YILDIZ TECHNICAL UNIVERSITY

Name, Surname :
Number :
Course Code : BLM1612
Course Name : CIRCUIT THEORY
Exam : ☐ Quiz ☐ Midterm ⊗ Final
Date : 12.02.2017

Please make sure to write your name and student number on each paper that you have used

Question Number	1	2	3	4	5	6	7	Total
Mark								

Note: Exam duration is 120 minutes only and max 2 papers can be used.

QUESTIONS

1. For the network of Fig. 1, find the total impedance $\mathbf{Z_T}$.

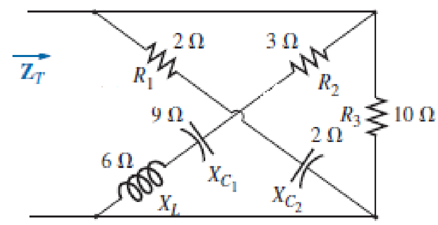


Fig. 1

2. Find the average power delivered to R_4 in Fig. 2.

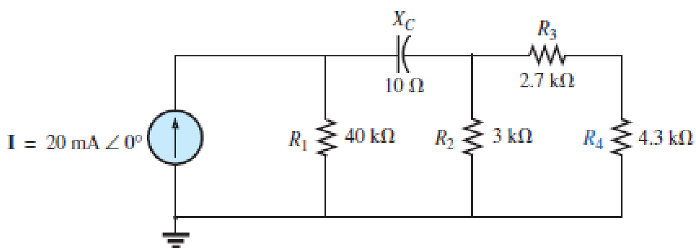


Fig. 2

3. For the network in Fig. 3, find the currents $\mathbf{I_1}$ and $\mathbf{V_1}$ in phasor form.

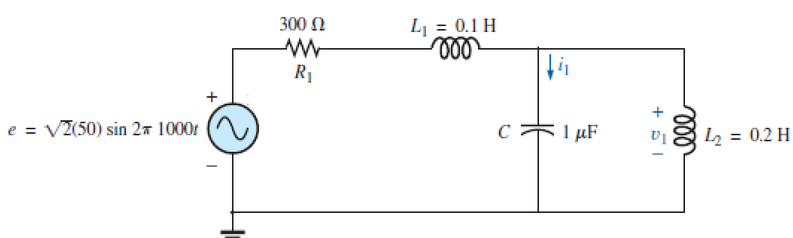


Fig. 3

4. Find the total inductance of the circuit of Fig. 4.

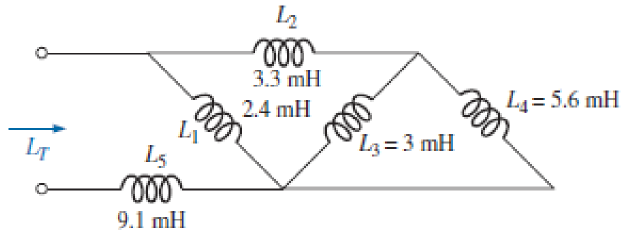


Fig. 4

5. For the circuit in Fig. 5; find the mathematical expression for the transient behavior of v_C, i_C, and v_R if the switch is closed at t = 0 s.

