Keep it 2010 - Greec model

$$\frac{\partial}{\partial t}\underbrace{(\alpha\rho_2 S_2)}_{1} + \frac{\partial}{\partial x}\underbrace{(\alpha\rho_2 u_2 S_2)}_{2} = S$$

gas - Solid

$\alpha\rho_2 = c\rho$, $S_2 c = S\chi$, $u_2 = u + (1-c)u_r$

$E = e + c(1-c)\frac{u_r^2}{2}$

$E_{u_r} = c(1-c)u_r$

1

$$\alpha\rho_2 S_2 = \cancel{c}\rho\,\frac{S\chi}{\cancel{c}} = \rho\chi S$$

2

$$\alpha\rho_2 S_2 u_2 = \cancel{c}\rho\cdot\frac{S\chi}{\cancel{c}}u_2$$

$$= \rho\chi S \overset{= u+(1-c)u_r}{u_2} = \rho\chi S u + \rho\chi S(1-c)u_r$$

$$\frac{\partial}{\partial t}(\rho\chi S) + \frac{\partial}{\partial x}(\rho\chi u S) + \frac{\partial}{\partial x}(\rho\chi S(1-c)u_r) = 0$$

$$\boxed{\frac{\partial}{\partial t}(\rho\chi) + \frac{\partial}{\partial x}(\rho\chi u) + \frac{\partial}{\partial x}\underbrace{(\rho\chi(1-c)u_r)} = -\frac{\rho\omega}{S}}$$

$$\frac{\partial}{\partial t}(\rho\chi) + \frac{\partial}{\partial x}(\rho\chi u) = -\frac{\rho}{S}\omega = \overbrace{-\frac{\rho}{S}\cdot\frac{1}{\tau(\chi)}\cdot e_\chi}^{E_{u_r}}$$

Continuity Eqn

$$\frac{\partial}{\partial t}(\alpha_2 \rho_2) + \frac{\partial}{\partial x}(\alpha_2 \rho_2 u_2) = 0 .$$

$$c_2 = \frac{\alpha_2 \rho_2}{\rho} = c \, , \quad u_2 = \underbrace{u + \underbrace{(1-c) u_r}} \; ; E = e + c(1-c)\frac{u_r^2}{2}$$

$$\Rightarrow \quad \frac{\partial}{\partial t}(c\rho) + \frac{\partial}{\partial x}\left(c\rho \left[\boxed{u} + (1-c) u_r \right] \right) = \Phi \qquad \text{"} \Phi = -\frac{\rho e_c}{\tau c}$$

$$\frac{\partial}{\partial t}(c\rho) + \frac{\partial}{\partial x}\left(\rho c u + \rho \underbrace{c(1-c) u_r}_{E_{ur}} \right) = 0$$

$$\frac{\partial}{\partial t}(c\rho) + \frac{\partial}{\partial x}\left(\rho c u + \rho E_{u_r} \right).$$

$$A k = \lambda k \qquad ; \boxed{\lambda = u}$$

Published Paper
AMC

$$\begin{bmatrix} u & 0 & \rho & 0 & 0 & 0 \\ 0 & u & 0 & 0 & 0 & 0 \\ \frac{a^2}{\rho} & P_\alpha/\rho & u & P_c/\rho & P_\varphi/\rho & P_s/\rho \\ 0 & 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 & u \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} = \begin{bmatrix} u k_1 \\ u k_2 \\ u k_3 \\ u k_4 \\ u k_5 \\ u k_6 \end{bmatrix}$$

eigenvectors
2010

$$\cancel{u k_1} + \rho k_3 = \cancel{u k_1} \qquad , k_3 = 0.$$

$$u k_2 = u k_2$$

$$\frac{a^2}{\rho} k_1 + \frac{P_\alpha}{\rho} \cancel{k_2} + \cancel{u k_3} + \frac{P_c}{\rho} \cancel{k_4} + \frac{P_\varphi}{\rho} \cancel{k_5} + \frac{P_s}{\rho} \cancel{k_6} = \cancel{u k_3}$$

$$u k_4 = u k_4 \qquad \boxed{a^2 k_1} + P_\alpha k_2 + P_c k_4 + P_\varphi k_5 + \boxed{P_s k_6} = 0.$$

$$u k_5 = u k_5$$

$$u k_6 = u k_6$$

$$\tilde{K}^{3,4}_1 = \left[-\frac{\partial P}{\partial s}, 0, 0, 0, 0, a^2 \right]$$

