Math2901: Group Assignment

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Question 1 Solutions

Let $A, B \subseteq \Omega$.

1.

Given that event A is independent of itself, this means that $\mathbb{P}(A|A) = \mathbb{P}(A)$. This implies $\mathbb{P}(A) = \mathbb{P}(A)$ and so $\mathbb{P}(A)$ must equal either 1 or 0.

2.

Given event A such that $\mathbb{P}(A) = 1$ or $\mathbb{P}(A) = 0$, we observe the conditional probability between events A and B.

For the case where $\mathbb{P}(A) = 1$:

$$\begin{aligned}
\mathbb{P}(B|A) &= \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} \\
&= \mathbb{P}(B \cap A) \\
&= \mathbb{P}(B).
\end{aligned}$$

We reach this result because the intersection with a guaranteed event will always be the probability of the other event.

For the case where $\mathbb{P}(A) = 0$:

$$\begin{aligned}
\mathbb{P}(A|B) &= \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \\
&= \frac{\mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B)}{\mathbb{P}(B)} \\
&= \frac{\mathbb{P}(B) - \mathbb{P}(A \cup B)}{\mathbb{P}(B)} \\
&= \frac{\mathbb{P}(B) - \mathbb{P}(B)}{\mathbb{P}(B)} \\
&= 0 \\
&= \mathbb{P}(A).
\end{aligned}$$

Since the union with an impossible event always returns the probability of the other event, we reach our result.

3.

Note, by the Total Law of Probability, $\mathbb{P}(A \cap B) \leq 0$.

Now observe,

$$
\begin{aligned}
\mathbb{P}(A \cap B) &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \\
1 &\geq \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \\
\mathbb{P}(A \cap B) \ &\geq \mathbb{P}(A) + \mathbb{P}(B) - 1.
\end{aligned}
$$

The inequality has been proven, thus we are done.

4.

Using the previous proven inequality,

$$
\begin{aligned}
\mathbb{P}(A_1 \cap A_2) \ &\geq \mathbb{P}(A_1) + \mathbb{P}(A_2) - 1 \\
\mathbb{P}(A_1 \cap A_2 \cap A_3) \ &\geq \mathbb{P}(A_1) + \mathbb{P}(A_2) - 1 \\
&= \mathbb{P}(A_1) + \mathbb{P}(A_2) - 1 + \mathbb{P}(A_3) - 1 \\
&= \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3) - 2 \\
&\vdots \\
\mathbb{P}(\bigcap_{i=1}^{n} A_i) \ &\geq \sum_{i=1}^{n} A_i - (n-1).
\end{aligned}
$$

We have thus proved the inequality.

Question 2 Solutions

Question 3 Solutions

Question 4 Solutions