Name: Justin Ngo
PSU ID: jvn5439
Professor: Mark Mahon
Class: CompEn 462
Date: 01.28.2025

Problem 1

1. True or False

   (a) False: Assuming that $N(A) = 0$ means that there is no set of vectors that span the Null Space, then the answer is false

   (b) True: the inverse of an orthogonal matrix is its transpose by definition

   (c) True: Convolution operation in time domain means you multiply in the frequency domain, so $s_1(t) * s_2(t) = f_1 * f_2 * f_3 * f_4$

   (d) False: A 3x3 matrix where the columns (b) form the space doesn't necessarily imply that all the values in each column is 0, which means that a_1, a_2, a_3 cant all be 0

   (e) False: if you set v = 4, then $\|v\| = \sqrt{1 + 4 + 9 + 16} = \sqrt{30}$, $4/30 = 2/15 \neq 0$

Problem 2

$$3x + 2y = 10$$
$$6x + 4y = b$$

Infinite Solution:

$$3x + 2y = 10$$
$$6x + 4y = 20$$
$$0 = 0$$
$$b = 20$$

No Solution:

$$3x + 2y = 10$$
$$6x + 4y = 30$$
$$0 = 20$$
$$b = 30$$

so long as $b \neq 20$ there should be no solution.

Problem 3

$x - y = 2$
$x + y = 4$
$2x + y = 8$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 4 \\ 2 & 1 & 8 \end{bmatrix} R_1 \leftrightarrow R_3 \begin{bmatrix} 2 & 1 & 8 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{bmatrix} R_1 - R_2 \to R_1 \begin{bmatrix} 1 & 0 & 4 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{bmatrix} R_2 - R_1 \to R_2$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} R_3 - R_1 \to R_3 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & -1 & -2 \end{bmatrix} R_3 + R_2 \to R_3 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \Rightarrow NULL$$

$$A^T = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix} A^T A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} A^T b = \begin{bmatrix} 22 \\ 10 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 6 & 2 & 22 \\ 2 & 3 & 10 \end{bmatrix} R_1/6 \to R_1 \begin{bmatrix} 1 & 1/3 & 11/3 \\ 2 & 3 & 10 \end{bmatrix} R_2 - 2R_1 \to R_2 \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 7/3 & 8/3 \end{bmatrix}$$

$$3R_2/7 \to R_2 \begin{bmatrix} 1 & 1/3 & 11/3 \\ 0 & 1 & 8/7 \end{bmatrix} R_1 - R_2/3 \to R_1 \begin{bmatrix} 1 & 0 & 23/7 \\ 0 & 1 & 8/7 \end{bmatrix}$$

$x = 23/7$
$y = 8/7$