$(A - \lambda I)k = 0$

$\boxed{\lambda = u - a}$

$a = u - \lambda$

as

$\lambda = u + a$

$$\begin{bmatrix} u-\lambda & 0 & \rho & 0 & 0 & 0 \\ 0 & u-\lambda & 0 & 0 & 0 & 0 \\ a^2/\rho & P_\alpha/\rho & u-\lambda & P_c/\rho & P_x/\rho & P_s/\rho \\ 0 & 0 & 0 & u-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & u-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & u-\lambda \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} = 0$$

$a k_1 + \rho k_3 = 0$ $\cancel{\#}$

$a k_2 = 0$

$\frac{a^2}{\rho} k_1 + \frac{P_\alpha}{\rho} k_2 + a k_3 + \frac{P_c}{\rho} k_4 + \frac{P_x}{\rho} k_5 + \frac{P_s}{\rho} k_6 \cancel{+ a k_1} = 0$

$a k_4 = 0$

$a k_5 = 0$

$a k_6 = 0$

$k_1 = 1, \Rightarrow k_3 = \frac{-a}{\rho}, \; k_2 = 0, k_4 = 0, k_5 = 0, k_6 = 0$

$k^{(1)} = \underbrace{\left[1, 0, -\frac{a}{\rho}, 0, 0, 0 \right]}$

β

Same as pg 94 Book. Since am using entropy NOT energy.

$$U = [\rho, \rho\alpha, \rho u, \rho c, \rho \chi, \rho E]$$

$$W = [\rho, \alpha, u, c, \chi, S]$$

$$\lambda = u.$$

$$\frac{\partial \lambda}{\partial \rho} = 0 = \frac{\partial \lambda}{\partial \alpha} = \frac{\partial \lambda}{\partial c} = \frac{\partial \lambda}{\partial \chi} = \frac{\partial \lambda}{\partial S}.$$

$$\frac{\partial \lambda}{\partial u} = 1.$$

$$[0, 0, 1, 0, 0, 0] \begin{bmatrix} -\frac{\partial \rho}{\partial s} \\ 0 \\ 0 \\ 0 \\ 0 \\ a^2 \end{bmatrix} = 0$$

Linearly degenerate.

$\lambda = u + a \qquad W = [\rho, \alpha, u, c, \chi, s]$.

$u - \lambda = a$

$\frac{\partial \lambda}{\partial \rho} = \frac{\partial a}{\partial \rho}$;

$\frac{\partial \lambda}{\partial \alpha} = \frac{\partial a}{\partial \alpha}$;

$\frac{\partial \lambda}{\partial u} = 1$.

$\frac{\partial \lambda}{\partial c} = \frac{\partial a}{\partial c}$

$\frac{\partial \lambda}{\partial \chi} = \frac{\partial a}{\partial \chi}$

$\frac{\partial \lambda}{\partial s} = \frac{\partial a}{\partial s}$

$$[a_\rho, a_\alpha, 1, a_c, a_\chi, a_s] \begin{bmatrix} 1 \\ 0 \\ -\frac{a}{\rho} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{\partial a}{\partial \rho} - \frac{a}{\rho}$$

$\neq 0$

Genuinely non-Lin

Source Terms.

[1] $\phi = \dfrac{-\rho}{\tau^{p}} e_{\alpha} = \dfrac{p_1 - p_2}{\tau}$

$$e_{\alpha} = -\frac{p_1 - p_2}{\rho} \Rightarrow \phi = \frac{-\rho}{\tau^{p}} \cdot -\frac{p_1 - p_2}{\rho} = \frac{p_1 - p_2}{\tau^{p}}$$

[2]

$$\psi = \frac{-\rho}{\tau^{c}} e_c \qquad e_c = e_1 + \frac{p_1}{\rho_1} - S_1 T_1 - e_2 - \frac{p_2}{\rho_2} + S_2 T_2 .$$

$$e_c = \left[e_1 + \underline{\frac{p_1}{\rho_1}} - S_1 T_1 \right] - \left[e_2 + \underline{\frac{p_2}{\rho_2}} \overset{\text{-ve.}}{-} S_2 T_2 \right]$$

[3]

$$\omega = \frac{-\rho}{\tau^{x}} e_x$$

$$e_x = s \left(T_1 - T_2 \right)$$

$$e = c\,e_1 + (1-c)\,e_2 \qquad c = \frac{\alpha \rho_1}{\rho} \;;\; (1-c) = \frac{(1-\alpha)\rho_2}{\rho}$$

$$e = \frac{s_0}{\gamma - 1}\,\frac{c}{\rho_0^{\gamma}}\,\exp\left(\frac{s}{cv_1}\right)\rho^{\gamma-1} \;+$$

$$(1-c)\,\frac{c_0^2}{2\alpha_0^2}\left[\left(\frac{\rho}{\rho_0}\right)^{\alpha_1} - 1\right]^2 - \frac{p_0(1-c)}{\rho} + \frac{cv_2\,T_0(1-c)}{\rho_0^{\beta}}\left[\exp\left(\frac{s}{cv_2}\right) - 1\right]\rho^{\beta}$$

