Workshop4. Normality

UNAM – FE Econometrics I

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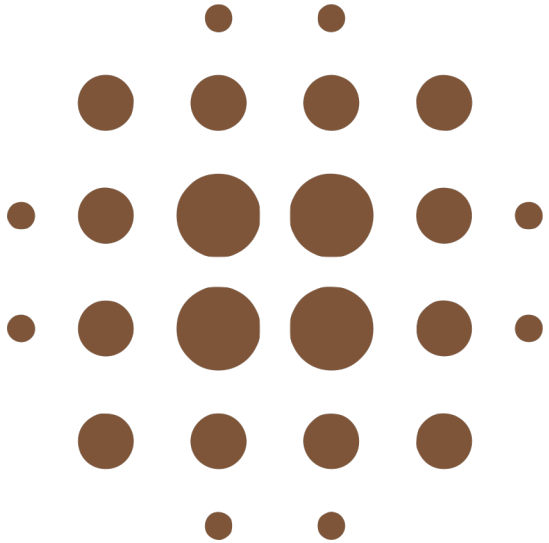
One of the main tasks in statistics relies on summaries and description from numerical information through metrics tht quantifies several aspects such as distribution from a dataset.

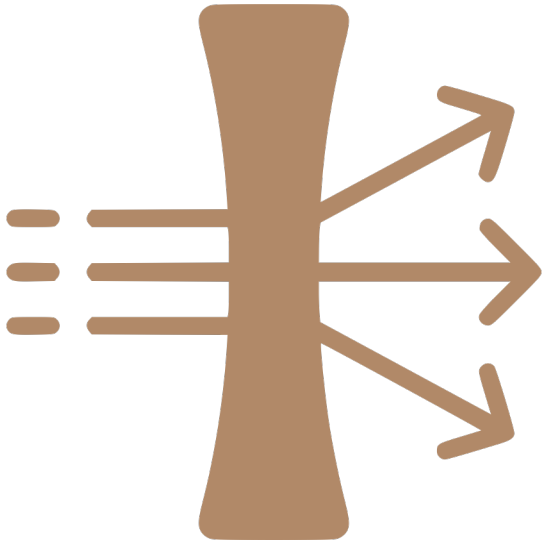
Introduction

Central tendency measures.



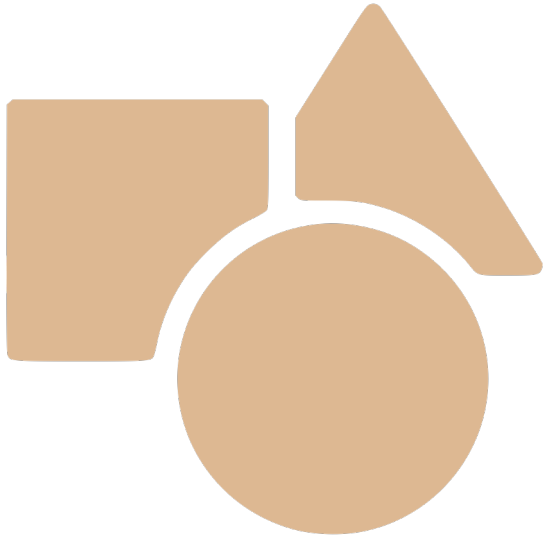
Single value that attempts to describe a whole dataset through the center of its distribution

Measures of dispersion



Spread of a dataset or variability of data (is my data homogeneus or heterogeneus?)

