Lab 7: RSA

50.042 Foundations of Cybersecurity

Hand-out: November 12 Hand-in: 11:59pm, November 19

1 Objective

By the end of this lab, you should be able to:

- · generate keys, encrypt, decrypt, sign and verify using RSA
- · explain the importance of padding in RSA digital signature

2 Part 0: Preparation

• Check whether you have PyCrypto installed or not.

```
>>> from Crypto.PublicKey import RSA
```

• If you do not have PyCrypto, install it using:

```
$ pip install pycrypto
```

- API for PyCrypto can be found at:
 - https://www.dlitz.net/software/pycrypto/api/current/

3 Part I: RSA Without Padding

- Use your previous Python script implementation of square and multiply to do large integer exponentiation.
- Encrypt message.txt using RSA:
 - Download the public key mykey.pem.pub from eDimension.
 - Use Crypto.PublicKey.RSA.importKey() function to import the public key as an RSA object in PyCrypto.

```
>>> key=open('mykey.pem.pub','r')
>>> rsakey=RSA.importKey(key)
```

You can use the ${\tt n}$ and ${\tt e}$ attributes to obtain the modulus and the exponent numbers of the public key.

```
>>> print(rsakey.n)
>>> print(rsakey.e)
```

- Define a function to encrypt a message using the following formula:

$$m \equiv x^e \bmod n \tag{1}$$

Use square and multiply algorithm to do the exponentiation.

- · Decrypt the encrypted message:
 - Download the private key mykey.pem.priv from eDimension.
 - Use Crypto.PublicKey.RSA.importKey() function to import the private key as an RSA object in PyCrypto. You can use use the n and d attributes to obtain the modulus and the exponent numbers of the private key.

```
>>> print(rsakey.n)
>>> print(rsakey.d)
```

- Define a function to decrypt the encrypted message using the following formula:

$$m \equiv x^d \bmod n \tag{2}$$

Use square and multiply algorithm to do the exponentation.

• Create a signature the plaintext message.txt using the **private key**. Rather than exponentiation of the actual message, signature using RSA is usually applied to the hash of the message. First, hash the plaintext using SHA-256 (use Crypto.Hash.SHA256), then exponentiate the digest using the following equation.

$$s \equiv x^d \bmod n \tag{3}$$

• Verify the signature using the **public key**. The resulting exponentiation must be the same as the hash value of the plaintext.

$$x' \equiv s^e \bmod n \tag{4}$$

4 Part II: Protocol Attack

RSA has an undesirable property, namely that it is malleable. An attacker can transform the ciphertext into another ciphertext which leads to a transformation of the plaintext.

- RSA Encryption Protocol Attack:
 - Encrypt an integer (e.g. 100) using the public key from previous part, e.g. y.
 - Choose a multiplier s equal to 2 and calculate

$$y_s \equiv s^e \bmod n \tag{5}$$

- Multiply the two numbers:

$$m \equiv y \times y_s \bmod n \tag{6}$$

 Decrypt using the private key from the previous part. You should display something as follows.

Part II-----Encrypting: 100

Result:

FRGXuy5uEmfkIgEy2y0e+7Age5/x2gDDD/m48XVxe9eEgGFU5Ru7669AGrR9rdxiIm2U/miColuSFRX2z/moF6v/Lz+o/FhABDzWWe4R4Xqt9rDdVNDeaQq4qoQE7PmsW/PTep/sim51Rt1TNWJO3jK6dpDIqiwtEOGDtpRGa8=

Modified to:

 $\label{lem:dpr87xC5fGMy86yvEb/8qyDxSccczfhZwJ48hyuEQ1lnwQDhaJTR6lNVFSaQXQdJzs4RxzPzWeZCf1CC0P8xa5yCl3Y+0iM1y6HMNic3/zSSY+ZJHKZvCw6tzWFhFffxInqCeh1Z3ExTlvVJPRBdxafr+kjSx4nBJ0j19fx5sV9tfu4RwAqJmJ2vu11wcZFPUfbSXRUU/FkfA5uYMihLv5ezf93p7n+ArijN31DFaNAKoqTe+G5kelbwy+I09B9iuy+AR7zByBm9C2WqtwbTVvmxpIa39uUdSsI17JfgI774ITwbdmqlHCVfWK6fzxDTUXzfLCG/R14By0WENRg2dQ==$

Decrypted: 200

- RSA Digital Signature Protocol Attack:
 - Obtain the public key from the previous section.
 - Generate a random signature s.
 - Compute an new digest from the signature using the **public key**.

$$x \equiv s^e \bmod n \tag{7}$$

- Send the signature s and the digest x.
- Verify the signature s for the digest x using the **public key**.

$$x' \equiv s^e \bmod n \tag{8}$$

- Since x' = x, it is a valid signature!

5 Hand-in 1

- · Demo encryption and decryption of RSA.
- · Demo signing a message and verifying it.
- Demo protocol attack to both encryption and digital signature.
- Explain the limitation of protocol attack.

6 Part III: Implementing RSA with Padding

The way to make RSA more secure is to use padding. In this implementation, we will use *Optimal Asymmetric Encryption Padding* (OAEP) for RSA encryption and *Probabilistic Signature Standard* (PSS) for RSA digital signature.

- Create an implementation of RSA with the following basic building blocks:
 - generate_RSA(bits=1024), which generate the private key and public key in PEM format. The input parameter is the number of bits in the key which has default value of 1024 bits. Hint: use RSA.generate() to generate the keys.
 - encrypt_RSA(public_key_file,message), which encrypt a string message using the public key stored in the file name public_key_file. The function returns the ciphertext in base 64. Use PyCrypto class Crypto.Cipher.PKCS1_OAEP to encrypt the message. Hint:
 - * Read the public key from a file.
 - * Use RSA.importKey() to import the key.
 - * Create a new PKCS1_{OAEP} object and use its encrypt method rather than using your own square and multiply. This will encrypt RSA with some padding following OAEP.
 - decrypt_RSA(private_key_file,cipher), which decrypt cipher text in base 64 using the private key stored in the file name private_key_file. The function returns the plaintext. Use PyCrypto class Crypto.Cipher.PKCS1_OAEP to decrypt the message. Hint: similar to encrypt but use decrypt method.
 - sign_data(private_key_file,data), which signs the data string using a private key stored in the file name private_key_file. The function returns a signature string in base 64. Use PyCrypto class Crypto.Signature.PKCS1_PSS. Use SHA-256 to create a digest of the data before signing.
 - verify_sign(public_key_file, sign, data), which verify the signature sign of a given data. The function returns either True or False. Use PyCrypto class Crypto.Signature.PKCS1_PSS instead of using the square and multiply routine you created. Use SHA-256 to create a digest of the data before verifying.
- Publish your public key on the internet. You can put it on your website, or via a social website.
- Ask your friend to encrypt a message using your public key. Decrypt the message using your private key.
- Sign the text mydata.txt using your private key. Send your data and its digital signature to a friend. Ask your friend to verify it.
- Similarly, get some data and its digital signature from a friend, and verify it.
- · Redo the protocol attack with the new RSA.

7 Hand-in 2

Demo decryption of a message from a friend using RSA.

- Explain the purpose of Optimal Asymetric Encryption Padding (OAEP) to encrypt and decrypt using RSA. Explain how it works.
- Explain the purpose of Probabilistic Signature Scheme (PSS) to sign and verify using RSA. Explain how it works.