NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2024

MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 22 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2.   Read the questions carefully.

3.   **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**

4.   Diagrams are not necessarily drawn to scale.

5.   You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.   Ensure that your calculator is in **DEGREE** mode.

7.   Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks**.

8.   Round off to **ONE DECIMAL PLACE** unless otherwise stated.

9.   It is in your own interest to write legibly and to present your work neatly.

10.   TWO blank pages (page 21 and 22) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
17	16	15	13	7	8	7	21	10	10	10	16	150

SECTION A

QUESTION 1

Solve for *x*:

(a) $(3x - 4)(x^2 + 6x + 9) = 0$

(3)

(b) $\sqrt{3x - 5} = -1$

(1)

(c) $\log_x 5 = 3$

(2)

(d) $4^{x+1} + 2^{2x} = 40$

(4)

(e) $\dfrac{8-4x}{x-2} = x$

(3)

(f)   Consider the equation: $(x+k)^2 = 2k+1$
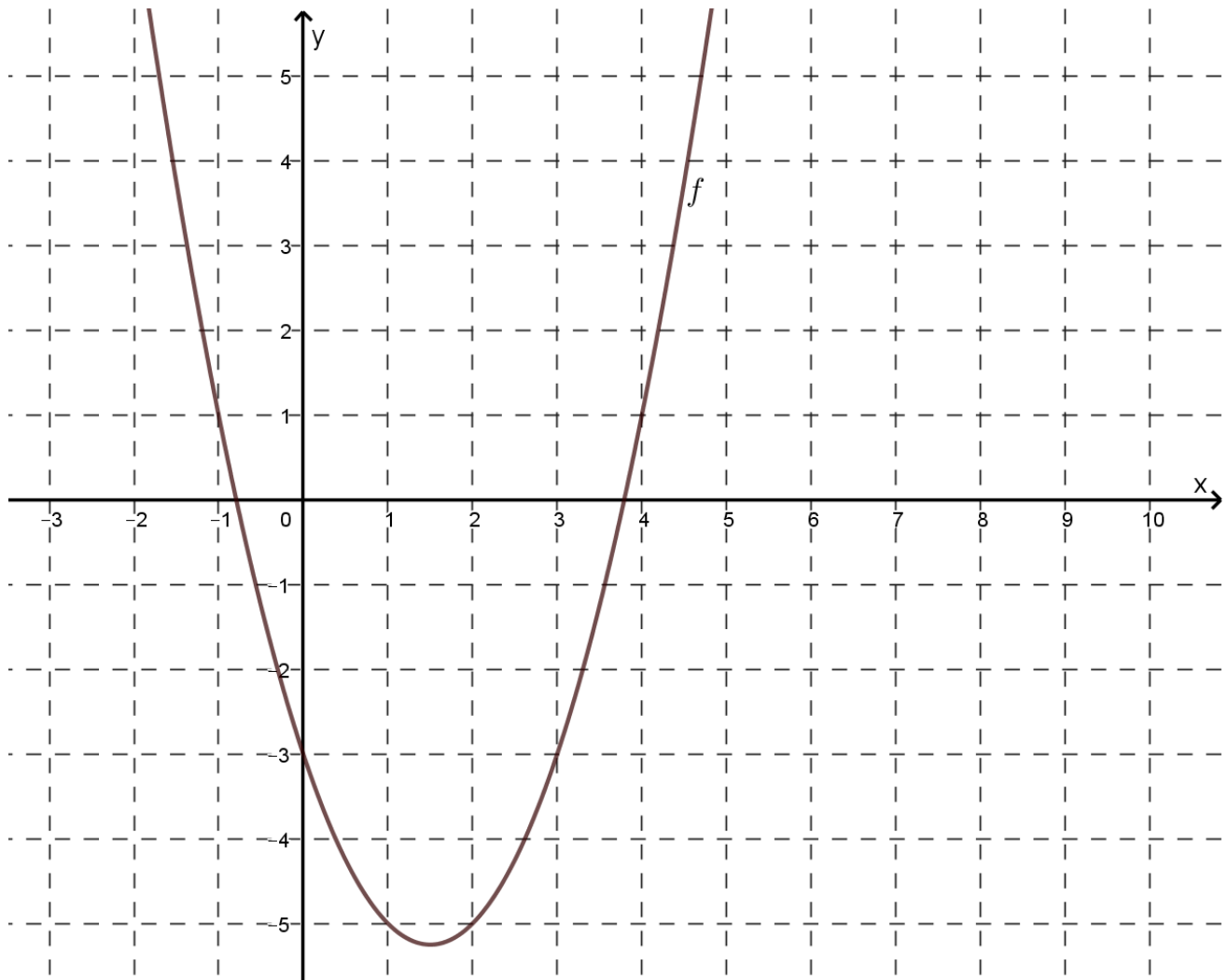
    (1)   Solve for x in terms of k.

