NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2024

MATHEMATICS: PAPER I
MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $\left(\frac{x}{4}-1\right)(x+2)=0$

$x=4 \quad or \quad x=-2$

(2) $2x^2-x-12=0$

$2x^2-x-12=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{1\pm\sqrt{(-1)^2-4(2)(-12)}}{2(2)}$

$x=-2,21 \qquad or \qquad x=2,71$

(3) $x^2+2x\le 0$

$x^2+2x\le 0$

$x(x+2)\le 0$

$CV: x=0 \ and \ x=-2$

$\therefore -2\le x\le 0$

(b) $x\left(\frac{1}{2}x+2\right)=-k^2$

$\frac{1}{2}x^2+2x+k^2=0$

$\Delta=b^2-4ac$

$0=(2)^2-4\left(\frac{1}{2}\right)(k^2)$

$0=4-2k^2$

$2k^2=4$

$k^2=2$

$k=\pm\sqrt{2}$

(two values)

ALT $x^2+4x+2k^2=0$

$\Delta=b^2-4ac$

$=(4)^2-4(1)(2k^2)$

$0=16-8k^2$

$8k^2=16$

$k^2=2$

$k=\pm\sqrt{2}$

(two values)

QUESTION 2

(a) $3^{x+2y} = \dfrac{9^{3y}}{27}$

$$3^{x+2y} = \frac{3^{6y}}{3^3}$$

$$3^{x+2y} = 3^{6y-3} \quad \text{(same base)}$$

$$x + 2y = 6y - 3$$

$$x = 4y - 3$$

(b) $\sqrt{x+8} - x = 2$

$$\sqrt{x+8} = 2 + x$$

$$x + 8 = (x+2)^2$$

$$x + 8 = x^2 + 4x + 4$$

$$0 = x^2 + 3x - 4$$

$$0 = (x+4)(x-1)$$

$$x \neq -4 \quad or \quad x = 1$$

(c) (1) $\log 6 + \log 3$

$= x + y$

(2) $\log 3^{\frac{1}{2}}$

$$= \frac{1}{2}\log 3$$

$$= \frac{1}{2}y$$

(3) $\log \dfrac{6}{3}$

$$= \log 6 - \log 3$$

$$= x - y$$

QUESTION 3

(a) $f(x) = x^2 + 1$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^2 + 1 - \left(x^2 + 1\right)}{h}$$
$$= \lim_{h\to 0}\frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$
$$= \lim_{h\to 0}\frac{2xh + h^2}{h}$$
$$= \lim_{h\to 0}\frac{h\left(2x + h\right)}{h}$$
$$= 2x \checkmark$$

notation

(b)

$$f(x) = 2x^4 + \frac{1}{\sqrt[4]{x}} - 4$$
$$= 2x^4 + x^{-\frac{1}{4}} - 4$$
$$f'(x) = 8x^3 - \frac{1}{4}x^{-\frac{5}{4}}$$

(c) (1) $x^2 - 3x - 4 = 0$

$\left(x-4\right)\left(x+1\right) = 0$

$x = 4$ of $x = -1$

$A\left(-1\ ;\ 0\right)$ en $B\left(4\ ;\ 0\right)$

(2) Determine the coordinates of D

$f'(x) = 2x - 3$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = -\frac{25}{4}$

$\therefore D\left(\frac{3}{2}; -\frac{25}{4}\right)$

ALT $x = -\frac{(-3)}{2(1)}$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = -\frac{25}{4}$

$\therefore D\left(\frac{3}{2}; -\frac{25}{4}\right)$

(3) $x < \frac{3}{2}$

(4) $m = -4$

$$2x - 3 = -4$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

QUESTION 4

(a) $(3x+1)-(5x-1)=2x-(3x+1)$ ✓✓

$$-2x+2=-x-1$$
$$-x=-3$$
$$x=3$$
$$14\,;10\,;6$$

(b) $T_n = a+(n-1)d$

$$=14+(n-1)(-4)$$
$$=18-4n$$

QUESTION 5

(a) $f(x) = x^3 + px^2 + 3x - 9 \quad sub \ E\left(-\frac{5}{3}; y\right)$

$$f'(x) = 3x^2 + 2px + 3$$
$$f''(x) = 6x + 2p$$
$$0 = 6\left(-\frac{5}{3}\right) + 2p$$
$$p = 5$$