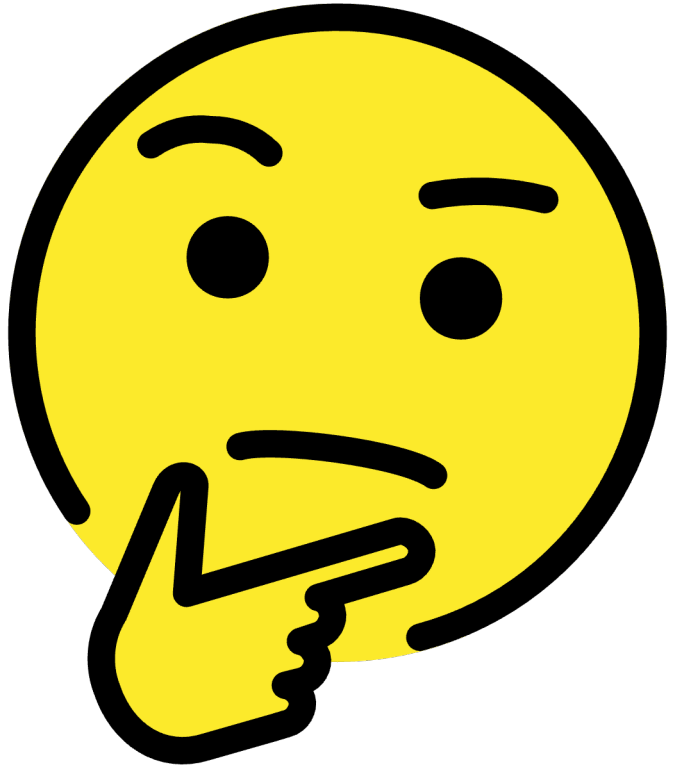
Measures of shape



Indicate how data is grouped according to their frequency

Okay but…How can we measure all this nonsense stuff?

Hint: we use statistical methods such as…



Position

They divide an ordered dataset in groups with the same quantity of individuals.

Quantiles, percentiles, quartiles, decil.

Dispersion

They indicate the degree of concentration of data with respect to measures of central tendency. They are used to quantify the variability of a dataset.

Standard Deviation, variance, coefficient of variation

Centralization

Each of these measurements give us a reference value for establishing how a dataset is centered.