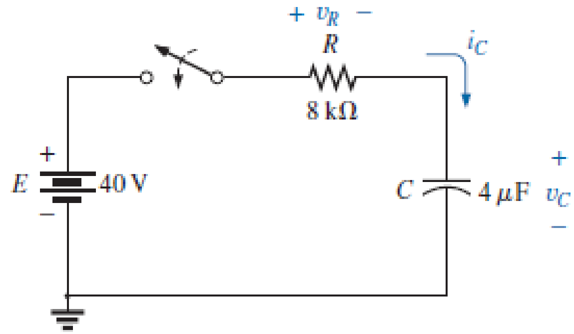


Fig. 5

6. Find the waveform for the average current if the voltage across the 2 μF capacitor is as shown in Fig. 6.

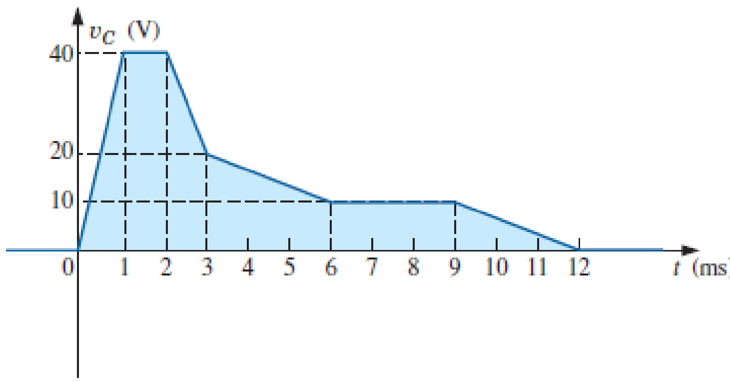


Fig. 6

7. For the multiple ladder configuration in Fig. 7, determine I.

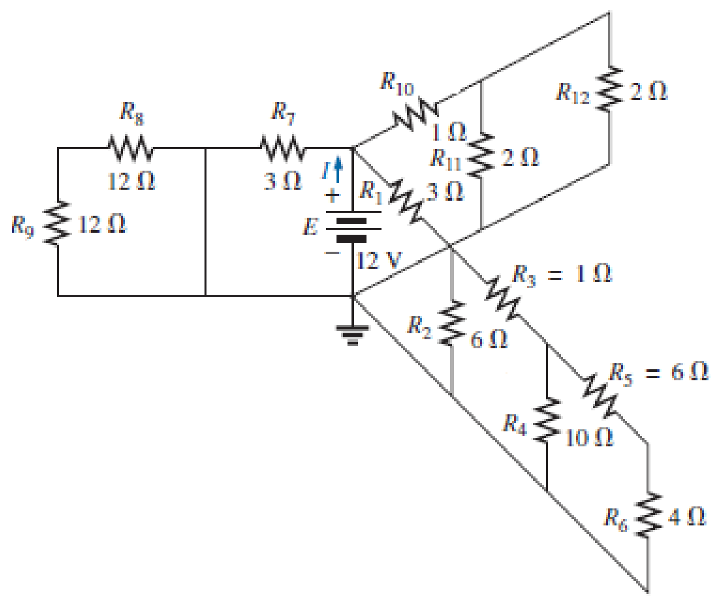
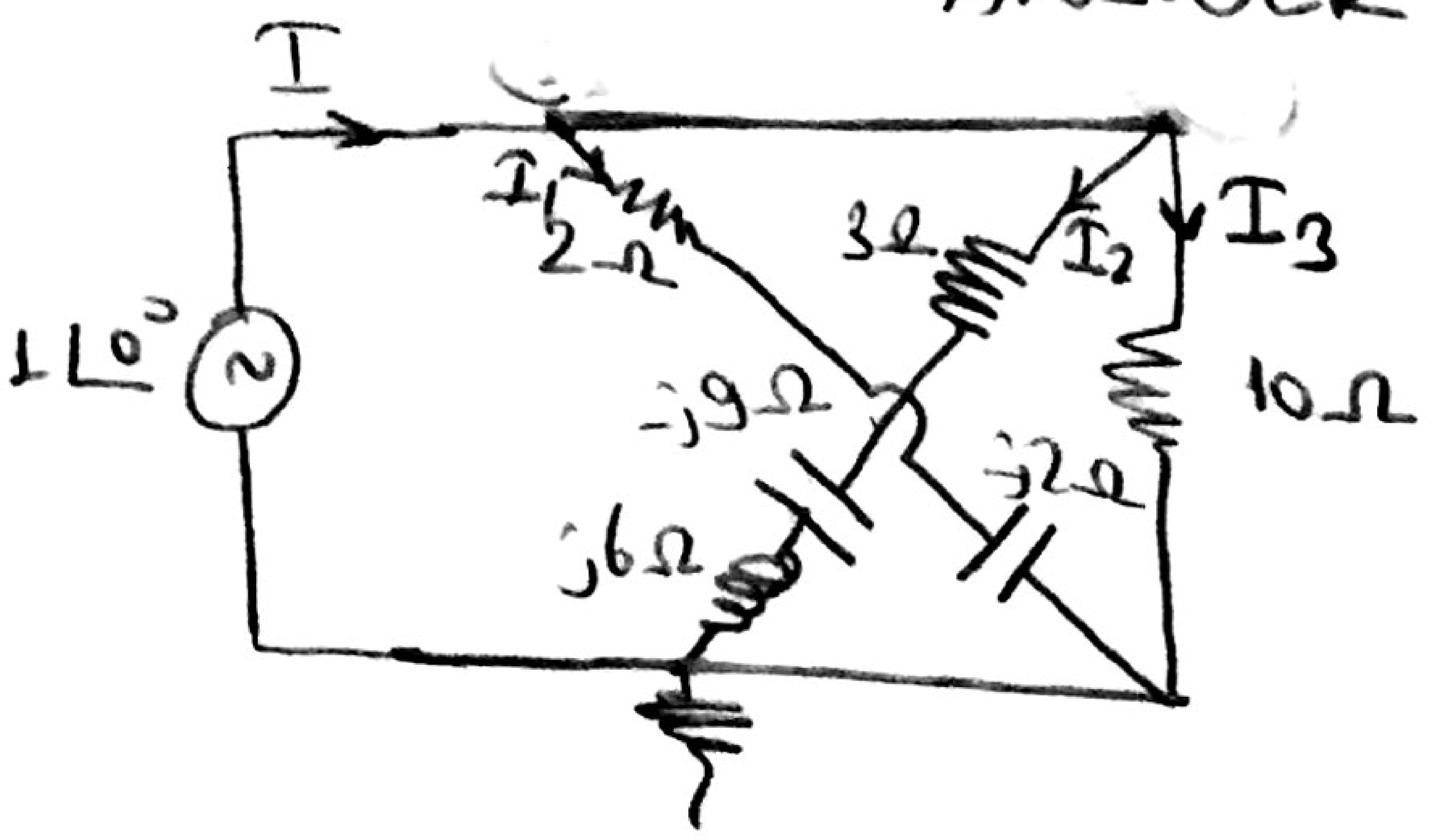