(b) (1) $f(x) = x^3 + 5x^2 + 3x - 9$

$$f(x) = x^3 + 5x^2 + 3x - 9$$
$$f'(x) = 3x^2 + 10x + 3$$
$$0 = 3x^2 + 10x + 3$$
$$0 = (3x + 1)(x + 3)$$
$$x = -3 \quad or \quad x = -\frac{1}{3}$$
$$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 + 5\left(-\frac{1}{3}\right)^2 + 3\left(-\frac{1}{3}\right) - 9$$
$$= -\frac{256}{27}$$
$$C\left(-\frac{1}{3}; -\frac{256}{27}\right) \quad \text{ALT} \quad C\left(-\frac{1}{3}; -9\frac{13}{27}\right)$$

(2) $0 = x^3 + 5x^2 + 3x - 9$

$$0 = (x - 1)(x + 3)^2$$

$x = 1$ of $x = -3$

B(1; 0)

QUESTION 6

(a) $$1+i_{eff}=\left(1+\frac{i}{n}\right)^{n}$$

$$1+i_{eff}=\left(1+\frac{0,06}{4}\right)^{4}$$

$$i_{eff}=0,06136$$ Rond af na 2 desimale plekke in V6(a).

$$\therefore i_{eff}=6,14\%\text{ p.a}$$

(b) (1) Deposit = R235 000

$$705\,000=\frac{x\left[1-\left(1+\frac{0,1125}{12}\right)^{-20\times12}\right]}{\frac{0,1125}{12}}$$ Incorrect formulae maxim in is Y-i deposit

$$x=7\,397,25$$

(2) $$B.O=705\;000\left(1+\frac{0,1125}{12}\right)^{11\times12}-\frac{7\,397,25\left[\left(1+\frac{0,1125}{12}\right)^{11\times12}-1\right]}{\frac{0,1125}{12}}$$

$$=501\,020,53$$

compound is correct – Present value

705 000 must be next to the correct bracket

ALT

$$B.O=7\,397,25\frac{\left[1-\left(1+\frac{0.1125}{12}\right)^{-108}\right]}{\frac{0.1125}{12}}$$

$$=501\,018,93$$

SECTION B

QUESTION 7

(a)

$$3 \;;\; 9 \;;\; 17 \;;\; 27 \;;\; 39;\ldots$$
$$6 \quad 8 \quad 10 \quad 12$$
$$2 \quad 2 \quad 2$$

$$\begin{aligned} 2a &= 2 \\ a &= 1 \\ 3a + b &= 6 \\ 3 + b &= 6 \\ b &= 3 \\ a + b + c &= 3 \\ 1 + 3 + c &= 3 \\ c &= -1 \end{aligned}$$

$$\therefore T_n = n^2 + 3n - 1$$

(b)

$$\begin{aligned} T_n &= n^2 + 3n - 1 \\ n^2 + 3n - 1 &= 161 \\ n^2 + 3n - 162 &= 0 \end{aligned}$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (3)^2 - 4(1)(-162) \\ &= 657 \end{aligned}$$

Roots: Irrational and unequal

Tlhogi is incorrect.

ALT $n^2 + 3n - 162 = 0$

$$n = -\frac{3 \pm \sqrt{3^2 - 4(1)(-162)}}{2}$$

$n = 14{,}32$ or $n = -11{,}32$

Tlhogi is incorrect.