(2)

    (2)   Hence, or otherwise, write down two positive values of k for which the roots are rational and real.

(2)
[17]

QUESTION 2

In the diagram below $f(x) = x^2 - 3x - 3$ has been drawn.

(a)   On the set of axes above sketch $g(x) = \log_3(x)$.

Show all asymptotes, and intercepts with the axes.

(3)

(b)   Determine the equation for $g^{-1}(x)$ in the form $y = \ldots$

(2)

(c)   (1)   *h* is formed if *g* is moved horizontally one unit to the right. Write down the equation of *h*(*x*).

(1)

      (2)   What is the equation of the asymptote of *h*?

(1)

(d)   (1)   Show that $f(x)$ can be written in the form $a(x-p)^2+q$ and hence write down the coordinates of the turning point of *f*.

(3)

      (2)   Write down the Domain and Range of *f*.

(3)

(e)   Use your graphs to give one value of *x* for which $f(x)-g(x)=-4$.

(1)

(f)   Show on your graph where you would read off the solution(s) to $f(x)<g(x)$.

(2)
[16]

QUESTION 3

(a)   Using first principles find $f'(x)$ if $f(x) = x^2 - x$.

(6)

(b)   Determine $g'(x)$ if $g(x) = 4x^3 - 2x^{-2} + \sqrt[8]{x}$.

(4)

(c)   Find the equation of the tangent to $h(x) = 3x^2 - 4x$ at $x = 1$.

(5)
[15]

QUESTION 4

(a)   Refer to the pattern $15 + 11 + 7 + 3 - 1 - 5 + \ldots - 209 - 213$ and determine the following:

   (1)   A general formula, T_n, of the pattern above.

(3)

   (2)   The sum of all the terms in the pattern above.

(4)

(b)   Determine the value of the following expression:

$$\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n + \sum_{n=2}^{20}\left(\frac{3}{5}\right)^n$$

(6)
[13]

QUESTION 5

(a)   In a shop there are six unique dresses.

   (1)   In how many ways can the six dresses be hung on a rail?

(1)

   (2)   If dress 1, dress 2 and dress 3 are hung side by side on the rail in any order, in how many ways can the six dresses be hung?

(3)

(b)   How many unique ways can the letters in the word 'DRESSES' be arranged?

(3)
[7]

QUESTION 6

You take out a home loan for R1 500 000 that needs to be repaid monthly over a period of twenty years. The interest rate on the loan is 11% per annum compounded monthly.

The first payment is made one month after the loan is granted.

(a)   What is the minimum monthly repayment that needs to be made to pay off the loan?

(4)

(b)   Determine the balance outstanding at the end of the ninth year after the payment at this point in time has been made.

(4)
[8]

76 marks

SECTION B

QUESTION 7

You receive the following information about an investment from the bank:

You must invest R25 000 on the first day of the investment and then pay in R600 at the end of every three months thereafter. The interest rate on the investment is 15% per annum compounded quarterly.

What will your investment be worth at the end of the last payment of the 8th year?

[7]

QUESTION 8

(a)   You are given the following functions:
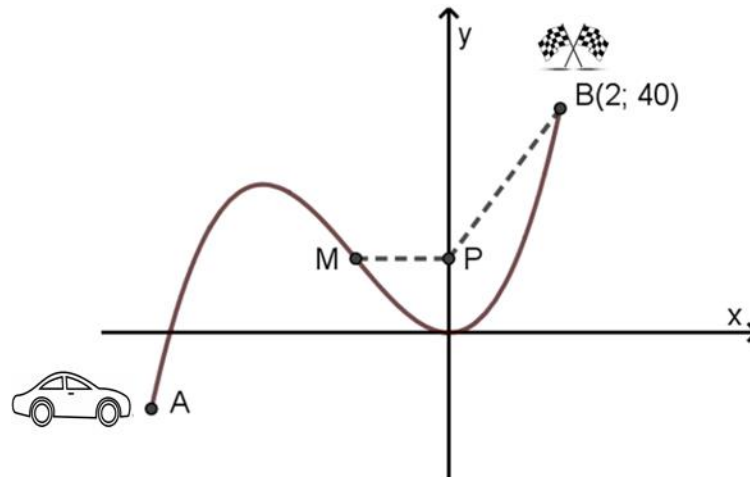
- $g(x) = 3x + c$
- $f(x) = -2x^2 + 15x$