Fig. 7

ANSWER KEY

(1)



$$Z_T = \frac{V_s}{I}$$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{1\angle 0°}{2-j2} = \frac{1\angle 0°}{2\sqrt{2}\angle -45°}$$

$$I_1 = 354\angle 45° \text{ mA}$$

$$I_2 = \frac{1\angle 0°}{3+j(6-9)} = \frac{1\angle 0°}{3-j3} = \frac{1\angle 0°}{3\sqrt{2}\angle -45°} = 236\angle 45° \text{ mA}$$

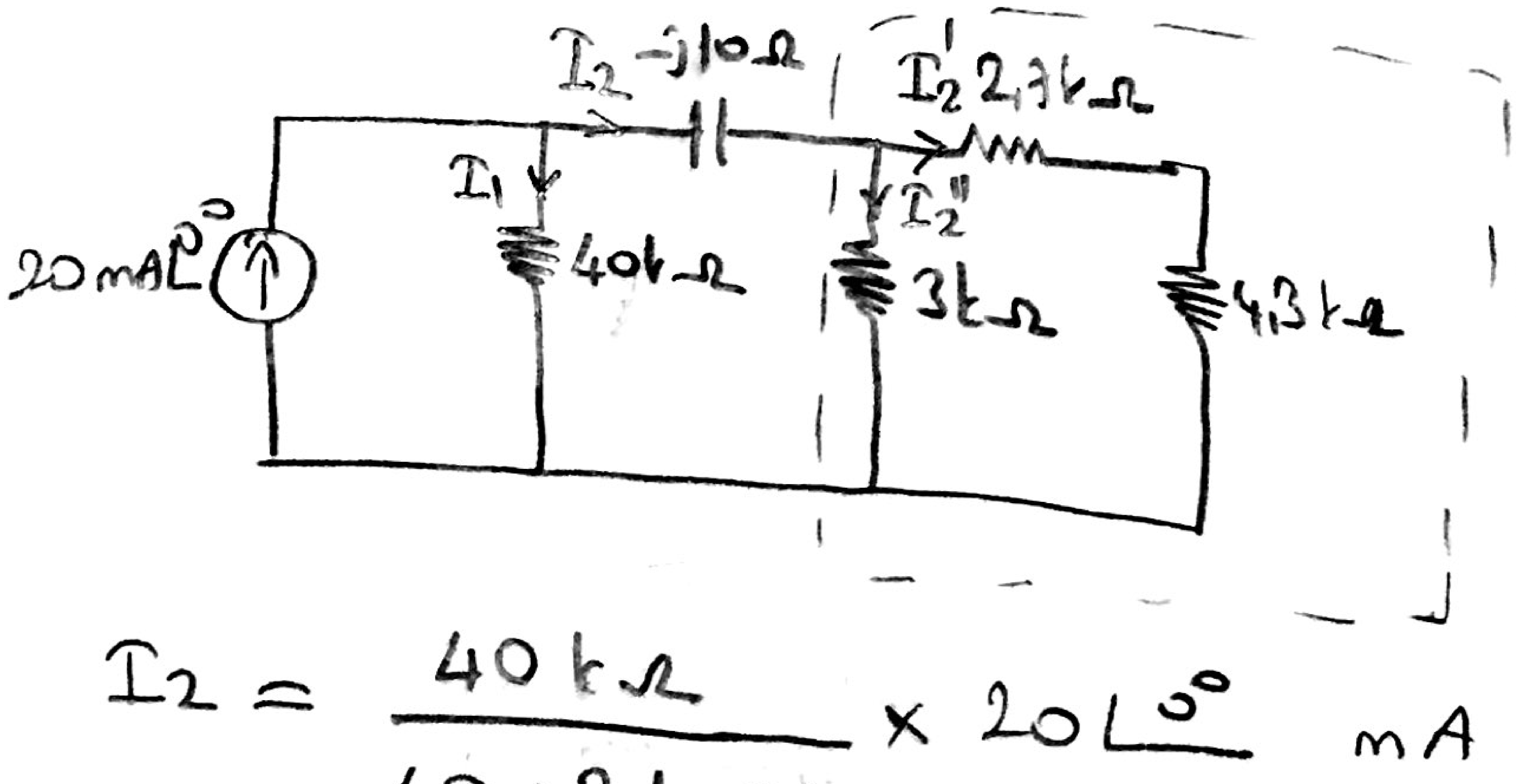
$$I_3 = \frac{1\angle 0°}{10} = 100\angle 0° \text{ mA.}$$

$$I = I_1 + I_2 + I_3 = (100+250+166) + j(250+166)$$

$$= 516 + j416$$

$$Z_T = \frac{V_s}{I} = \frac{1\angle 0°}{(516+j416)\times 10^{-3}} = 1{,}52\angle -38{,}89° \ \Omega$$

(2)



$$R_{eq} = 3k\Omega \parallel 7k\Omega$$
$$= 2{,}1\ k\Omega$$

$$I_2 = \frac{40\ k\Omega}{40+2{,}1-j10} \times 20\angle 0° \text{ mA}$$

$$= \frac{40.000}{40.002{,}1-j10} \cong 19, \text{ mA}$$

$$I_2' = \frac{3}{10} \times 19, \text{ mA} \cong 5{,}7 \text{ mA}$$

$$P \simeq R.(I_2')^2 = 4{,}3\times10^3 . (5{,}7\times10^{-3})^2 = 139{,}7 \text{ m Watt}$$