$$+\; e_0(1-c)$$

$$e = \frac{s_0}{(\gamma-1)\rho_0^{\gamma}}\,\frac{\alpha\rho_1}{\rho}\,\exp\left(\frac{s}{cv_1}\right)\rho^{\gamma-1} \;+$$

$$\frac{(1-\alpha)\rho_2}{\rho}\,\frac{c_0^2}{2\alpha_0^2}\left[\left(\frac{\rho}{\rho_0}\right)^{\alpha_0} - 1\right] + \frac{p_0}{\rho}\,\frac{(1-\alpha)\rho_2}{\rho} + \frac{cv_2 T_0}{\rho_0^{\beta}}\,\frac{(1-\alpha)\rho_2}{\rho}\left[\exp\left(\frac{s}{cv_2}\right) - 1\right]$$

$$\rho^{\beta} + e_0\,\frac{(1-\alpha)\rho_2}{\rho}$$

$$e_\alpha = \frac{S_0}{(\gamma-1)\rho_0^\gamma} \, 1 \cdot \rho_1 \exp\left(\frac{s}{c_{v_1}}\right) \rho^{\boxed{\gamma-2}}$$

$$-\rho_2 \frac{c_0^2}{2\alpha_0^2}\left[\left(\frac{\rho}{\rho_0}\right)^{\alpha_0} - 1\right] - \frac{P_0\rho_2}{\rho^2} - \frac{c_{v_2} T_0 \rho_2}{\rho_0^\beta}\left[\exp\left(\frac{s}{c_{v_2}}\right)-1\right]\rho^{\beta-1}$$

$$-\frac{e_0\rho_2}{\rho}$$

also: $e_\alpha = \dfrac{P_2-P_1}{\rho} = \dfrac{\rho_2^2}{\rho}\dfrac{\partial e_2}{\partial \rho_2} - \dfrac{\rho_1^2}{\rho}\dfrac{\partial e_1}{\partial\rho_1}$

$u_2 - u_1 = u_r$ $\quad c = \frac{\alpha_2 \rho_2}{\rho}$

$u_r \neq 0$

Keep it

$u_r \cdot u_r$

$\vec{U}_r = \vec{V}_1 - \vec{V}_1$

- equations of mass, momentum and energy for a mixture of gas-liquid

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + \boxed{P + \rho c(1-c)u_r^2}\right) + \frac{\partial}{\partial y}\left(\rho uv\right) + \frac{\partial}{\partial z}\left(\rho uw\right) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}\left(\rho uv\right) + \frac{\partial}{\partial y}\left(\rho v^2 + \left[P + \rho c(1-c)u_r^2\right]\right) + \frac{\partial}{\partial z}\left(\rho vw\right) = 0$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}\left(\rho uw\right) + \frac{\partial}{\partial y}\left(\rho vw\right) + \frac{\partial}{\partial z}\left(\rho w^2 + P + \rho c(1-c)u_r^2\right) = 0$$

$\vec{V}_1 \vec{V}$ $\quad (V_1 - V_1) \cdot U_r$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho E\right) + \frac{\partial}{\partial x}\left(\rho(E+P)u + \rho c(1-c)u_r\left(uu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &+ \\ \frac{\partial}{\partial y}\left(\rho(E+P)v + \rho c(1-c)u_r\left(vu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &+ \\ \frac{\partial}{\partial z}\left(\rho(E+P)w + \rho c(1-c)u_r\left(wu_r + (1-2c)\frac{u_r^2}{2}\right) + \frac{\partial e}{\partial c}\right) &= 0\end{aligned}$$

$\frac{\rho_1}{\rho}$

$E + \frac{1}{2}(V_r V_r)$ $\quad \alpha_1 \rho$ $\quad \alpha_2 \rho$ $\quad \rho$

$\rho u w E$

$$\frac{\partial \alpha_1 \rho_1}{\partial t} + \nabla \cdot \rho \alpha_1 V_1 = 0$$

$$\frac{\partial \alpha_2 \rho_2}{\partial t} + \nabla \cdot \rho \alpha_2 V_2 = 0$$

$$\left(\frac{\rho_1 V_1 V_1}{\rho \; 2} + \frac{\rho_2 V_2 V_2}{\rho}\right) - V_1 V_1$$

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \left(\alpha_1 \rho_1 V_1 + \alpha_2 \rho_2 V_2\right)$$

$$+ \nabla \cdot \bar{\rho} \vec{V}$$

$$\bar{\rho} = \alpha_1 \rho_1 + \alpha_2 \rho_1 \qquad \bar{V} = \frac{\alpha_1 \rho_1 V_1 + \alpha_2 \rho_2 V_2}{\bar{\rho}}$$