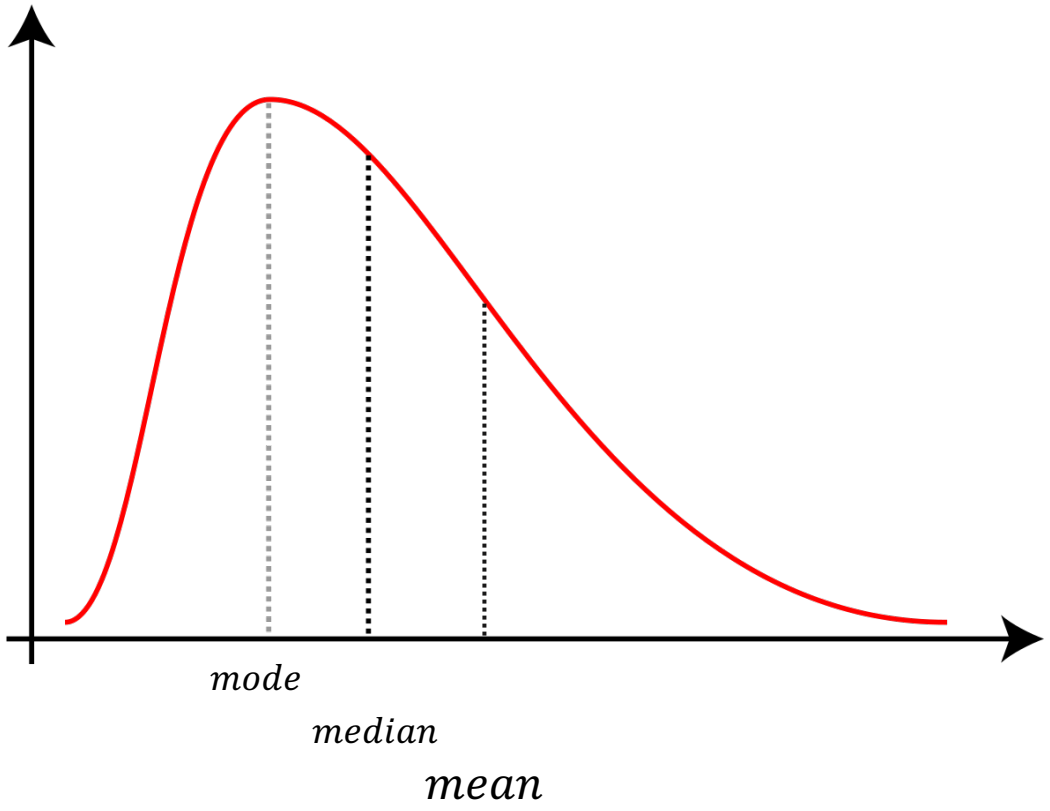
Mean, median, mode

Shape

They focus on the ditribution shape according to their symmetry

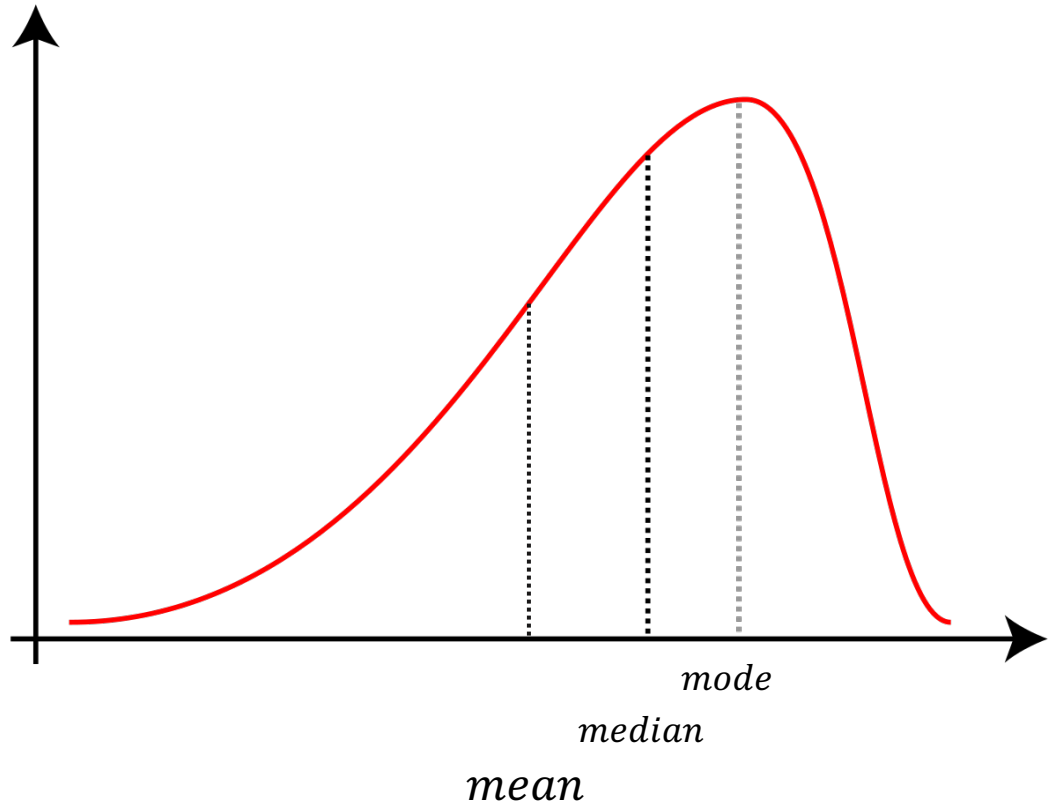
Assymmetry, kurtosis

Mean, median and mode



$mode < median < mean$

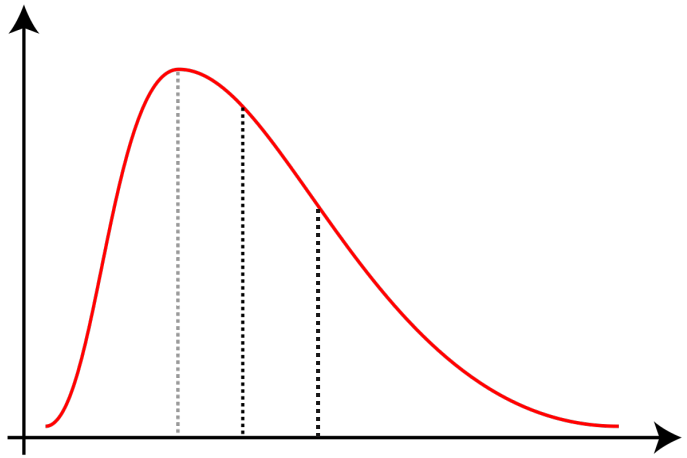
Positively skewed



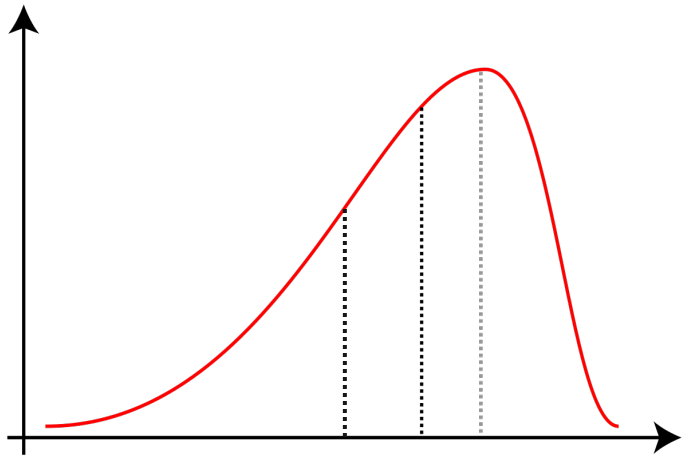
$mean < median < mode$

Negatively skewed

Imagine house prices ranging from $10k to $1,000,000 with average being $500,000

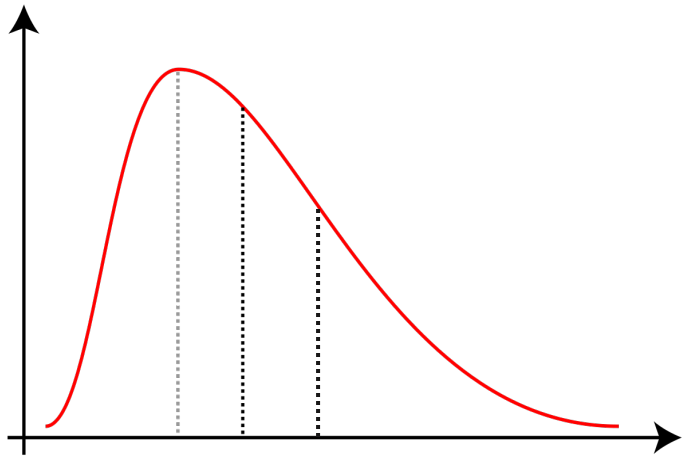


Most of our houses are being sold at a price less than the average

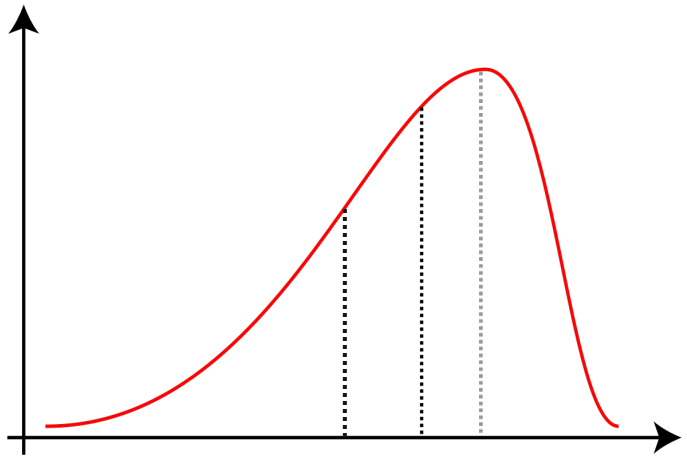