QUESTION 8

(a) (1) $1+\left(\frac{x+2}{2}\right)^1+\left(\frac{x+2}{2}\right)^2+....$

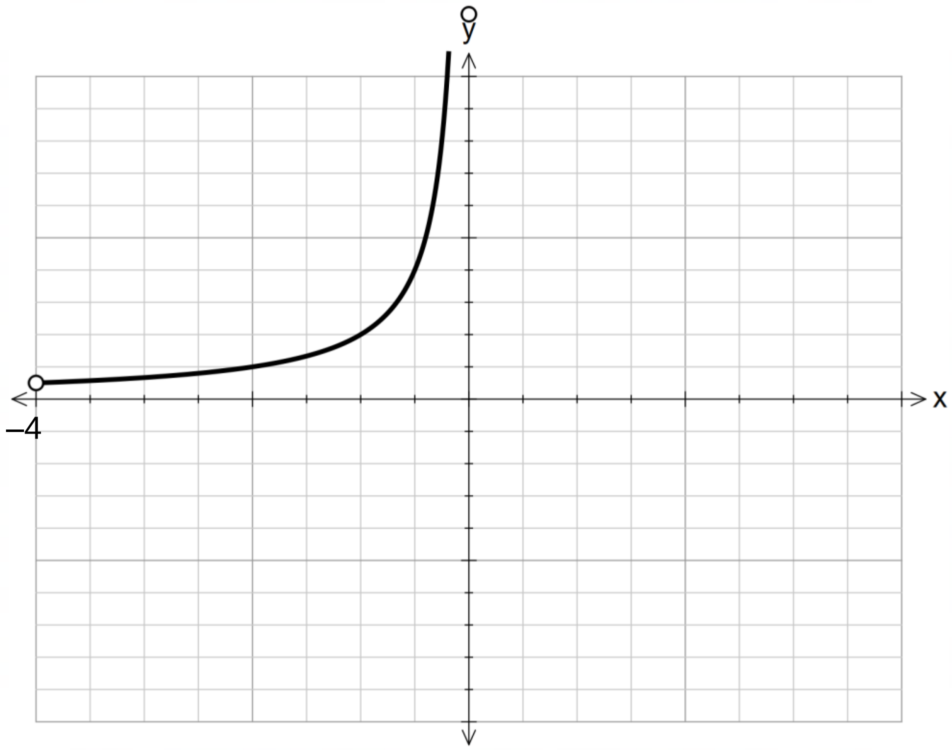
(2) $-1<r<1$ and

$$-1<\left(\frac{x+2}{2}\right)<1$$
$$-2<x+2<2$$
$$-4<x<0$$

(3) $1+\left(\frac{x+2}{2}\right)^1+\left(\frac{x+2}{2}\right)^2+....$

$$s_\infty=\frac{1}{1-\left(1+\frac{x}{2}\right)}$$
$$=-\frac{2}{x}$$

(4) Shape restriction asymptote Show a point that lies on the curve or implied



(b) (1) $0 = -\frac{20}{11449}x^2 + 20$

$x = \pm 107$

$x = 107$

$\therefore a = 107$

(2) $r = \frac{2}{3}$

$\text{diameter} = 36 \text{ P}$

$\pi r^2 = 324$

$r = 18$

$S_n = 36 + 24 + 16 + ...$

$S_n < 107$

$\frac{36\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1} < 107$

$\left(\frac{2}{3}\right)^n > \frac{1}{108}$

$n < 11.5$

$\therefore n = 11$

ALT $\frac{36\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1} = 107$

$\left(\frac{2}{3}\right)^n = \frac{1}{108}$

$n = 11,55$

$\therefore n = 11$

QUESTION 9

(a) (1) $x \in \mathbb{R} \quad x \neq 2$

(2) $f(x) = a(x-2)^2 + 3 \quad sub\,(3;5)$

$$5 = a(3-2)^2 + 3$$
$$5 = a + 3$$
$$a = 2$$
$$f(x) = 2(x-2)^2 + 3$$

(3) $g(x) = \dfrac{k}{x-2} + 3 \quad \text{sub}(3;5)$

$$5 = \frac{k}{3-2} + 3$$
$$\text{k} = 2$$

$$g(x) = \frac{2}{x-2} + 3 \qquad\qquad f(x) = 2(x-2)^2 + 3$$

$$\text{AB} = 2(0-2)^2 + 3 - \left[\frac{2}{0-2} + 3\right]$$
$$= 11 - 2$$
$$= 9$$

(4) $0 = \dfrac{2}{x-2} + 3$

$$0 = 2 + 3x - 6$$
$$-3x = -4$$
$$x = \frac{4}{3}$$

$$x < \frac{4}{3} \quad \text{of} \quad x > 2$$

(b) $h\left(\frac{31}{10}\right) = -1$

$$\begin{aligned} \log_k(4+m) &= 0 \\ 4+m &= k^0 \\ 4+m &= 1 \\ m &= -3 \end{aligned}$$

$$\begin{aligned} -1 &= \log_k\left(\frac{31}{10} - 3\right) \\ \log_k\left(\frac{1}{10}\right) &= -1 \\ \frac{1}{10} &= k^{-1} \\ 10 &= k \end{aligned}$$

