

In[1]:= **CIP – CSL3 derivation;**

In[2]:= **Reference : Xiao, 2001;**



In[2]:= **ClearAll["Global`*"]**

In[3]:= **FL[x_] := c10 + c11 (x - ξ) + c12 (x - ξ)² + c13 (x - ξ)³**
FR[x_] := cr0 + cr1 (x - ξ) + cr2 (x - ξ)² + cr3 (x - ξ)³

In[5]:= **Constrains;**

In[6]:= **Left – Bias;**

LBiasEqn = {
 $\text{FL}[\xi] == f_i,$
 $\text{FL}[\xi - h] == f_{i-1},$
 $\int_{\xi-h}^{\xi} \text{FL}[x] \, dx == \rho_{i-\frac{1}{2}},$
 $\left(D[\text{FL}[x], x] /. x \rightarrow \xi - \frac{h}{2} \right) == d_{i-\frac{1}{2}}$
};

Right – Bias;

RBiasEqn = {
 $\text{FR}[\xi] == f_i,$
 $\text{FR}[\xi + h] == f_{i+1},$
 $\int_{\xi}^{\xi+h} \text{FR}[x] \, dx == \rho_{i+\frac{1}{2}},$
 $\left(D[\text{FR}[x], x] /. x \rightarrow \xi + \frac{h}{2} \right) == d_{i+\frac{1}{2}}$
};
LBiasEqnSol = Solve[LBiasEqn, {c10, c11, c12, c13}];
RBiasEqnSol = Solve[RBiasEqn, {cr0, cr1, cr2, cr3}];

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In[12]:= Style[Row[{
  Column[LBiasEqnSol // Flatten, Spacings → 2] // ExpandAll,
  Column[RBiasEqnSol // Flatten, Spacings → 2] // ExpandAll
}, Spacer[20]], FontSize → 16] // TraditionalForm
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Out[12]//TraditionalForm=

$$\text{cl0} \rightarrow f_i$$

$$\text{cr0} \rightarrow f_i$$

$$\text{cl1} \rightarrow -2 d_{i-\frac{1}{2}} + \frac{6 f_i}{h} - \frac{6 \rho_{i-\frac{1}{2}}}{h^2}$$

$$\text{cr1} \rightarrow -2 d_{i+\frac{1}{2}} - \frac{6 f_i}{h} + \frac{6 \rho_{i+\frac{1}{2}}}{h^2}$$

$$\text{cl2} \rightarrow -\frac{6 d_{i-\frac{1}{2}}}{h} - \frac{3 f_{i-1}}{h^2} + \frac{9 f_i}{h^2} - \frac{6 \rho_{i-\frac{1}{2}}}{h^3}$$

$$\text{cr2} \rightarrow \frac{6 d_{i+\frac{1}{2}}}{h} + \frac{9 f_i}{h^2} - \frac{3 f_{i+1}}{h^2} - \frac{6 \rho_{i+\frac{1}{2}}}{h^3}$$

$$\text{cl3} \rightarrow -\frac{4 d_{i-\frac{1}{2}}}{h^2} - \frac{4 f_{i-1}}{h^3} + \frac{4 f_i}{h^3}$$

$$\text{cr3} \rightarrow -\frac{4 d_{i+\frac{1}{2}}}{h^2} - \frac{4 f_i}{h^3} + \frac{4 f_{i+1}}{h^3}$$