Most of our houses are being sold at a price greater than the average

In finance, if we want to know how the returns in our portfolio behaves, we state that…



Many small losses and few extreme gains

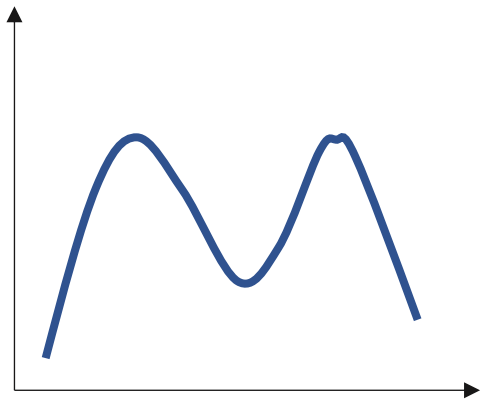


Many small gains and few extreme losses

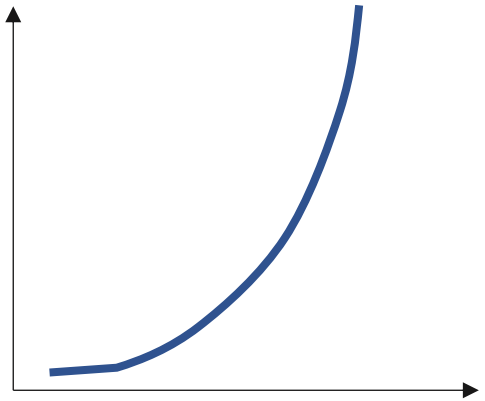
It describes the probabilistic shape (or behavior) of a random variable X associated with a random experiment which can be

$$f(x) \text{ or } Fx$$

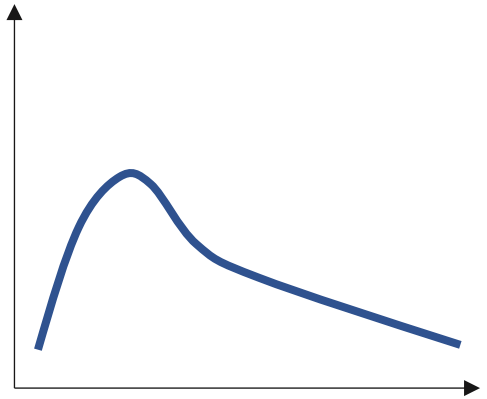
Thus, we can say that a distribution function allows to study the behavior of our data.