ALT

$$\begin{aligned} \log_k(4+m) &= 0 \\ 4+m &= k^0 \\ 4+m &= 1 \\ m &= -3 \end{aligned}$$

$$\begin{aligned} \log_k(y+m) &= x \\ y &= k^x - m \\ \frac{31}{10} &= k^{-1} + 3 \\ \frac{1}{k} &= \frac{1}{10} \\ k &= 10 \end{aligned}$$

QUESTION 10

(a) $f'(x) = 2x - 14$

$f'(8) = 2$
$f(8) = -5$

$y = 2x + c$ sub $(8; -5)$ ALT $y + 5 = 2(x - 8)$
$-5 = 2(8) + c$ $y + 5 = 2x - 16$
$c = -21$ $y = 2x - 21$
$y = 2x - 21$

(b) $g'(x) = -2x + 10$ if the diff f(x)=2 and equal slope then BD (Zero)

$-2x + 10 = 2$
$x = 4$
$B(4; -13)$

$g(x) = -x^2 + 10x - 19 + k$ sub $B(4; -13)$
$-13 = -(4)^2 + 10(4) - 19 + k$
$k = -18$

QUESTION 11

$3OG = EF = 2OE \quad or \ \dfrac{OG}{OE} = \dfrac{2}{3}$

$y = -\dfrac{2}{3}x + \dfrac{2}{3}k$ ALT if $\text{B}(x\,;y)$ then $\dfrac{y}{k-x} = \dfrac{2}{3} \quad \therefore y = \dfrac{2}{3}(k-x)$

$\text{A} = 2x.y$

$\text{A} = 2x\left(-\dfrac{2}{3}x + \dfrac{2}{3}k\right)$

$= -\dfrac{4}{3}x^2 + \dfrac{4}{3}xk$

$\dfrac{d\text{A}}{dx} = -\dfrac{8}{3}x + \dfrac{4}{3}k = 0$

$8x = 4k$

$x = \dfrac{1}{2}k$

$\text{A} = 2.\dfrac{1}{2}k\left(-\dfrac{2}{3}.\dfrac{1}{2}k + \dfrac{2}{3}k\right)$

$= k\left(-\dfrac{1}{3}k + \dfrac{2}{3}k\right)$

$= \dfrac{1}{3}k^2$

QUESTION 12

(a) (1) $10 \times 9 \times 8 \times 7$
$9 \times 8 \times 7 \times 6$
$10!/6!$
10P4

(2) Exactly two 9s in the PIN

9 9 - -
- - 9 9
9 - - 9
- 9 - 9 for cases
9 - - 9
- 9 9 -

$\therefore 6 \times 9 \times 9$

$= P$ (Exactly two 9s in the PIN)

$$= \frac{6 \times 9 \times 9}{10 \times 10 \times 10 \times 10}$$

$$= \frac{243}{5000} \quad \text{or} \quad 0{,}05$$

(n(s) must make sense 9"4 something with 4 position to get P(A))

ALT

Two 9s in 4 positions:

$4C_2 = 6$

$\therefore (1 \times 1 \times 9 \times 9) \times 6$

$= P$ (Exactly two 9s in the PIN)

$$= \frac{(1 \times 1 \times 9 \times 9) \times 6}{10 \times 10 \times 10 \times 10}$$

$$= \frac{243}{5000} \quad \text{or} \quad 0{,}05$$

(b) $4\left(\frac{1}{4}\right) = 5P(A \cap B)$ ✓

$$P(A \cap B) = \frac{1}{5}$$

$$y = \frac{1}{5} \times 80$$

$$= 16$$

$$z + y = 20$$

$$z = 4$$

Slegs A is 24

$$x + z = 40$$

$$x = 36$$

ALT

$P(A) = \frac{1}{2} \quad \therefore n(A) = 40 \quad \therefore x + 40 + z = 80$

Slegs A is 24

$x + z = 40 \qquad y = 16 \qquad z = 4$

$x = 36$

ALT

$4(y + z) = 5y \quad \therefore y = 4z \quad$ and $\quad y + z = 20 \quad$ but $\quad x + z = 40$

Hence $5z = 20$

$$\therefore z = 4$$

$$y = 16$$

$$x + z = 40$$

$$\therefore \quad x = 36$$

Total: 150 marks