Final Exam

BLM 1541: Probability and Statistics — Fall 2016

Print family (or last) name: —————————————————————————————

Print given (or first) name: ——————————————————————————————

Print student number: ———————————————————— Group: ——— Order: ———

I have read and understand all of the instructions below, and I will obey the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.

- There are 8 questions. *You have to solve 2 mandatory questions. You should choose 3 out of 5 optional questions.* If you solve more than 3 optional questions, the first 3 optional questions you solved will be graded.

- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.

- This exam will be 1 hour and 25 minutes in length.

- This is a closed-book and closed-note exam. Electronic devices (e.g., cellphone, smart watch) are not allowed. Single page A4 note sheet is allowed. *Don't forget to attach your A4 note sheet into your solution; otherwise your solution will not be graded.*

- For all problems, follow these instructions:

  o Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.

  o PMF stands for probability mass function; CDF stands for cumulative distribution function; $var(X)$ stands for the variance of the random variable X; $cov(X, Y)$ stands for the covariance between the random variables X and Y.

  o For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1 Mandatory	2 Mandatory	3 Optional	4 Optional	5 Optional	6 Optional	7 Optional	8 Optional	Total
	20	20	20	20	20	20	20	20	100
Points									

1. **[20 points][Mandatory]** Experiment A has three mutually exclusive possible outcomes $\{A_1, A_2, A_3\}$ and experiment B has two mutually exclusive possible outcomes $\{B_1, B_2\}$. The joint probabilities are

$$P(A_1 B_1) = 0.2 \qquad P(A_1 B_2) = 0.1$$
$$P(A_2 B_1) = 0.1 \qquad P(A_2 B_2) = 0.2$$
$$P(A_3 B_1) = 0.1 \qquad P(A_3 B_2) = 0.3$$

Based on these joint probabilities, answer the following questions.

a) Determine the probability $P(A_2)$.

b) Determine the probability $P(B_2)$.

c) Determine the conditional probability $P(A_2|B_2)$.

d) Determine the conditional probability $P(B_2|A_2)$.

2. **[20 points][Mandatory]** A set of 3 observations $\{0.4, 0.7, 0.9\}$ is collected from a continuous random variable whose distribution follows thee PDF

$$f_X(x) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Estimate the parameter θ using "method of moments" technique.

Hint: Find the estimator of θ, then use the observation samples.

b) Estimate the parameter θ using "method of maximum likelihood" technique.

Hint: Find the estimator of θ, then use the observation samples.

3. **[20 points][Optional]** Answer the following questions for the data:

$$22, 18, 17, 18, 15$$

a) Compute the sample mean, variance, and standard deviation.

b) Estimate the standard error of the sample mean

d) Compute first quartile, median, and third quartile

e) Compute the interquartile range IQR. Are there any outliers? If so what are they?

4. **[20 points][Optional]** If A, B are independent events with $P(A) \neq 0$ and $P(B) \neq 1$, is it possible to have $P(A \cap B) = P(A)$? Justify your answer.

5. **[20 points][Optional]** Let X and Y are two continuous random variables jointly distributed with density

$$f_{XY}(x,y) = \begin{cases} kx + y, & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Answer the following questions.

a) Find the value of k if $f_{XY}(x,y)$ is a joint probability density function.

a) Find the probability density function of X, $f_X(x) =$?

b) Find the conditional probability density function of Y given X, $f_{Y|X}(y|x) =$?

6. **[20 points][Optional]** An urn contains red, black, and white balls. The proportion of red is 0.4, the proportion of black is 0.4, and the proportion of white is 0.2. If 5 balls are drawn with replacement, what is the probability of 2 red, 2 black, and 1 white in any order?

7. **[20 points][Optional]** Let's given any random variables X, Y, Z, and any non-random numbers a, b, c, d, e. Find the covariance $Cov(aX + bY + c, dZ + e)$ in terms of $Cov(X, Z)$ and $Cov(Y, Z)$.

8. **[20 points][Optional]** A manufacturing plant makes radios that each contain an integrated circuit (IC) supplied by three sources A, B, and C. The probability that the IC in a radio came from one of the sources is the same for all sources. ICs are known to be defective with probabilities 0.001, 0.003, and 0.002 for sources A, B, and C, respectively.

a) What is the probability any given radio will contain a defective IC?

b) If a radio contains a defective IC, find the probability that it came from source A.