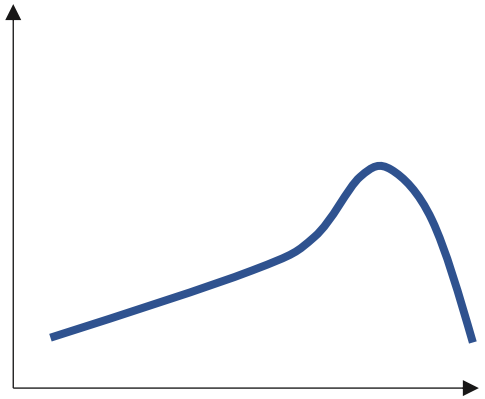
Bimodal distribution



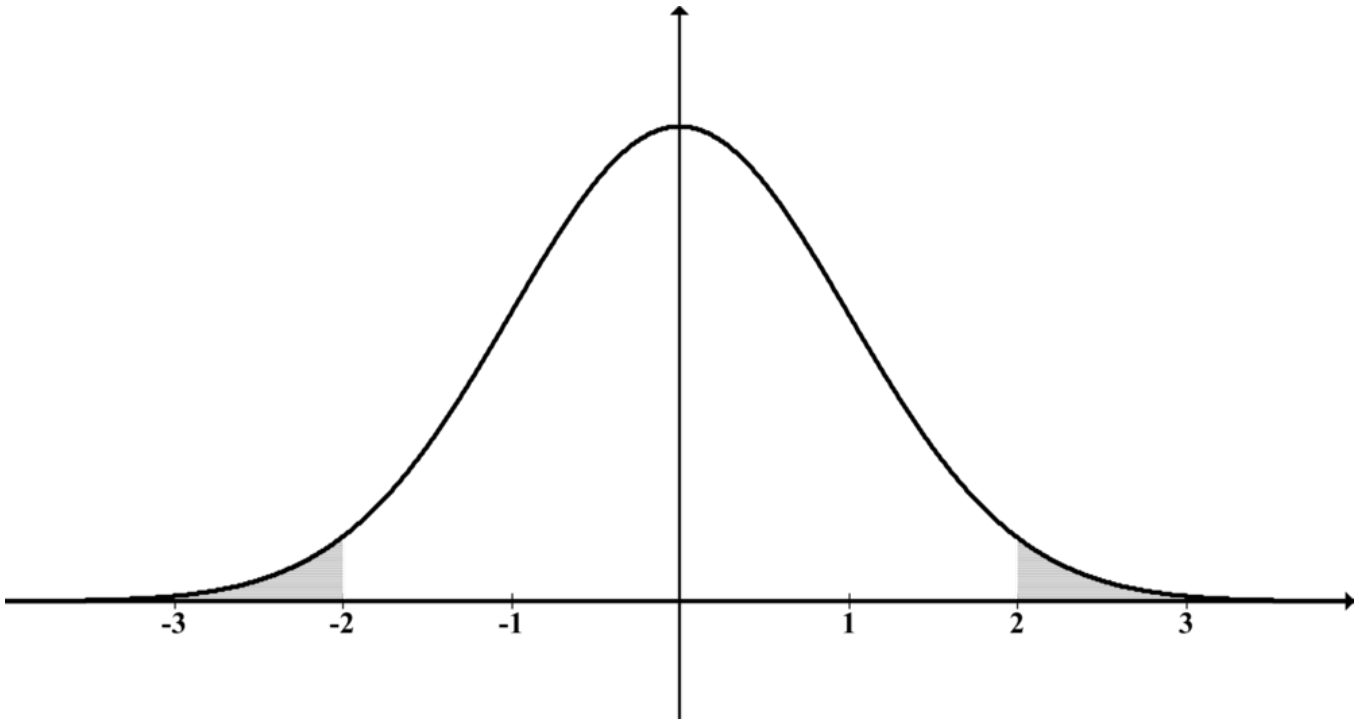
J distribution



Positively skewed distribution



Negatively skewed distribution



Undoubtely, the normal distribution is the most well-known probablity function. It was discovered as an approximation of the a binomial dist. by Abraham de Moivre (1667-1754)

Two parameters: mean and standard deviation.