If g is a tangent to f, then calculate the coordinates of the point where g cuts the x-axis.

(5)

(b)   In the diagram below the section of the rally track that starts at A, goes through M
       and finishes at B, is modelled by $h(x) = ax^3 + bx^2$.

       Key information:
       - M is the point of inflection for h and P lies on the y-axis.
       - MP is parallel to the x-axis and 1 kilometre long.
       - B(2; 40) lies on h.

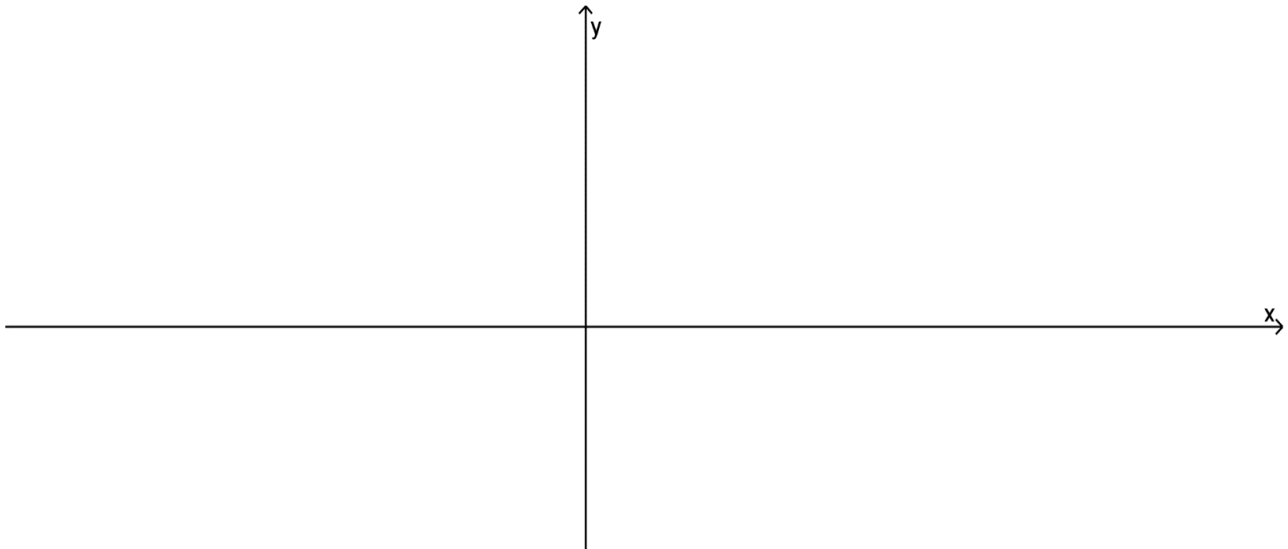   (1)   Determine the value of a and b.

(7)

       (2)   Determine the straight-line distance PB.

(3)

(c)   (1)   Sketch the graph of $f(x) = ax^3 + bx^2 + cx + d$ if:
              (Label the turning points and the point of inflection)

- $f'(-3) = 0 = f'(5)$
- $f(-3) = 5$ and $f(5) = -3$
- f has three unequal roots.

(4)

       (2)   If $g(x) = f(x) + t$, then for what value(s) of t will g have one real root?

