

Recovery Exam

BLM 1541: Probability and Statistics — Fall 2016

Print family (or last) name: _____

Print given (or first) name: _____

Print student number: _____ Group: _____ Order: _____

I have read and understand all of the instructions below, and I will obey the Academic Integrity Code of Yıldız Technical University.

Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book and closed-note exam. Electronic devices (e.g., cellphone, smart watch) are not allowed. Single page A4 note sheet is allowed. *Don't forget to attach your A4 note sheet into your solution; otherwise your solution will not be graded.*
- For all problems, follow these instructions:
 - Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
 - PMF stands for probability mass function; CDF stands for cumulative distribution function; $\text{var}(X)$ stands for the variance of the random variable X ; $\text{cov}(X, Y)$ stands for the covariance between the random variables X and Y .
 - For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	Total
	20	20	20	20	20	20	100
Points							

1. **[20 points]** An example of Gaussian mixture PDF is

$$p_X(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2}} + \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x+1)^2}{2}} \text{ for all } -\infty < x < \infty$$

Determine its mean and variance.

Hint: $\int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu$ and $\int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \sigma^2 + \mu^2$.

2. **[20 points]** Let X be a random variable with the following probability distribution

$$f_X(x) = \begin{cases} (\theta + 1)x^\theta, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the maximum likelihood estimator of θ , based on the random samples of size n .

3. **[20 points]** Answer the following questions for the following data:

17.2, 22.1, 18.5, 17.2, 18.6

a) Compute the sample mean, variance, and standard deviation

b) Estimate the standard error of the sample mean.

d) Compute the interquartile range. Are there any outliers?

4. **[20 points]** We define the function $f_X(\cdot)$ as follows

$$f_X(x) = \begin{cases} K \exp(ax), & \text{if } x < 0 \\ K \exp(-bx), & \text{otherwise} \end{cases}$$

where $a > 0$, $b > 0$, and K is a positive constant.

a) Find K such that $f_X(\cdot)$ is a valid probability density function (PDF).

b) Find the corresponding cumulative distribution function (CDF).

5. **[20 points]** An average scanned image occupies 0.6 megabytes of memory with a standard deviation of 0.4 megabytes. If you plan to publish 80 images on your web site, what is the probability that their total size is between 47 megabytes and 50 megabytes?

If you need cumulative distribution tables just write that. We will accept.

6. **[20 points]** If X and Y have the joint probability mass function (PMF)

$$P(X = m, Y = n) = p_1 p_2 (1 - p_1)^m (1 - p_2)^n \text{ for all } m = 0, 1, 2, 3, \dots \text{ and } n = 0, 1, 2, 3, \dots$$

where $0 < p_1 < 1$ and $0 < p_2 < 1$, then answer the following questions. Find the conditional probability $P(X = m | Y = n) = \frac{P(X=m, Y=n)}{P(Y=n)}$.

Hint: $\sum_{k=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}$ and therefore $\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}$ for $|r| < 1$.