{\partial x}(\rho u_x^2 + \rho c(1-c)u_{r,x}^2 + P) + \partial_y(\rho u_x u_y + \rho c(1-c)u_{r,x}u_{r,y}) = S_2$$

$$\frac{\partial}{\partial t}(\rho u_y) + \frac{\partial}{\partial y}(\rho u_y^2 + \rho c(1-c)u_{r,y}^2 + P) + \partial_x(\rho u_x u_y + \rho c(1-c)u_{r,x}u_{r,y}) = S_3$$

ρ is the mixture density, u is the mixture velocity in x and y direction, c is the gas mass void fraction, P is the pressure which is the same between the gas and liquid, u_r is the relative velocity between the two phases and $\psi(P)$ is a function that recounts the gas and liquid phases through the momentum equations.

Our system model

relative velocity conservation:

$$\frac{\partial}{\partial t}\left(u_{r,x}\right)+\frac{\partial}{\partial x}\left(u_x u_{r,x}+(1-2c)\frac{u_{r,x}^2}{2}+\psi(P)\right)+$$

$$\frac{\partial}{\partial y}\left(u_x u_{r,y}+u_y u_{r,x}+(1-2c)u_{r,x}u_{r,y}\right)=S_4$$

$$\frac{\partial}{\partial t}\left(u_{r,y}\right)+\frac{\partial}{\partial y}\left(u_y u_{r,y}+(1-2c)\frac{u_{r,y}^2}{2}+\psi(P)\right)+$$

$$\frac{\partial}{\partial x}\left(u_x u_{r,y}+u_y u_{r,x}+(1-2c)u_{r,x}u_{r,y}\right)=S_5$$

Courant-Friedrichs-Lewy number

With this model the CFL number has to be < 1

- easy in 1D and structured meshes
- difficult in 2D and unstructured meshes
- Δt can be calculated from the eigenvalues of the Jacobien matrices
- 2 Solutions in 2D: Splitting and non-splitting scheme

Implementation of the splitting scheme

Rewrite:

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=\frac{\partial U}{\partial t}+J_x(U)\frac{\partial U}{\partial x}+J_y(U)\frac{\partial U}{\partial y}=S(U), \quad (1)$$

with the fluxes $F(U)$ and $G(U)$ and the Jacobien $J_x = \frac{\partial F}{\partial U}$ and $J_y = \frac{\partial G}{\partial U}$.

In the splitting scheme the fields are updated first in one direction with Lax-Friedrichs scheme according to

$$F^{LF}_{i+1/2} = \frac{1}{2}\left(F^n_i + F^n_{i+1}\right) + \frac{\Delta x}{2\Delta t}\left(U^n_i - U^n_{i+1}\right). \quad (2)$$

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Implementation of the splitting scheme

This is followed by the update step

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}\left(F_{i-1/2}^{n,LF} - F_{i+1/2}^{n,LF}\right), \tag{3}$$

and with the new values in the second direction. In such case, the scheme converges for

$$\Delta t = C_{cfl} \cdot \min\left(\frac{\Delta x}{\lambda_x}, \frac{\Delta y}{\lambda_y}\right), \tag{4}$$

with $\lambda_{x/y}$ being the largest eigenvalue of J_x and J_y respectively and $C_{cfl} \leq 1$.

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Implementation of the non-splitting scheme

In the non-splitting scheme the two-dimensional Lax-Friedrichs flux is updated in one step using

$$\Delta t = \frac{C_{cfl}}{\frac{\lambda_{max;x}}{\Delta x} + \frac{\lambda_{max;y}}{\Delta y}}. \tag{5}$$

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We compared our results to OpenFOAM

- OpenFOAM solves the standard mixture model
- no information about the relative velocity
- Kurganov and Tadmor central-upwind scheme
- CFL number is calculated in a different way

Results

Our test case is a rarefaction wave: Initial conditions

- left side velocity is negative
- right side velocity is positive
- density is uniform

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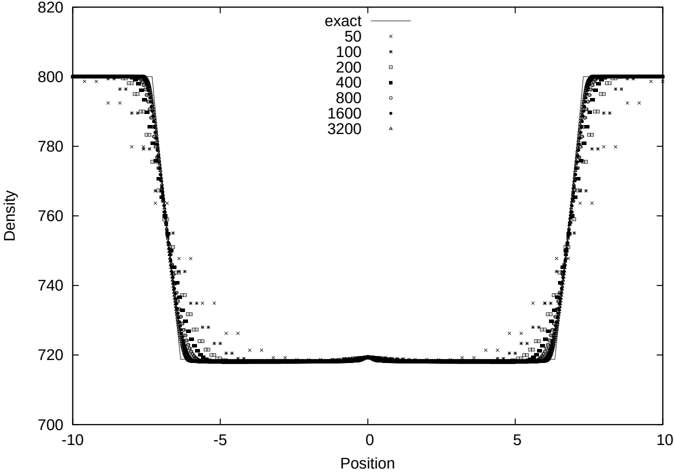


Figure : Rarefaction-waves propagation within an air-water mixture: mixture density for different lattice sizes with $C_{cfl} = 0.9$, compared to the exact results in one-dimension.