(2)
[21]

QUESTION 9

(a)   In an arithmetic sequence the first term is $-\log_3 x$ and the common difference is $\log_3 x$. Determine the value of x if $T_6 = 12$.
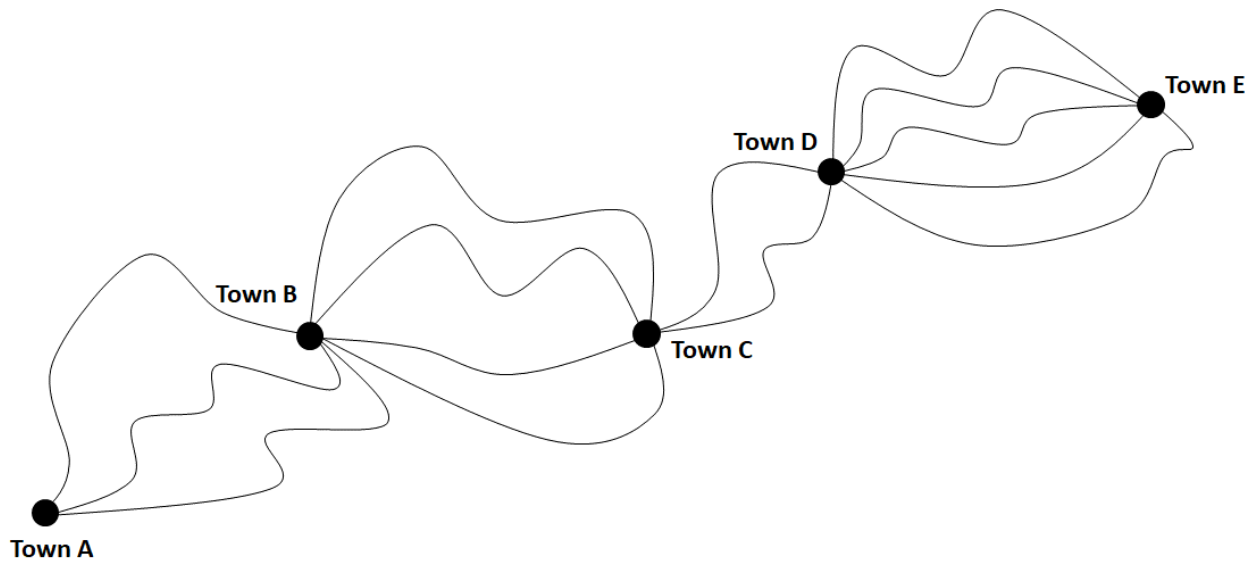
(4)

(b)   If $5x + 1$; $2x + 6$ and $4x - 3$ are the first three terms in a converging geometric sequence then determine the value(s) of x.
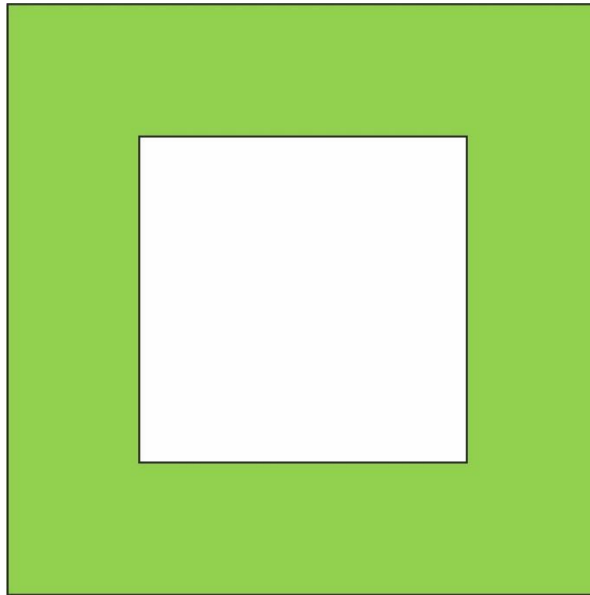
(6)
[10]

QUESTION 10

(a)　The picture below shows the roads that link Town A, Town B, Town C, Town D and Town E.

　　(1)　If you start at Town A, how many different ways could you drive to Town E?

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(2)

　　(2)　The traffic police set up three roadblocks: one between Town A and Town B, one between Town B and Town C and one between Town C and Town D. What is the probability that you will go through at least one roadblock if you travel from Town A to Town E?

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(3)

　　　　　　　　　　　　　　　　　　　　　　　　PLEASE TURN OVER

(b)   In the diagram below, two squares are shown.
        The larger square has an area of 1,5 times the area of the smaller square.

        If two darts are thrown, what is the probability that one dart lands in the green shaded area and the other does not?

        You can assume that both darts land within the larger square.