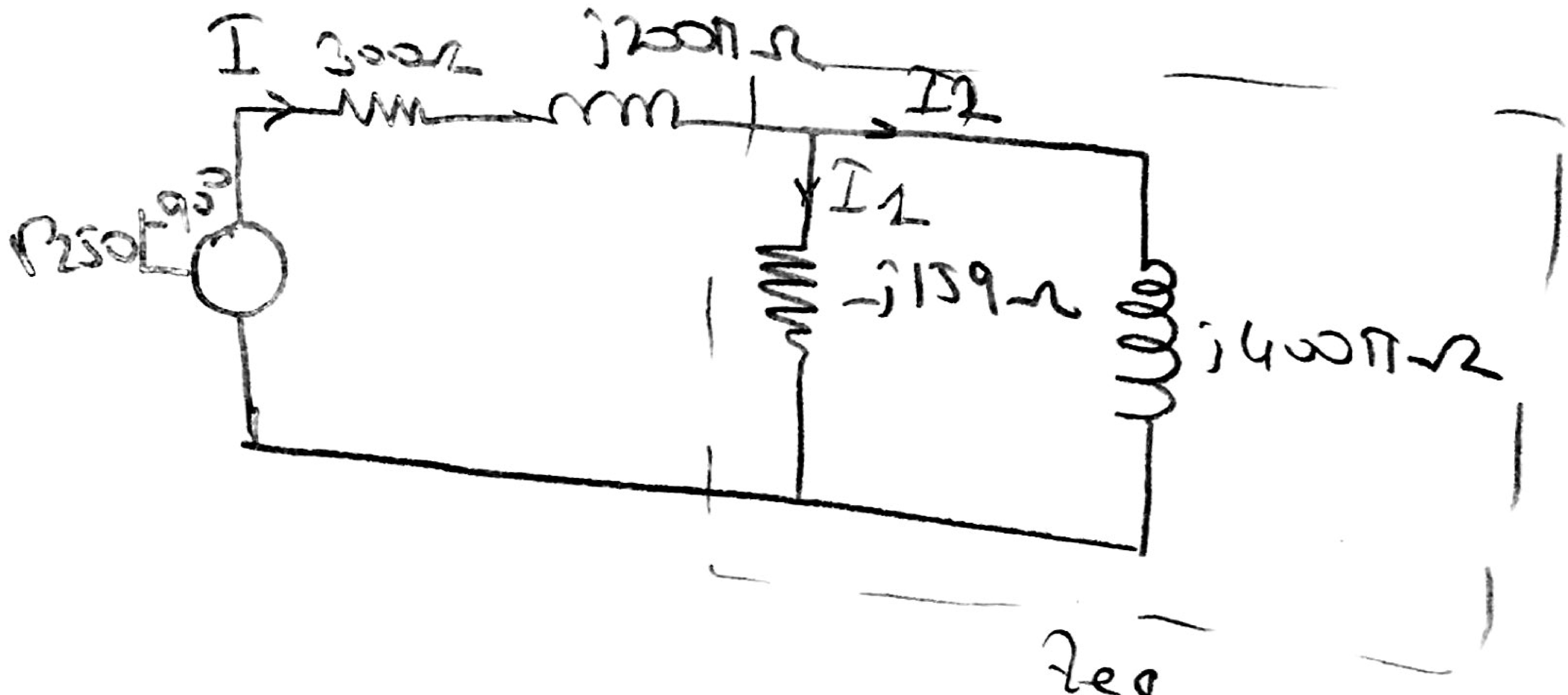
(3) $e = \sqrt{2}.50.\sin 2\pi 1000t = \sqrt{2}.50.\cos(2\pi 1000t - 90°)$

$$\vec{E} = \sqrt{2}.50 \angle -90°$$

$$jX_{L1} = 0{,}1\times 2\pi.1000 = j200\pi \ \Omega$$

$$-jX_C = \frac{-j}{2\pi.1000t.1\times10^{-6}} = -j159 \ \Omega$$

$$jX_{L2} = 0{,}2\times 2\pi.1000 = j400\pi \ \Omega$$



$$Z_{eq} = \frac{-j159\times j400\pi}{j400\pi - j159} = \frac{199{,}704}{-j1097}$$

$$Z_{eq} = j.182{,}04 \ \Omega$$

$$I = \frac{\sqrt{2}50\angle -90°}{300+j(200\pi+182{,}04)} = \frac{\sqrt{2}50\angle -90}{300+j(810{,}04)}$$