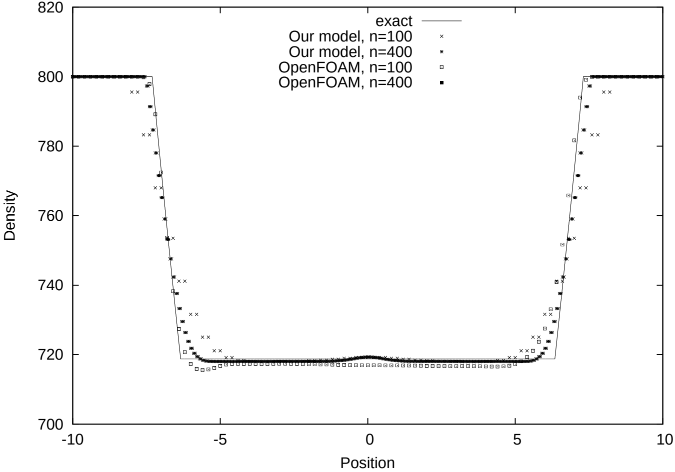
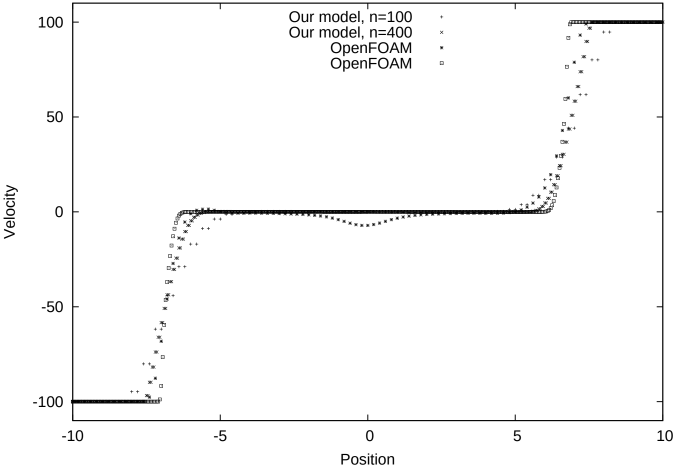


Figure : Rarefaction-waves propagation within an air-water mixture: mixture density profile computed with the mixture model of OpenFOAM and with the current model.



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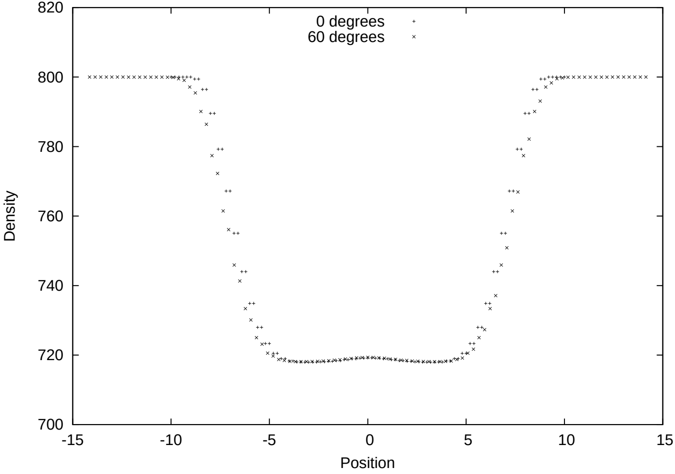

Figure : Rarefaction-waves propagation within an air-water mixture: mixture density profile for a rarefaction wave going in x-direction and with an angle of 60 degrees between the wave and the x-direction.

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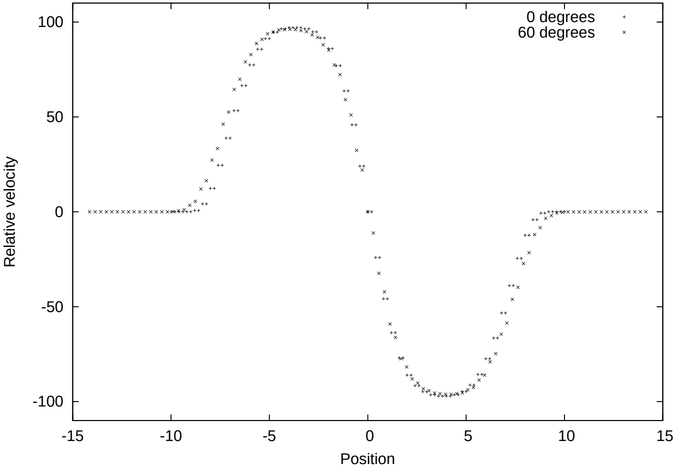

Figure : Rarefaction-waves propagation within an air-water mixture: relative velocity profile for a rarefaction wave going in x-direction and with an angle of 60 degrees between the wave and the x-direction.

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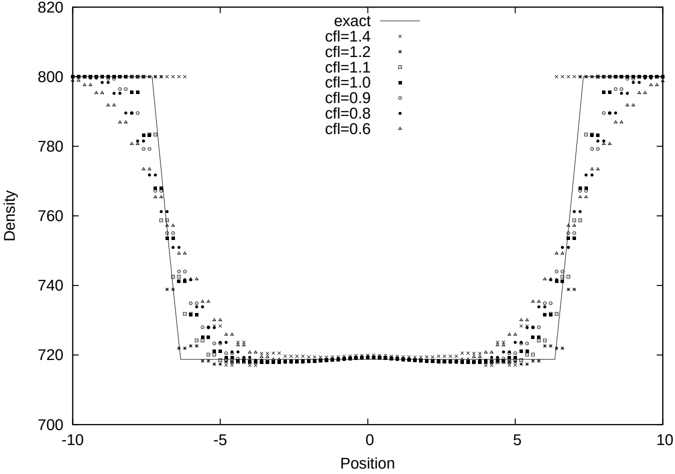

Figure : Rarefaction-waves propagation within an air-water mixture: mixture density profile for a rarefaction wave going in x-direction with different CFL numbers

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Conclusions and current work

- We get good agreement between OpenFOAM and our model
- Rarefactions can be reproduced
- Our model is very sensitive to different CFL numbers within the non-splitting scheme
- Use different flux schemes
- extend our model to unstructured meshes
- implement it in OpenFOAM