When normal distribution has $mean = 0$ and $variance = 1$ it is called standard normal distribution.

Central Limit Theorem

CLT is a statistical theorem which states that, given a large enough sample of a population, the distribution of sample means will lead to a normal distribution.

Furthermore CLT establishes that the larger the sample, the more the sample mean approaches the population mean.

Due to CTL, it has been proven that if there are many independent random variables with identical distributions the distribution of its sum tends to be normal.

CLT is the ground for the assumption of normality

Classic linear regression model assumes that every U_j is normally distributed with

$$Mean: E(u_i) = 0$$

$$Variance: E[u_i - E(u_i)]^2 = E(u_i^2) = \sigma^2$$

$$Cov: E\{[u_i - E(u_i)][u_j - E(u_j)]\} = E(u_i u_j) = 0; \qquad i \neq j$$

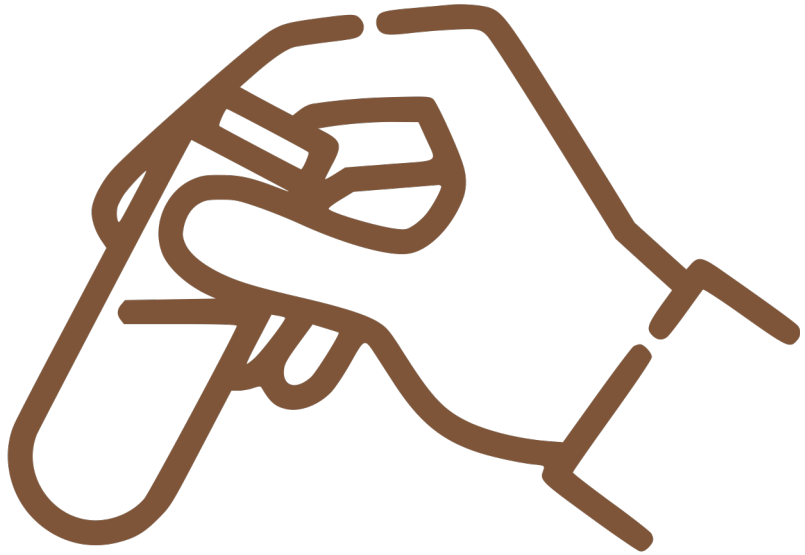
They are expressed in a compact form as:

$$u_i \sim N(0, \sigma^2)$$

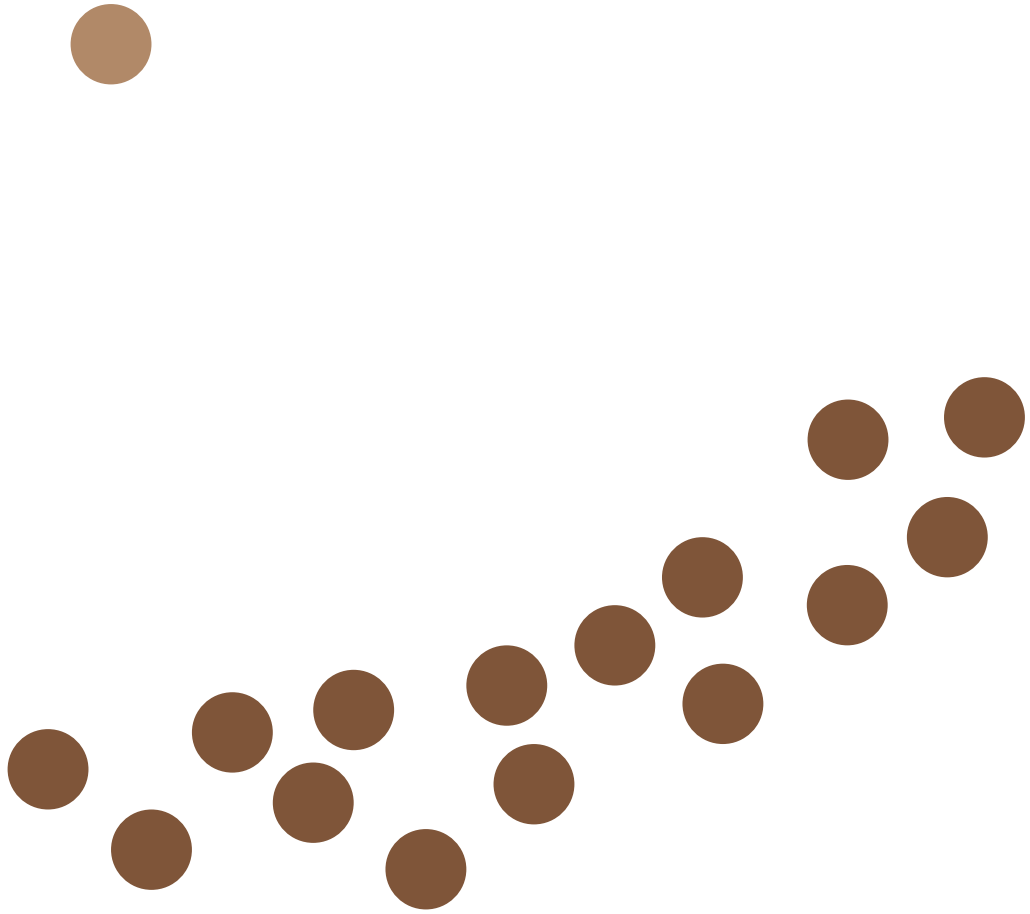
Why is it important the assumption of normality?