$$I = \frac{\sqrt{2}50\angle -90}{863{,}8\angle 69{,}67} \simeq 82 \angle -159{,}67° \text{ mA}$$

$$I_1 = \frac{j400\pi}{j400\pi - j159} \times I$$

$$= \frac{j400\pi}{j1097} \cdot 82 \angle -159{,}67 \sim 102{,}86 \angle -159{,}67 \text{ mA}$$

$$V_1 = j400\pi \times . \left(I - I_1 \right)$$

$$= j400\pi \times . \left(-20{,}85 \angle -159{,}67 \right) \times 10^{-3}$$

$$= 400\pi \angle 90^\circ \cdot 20{,}85 . \angle -159{,}67 - 90 \times$$

$$= 26{,}19 . \angle -159{,}67 \text{ V}$$

(4)

$$L_T = L_1 \parallel \left(L_2 + \left(L_3 \parallel L_4 \right) \right) + L_5$$

$$= 2{,}4 \parallel \left(3{,}3 + \frac{3 \times 5{,}6}{8{,}6} \right) + 9{,}1$$

$$= \frac{2{,}4 \times 5{,}25}{7{,}65} + 9{,}1 = \underline{\underline{10{,}75 \text{ mH}}}$$

(5) $V_c(t) = V_{cn}(t) + V_{cf}(t)$

$V_{cf}(t) = V_c(t \to \infty) = E$ since C behaves as if open circuit.

$V_{cn}(t) = A . e^{-t/\tau}$ $\qquad \tau = R . C = 8 \times 10^{3} \times 4 \times 10^{-6}$

$\tau = 32 \times 10^{-3}$

$V_{cn}(t) = A . e^{-t/32 \times 10^{-3}} = A . e^{-31{,}25 t}$

$$V_C(t) = E + A_1 e^{-31{,}25t}$$

At $t = 0$ $\quad V_C(0) = 0$ then

$$0 = E + A \Rightarrow A = -E$$

$$V_C(t) = E \cdot \left(1 - e^{-31{,}25t}\right) V = 40\left(1 - e^{-31{,}25t}\right)$$

$$I_C = \frac{40\, e^{-31{,}25t}}{8 \times 10^3} A = 5\, e^{-31{,}25t}\ mA$$

$$V_R = 40\, e^{-31{,}25t}\ V$$

(6) $\quad i_C = C \cdot \dfrac{dV_C}{dt} = C \cdot \text{slope} = C \cdot m.$

$0 < t < 1$ ms

$$m = \frac{40 - 0}{(1 - 0) \times 10^{-3}} = 40 \times 10^3$$

$$I_C = 2 \times 10^{-6} \times 40 \times 10^3 = 80\ mA$$

$1 < t < 2$ ms.

$$m = 0 \Rightarrow i_C = 0$$

$2 < t < 3$ ms.

$$m = \frac{20 - 40}{(3 - 2) \times 10^{-3}} = -20 \times 10^3$$

$$i_C = 2 \times 10^{-6} \times (-20 \times 10^3) = -40\ mA.$$

$3 < t < 6$ ms

$$m = \frac{10 - 20}{(6 - 3)\, 10^{-3}} = -\frac{10}{3} \times 10^3$$

$$i_C = 2 \times 10^{-6} \times \left(-\frac{10}{3} \times 10^3\right) = -6{,}67\ mA$$