(5)
[10]

QUESTION 11

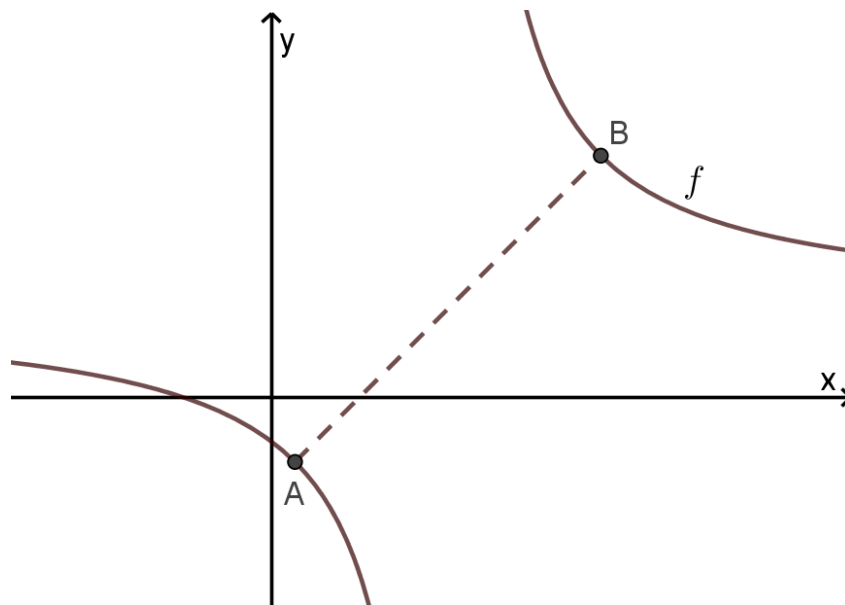(a)   Solve for x if $\dfrac{10}{x^2 - 5x - 6} \geq 0$.

(4)

(b)   Solve for x and y if $\sqrt{2x+1} - 2 = y$ and $\dfrac{\sqrt{x}}{\sqrt{y}} = 2$.

(6)
[10]

QUESTION 12

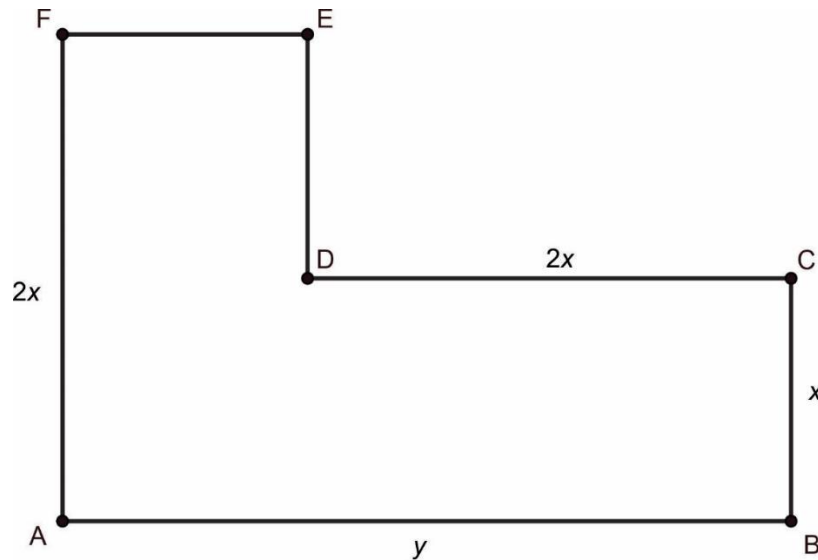(a)   The graph of $f(x) = \dfrac{(x-2)+3}{(x-2)}$ has been sketched below.

(1)   Determine the equation of line AB, if AB has a minimum length.

(5)

(2)   Hence, determine the coordinates of point A. (Give your answer in the simplest root form where applicable.)

(4)

(b)   You need to fence your vegetable garden ABCDEF as shown below.

- FE//DC//AB and AF//DE//BC.
- AF is perpendicular to FE.
- AB = y; AF = DC = $2x$ and BC = x.

F ● ———————————— ● E

D ● ——— $2x$ ——— ● C

$2x$

x

A ● ——————— y ——————— ● B

If you have 20 metres of fencing, then determine the value for x so that the maximum area is enclosed by the fence in the shape used above.

(7)
[16]

┌─────────────┐
│ **74 marks** │
└─────────────┘

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.