~~$V = \alpha_1 V_1 + \alpha_2 V_2$~~

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0.$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + P) + \frac{\partial}{\partial y}(\rho u v) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v^2 + P) = 0.$$

$$\frac{\partial}{\partial t}(\rho s) + \frac{\partial}{\partial x}(\rho u s) + \frac{\partial}{\partial y}(\rho v s) = 0.$$

$$\frac{\partial}{\partial t}(\rho \alpha) + \frac{\partial}{\partial x}(\rho u \alpha) + \frac{\partial}{\partial y}(\rho v \alpha) = 0$$

$$\frac{\partial}{\partial t}(\rho c) + \frac{\partial}{\partial x}(\rho u c) + \frac{\partial}{\partial y}(\rho v c) = 0.$$

$$\frac{\partial}{\partial t}(\rho X) + \frac{\partial}{\partial x}(\rho u X) + \frac{\partial}{\partial y}(\rho v X) = 0$$

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ — direction indi

1

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

2

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + \rho c(1-c)u_r^2 + P\right) + \frac{\partial}{\partial y}(\rho u v) = 0$$

3

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}\left(\rho v^2 + \rho c(1-c)u_r^2 + P\right) = 0$$

4

$u u_r \quad v u_r$

$$\frac{\partial}{\partial t}\left(u_r^x\right) + \frac{\partial}{\partial x}\left[\left(u u_r^x + \cancel{v u_r^y}\right) + \boxed{\frac{1-2c}{2}u_r^2 + e_c}^{\,E_c}\right]$$

$$+ \frac{\partial}{\partial y}\left[\left(v u_r^x + \cancel{v u_r^y}\right) + \cancel{\frac{1-2c}{2}u_r^2} + \boxed{e_c}^{\,E_c}\right] = 0$$

5

$$\frac{\partial}{\partial t}\left(u_r^y\right) + \frac{\partial}{\partial x}\left[\left(u v_r^x + \cancel{u u_r^y}\right) + \cancel{\frac{1-2c}{2}u_r^2} + \overset{E_c}{e_c}\right)\Big]$$

$$+ \frac{\partial}{\partial y}\left[\left(v v_r^x + \cancel{v u_r^y}\right) + \frac{1-2c}{2}u_r^2 + \overset{E_c}{e_c}\right] = 0$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x}\left[\rho u(E+P) + \rho u\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c\cdot(1-c)u_{r_x}\right]$$

$$+ \frac{\partial}{\partial y}\left[\rho v(E+P) + \rho v\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c(1-c)u_{r_y}\right]$$

$$+ \frac{\partial}{\partial z}\left[\rho w(E+P) + \rho\, w\, c(1-c)u_r^2 + \rho\left((1-2c)u_r^2 + e_c\right)c(1-c)u_{r_z}\right] = 0$$

$$\vec{V}_r = \vec{V}_1 - \vec{V}_a$$
$$= (u_1 - u_a)i$$
$$+ (v_1 - v_a)j$$
$$+ (w_1 - w_a)k$$

(1)

$$\lambda = u + a_1 + y(a_1).$$

$$a_m = \left[\frac{1}{\sqrt{2}}\left(\right.\right.$$

1

$$a_1 = \frac{1}{\sqrt{2}}\left[-\varsigma_2^0 + \sqrt{(\varsigma_2^0)^2 - 4\varsigma_0^0}\right]^{1/2}.$$

$$\varsigma_0^0 = c(1-c)(P_p\, e_{cc} - P_c\, e_{cp}).$$

$$\varsigma_2^0 = -(P_p + c(1-c)\, e_{cc})$$

2

$$y(a_1) = -\frac{-\varsigma_1^1 + \varsigma_3^1 a_1^2}{2(\varsigma_2^0 + 2a_1^2)}$$

δ_{max}

$\varsigma_0^0 = sig1$

$\varsigma_1^0 = sig2$

$\varsigma_1^1 = sig3$

$\varsigma_3^1 = sig4$

2006-RAV.

$\cancel{\lambda = \Delta a_m + b a_m}$

(1)

$$\xi^1_1 = 2\left(2\, p_c\,(1-c)\, \underset{\sim}{\dot{e}_{cp}} - (1-2c)\dot{p}_p\right) u_r$$

added in 2009 after italy check -

$$\xi^1_3 = 2(1-2c)u_r$$

$$\cancel{\xi^0_2 = -\left(\dot{p}_p + c(1-c)\underset{\sim}{\dot{e}_{cc}}\right).}$$

p_p

p_c

e_{cc}

e_{cp}