Normality in residuals ensure that estimator from ordinary least squares is consistent and efficient.

Tests such as T and F are calculated from the assumption of normal distribution.



Practice is much harder than an academic stone engraving sentence.



Why is it important the assumption of normality?

If there is a highly abnormal data point, caused for a special situation that is out of model's reach, it can lead to a modification of residuals' distribution

A data point is considered as an outlier, if the value of a data point for a variable differs substantially from the pattern from the rest of variables

What can we do if we find an outlier?

We use elemapi2.dta which contains information about basic academic performance in USA

Let's prove that (`api00`) depends on free meals percentage given to students (`meals`), students currently learning English (`ell`) and percentage professors with recent accreditations (`emer`)

We use `predict` to generate residuals.

STATA COMMANDS

```
1. regress api00 meals ell emer
2. predict r, resid
```

STATA COMMANDS

We use the `kdensity` command to make a density kernel graph with *`normal`* as an option for it to overlap a normal distribution.

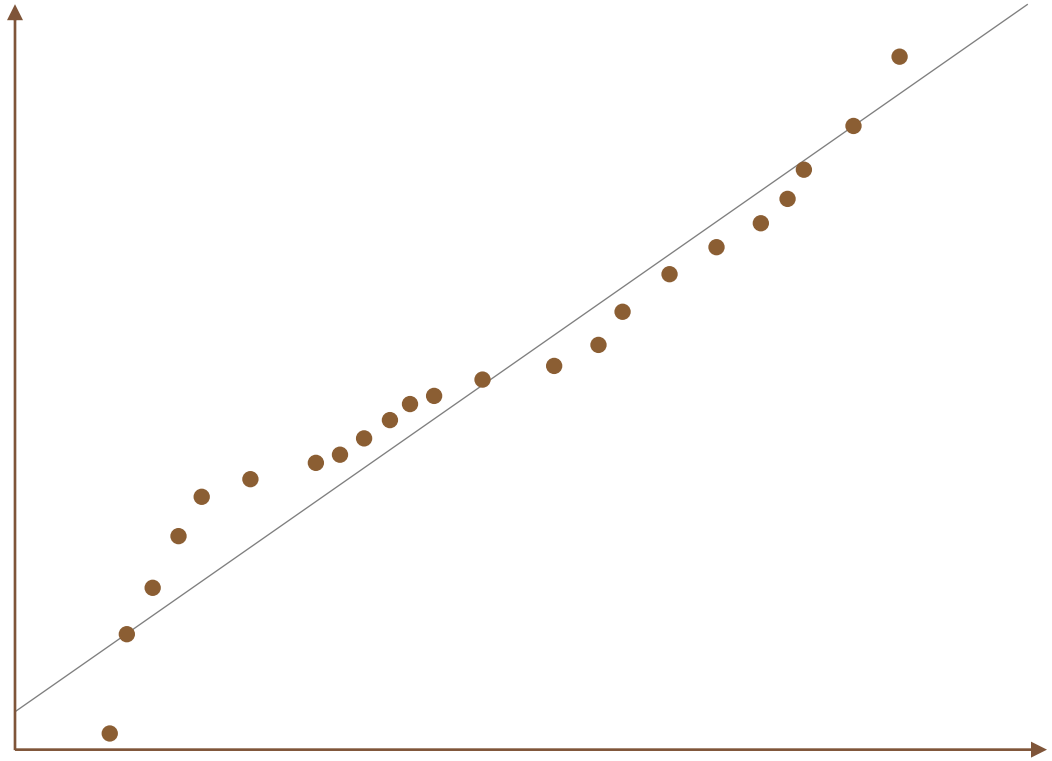
Kernel density is a non-parametric estimation method for a random variable density

```
3. kdensity r, normal
```