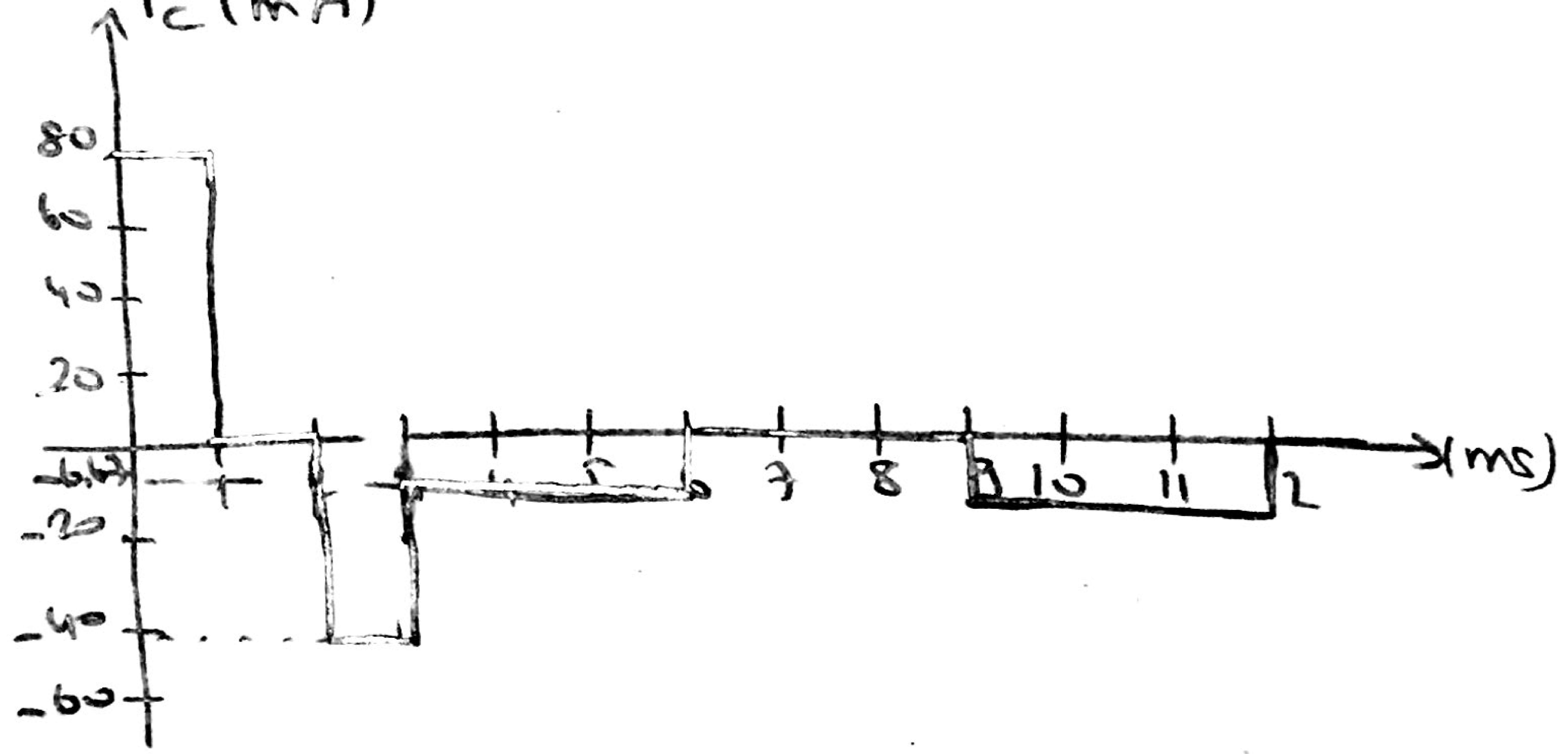
$6 < t < 9$ ms

$m = 0 \Rightarrow i_c = 0$

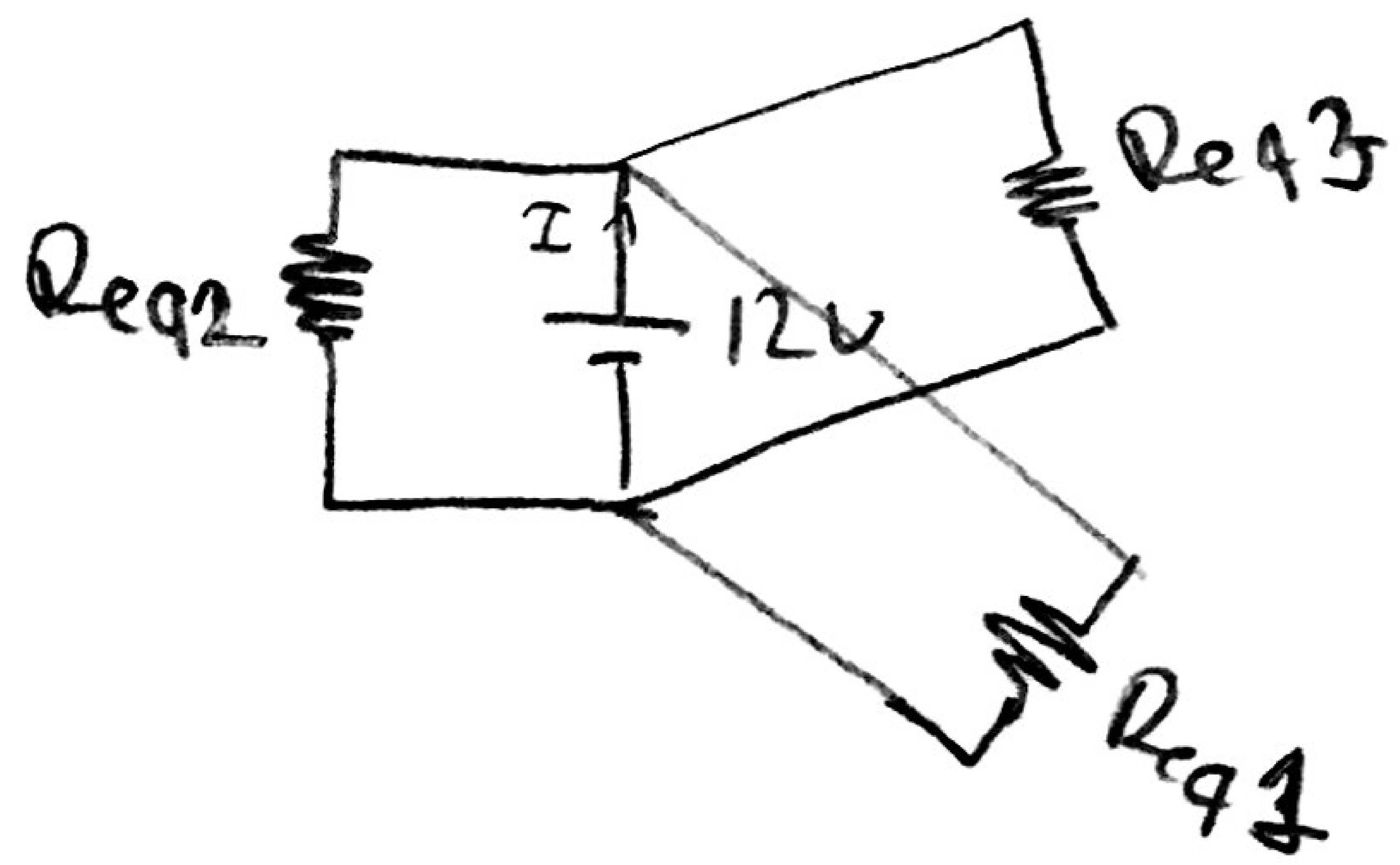
$9 < t < 12$ ms

$$m = \frac{0 - 10}{(12 - 9) \times 10^{-3}} = -\frac{10}{3} \times 10^{3}$$

$$i_c = 2 \times 10^{-6} \times \left(-\frac{10}{3} \times 10^{3}\right) = -6{,}67 \text{ mA}$$



(7)



$$R_{eq1} = R_1 + R_2 \| (R_3 + R_4 \| (R_5 + R_6)$$

$$= 3 + 6 \| (1 + 10 \| (6 + 4))$$

$$= 6\ \Omega$$

$$R_{eq2} = R_7 = 3\ \Omega$$

$$R_{eq3} = R_{10} + R_{11} \| R_{12} = 1 + 2 \| 2 = 2\ \Omega$$

$$\frac{1}{Req} = \frac{1}{Req_1} + \frac{1}{Req_2} + \frac{1}{Req_3} = \frac{1}{6} + \frac{1}{\underset{(2)}{3}} + \frac{1}{\underset{(3)}{2}}$$

$$Req = 1\ \Omega$$

$$I = \frac{E}{Req} = \frac{12}{1} = 12\ A.$$