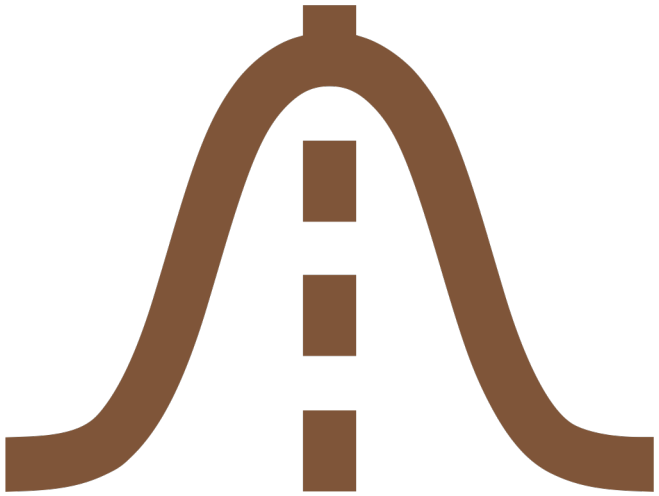
The Standardized Normal Probability Diagram is a graph technique to prove normality.

It indicates if a dataset has an approximated normal bell shape

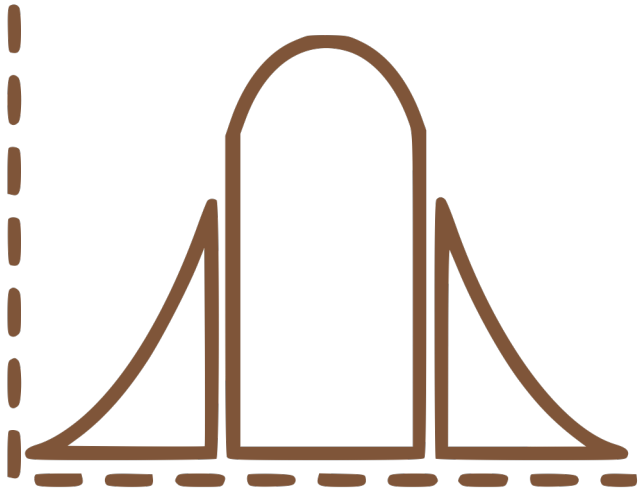
How to read it?: if distribution is normal, then points will be close to the straight line



On the other hand, the Normal Probability Graph is used to answer questions such as:



Is data normally distributed?



What is the nature of standar deviation of nor mality (long tails, shorter tails than expected)?

STATA COMMANDS

We can use `pnorm` to show a standardized normal probability diagram

```
4. pnorm r
```

The test of the assumption that residuals distributes as a normal shape is carried through means of residuals from the regression

Use the `sktest` command. Null hypothesis indicates sample skewed and sample kurtosis distributes as a normal shape

STATA COMMANDS

```
5. Sktest r
```

Non-parametric test do not assume the shape of data

They are also known as distribution free.

Kolmogorov-Smirnov is one example. It is used to determine goodness-of-fit from two probability distributions

This test is sensitive to value that are closed to the median rather than extreme tail values

STATA COMMANDS

Command syntax is made from scratch

With this command we would prove equality between two distributions. In `varname` we put data that will be proven and on the right side we evaluate with an accumulated normal distribution

```
6.  ksmirnov varname=normal((varname-(mean))/varname(sd))
```

To apply this test, we need to build statistic Z from normal distribution.

Null hypothesis states that distributions are equal. In our example:

- 0.426
- 0.797
- 0.787

The, we cannot reject null hypothesis…what does that mean?

STATA COMMANDS

```
7.  sum r

8.  ksmirnov r = normal((r-7.37e-08)/57.60224)
```

On the other hand, Shapiro-Francia test shows the squared correlation among ordered values in a sample and ordered approximated quartiles that are expected in a normal distribution.

Null hypothesis states that sample approximates to a normal distribution.

What can we conclude from this test?

STATA COMMANDS

```
8.  sfrancia r
```

References

- **Salvatore, D., & Sarmiento, J. C.** (1983). *Econometría* (No. HB141 S39). McGraw-Hill.
- **Gujarati, D. N.** (2009). Basic econometrics. Tata McGraw-Hill Education.
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